An Economic Model of Acculturation under Strategic Complements and Substitutes

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Abstract

We propose a cultural transmission model based on the co-evolution of cultural traits, behaviors, and socialization levels. Cultural traits affect agents' behavior during their interaction in a strategic environment. In turn, behaviors affect both how much parents direct socialize their children and the trait they decide to transmit. We characterize the co-evolution of cultural traits and behaviors, and their long-run outcomes, in terms of well-established acculturation processes: *assimilation*, *integration*, *marginalization*, *separation*. We show how the occurrence of each process depends on the nature of strategic environment (complements or substitutes), the cost of transmitting traits, and the size of the majority.

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From any given set of rules of conduct of the element will arise a steady structure only in an environment in which there prevails a certain probability of encountering the sort of circumstances to which the rules of conduct are adapted. A change of environment may require, if the whole is to persist, a change in the order of the group and therefore in the rules of conduct of the individuals. [Hayek 1967: 71]

1 Introduction

It is well known that culture has a huge impact on behaviors and economic outcomes.[1] At the same time, there is evidence that material incentives set by different socio-economic environments, and the behaviors that they promote, shape the evolution of cultural traits.[2] Both processes are likely to be at play when groups of individuals having different cultures come into continuous contact and interact, with possible consequences on their cultural traits, behaviors, and socialization attitude. Broadly speaking, these changes are known as *acculturation* (Redfield et al., 1936), and some key patterns have been identified: *assimilation*, *integration*, *marginalization*, *separation* (Berry, 1997).

Having a comprehensive understanding of the forces behind these acculturation processes is key to study and handle a number of economic and social problems. Furthermore, the outcome may be beneficial or detrimental, depending on the nature of the interaction and the desired objective. For example, processes of homogenization of cultures—as in assimilation and integration—can enhance productivity by reducing the inefficiencies due to miscoordination (Fredrickson, 1999). At the same time, preserving or reaching a certain level of cultural heterogeneity—as in separation and marginalization—can be economically beneficial[3] even if it could be socially and politically hard to manage (Desmet et al., 2017; Arbatlı et al., 2020; Tabellini, 2020). Thus, to understand and regulate these phenomena, it is necessary to identify under which socio-economic conditions each acculturation process is likely to manifest itself.

Aimed by this goal, a first model we provide capable of characterizing the different

[1]See for example Akerlof and Kranton (2000) for the effect of identity on economic behavior or Guiso et al. (2006) for a review of the interaction between culture and economic outcomes.

[2]The effect has been shown to arise due to exogenous institutions (see Section 3.2 of Alesina and Giuliano, 2015, for a review) such as the lack of private property (Alesina and Fuchs-Schündeln, 2007), the presence of an empire (Becker et al., 2016), or centralized formal institutions (Lowes et al., 2017); but also due to exogenous experiences such as natural disasters (see e.g. Callen, 2015). Moreover, the idea that different socio-economic environments affect the emergence of different cultures and rules of conduct goes back to Hayek (1967).

[3]It was shown that cultural heterogeneity favours the division of labour (Bernstein and Swirski, 1982), foster innovations (Hunt and Gauthier-Loiselle, 2010; Terry et al., 2022), and promotes consumption of different goods (Charles et al., 2009).

acculturation outcomes and processes in terms of the co-evolution of cultural traits, behaviors, and socialization levels as driven by both economic and social forces.[4] Then, we use it to offer a comprehensive understanding of how the acculturation outcomes and processes depend on: (*i*) the nature of strategic environment (complements vs substitutes); (*ii*) the stability of strategic environment; and (*iii*) the (opportunity) cost of parental transmission.

We consider two cultural groups (e.g., natives and immigrants) who interact in a socio-economic environment and whose cultural traits are transmitted and evolves intergenerationally. Economic environments are modeled as strategic interactions whose material payoffs induce actions to be either complements or substitutes. For simplicity, we focus on games with only two actions—e.g. participating in religious activities, exerting low or high work effort at the workplace, speaking the majority or the minority language, eating traditional or ethnic food, playing football or basketball, being aggressive when fighting for being the first in a line or not etc. Moreover, we consider both the case in which the strategic environment is simple—i.e., the payoffs are fixed—and the one in which is complex—i.e., the payoffs change within each round of interaction.

We interpret cultural traits as personal norms that make agents more or less identified with each action (as in Akerlof and Kranton, 2000), so that agents' total payoffs depend both on material incentives and cultural identities. We model the interaction between cultural traits and material payoffs by specifying key properties of a general payoff function parameterized by the agent's cultural trait. Providing rather mild assumptions, we show the existence of thresholds on traits' strength such that agents chooses an action over the other. A trait is defined **strong** (given material incentives) when it makes one of the two actions strictly dominant, otherwise it is defined **weak**. Lastly, if a trait, independently of the strength of material payoffs, is always either strong or weak we said that it is **extreme** or **neutral**, respectively.

The cultural dynamics is induced by the intergenerational transmission of traits, along the lines of the seminal contribution of Bisin and Verdier (2001) (see also Bisin and Verdier, 2011, 2023a, for two surveys). Children acquire their cultural trait interacting both with parents—i.e., *vertical socialization*—and observing the average behavior in the society—i.e., *oblique socialization*. Differently from the previous literature, parents optimally choose both the socialization effort and the role model to display to the children. Parents are moved by altruism toward the future payoffs of children, face costs to directly

[4] Since, as stated by Redfield et al. (1936), "acculturation is to be distinguished from *cultural-change*, of which it is but one aspect", we focus not only the transmission and thus the dynamics of cultural traits but also on the dynamics of behavior and socialization itself. As we will show all these quantities depend (directly or indirectly) on the exogenous economic incentives and initial cultural diversity.

socialize them and to declare a role model different from own trait. In doing so, we internalize the role of the strategic environment in the transmission processes and we show that if there are no transmission costs and parents are moved only by altruism the traits children acquire are equal to the average action played by parents in the previous generation; whereas, with positive transmission costs, the acquired traits are generically convex combinations of the average behavior of the two groups in the previous generation. When the environment in which agents interact displays strategic substitutes, parental effort is always decreasing in own population share—i.e., *cultural substitution*. Conversely, if agents interact in an environment with strategic complements, parental effort might have a inverted U-shaped relationship with own population shares—that is, *cultural complementarity* for low population shares and *cultural substitution* for high population shares. The material incentives are key, because only under strategic complementarity a minority (with weak traits) face material gains in coping the majority (with straong traits) and coordinating in their equilibrium. The result is consistent with empirical findings in Bisin et al. (2004b); Cohen-Zada (2006).

We provide a taxonomy of steady states and long-run dynamics through new definitions of *acculturation outcomes* and *processes* that take into account both the cultural traits and agents' behaviour. In particular, two groups are **assimilated** (**separated**), if they have strong traits in favour of the same (different) action, thus inducing the same (different) behavior among groups. Similarly, two groups are **integrated** (**marginalized**) if they have, respectively, a strong and weak trait in favour of the same (different) action; in such a case, agents with the strong trait always behave accordingly, whereas the others change their behavior depending on the strategic environment and the agents they interact with. Lastly, if both groups converge to weak traits, we talk about **identity erosion**. Notably, we show that, given our definition, the levels of inter-group socialization and the distance between role models in the different acculturation outcomes and processes are, as a result, endogenously consistent with the definition of Berry (1997).

At first, we analyse the situation in which strategic environment is simple, that is the payoffs are are fixed across interactions. Starting from a benchmark without parental costs, we show that if the initial level of cultural diversity or homogeneity is high—that is, strong traits of the same or opposite type—, then, regardless of whether the environment is with strategic complements or substitutes, the society remains separated or assimilated—that is, traits remain strong and agents of the two groups always play the same or the opposite action, respectively. Similarly, if cultural traits of both groups are weak their effect vanishes and, in the long run, agents care only about material payoffs.

Conversely, if the initial level of cultural diversity or homogeneity is mild—that is, one group has strong traits and the other has weak traits—, then, the cultural dynamics depends on both the strategic environment, the cost of socialization and the size of majority. In particular, under strategic complements we can observe processes of assimilation and integration, whereas, in environments with strategic substitutes, we can observe marginalization and separation. Notably, the larger the group with a strong trait, the easier it is to observe either full homogeneity or diversity (assimilated or separated). Conversely, the larger the group with a weak trait, the easier it is to observe weak homogeneity or diversity (marginalized or integrated).

If we take into account positive parental costs, the results remain qualitatively consistent for low costs. However, we show that if the cost of direct socialization is very high the effect of oblique socialization can dominate on vertical socialization and we cannot observe cultural diversity in steady state.

We conclude the analysis by considering the case in which the strategic environment is complex—that is, material payoffs can vary within each generation and the thresholds for a cultural trait to be weak or strong are uniformly distributed. We show that the number of possible acculturation processes shrinks and the cultural dynamics is mostly driven by the strategic environment and that initial conditions play a very marginal role. With strategic complements there is always assimilation to homogeneous extreme traits, whereas, with strategic substitutes, traits either become heterogeneous or converge to neutrality, depending on the cost of socialization and initial levels of diversity. In particular, when there is no cost of socialization there is always a process of separation to heterogeneous extreme traits. Conversely, for high costs of socialization there is always a process of identity erosion to neutral traits.

This paper contributes to the theoretical literature about cultural transmission (refer to Bisin and Verdier, 2011, 2023a, and references therein) and, in particular, to those papers that consider the interaction of culture and strategic environment (e.g., Hauk and Saez-Marti, 2002; Bisin et al., 2004a; Tabellini, 2008; Della Lena et al., 2023).[5] This paper introduce in the literature the possibility of parents of choosing both the socialization effort and the role model to transmit, to maximize children future expected payoffs.[6]

[5]The study of cultural transmission stems form Cavalli-Sforza and Feldman (1981). In economic literature, starting from Bisin and Verdier (2001), the study of cultural transmission developed in several directions. Among others, Buechel et al. (2014) and Panebianco (2014) consider the role of network structure in the transmission of continuous traits, Vaughan (2013) introduces the role of peers in horizontal socialization, Cheung and Wu (2018) generalize the standard framework with probabilistic transmission to continuous traits, Della Lena and Panebianco (2021) analyze the effect of incomplete information in both parental transmission and long-run cultural dynamics, Bisin and Verdier (2023b) analyze the joint evolution of culture and institutions.

[6]This idea is present also in the original Bisin and Verdier (2001) where, assuming imperfect empathy and dicothomous traits, parents always transmit their own trait. Moreover, they do not explicitly model the strategic environment.

We show that interacting in an environment characterized by cultural complements is a necessary condition for observing the inverted U-shaped relationship between direct socialization and population share, observed in empirical literature on cultural transmission (Bisin et al., 2004b; Cohen-Zada, 2006). Moreover, this is also the first cultural transmission paper that analyses both games with strategic complements and substitution and allows the payoffs to be randomly drawn and change at different interactions.

The paper informs the literature that studies the interaction among different ethnic or cultural groups, in particular between immigrants and natives. This literature has shown that the assimilation and integration of immigrants depend on groups' cultural identity and cultural transmission (Bisin et al., 2011; Panebianco, 2014; Bisin et al., 2016), on the religious and cultural uses (Carvalho, 2013), on their cultural leaders (Prummer and Siedlarek, 2017; Verdier and Zenou, 2018), but also on the pattern of interactions such as social networks, geographical locations or the role of majority (Kuran and Sandholm, 2008; Verdier and Zenou, 2017; Sato and Zenou, 2020; Itoh et al., 2021). Moreover, some recent empirical papers show the importance of economic incentives (Algan et al., 2022) and intergenerational stability of the environment (Giuliano and Nunn, 2021) for the transmission of cultural trait. To the best of our knowledge, this is the first paper that provides a comprehensive understanding on which socio-economic forces may lead to different long-run acculturation outcomes trough a theoretical model that explicitly considers the join dynamics of cultural traits, behavior, and socialization as dependent on the nature of the strategic environment (complements or substitutes).

The paper is organized as follows. We set up the model in Section 2, analyzing the interaction between cultural traits and strategic environment in Section 2.1 and the intergenerational transmission of cultural traits in Section 2.2. In Section 3, we study the cultural dynamics and characterize the long-run outcomes to acculturation outcomes and processes. In particular, in Section 3.1, we characterize the acculturation outcomes and processes under fixed material payoffs (i.e., simple environment) and as dependent on the type of strategic interaction, socialization costs, and size of the majority. Lastly, in Section 3.2, we consider the case in which material payoffs can vary within each generation (i.e., complex environment). We conclude in Section 4. In Appendix A, we provide a general discussion about complements and substitutes in 2×2 games (A.1) and we specify the results of Section 2.1 for the particular case of separable payoffs and linear cultural component (A.2). All the proofs are in Appendix B.

2 The Model

We consider an overlapping generation model where, in each period $t \in \mathbb{N}_0$, a continuum of agents $I = [0, 1]$ is divided into two communities: a majority, the *L*arge group $L = [0, \eta]$ with $\eta \in \left[\frac{1}{2}, 1\right)$, and a minority, the *S*mall group $S = (\eta, 1]$. The size of the community to which agent i belongs is denoted with η_i. Within each generation t, agents are active in two different sub-periods, youth and adult age. Each agent is characterized by a cultural trait that affects strategic decisions in the adult age and is transmitted from adults of one generation to the youth of the next one.

As illustrated in Section 2.1, during the adult age of generation t, each agent $i \in I$ with cultural trait $x_{i,t} \in [0, 1]$ is randomly matched to play an infinite number of symmetric 2×2 games. Each game payoff has both a material and a cultural component. Material payoffs can vary within each period and are drawn according to a given probability distribution which restricts them to exhibit either strategic complementarity or substitutability. The cultural payoffs are such that an agent obtains a higher (lower) payoff for playing an action that is (not) in line with her cultural trait. The interaction determines the average action played by agent i in t, denoted as $\bar{a}_{i,t}$.

At the end of the adult age of generation t, each agent reproduces asexually giving birth to one child of the next generation $t + 1$. The youth of the new generation overlaps with the adults of the previous one, allowing for the transmission of cultural traits, as described in Section 2.2. In particular, each parent optimally chooses both the direct socialization effort $\tau_{i,t} \in [0, 1]$ and the role model $\theta_{i,t} \in [0, 1]$ to transmit to her own child as to maximize her expected future welfare, and while being subject to costs for direct socialization and for choosing a role model that differs from her own cultural trait. The offspring of each parent i acquires the cultural trait $x^o_{i,t} \equiv x_{i,t+1}$ as a combination of the role model $\theta_{i,t}$—*vertical socialization*—and the average behavior in the previous generation, $\bar{a}_t$—*oblique socialization*—, the weight of each component being dependent by the socialization effort.

In our analysis, homogeneity of traits within each community is assumed in $t = 0$ and, given the model, remains valid for all t also for actions, role models, and socialization levels. The evolution of cultural traits from generation t to generation $t + 1$ can thus be represented at the community level. The following table, where $i = \text{L}, \text{S}$ is each community representative agent, summarizes the dynamics of cultural traits $\mathbf{x}_t := (x_{\text{L},t}, x_{\text{S},t})$, as dependent by average actions $\bar{\boldsymbol{a}}_t := (\bar{a}_{\text{L},t}, \bar{a}_{\text{S},t})$, role models $\boldsymbol{\theta}_t := (\theta_{\text{L},t}, \theta_{\text{S},t})$, and socialization effort $\boldsymbol{\tau}_t := (\tau_{\text{L},t}, \tau_{\text{S},t})$.

	YOUTH t		ADULT AGE t YOUTH $t+1$	
gen. t	$\mathbf{x}_t$	$\longrightarrow$	$\bar{\boldsymbol{a}}_t$	
			$\downarrow$ Cult. Transm. with $\boldsymbol{\theta}_t, \boldsymbol{\tau}_t$	
gen. $t+1$			$\mathbf{x}_t^o \equiv \mathbf{x}_{t+1}$	$\longrightarrow$

2.1 Cultural Traits and Strategic Environments

In this section, we model how cultural traits affect agents' payoffs, and thus agents' behavior. As this phenomenon occurs within the adult age, we drop the time index to ease the notation.

During their adult age, agents are randomly matched to play an infinite number of symmetric 2×2 games. In each game, agents belonging to each community can play either as row player, r, or column player, c; the set of players of each game is thus $J = \{r, c\}$. The cultural trait of the community of player $j \in J$ is denoted by x_j. Each player j can choose between two actions, $a_j \in \{0, 1\}$ —e.g. exerting a low versus a high work effort at the workplace, speaking the majority versus the minority language, eating traditional versus ethnic food, playing football versus basketball, being aggressive when fighting for being the first in a line or not etc. The cultural trait x_j represents a measure of j's relative identification with action 1 rather than with action 0 and modifies payoffs as follows.[7]

We assume each player j payoff depends on a material component, the usual $\pi(a_j, a_{-j})$, and on a cultural component, $\rho(a_j, x_j)$. The latter reflects the consistency between actions and cultural traits. The overall payoff function for player j in adult age evaluated at (a_j, a_{-j}), given the trait x_j, is[8]

$$u\big(a_j, a_{-j}; x_j\big) := \underbrace{\pi(a_j, a_{-j})}_{material} + \underbrace{\rho(a_j, x_j)}_{cultural}. \tag{1}$$

The material payoff $\pi(a_j, a_{-j})$ is symmetric and represents the utility that agents would obtain without the cultural component (or, as we shall define later, when the trait is neu-

[7]Defining the cultural identification of j with actions 1 and 0 as $\mathrm{I}_j^1 \in [0, 1]$ and $\mathrm{I}_j^0 \in [0, 1]$, respectively, then $x_j = \frac{1}{2} + \frac{\mathrm{I}_j^1 - \mathrm{I}_j^0}{2}$.

[8]In the main body of the paper, to simplify the analysis, we employ separability of the two components. However, in the Appendix B, we prove the result of this section, Proposition 1, for a general aggregating function $v(\pi, \rho)$. In Appendix A.2 we show how the payoff function we propose generalizes, among others, Bisin et al. (2004a); Akerlof and Kranton (2005); Tabellini (2008); Kimbrough and Vostroknutov (2016).

tral). We are interested in the role of cultural traits when material payoffs alone imply multiple equilibria, thus we restrict them to exhibit either strategic complementarity—as in coordination games (e.g., Stag Hunt)—or strategic substitutability—as in anti-coordination games (e.g., Chicken Game). [9]

The cultural component $\rho(a_j, x_j)$ represents an evaluation of action a_j given the cultural trait x_j. For each agent, we assume that the more (less) her action is in line with her cultural trait, the higher (lower) her cultural payoff. Overall, the cultural component ρ is assumed to satisfy the following properties.

A1 **Smoothness**: *For each* $a_j \in \{0, 1\}$*,* $\rho(a_j, x_j)$ *is smooth, and thus continuous, in* $x_j \in [0, 1]$.

The assumption of continuity is quite natural due to the continuous nature of the cultural trait. Smoothness will prove useful only when characterizing optimal parental choices during the cultural transmission.

A2 **Neutrality**: $\rho(1, \frac{1}{2}) = \rho(0, \frac{1}{2})$.

The cultural trait $x_j = \frac{1}{2}$ is said to be **neutral**, implying that j's culture is equally identified with both actions and, thus, leads to equal cultural payoff. With a neutral trait the payoff function coincides with $\pi(a_j, a_{-j})$ up to a constant so that behavior depends only on the material component.

A3 **Monotonicity**: *For each* $a_j \in \{0, 1\}$*,* $\rho(a_j, x_j)$ *is strictly decreasing in* $d_j := (x_j - a_j)^2$.

Thus, Monotonicity implies the more j's culture is identified with action 1 (0), that is, that the closer a cultural trait x_j is to 1 (0), the higher the benefit for playing action 1 (0) and the penalty for playing action 0 (1).

A4 **Extremism**: *For all material components* π*,* $u\big(1, a_{-j}; 1\big) > u\big(0, a_{-j}; 1\big)$ *and* $u\big(1, a_{-j}; 0\big) < u\big(0, a_{-j}; 0\big)$*, for all* $a_{-j} \in \{0, 1\}$.

Under extremism, when the trait is 1 (0), action 1 (0) is strictly dominant for all possible material components π. The cultural trait 1 (0), for which the culture is fully identified with corresponding action 1 (0), is said to be **extreme** because, by $A4$, it leads to a dominant strategy independently of the material component. An example of extreme traits are the strong religious norms that lead agents to behave accordingly.[10]

[9] A 2×2 game displays **strategic complementarity** if each action is a best reply to itself. Conversely, it displays **strategic substitutability** if each action is the best reply to the other one. In Appendix A, we extend the analysis to all symmetric games.

[10] E.g. veiling, see Carvalho (2016) for an analysis of how veiling might affect integration dynamics.

We turn to the implication of having a cultural component on the equilibrium analysis. Before characterizing Nash equilibria, the next proposition shows that under continuity, neutrality, monotonicity, and extremism ($A1$-$A4$), there always exist thresholds of cultural traits implying that action 1 or 0 is strictly dominant.

Proposition 1 *Consider a game with payoffs described by equation (1) and π exhibiting strategic* ***complementarity*** *or* ***substitutability****. If the cultural component ρ and the utility u satisfy A1-A4 then, for each π, there exists an $\hat{x}_1(\pi) \in \left(\frac{1}{2}, 1\right)$ such that for all $x_j > \hat{x}_1$ action 1 is strictly dominant for player j and an $\hat{x}_0(\pi) \in \left(0, \frac{1}{2}\right)$ such that for all $x_j < \hat{x}_0$ action 0 is strictly dominant for player j.*

Proposition 1 shows that the relative position of traits (x_r, x_c) with respect to the thresholds $(\hat{x}_0, \hat{x}_1)$ determines if action 0 or 1, respectively, is dominant.[11] The thresholds $\hat{x}_0$ and $\hat{x}_1$ depend on the material component π and represent the relative strengths of material and cultural component. The closer they are to $\frac{1}{2}$, the stronger is the cultural component as even a weak identification with an action makes it dominant. Even for symmetric material payoffs π, the thresholds need not to be symmetrically around $\frac{1}{2}$ as the cultural component could be stronger in favour of action 1, shifting $\hat{x}_0$ toward 0 and $\hat{x}_1$ toward $\frac{1}{2}$, or of action 0, shifting the thresholds in the opposite way.

Similarly, changing material payoffs shifts the threshold differently, also depending on the strategic environment. For games with strategic complementarity, the threshold $\hat{x}_1$ moves closer to (farther from) 1 when incentives to deviate from the equilibrium $(0, 0)$ are weaker (stronger), giving relatively more (less) strength to material payoffs, while $\hat{x}_0$ is changed by the the incentives to deviate from the equilibrium $(1, 1)$.[12] For games with strategic substitutability, the threshold $\hat{x}_1$ moves closer to (farther than) 1 when the incentives to anti-coordinate and play action 0 when the other plays action 1 are stronger (weaker), also here giving relatively more (less) strength to material payoffs, while $\hat{x}_0$ is changed by the incentives to anti-coordinate and play action 1 when the other plays action 0.

When a cultural trait implies that an action is strictly dominant, we shall say that the trait is **strong**. A trait x_j can be strong either in the direction of action 1, when $x_j > \hat{x}_1$, or of action 0, when $x_j < \hat{x}_0$. As each threshold depends on the material payoff π, the strength of a trait is defined relatively to a specif material payoff π. The traits $x_j = 1$ and $x_j = 0$ are **extreme** because they are strong for all specifications of π, a consequence

[11] To ease the notation the dependence of the threshold $(\hat{x}_0, \hat{x}_1)$ on the material component π is often omitted.

[12] Relatedly, in Appendix A.2, it is shown that having e.g. $\hat{x}_1$ is father from 1 corresponds to $(1, 1)$ being risk-dominant. This is no coincidence as also risk dominance of one equilibrium is determined by the incentives to deviate from the other.

of $A4$ that we could remove. When $\hat{x}_0 < x_j < \hat{x}_1$, then the player has not dominant strategies. In these cases, we say that the cultural traits are **weak** with respect to that material payoff. The trait $x_j = \frac{1}{2}$ is weak for all specifications of π, as implied by $A1$, as thus named **neutral**.

The implication of Proposition 1 on Nash equilibria is rather straightforward. In Figure 1, we depict the (pure strategy) Nash Equilibria as depending on the relative strength of cultural trait and material incentives for a specific choice of the material payoff π exhibiting strategic complementarity (left, panel (a)) or strategic substitutability (right, panel (b)).[13]

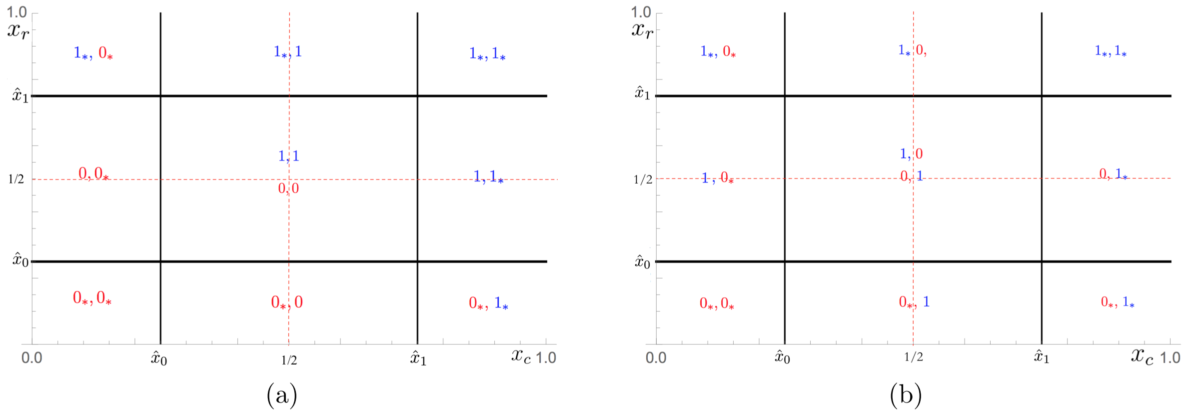


Figure 1: Nash equilibria as a function of cultural traits (x_r, x_c) in strategic environments with strategic (a) complements and (b) substitutes, given $\hat{x}_0, \hat{x}_1$. The star "$_*$" denotes that the strategy is dominant.

Average Behavior During the adult age, agents are matched randomly—with frequency η against a player of community L and $1-\eta$ against a player of community S—to play different games, all exhibiting strategic complementarity or substitutability, according to a distribution γ and independently from the drawn player. When a trait affects decisions only for a specific type of interaction, γ has a degenerate support on one material payoff and we shall say that the environment is **simple**, otherwise the environment

[13]For x_r and x_c in the neighborhood of the neutral trait $\frac{1}{2}$, the games have the same equilibria as the corresponding game without the cultural component, whereas as x_r or x_c move away from $\frac{1}{2}$ and become strong, the set of equilibria changes and the equilibrium becomes unique. In these cases the equilibrium is determined, given the strategic nature of material payoffs, by the behavior of the agent with the strong trait.

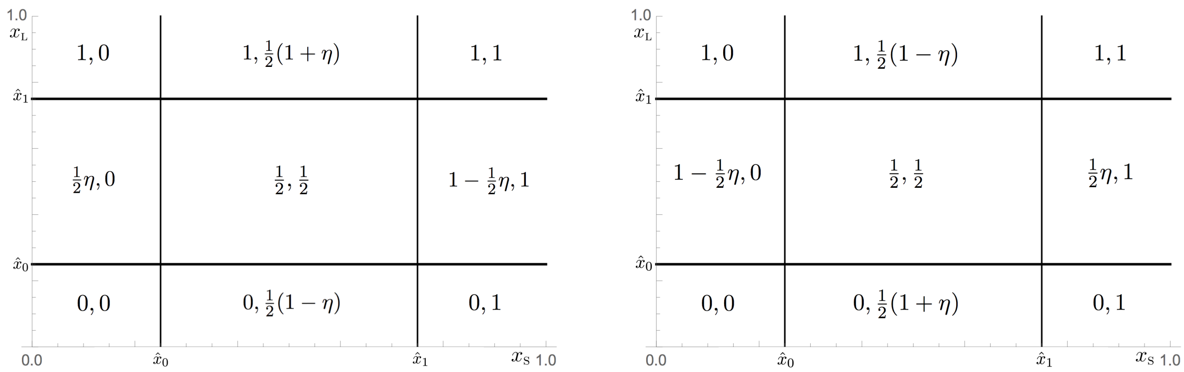
Note also that the Nash equilibria that would be present without the cultural component, for example to coordinate in case of strategic complements, are still present unless one player cultural trait opposes it by dictating another action. For example, as shown in panel (a), the equilibrium $(1, 1)$ is always present under strategic complementarity unless at least one player has a strong cultural trait in favor of action 0, $x_j < \hat{x}_0$ for either $j = r, c$.

is **complex**.[14]

We further assume that, whenever there are multiple equilibria, the two actions are played with the same frequency, as if agents are coordinating half of the times on each equilibrium. As we shall see, this implies that whenever the trait does not help to select an action there is a push towards neutrality.

In Figure 2, we show average equilibrium behavior for agents of both communities $\bar{\boldsymbol{a}} := (\bar{a}_{\text{L}}, \bar{a}_{\text{S}}) = (\mathbb{E}_\eta[a_{\text{L}}], \mathbb{E}_\eta[a_{\text{L}}])$ for fixed material payoffs, i.e. a simple environment, and corresponding thresholds $(\hat{x}_1, \hat{x}_0)$ Material payoffs exhibit strategic complementarity in panel (a) and strategic substitutability in panel (b).

When the environment is complex, i.e. γ has non-degenerate support, on different material payoffs, the vector of average actions is $(\bar{a}_{\text{L}}, \bar{a}_{\text{S}}) = (\mathbb{E}_{\eta,\gamma}[a_{\text{L}}], \mathbb{E}_{\eta,\gamma}[a_{\text{S}}])$. Similarly we can define the average behavior for the whole society as $\bar{a} = \eta\bar{a}_{\text{L}} + (1-\eta)\bar{a}_{\text{S}}$.



(a) (b)

Figure 2: Average actions $\bar{a}_{\text{L}}, \bar{a}_{\text{S}}$, in the space $(x_{\text{S}}, x_{\text{L}})$ for a specific choice of the material component π. In (a) the environments has strategic complements, in (b) the environment has strategic substitutes.

Prescriptive and descriptive norms Having differentiated between cultural traits and actions allows us to distinguish between prescriptive and descriptive norms. Cultural traits in $\mathbf{x}$ represent the **prescriptive norms** of the two groups —i.e. cultural/moral values prescribing the action that an agent *should* do—, whereas the average actions in $\bar{\mathbf{a}}$ represent the **descriptive norms** —i.e. the frequency with which each given behavior occur and thus what agents *actually* do. We refer to Bell et al. (1988) for further discussion about prescriptive and descriptive norms in decision making. In the following section, we examine the transmission of prescriptive norms—i.e., cultural traits—from one generation to another and analyze how their formation is influenced by both prescriptive

[14] Note that the notion of **strong** (**weak**) traits can be defined also for complex environment γ. A trait is defined **strong** (**weak**) if it (not) induces a dominant action for each material payoff in the support of γ.

and descriptive norms, as described by the social science literature. (e.g., Bicchieri et al., 2014).

2.2 Cultural Transmission

In this section, we model how traits are transmitted from parents and acquired by their children.

During the cultural transmission process, each parent $i \in I$ directly socializes her child choosing a role model, or *displayed trait*, $\theta_i \in [0,1]$ and exerting a costly vertical socialization effort $\tau_i \in [0,1]$. The role model θ_i can be thought as a **displayed prescriptive norm** that parents use to educate the children.

The offspring of i acquires their cultural trait, $x_i^o \in [0,1]$, as the result of observing both parental role model θ_i—i.e., *vertical socialization*—and the average behaviour observed in the previous generation, $\bar{a}$—i.e., *oblique socialization*:

$$x_i^o = \underbrace{\tau_i \, \theta_i}_{vertical\ soc.} + \underbrace{(1-\tau_i)\ \bar{a}}_{oblique\ soc.}. \tag{2}$$

Parents optimal decisions about the role model to transmit and the socialization effort to exert are moved by: (i) altruism toward their offspring; (ii) costly directly socialization effort; and (iii) cultural resilience. Each parent $i \in I$ is assumed to maximize their subjective expectation about offspring's future utility, anticipating the future child's trait x_i^o as in (2) and given their expectations about actions played in the society $\{a_j^o\}_{\eta,\gamma}$ (altruism). Moreover, parents are subject to a quadratic cost of direct socialization, as dependent on a parameter $c \geq 0$ (socialization cost), and of displaying a trait different from x_i, as dependent on a parameter $\lambda \geq 0$ (cultural resilience) Formally, each parent i solves the following problem

$$\max_{(\theta_i,\tau_i)\in[0,1]\times[0,1]} \quad U_i = \underbrace{\mathbb{E}_i \left[\int_\gamma \int_{j \in I/\{i\}} u\big(a_i^o, a_j^o; x_i^o\big)\ dj d\gamma \right]}_{altruism/child's\ welfare} - \underbrace{c(\tau_i)^2}_{socialization\ cost} - \underbrace{\lambda(\theta_i - x_i)^2}_{cultural\ resilience}, \tag{3}$$

where x_i^o is set by (2).

To capture the limited farsightedness that parents might have in anticipating future strategic environments, we assume *adaptive expectations* about the actions played in the new generation, namely, for each $i \in I$, $\mathbb{E}_i[\mathbf{a}^o] = \mathbf{a}$.[15] For tractability, we also assume

[15] As we shall see adaptive expectation coincides with rational expectation at the states states of out cultural dynamics. Note that assuming rational expectation leads to the issue of multiplicity of equilibria,

a specific choice of cultural payoff (quadratic): $u(a_i, a_{-i}; x_i) = \pi(a_i, a_{-i}) - \psi\,(a_i - x_i)^2$, where ψ measures the strength of the cultural component.[16] The parenting socialization cost and role model cost relative to the strength of the cultural component ψ are, respectively, $\tilde{\lambda} = \frac{\lambda}{\psi}$ and $\tilde{c} = \frac{c}{\psi}$.

The next proposition characterizes the cultural transmission choices.

Proposition 2 *For each $i \in I$, consider the cultural transmission mechanism described by equations (2)-(3) and assume adaptive expectations, $\mathbb{E}^i[\mathbf{a}^o] = \mathbf{a}$*

(i) The optimal parental choice of role models and direct socialization (θ_i^*, τ_i^*) satisfies

$$\begin{cases} \theta_i^* = F_i^\theta(\tau_i^*) \\ \tau_i^* = F_i^\tau(\theta_i^*) \end{cases}, \tag{4}$$

with

$$F_i^\theta(\tau) := \left[\frac{\tau^2}{\tilde{\lambda} + \tau^2}\bar{a}_i + \frac{\tilde{\lambda}}{\tilde{\lambda} + \tau^2}x_i + \frac{\tau(1-\tau)}{\tilde{\lambda} + \tau^2}\left(\bar{a}_i - \bar{a}\right)\right]_0^1; \tag{5}$$

and

$$F_i^\tau(\theta) := \left[\frac{(\bar{a}_i - \bar{a})(\theta - \bar{a})}{\tilde{c} + (\theta - \bar{a})^2}\right]_0^1. \tag{6}$$

(ii) Given the optimal choices in (5) and (6), there exists a $\rho_i \in [0, 1]$ such that the trait acquired by i's offspring can be written as

$$x_i^o = \rho_i\ \bar{a}_i + (1 - \rho_i)\ \bar{a}. \tag{7}$$

Moreover, if $\bar{\lambda} = \bar{c} = 0$, then $\rho_i = 1$.

(iii) Let assume, without loss of generality, that $\bar{a}_i \geq \bar{a}$. Then,

(a) if $x_i \geq \bar{a}$, then $(\theta_i^* \geq x_i,\ \tau_i^* > 0)$ and $x_i^o > \bar{a}$;

(b) if $x_i \leq \bar{a}$ then either $(\theta_i^* > \bar{a},\ \tau_i^* > 0)$ and $x_i^o > \bar{a}$, or $(\theta_i^*, \tau_i^*) = (x_i, 0)$ and $x_i^o = \bar{a}$.

To properly understand Proposition 2, recall that parental utility deepens on three forces: altruism, cost of direct socialization, and cultural resilience. When the cost of socialization and cultural resilience are zero —i.e. $\tilde{\lambda} = \tilde{c} = 0$— parental decisions are

thus adaptive expectation solves this issue.

[16] Although the characterization of cultural transmission is proved in the proof of Proposition 2 in Appendix B for a general payoff as in (1), here we concentrate on a simplified version which keeps all the relevant features and discuss the effect of the more general payoff in a paragraph at the end of the section.

driven solely by the objective of maximizing children's adult-age expected utility, i.e. altruism. In such a case, parents are always successful in transmitting a trait equal to the average optimal action they play during their adult life, $\bar{a}_i$. Indeed, the later maximizes the cultural utility ρ given that parents expect their children to have their same behaviour. Conversely, when the costs of socialization and cultural resilience are positive—i.e., $\tilde{\lambda} > 0$ and $\tilde{c} > 0$—parents face trade-offs between transmitting the optimal altruistic trait, $\bar{a}_i$, and minimizing the cost of socialization and cultural resilience.

Proposition 2 (i) characterizes this trade-off and gives the optimal combination of role model θ_i^* and socialization effort τ_i^* each parent chooses to educate their offspring.

The role model θ_i^*, when interior, positively depends on the weighted average expected action $\bar{a}_i$, on the parental trait x_i, and on the distance between this average action and the overall average action in the society, that is $(\bar{a}_i - \bar{a})$. The first effect is due to the altruism: parents want to transmit a trait in line with the average action $\bar{a}_i$ which maximizes the cultural utility ρ, given the expectations about future actions. The second effect is due to cultural resilience, anchoring the role model to their own trait to minimize the cost of departing from it. The third effect depends on the fact that parents try to contrast the effect of oblique socialization through a more extreme role model, whenever that is not in line with their objective. When $\tilde{\lambda}$ is small enough, The need to contrast the oblique socialization is responsible for a border role model which can go to 1 or 0 depending on the sign of $(\bar{a}_i - \bar{a})$.

The socialization effort τ_i^*, when interior, positively depends on the distance between own average action $\bar{a}_i$ and average behavior in the society $\bar{a}$. Notably, when all the agents in the society plays the same action –i.e., $\bar{a}_i = \bar{a}$– and when the optimal role model does not help to contrast the effect of oblique socialization –i.e. $sign(\bar{a}_i - \bar{a}) \neq sign(\theta_i - \bar{a})$– parents do not exert any socialization effort and let children been socialized by the society leading to $\tau_i^* = 0$, a border solution.

Proposition 2 (ii) shows that the trait acquired by the offspring is always a convex combination of parent average action and the average behavior in the society. In particular, if parental costs are null ($\lambda = 0$ and $c = 0$) each parent i can always find a combination of θ_i^* and τ_i^* such that the oblique socialization effect is crowded out and $x_i^o = \bar{a}_i$. In such a case, each parent can freely substitute a higher level of direct socialization with a more extreme role model in the space $[0, 1]^2$. If instead transmission costs are positive parents are not generically able to avoid the effect of oblique socialization in the children's traits formation, and the child's trait x_i^o will be closer to either parental action $\bar{a}_i$ or average behavior in the society $\bar{a}$ depending on the intensity of the cultural resilience and the cost of direct socialization. For example, if the parenting relative costs $\tilde{\lambda}$ and/or $\tilde{c}$ are small enough parent i may choose extreme role model $\theta_i^* = 1$ (if $\bar{a}_i > \bar{a}$) and/or full vertical socialization $\tau_i = 1$ and the trait of the offspring of parent i is close to $\bar{a}_i$. The larger the parenting costs the closer the new generation's traits to the average behaviour $\bar{a}$.

Figure 3 below shows how the optimal role model θ^* (in red) and offspring acquired traits x_i^o (in blue) depends on the ordering of parental traits, x_i, parental average action $\bar{a}_i$, and average behavior in the society, $\bar{a}$.

In particular, Proposition 2 (iii) implies that, if the parental trait x_i is far enough from $\bar{a}$ on the opposite side of $\bar{a}_i$ (see Figure 3 (b'')), parents might find optimal to not socialize the offspring choosing $\tau_i^* = 0$ and take advantage of the oblique socialization, so that $x_i^o = \bar{a}$. In such a case, the cost of directly socializing the child is not justified by the success of the transmission and parents prefer to let the offspring acquire traits only observing the average behaviour in the society. Otherwise, parents always exert positive socialization effort and the offspring trait is a convex combination of parental average action and society average behavior as discussed in equation (7) (see Figure 3 (a'), (a''), and (b')).

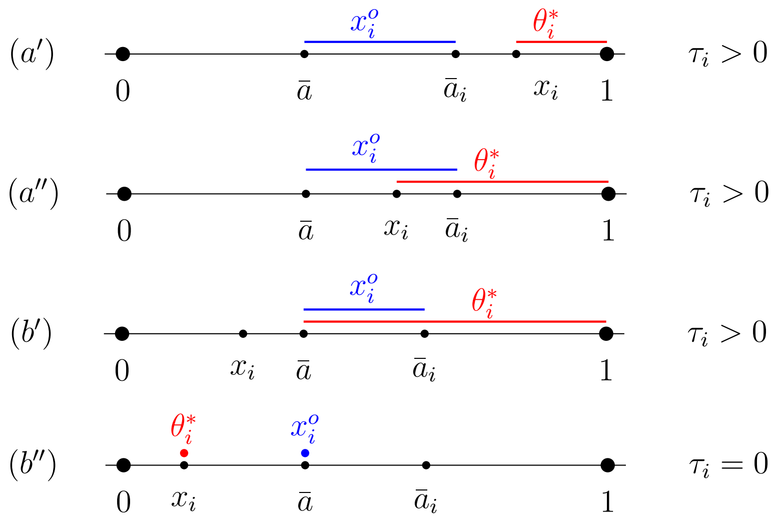


Figure 3: Space of the optimal role model θ_i^* (red) and offspring cultural trait x_i^o (blue), depending on parental trait x_i, when $\bar{a}_i > \bar{a}$ and transmission costs are positive. In (d) $\tau_i^* = 0$.

Effect of the group size We now analyse how socialization effort is affected by the share of own type in the population.

A common result in the previous literature, in which strategic environment does not play a role (e.g. Bisin and Verdier, 2011, 2023a), is that socialization efforts are decreasing in parent's type share in the population, due to a substitution effect between vertical and oblique socialization. We show that in our model the presence of strategic environments affect parental incentives to socialize their child and may lead to cultural complementarity in environments exhibiting strategic complementarity and for small population shares.[17]

In our context, having named $\eta_i \in [0,1]$ the population share parent i's type, we say that a socialization effort τ_i displays *cultural substitution* if $\frac{\partial \tau_i^*}{\partial \eta_i} < 0$, whereas it *displays cultural complementarity* $\frac{\partial \tau_i^*}{\partial \eta_i} > 0$. In general, both the socialization effort and the role model are affected by the population shares. To provide clear-cut results, we first concentrate on the variation of socialization levels by setting the cost for setting a role model to zero, $\lambda = 0$, so that the optimal role model θ_i^* is set the an extreme level and does not change with respect to the populations shares. This is the content of the following corollary.

Proposition 3 *Fix a material payoff π and consider no costs for setting the role models, $\lambda = 0$. For each $i \in [0, \eta_i]$, if x_i is strong, then τ_i^* displays cultural substitution. If x_i is weak and, for all $j \in [\eta_i, 1]$, x_j is strong then*

- *under strategic substitutes, τ_i^* displays cultural substitution;*
- *under strategic complements, τ_i^* displays cultural substitution when η_i is large and it displays cultural complementarity when η_i is small.*

Proposition 3 shows that the effect of population shares on socialization effort crucially depends on the strategic environment and the strength of cultural traits.

First note that with a fixed role model, the strength of the optimal socialization level in (6) is proportional to the difference between the average action of group i, i.e. the target $\bar{a}_i$ which parents want to transmit, and the average $\bar{a}$. That difference $|\bar{a}_i - \bar{a}|$ can be rewritten as $(1-\eta_i)|\bar{a}_i - \bar{a}_{j\neq i}|$ and thus depends on the size η_i both directly (increasing η_i decreases the difference between the group average and the total average) and indirectly through possible changes in average groups actions. The first effect is the typical one which creates cultural substitution and is always at play. The second effect is at play only when the group average action changes with group sizes. The latter occurs only when one group has a weak trait and the other has a strong trait, so that the times the two coordinate depend on their relative size. Moreover, the sign of the effect depends on the strategic environment. The total effect could be reinforced in the direction of substitution or even change to cultural complementarity.

When agent i has a strong trait, either the first effect is the only one at play (when also $j \neq i$ has a strong trait and thus $\bar{a}_j$ does not change) or it always dominates (when $j \neq i$ has a weak trait).

[17] If the strategic environment does not play a role, cultural complementarity is possible when the transmission technology depends on the own group share—see, for example, Bisin and Verdier (2001) and Della Lena and Panebianco (2021). Conversely, we refer to Della Lena et al. (2023) for an analysis of how cultural complementarity/substitution depends on the strategic environment in a dichotomous traits setting.

On the contrary, when agent i has a weak trait while agents in the other group have a strong trait jthe strategic environment turns out to be crucial.[18] Take an environment with strategic substitutes, under a small η_i anti-coordination incentives are such that most of the times and agent in i plays an action different from the one played by an agent with a strong trait, making i's average action $\bar{a}_i$ very far from the action played by population $j \neq i$. An increase in η_i moves both $\bar{a}_i$ and $\bar{a}_{j\neq i}$ closer to each other and thus there is a further reason not to vertically socialize children. The total effect is even stronger than the typical one which in our context is due to $\bar{a}_i$ converging to $\bar{a}$. Symmetrically, a decrease in η_i increases the effort of agents in i to vertically socialize.

Conversely, with strategic complements, an increase in i's population share η_i makes i's average action, $\bar{a}_i$, further away from the average action of the other group $\bar{a}_{j\neq i}$ because, on average, there are less and less incentives to coordinate. This creates cultural complementarity, the larger the population the higher the need to differentiate from the other population and vertically socialize. The product of the two effects results in cultural complementarity for a small population share and in cultural substitution for a large population share, leading to an inverted U-shaped relationship between the socialization effort, τ_i, and population share, η_i.[19]

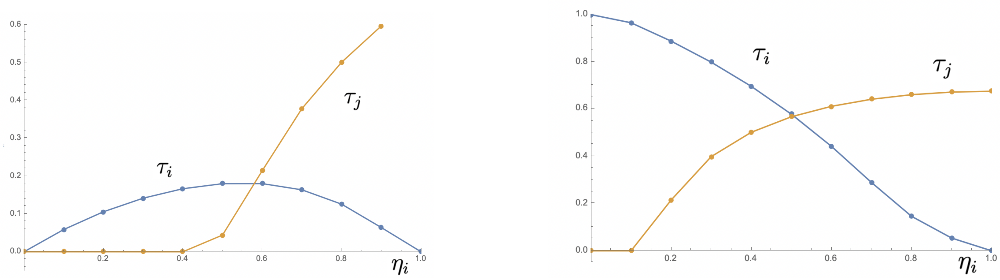
The overall effects seem to be robust to the introduction of role model costs, $\lambda > 0$, as shown in Figure 4 where the socialization efforts τ_i^* and τ_j^*, with $i \in [0, \eta_i]$ and $j \in [\eta_i, 1]$ are shown as a function of i's population share η_i when group i has a weak trait while groups j has a strong trait, both in environments with strategic complements (Panel (a)) and strategic substitutes (Panel (b)). We can see that τ_j^* (in orange) is always increasing in η_i, which means that is decreasing in its own presence in the society ($\eta_j = 1 - \eta_i$) and always displays cultural substitution. Whereas, τ_i^* (in blue) has a inverted U-shaped relationship in environments with complements (Panel (a)) and always decreasing in environments with substitutes (Panel (b)).

2.3 Lon-run cultural dynamics

Having explicitly described the cultural transmission process, we can now study the long-run cultural dynamics. Since, for each group $i = L, S$, agent i born in $t+1$ is the offspring of agent i born in time t, so that $x_{i,t+1} \equiv x_{i,t}^o$, the dynamics of cultural traits is described by equation (7): $x_{i,t+1} = \rho_{i,t}\bar{a}_{i,t} + (1-\rho_{i,t})\bar{a}_t$, where the weight $\rho_{i,t}$ and the average actions depend on both groups parent trait $(x_{\text{L},t}, x_{\text{S},t})$. Making explicit this dependence, we can

[18] If traits of both groups are weak, then average actions are equal and all agents decide to not exert any socialization effort and let the children being socialized by the society.

[19] Bisin et al. (2004b); Cohen-Zada (2006) show empirically the same inverted U-shaped relationship between parental socialization efforts and religious share. See Section 4 for a further discussion.



(a) Socialization effort under complements.

(b) Socialization effort under substitutes.

Figure 4: Optimal socialization efforts for generic groups i (weak trait, $x_i = 0.6$) and j (strong trait, $x_j = 0.1$, and thus $\bar{a}_j = 0$) depending on η_i, the share of i agents in the society, under costs $\tilde{c} = 0.1$, $\tilde{\lambda} = 0.5$. The action $\bar{a}_i$ of the group i (weak trait) depends on the strategic environment: $\bar{a}_i = \frac{1}{2}\eta_i$ with strategic complements (Panel (a)) and $\bar{a}_i = 1 - \frac{1}{2}\eta_i$, with strategic substitutes (Panel (b)).

define $\boldsymbol{\rho}(\mathbf{x}_t) := (\rho_{\text{L}}(\mathbf{x}_t), \rho_{\text{S}}(\mathbf{x}_t))$ and write the cultural dynamics as

$$\mathbf{x}_{t+1} \equiv \mathbf{x}_t^o = \boldsymbol{\rho}(\mathbf{x}_t) \cdot \bar{\boldsymbol{a}}(\mathbf{x}_t) + (1 - \boldsymbol{\rho}(\mathbf{x}_t)) \cdot \bar{a}(\mathbf{x}_t). \tag{8}$$

In next section, we study the cultural dynamics characterized by the latter and provide a taxonomy of traits trajectories and steady states in terms of acculturation processes and outcomes, respectively.

3 Acculturation

When two cultural groups interact, the process of change in cultural traits, behavior and socialization is known as *acculturation*.[20] In the original definitions of Berry (1997), employed in economics by Verdier and Zenou (2017), the possible acculturation processes differ over two dimensions: *cultural maintenance*; and *contact* between groups. Their levels defines four acculturation processes: *assimilation* (high contact, null maintenance), *integration* (normal contact, low maintenance), *marginalization* (low contact, normal maintenance), and *separation* (null contact, high maintenance).

We now present a taxonomy to characterize cultural steady states —i.e. acculturation outcomes— and cultural dynamics —i.e., acculturation processes— in terms of the strength and the type of traits of the two cultural groups.[21] In doing so, our definitions

[20]According to Redfield et al. (1936) "Acculturation comprehends those phenomena which result when groups of individual having different cultures come into continuous first-hand contact, with subsequent changes in the original cultural patterns of either or both groups."

[21]Recall from Section 2.1 that a cultural trait x_i is **strong** (**weak**) with respect to a material payoff π if agents belonging to i have (not) a dominant strategy for playing either 1 ($x_i > \hat{x}_1$) or 0 ($x_i < \hat{x}_0$)

take into account both the traits and the resulting induced behaviors. Then, we derive implications of our taxonomy on both *cultural maintenance* and *contact* between groups, which for us are endogenously determined during the cultural transmission process, and replicate the taxonomy used by Berry (1997).

Definition 1 (Strong Homogeneity/Diversity) *Two cultural groups are* ***assimilated*** *(****separated****) if they have strong traits of the same (different) type. A process of* ***assimilation*** *(****separation****) occurs if one group acquires a strong cultural trait of the same (different) type of the other.*

If groups are assimilated, then all the agents are strongly identified with the same action and always play it, independently of the strategic environment. Notably, when the two groups are assimilated, even if cultural traits (prescriptive norms) might be slightly different, the induced behavior, and thus the same descriptive norm, coincides.

Conversely, if groups are separated, agents of the two groups always play and are strongly identified with opposite actions, thus, they have also opposite descriptive norms.

Both for assimilated and separated outcome, we have a process, named assimilation or separation, respectively, when we end up there starting from a different configuration of traits.

Definition 2 (Weak Homogeneity/Diversity) *Two cultural groups are* ***integrated*** *(****marginalized****) if they have, respectively, a strong and weak trait of the same (different) type. A process of* ***integration*** *(****marginalization****) occurs if one group acquires a weak trait of the same (different) type of the strong trait of the other.*

In both integrated and marginalized societies, one cultural group has a strong trait whereas the other has a weak trait. The difference between the two depends on the group with the weak trait. In particular, if the type of trait is aligned the one of the other group (which is strong), then we say that the group is integrated; if instead the trait is of the opposite type, then we say that the group is marginalized. When the latter occurs, agents do not identify themselves with the same type of the other group (as in integration or assimilation) but they are not even strongly identified with the opposite trait (as in separation). From the point of view of action played, integrated or marginalized societies do not differ. In both, the agent with strong trait always play the same dominant

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A cultural trait is **extreme** (**neutral**) if it is strong (weak) in any possible strategic environments. Moreover, we say two cultural traits are of the **same type** if both are greater or lower than $\frac{1}{2}$. Likewise, when agents face games whose material payoffs are distributed according to a non-degenerate γ, a trait is defined **strong** (**weak**) if it (not) induces a dominant action for each material payoff in the support of γ.

action and the agent with the weak trait coordinate or anti-coordinate, depending on the strategic environment. However, it is important to distinguish between the two because they can lead to different acculturation processes when material incentives change, which is very relevant when the material incentives vary in every period (the so-called complex environment discussed in Section 3.2 or when they are changed from a period to another to the the implementation of a specific policy (as discussed in the policy analysis of Section 4). For instance, a society can undergo a process of separation starting from being marginalized but not starting from being integrated.

Definition 3 *Two cultural groups are said* ***without identities*** *if they have weak traits. The process of* ***identity erosion*** *occurs if traits of the two groups became weak in the long-run.*

Lastly, when agents have weak traits we say the society is without identities because only material incentives matter and agents do not have a strongly relative identification with either of the two actions. Table 1 below summarizes the different acculturation outcomes as depending on the types and the strength of cultural traits and assuming that the other group has a strong trait.

	Weak Trait	Strong Trait
Same Type	**Integrated** (weak cultural homogeneity)	**Assimilated** (strong cultural homogeneity)
Different Type	**Marginalized** (weak cultural diversity)	**Separated** (strong cultural diversity)

Table 1: Acculturation Outcomes and defined depending on the type and strength of the trait of one group, given the other group has a strong trait.

In our model, the first dimension considered by Berry (1997) —that is, *cultural maintenance*— can be captured the distance of the role models, that is displayed prescriptive, to the overall population descriptive norm (i.e. average action), that is, $\Delta_i := |\theta_i - \bar{a}|$ for $i = \text{L}, \text{S}$.

Similarly, the second dimension —that is, *contact* between groups— is reflected, endogenously, by the parental socialization efforts $\boldsymbol{\tau} = (\tau_{\text{L}}, \tau_{\text{S}})$. Indeed, the more effort a generic parent i exerts to directly socialize the child, the lower the child's inter-group interactions.

The next proposition provides sufficient conditions for which the ordering of distance in the transmitted role models and parental socialization efforts in the different acculturation outcome are consistent with the definitions of Berry (1997).

Proposition 4 *Consider the steady states traits of the cultural dynamics in* (8). *If* $\tilde{c}$ *and* $\tilde{\lambda}$ *high enough, in particular higher than* 1*, or if either* λ *or* c *are equal to zero, then for each group* $i = \text{L}, \text{S}$ *it holds* $0 = \tau_i^{ass.} = \tau_i^{no\ id.} \leq \tau_i^{int.} \leq \tau_i^{mar.} \leq \tau_i^{sep.}$ *and* $0 = \Delta_i^{ass.} = \Delta_i^{no\ id.} \leq \Delta_i^{int.} \leq \Delta_i^{mar.} \leq \Delta_i^{sep.}$.

Consistently with Berry (1997), if two groups are assimilated, agents seek to have interactions with the other group and, in this case, the effect of cultural substitution induced by cultural distance is null and the strength of vertical socialization under assimilation is minimum —that is, $\boldsymbol{\tau}^{ass.} = (0, 0)$. Moreover, in steady state the two role models are the same– that is, $\Delta_i^{ass.} = 0$ —so that the two groups are fully identified with the same action and culture.

When groups are separated, agents belonging to the two groups tend to transmit very different role models and to avoid across-culture interactions. Thus, the strength of vertical socialization, $\boldsymbol{\tau}^{sep.}$ and $\Delta_i^{sep.}$ are maximal.

Lastly, in both integrated and marginalized societies, both the distance between role models—$\Delta_i^{int.}$ and $\Delta_i^{mar.}$, respectively—and the strength of oblique socialization—$\boldsymbol{\tau}^{int.}$ and $\boldsymbol{\tau}^{mar.}$, respectively—are lower than in separated societies but higher than in assimilated ones. Integrated societies have lower horizontal socialization levels and role models closer to the population descriptive norm than marginalized ones.

We proceed with the characterization of acculturation process in dependence of the strategic environment for cases when material incentives are constant in every period, the so-called fixed environment in the next section 3.1, or when the change within the same type of strategic uncertainty, as in a so-called complex environment in Section 3.2.

3.1 Acculturation in Simple Environments

In this section, we present the results about acculturation as implied by our model of cultural dynamics when the environment in which agents interact is simple. Namely, we consider strategic environments with a fixed material payoff π, and thus fixed thresholds $\hat{x}_0$ and $\hat{x}_1$, exhibiting either strategic complementarity of strategic substitutability.

No parental transmission costs $\lambda = c = 0$

As a benchmark, we first consider the particular case where parents do not face any cost of direct socialization, $c = 0$, or to transmits role models different from own traits, $\lambda = 0$.

In such a case, altruism is the only driver of cultural transmission, the effect of oblique socialization can be neutralized at no costs, and each parent i chooses a combination of θ_i and τ_i such that the child acquires the trait which maximizes the expected future welfare in that environment, namely the average action played by the group. Therefore, also in a steady state traits are in line with induced behaviors, that is, $x_i^* = \bar{a}_i^*$ for both groups $i = \text{L}, \text{S}$. The following proposition characterizes the steady states of cultural dynamics in terms of acculturation outcomes, as depending on initial traits' distribution $\mathbf{x}_0$.

Proposition 5 *Consider the cultural dynamics* (8) *under a fixed material payoff* π*, thresholds* $\hat{x}_0$ *and* $\hat{x}_1$*, and with no parental transmission costs,* $c = 0$ *and* $\lambda = 0$*.*

(i) *If initial traits* $\mathbf{x}_0$ *are strong and of the same (a different) type, then the two groups became assimilated (separated) at the extreme;*

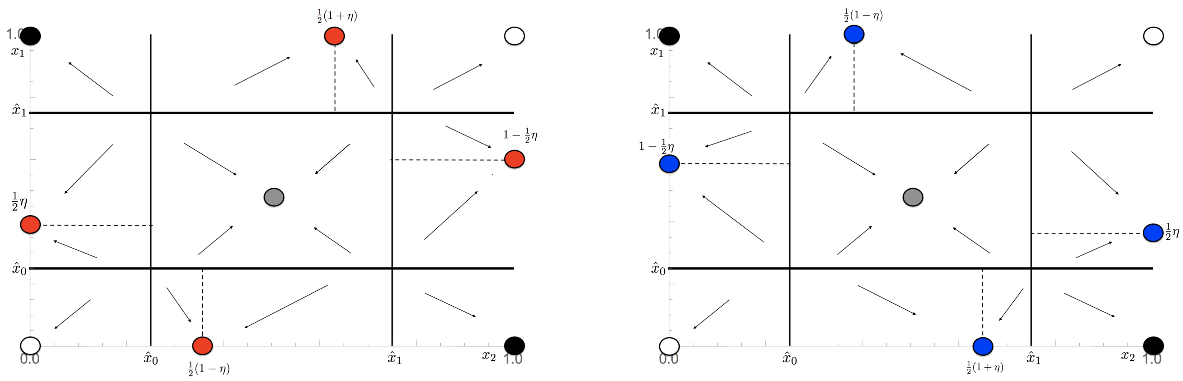
(ii) *If initial traits* $\mathbf{x}_0$ *are weak, then the two groups became neutral;*

(iii) *If initial traits* $\mathbf{x}_0$ *are one weak and one strong and the environments displays strategic complements, then the two groups became either integrated or assimilated at the extreme, depending on the values of* $\hat{x}_0, \hat{x}_1, \eta$*;*

(iv) *If traits* $\mathbf{x}_0$ *are one weak and one strong and the environments displays strategic substitutes, then the two groups became either marginalized or separated at the extreme, depending on the values of* $\hat{x}_0, \hat{x}_1, \eta$*.*

Under null parental costs, parents are always able to socialize their children to their group average behaviour. For this reason, traits move towards own behavior. When a trait is strong, it implies that agents plays the corresponding behaviour as dominant strategy, which reinforce the trait to became extreme. When a trait is weak, behaviour is influenced by the trait of the other group, through strategic nature of the environment. It follow that, when both traits are strong their long-run values do not depend on the strategic environment and the initial strong cultural diversity—i.e., separated societies—or homogeneity—i.e., assimilated societies—are reinforced in the long-run, as shown by Proposition 5 (i) and illustrated in black (separated) and white (assimilated) dots in Figure 5 (a) and (b). Conversely, if initial cultural traits of both groups are not strong enough to affect behavior—i.e., weak—, traits are eroded and became both neutral, as shown by Proposition 5 (ii) and illustrated by grey dots in Figure 5 (a) and (b). Notice, however, that the induced behavior depends on the strategic nature of the environment.

When the society is initially composed by one group with strong traits—think for example of a cultural dominant majority or of a very traditional minority—and the other with a weak trait, the strategic environment affects both the long-run trait and behaviour of the weak group. Under strategic complements, the weak group has material incentives to coordinate to the behavior of the strong group, thus pushing traits to undergo a processes of cultural homogenization and became integrated (Proposition 5 (*iii*) and red dots in Figure 5 (*a*)). Under strategic substitutes, the weak group has material incentives to anti-coordinate to the behavior of the strong group, thus pushing traits to undergo a processes of cultural divergence and became marginalized (Proposition 5 (*iv*) and blue dots in Figure 5 (*b*)).



(a) Strategic Complements (b) Strategic Substitutes

Figure 5: Cultural dynamics with no parental costs ($c = \lambda = 0$) in the space $x_{\text{S}}, x_{\text{L}}$ and fixed material payoff π inducing $\hat{x}_0$ and $\hat{x}_1$. Material payoffs are assumed to be strong enough to ensure the existence of acculturation outcomes with integration and marginalization (i.e., $\hat{x}_0 < \frac{1}{2}\eta$ or $\hat{x}_1 > 1 - \frac{1}{2}\eta$) .

Note that, these steady states with one strong and one weak trait—i.e. integrated and marginalized—may exist only if material incentives are strong enough for the weak group to keep playing both actions in in-group interactions. Otherwise, the weak acquires a strong trait of the same (different) type of the other group in environments with complements (substitutes) leading to a process of assimilation (separation), as also stated in Proposition 5 (*iii*) (Proposition 5 (*iv*)).

Threshold values of material incentives are reflected in average behavior of the weak group, induced by the interaction with the strong group, being larger or smaller than the thresholds $\hat{x}_1$ or $\hat{x}_0$. Let us consider the integration case when the strong group plays 1 (0): here what matters is that there are enough material incentives for the group with a weak trait to keep coordinating on $(0, 0)$ ($(1, 1)$) when playing against themselves so that their average behaviour is smaller than $\hat{x}_1$ (larger than $\hat{x}_0$). Conversely, if we consider the marginalization case when the strong group plays 1 (0) what matters is that there are enough material incentives for the group with a weak trait to play 1 ((0)) when playing among themselves so that their average behaviour is larger than $\hat{x}_0$ (smaller than $\hat{x}_1$).

Figure 5 represents the case in which the incentives set by strategic environment are strong enough that integrated and marginalized outcomes exist within the corresponding strategic environment.[22]

The following remark summarizes the main take-away message of Proposition 5.

Remark 1 *In a simple environment with no parental transmission costs, strong cultural homogeneity and diversity are always preserved. Moreover, processes of integration and assimilation can be observed only with strategic complements while processes of marginalization and separation can be observed only with strategic substitutes.*

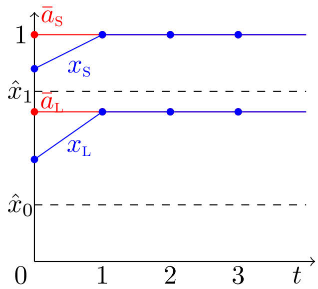
Figure 6 shows how dynamics of cultural traits changes in a society with a strong a weak traits if there is a shock on the material payoffs which moves the thresholds $\hat{x}_0$ and/or $\hat{x}_1$, for example due to the implementation of a policy which changes material incentives. A weakening of material incentives in the case of strategic complements, corresponding to a shifts of $\hat{x}_1$ downwards, is represented in moving from panel (a) to panel (b). Starting from a situation which leads to integration (panel (a)), the weakening of incentives to coordinate on the equilibrium $(0, 0)$ amounts to a decrease of $\hat{x}_1$ (the minimum trait such that action 1 is dominant) and can create assimilation because can induce a behaviour solely due to the cultural component for both populations. A similar effect is shown for strategic substitutes, moving from panel (c), exhibiting marginalization, to panel (d), exhibiting separation. The change in the acculturation process is due to a weakening of the material incentives to anticoordinate and thus a strengthening to best-reply with a zero even when its own population plays action zero. This initial change, which is triggered by a change of material payoffs, is reinforced as the cultural component in favour of action 0 moves to an extreme value. Finally, note that in both situations a subsequent restoring of the initial value of material payoff does not restore integration or marginalization. so even temporary policies have permanent effects

Lastly, it is worth mentioning that a similar reasoning can be done by moving the population shares while keeping the material payoffs and, thus, the thresholds $\hat{x}_0$ and $\hat{x}_1$ fixed. Indeed, increasing the size of the group with strong traits, it would increase the frequency with which the agents with the weak trait coordinate (anti-coordinates) to the behavior of the strong group, in environments with strategic complements (substitutes). Therefore, given the thresholds $\hat{x}_1$ ($\hat{x}_0$), if the share in the population of the strong group is sufficiently large, the average behavior becomes larger (smaller) than $\hat{x}_1$ ($\hat{x}_0$), leading to a process of assimilation or separation, depending on the strategic environment. These considerations on the population shares lead to the following remark where conditions on

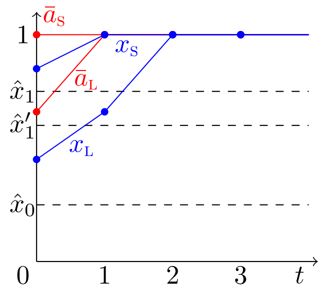
[22]Instead, having assumed extremism ($A4$) assimilated, separated, and neutral steady states always exist.

the nature of material payoffs are related to the relative size of each group.

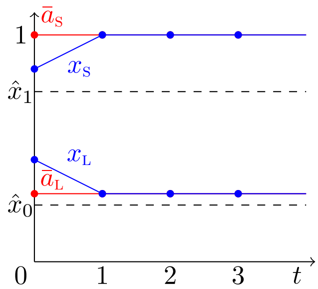
Remark 2 *In a simple environment with no transmission costs, it is more likely to observe processes of marginalization or integration—as opposed to assimilation or separation—when the minority is the group with strong trait.*



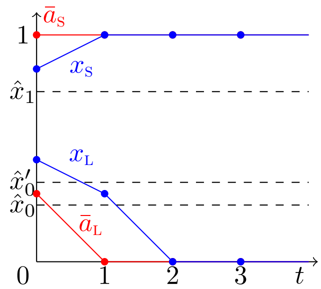
(a) Strategic Complements and $\hat{x}_1 > \bar{a}_L = 1 - \frac{1}{2}\eta$



(b) Strategic Complements and $\hat{x}_1 < \bar{a}_L = 1 - \frac{1}{2}\eta$



(c) Strategic Substitutes and $\hat{x}_0 < \bar{a}_L = \frac{1}{2}\eta$



(d) Strategic Substitutes and $\hat{x}_0 < \bar{a}_L = \frac{1}{2}\eta$

Figure 6: Dynamics of cultural traits and behavior starting from society where agents in group L or S have weak and strong traits, respectively.

Positive parental trasmission costs $\lambda > 0$, $c > 0$

Let us now consider the case in which parents face costs of direct socialization, $c > 0$, and to transmit a role model different from own trait, $\lambda > 0$. Under positive costs, parents are not generically able to fully transmit the optimal altruistic trait, In particular, children of both groups are also influenced by the population average behaviour, through the oblique socialization which has a coordinating effect. Overall, the cultural dynamics is (8) with $\rho_i(\mathbf{x}_t) < 1$, for both $i = L, S$. Although the dynamics does change, with trait adapting more slowly, most of the results in Proposition 5 remain qualitatively unchanged. Since introducing positive parental costs creates a coordinating force, in environments with complements this is aligned with the incentives coming from agents' interaction and one continues to observe processes of cultural homogenization, such as integration and assimilation. On the contrary, in environments with substitutes, the coordinating forces who drive cultural transmission contrasts the incentives coming from agents' interaction. If the costs are high enough, coordinating forces are so strong to make the typical cultural dynamics associated with strategic substitutes, marginalization and separation, not possible.

The next proposition formalizes this instance and characterizes the differences in the steady states and long-run cultural dynamics from the case without parental costs.

Proposition 6 *Consider the cultural dynamics* (8) *under a fixed material payoff* π *and thresholds* $\hat{x}_0$ *and* $\hat{x}_1$ *and with positive parental transmission costs,* $c > 0$ *and* $\lambda > 0$.

- (i) *In the long run traits become extreme if and only if groups are assimilated.*
- (ii) *If the cost of direct socialization* c *is large enough, the two groups cannot be separated or marginalized in the long run, independently of the strategic environment.*

Proposition 6 (i) states that the effect of oblique socialization, which cannot be counterbalanced when parents face positive transmission costs, makes it impossible for the trait to became extreme but different. This also implies that, when traits are not assimilated—i.e., both strong and of the same type—there always exists material incentives strong enough, with $\hat{x}_0$ and $\hat{x}_1$ moving to 0 and 1 respectively, such that traits became neutral in the long run and only material payoffs play a role, as in a process of identity erosion.

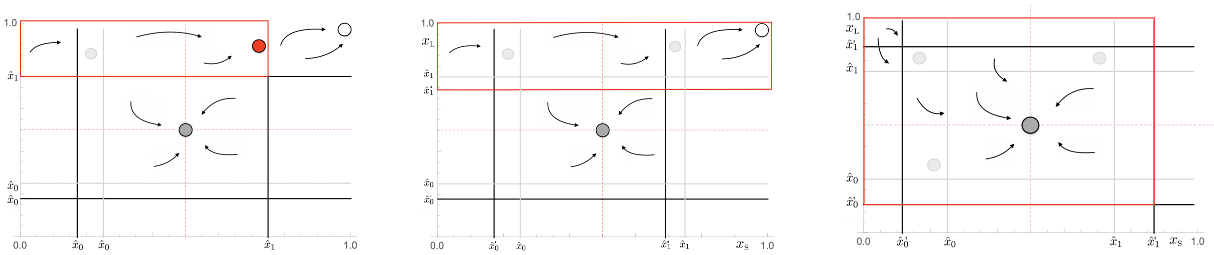
Proposition 6 (ii) shows that when the direct transmission cost c is sufficiently high, oblique socialization causes the steady states with cultural diversity to no longer exist—i.e., separated and marginalized societies—,independently of the strategic environment.[23] Moreover, one can also show that the threshold on the cost c, above which steady states with diversity do not exists, is higher with substitutes than with complements. This occurs because in environments with strategic complements, once a trait becomes weak, the coordination incentives lead to the homogenization of traits. Conversely, in environments with strategic substitutes, achieving cultural homogeneity requires such high costs that not only one trait must be weak, but it must also be of the same type as that of the other group.

Lastly, having in mind the results of Proposition 3, we also know that cultural complementarity is observable only if there is a group with a weak trait that has material incentives to coordinate with the other group.

[23]Note that the effect of oblique socialization in our model is similar to the one of conformism in Desmet and Wacziarg (2021).

Remark 3 *In a simple environment, if the parental socialization cost is high enough, then we can have process of integration and assimilation also with strategic substitutes. Moreover, cultural complementary is observable only in processes of integration that occur in environments with strategic complements.*

Figure 7 shows how, starting from a separated society, the absence of extreme traits can lead to processes of Integration (Panel (a)), Assimilation (Panel (b)), and Identity erosion (Panel (c)), depending on material incentives—i.e., thresholds $\hat{x}_0$ and $\hat{x}_1$.



(a) Integration (b) Assimilation (c) Identity Erosion

Figure 7: Cultural dynamics, starting from separated society, with positive parental costs ($c > 0$ and $\lambda > 0$) in a simple environment, and converging to different acculturation outcomes, depending on the thresholds $\hat{x}_0$ and $\hat{x}_1$, induced by different fixed material payoffs.

3.2 Acculturation in Complex Environments

In the previous section, we have characterized the long-run dynamics and steady states when the material payoffs agents face in their adult age are fixed. We relax this assumption allowing material payoffs and, thus, the threshold $\hat{x}_0, \hat{x}_1$, to vary. For simplicity, we assume the distribution of payoffs γ is such that thresholds $(\hat{x}_0, \hat{x}_1)$ are uniformly distributed in $(0, \frac{1}{2}) \times (\frac{1}{2}, 1)$.[24]

Here, we consider a complex environment where a trait may be either strong or weak depending on the realization of the payoffs in the specific interaction and the only traits which are always strong or weak are extreme traits and neutral traits.

The next proposition characterizes the cultural dynamics in an environment with strategic complements when payoff are random and uniformly distributed.

Proposition 7 *Consider the cultural dynamics in (8) in a* ***complex*** *environment with strategic* ***complementarity****. If material payoffs are such that $(\hat{x}_0, \hat{x}_1)$ is uniformly dis-*

[24]Note that the extrema 0 and 1 are not included to allow a strong enough cultural trait to satisfy the assumption of extremism, $A4$. $\frac{1}{2}$ is not included to allow the strategic environment to have an effect.

tributed in $(0, \frac{1}{2}) \times (\frac{1}{2}, 1)$*, then* ***cultural homogeneity*** *prevails and traits converge to be* ***extreme****, either to* $(0, 0)$ *or to* $(1, 1)$*, depending on initial conditions.*

Proposition 7 shows that, when thresholds are uniformly randomly distributed the traits always converges to extreme homogeneous traits. The only stable steady states of the cultural dynamics (8) are those where the traits become extreme. If the initial conditions is such that traits are different, or one is close to be neutral, this corresponds to an assimilation process.

This is due to the fact that with strategic complements the material incentives and the effect of oblique socialization go in the same direction, pushing toward homogeneity. In this process, a crucial role is played by the fact that, when the realized payoffs of a game result in one group having a weak trait, the material incentives drive agents to coordinate their actions with the other group, even if the traits are of different types. This, coupled with the uniform distribution of payoffs, ensures that when traits are different, the distance between the average actions played is always smaller than the distance between the traits, preventing the existence of steady states with traits of different types. Similarly, when traits are of the same type they reinforce each other, converging together to extremity.

Let us now consider environments with strategic substitutes. In such a case the effect of oblique socialization and material incentives go in opposite directions, thus, the cultural dynamics is much more complex and depends on the relative strength of the two.

Proposition 8 *Consider the cultural dynamics in* (8) *in a* ***complex*** *environment with strategic* ***substitutes****. If material payoffs are such that* $(\hat{x}_0, \hat{x}_1)$ *is uniformly distributed in* $(0, \frac{1}{2}) \times (\frac{1}{2}, 1)$*, then it is not possible to observe any stable cultural homogeneity other than neutral traits. Moreover,*

(i) If $c = 0$ *and* $\lambda = 0$*, then* ***cultural heterogeneity*** *prevails and traits converge to be* ***extreme*** *either to* $(1, 0)$ *or to* $(0, 1)$*, depending on initial conditions;*

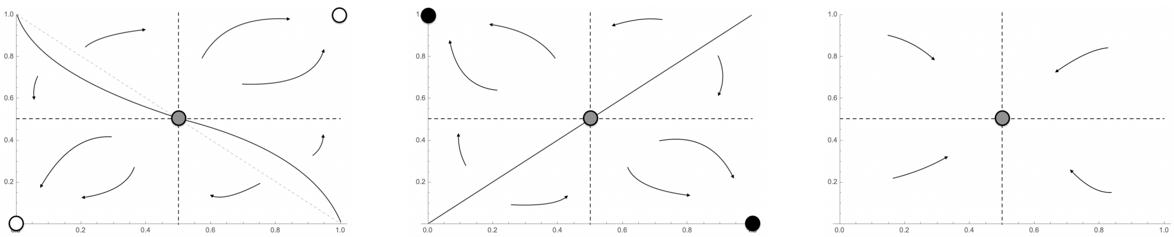
(ii) If $c \to +\infty$*, then for any* λ *traits converge to be* ***neutral****.*

Proposition 8 shows that complex environments with uniformly distributed payoffs prevent traits to became of the same type in environments with strategic substitutes, unless they are both neutral. This is an important difference with respect to the case of fixed payoffs (Section 3.1), where, if the socialization cost is high enough, the cultural dynamics in environments with strategic substitutes behaves as in environments with strategic complements.

The level of heterogeneity of cultural traits at steady state depends on the transmission costs. In particular, we characterize the two extremes. When parenting costs are null, as in (i), parents are able to fully transmit the desired traits and the oblique socialization does not affect cultural dynamics, causing traits to become extreme and of different type. When direct socialization cost is high enough, as in (ii), parents cannot transmit their own trait are the imitating effect of oblique socialization dominates so that the cultural traits of the two groups are eroded and reach neutrality.

Remark 4 *In a complex environment we can observe, at the steady state, cultural homogeneity only with strategic complements and cultural diversity only with strategic substitutes.*

Figure 8 qualitatively shows the dynamics of traits described in Proposition 7 and Proposition 8.



(a) Strategic complements (b) Strategic substitutes and $c = 0$ (c) Strategic substitutes and $c = \infty$

Figure 8: Cultural dynamics for $(\hat{x}_0, \hat{x}_1)$ uniformly distributed in the space $\left[0, \frac{1}{2}\right] \times \left[\frac{1}{2}, 1\right]$.

4 Final Discussion

With this work, we provide a unified theoretical framework capable of studying the different acculturation patterns which may occur when two cultural groups interact in a strategic environment, and we relate them to the social and economic incentives faced by agents.

We consider two cultural groups whose agents interact in strategic environments. Agents' behavior is affected by both material payoffs and cultural traits that are acquired in the young age and are intergenerationally transmitted.

In our model, the results about the different long-run cultural configurations depend on three main dimensions: (i) the nature of strategic environment, whether it is characterized by complements or substitutes; (*ii*) the stability of payoffs; and (*iii*) the costs of parental transmission.

The paper shows that when agents face environments with strategic complements, there is general tendency toward the homogenization of traits; when, instead, agents face environments with strategic substitutes traits tend to become heterogeneous as long as the cost of socialization is low enough. However, when the payoffs agents face are fixed, the initial level of cultural diversity and strength of traits play a major role. Indeed, if traits are both strong (with respect to the strategic environment) the society, independently of having complements or substitutes, may remain assimilated or separated—i.e., traits remain strong and agents of the two groups always play the same or the opposite action, respectively. Similarly, if cultural traits of both groups are weak their effect vanishes and, in the long run, agents care only about material payoffs.

Interesting cases occur when agents of one group have a strong cultural trait that induce homogeneous behavior but agents of the other group have weak traits and may behave differently in within-group or intra-group interactions. For example, in a processes of integration —which occurs with strategic complements or with strategic substitutes only if the vertical socialization cost is high—agents belonging to the minority conform to the majority when interact with them (e.g. using the same language) but, at the same time, they preserve a heterogeneous behavior when interacting among themselves (e.g. they may use their original language). Thus, in our model it is possible to have long-run integration while preserving a certain level of multiculturalism, a feature that cannot occur in models where trait and behavior coincide at the the steady state (e.g., Kuran and Sandholm, 2008). Similarly, when there is a processes of marginalization—which occur only with strategic substitutes—agents with weak traits do not conform to the others but neither develop a strong opposite trait, as they do in separation.

The paper also shows that parental investment in direct socialization may display non-monotonic behavior, with respect to population shares, when there is a group with a weak trait that has material incentives to coordinate with the other group. This prediction is confirmed by empirical literature on cultural transmission (e.g., Bisin et al., 2004b; Cohen-Zada, 2006), specifically in the context of religious traits. Indeed, we can argue that when a religious trait is weak, the decision to socialize the child with one's own trait or not mostly depends on coordination motives within the society.

The paper further shows that, when the environment is complex and the payoffs that agents face change at across interactions the cultural dynamics may be severely affected, precisely due to the role of the cases withe diversity of traits. In environments with strategic complements there is always, and independently of the initial cultural diversity, a convergence to assimilated extreme traits not leaving any room for multiculturalism nor for any heterogeneity in traits and behavior. In environments with substitutes, in complex environments with uniformly distributed payoffs, traits are always either of different types or neutral, depending on the cost of socialization and the initial conditions. This contrasts with what happens in simple environments with substitutes, where we can have traits of the same type both when the cost of socialization is very high or due to specific initial conditions.

Overall, our results show that when material payoffs change within the same generation change—i.e., complex environments—the cultural dynamics is mostly driven by the strategic nature of material incentives and the initial conditions play a marginal (or null) role compared to when the payoffs are fixed—i.e., simple environments—where our model predicts more cultural persistence. This result is complementary to Giuliano and Nunn (2021) which consider change of payoffs across generation and show that it leads to less-persistent cultural traits, as opposed to the cultural persistence observed when payoffs do not change across generations.

The following tables summarize the predictions of our model depending on whether the strategic environments displays strategic complements or substitutes and if the environment is simple or complex, with uniformly distributed payoffs. In particular, Table ?? refers to the case in which parental costs are small, whereas Table ?? refers to the case in which parental costs are high.

Environment	**Simple**	**Complex**
Complements	Assimilation and/or Integration Strong cultural heterogeneity is preserved	Assimilation
Substitutes	Separation and/or Marginalization Strong cultural homogeneity is preserved	Cultural Diversity

Table 2: Lung-run cultural outcomes when there are **small** parental costs and at least one strong trait at $t = 0$.

Environment	**Simple**	**Complex**
Complements	Assimilation and/or Integration	Assimilation
Substitutes	Assimilation and/or Integration	Identity Erosion

Table 3: Lung-run cultural outcomes when there are **high** parental costs and at least one strong trait at $t = 0$.

Policy Implications As previously discussed, in this paper we do not assume that an acculturation outcome or process is more desirable than another one. Indeed, the different acculturation outcomes might be more or less desirable depending on how welfare is measured—e.g., taking into account only material or the whole payoff—and on features not included in the model—such as the objective function of the policy maker. However, the paper sheds light on the conditions under which different acculturation outcomes arise and, thus, does inform the policy maker about how to act on them.

In this context, (i) the nature of strategic environment, (ii) the stability of payoffs, and (iii) transmission costs, represent potential channels for policy intervention, subject to their feasibility and the constraints faced by policy makers.

In particular, dimension (i) refers to whether agents possess economic incentives to align their efforts in similar activities or if there are incentives for specialization in distinct areas. Notably, certain economic interactions exhibit synergies and positive network externalities, promoting coordination—e.g., using the same language, adopting the same technology, or adopting similar social norms—, while others manifest congestion effects, encouraging specialization—e.g. job division, heterogeneous educational choices, consumption of differentiated products. At the same time the policy maker can, up to a certain constraints, shape the incentives promoting certain level of coordination or anti-coordination—e.g., incentivizing high work ethics, promoting the adoption of the native language through free language courses, providing subsidies for ethnic, cultural, or economic activities, etc.

In the context of dimension (ii), the stability of the economic environment is contingent on several factors. For instance, it may depend on the frequent changes in policies related to taxation and/or subsidies. But also on things not fully under the control of the policy maker as the high price and wage volatility characterizing periods of financial instability and crisis, or even frequent technological shocks and innovations. In such a case, the policymaker cannot directly intervene on this dimension but should consider its impact on cultural dynamics, as shown by the model.

Finally, dimension (iii) can refer to all those policies that affect the opportunity cost of direct socialization for parents. For example, policies that increase the opportunity cost of direct socialization include promoting parental engagement in full-time employment, initiatives that support the establishment of kindergartens and after-school programs, as well as subsidies aimed at assisting babysitters.

Limitations and future works Let us first notice that the paper considers a form of *imperfect* parental altruism, in the intergenerational transmission of preference.[25] Indeed, assuming adaptive expectation about future generation actions, parents do not fully internalize the effect of their parental choices on the offspring future welfare. We do this choice both to capture the limited farsighted of parents, allowing for a more tractable solution, and also for a more clear exposition of the possible long-run outcomes of cultural dynamics.

In particular, if cultural resilience is large enough, allowing for full altruism, as oppose to imperfect altruism, results do not change. In such a case, parents are bounded to transmit a trait not too far from own trait and, thus, the offspring would play as optimal action the same action of the parents, which would also be the target trait parent want altruistically transmit. Conversely, if the cultural resilience is low, parents face a coordination/anti-coordination problem that may give them incentives to transmits traits that would induce a different actions. Thus, the strategic environment would play a more prominent role. For example, in an environment with complements, parents of a separated minority may aim to transmit a trait that induces actions aligned with the majority to gain a higher payoff through coordination. While we acknowledge the possibility of such sophisticated reasoning, we believe that, on average, most parents lack such foresight in transmittig their traits.[26]

As a future work, the analysis can be extended in several dimensions. For example, we believe it is worth studying the cultural dynamics in environments with wider payoffs distributions. One possibility is to consider case where material payoffs associated to the same action can exhibit strategic complementarity in some interactions and substitutability in others, for example related to effort in team-works under different technologies or institutional frameworks. Another possibility is to consider social dilemmas as the prisoner dilemma or public good games. Another potentially interesting extension is to analyze how the interaction of more than two groups affects the acculturation dynamics. For example, Fouka et al. (2022) empirically shows that the arrival of a new minority group may induce higher assimilation of existing minorities.

Lastly, we believe that a further effort should be exerted to incorporate political motives into the analysis. Either extending the analysis in the direction of Bisin and Verdier

[25] Refer to the seminal paper Doepke and Zilibotti (2017) for an analysis of intergenerational transmission of preferences when parents, motivated by (perfect) altruistic and paternalism, can choose their parenting style.

[26] It is worth noting that the potential long-run outcomes under parental *perfect* altruism are the same of those under imperfect altruism. The main difference is the more detailed characterization of initial conditions, as depending on the payoffs of the coordination/anti-coordination games parents might face. Therefore, to study the effect of the strategic environment in the cultural dynamics, while maintaining result characterization clarity, we opted for imperfect altruism.

(2023b) and Bisin et al. (2021), studying the joint evolution of culture and political institutions, or including the political dimension and its interaction with economic and social forces, taking into account the interaction of economic, social, and political issues to study, for example, the impact of different degrees of cultural diversity on economic production together with the probability of having clashes and conflicts.

A Appendix

In this appendix, we provide a specific example of cultural and material payoffs which can be used to characterize the thresholds $\hat{x}_0$ and $\hat{x}_1$.

A.1 A classification of material payoffs

Let us consider generic 2×2 symmetric games represented by the following (material) payoff matrix

		Agent c	
		1	0
Agent r	1	α, α	$\beta + D_0, \alpha + D_1$
	0	$\alpha + D_1, \beta + D_0$	β, β

where $D_1 := \pi(1,1) - \pi(0,1)$ and $D_0 := \pi(0,0) - \pi(1,0)$ are the material incentives to deviate from $(1,1)$ and $(0,0)$, respectively. We further assume without loss of generality that $\alpha > \beta$. We can classify the game described in the matrix according to the the ordering of D_1 and D_0.

- **Coordination game**: $D_1 < 0$ and $D_0 < 0$
- **Anti-coordination game**: $D_1 > 0$ and $D_0 > 0$
- **Prisoner's Dilemma**: $D_0 < 0 < D_1$
- **Efficient Dominant Strategy**: $D_1 < 0 < D_0$

Figure 9 show the classification of games.

Let us now define **strategic complementarity** and **substitution** in 2×2 games.

Definition 4 *A 2×2 game displays*

- ***strategic complements*** *when $BR_j(1) = 1$ and $BR_j(0) = 0$, for all $j = r, c$;*
- ***strategic substitutes*** *when $BR_j(1) = 0$ and $BR_j(0) = 1$, for all $j = r, c$.*

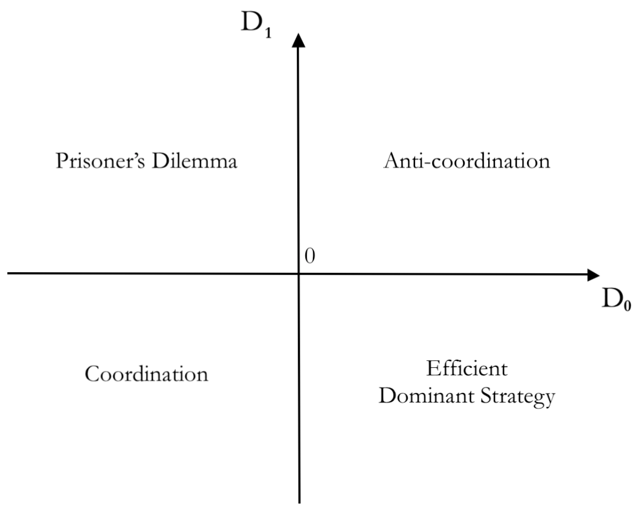


Figure 9: Classification of 2×2 symmetric games depending on D_1 and D_0, when $\pi(1,1) > \pi(0,0)$.

By inspection of the definition above we can easily see that Coordination games always display **strategic complements**. Conversely, Anti-coordination games always display **strategic substitutes**.

In Prisoner's Dilemmas there are incentives to deviate from $(1,1)$, i.e., $D_1 > 0$, but not from $(0,0)$, i.e., $D_0 < 0$. Vice versa for Efficient Dominant Strategy games. Thus, Prisoner's Dilemmas and Efficient Dominant Strategy games have strategic complements with respect to one action and strategic substitutes with respect the other. We can talk about strategic complementarity (or substitution) in Prisoner's Dilemmas and Efficient Dominant Strategy games if the effect of complementarity for one action is stronger (weaker) than the effect of substitution for the other (e.g., Tabellini (2008) consider the case of Prisoner's Dilemmas with strategic complements). In what follows we focus on Prisoner's Dilemma, mirror results and discussion can be obtained in the case of Efficient Dominant Strategy games. Let us define, depending on the relative magnitude of the complements and substitutes forces:

- **Prisoner's Dilemma** with **strategic complements** if $|D_0| > |D_1|$;
- **Prisoner's Dilemma** with **strategic substitutes** if $|D_1| > |D_0|$.

Figure 10 shows how 2×2 games displays strategic complements or substitutes as depending on D_1 and D_0.

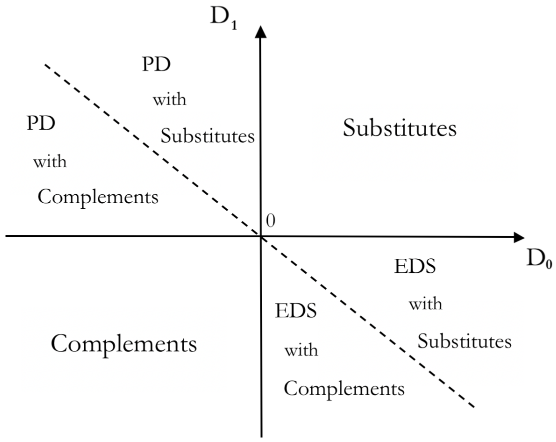
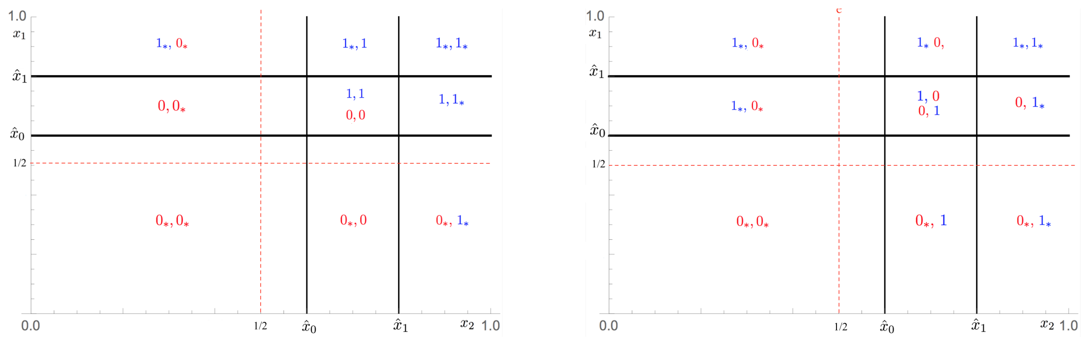


Figure 10: Complements and Substitutes in 2×2 symmetric games depending on D_1 and D_0.



(a) (b)

Figure 11: Nash Equilibrium in Prisoner's Dilemma with strategic (a) Complements and (b) Substitute, as depending on the cultural traits.

A.2 Separable payoffs with linear cultural component

We now show the functional form of the thresholds $\hat{x}_1$ and $\hat{x}_0$ when the function $v(.)$ is separable and the cultural component $\rho(.)$ depends quadratically on the distance between action and trait.

$$u^a\left(a_j, a_{-j}; x_i\right) = \pi(a_j, a_{-j}) - \psi(a_j - x_j)^2, \tag{A.1}$$

with $\psi \in \mathbb{R}_+$. The matrix of ultimate payoffs is

		Agent c	
		1	0
Agent r	1	$\alpha - \psi(1-x_r)^2, \alpha - \psi(1-x_c)^2$	$\beta + D_0 - \psi(1-x_r)^2, \alpha + D_1 - \psi x_c^2$
	0	$\alpha + D_1 - \psi x_r^2, \beta + D_0 - \psi(1-x_r)^2$	$\beta - \psi x_r^2, \beta - \psi x_c^2$

Let us now consider the condition, for a generic j, on x_j such that actions 1 or 0 become dominant. Note that the following condition are the same of the case in which the cultural component $\rho(.)$ depends linearly on the distance between action and trait, as in Akerlof and Kranton (2005); Kimbrough and Vostroknutov (2016).

Action $a_j = 1$ is dominant if and only if

$$\alpha - \psi(1 - x_j) > \alpha + D_1 - \psi x_j \quad \text{and} \quad \beta + D_0 - \psi(1 - x_j) > \beta - \psi x_j$$

$$\Rightarrow \quad -\psi + \psi x_j > +D_1 - \psi x_j \quad \text{and} \quad +D_0 - \psi + \psi x_j > -\psi x_j$$

$$\Rightarrow \quad x_j > \frac{1}{2} + \frac{D_1}{2\psi} \quad \text{and} \quad x_j > \frac{1}{2} - \frac{D_0}{2\psi}.$$

Therefore, Action $a_j = 0$ is dominant if and only if

$$\Rightarrow \quad x_j < \frac{1}{2} + \frac{D_1}{2\psi} \quad \text{and} \quad x_j < \frac{1}{2} - \frac{D_0}{2\psi}.$$

Given the ordering of D_1 and D_0 in different classes of games, we can easily verify that:

- In games with **Complements** ($D_1 < 0$ and $D_0 < 0$)

$$\hat{x}_1 = \frac{1}{2} - \frac{D_0}{2\psi} > \frac{1}{2} \quad \text{and} \quad \hat{x}_0 = \frac{1}{2} + \frac{D_1}{2\psi} < \frac{1}{2}$$

- In games with **Substitutes** ($D_1 > 0$ and $D_0 > 0$)

$$\hat{x}_1 = \frac{1}{2} + \frac{D_1}{2\psi} > \frac{1}{2} \quad \text{and} \quad \hat{x}_0 = \frac{1}{2} - \frac{D_0}{2\psi} < \frac{1}{2}$$

- In **Prisoner' Dilemmas** with **Complements** ($D_0 < 0 < D_1$ with $|D_0| > |D_1|$)

$$\hat{x}_1 = \frac{1}{2} - \frac{D_0}{2\psi} > \frac{1}{2} \quad \text{and} \quad \hat{x}_0 = \frac{1}{2} + \frac{D_1}{2\psi} < \frac{1}{2}$$

- In **Prisoner' Dilemmas** with **Substitutes** ($D_0 < 0 < D_1$ with $|D_1| > |D_0|$)

$$\hat{x}_1 = \frac{1}{2} + \frac{D_1}{2\psi} > \frac{1}{2} \quad \text{and} \quad \hat{x}_0 = \frac{1}{2} - \frac{D_0}{2\psi} < \frac{1}{2}.$$

Risk dominance of strong traits In the case of games with complements, having a specific choice of the effect of traits on payoffs makes it possible to related the position of the two thresholds $\hat{x}_1$ and $\hat{x}_0$ with respect to the neutrality trait level $\frac{1}{2}$ and the corresponding strategy profile, respectively $(1,1)$ or $(0,0)$ being risk dominant for the material part of the payoff.

Recall that for a symmetric coordination game the strategy profile $(1,1)$ risk dominates $(0,0)$ if, assigning an equal probability to the action of the opponent, playing 1 gives a higher payoffs than playing 0. With the material payoffs at the beginning of the Appendix we have

$$\frac{\alpha}{2} + \frac{\beta}{2} + \frac{D_0}{2} > \frac{\alpha}{2} + \frac{D_1}{2} + \frac{\beta}{2}$$

which, being both incentive to deviates from coordination D_0 and D_1 negative, leads to

$$|D_0| < |D_1|.$$

But under the same condition, by definition of the two thresholds $\hat{x}_1$ and $\hat{x}_0$, it holds that the former threshold is closer to the neutrality level than the latter,

$$\left|\hat{x}_1 - \frac{1}{2}\right| < \left|\hat{x}_0 - \frac{1}{2}\right|,$$

implying a larger region with strong traits close to $(1,1)$ than to $(0,0)$.

The same correspondence holds between $(0,0)$ being risk dominant over $(1,1)$ and the region of strong norms around $(0,0)$ being larger that the region of strong norms around $(1,1)$ ($\hat{x}_0$ closer to the middle than $\hat{x}_1$).

The correspondence relies on the fact that both concepts are related to the consequence of not coordinating on the other equilibrium. Indeed, both one strategy profile being risk dominant and the corresponding trait being strong for more values, depend on the size of the payoff loss for not coordinating on the other equilibrium. Consider for example the strategy profile $(1,1)$. It is risk dominant when the loss for playing 1 when the opponent plays 0, $|D_0|$, is smaller that the loss for playing 0 when the opponent plays 1, $|D_1|$. Similarly with a small $|D_0|$, playing 1 when the other pays 0 becomes the best reply for lower levels of the trait in favour of action 1, thus the region in which trait 1 is strong is larger.

B Proofs

Proof of Proposition 1

Fix the strategy of player $-j$, a_{-j}, and consider agent j payoff difference for playing 1 or 0:

$$\Delta u^a(a_{-j}; x_j) := u^a(1, a_{-j}; x_j) - u^a(0, a_{-j}; x_j).$$

A positive (negative) $\Delta u^a(a_{-j}; x_j)$ implies that 1 (0) is the best reply to a_{-j} for the cultural level x_j. To prove the lemma, we show that for each fixed material payoff π exhibiting either complementarity or substitutability, there exist an $\hat{x}_1 \in \left(\frac{1}{2}, 1\right)$ such that for all $x_j > \hat{x}_1$ it holds $\Delta u^a(1; x_j) > 0$ and $\Delta u^a(0; x_j) > 0$ as well as an $\hat{x}_0 \in \left(0, \frac{1}{2}\right)$ such that for all $x_j < \hat{x}_0$ it holds $\Delta u^a(1; x_j) < 0$ and $\Delta u^a(0; x_j) < 0$.

For all π, $A1$ (continuity) and continuity of the utility v imply that both $\Delta u^a(1; x_j)$ and $\Delta u^a(0; x_j)$ are continuous. $A3$ (monotonicity) and v being increasing in both material and cultural component imply that both $\Delta u^a(1; x_j)$ and $\Delta u^a(0; x_j)$ are increasing in x_j (as difference of an increasing and a decreasing function).[27] $A4$ (extremism) implies that $\Delta u^a(1; 0) < 0$ and $\Delta u^a(0; 0) < 0$, as well as $\Delta u^a(1; 1) > 0$ and $\Delta u^a(0; 1) > 0$.

From here we distinguish the case of strategic complementarity and substitutability. Let us start with a π exhibiting strategic complementarity. By $A2$ (neutrality)

$$\Delta u^a\,(1; 1/2) > 0 \quad \text{and} \quad \Delta u^a\,(0; 1/2) < 0.$$

The latter together with the other properties shown above -continuity, being increasing, negativity at 0, positivity at 1- imply that there exists an $\hat{x}_1 \in \left(\frac{1}{2}, 1\right)$ such that for all $x_j > \hat{x}_1$ it hold $\Delta u^a(1; x_j) > 0$ and $\Delta u^a(0; x_j) > 0$, as well as an $\hat{x}_0 \in \left(0, \frac{1}{2}\right)$ such that for all $x_j < \hat{x}_0$ it holds $\Delta u^a(1; x_j) < 0$ and $\Delta u^a(0; x_j) < 0$. The threshold $\hat{x}_0$ is found when $\Delta u^a(1; x_j)$ becomes positive, the threshold $\hat{x}_1$ is found when $\Delta u^a(0; x_j)$ becomes positive.

A similar argument applies when π exhibits strategic substitutability. Now, by $A2$ (neutrality)

$$\Delta u^a\,(1; 1/2) < 0 \quad \text{and} \quad \Delta u^a\,(0; 1/2) > 0.$$

The threshold $\hat{x}_0$ is found when $\Delta u^a(0; x_j)$ becomes positive, the threshold $\hat{x}_1$ is found when $\Delta u^a(1; x_j)$ becomes positive.

Finally, note that by symmetry of each material component π and by homogeneity of the cultural component and utility v, the thresholds are the same for all players.

■

[27] Note that A3 implies that $\rho(1, x_j)$ is increasing in x_j and $\rho(0, x_j)$ is decreasing in x_j.

Proof of Nash Equilibria depicted in Figure 1

As in the proof of Proposition 1, we evaluate the sign of the payoffs differences $\Delta u^a(0;x)$ and $\Delta u^a(1;x)$ for different values of the cultural trait x.

Let us consider a π exhibiting strategic complementarity. In the proof of Proposition 1 it is shown that if $x > \hat{x}_0$, then $\Delta u^a(1;x) > 0$. It follows that if both players $j = r, c$ have $x_j > \hat{x}_0$, then $(1,1)$ is a Nash equilibrium. Similarly, if $x < \hat{x}_1$, then $\Delta u^a(0;x) < 0$. It follows that if both players $j = r, c$ have $x_j < \hat{x}_0$, then $(0,0)$ is a Nash equilibrium. Reasoning along the same lines, for $(1,0)$ to be a Nash equilibrium, it has to be $x_r > \hat{x}_1$, so that $\Delta u^a(0;x_r) > 0$, and $x_c < \hat{x}_0$, so that $\Delta u^a(1;x_c) < 0$. Finally, for $(0,1)$ to be a Nash equilibrium, it has to be $x_r < \hat{x}_0$, so that $\Delta u^a(1;x_r) < 0$, and $x_c > \hat{x}_1$, so that $\Delta u^a(0;x_c) > 0$.

The proof for a π exhibiting strategic substitutabilty proceeds along the same lines.

■

Proof of Proposition 2

Assuming adaptive expectations, we shall characterize the solutions of

$$\max_{(\theta_i,\tau_i)\in[0,1]\times[0,1]} \quad U_i = \mathbb{E}_i\left[\int_\gamma \int_{j\in I/\{i\}} u\big(a_i(a_j), a_j; x_i^o\big)\ dj\ d\gamma\right] - c(\tau_i)^2 - \lambda(\theta_i - x_i)^2, \quad \text{(B.1)}$$

where

$$\begin{cases} x_i^o = \tau_i\theta_i + (1-\tau_i)\bar{a}, \\ a_i(a_j) = Argmax_a\{u(a, a_j; x_i^o)\}, \ \ \forall a_j \in \{a_j\}_{\eta,\gamma} \end{cases},$$

and the trait x_i, the actions played by players $j \in I$, $\{a_j\}_{\eta,\gamma}$, and the population average $\bar{a}$ are taken as given.

(*i*) Continuity of the payoff in the trait, Assumption A1, and compactness of the choice set imply, via the Weierstrass's Theorem, that (B.1) has a solution.

To characterize the solution, let us investigate the sign of the objective function partial derivatives:

$$U_{\theta_i}(\theta_i,\tau_i) = \frac{\partial U_i(\theta_i,\tau_i)}{\partial\theta_i} \ \text{ and } \ U_{\tau_i}(\theta_i,\tau_i) = \frac{\partial U_i(\theta_i,\tau_i)}{\partial\tau_i}.$$

At an internal solution $(\theta_i^*, \tau_i^*) \in (0,1)\times(0,1)$ both derivatives need to be zero. We shall first characterize the zeros of $U_{\theta_i}(\theta_i,\tau_i)$ for a given value of τ_i as $\theta_i(\tau_i)$ and the zeros of $U_{\tau_i}(\theta_i,\tau_i)$ for a given value of θ_i as $\tau_i(\theta_i)$, so that solutions in $(0,1)\times(0,1)$

of

$$\begin{cases} \theta_i = \theta_i(\tau_i) \\ \tau_i = \tau_i(\theta_i) \end{cases} \tag{B.2}$$

are candidates to solve (B.1). Then, we shall exploit the monotonicity of $U_{\theta_i}(\theta_i, \tau_i)$ in θ_i, given τ_i, and of $U_{\tau_i}(\theta_i, \tau_i)$ in τ_i, given θ_i, to show that internal solutions are indeed maxima and that border solutions can be found by solving the same system (B.2) when restricting the functions $\theta_i(\tau_i)$ and $\tau_i(\theta_i)$ to assume values in $[0, 1]$.

Let us first compute both partial derivatives. It holds:

$$U_{\theta_i} = \int_{j\in I} \frac{\partial u(a_i(a_j), a_j; x_i^o)}{\partial x_i^o} \frac{\partial x_i^o}{\partial \theta_i}\, dj - 2\lambda(\theta_i - x_i),$$

$$U_{\tau_i} = \int_{j\in I} \frac{\partial u_i(a_i(a_j), a_j; x_i^o)}{\partial x_i^o} \frac{\partial x_{i^o}}{\partial \tau_i}\, dj - 2c\tau_i.$$

Let us define a_{L} the action of a player in group L, $d_i^{\text{L}} := (x_i^o - a_i(a_{\text{L}}))^2$ where $a_i(a_{\text{L}})$ is the best reply of agent i when facing action a_{L} and, similarly, $d_i^{\text{S}} := (x_i^o - a_i(a_{\text{S}}))^2$. Taking the integral and exploiting the separability of stage game payoffs u in material and cultural components, so that

$$\frac{\partial u(a_i(a_j), a_j; x_i^o)}{\partial \rho} = 1,$$

the two partial derivatives can be written as

$$\frac{U_{\theta_i}(\theta_i, \tau_i)}{2} = \left(\eta \frac{\partial \rho}{\partial d_i^{\text{L}}} (x_i^o - a_i(a_{\text{L}})) + (1-\eta) \frac{\partial \rho}{\partial d_i^{\text{S}}} (x_i^o - a_i(a_{\text{S}}))\right)\tau_i - \lambda(\theta_i - x_i),$$

$$\frac{U_{\tau_i}(\theta_i, \tau_i)}{2} = \left(\eta \frac{\partial \rho}{\partial d_i^{\text{L}}} (x_i^o - a_i(a_{\text{L}})) + (1-\eta) \frac{\partial \rho}{\partial d_i^{\text{S}}} (x_i^o - a_i(a_{\text{S}}))\right)(\theta_{i,t} - \bar{a}) - c\tau_i.$$

Define $\psi_i^j := -\frac{\partial \rho}{\partial d_i^j}$ for the generic $j = \text{L}, \text{S}$, which is positive given that we have assumed cultural payoff to be decreasing in the distance between actions and traits, Assumption $A3$, and depends both on endogenous θ_i and τ_i, through x_i^0 in ρ. If, for example,

$$\rho(a_i(a_j), x_i^o) = -\psi|x_i^o - a_i(a_j)| = -\psi\sqrt{d_i^j},$$

then

$$\psi_i^j = \frac{1}{2}\frac{1}{\sqrt{d_i^j}} = \frac{1}{2}\frac{1}{|x_i^o - a_i(a_j)|}.$$

If, otherwise,

$$\rho(a_i(a_j), x_i^o) = -\psi(x_i^o - a_i(a_j))^2 = -\psi d_i^j,$$

then

$$\psi_i^j = \psi.$$

Using ψ_i^j and re-arrenging, we can re-write

$$\frac{U_{\theta_i}(\theta_i, \tau_i)}{2} = \Big(\psi_i^{\text{L}}\eta a_i(a_{\text{L}}) + \psi_i^{\text{S}}(1-\eta)a_i(a_{\text{S}}) - \big(\psi_i^{\text{L}}\eta + \psi_i^{\text{S}}(1-\eta)\big)x_i^o\Big)\tau_i - \lambda(\theta_i - x_i),$$

$$\frac{U_{\tau_i}(\theta_i, \tau_i)}{2} = \Big(\psi_i^{\text{L}}\eta a_i(a_{\text{L}}) + \psi^{\text{S}}(1-\eta)a_i(a_{\text{S}}) - \big(\psi_i^{\text{L}}\eta + \psi_i^{\text{S}}(1-\eta)\big)x_i^o\Big)(\theta_i - \bar{a}) - c\tau_i,$$

and substituting for x_i^o

$$U_{\theta_i}(\theta_i, \tau_i) = \Big(\psi_i^{\text{L}}\eta a_i(a_{\text{L}}) + \psi_i^{\text{S}}(1-\eta)a_i(a_{\text{S}}) - \big(\psi_i^{\text{L}}\eta + \psi_i^{\text{S}}(1-\eta)\big)\big(\tau_i\theta_i + (1-\tau_i)\bar{a}\big)\Big)\tau_i - \lambda(\theta_i - x_i),$$

$$U_{\tau_i}(\theta_i, \tau_i) = \Big(\psi_i^{\text{L}}\eta a_i(a_{\text{L}}) + \psi_i^{\text{S}}(1-\eta)a_i(a_{\text{S}}) - \big(\psi_i^{\text{L}}\eta + \psi_i^{\text{S}}(1-\eta)\big)\big(\tau_i\theta_i + (1-\tau_i)\bar{a}\big)\Big)(\theta_i - \bar{a}) - c\tau_i.$$

Now define $\psi_i := \psi_i^{\text{L}}\eta + \psi_i^{\text{S}}(1-\eta)$ and $\tilde{a}_i := \tilde{\psi}_i^{\text{L}} a_i(a_{\text{L}}) + \tilde{\psi}_i^{\text{S}} a_i(a_{\text{S}})$ with $\tilde{\psi}_i^{\text{S}} := \frac{\psi_i^{\text{S}}(1-\eta)}{\psi_i}$ and $\tilde{\psi}_i^{\text{L}} := \frac{\psi_i^{\text{L}}\eta}{\psi_i}$. Note that $\tilde{a}_i$ is a transformation of $\bar{a}_i$ that takes into account the different sensitivity of the cultural component to action plays in response to the majority or to the minority. The higher the sensitivity the larger the weight in computing the average action. In terms of ψ_i and $\tilde{a}_i$ one has a much more compact version of the two partial derivatives

$$\frac{U_{\theta_i}(\theta_i, \tau_i)}{2} = \psi_i\Big(\tilde{a}_i - \big(\tau_i\theta_i + (1-\tau_i)\bar{a}\big)\Big)\tau_i - \lambda(\theta_i - x_i),$$

$$\frac{U_{\tau_i}(\theta_i, \tau_i)}{2} = \psi_i\Big(\tilde{a}_i - \big(\tau_i\theta_i + (1-\tau_i)\bar{a}\big)\Big)(\theta_i - \bar{a}) - c\tau_i.$$

We are interested in the sign and in the zeros of the two partial derivatives. First note that for cultural components that are quadratic with respect to the distance, $\rho(a_i, x_i) = -\psi(x_i - a_i)^2$, it holds $\psi_i = \psi_i^{\text{L}} = \psi_i^{\text{S}} = \psi$, an exogenous parameter, and thus $\tilde{a}_i = \bar{a}_i$. It follows that, for each τ_i, the zeros of $U_{\theta_i}(\theta_i, \tau_i)$ are given by

$$f_i^\theta(\tau_i) = \frac{\psi\tau_i}{\lambda + \psi\tau_i^2}\bar{a}_i + \frac{\lambda}{\lambda + \psi\tau_i^2}x_i - \frac{\psi}{\lambda + \psi\tau_i^2}\tau_i(1-\tau_i)\bar{a}.$$

Similarly, for each θ_i the zeros of $U_{\tau_i}(\theta_i, \tau_i)$ are given by[28]

$$f_i^\tau(\theta_i) = \frac{\psi(\bar{a}_i - \bar{a})(\theta_i - \bar{a})}{c + \psi(\theta_i - \bar{a})^2}.$$

From now on we continue with the quadratic cultural component $\rho(a_i, x_i) = -\psi(x_i - a_i)^2$. In this case each partial derivative is linear and decreasing in one of the two variable, $U_{\theta_i} = f_i^\theta(\tau_i) - \theta_i$ and $U_{\tau_i} = f_i^\tau(\theta_i) - \tau_i$, so that

$$U_{\theta_i}(\theta_i, \tau_i) \gtreqless 0 \text{ for } \theta_i \lesseqgtr f_i^\theta(\tau_i)$$

and

$$U_{\tau_i}(\theta_i, \tau_i) \gtreqless 0 \text{ for } \tau_i \lesseqgtr f_i^\tau(\theta_i).$$

For a given τ_i, the value $f_i^\theta(\tau_i)$ is a candidate to be an optimal choice to maximize the objective function. If, for a given τ_i, $f_i^\theta(\tau_i) \in (0, 1)$, then the sign of $U_{\theta_i}(\theta_i, \tau_i)$ around $f_i^\theta(\tau_i)$ implies that it cannot be a minimizer. If, instead, $f_i^\theta(\tau_i) \le 0$, then the sign of $U_{\theta_i}(\theta_i, \tau_i)$ implies that the optimal choice of θ_i can only be $\theta_i = 0$. Similarly, if $f_i^\theta(\tau_i) \ge 1$, then the optimal can only be $\theta_i = 1$. Summarizing for a given τ_i the maximum can only be achieved at

$$\theta_i = F_i^\theta(\tau_i) := \left[f_i^\theta(\tau_i)\right]_0^1$$

Exploiting the sign of $U_{\tau_i}(\theta_i, \tau_i)$, the same reasoning holds for the optimal choice of τ_i given θ_i.

Thus, each solution (θ_i^*, τ_i^*) of (B.1) solves[29]

$$\begin{cases} \theta_i = F_i^\theta(\tau_i) \\ \tau_i = F_i^\tau(\theta_i) \end{cases} \tag{B.3}$$

with

$$F_i^\theta(\tau_i) := \left[\frac{\psi_i \tau_i}{\lambda + \psi_i \tau_i^2}\tilde{a}_i + \frac{\lambda}{\lambda + \psi \tau_i^2}x_i - \frac{\psi_i}{\lambda + \psi_i \tau_i^2}\tau_i(1 - \tau_i)\bar{a}\right]_0^1 \tag{B.4}$$

and

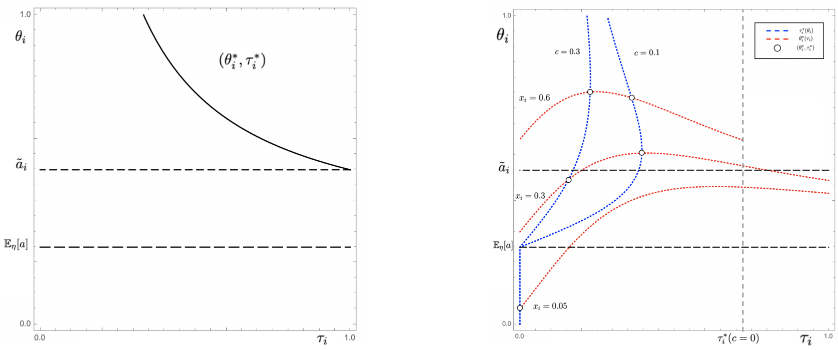
$$F_i^\tau(\theta_i) := \left[\frac{\psi_i(\tilde{a}_i - \bar{a})(\theta_i - \bar{a})}{c + \psi_i(\theta_i - \bar{a})^2}\right]_0^1. \tag{B.5}$$

[28]The same expressions would hold also for non-quadratic cultural components, using ψ_i in place of ψ and $\tilde{a}_i$ in place of $\bar{a}_i$. However, given the dependence of ψ_i and $\tilde{a}_i$ on θ_i and τ_i, they would characterize both $f_i^\theta(\tau_i)$ and $f_i^\tau(\theta_i)$ only implicitly.

[29]Note that the existence of a solution of (B.3) is also ensured by Brouwer's Theorem, due to the continuity in (θ_i, τ_i) of the partial derivatives, and thus of their restrictions to the interval $[0, 1]$.

Finally note that defining $\tilde{\lambda} = \frac{\lambda}{\psi}$ and $\tilde{c} = \frac{c}{\psi}$ and re-arranging the two functions can be rewritten as in (5) and (6), hence the first part of the statement.

Figure 12 below shows the optimal choices of socialization effort and parental role models depending on socialization costs and parental traits.



(a) $c = 0,\ \lambda = 0$ (b) $c > 0,\ \lambda > 0$

Figure 12: Solution of the optimal parental choices. (a) Locus of optimal (θ^*, τ^*) wit no parenting cost. (b) Optimal solutions θ^*, τ^* with positive parenting cost.

(ii) Without loss of generalities, let us assume that $\bar{a}_i > \bar{a}$. We first prove that $x_i^o \geq \bar{a}$. Recall equation (2):

$$x_i^o = \tau_i^* \theta_i^* + (1 - \tau_i^*)\bar{a}. \tag{B.6}$$

If $sign\Big(\bar{a}_i - \bar{a}\Big) = sign\Big(\theta_i - \bar{a}\Big)$, then, having assumed $\bar{a}_i > \bar{a}$ implies also $\theta_i > \bar{a}$. The latter and (B.6) imply the desired result.
If instead $sign\Big(\bar{a}_i - \bar{a}\Big) \neq sign\Big(\theta_i - \bar{a}\Big)$, then (B.5) implies $\tau_i^* = 0$ and thus $x_i^o = \bar{a}$. Thus, $x_i^o \geq \bar{a}$ also in this case.

Next, we prove that $x_i^o \leq \bar{a}_i$. Each parent utility depends on three components: (i) Altruism, which is maximized when the child acquires a trait x_i^o as close as possible to the average action $\bar{a}_i$; (ii) Cultural Resilience, for which the parent wants to declare a θ_i as close as possible to own trait x_i; and (iii) Direct Socialization, for which parents want to minimize the exerted effort τ_i. All the three forces together imply that any combination of θ_i and τ_i that leads to $x_i^o > \bar{a}_i$ is dominated.
Let us clarify the last statement.
Since we have assumed $\bar{a}_i > \bar{a}$, if $x_i^o > \bar{a}_i$, then from (B.6) it follows that $\theta_i^* > x_i^o > \bar{a}_i$. Now there are three possibilities: 1) $\theta_i^* = x_i$; 2) $\theta_i^* > x_i$; 3) $\theta_i^* < x_i$.

1) Having $x_i^0 > \bar{a}_i$ is dominated because parent i can improve his utility keeping

$\theta_i^* = x_i$ and decreasing the socialization cost τ_i so that the child is more exposed to oblique socialization and the trait x_i^o becomes closer to $\bar{a}_i$. This means that the Altruism (i) and Direct Socialization (iii) parts of parental utility increase, while the cultural resilience part (iii) remain maximized by $\theta_i^* = x_i$.

2) Having $x_i^0 > \bar{a}_i$ and is $\theta_i^* > x_i$ is dominated by case 1), itself dominated. Indeed, when $\theta_i^* > x_i$ and $x_i^o > \tilde{a}_i$, if parent i chooses $\theta_i^* = x_i$, then he gains more utility from both Altruism (i)—bringing the trait of the child closer to the optimal action—and from Cultural Resilience (ii)—not paying the cost of declaring a role model different from the true parental trait.

3) Having $x_i^0 > \bar{a}_i$ is dominated because, by reducing τ_i and paying a smaller cost of Direct Socialization (iii), the parent can bring the child trait closer to $\bar{a}_i$ and thus increasing utility stemming from Altruism (i).

(iii) Without loss of generalities, let us assume that $\bar{a}_i > \bar{a}$—so that $sign\Big(\bar{a}_i - \bar{a}\Big) = +1$. Let us start proving point (a). Due to (B.5), $\tau_i^* = 0$, and thus $x_i^o = \bar{a}$, can arise only if $sign\Big(\bar{a}_i - \bar{a}\Big) \neq sign\Big(\theta_i - \bar{a}\Big)$. Having assumed $sign\Big(\bar{a}_i - \bar{a}\Big) = +1$, let us derive the conditions for which $sign\Big(\theta_i - \bar{a}\Big) = -1$.

$$
\begin{aligned}
sign\Big(\theta_i - \bar{a}\Big) &= \\
sign\left(\frac{\tau_i}{\tilde{\lambda} + \tau_i^2}\bar{a}_i + \frac{\tilde{\lambda}}{\tilde{\lambda} + \tau_i^2}x_i - \frac{1}{\tilde{\lambda} + \tau_i^2}\tau_i(1-\tau_i)\bar{a} - \bar{a}\right) &= \\
sign\left(\tau_i\bar{a}_i + \tilde{\lambda}x_i - (\tau_i(1-\tau_i) + \tilde{\lambda} + \tau_i^2)\bar{a}\right) &= \\
sign\left(\tau_i\bar{a}_i + \tilde{\lambda}x_i - (\tau_i + \tilde{\lambda})\bar{a}\right) &= \\
sign\left(\frac{\tau_i}{\tau_i + \tilde{\lambda}}\bar{a}_i + \frac{\tilde{\lambda}}{\tau_i + \tilde{\lambda}}x_i - \bar{a}\right) &= \\
sign\left(\tau_i(\bar{a}_i - \bar{a}) + \tilde{\lambda}(x_i - \bar{a})\right) &= -1 \quad only\ if \quad x_i < \bar{a} - \frac{\tau_i}{\tilde{\lambda}}(\bar{a}_i - \bar{a})
\end{aligned}
$$

When $\tau_i = 0$, the latter occurs if $x_i < \bar{a}$ Therefore, when $\bar{a}_i > \bar{a}$, $sign\Big(\theta_i - \bar{a}\Big) = -1$ only if $x_i \leq \bar{a}_i$, which implies that if $x_i > \bar{a}_i$, $\tau_i > 0$ and $\theta_i \geq \bar{a}$. In such a case, equation (7) implies that $x_i^o \geq \bar{a}$. Note also that when $x_i > \bar{a}$ any $\theta_i \in [\bar{a}, x_i)$ is dominated by $\theta_i = x_i$, because moving from any $\theta_i < x_i$ to $\theta_i = x_i$ increases the parental utility for two reasons: reduces the cultural resilience and the altruism (being closed to $\bar{a}_i$).

Let us now consider point (b). We already know that, if $\bar{a}_i > \bar{a}$, $sign\Big(\theta_i - \bar{a}\Big) = -1$ only if $x_i \leq \bar{a}_i$. However, if the cultural resilience cost is small enough (or x_i is relatively close to $\bar{a}$), parents might still find optimal to choose $\theta_i > \bar{a}$ because the marginal benefit from a child trait x_i^o closer to $\bar{a}_i$ is larger then the cost of directly socialization declaring a role model θ_i different from own trait x_i exert positive socialization effort. Conversely, if the cultural resilience cost is large enough (or x_i is relatively far away to $\bar{a}$, from the left side), the parent i finds optimal to not exert any socialization effort and let the child socialized by the society (oblique socialization).

■

Proof of Proposition 3

First, we show that when the cost of parent i for setting θ_i differently from x_i is zero, $\lambda = 0$, while the cultural transmission cost is positive, $c > 0$, then for any population size η_i the optimal choice of parent i is to have an extreme role model θ_i^*, either 0 or 1 depending on the sign of $\bar{a}_i - \bar{a}$, and an interior optimal socialization level

$$\tau^* = F_i^\tau(\theta_i^*) = \frac{(\bar{a}_i - \bar{a})(\theta_i^* - \bar{a})}{c + (\theta_i^* - \bar{a})^2} \in (0, 1), \;\; \theta_i^* \in \{0, 1\}.$$

Without loss of generality, take $\bar{a}_i > \bar{a}$. Then, the candidate to be an optimal choice is $\theta_i^* = 1$ and, thus

$$F_i^\tau(1) = \frac{(\bar{a}_i - \bar{a})(1 - \bar{a})}{c + (1 - \bar{a})^2} = f_i^\tau(1) \in (0, 1). \tag{B.7}$$

At this level of socialization, the value of θ_i which equates the partial derivative U_θ to zero is

$$f_i^\theta(\tau^*)|_{\lambda=0} = \bar{a}_i + \frac{1 - \tau_i^*}{\tau_i^*}(\bar{a}_i - \bar{a})|_{\tau_i^* = F_i^\tau(1)} = 1 + \frac{\tilde{c}}{(\bar{a}_i - \bar{a})(1 - \bar{a})} > 1\,.$$

(see the proof of Proposition 2 for details on the partial derivative and on the function f_i^θ). Since

$$U_{\theta_i}(\theta_i, \tau_i) = f_i^\theta(\tau_i) - \theta_i \gtreqless 0 \;\; \text{for} \;\; \theta_i \lesseqgtr f_i^\theta(\tau_i),$$

then $U_{\theta_i}(\theta, \tau_i^*) > 0$, for all $\theta \in [0, 1]$ and the optimal role model is thus $\theta_i^* = 1$. The latter, together with $U_{\tau_i}(\theta, \tau) = 0$ at $\theta = 1$ and $\tau = f_i^\tau(1)$ concludes the prove of optimality. The case with $\bar{a}_i < \bar{a}$ is proved in the same way changing the optimal role model to $\theta_i^* = 0$.

Having the optimal socialization level τ_i^*, we can proceed with the evaluation of its change with respect to η_i. Without loss of generality, we continue to consider the case $\bar{a}_i > \bar{a}$. The latter is implied by different values of x_i and x_j, when j does not belong to the community of i. In all such cases, $\bar{a}_i > \bar{a}$ is implied by $\bar{a}_i > \bar{a}_j$.

Strong-Strong When both x_i and x_j are strong, optimal strategies do not depend on the nature of material payoff π. Having $\bar{a}_i > \bar{a}_j$ implies that $\bar{a}_i = 1$ while $\bar{a}_j = 0$, so that $\bar{a} = \eta_i$. Computing the optimal socialization level in (B.7) gives

$$\tau_i^*(\eta_i) = \frac{(1-\eta_i)^2}{\tilde{c} + (1-\eta_i)^2},$$

which is clearly decreasing in η_i.

Strong-Weak When x_i is strong while x_j is weak, the optimal choice of parents in population j depends on the nature of the material payoff. Strategic complementarity of π and $\bar{a}_i > \bar{a}_j$ imply $\bar{a}_i = 1$ while $\bar{a}_j = \frac{1+\eta_i}{2}$, so that $\bar{a} = \eta_i + \frac{1-\eta_i^2}{2}$. Computing the optimal socialization level in (B.7) gives

$$\tau_i^*(\eta_i) = \frac{(1-\eta_i)^4}{\tilde{c} + (1-\eta_i)^4},$$

which is clearly decreasing in η_i.
Strategic substitutability of π and $\bar{a}_i > \bar{a}_j$ imply $\bar{a}_i = 1$ while $\bar{a}_j = \frac{1-\eta_i}{2}$, so that $\bar{a} = \eta_i + \frac{(1-\eta_i)^2}{2}$. Computing the optimal socialization level in (B.7) gives

$$\tau_i^*(\eta_i) = \frac{(1-\eta_i^2)^2}{4\tilde{c} + (1-\eta_i^2)^2},$$

which is also clearly decreasing in η_i.

Weak-Strong When x_i is weak while x_j is strong, the optimal choice of parents in population j depends on the nature of the material payoff. Strategic complementarity of π and $\bar{a}_i > \bar{a}_j$ imply $\bar{a}_i = \frac{\eta_i}{2}$ while $\bar{a}_j = 0$, so that $\bar{a} = \frac{\eta_i^2}{2}$. Computing the optimal socialization level in (B.7) gives

$$\tau_i^*(\eta_i) = \frac{\eta_i(1-\eta_i)\left(1-\frac{\eta_i^2}{2}\right)}{\tilde{c} + \left(1-\frac{\eta_i}{2}\right)^2},$$

which is increasing (decreasing) in η_i when η_i is close to zero (one) (in fact, τ_i^* is positive for $\eta_i \in [0,1]$ and zero only in $\eta_i = 0$ and in $\eta_i = 1$).

Strategic substitutability of π and $\bar{a}_i > \bar{a}_j$ imply $\bar{a}_i = 1 - \frac{\eta_i}{2}$ while $\bar{a}_j = 0$, so that $\bar{a} = \eta_i\left(1-\frac{\eta_i}{2}\right)$. Computing the optimal socialization level in (B.7) gives

$$\tau_i^*(\eta_i) = \frac{(1-\eta_i\left(1-\frac{\eta_i}{2}\right))^2}{\tilde{c}+(1-\eta_i\left(1-\frac{\eta_i}{2}\right))^2} - \frac{\eta_i}{2}\frac{1-\eta_i\left(1-\frac{\eta_i}{2}\right)}{\tilde{c}+(1-\eta_i\left(1-\frac{\eta_i}{2}\right))^2}.$$

The first term is clearly decreasing in η_i. The second term can be rewritten as the product

$$\frac{-\eta_i}{2\left(1-\eta_i\left(1-\frac{\eta_i}{2}\right)\right)}\frac{1}{\frac{\tilde{c}}{\left(1-\eta_i\left(1-\frac{\eta_i}{2}\right)\right)^2}+1},$$

whose two terms are both decreasing in η_i.

Given $\bar{a}_i > \bar{a}_j$, no other cases are possible. The proof for $\bar{a}_i < \bar{a}_j$ proceeds along the same lines.

■

Preliminaries for the Proofs of Propositions in Section 3

Before characterizing the long-run acculturation outcomes and process we need to state the following auxiliary proposition.

Proposition 9 *Consider the cultural dynamics in equation (8). For all strategic environments,*

(*i*) *Independently of γ, if $\mathbf{x}^* := \lim_{t\to\infty}\mathbf{x}_t$ exits it satisfies*

$$\begin{cases} x_{\text{L}}^* = \phi_{\text{L}}^*\bar{a}_{\text{L}}^* + (1-\phi_{\text{L}}^*)\bar{a}_{\text{S}}^* \\ x_{\text{S}}^* = \phi_{\text{S}}^*\bar{a}_{\text{S}}^* + (1-\phi_{\text{S}}^*)\bar{a}_{\text{L}}^* \end{cases} \tag{B.8}$$

where for each $i = L, S$ $\phi_i^* = \frac{\tau_i^{2*}+\lambda(1-\tau_i^*)\eta_i}{\tau_i^{2*}+\lambda(1-\tau_i^*)} \in (0,1)$.

(*ii*) *If γ is a point distribution, then the dynamics always converges to an asymptotically stable steady state.*

Proof of Proposition 9

- Let us first consider the case in which, for each i, θ_i is interior.

  Let us define $p_i := \frac{\tau_i^2}{\lambda+\tau_i^2}$. At the steady state, the socialization process with interior θ_i satisfies

$$\begin{cases} x_{\text{L}} = p_{\text{L}}\bar{a}_{\text{L}} + (1-p_{\text{L}})\tau_{\text{L}}x_{\text{L}} + (1-p_{\text{L}})(1-\tau_{\text{L}})(\eta\bar{a}_{\text{L}} + (1-\eta)\bar{a}_{\text{S}}) \\ x_{\text{S}} = p_{\text{S}}\bar{a}_{\text{S}} + (1-p_{\text{S}})\tau_{\text{S}}x_{\text{S}} + (1-p_{\text{S}})(1-\tau_{\text{S}})(\eta\bar{a}_{\text{L}} + (1-\eta)\bar{a}_{\text{S}}) \end{cases}$$

$$\begin{cases} x_{\text{L}} = p_{\text{L}}\bar{a}_{\text{L}} + (1-p_{\text{L}})\tau_{\text{L}}x_{\text{L}} + (1-p_{\text{L}})(1-\tau_{\text{L}})\bar{a} \\ x_{\text{S}} = p_{\text{S}}\bar{a}_{\text{S}} + (1-p_{\text{S}})\tau_{\text{S}}x_{\text{S}} + (1-p_{\text{S}})(1-\tau_{\text{S}})\bar{a} \end{cases}$$

$$\begin{cases} x_{\text{L}} = \frac{p_{\text{L}}}{1-(1-p_{\text{L}})\tau_{\text{L}}}\bar{a}_{\text{L}} + \frac{(1-p_{\text{L}})(1-\tau_{\text{L}})}{1-(1-p_{\text{L}})\tau_{\text{L}}}\bar{a} \\ x_{\text{S}} = \frac{p_{\text{S}}}{1-(1-p_{\text{S}})\tau_{\text{S}}}\bar{a}_{\text{S}} + \frac{(1-p_{\text{S}})(1-\tau_{\text{S}})}{1-(1-p_{\text{S}})\tau_{\text{S}}}\bar{a} \end{cases}$$

$$\begin{cases} x_{\text{L}} = (p_{\text{L}} + (1-p_{\text{L}})(1-\tau_{\text{L}})\eta)\bar{a}_{\text{L}} + (1-p_{\text{L}})\tau_{\text{L}}x_{\text{L}} + (1-p_{\text{L}})(1-\tau_{\text{L}})(1-\eta)\bar{a}_{\text{S}} \\ x_{\text{S}} = (p_{\text{S}} + (1-p_{\text{S}})(1-\tau_{\text{S}})(1-\eta)\bar{a}_{\text{S}} + (1-p_{\text{S}})\tau_{\text{S}}x_{\text{S}} + (1-p_{\text{S}})(1-\tau_{\text{S}})\eta\bar{a}_{\text{L}} \end{cases} \tag{B.9}$$

$$\begin{cases} x_{\text{L}} = \frac{(p_{\text{L}}+(1-p_{\text{L}})(1-\tau_{\text{L}})\eta)}{1-(1-p_{\text{L}})\tau_{\text{L}}}\bar{a}_{\text{L}} + \frac{(1-p_{\text{L}})(1-\tau_{\text{L}})(1-\eta)}{1-(1-p_{\text{L}})\tau_{\text{L}}}\bar{a}_{\text{S}} \\ x_{\text{S}} = \frac{(p_{\text{S}}+(1-p_{\text{S}})(1-\tau_{\text{S}})(1-\eta)}{1-(1-p_{\text{S}})\tau_{\text{S}}}\bar{a}_{\text{S}} + \frac{(1-p_{\text{S}})(1-\tau_{\text{S}})\eta}{1-(1-p_{\text{S}})\tau_{\text{S}}}\bar{a}_{\text{L}} \end{cases}$$

$$\Rightarrow \begin{cases} x_{\text{L}} = \phi_{\text{L}}\bar{a}_{\text{L}} + (1-\phi_{\text{L}})\bar{a}_{\text{S}} \\ x_{\text{S}} = \phi_{\text{S}}\bar{a}_{\text{S}} + (1-\phi_{\text{S}})\bar{a}_{\text{L}} \end{cases} \tag{B.10}$$

with

$$\phi_i = \frac{p_i + (1-p_i)(1-\tau_i)\eta_i}{1-(1-p_i)\tau_i} = \frac{\tau_i^2 + \lambda(1-\tau)\eta}{\tau_i^2 + \lambda(1-\tau)} \in (0,1).$$

Similarly, for the steady state role model holds

$$\begin{cases} \theta_{\text{L}} = \frac{\tau_{\text{L}}}{\tilde{\lambda}+\tau_{\text{L}}^2}\bar{a}_{\text{L}} + \frac{\tilde{\lambda}}{\tilde{\lambda}+\tau_{\text{L}}^2}\left(\tau_{\text{L}}\theta_{\text{L}} + (1-\tau_{\text{L}})\mathbb{E}_\eta[a]\right) + \frac{\tau_{\text{L}}(1-\tau_{\text{L}})}{\tilde{\lambda}+\tau_{\text{L}}^2}(\bar{a}_{\text{L}} - \mathbb{E}_\eta[a]) \\ \theta_{\text{S}} = \frac{\tau_{\text{S}}}{\tilde{\lambda}+\tau_{\text{L}}^2}\bar{a}_{\text{S}} + \frac{\lambda}{\tilde{\lambda}+\tau_{\text{S}}^2}\left(\tau_{\text{S}}\theta_{\text{L}} + (1-\tau_{\text{S}})\mathbb{E}_\eta[a]\right) + \frac{\tau_{\text{S}}(1-\tau_{\text{S}})}{\tilde{\lambda}+\tau_{\text{S}}^2}(\bar{a}_{\text{S}} - \mathbb{E}_\eta[a]) \end{cases}$$

$$\Rightarrow \begin{cases} \theta_{\text{L}} = \rho'_{\text{L}}\bar{a}_{\text{L}} + (1-\rho'_{\text{L}})\mathbb{E}_\eta[a] \\ \theta_{\text{S}} = \rho'_{\text{S}}\bar{a}_{\text{S}} + (1-\rho'_{\text{S}})\mathbb{E}_\eta[a] \end{cases} \Rightarrow \begin{cases} \theta_{\text{L}} = \bar{a} + \rho'_{\text{L}}(\bar{a}_{\text{L}} - \bar{a}) \\ \theta_{\text{S}} = \bar{a} + \rho'_{\text{S}}(\bar{a}_{\text{S}} - \bar{a}) \end{cases} \tag{B.11}$$

with

$$\rho'_i = \frac{\tau_i(2-\tau_i)}{\tau_i^2 + \lambda(1-\tau_i)}.$$

Alternatively, in terms of $\bar{a}_{\text{L}}$ and $\bar{a}_{\text{S}}$

$$\Rightarrow \begin{cases} \theta_{\text{L}} = \phi'_{\text{L}}\bar{a}_{\text{L}} + (1-\phi'_{\text{L}})\bar{a}_{\text{S}} \\ \theta_{\text{S}} = \phi'_{\text{S}}\bar{a}_{\text{S}} + (1-\phi'_{\text{S}})\bar{a}_{\text{L}} \end{cases} \tag{B.12}$$

with

$$\phi_i' = \frac{\tau_i(2-\eta_i(1+\lambda)) - \tau_i^2(1-\eta_i) + \lambda\eta_i}{\tau_i^2 + \lambda(1-\tau_i)}$$

We can thus easily verify that if $\bar{a}_{\text{L}} = \bar{a}_{\text{S}}$ then, $x_{\text{L}} = x_{\text{S}} = \theta_{\text{L}} = \theta_{\text{S}}$. Instead, if w.l.g. $\bar{a}_{\text{L}} > \bar{a}_{\text{S}}$, due to the fact that for $\tau_i \in (0,1)$, $\tau^2 < \tau$ and thus $\phi_i' > \phi_i$ as well as $\rho_i' > \phi_i'$, we have that $\bar{a}_{\text{S}} \leq x_{\text{S}} \leq \mathbb{E}_\eta[a] \leq x_{\text{L}} \leq \bar{a}_{\text{L}}$. and $\theta_{\text{S}} \leq x_{\text{S}} \leq \mathbb{E}_\eta[a] \leq x_{\text{L}} \leq \theta_{\text{L}}$.

- More in general, it could happen that at a steady state θ is not interior. The latter occurs when the θ found as combination of action, average actions, and traits is outside the interval $[0,1]$. Repeating the decomposition above, the latter occurs only when $\lambda < \tau$, otherwise θ found as combination of action, average actions, and traits is inside the interval $[min\{\bar{a}_{\text{S}}, \bar{a}_{\text{L}}\}, max\{\bar{a}_{\text{S}}, \bar{a}_{\text{L}}\}]$ and thus inside $[0,1]$.

  The condition $\lambda < \tau$ is necessary for a border steady state, but not sufficient when actions are interior. Consider for example $\bar{a}_{\text{S}} = \frac{1}{2}(1+\eta)$ and $\bar{a}_{\text{L}} = 1$. Having $\theta_{\text{S}} = 0$ requires not only $\lambda < \tau$ but also that λ is particularly small. Instead having $\theta_{\text{S}} = 1$ requires only $\lambda \leq \tau$. (which is, however, an implicit condition on the parameters)

  However, we can say that if $\lambda > 1 \leq \tau$, then the steady state values of the role model are interior.

$$x_i^o = (1-\tau_i)\mathbb{E}_\eta[a]$$

Convergence and stability with γ point distribution To prove the convergence and stability when γ is a point distribution, let us consider that the dynamics (8), inside each region identified by thresholds $(\hat{x}_1, \hat{x}_0)$.

We can also define $\boldsymbol{P} := \begin{bmatrix} (p_{\text{L}} + (1-p_{\text{L}})(1-\tau_{\text{L}})\eta) & (1-p_{\text{L}})(1-\tau_{\text{L}})(1-\eta) \\ (1-p_{\text{S}})(1-\tau_{\text{S}})\eta & (p_{\text{S}} + (1-p_{\text{S}})(1-\tau_{\text{S}})(1-\eta) \end{bmatrix}$ and $\boldsymbol{D} := \begin{bmatrix} (1-p_{\text{L}})\tau_{\text{L}} & 0 \\ 0 & (1-p_{\text{S}})\tau_{\text{S}} \end{bmatrix}$.

Thus we can write equation (8) as

$$\mathbf{x}^o = \boldsymbol{P}\bar{\boldsymbol{a}} + \boldsymbol{D}\mathbf{x}$$

Note that if $\mathbf{x}^o \neq \mathbf{x}$ then matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ vary across generation through the effect of parental socialization. Iterating the process we get

$$\begin{aligned}\mathbf{x}_{t+1} =& \boldsymbol{P}_t\bar{\boldsymbol{a}} + \boldsymbol{D}_t\mathbf{x}_t \\ =& \boldsymbol{P}_t\bar{\boldsymbol{a}} + \boldsymbol{D}_t\boldsymbol{P}_{t-1}\bar{\boldsymbol{a}} + \boldsymbol{D}_t\boldsymbol{D}_{t-1}\mathbf{x}_{t-1} \\ =& \boldsymbol{P}_t\bar{\boldsymbol{a}} + \boldsymbol{D}_t\boldsymbol{P}_{t-1}\bar{\boldsymbol{a}} + \boldsymbol{D}_t\boldsymbol{D}_{t-1}\boldsymbol{P}_{t-2}\bar{\boldsymbol{a}} + \boldsymbol{D}_t\boldsymbol{D}_{t-1}\boldsymbol{D}_{t-2}\mathbf{x}_{t-2} \\ =& \dots \\ =& \sum_{\mathrm{T}=0}^{t-1}\prod_{\tau=0}^{t-1-\mathrm{T}}\boldsymbol{D}_{t-\tau}\boldsymbol{P}_{t-\mathrm{T}-1}\bar{\boldsymbol{a}} + \prod_{\tau=0}^{t}\boldsymbol{D}_{t-\tau}\mathbf{x}_0\end{aligned}$$

Because all matrices $\boldsymbol{D}_t$ are always diagonal substochastic the process converges. Therefore

$$\mathbf{x} := \lim_{t\to\infty}\mathbf{x}_t = \sum_{\tau=0}^{\infty}\boldsymbol{D}^{\tau}\boldsymbol{P}\bar{\boldsymbol{a}} = (\boldsymbol{I}-\boldsymbol{D})^{-1}\boldsymbol{P}\bar{\boldsymbol{a}} = \boldsymbol{\Phi}\bar{\boldsymbol{a}}$$

which is equivalent to (B.8).

We now need to show that the dynamics always end in a region where the same average actions are played, so that the previous argument holds in all the space $[0,1]^2$, because even if we start from a region we move to another, from a certain t on $\mathbf{x}_t$ remain inside the same region where $\bar{\boldsymbol{a}}$ is fixed and we can use the previous argument to prove stability and convergence.

The space $[0,1]^2$ can be partitioned in 9 regions: $(\hat{x}_1,1]^2$, $[0,\hat{x}_0)^2$, $[\hat{x}_0,\hat{x}_1]^2$, $[0,\hat{x}_0)\times(\hat{x}_1,1]$, $(\hat{x}_1,1]\times[0,\hat{x}_0)$, $[0,\hat{x}_0)\times[\hat{x}_0,\hat{x}_1]$, $(\hat{x}_1,1]\times[\hat{x}_0,\hat{x}_1]$, $[\hat{x}_0,\hat{x}_1]\times[0,\hat{x}_0)$, $[\hat{x}_0,\hat{x}_1]\times(\hat{x}_1,1]$.

Recall that γ is a point distribution and that $\hat{x}_0,\hat{x}_1$ remain fixed over time. Inside each of these regions the dynamics of $\mathbf{x}_t$ push toward the convex combination of average actions played in each region as described by (B.8), if it belongs to the same region then $\bar{\boldsymbol{a}}$ is fixed and the process converges, as previously proven. Otherwise the dynamics jumps in another region and the same argument applies. Therefore, to show that the dynamics always converges to a stable steady state, Independently of the initial conditions $\mathbf{x}_0$ and thresholds $\hat{x}_0,\hat{x}_1$, we need to prove two things: (i) There is at least one region such that if you jump inside you never go out; and, (ii) once you jump outside from one of the above regions you never come back.

(i) Let us consider the regions $[\hat{x}_1,1]^2$, $[0,\hat{x}_0]^2$, $[\hat{x}_0,\hat{x}_1]^2$. In these regions traits of two groups are either both strong or week, thus, environment and social effect go in the same direction and traits move towards the average played actions $(1,1)$, $(0,0)$, $\left(\frac{1}{2}.\frac{1}{2}\right)$, respectively, thus the process cannot go out from any of these regions.

(ii) To see that if you exit from one region you never come back is enough to see the ordering of average actions in Figure 2 and recall that the traits are always a convex combinations of role models which are convex combinations of average actions, as shown in equations (7).

■

Proof of Proposition 4

Let us define $\Delta_{a,i} := |\bar{a}_i - \bar{a}|$ and assume without loss of generalities it being greater than 0. Notice that, given the definitions 1-3, for $i = L, S$, the following ordering holds $0 = \Delta_{a,i}^{ass.} = \Delta_{a,i}^{no\ id.} < \Delta_{a,i}^{int.} < \Delta_{a,i}^{mar.} < \Delta_{a,i}^{sep.}$.

We have seen in equation (B.11) that at steady state

$$\theta_i^*(\tau_i) = \rho_i' \bar{a}_i + (1 - \rho_i')\bar{a} \tag{B.13}$$

with $\rho_i' = \frac{\tau_i}{\tau_i^2 + \tilde{\lambda}(1-\tau_i)}$. Thus,

$$\Delta_{\theta,i}^*(\tau_i) \equiv \theta_i^*(\tau_i) - \bar{a} = \rho_i' \Delta_{a,i} \tag{B.14}$$

Moreover,

$$\tau_i^*(\theta_i) = \frac{\Delta_{a,i}(\theta_i - \bar{a})}{\tilde{c} + (\theta_i - \bar{a})^2}. \tag{B.15}$$

Let us start noticing that, in steady states with assimilated groups or and no identities agents plays the same actions and, for all i, $\Delta_\theta^{ass.} = \tau_i^{ass.} = \Delta_{a,i}^{ass.} = \Delta_\theta^{no.id.} = \tau_i^{no.id.} = \Delta_{a,i}^{no.id.} = 0$.

Let us now consider the ordering of Δ_θ in the steady states with other acculturation outcomes. From equation (B.14), we can easily see that a sufficient condition for $\Delta_{\theta,i}$ for being ordered as $\Delta_{a,i}$ is that for $i = L, S$, $\frac{\partial \rho_i'}{\partial \Delta_{a,i}} \geq 0$. Notice also that $\frac{\partial \rho_i'}{\partial \Delta_{a,i}} = \frac{\partial \rho_i'}{\partial \tau_i} \frac{d\tau_i}{d\Delta_{a,i}}$, with $\frac{\partial \rho_i'}{\partial \tau_i} = -\frac{\tau_i^2 - \tilde{\tilde{\lambda}}}{\tau_i^2 + \tilde{\lambda}(1-\tau_i)} > 0$ if and only if $\tilde{\lambda} > \tau_i^2$. Thus, if $\tilde{\lambda}$ is large enough and $\frac{d\tau_i}{d\Delta_{a,i}} > 0$ then the ordering of $\Delta_{\theta,i}$ in different acculturation outcomes directly follows the ordering of $\Delta_{a,i}$.

Let us now verify that $\frac{d\tau_i}{d\Delta_{a,i}} > 0$ when $\tilde{\lambda}$ and c are large enough. Using the Implicit Function Theorem we can write

$$\frac{d\tau_i}{d\Delta_{a,i}} = \frac{\partial \tau_i}{\partial \Delta_{a,i}} + \frac{\partial \tau_i}{\partial \theta_i} \frac{d\theta_i}{d\Delta_{a,i}}$$

$$\Rightarrow \frac{d\tau_i}{d\Delta_{a,i}} = \left(1 - \frac{\partial \tau_i}{\partial \theta_i}\frac{\partial \theta_i}{\partial \tau_i}\right)^{-1}\left(\underbrace{\frac{\partial \tau_i}{\partial \Delta_{a,i}}}_{+} + \frac{\partial \tau_i}{\partial \theta_i}\frac{\partial \theta_i}{\partial \Delta_{a,i}}\right)$$

where

$$\frac{\partial \tau_i}{\partial \theta_i} = \frac{\Delta_{a,i}(c - (\theta_i - \bar{a})^2)}{(c + (\theta_i - \bar{a})^2)^2} \geq 0 \quad iff \quad c > (\theta_i - \bar{a})^2$$

and

$$\frac{\partial \theta_i}{\partial \tau_i} = \frac{\Delta_{a,i}(\tilde{\lambda} - \tau_i^2)}{(\tilde{\lambda} + \tau_i^2 - \tilde{\lambda}\tau_i)^2} \geq 0 \quad iff \quad \tilde{\lambda} > \tau_i^2$$

This means that given any $\tilde{\lambda}$, if $\tilde{c}$ is large enough the indirect effect never dominates the direct effect, so that τ_i positively deepens on the distance in actions $\Delta_{a,i}$. Then, the ordering of τ_i in the different acculturation outcomes directly follows the ordering of $\Delta_{a,i}$.

When $\tilde{\lambda} = 0$, the optimal role model θ_i is at the border, thus equation (B.15) shows the ordering always holds τ_i. Similarly, $\tilde{c} = 0$, the optimal socialization effort τ_i is at the border, thus equation (B.14) shows the ordering always holds θ_i.

■

Proof of Proposition 5

When $c = \lambda = 0$ then $\tau_{i,t} = 0$ and, thus, at steady states traits are equal to the average action played — i.e., from (B.8), $\mathbf{x}^* = \bar{\boldsymbol{a}}^*$. Given the average actions played in the two different strategic environments in Figure 2, it is trivial to see that steady states where the two groups are assimilated to extreme — i.e., $\mathbf{x}^* = \bar{\boldsymbol{a}}^* = (0,0)$ or $\mathbf{x}^* = \bar{\boldsymbol{a}}^* = (1,1)$ —, neutral — i.e., $\mathbf{x}^* = \bar{\boldsymbol{a}}^* = \left(\frac{1}{2}, \frac{1}{2}\right)$ —, or separated to extreme — i.e., $\mathbf{x}^* = \bar{\boldsymbol{a}}^* = (1,0)$ or $\mathbf{x}^* = \bar{\boldsymbol{a}}^* = (1,0)$ — always exist for any possible $\hat{x}_1, \hat{x}_1$.

For integrated and marginalized outcomes, we can easily verify that they can exists only in environments with strategic complements and substitutes respectively. Indeed, given the average actions, as reported in Figure 2a, in environments with complements the other possible steady states are $\left(1, \frac{1}{2}(1+\eta)\right)$ and $\left(1 - \frac{1}{2}\eta, 1\right)$, that belong to the space $\left[\frac{1}{2}, 1\right]^2$ and $\left(\frac{1}{2}\eta, 0\right)$ and $\left(0, \frac{1}{2}(1-\eta)\right)$, that belong to the space $\left[0, \frac{1}{2}\right]^2$, so that the traits are always of the same type and thus the two groups cannot be marginalized. The existence of these steady states depends on $\hat{x}_1, \hat{x}_0$, that describe the regions in which those the average actions are played. The conditions on $\hat{x}_0$ and $\hat{x}_1$ stem from the fact that $\frac{1}{2}\eta \geq \frac{1}{2}(1-\eta)$. The same argument applies for marginalized outcomes that can exists only in environments with strategic substitutes.

■

Proof of Proposition 6

To prove point (i) recall now that

$$\begin{cases} x_{\text{L},t} = \phi_{\text{L}}\bar{a}_{\text{L}} + (1-\phi_{\text{L}})\bar{a}_{\text{S}} \\ x_{\text{S},t} = \phi_{\text{S}}\bar{a}_{\text{S}} + (1-\phi_{\text{S}})\bar{a}_{\text{L}} \end{cases}$$

with $\phi_{i,t} = \frac{\tau_{i,t}^2+\lambda(1-\tau_{i,t})\eta}{\tau_{i,t}^2+\lambda(1-\tau_{i,t})}$. Note also that $\phi_{i,t} < 1$ if $c > 0$ and $\lambda > 0$ ($\eta \in [\frac{1}{2}, 1)$). Therefore, it trivially follows that to have extreme traits—i.e., $x_{\text{L},t} \in \{0,1\}$ and $x_{\text{S},t} \in \{0,1\}$—$\bar{a}_{\text{L}} = \bar{a}_{\text{S}} \in \{0,1\}$.

To prove point (ii) it is sufficient to show that in both environments with strategic complements and substitutes there exist thresholds on c such that $sign(x_{\text{L},t}-\frac{1}{2}) = sign(x_{\text{S},t}-\frac{1}{2})$ for all $t \neq 0$. Recall that for each $i \in I$, $x_{i,t} = \rho_{i,t}\bar{a}_{i,t} + (1-\rho_{i,t})\bar{a}_t$ with $\rho_{i,t} = \frac{\tau_i^2}{\tau_{i,t}^2+\lambda(1-\tau_{i,t})}$ and

$$\frac{\partial \rho_{i,t}}{\partial c} = \underbrace{\frac{\partial \rho_{i,t}}{\partial \tau_{i,t}}}_{>0}\underbrace{\frac{\partial \tau_{i,t}}{\partial c}}_{<0} < 0$$

Therefore, assuming without loss of generalities that $\bar{a}_t > \frac{1}{2}$ we can easily see that if $\bar{a}_{i,t} > \frac{1}{2}$ then also $x_{i,t+1} > \frac{1}{2}$, whereas if $\bar{a}_{i,t} < \frac{1}{2}$ then, by continuity, there exist a $\bar{c}$ such that $x_{i,t+1} > \frac{1}{2}$ if and only if $c > \bar{c}$. Therefore for high enough c the cultural dynamics always push away from the regions in which $sign(x_{\text{L},t} - \frac{1}{2}) \neq sign(x_{\text{L},t} - \frac{1}{2})$.

■

Proof of Proposition 7

Let us state and prove the following technical proposition, from which Proposition 7 trivially follows.

Proposition 10 *Consider an environment with strategic complements and random payoffs — i.e., $\hat{x}_0, \hat{x}_1$ uniformly distributed in $[0, \frac{1}{2}] \times [\frac{1}{2}, 1]$. For any choice of majority size η and transmission costs c and λ:*

(i) There exists three steady states: $(0,0)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, and $(1,1)$.

(ii) Only $(0,0)$ and $(1,1)$ are asymptotically stable.

(iii) The basin of attraction of $(0,0)$ includes $[0, \frac{1}{2}]^2 \setminus \left\{\left(\frac{1}{2}, \frac{1}{2}\right)\right\}$ and the basin of attraction of $(1,1)$ includes $[\frac{1}{2}, 1]^2 \setminus \left\{\left(\frac{1}{2}, \frac{1}{2}\right)\right\}$

Proof. To verify that $(1,1)$, $(0,0)$, and $\left(\frac{1}{2},\frac{1}{2}\right)$ are steady states it is enough to observe that for all choices of the thresholds $\hat{x}_1$ and $\hat{x}_0$ they imply, respectively $\bar{a}_{\text{L}}=\bar{a}_{\text{S}}=\bar{a}_{\text{L}}=1$ or $\bar{a}_{\text{L}}=\bar{a}_{\text{S}}=\bar{a}_{\text{L}}=0$ or $\bar{a}_{\text{L}}=\bar{a}_{\text{S}}=\bar{a}_{\text{L}}=\frac{1}{2}$. As a result, parents i can always rely on the oblique socialization and choose $\tau_i=0$ to achieve the desired trait $x_i^o=\bar{a}_i=x_i$.

In order to study the dynamics, we exploit (7) which gives the date $t+1$ trait as a convex combination of average traits $\bar{a}_{i,t}=\mathbb{E}_{\eta,\gamma}[a_{i,t}]$. Let's first focus on the region $\{(x_{\text{L},t},x_{\text{S},t}) : x_{\text{L},t}\in\left(\frac{1}{2},1\right), x_{\text{S},t}\in\left(\frac{1}{2},1\right)\}$. Discarding measure zero events, the average action $\bar{a}_{\text{L},t}$ is

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]=\gamma(\hat{x}_1<x_{\text{L},t})+\gamma(\hat{x}_1>x_{\text{L},t}\ \wedge\ \hat{x}_1<x_{\text{S},t})\left(1-\frac{1}{2}\eta\right)+\gamma(\hat{x}_1>x_{\text{L},t}\ \wedge\ \hat{x}_1>x_{\text{S},t})\frac{1}{2}$$

Since the game exhibits strategic complementarity and payoffs are randomly distributed such that $\hat{x}_1$ is uniform in $\left(\frac{1}{2},1\right)$ it holds

$$\begin{aligned}
\gamma(\hat{x}_1<x_{\text{L}})&=2\left(x_{\text{L}}-\frac{1}{2}\right),\\
\gamma(\hat{x}_1>x_{\text{L}}\wedge\hat{x}_1<x_{\text{S}})&=4\left(1-x_{\text{L}}\right)\left(x_{\text{S}}-\frac{1}{2}\right),\\
\gamma(\hat{x}_1>x_{\text{L}}\wedge\hat{x}_1>x_{\text{S}})&=4\left(1-x_{\text{L}}\right)\left(1-x_{\text{S}}\right).
\end{aligned}$$

As a result

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]&=2\left(x_{\text{L},t}-\frac{1}{2}\right)+4\left(1-x_{\text{L},t}\right)\left(x_{\text{S},t}-\frac{1}{2}\right)\left(1-\frac{1}{2}\eta\right)+4\left(1-x_{\text{L},t}\right)\left(1-x_{\text{S},t}\right)\frac{1}{2}\\
&=-1+2x_{\text{L},t}+2x_{\text{S},t}-2x_{\text{L},t}x_{\text{S},t}+(1-x_{\text{L},t}-2x_{\text{S},t}+2x_{\text{L},t}x_{\text{S},t})\eta.\\
&\quad x_{\text{L},t}+(2x_{\text{S},t}-1)(1-x_{\text{L},t})(1-\eta)\,.
\end{aligned}$$

A similar computation gives $\bar{a}_{\text{S},t}=\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]$.

$$\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]=x_{\text{S},t}+(2x_{\text{L},t}-1)(1-x_{\text{S},t})\eta\,.$$

Next, we shall show that if $x_{\text{L},t}\in(\frac{1}{2},1)$ and $x_{\text{S},t}\in(\frac{1}{2},1)$, then $\min\{x_{\text{L},t},x_{\text{S},t}\}<\min\{x_{\text{L},t+1},x_{\text{S},t+1}\}\leq 1$ for all $t\in\mathbb{N}$.

We first prove that $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]>x_{\text{L},t}$ and $\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]>x_{\text{S},t}$.

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] > x_{\text{L},t}$:

$$x_{\text{L},t} + (2x_{\text{S},t} - 1)(1 - x_{\text{L},t})(1 - \eta)\,. > x_{\text{L},t}$$
$$\underbrace{(2x_{\text{S},t} - 1)}_{>0}\underbrace{(1 - x_{\text{L},t})}_{>0}\underbrace{(1 - \eta)}_{>0} > 0, \qquad \textit{always satisfied.}$$

By symmetry (the two groups have different sizes but it is enough to replace η with $(1-\eta)$ for the minority), we can repeat the reasoning for $x_{\text{S},t}$ and show that $\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] > x_{\text{S},t}$.

By eq. (7) of Proposition 2, both $x_{\text{L},t+1}$ and $x_{\text{S},t+1}$ are a convex combination of $\bar{a}_{\text{L},t}$ and $\bar{a}_{\text{S},t}$. Assume w.l.o.g. $x_{\text{L},t} \geq x_{\text{S},t}$, then both $\bar{a}_{\text{L},t} > x_{\text{L},t} \geq x_{\text{S},t}$ and $\bar{a}_{\text{S},t} > x_{\text{S},t}$, so that by eq. (7) also $x_{\text{L},t+1} > x_{\text{S},t}$ and $x_{\text{S},t+1} > x_{\text{S},t}$. More in general, allowing also for $x_{\text{S},t} \geq x_{\text{L},t}$, we have $\min\{x_{\text{L},t+1}, x_{\text{S},t+1}\} > \min\{x_{\text{L},t}, x_{\text{S},t}\}$ for all $t \in \mathbb{N}$.

The same strict inequality holds also at the borders of the box $\left[\frac{1}{2}, 1\right]^2$, unless we are in one of the two steady states $(1,1)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$. Assume for example $x_{\text{L},t} = 1$ and $x_{\text{S},t} \in \left[\frac{1}{2}, 1)\right)$. Then,

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = 1 \geq \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] > x_{\text{S},t} = \min\{x_{\text{L},t}, x_{\text{S},t}\}.$$

Thus, by (7) and using the same argument $\min\{x_{\text{L},t+1}, x_{\text{S},t+1}\} > \min\{x_{\text{L},t}, x_{\text{S},t}\}$. Similarly when $x_{\text{L},t} = \frac{1}{2}$ and $x_{\text{S},t} \in \left(\frac{1}{2}, 1]\right)$, both

$$\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] = x_{\text{S},t} > \frac{1}{2} = \min\{x_{\text{L},t}, x_{\text{S},t}\}.$$

and

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] > x_{\text{L},t} = \frac{1}{2} = \min\{x_{\text{L},t}, x_{\text{S},t}\}.$$

So that, again, $\min\{x_{\text{L},t+1}, x_{\text{S},t+1}\} > \min\{x_{\text{L},t}, x_{\text{S},t}\}$. The same argument applies to the other two "borders".

We have shown that in the set $\left[\frac{1}{2}, 1\right]^2 \setminus \left\{\left(\frac{1}{2}, \frac{1}{2}\right), (1,1)\right\}$

$$\min\{x_{\text{L},t+1}, x_{\text{S},t+1}\} > \min\{x_{\text{L},t}, x_{\text{S},t}\}$$

The latter together with $\min\{x_{\text{L},t+1}, x_{\text{S},t+1}\} \leq 1$, which holds by construction, proves that $(1,1)$ is asymptotically stable with a basin of attraction that with includes $\left[\frac{1}{2}, 1\right]^2 \setminus \left\{\left(\frac{1}{2}, \frac{1}{2}\right)\right\}$.

The proof of the asymptotic stability of $(0,0)$, as well as the fact that there are no steady states in $\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in \left[0, \frac{1}{2}\right), x_{\text{S},t} \in \left[0, \frac{1}{2}\right)\} \setminus \{(0,0)\}$, proceeds along the same lines.

Note that we have also shown that $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not asymptotically stable: for all its neighbourhood there exists some initial traits which do not converge to it (but instead converge to $(1,1)$ or $(0,0)$).

We now turn to the region of initial conditions $\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in \left(\frac{1}{2}, 1\right], x_{\text{S},t} \in \left[0, \frac{1}{2}\right)\}$ to show that it does not contain steady states, so that the dynamics either converges to $\left(\frac{1}{2}, \frac{1}{2}\right)$ or moves in another region.

Assume $x_{\text{L},t} \in (\frac{1}{2}, 1]$ and $x_{\text{S},t} \in [0, \frac{1}{2})$, then, discarding measure zero events,

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = \gamma(\hat{x}_1 < x_{\text{L},t}) + \gamma(\hat{x}_1 > x_{\text{L},t} \ \wedge \ \hat{x}_0 > x_{\text{S},t})\frac{1}{2}\eta + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2}$$

with

$$\begin{aligned}
&\gamma(\hat{x}_1 < x_{\text{L}}) = 2\left(x_{\text{L}} - \frac{1}{2}\right) \\
&\gamma(\hat{x}_1 > x_{\text{L}} \ \wedge \ \hat{x}_0 > x_{\text{S}}) = 2(1 - x_{\text{L}})(1 - 2x_{\text{S}}) \\
&\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 < x_{\text{S}}) = 2(1 - x_{\text{L}})2x_{\text{S}}.
\end{aligned}$$

As a result

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] &= 2\left(x_{\text{L},t} - \frac{1}{2}\right) + 2(1 - x_{\text{L},t})(1 - 2x_{\text{S},t})\frac{1}{2}\eta + 2(1 - x_{\text{L},t})2x_{\text{S},t}\frac{1}{2} \\
&= 2x_{\text{L},t} - 1 + \eta(1 - x_{\text{L},t})(1 - 2x_{\text{S},t}) + 2(1 - x_{\text{L},t})x_{\text{S},t}.
\end{aligned}$$

Similarly, we can compute

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] &= \gamma(\hat{x}_1 < x_{\text{L},t} \ \wedge \ \hat{x}_0 < x_{\text{S},t})\frac{1}{2}(1 + \eta) + (\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2}\gamma \\
&= 2(1 + \eta)\left(x_{\text{L},t} - \frac{1}{2}\right)x_{\text{S},t} + 2(1 - x_{\text{L},t})x_{\text{S},t}.
\end{aligned}$$

Below, we shall show that in the set of traits $\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L}} \in \left(\frac{1}{2}, 1\right], x_{\text{S}} \in \left[0, \frac{1}{2}\right)\}$ it holds $x_{\text{S},t} < \mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] < x_{\text{L},t}$ and $x_{\text{S},t} < \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] < x_{\text{L},t}$. Using equation (7), the former inequality implies $x_{\text{S},t} < x_{\text{L},t+1} < x_{\text{L},t}$ while the latter implies $x_{\text{S},t} < x_{\text{S},t+1} < x_{\text{L},t}$. Finally, having shown that $x_{\text{L},t+1} < x_{\text{L},t}$ and $x_{\text{S},t+1} > x_{\text{S},t}$ proves the result. (in what follows we remove the time index to simplify the notation)

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\mathrm{L}}] < x_{\mathrm{L}}$:

$$\begin{aligned} 2x_{\mathrm{L}} - 1 + \eta(1 - x_{\mathrm{L}})(1 - 2x_{\mathrm{S}}) + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} &< x_{\mathrm{L}} \\ x_{\mathrm{L}} - 1 + \eta(1 - x_{\mathrm{L}})(1 - 2x_{\mathrm{S}}) + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} &< 0 \\ (1 - x_{\mathrm{L}})(-1 + \eta(1 - 2x_{\mathrm{S}}) + 2x_{\mathrm{S}}) &< 0 \end{aligned}$$

$$\underbrace{(1 - x_{\mathrm{L}})}_{>0}\underbrace{(1 - \eta)}_{>0}\underbrace{(2x_{\mathrm{S}} - 1)}_{<0} < 0, \qquad \textit{always satisfied.}$$

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\mathrm{S}}] > x_{\mathrm{S}}$:

$$\begin{aligned} 2(1 + \eta)\left(x_{\mathrm{L}} - \frac{1}{2}\right)x_{\mathrm{S}} + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} &> x_{\mathrm{S}} \\ \left(2(1 + \eta)\left(x_{\mathrm{L}} - \frac{1}{2}\right) + 2(1 - x_{\mathrm{L}})\right)x_{\mathrm{S}} &> x_{\mathrm{S}} \\ (+2\eta x_{\mathrm{L}} - \eta + 1)\,x_{\mathrm{S}} &> x_{\mathrm{S}} \end{aligned}$$

$$\underbrace{(\eta\underbrace{(2x_{\mathrm{L}} - 1)}_{>0} + 1)}_{>1}\,x_{\mathrm{S}} > x_{\mathrm{S}}, \qquad \textit{always satisfied.}$$

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\mathrm{L}}] > x_{\mathrm{S}}$

$$\begin{aligned} 2x_{\mathrm{L}} - 1 + \eta(1 - x_{\mathrm{L}})(1 - 2x_{\mathrm{S}}) + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} &> x_{\mathrm{S}} \\ 2x_{\mathrm{L}} - 1 + \eta(1 - x_{\mathrm{L}})(1 - 2x_{\mathrm{S}}) + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} - x_{\mathrm{S}} &> 0 \\ 2x_{\mathrm{L}} - 1 + (1 - x_{\mathrm{L}})(\eta(1 - 2x_{\mathrm{S}}) + 2x_{\mathrm{S}}) - x_{\mathrm{S}} &> 0 \end{aligned}$$

$$2x_{\mathrm{L}} - 1 - x_{\mathrm{S}} + (1 - x_{\mathrm{L}})(\eta + 2x_{\mathrm{S}}(1 - \eta)) > 0, \quad \textit{always satisfied.}$$

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\mathrm{S}}] < x_{\mathrm{L}}$:

$$\begin{aligned} 2(1 + \eta)\left(x_{\mathrm{L}} - \frac{1}{2}\right)x_{\mathrm{S}} + 2(1 - x_{\mathrm{L}})x_{\mathrm{S}} &< x_{\mathrm{L}} \\ 2x_{\mathrm{L}}x_{\mathrm{S}} - x_{\mathrm{S}} + 2\eta x_{\mathrm{L}}x_{\mathrm{S}} - \eta x_{\mathrm{S}} + 2x_{\mathrm{S}} - 2x_{\mathrm{L}}x_{\mathrm{S}} &< x_{\mathrm{L}} \\ x_{\mathrm{S}} + 2\eta x_{\mathrm{L}}x_{\mathrm{S}} - \eta x_{\mathrm{S}} &< x_{\mathrm{L}} \\ x_{\mathrm{S}}(1 - \eta + 2\eta x_{\mathrm{L}}) &< x_{\mathrm{L}} \\ x_{\mathrm{S}} - x_{\mathrm{S}}\eta + 2\eta x_{\mathrm{L}}x_{\mathrm{S}} - x_{\mathrm{L}} &< 0 \\ (x_{\mathrm{S}} - x_{\mathrm{L}}) + x_{\mathrm{S}}\eta(2\eta x_{\mathrm{L}} - 1) &< 0. \end{aligned}$$

The left-hand side is maximized by $\eta = 1$. Thus, we should verify that

$$\begin{aligned}(x_{\rm S} - x_{\rm L}) + x_{\rm S}(2x_{\rm L} - 1) &< 0 \\ 2x_{\rm L}x_{\rm S} - x_{\rm L} &< 0 \\ x_{\rm L}\underbrace{(2x_{\rm S} - 1)}_{<0} &< 0, \quad \textit{always satisfied.}\end{aligned}$$

The proof that there are no steady states in $\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in \left[0, \frac{1}{2}\right), x_{\text{S},t} \in \left(\frac{1}{2}, 1\right]\}$ proceeds along the same lines.

■

Proof of Proposition 8

Let us state the following technical proposition, from which Proposition 8 trivially follows, and prove it.

Proposition 11 *Consider environments with strategic substitutes and random payoffs — i.e., $\hat{x}_0, \hat{x}_1$ uniform distributed in $[0, \frac{1}{2}] \times [\frac{1}{2}, 1]$. For any η, c, and λ, there exists three steady states, namely $\left(\frac{1}{2}, \frac{1}{2}\right)$, $(0,0)$, and $(1,1)$, and traits either converge to be neutral or for any time T there exists a time $t' > T$ in which traits are of different type. In particular, both $(0,0)$ and $(1,1)$ are unstable and there are no other attractors where traits are of the same type. Moreover,*

(i) For $c = 0$ and $\lambda = 0$,

- *The steady state $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a saddle.*
- *There exists two other steady states, $(0,1)$ and $(1,0)$, which are asymptotically stable.*
- *The basin of attraction of $(0,1)$ includes $[0, \frac{1}{2}) \times (\frac{1}{2}, 1]$ and the basin of attraction of $(1,0)$ includes $(\frac{1}{2}, 1] \times [0, \frac{1}{2})$.*

(ii) For $c = +\infty$ and any λ or for $\lambda = +\infty$ and any $c > 0$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ is the unique asymptotically stable state (GLOBALLY STABLE?)

Proof. To verify that $(1,1)$, $(0,0)$, and $\left(\frac{1}{2}, \frac{1}{2}\right)$ are steady states it is enough to observe that for all choices of the thresholds $\hat{x}_1$ and $\hat{x}_0$ they imply, respectively $\bar{a}_{\rm L} = \bar{a}_{\rm S} = \bar{a}_{\rm L} = 1$ or $\bar{a}_{\rm L} = \bar{a}_{\rm S} = \bar{a}_{\rm L} = 0$ or $\bar{a}_{\rm L} = \bar{a}_{\rm S} = \bar{a}_{\rm L} = \frac{1}{2}$. As a result, parents i can always rely on the oblique socialization and choose $\tau_i = 0$ to achieve the desired trait $x_i^o = \bar{a}_i = x_i$.

Before showing the existence of other fixed points, and assessing their stability, we prove that $(1,1)$ and $(0,0)$ are unstable.

Let us concentrate on $(1,1)$ first and consider the region $\left\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in \left(\frac{1}{2}, 1\right), x_{\text{L},t} \in \left(\frac{1}{2}, 1\right)\right\}$. The average action $\bar{a}_{\text{L},t} = \mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]$ is, discarding zero measure events,

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = \gamma(\hat{x}_1 < x_{\text{L},t}) + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_1 < x_{\text{S},t})\frac{1}{2}\eta + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_1 > x_{\text{S},t})\frac{1}{2}.$$

Since payoffs are such that the thresholds are uniformly distributed in the region characterized by strategic substitutability, we obtain

$$\begin{aligned}
&\gamma(\hat{x}_1 < x_{\text{L}}) = 2x_{\text{L}} - 1, \\
&\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 < x_{\text{S}}) = 4(1 - x_{\text{L}})\left(x_{\text{S}} - \frac{1}{2}\right), \\
&\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_1 > x_{\text{S}}) = 4(1 - x_{\text{L}})(1 - x_{\text{S}}),
\end{aligned}$$

so that

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] &= (2x_{\text{L},t} - 1) + 4(1 - x_{\text{L},t})\left(x_{\text{S},t} - \frac{1}{2}\right)\frac{1}{2}\eta + 4(1 - x_{\text{L},t})(1 - x_{\text{S},t})\frac{1}{2} \\
&= x_{\text{L},t} + (1 - x_{\text{L},t})(1 - 2x_{\text{S},t})(1 - \eta).
\end{aligned}$$

By symmetry

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] &= (2x_{\text{S},t} - 1) + 4(1 - x_{\text{S},t})\left(x_{\text{L},t} - \frac{1}{2}\right)\frac{1 - \eta}{2} + 4(1 - x_{\text{L},t})(1 - x_{\text{S},t})\frac{1}{2} \\
&= x_{\text{S},t} + (1 - x_{\text{S},t})(1 - 2x_{\text{L},t})\eta.
\end{aligned}$$

By eq. (7) both $x_{\text{L},t+1} = x^o_{\text{L},t}$ and $x_{\text{S},t+1} = x^o_{\text{S},t}$ are convex combinations of $\mathbb{E}_{\eta,\gamma}[a_{\text{L}}]$ and $\mathbb{E}_{\eta,\gamma}[a_{\text{S}}]$. Thus, $\mathbb{E}_{\eta,\gamma}[a_{\text{L}}] < x_{\text{L}}$ and $\mathbb{E}_{\eta,\gamma}[a_{\text{S}}] < x_{\text{S}}$ are sufficient to ensure that $\max\{x_{\text{L},t+1}, x_{2,t+1}\} < \max\{x_{\text{L},t}, x_{\text{S},t}\}$.

Let us verify that $\mathbb{E}_{\eta,\gamma}[a_{\text{L}}] < x_{\text{L}}$:

$$x_{\text{L}} + (1 - x_{\text{L}})(1 - 2x_{\text{S}})(1 - \eta) < x_{\text{L}}$$

$$\underbrace{(1 - x_{\text{L}})}_{>0}\underbrace{(1 - 2x_{\text{S}})}_{<0}\underbrace{(1 - \eta)}_{>0} < 0, \quad \textit{always.}$$

Similarly, it can be easily verified that $\mathbb{E}_{\eta,\gamma}[a_{\text{S}}] < x_{\text{S}}$.

Next we show that the strict inequality $\max\{x_{\text{L},t+1}, x_{2,t+1}\} < \max\{x_{\text{L},t}, x_{\text{S},t}\}$ holds also at the borders of the box $\left[\frac{1}{2}, 1\right]^2$, unless we are in one of the two steady states. Assume for example $x_{\mathrm{L},t} = 1$ and $x_{\mathrm{S},t} \in \left[\frac{1}{2}, 1\right)$. Then,

$$\bar{a}_{\mathrm{L},t} = x_{\mathrm{L},t} = 1 \leq \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$$

and

$$\bar{a}_{\mathrm{S},t} = x_{\mathrm{S},t} - (1 - x_{\mathrm{S},t})\eta < 1 = \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$$

Although the first inequality is not strict, we do have (for any pair of positive finite costs) that

$$x^o_{i,t} \in (\bar{a}_{\mathrm{S},t}, \bar{a}_{\mathrm{L},t} = 1) \;\; i = \mathrm{S}, \mathrm{L}$$

so that the strict inequality $\max\{x_{\mathrm{L},t+1}, x_{2,t+1}\} < \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$ still holds.

Similarly when $x_{\mathrm{L},t} = \frac{1}{2}$ and $x_{\mathrm{S},t} \in \left(\frac{1}{2}, 1\right]$, both

$$\bar{a}_{\mathrm{L},t} = \frac{1}{2} + \frac{1}{2}(1 - 2x_{\mathrm{S},t})(1 - \eta) < \frac{1}{2} < x_{\mathrm{S},t} = \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$$

and

$$\bar{a}_{\mathrm{S},t} = x_{\mathrm{S},t} = \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}.$$

Also here, although the first inequality is not strict, we do have (for any pair of positive finite costs) that

$$x^o_{i,t} \in (\bar{a}_{\mathrm{L},t}, \bar{a}_{\mathrm{S},t} = 1) \;\; i = \mathrm{S}, \mathrm{L}$$

so that the strict inequality $\max\{x_{\mathrm{L},t+1}, x_{2,t+1}\} < \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$ still holds.

A similar arguments holds for the other two edges of the box $\left[\frac{1}{2}, 1\right]^2$.

Overall, we have shown that in $\left[\frac{1}{2}, 1\right]^2 \setminus \left\{\left(\frac{1}{2}, \frac{1}{2}\right), (1, 1)\right)$ it holds $\max\{x_{\mathrm{L},t+1}, x_{2,t+1}\} < \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$. Therefore, either traits converge to the neutral $\left(\frac{1}{2}, \frac{1}{2}\right)$ or they eventually become of different type. No attractor with both traits of the same type as trait 1 exists.

By symmetry, a similar argument holds when $\left\{(x_{\mathrm{L},t}, x_{\mathrm{S},t}) : x_{\mathrm{L},t} \in \left[0, \frac{1}{2}\right], x_{\mathrm{S},t} \in \left[0, \frac{1}{2}\right]\right\} \setminus \left\{(0, 0), \left(\frac{1}{2}, \frac{1}{2}\right)\right\}$, leading to $\max\{x_{\mathrm{L},t+1}, x_{2,t+1}\} > \max\{x_{\mathrm{L},t}, x_{\mathrm{S},t}\}$.

We now turn to the existence of other fixed points and to their stability, as well as to the stability of $\left(\frac{1}{2}, \frac{1}{2}\right)$, by analyzing the cultural traits dynamics in the region where $x_{\mathrm{L},t} \in (\frac{1}{2}, 1)$ and $x_{\mathrm{S},t} \in (0, \frac{1}{2})$ and, symmetrically, $x_{\mathrm{L},t} \in (0, \frac{1}{2})$ and $x_{\mathrm{S},t} \in (\frac{1}{2}, 1)$ for specific choices of transmission costs c and λ.

First, we compute average payoffs in the region $\left\{(x_{\mathrm{L},t}, x_{\mathrm{S},t}) : x_{\mathrm{L},t} \in (\frac{1}{2}, 1], x_{\mathrm{S},t} \in [0, \frac{1}{2})\right\}$.

For $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]$ we have

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = \gamma(\hat{x}_1 < x_{\text{L},t}) + \gamma(\hat{x}_1 > x_{\text{L},t} \ \wedge \ \hat{x}_0 > x_{\text{S},t}) \left(1 - \frac{\eta}{2}\right) + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2}.$$

Since payoffs are uniformly distributed in the region characterized by strategic substitutability:

$$\begin{aligned}
&\gamma(\hat{x}_1 < x_{\text{L}}) = 2x_{\text{L}} - 1,\\
&\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 > x_{\text{S}}) = 4(1 - x_{\text{L}}) \left(\frac{1}{2} - x_{\text{S}}\right),\\
&\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 < x_{\text{S}}) = 4(1 - x_{\text{L}})x_{\text{S}},
\end{aligned}$$

then

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] &= 2x_{\text{L},t} - 1 + 2(1 - x_{\text{L},t})(1 - 2x_{\text{S},t}) \left(1 - \frac{\eta}{2}\right) + 2(1 - x_{\text{L},t})x_{\text{S},t}\\
&= x_{\text{L},t} + (1 - x_{\text{L},t}) (1 - 2x_{\text{S},t}) (1 - \eta).
\end{aligned}$$

Turning to $\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]$, we have

$$\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] = \gamma(\hat{x}_1 < x_{\text{L},t} \ \wedge \ \hat{x}_0 < x_{\text{S},t})\frac{1-\eta}{2} + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2},$$

so that, computing the probabilities of having norms within the given bounds,

$$\begin{aligned}
\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] &= 4\left(x_{\text{L},t} - \frac{1}{2}\right) x_{\text{S}}\frac{1-\eta}{2} + 4(1 - x_{\text{L},t})x_{\text{S},t}\frac{1}{2}\\
&= x_{\text{S},t} + (1 - 2x_{\text{L},t})x_{\text{S},t}\eta.
\end{aligned}$$

Similar expressions are found in the region $\left\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in [0, \frac{1}{2}), x_{\text{S},t} \in (\frac{1}{2}, 1]\right\}$.

Not being able to characterize the dynamics in these two regions for any parameterization of the cost of acculturation, next we turn to the analysis for specific costs.

$c = 0$ **and** $\lambda = 0$ Under no cost of acculturation, the dynamics of cultural trait is

$$\begin{cases} x_{\text{L},t+1} = \mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = f_{\text{L}}(x_{\text{L},t}, x_{\text{S},t}) \\ x_{\text{S},t+1} = \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] = f_{\text{S}}(x_{\text{L},t}, x_{\text{S},t}) \end{cases},$$

where the functional form $f = (f_{\text{L}}, f_{\text{S}})$ of the average actions depends on the types of traits. Using the functional form of the average action given above, it can be derived that both maps f_{L} and f_{S} are C^1.

First, we shall prove that $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a saddle. Computing the Jacobian of f and evaluating it in $\left(\frac{1}{2}, \frac{1}{2}\right)$, we get

$$J\left(\frac{1}{2}, \frac{1}{2}\right) = \begin{pmatrix} 1 & -(1-\eta) \\ \eta & 1 \end{pmatrix}$$

whose eigenvalues are $\lambda_1 = 1 + \sqrt{\eta(1-\eta)} > 1$ and $\lambda_2 = 1 - \sqrt{\eta(1-\eta)} < 1$, leading to the wanted result.

Next, we shall prove that $(1, 0)$ is an asymptotically stable steady state. The fact that it is a steady state follows trivially by substitution (using functional form of the map in the region $\left\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in (\frac{1}{2}, 1], x_{\text{S},t} \in [0, \frac{1}{2})\right\}$. For the asymptotic stability we compute the Jacobian in $(1, 0)$, finding

$$J(1,0) = \begin{pmatrix} \eta & 0 \\ 0 & 1-\eta \end{pmatrix},$$

and note that both eigenvalues are in $(-1, 1)$. A similar results holds for the asymptotic stability of $(0, 1)$.

Finally, to conclude the proof of (ii), we note that in the region $\left\{(x_{\text{L},t}, x_{\text{S},t}) : x_{\text{L},t} \in (\frac{1}{2}, 1], x_{\text{S},t} \in [0, \frac{1}{2})\right\}$ for all t

$$x_{\text{L},t+1} - x_{\text{L},t} = f_{\text{L}}(x_{\text{L},t}, x_{\text{S},t}) - x_{\text{L},t} = \underbrace{(1 - x_{\text{L},t})}_{\geq 0}\underbrace{(1 - 2x_{\text{S},t})}_{>0}\underbrace{(1-\eta)}_{>0} \geq 0,$$

with equality only for $x_{\text{L},t} = 1$, and

$$x_{\text{S},t+1} - x_{\text{S},t} = f_{\text{S}}(x_{\text{L},t}, x_{\text{S},t}) - x_{\text{S},t} = \underbrace{(1 - 2x_{\text{L},t})}_{<0}\underbrace{x_{\text{S},t}\eta}_{\geq 0} \leq 0,$$

with equality only for $x_{\text{S},t} = 0$, showing that starting from the region there is convergence to $(1, 0)$. By symmetry. similar result holds for $(0, 1)$.

$c = +\infty$ **and** $\lambda \geq 0$ Under infinite socialization costs, the optimal socialization parameter is set to $\tau_{i,t} = 0$ for each i and t, so that the new trait of both generations is equal to the average action of the two populations. For each t

$$x_{\text{L},t+1} = x_{\text{S},t+1} = \eta\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] + (1-\eta)\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}],$$

where the actual expression for the average actions are the same as those already computed for the other cases above. The two traits become equal in one period and any initial cultural heterogeneity is lost. Thus the only steady states have equal traits. We have already proved for the general case that both $(0,0)$ and $(1,1)$ are unstable. The steady state $\left(\frac{1}{2}, \frac{1}{2}\right)$ is instead globally stable, as it can be easily checked by showing that for the homogeneous trait $x_t = x_{\text{L},t} = x_{\text{S},t}$ it holds

$$x_{t+1} - x_t \leq 0,$$

when $x_t \in \left[\frac{1}{2}, 1\right]$, with equality only when $x = \frac{1}{2}$, and

$$x_{t+1} - x_t \geq 0,$$

when $x_t \in \left[0, \frac{1}{2}\right]$, with equality only when $x = \frac{1}{2}$.

$c > 0$ **and** $\lambda = +\infty$ With infinite costs for choosing a role models which differ from the trait, each parent choice is $\theta_{i,t} = x_{i,t}$ so that the dynamics of traits becomes:

$$x_{i,t+1} = \tau_{i,t} x_{i,t} + (1 - \tau_{i,t}) \left(\eta \mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] + (1-\eta) \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]\right), \quad i = \text{L}, \text{S}.$$

The dynamics is similar to the one with infinite socialization cost, in that it is driven by the average action but is delayed do to a typo-specific memory of the previous trait. Moreover, the only steady states are those with a homogeneous trait: $(0,0)$, $(1,1)$, and $\left(\frac{1}{2}, \frac{1}{2}\right)$. As with the general case, the only stable steady state can be the latter.

When $\eta = \frac{1}{2}$, $f_{\text{L},t+1} = f_{2,t+1}$ so that $p_{\text{L},t+1} = p_{\text{S},t+1} = \frac{2c+\Delta_t^2}{4c+\Delta_t^2}$ $p_{\text{L},t+1} = 1 - \frac{1}{2}f_{\text{L},t+1}$ and $p_{\text{S},t+1} = 1 - p_{\text{L},t+1} = \frac{1}{2}f_{\text{L},t+1}$. The norm dynamics simplifies to

$$\begin{cases} x_{\text{L},t+1} = (1-\frac{1}{2}f_{\text{L},t+1})((1-\lambda)x_{\text{L},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]) + \frac{1}{2}f_{\text{L},t+1}((1-\lambda)x_{\text{S},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]) \\ x_{\text{S},t+1} = \frac{1}{2}f_{\text{L},t+1}((1-\lambda)x_{\text{L},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]) + (1-\frac{1}{2}f_{\text{L},t+1})((1-\lambda)x_{\text{S},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]) \end{cases} \tag{B.16}$$

$$\begin{cases} \Rightarrow x_{\text{L},t+1} = \frac{2c+\Delta_t^2}{4c+\Delta_t^2}((1-\lambda)x_{\text{L},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]) + (1-\frac{2c+\Delta_t^2}{4c+\Delta_t^2})((1-\lambda)x_{\text{S},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]) \\ x_{\text{S},t+1} = (1-\frac{2c+\Delta_t^2}{4c+\Delta_t^2})((1-\lambda)x_{\text{L},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]) + \frac{2c+\Delta_t^2}{4c+\Delta_t^2}((1-\lambda)x_{\text{S},t} + \lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]) \end{cases} \tag{B.17}$$

and, adding up the two equations,

$$x_{\text{L},t+1} + x_{\text{S},t+1} = (1-\lambda)(x_{\text{L},t} + x_{\text{S},t}) + \lambda(\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]] + \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]]). \tag{B.18}$$

Next we shall show that if $x_{\text{L},t} \in (\frac{1}{2}, 1)$ and $x_{\text{S},t} \in (0, \frac{1}{2})$, or $x_{\text{L},t} \in (\frac{1}{2}, 1)$ and $x_{\text{S},t} \in (0, \frac{1}{2})$, the term $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]] + \mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]$ depends only on the sum $z_t = x_{\text{L},t} + x_{\text{S},t}$, so that (B.18) can be used to characterize the dynamics of z_t.

Let us start from the region where $x_{\text{L},t} \in (\frac{1}{2}, 1)$ and $x_{\text{S},t} \in (0, \frac{1}{2})$. For $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]$ we have

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] = \gamma(\hat{x}_1 < x_{\text{L},t}) + \gamma(\hat{x}_1 > x_{\text{L},t} \ \wedge \ \hat{x}_0 > x_{\text{S},t})(1 - \frac{1}{4}) + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2}.$$

Since payoffs are uniformly distributed in the region characterized by strategic substitutability

- $\gamma(\hat{x}_1 < x_{\text{L}}) = 2x_{\text{L}} - 1$
- $\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 > x_{\text{S}}) = 4(1 - x_{\text{L}})(\frac{1}{2} - x_{\text{S}})$
- $\gamma(\hat{x}_1 > x_{\text{L}} \wedge \hat{x}_0 < x_{\text{S}}) = 4(1 - x_{\text{L}})x_{\text{S}}$

then

$$\begin{aligned} \mathbb{E}_{\eta,\gamma}[a_{\text{L},t}] &= 2x_{\text{L},t} - 1 + \frac{3}{2}(1 - x_{\text{L},t})(1 - 2x_{\text{S},t}) + 2(1 - x_{\text{L},t})x_{\text{S},t} \\ &= \frac{1}{2} - x_{\text{S},t} + x_{\text{L},t}(\frac{1}{2} + x_{\text{S},t}) \end{aligned}$$

Turning to $\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]$, we have

$$\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] = \gamma(\hat{x}_1 < x_{\text{L},t} \ \wedge \ \hat{x}_0 < x_{\text{S},t})\frac{1}{4} + \gamma(\hat{x}_1 > x_{\text{L},t} \wedge \hat{x}_0 < x_{\text{S},t})\frac{1}{2}$$

so that, computing the probabilities of having norms within the given bounds,

$$\begin{aligned}\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] &= 4\left(x_{\text{L},t}-\frac{1}{2}\right)x_{\text{S}}\frac{1}{4}+4(1-x_{\text{L},t})x_{\text{S},t}\frac{1}{2}\\ &= \left(\frac{3}{2}-x_{\text{L},t}\right)x_{\text{S},t}.\end{aligned}$$

Importantly

$$\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]+\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}] = \frac{1}{2}\left(1+x_{\text{L},t}+x_{\text{S},t}\right).$$

Swapping the role of $x_{\text{L},t}$ and $x_{\text{S},t}$, we get the values of $\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]$ and $\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]$, and their sum, also in the region where $x_{\text{L},t} \in (0, \frac{1}{2})$ and $x_{\text{S},t} \in (\frac{1}{2}, 1)$.

Using the sum of average payoffs in (B.18), we obtain the dynamics for $z_t = x_{\text{L},t}+x_{\text{S},t}$:

$$z_{t+1} = (1-\lambda)z_t+\frac{\lambda}{2}\left(1+z_t\right) = \left(1-\frac{\lambda}{2}\right)z_t+\frac{\lambda}{2} \tag{B.19}$$

The latter has a unique, and globally stable, steady state $z^* = 1$, implying that we can restrict the stability analysis of the traits' dynamics on the line $x_{\text{L},t}+x_{\text{S},t} = 1$.

We turn to the analysis of the cultural traits dynamics on the line $x_{\text{L},t}+x_{\text{S},t} = 1$. Without loss of generality we also impose $x_{\text{L}} \in (\frac{1}{2}, 1)$. From the dynamics in equation (B.17) we obtain

$$x_{\text{L},t+1} = (1-\frac{1}{2}f_{\text{L},t+1})[(1-\lambda)x_{\text{L},t}+\lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}]]+\frac{1}{2}f_{\text{L},t+1}[(1-\lambda)(1-x_{\text{L},t})+\lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}]]$$

$$\Rightarrow x_{\text{L},t+1} = \frac{2c+\Delta_t^2}{4c+\Delta_t^2}((1-\lambda)x_{\text{L},t}+\lambda\mathbb{E}_{\eta,\gamma}[a_{\text{L},t}])+(1-\frac{2c+\Delta_t^2}{4c+\Delta_t^2})((1-\lambda)(1-x_{\text{L},t})+\lambda\mathbb{E}_{\eta,\gamma}[a_{\text{S},t}])$$

$$x_{\text{L},t+1} = (1-\lambda)(1-x_{\text{L},t})+\lambda\bar{a}_{\text{S},t}+\frac{2c+\Delta_t^2}{4c+\Delta_t^2}((1-\lambda)(2x_{\text{L},t}-1)+\lambda(\bar{a}_{\text{L},t}-\bar{a}_{\text{S},t}))$$

$$x_{\text{L},t+1} = (1-\lambda)(1-x_{\text{L},t})+\lambda\bar{a}_{\text{S},t}+\frac{2c+\Delta_t^2}{4c+\Delta_t^2}\Delta_t \tag{B.20}$$

$$\begin{cases}\bar{a}_{\text{L},t} = \frac{5}{2}x_{\text{L},t}-\frac{1}{2}-x_{\text{L},t}^2\\ \bar{a}_{\text{S},t} = \left(\frac{3}{2}-x_{\text{L},t}\right)\left(1-x_{\text{L},t}\right)\\ \Delta_t = (1-\lambda)(2x_{\text{L},t}-1)+\lambda(\bar{a}_{\text{L},t}-\bar{a}_{\text{S},t})) = (2x_{\text{L},t}-1)(1+(1-x_{\text{L},t})\lambda)\end{cases}$$

- If $c = 0$ The dynamics in equation (B.20) becomes

$$x_{\mathrm{L},t+1} = \frac{x_{\mathrm{L},t}\big(2 + (3 - 2x_{\mathrm{L},t})\lambda\big) - \lambda}{2} \tag{B.21}$$

whose fixed points are

$$x^*_{\mathrm{L}} = \frac{1}{2} \quad \text{and} \quad x^*_{\mathrm{L}} = 1.$$

The Jacobian at the steady state,

$$1 + \left(\frac{3}{2} - 2x^*_{\mathrm{L}}\right)\lambda,$$

implies that, for all values of λ, $x^*_{\mathrm{L}} = \frac{1}{2}$ is unstable, whereas $x^*_{\mathrm{L}} = 1$ is stable. The higher λ the higher the speed of convergence.

- If $c > 0$ The dynamics in equation (B.20) can be written as

$$x_{\mathrm{L},t+1} = \frac{1}{2} + \frac{\Delta_t^2\left(x_{\mathrm{L},t}\big(2 + (3 - 2x_{\mathrm{L},t})\lambda\big) - \lambda - 1\right)}{2\big(4c + \Delta_t^2\big)} \tag{B.22}$$

or, defining $y_{\mathrm{L},t} = x_{\mathrm{L},t} - \frac{1}{2}$, and $\Delta^2(y_{\mathrm{L},t}) := (2(y_{\mathrm{L},t} + \frac{1}{2}) - 1)^2(1 + (1 - (y_{\mathrm{L},t} + \frac{1}{2}))\lambda)^2 = 4y^2_{\mathrm{L},t}(1 + (\frac{1}{2} - y_{\mathrm{L},t})\lambda)^2$

$$\begin{aligned} y_{\mathrm{L},t+1} =& \frac{\Delta^2(y_{\mathrm{L},t})\left((2y_{\mathrm{L},t} + 1)\big(1 + (1 - y_{\mathrm{L},t})\lambda\big) - \lambda - 1\right)}{2\big(4c + \Delta^2(y_{\mathrm{L},t})\big)} \\ =& \underbrace{\frac{4y^2_{\mathrm{L},t}(1 + (\frac{1}{2} - y_{\mathrm{L},t})\lambda)^2\left((2y_{\mathrm{L},t} + 1)\big(1 + (1 - y_{\mathrm{L},t})\lambda\big) - \lambda - 1\right)}{2\big(4c + 4y^2_{\mathrm{L},t}(1 + (\frac{1}{2} - y_{\mathrm{L},t})\lambda)^2\big)}}_{=:\varphi(y_{\mathrm{L},t})} \end{aligned}$$

At steady state it should hold that $y_{\mathrm{L},t+1} - y_{\mathrm{L},t} = 0$. Thus,

$$\frac{\Delta^2(y_{\text{L},t})\Big((2y_{\text{L}}+1)\big(1+(1-y_{\text{L}})\lambda\big)-\lambda-1\Big)}{2\big(4c+\Delta^2(y_{\text{L}})\big)}-y_{\text{L}}=0$$

$$4y_{\text{L},t}^2\left(1+\left(\frac{1}{2}-y_{\text{L}}\right)\lambda\right)^2\Big((2y_{\text{L}}+1)\big(1+(1-y_{\text{L}})\lambda\big)-\lambda-1-2y_{\text{L}}\Big)-8cy_{\text{L}}=0$$

A first solution is $y_{\text{L}}^0=0$. The other solution solves:

$$y_{\text{L}}\left(1+\left(\frac{1}{2}-y_{\text{L}}\right)\lambda\right)^2\Big((2y_{\text{L}}+1)(1-y_{\text{L}})\lambda-\lambda\Big)-2c=0$$

$$y_{\text{L},t}^2\left(1+\left(\frac{1}{2}-y_{\text{L}}\right)\lambda\right)^2\lambda\underbrace{\Big(1-2y_{\text{L}}\Big)}_{>0}-2c=0$$

$$y_{\text{L},t}^2\left(\frac{1}{2}+\frac{1}{\lambda}-y_{\text{L}}\right)^2\lambda^3\underbrace{\left(\frac{1}{2}-y_{\text{L}}\right)}_{>0}=c \tag{B.23}$$

In what follows, by studying L.H.S as a function of y_{L}, we show that the equation can have 2, 1 or 0 solution in the interval $(0,\frac{1}{2})$, depending on c and λ. The L.H.S is a polynomial of degree 5 with three roots: $y_{\text{L}}^0=0$, $y_{\text{L}}^1=\frac{1}{2}$, $y_{\text{L}}^2=\frac{1}{2}+\frac{1}{\lambda}$ and it is non-negative for $y_{\text{L}}\le\frac{1}{2}$ and non-positive for $y_{\text{L}}\ge\frac{1}{2}$. Therefore, it changes concavity three times once in $[0,\frac{1}{2}]$ twice for $y_{\text{L}}>\frac{1}{2}$. As a result, in the interval $[0,\frac{1}{2}]$ the L.H.S may equal to c, twice, once, or never. Therefore, there exist a threshold values $\bar{c}$ such that if $c<\bar{c}$ equation (B.23) has two solutions, if $c=\bar{c}$ the two solution coincides, and if $c>\bar{c}$ there are no solutions. Since for $y_{\text{L}}\in[0,\frac{1}{2}]$, the polynomial is increasing in λ, then also $\bar{c}$ is increasing in λ. Note also that when the two solutions $0<y_{\text{L}}^1<y_{\text{L}}^2<\frac{1}{2}$ exist their position depends on c and on λ. In particular, $\frac{\partial y_{\text{L}}^1}{\partial\lambda}<0$ $\frac{\partial y_{\text{L}}^1}{\partial c}>0$ $\frac{\partial y_{\text{L}}^2}{\partial\lambda}>0$ $\frac{\partial y_{\text{L}}^2}{\partial c}<0$.

Finally, to prove stability we study the sign of $\varphi(y_{\text{L}})-y_{\text{L}}$. In fact, if $\varphi(y_{\text{L}})-y_{\text{L}}\gtreqless 0$ then $y_{\text{L},t+1}\gtreqless y_{\text{L},t}$. Thus, performing the same analysis we easily see that: if $c>\bar{c}$ then y_{L}^0 is globally stable; if $c<\bar{c}$ then both y_{L}^0 and y_{L}^2 are locally stable with basin of attractions $(0,y_{\text{L}}^1)$ and $(y_{\text{L}}^1,\frac{1}{2})$, respectively; if $c=\bar{c}$ then y_{L}^1 is locally stable and $y_{\text{L}}^2=y_{\text{L}}^1$ is meta-stable with basin of attractions $(0,y_{\text{L}}^1)$ and $(y_{\text{L}}^1,\frac{1}{2})$, respectively.

Recalling that $x_{\text{L}}=y_{\text{L}}+\frac{1}{2}$ we get the possible steady states and their stability.

Applying the same reasoning to spaces $(0,\frac{1}{2})^2$ and $(0,\frac{1}{2})\times(\frac{1}{2},1)$ the proof is concluded.

■

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