DISCLOSURE IN INSURANCE MARKETS WITH LIMITED SCREENING

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We investigate the impact of information disclosure, via a statistical instrument, on consumer welfare in competitive insurance markets with limited screening. We demonstrate that, under natural constraints on information disclosure, no statistical instrument is "safe" to implement. There always exists a nonnegligible set of prior beliefs about the risk types of consumers, compatible with an observed market allocation, under which additional information disclosure strictly worsens welfare.

KEYWORDS: insurance market, adverse selection, information disclosure, screening, regulation, privacy.

1. INTRODUCTION

Regulation of information provision in insurance markets is largely shaped by two forces. On the one hand, the political and ideological pressure, stemming from concerns for nondiscrimination, equal opportunity, civil rights, and privacy protection, advocates disregarding the consumers' individual characteristics. On the other hand, the economic argument of adverse selection suggests that informational asymmetries are detrimental for proper functioning of insurance markets. The latter is particularly severe in competitive markets, where insurers have no profit capacity to absorb distortions created by the regulation. This concern for adverse selection has played a major role in many regulatory decisions, for example, granting to the equal treatment laws significant exceptions that are sometimes exploitable beyond their original purpose (Siegelman, 2004).

Numerous empirical studies (e.g., Buchmueller and DiNardo, 2002, Schwarze and Wein, 2005, Simon, 2005, Chiappori et al., 2006, He, 2009, Einav et al., 2010b, Bundorf et al., 2012) find mixed evidence of adverse selection in insurance markets.[1] In cases where adverse selection is detected, the conclusion is that it would be socially desirable to reduce the information asymmetry between insurers and consumers.[2] This conclusion is also grounded in the theoretical literature on competitive insurance contracts (Crocker and Snow, 2013, Farinha Luz et al., 2023), which argues that a reduction of informational asymmetries is welfare improving.[3]

In this paper, we argue that empirical evidence of adverse selection does not automatically mandate the desirability of consumer information provision to the insurers. To the contrary, we demonstrate that, under realistic conditions, regulators cannot dismiss the possibility that disclosure of information related to consumer risk will worsen the welfare even when the market is competitive. In other words, we emphasize the need for regulators to exercise caution when disclosing consumers' private information.

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[1]For surveys of the theory, methodology, and evidence, see Einav et al. (2010a), Chiappori and Salanié (2013), and Geruso and Layton (2017).

[2]For instance, Einav et al. (2010b) find that the welfare loss due to information asymmetries in the UK annuity market is "£127 million per year or about 2 percent of annuitized wealth."

[3]The literature also studies alternative ways of regulation that do not involve information disclosure, such as government mandates (e.g., Einav et al., 2010b, Azevedo and Gottlieb, 2017, Cabral et al., 2022).

The intuition for our result is that, when insurers compete away their profits, there are two effects associated with information disclosure. On the one hand, more information alleviates the problem of adverse selection, as lower-risk consumers can be identified as such and offered more favorable premiums. On the other hand, more information may result in misclassification, as lower-risk consumers can be misclassified into a higher risk group, face higher premiums, and decide to reduce their coverage or completely self-insure. In Crocker and Snow (2013) and Farinha Luz et al. (2023), the positive effect dominates the negative. This is because they allow for unrestricted screening, so each type of consumer receives a personalized contract, and the reduction in information asymmetry can only be beneficial as it relaxes the incentive compatibility constraints in the contract design. In contrast, in this paper we assume limited screening. This is a practical assumption, as in reality insurance contracts rarely feature a continuum of options that consumers can use to signal their types. When screening is limited, the negative effect of information disclosure can no longer be ignored and sometimes will outweigh the positive effect.

More specifically, we study the implications of limited screening on the demand for insurance in an otherwise standard insurance setting. The market is competitive, and consumers are risk averse and heterogeneous (with a continuum of types) in both the underlying risk and their risk preferences, which have been identified as two major sources of private information (Finkelstein and McGarry, 2006, Einav et al., 2007, Cutler et al., 2008). The insurers are risk neutral and have consistent beliefs under unraveling, as in Bisin and Gottardi (2006), Azevedo and Gottlieb (2017), and Farinha Luz et al. (2023). Disclosure of additional information is modeled as a "monotone instrument" that classifies consumer types into two groups—high-risk and low-risk—such that higher-risk consumers are more likely to be assigned into the high-risk group. Examples of such monotone classifications are abundant and include, for instance, risk tiers in auto insurance where drivers are assigned into a tier based on their individual risk score, or occupation-based risk classifications in life insurance where different occupations are associated with different risk categories.

When a regulator makes decisions, she is typically unable to see the detailed information on consumer preferences and their risk types. We assume that the regulator sees only what we call an *observable situation*: insurance contracts, and numbers of consumers who sign these contracts. Our analysis demonstrates that, irrespective of the current observable situation, if the insurers receive additional information in the form of a monotone statistical instrument, there *always* exists a nonnegligible set of unobserved prior beliefs over consumer risk types consistent with the observable situation, such that under any such prior belief, the additional information disclosure about consumer risk-related characteristics strictly worsens the welfare.

For illustrative purposes, we model the instrument as binary, so that additional information classifies consumers into two groups and the classification is correlated with risk. We discuss in Section 4 that our main result extends to arbitrary number of classification groups, as long as the instrument is monotone with respect to consumer types.

Related Literature. Our paper is related to multiple strands of literature on information asymmetry in insurance markets. One strand of literature characterizes equilibria focusing on the trade-off between tailoring contracts to individual information and the resulting price dispersion (Handel et al., 2015, Farinha Luz et al., 2023). The key difference of our approach from Farinha Luz et al. (2023) is that we assume that a rich set of screening contracts is unavailable. This is also our key distinction to Veiga (2024) who studies the implications of price dispersion on welfare when a continuum of information regulation policies is allowed.

In addressing inefficiencies linked to information disclosure, we contribute to the literature on information regulation related to privacy. Typically this literature studies allocations with monopolistic firms, as in Eilat et al. (2021) and Bird and Neeman (2022). In such settings, information disclosure is associated with the loss of consumer information rents, prompting a regulator to limit the amount of disclosed information. This channel is absent in our setting with competitive insurance markets. Therefore, we see our approach as complementary to information design for pricing within monopolistic markets (see also Bergemann et al., 2015, Roesler and Szentes, 2017, Hidir and Vellodi, 2021).

Our paper is related to the literature on information provision in the Akerlof's (1970) framework. While Levin (2001) shows that publicly releasing private information improves allocation in a setting without screening, provided that the markets are "well behaved", we emphasize the challenges in insurance markets associated with limited screening possibilities.

Another related paper is Garcia and Tsur (2021), who characterize optimal information provision in competitive insurance markets with limited screening, addressing the trade-off between risk sharing and contract adaptation. They show that full release of private information is generally inefficient, and optimal design matches consumer risk types in a negative assortative manner. In contrast, we do not take the information design perspective, and consider only monotone contracts, thus ruling out negative assortative matching. Our contribution lies in demonstrating the potential negative impact on welfare through information disclosure.

2. MODEL

Consider a competitive insurance market with a population of risk-averse consumers who buy insurance against potential losses they may incur. Each consumer is characterized by a type $(\mu,\nu)\in[0,\bar{\nu}]^2$, where μ is the mean loss and ν is the consumer's willingness to pay for the full cover insurance.[4] When a full cover insurance contract is offered at premium p, the consumer is willing to buy it if and only if $\nu>p$, and the insurer makes profit if and only if $\mu<p$. The consumer's monetary gain from this contract is $\max\{\nu-p,0\}$.

Let $F(\mu,\nu)$ be a prior joint distribution of types in the population of consumers. Let $\mathcal{F}$ be the set of joint distributions on $[0,\bar{\nu}]^2$ that satisfy

$$\begin{aligned} &\mathbb{P}(\mu=0)<1, \quad \text{and} \\ &\mathbb{P}(\mu<\nu|\nu)=1 \quad \text{for all } \nu\in(0,\bar{\nu}]. \end{aligned} \tag{A$_1$}$$

The first assumption rules out a triviality. The second assumption reflects that consumers are strictly risk averse.

Let $s\in S=\{s_1,\dots,s_K\}$ be an observable signal about the consumer's type. Suppose that each s_k induces a posterior joint distribution $F_k(\mu,\nu)\in\mathcal{F}$ conditional on that signal.

Several insurers compete for consumers. They do not observe the consumer's type. Instead, they know its prior distribution F, observe a signal s_k, and derive the posterior distribution F_k conditional on s_k. For clarity of exposition, suppose that the contracts offered by the insurers fully cover the losses, so the only variable that the insurers choose is the premium.[5] An insurer's profit from offering insurance at premium p_k to a consumer with signal s_k is given by

$$\pi(p_k|F_k)=\int_{(\mu,\nu):\nu\geq p_k}(p_k-\mu)F_k(\mathrm{d}\mu,\mathrm{d}\nu)=\int_{p_k}^{\bar{\nu}}\left(p_k-\mathbb{E}_{F_k}[\mu|\nu]\right)F_k(\mathrm{d}\nu), \tag{1}$$

[4]Suppose that a consumer has an initial wealth w and a strictly increasing and concave utility function u, and faces a random loss ξ. He is willing to pay premium p for the full cover insurance if and only if $u(w-p)>\mathbb{E}[u(w-\xi)]$. The maximum premium that he is willing to pay is $w-u^{-1}(\mathbb{E}[u(w-\xi)])$. The type of this consumer is summarized by the pair $(\mu,\nu)=(\mathbb{E}[\xi],w-u^{-1}(\mathbb{E}[u(w-\xi)]))$.

[5]As we discuss in Section 4, our results extend straightforwardly if we allow for richer contracts, as long as the menu of contracts is finite.

where $F_k(\nu)$ denotes the posterior marginal distribution of ν, and $\mathbb{E}_{F_k}[\mu|\nu]$ denotes the posterior conditional mean value of μ given ν.

As the insurers compete, they make zero profit. That is, in equilibrium, each insurer offers an actuarily fair premium p_k to consumers with signal s_k, for each $k=1,...,K$. Moreover, no insurer can benefit by deviation to a smaller premium, that is, any price smaller than p_k must yield a strict loss. Formally, a profile of premiums $\mathbf{p}=(p_1,...,p_k)$ is an *equilibrium* if, for all $k=1,...,K$,

$$\pi(p_k|F_k)=0 \text{ and } \pi(p'|F_k)<0 \text{ for all } p'<p_k. \tag{2}$$

Observe that there exists a unique equilibrium. This immediately follows from the continuity of $\pi(p|F_k)$ in p, and that, by (1) and (A_1), we have

$$\pi(0|F_k)=-\int_0^{\bar{\nu}}\mathbb{E}_{F_k}[\mu|\nu]F_k(\mathrm{d}\nu)<0 \quad \text{and} \quad \pi(\bar{\nu}|F_k)=0. \tag{3}$$

Given an equilibrium $\mathbf{p}=(p_1,...,p_K)$, for each $k=1,...,K$, let z_k be the fraction of the consumers in group k who buy insurance in that equilibrium:

$$z_k=\mathbb{P}(\nu>p_k)=1-F_k(p_k).$$

The profile $\mathbf{z}=(z_1,...,z_K)$, which captures the consumer's participation in equilibrium, will be a useful metric for our further analysis.

We now introduce the notion of *observable situation*. A pair (p_k,z_k) of a premium and a fraction consumers in group k who buy insurance at that premium is called *observable* if $z_k>0$, and (p_k,z_k) can occur in equilibrium under some posterior F_k. To put it formally, given a pair $(p_k,z_k)\in\mathbb{R}_+\times(0,1]$, let $\mathcal{F}(p_k,z_k)$ be the set of all distributions in $\mathcal{F}$ that support (p_k,z_k) in equilibrium:

$$\mathcal{F}(p_k,z_k)=\{F_k\in\mathcal{F}: p_k \text{ satisfies (2), and } z_k=1-F_k(p_k)\}.$$

If $\mathcal{F}(p_k,z_k)$ is nonempty, then we will call (p_k,z_k) observable. Let $\mathcal{S}$ be the set of all such observable pairs:

$$\mathcal{S}=\{(p_k,z_k)\in\mathbb{R}_+\times(0,1]:\mathcal{F}(p_k,z_k)\neq\varnothing\}.$$

An *observable situation* is a profile $(\mathbf{p},\mathbf{z})=((p_1,z_1),...,(p_K,z_K))$ such that each (p_k,z_k) is observable, that is, $(\mathbf{p},\mathbf{z})\in\mathcal{S}^K$.

Note that a situation $((p_1,z_1),...,(p_K,z_K))$ is deemed as observable only if there is trade $(z_k>0)$ in each category $k=1,...,K$. In other words, we only observe categories where trade occurs. Implicitly, there could exist more categories, $k=K+1,...$, but we know nothing about them because there is no trade in these categories. This assumption also rules out the case of complete market collapse, where nobody at all buys insurance.

3. EFFECT OF ADDITIONAL INFORMATION

Consider a regulator who decides whether or not to allow the insurers to use additional information about the consumer's risk type. This additional information is conveyed via a binary random variable $t\in\{t_H,t_L\}$ correlated with the mean loss μ, whose conditional distribution is given by $\lambda(\mu)=\mathbb{P}[t=t_H|\mu]$ for each $\mu\in[0,\bar{\nu}]$. We will refer to t as a (statistical) instrument.

We restrict attention to monotone instruments. An instrument t is *monotone* if $t = t_H$ is strictly more likely when the insurer's mean loss μ is higher:

$$\lambda(\mu) \text{ is strictly increasing.} \tag{A$_2$}$$

The monotonicity assumption allows for an interpretation of signals t_H and t_L as the assignments of the consumers to high and low risk group, respectively. This assumption is justified in practice for the following reasons. If an instrument is not monotone, it means that some lower-risk consumers are more likely to be assigned to the high-risk group than some higher-risk consumers. This assignment may be considered unfair and not permitted by regulations. In addition, consumers may have incentives to increase their risk μ, which would be a first-order concern if moral hazard was added to our model.

We assume that the regulator does knows neither the prior distribution F, nor the profile of the posteriors $\mathbf{F} = (F_1, ..., F_K)$. In particular, the consumers' preferences and risk attitudes are unobservable. The regulator only observes the equilibrium profile of premiums, $\mathbf{p} = (p_1, ..., p_K)$, and the profile of fractions of consumers, $\mathbf{z} = (z_1, ..., z_K)$, who buy insurance at those premiums. In other words, for each observable situation $(\mathbf{p}, \mathbf{z}) = ((p_1, z_1), ..., (p_K, z_K))$, the regulator contemplates the possibility of every posterior $F_k \in \mathcal{F}(p_k, z_k)$ for each $k = 1, ..., K$, under which $(\mathbf{p}, \mathbf{z})$ emerges in equilibrium.

When the insurers are allowed to use the instrument t, the consumers in each group k are further divided into two categories, H (high-risk) and L (low-risk), with two new posterior distributions, $F_{k,H}$ and $F_{k,L}$, conditional on t_H and t_L, respectively. This leads to a new equilibrium with two premiums, $p_{k,H}$ and $p_{k,L}$, for the high-risk and low-risk consumers in each group $k = 1, ..., K$:

$$\pi(p_{k,}|F_{k,j}) = 0 \text{ and } \pi(p'|F_{k,j}) < 0 \text{ for all } p' < p_{k,j} \text{ and each } j = H, L. \tag{4}$$

We now compare the monetary welfare before and after the instrument t is introduced. For each group $k = 1, ..., K$, the original welfare is given by

$$W_0(F_k) = \int_{(\mu,\nu)} \max\{\nu - p_k, 0\} F_k(\mathrm{d}\mu, \mathrm{d}\nu) = \int_{p_k}^{\bar{\nu}} (\nu - p_k) F_k(\mathrm{d}\nu). \tag{5}$$

The welfare with the instrument t is given by

$$\begin{aligned} W_t(F_k) &= \int_{(\mu,\nu)} \Big(\max\{\nu - p_{k,H}, 0\}\lambda(\mu) + \max\{\nu - p_{k,L}, 0\}(1 - \lambda(\mu)) \Big) F_k(\mathrm{d}\mu, \mathrm{d}\nu) \\ &= \int_{p_{k,H}}^{\bar{\nu}} (\nu - p_{k,H}) \hat{\lambda}_k(\nu) F_k(\mathrm{d}\nu) + \int_{p_{k,L}}^{\bar{\nu}} (\nu - p_{k,L})(1 - \hat{\lambda}_k(\nu)) F_k(\mathrm{d}\nu), \end{aligned} \tag{6}$$

where

$$\hat{\lambda}_k(\nu) = \int_0^{\bar{\nu}} \lambda(\mu) F_k(\mathrm{d}\mu|\nu).$$

Given observable situation, we say that an instrument is *potentially Pareto damaging* in that situation if there exists an open set[6] of posterior distributions such that the welfare of each group $k = 1, ..., K$ strictly diminishes as a result of the introduction of this instrument.

[6] E.g., in the topology of uniform convergence.

DEFINITION 1: Given an observable situation $(\mathbf{p},\mathbf{z}) = ((p_1,z_1),...,(p_K,z_K))$, an instrument t is *potentially Pareto damaging in* $(\mathbf{p},\mathbf{z})$ if

$$W_t(F_k) < W_0(F_k), \text{ for all } F_k \text{ in an open subset of } \mathcal{F}(p_k,z_k) \text{ and all } k = 1,...,K.$$

Our main result shows that no monotone instrument is "safe" to implement. Regardless of a situation on the insurance market, every monotone instrument can potentially reduce the welfare of all the consumer groups.

THEOREM 1: *For every observable situation, every monotone instrument is potentially Pareto damaging in that situation.*

The proof is in the Appendix. The intuition is as follows. Fix any consumer group k. The introduction of a monotone instrument strictly raises the premium for consumers classified as high-risk, from an initial value p_k up to some new value $p_{k,H}$. At the same time, it strictly lowers the premium for consumers classified as low-risk, from p_k down to some new value $p_{k,L}$. High-risk consumers whose willingness to pay is between p_k and $p_{k,H}$ no longer buy insurance, thus contributing to welfare loss, whereas low-risk consumers whose willingness to pay is between $p_{k,L}$ and p_k switch from not buying to buying insurance, thus contributing to welfare gain. However, the observable situation does not provide any information about relative mass of the consumers who contribute to welfare gain to those who contribute to welfare loss. There always exists an open set of posteriors F_k under which the latter dominates the former.

4. DISCUSSION

In a scenario where contracts pool multiple risk types due to either regulatory or practical constraints, we demonstrate that the disclosure of correlated information, modeled as a monotone instrument, can detrimentally affect welfare. We show that, even in fully competitive markets, there exists an open set of unobserved prior distributions over consumer risk types consistent with the observable situation, such that information disclosure strictly worsens welfare. Therefore, this note highlights the potential negative consequences of information disclosure in situations where contracts are limited, even in competitive insurance markets.

We now discuss several assumptions and their role for our result.

Non-binary Instruments. Our results extend to instruments that categorize consumers into $n \geq 2$ categories, as long as the likelihood ratio of the underlying risk between each pair of categories is weakly monotone, and for some pair it is strictly monotone. Our proof applies verbatim for each pair of categories, and the intuition for this result does not change.

Non-monotone Instruments. We assume strict monotonicity of instruments for Theorem 1 to hold. The key part of the proof is that, after the introduction of the instrument, a positive mass of consumers who previously bought insurance are now categorized as high-risk, and find the new premium too high. Such a premium and such a mass of consumers need not exist if the instrument is only weakly monotone, so that it could treat all types in some set identically. For the same reason, an instrument with infinite categories (e.g., full disclosure of the type) may invalidate our results, as there is zero mass of consumers in each category. Finally, when the monotonicity assumption is not imposed at all, Garcia and Tsur (2021) show that a welfare improvement can always be achieved by an instrument that pools high-risk and low-risk types pairwise in a negative assortative fashion.

Limited Screening and Partial Cover Contracts. We have only considered the simplest contracts that offer full cover to consumers, and do not permit any screening, except for the consumers' participation decision. Our results extend to the model with a finite menu of contracts that allow for limited screening. The proof follows the same steps, but the construction of the prior distribution of consumer types is more intricate. The support of types is concentrated in small intervals to the right of the threshold types (who are indifferent between adjacent contracts in the menu). As a result, when the instrument is introduced, the consumers switch only downwards, to contracts with less cover, thus causing the welfare to decrease.

APPENDIX: PROOF OF THEOREM 1

It suffices to show that, for each group $k = 1, \ldots, K$ and each observable pair $(p_k, z_k) \in \mathcal{S}$, we can find a posterior distribution $\hat{F}_k$ such that $W_t(\hat{F}_k) - W_0(\hat{F}_k) < 0$. Because the inequality is strict, by the continuity of the welfare functions in F_k, there exists an open neighborhood of $\hat{F}_k$ in $\mathcal{F}(p_k, z_k)$ such that $W_t(F_k) < W_0$ for all F_k in that neighborhood.

In what follows, we fix a consumer group $k \in \{1, \ldots, K\}$ and focus the analysis on this group. For exposition, we omit the subscript k from the notation.

Let (p^*, z^*) be an observable pair, so $(p^*, z^*) \in \mathcal{S}$. By (3), $\pi(0|F) < 0$ for all $F \in \mathcal{F}$, so $p^* = 0$ cannot be the equilibrium. By definition of observable pair, $z^* = 1 - F(p^*) > 0$, so $F(p^*) < F(\bar{\nu}) = 1$. We conclude that

$$0 < p^* < \bar{\nu}. \tag{7}$$

Consider a joint distribution $\hat{F}(\mu, \nu)$ whose marginal distribution $\hat{F}(\nu)$ and conditional distribution $\hat{F}(\mu|\nu)$ are given by:

$$\hat{F}(\nu) = \begin{cases} a, & \text{if } \nu \in [0, p^*), \\ a + b(\nu - p^*), & \text{if } \nu \in [p^*, \bar{\nu}], \end{cases}$$

$$\hat{F}(\mu|\nu) \text{ assigns probability } 1 \text{ to } \mu = c\nu \text{ for each } \nu \in [0, \bar{\nu}],$$

where

$$a = 1 - z^*, \quad b = \frac{z^*}{\bar{\nu} - p^*}, \quad \text{and} \quad c = \frac{2p^*}{\bar{\nu} + p^*}.$$

Observe that $\hat{F}(\nu)$ is a probability distribution. Indeed, by construction and (7), $\hat{F}(\nu)$ is increasing and right-continuous, and it satisfies $\hat{F}(0) = a > 0$ and

$$\hat{F}(\bar{\nu}) = a + b(\bar{\nu} - p^*) = 1 - z^* + \frac{z^*}{\bar{\nu} - p^*}(\bar{\nu} - p^*) = 1.$$

Given a function $\hat{\lambda} : [0, \bar{\nu}] \to [0, 1]$, let

$$\pi_{\hat{\lambda}}(p) = \int_{(\mu,\nu):\nu > p} (p - \mu)\hat{\lambda}(\nu)\hat{F}(\mathrm{d}\mu, \mathrm{d}\nu). \tag{8}$$

Before proceeding with the proof of Theorem 1, we prove a lemma.

LEMMA 1:

$$\pi_{\hat{\lambda}}(p) = b\int_{\max\{p^*,p\}}^{\bar{\nu}} (p - c\nu)\hat{\lambda}(\nu)\mathrm{d}\nu. \tag{9}$$

Moreover:

(a) if $\hat{\lambda}(\nu)$ is positive for all $\nu \in (0,\bar{\nu})$, then $\pi_{\hat{\lambda}}(p)$ is strictly increasing for $p \in [0,p^*]$;

(b) if $\hat{\lambda}(\nu)$ is positive for all $\nu \in (0,\bar{\nu})$ and weakly increasing, then $\pi_{\hat{\lambda}}(p)$ is strictly quasiconcave for $p \in [0,\bar{\nu}]$.

PROOF: Since $\hat{F}$ assigns probability $a > 0$ to $\nu = 0$ and has constant density $b > 0$ for $\nu \in [p^*,\bar{\nu}]$ and zero density elsewhere, it is immediate that

$$\pi_{\hat{\lambda}}(p) = \begin{cases} (0 - c\nu)\hat{\lambda}(\nu)a|_{\nu=0} + \int_{p^*}^{\bar{\nu}} (p - c\nu)\hat{\lambda}(\nu)b\mathrm{d}\nu, & \text{if } p = 0, \\ \int_{p^*}^{\bar{\nu}} (p - c\nu)\hat{\lambda}(\nu)b\mathrm{d}\nu, & \text{if } 0 < p \le p^*, \\ \int_{p}^{\bar{\nu}} (p - c\nu)\hat{\lambda}(\nu)b\mathrm{d}\nu, & \text{if } p^* < p \le \bar{\nu}, \end{cases}$$

which yields (9).

Suppose that $\hat{\lambda}(\nu)$ is positive. By (7), $b > 0$. Hence, $b\int_{p^*}^{\bar{\nu}} \hat{\lambda}(\nu)\mathrm{d}\nu > 0$. Thus, by (9), $\pi_{\hat{\lambda}}(p)$ is linear and strictly increasing on $[0,p^*]$.

Suppose in addition that $\hat{\lambda}(\nu)$ is weakly increasing. By (7), $b > 0$ and $1 - c > 0$. Thus, we obtain that the derivative of $\pi_{\hat{\lambda}}(p)$, given by

$$\pi'_{\hat{\lambda}}(p) = b\int_{p^*}^{\bar{\nu}} \hat{\lambda}(\nu)\mathrm{d}\nu - b(1-c)p\hat{\lambda}(p), \quad \text{for all } p \in (p^*,\bar{\nu}],$$

is strictly decreasing on the interval $(p^*,\bar{\nu}]$. So, $\pi_{\hat{\lambda}}(p)$ is strictly concave on $(p^*,\bar{\nu}]$. As we have found that $\pi_{\hat{\lambda}}(p)$ is strictly increasing on $[0,p^*]$ and strictly concave on $(p^*,\bar{\nu}]$, we conclude that $\pi_{\hat{\lambda}}(p)$ is strictly quasiconcave on $[0,\bar{\nu}]$. *Q.E.D.*

We now return to the proof of Theorem 1. Let t be a monotone instrument that categorizes each type (μ,ν) as H (high-risk) or L (low-risk) with probabilities $\lambda(\mu)$ and $1-\lambda(\mu)$, respectively. We will write $j = H, L$ to denote the categories H and L induced by the instrument t. We will also write $j = 0$ to denote the original pooling category before the instrument is introduced.

Because under $\hat{F}$ the mean loss μ is equal to $c\nu$ with certainty, we can substitute $\mu = c\nu$ and let

$$\hat{\lambda}_H(\nu) = \lambda(c\nu) \quad \text{and} \quad \hat{\lambda}_L(\nu) = 1 - \lambda(c\nu).$$

Note that, by (A_2), $\hat{\lambda}_H(\nu)$ is strictly increasing and $\hat{\lambda}_L(\nu)$ is strictly decreasing, and both are strictly positive for all $\nu \in (0,\bar{\nu})$. Also, let

$$\hat{\lambda}_0(\nu) = 1.$$

Let p_H^* and p_L^* be the equilibrium premiums for categories H and L. By definition of equilibrium and (8), for each $j \in \{H,L\}$, p_j^* must satisfy

$$\pi_{\hat{\lambda}_j}(p_j^*) = 0, \text{ and } \pi_{\hat{\lambda}_j}(p) < 0 \text{ for all } p < p_j^*.$$

For $j = 0$, we apply Lemma 1 with $\hat{\lambda} = \hat{\lambda}_0$. By (8), $\pi(p^*|\hat{F}) = \pi_{\hat{\lambda}_0}(p^*)$. By (9) and the equilibrium condition that $\pi(p^*|\hat{F}) = 0$, we have

$$b\int_{p^*}^{\bar{\nu}}(p^* - c\nu)\mathrm{d}\nu = 0. \tag{10}$$

For $j = L$, we apply Lemma 1 with $\hat{\lambda} = \hat{\lambda}_L$. By (9) and the equilibrium condition that $\pi_{\hat{\lambda}_L}(p_L^*) = 0$, we have

$$b\int_{p_L^*}^{\bar{\nu}}(p_L^* - c\nu)\hat{\lambda}_L(\nu)\mathrm{d}\nu = 0. \tag{11}$$

As $\hat{\lambda}_L(\nu)$ is positive, by Lemma 1(a), $\pi_{\hat{\lambda}_L}(p)$ is strictly increasing on $[0, p^*]$. By (9), we have

$$\pi_{\hat{\lambda}_L}(0) = b\int_{p^*}^{\bar{\nu}}(0 - c\nu)\hat{\lambda}_L(\nu)\mathrm{d}\nu < 0.$$

Next, we have

$$\begin{aligned}\pi_{\hat{\lambda}_L}(p^*) = b\int_{p^*}^{\bar{\nu}}(p^* - c\nu)\hat{\lambda}_L(\nu)\mathrm{d}\nu &> b\int_{p^*}^{\bar{\nu}}(p^* - c\nu)\hat{\lambda}_L(\min\{p^*/c, \bar{\nu}\})\mathrm{d}\nu \\ = \hat{\lambda}_L(\min\{p^*/c, \bar{\nu}\})b\int_{p^*}^{\bar{\nu}}(p^* - c\nu)\mathrm{d}\nu &= 0,\end{aligned} \tag{12}$$

where the first equality is by (9), the inequality is because $\hat{\lambda}_L(\nu) > \hat{\lambda}_L(\min\{p^*/c, \bar{\nu}\})$ when $c\nu < p^*$, and $\hat{\lambda}_L(\nu) < \hat{\lambda}_L(\min\{p^*/c, \bar{\nu}\})$ when $c\nu > p^*$, the second equality is by rearrangement, and the third equality is by (10). We thus obtain that $\pi_{\hat{\lambda}_L}$ is strictly increasing on $[0, p^*]$, with $\pi_{\hat{\lambda}_L}(0) < 0 \leq \pi_{\hat{\lambda}_L}(p^*)$, implying that $p_L^* \in (0, p^*)$.

For $j = H$, we apply Lemma 1 with $\hat{\lambda} = \hat{\lambda}_H$. By (9) and the equilibrium condition that $\pi_{\hat{\lambda}_H}(p_H^*) = 0$, we have

$$b\int_{p_H^*}^{\bar{\nu}}(p_H^* - c\nu)\hat{\lambda}_H(\nu)\mathrm{d}\nu = 0. \tag{13}$$

As $\hat{\lambda}_H(\nu)$ is positive and strictly increasing, by Lemma 1(b), $\pi_{\hat{\lambda}_H}(p)$ is strictly quasiconcave on $[0, \bar{\nu}]$. By (8), we have $\pi_{\hat{\lambda}_H}(\bar{\nu}) = 0$. Next, symmetrically to (12), we have

$$\pi_{\hat{\lambda}_H}(p^*) = b\int_{p^*}^{\bar{\nu}}(p^* - c\nu)\hat{\lambda}_H(\nu)\mathrm{d}\nu < 0.$$

We thus obtain that $\pi_{\hat{\lambda}_H}$ is strictly quasiconcave on $[0, \bar{\nu}]$, with $\pi_{\hat{\lambda}_H}(p^*) < 0 = \pi_{\hat{\lambda}_H}(\bar{\nu})$, implying that $p_H^* \in (p^*, \bar{\nu}]$.

Summarizing the above, we obtain

$$0 < p_L^* < p^* < p_H^* \leq \bar{\nu}. \tag{14}$$

Finally, we have

$$W_t(\hat{F}) - W_0(\hat{F}) =$$

$$= \int_{p_H^*}^{\bar{\nu}} (\nu - p_H^*)\hat{\lambda}_H(\nu)\hat{F}(\mathrm{d}\nu) + \int_{p_L^*}^{\bar{\nu}} (\nu - p_L^*)\hat{\lambda}_L(\nu)\hat{F}(\mathrm{d}\nu) - \int_{p^*}^{\bar{\nu}} (\nu - p^*)\hat{F}(\mathrm{d}\nu)$$

$$= b\left(\int_{p_H^*}^{\bar{\nu}} (\nu - p_H^*)\hat{\lambda}_H(\nu)\mathrm{d}\nu + \int_{p^*}^{\bar{\nu}} (\nu - p_L^*)\hat{\lambda}_L(\nu)\mathrm{d}\nu - \int_{p^*}^{\bar{\nu}} (\nu - p^*)\mathrm{d}\nu\right)$$

$$= b\left(\int_{p_H^*}^{\bar{\nu}} (\nu - c\nu)\hat{\lambda}_H(\nu)\mathrm{d}\nu + \int_{p^*}^{\bar{\nu}} (\nu - c\nu)\hat{\lambda}_L(\nu)\mathrm{d}\nu - \int_{p^*}^{\bar{\nu}} (\nu - c\nu)\mathrm{d}\nu\right)$$

$$= b(1-c)\left(\int_{p^*}^{\bar{\nu}} \nu(\hat{\lambda}_H(\nu) + \hat{\lambda}_L(\nu) - 1)\mathrm{d}\nu - \int_{p^*}^{p_H^*} \nu\hat{\lambda}_H(\nu)\mathrm{d}\nu\right)$$

$$= -b(1-c)\int_{p^*}^{p_H^*} \nu\hat{\lambda}_H(\nu)\mathrm{d}\nu < 0.$$

The first equality is by (5) and (6); the second equality is by (14) and because $\hat{F}$ has support on $\{0\} \cup [p^*, \bar{\nu}]$ with density b on $[p^*, \bar{\nu}]$; the third equality is by (10), (11), and (13); the fourth equality by rearrangement; the fifth equality is by definition of $\hat{\lambda}_H$ and $\hat{\lambda}_L$; and the inequality is because $\hat{\lambda}_H$ is strictly increasing, and $b > 0$ and $1 - c > 0$ by (7). *Q.E.D.*

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