

Dynamically Consistent Intergenerational Welfare

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Abstract

Dynamic consistency is crucial for credible evaluation of intergenerational choice plans that inherently lack commitment. We offer a general characterization for dynamically consistent intergenerational welfare aggregation. The aggregation is characterized by envy-guilt asymmetry in discounting with respect to future generations' utility: Higher utility than future generations' utility is discounted differently than lower utility than future generations' utility. This offers a simple and tractable characterization for the dynamically consistent choice rules.

Keywords: Dynamic consistency, discounting, social discount factor, preference aggregation

1 Introduction

The most used model of intertemporal choice is the exponential discounted utility that was proposed by Samuelson (1937). This evaluates a utility stream (x_0, x_1, \dots) with the recursive formulation

$$V_t(x_t, x_{t+1}, \dots) = x_t + \delta(V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t) = (1 - \delta) \sum_{t'=t}^{\infty} \delta^{t'-t} x_{t'}$$

where $\delta \in (0, 1)$ is the discount factor. This model is analytically and axiomatically tractable (Koopmans, 1960) and has become the standard model for intertemporal choice.

However, applying exponential discounting to an intergenerational choice problem is difficult since the evaluation of utility streams depends significantly on the exact discount rate. Even small changes in the discount factor can lead to significantly different evaluations. This problem has been raised by Weitzman (2001) who observed that there is a significant amount of disagreement on the discount factor even among experts.

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The intergenerational choice problem is additionally difficult due to the inherent instability of the choice plan across different generations. Each generation can reevaluate the choice plan since there is no intergenerational commitment to the plan. Hence, the dynamic consistency of the choice plan is crucial for its credibility. However, the previous literature (Zuber, 2011; Jackson and Yariv, 2015) has shown that dynamic consistency and unanimity, that is Pareto efficiency, on the possible discounted utilities lead to a dictatorial choice rule.

Here, we relax the assumption of unanimity on every possible discounted utility to unanimity for every generation. We offer a general characterization for these decision rules under dynamic consistency: These decision rules are characterized by envy or guilt of each generation for future generations as in Fehr and Schmidt's (1999) other-regarding preferences. Here, higher utility than future generations' utility is discounted differently than lower utility than future generations' utility. Formally, a utility stream (x_0, x_1, \dots) is evaluated with the recursive formulation

$$\begin{aligned} &V_t(x_t, x_{t+1}, \dots) && (1) \\ &= x_t + \delta_t^+ \min \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\} + \delta_t^- \max \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\} \end{aligned}$$

where the first term captures the selfish utility, the second term captures the guilt for a higher utility than future generations that is discounted with $\delta_t^+ \in (0, 1)$, and the third term captures the envy for a lower utility than future generations that is discounted with $\delta_t^- \in (0, 1)$. The recursive solution $V_0(x_0, x_1, \dots)$ captures the value of the utility stream.

More specifically, we consider a social order over utility streams (x_0, x_1, \dots) where the indices represent generations. We make three main assumptions. The first assumption is the co-cardinality axiom that specifies the degree of comparability across different generations' utilities. It roughly assumes that the utility of different generations is measured in the same units and is comparable. Co-cardinality is a standard idea in the literature on welfare economics and social choice (it was introduced by d'Aspremont and Gevers (1977), Sen (1979), and Chambers and Echenique (2018)). Second, we assume that the social order respects unanimous improvements for every generation. That is the choice rule follows Pareto efficiency across the generations. Third, we assume that the social order is dynamically

consistent. This assumes that the future generations will not want to reevaluate the choice rule.

Under standard continuity assumptions, we show that each generation t has envy-guilt discount factors $\delta_t^+, \delta_t^- \in (0, 1)$ that satisfy (1) and the recursive solution V_0 represents the social order. This result provides a general characterization for aggregating utility streams over multiple generations under dynamic consistency.

Additionally, we consider stationary social orders as introduced in Koopmans (1960). The stationarity axiom captures that the passage of time does not affect the preferences and so the discounting is the same for each generation. We show that under stationarity, the social order is convex or concave. That is, there is uncertainty about the discount factor and the social order is uncertainty averse or uncertainty loving towards the uncertainty. Formally, there exist discount factors $\delta^1 \leq \delta^2$ such that each utility stream (x_0, x_1, \dots) is evaluated with the recursive function

$$V(x_0, x_1, \dots) = \min_{\delta \in [\delta^1, \delta^2]} (1 - \delta)x_0 + \delta V(x_1, x_2, \dots)$$

or each utility stream (x_0, x_1, \dots) is evaluated with the recursive function

$$V(x_0, x_1, \dots) = \max_{\delta \in [\delta^1, \delta^2]} (1 - \delta)x_0 + \delta V(x_1, x_2, \dots).$$

This result highlights a contrast to the multi-utilitarian choice rule as proposed by Chambers and Echenique (2018). In the multi-utilitarian choice rule, there exists a collection Σ of probability measures on discount factors in $(0, 1)$ such that the utility stream (x_0, x_1, \dots) is evaluated using

$$\min_{\mu \in \Sigma} \int_{(0,1)} \left((1 - \delta) \sum_{t=0}^{\infty} \delta^t x_t \right) \mu(d\delta).$$

We show that in order to maintain dynamic consistency, we need to change the order of summation and minimization. This considers each discount factor for each generation independently of the previous generations.

Our results contribute to the literature on intergenerational choice. Previously, Zuber (2011) and Jackson and Yariv (2015) have shown that preferences of exponential discounted utility maximizers cannot be aggregated while satisfying Pareto efficiency for the exponential discounters and dynamic consistency unless the choice rule is a dictatorship. Here, we relax

the Pareto efficiency to each generation to circumvent this impossibility result. Chambers and Echenique (2018) provides a characterization for dynamically inconsistent choice rules under Pareto efficiency for a set of exponential discounters.

Feng and Ke (2018) studies a relaxation of Pareto efficiency to apply only to future generations when the planner and each generation uses exponentially discounted utility. They show that in this case, the social planner is not necessarily dictatorial. We relax the Pareto efficiency further and provide a full characterization for dynamically consistent choice rules.

Technically, our results apply the results from the context of intertemporal choice in Mononen (2024) to intergenerational choice with preferences over utility streams. These type of preferences have been studied in the intertemporal context also in Wakai (2008) and from a programming perspective in Drugeon et al. (2019). The envy-guilt asymmetry is based on Fehr and Schmidt's (1999) other-regarding preferences that were axiomatized in Rohde (2010) but in our model, the reference utility is the future generations' utility.

The remainder of the paper proceeds as follows: Section 2.1 axiomatizes the dynamically consistent intergenerational welfare aggregation. Section 2.2 considers a stationary version of this model. Section 3 concludes. The proofs for the results are in Section 4.

2 Model

We use the setting from Chambers and Echenique (2018). The objects of choice are bounded utility streams of $x = (x_t)_{t=0}^{\infty}$ of real numbers. The space of all utility streams is ℓ^{∞} . Here, the indices $t = 0, 1, 2, \dots$ are referred to as generations and the interpretation of x_t is the utility that generation t obtains with stream x . When $\theta \in \mathbb{R}$ is a scalar, we often abuse notation and use θ to denote the constant sequence (θ, θ, \dots) . Our primitive is a binary relation \succsim on ℓ^{∞} . As usual, \succ and \sim denote the asymmetric and symmetric parts of \succsim respectively.

For $a, x \in \ell^{\infty}, t \in \mathbb{N}$, $(a_0, \dots, a_{t-1}, x_t, x_{t+1}, \dots)$ denotes the utility stream where the utility in the generations $t' < t$ is $a_{t'}$ and in the generations $t' \geq t$ is $x_{t'}$.

Remark Our approach to modeling social order directly on utility streams captures the debate for intergenerational preference aggregation for example in climate change. Here,

the disagreement is largely about how to discount the utility streams and not about how to measure the outcomes in utilities (Varian, 2006; Nordhaus, 2007; Chambers and Echenique, 2018).

2.1 Dynamically Consistent Intergenerational Welfare

Our first axioms are standard axioms that \succsim is a weak order and satisfies continuity.

Axiom 1 (Weak Order) \succsim is complete and transitive.

Axiom 2 (Continuity) For each $x, y \in \ell^\infty$ such that $x \succ y$, there exists $\theta > 0$ such that

$$x - \theta \succ y \text{ and } x \succ y + \theta.$$

The next axiom captures that the social order is over utility streams. It assumes that the utilities across generations are comparable and that the social order is invariant to positive affine transformations of utilities. This axiom was discussed extensively in Chambers and Echenique (2018).

Axiom 3 (Co-Cardinality) For each $x, y \in \ell^\infty$, scalar $\eta > 0$, and constant sequence θ ,

$$x \succsim y \iff \eta x + \theta \succsim \eta y + \theta.$$

The next axiom assumes that the preferences respect unanimous improvements for every generation. This axiom is the major difference to the previous literature. The previous literature has assumed stronger unanimity with respect to a set of discount factors.¹ However, this Pareto efficiency for a set of discount factors is restrictive under dynamic consistency as observed in Zuber (2011) and Jackson and Yariv (2015).

Axiom 4 (Generation-wise Unanimity) For all $x, y \in \ell^\infty$, if for all $t \in \mathbb{N}$, $x_t \geq y_t$ and for some $t' \in \mathbb{N}$, $x_{t'} > y_{t'}$, then $x \succ y$.

Next, we assume the history independence axiom from Zuber (2011). This is the dynamic consistency axiom from Epstein and Schneider (2003) when applied to our setting. This guarantees that temporal choices are time-consistent.²

¹Formally, there is a set $D \subset (0, 1)$ of possible discount factors such that for all $x, y \in \ell^\infty$ if for all $\delta \in D$, $\sum_t \delta^t x_t \geq \sum_t \delta^t y_t$, then $x \succsim y$.

²This is a weaker condition than the time consistency axiom in Jackson and Yariv (2015) that assumes independence for any common utility at any generation.

Axiom 5 (History Independence) For all $t \in \mathbb{N}, a, b, x, y \in \ell^\infty$,

$$\begin{aligned} & (a_0, \dots, a_{t-1}, x_t, x_{t+1}, \dots) \succsim (a_0, \dots, a_{t-1}, y_t, y_{t+1}, \dots) \\ \iff & (b_0, \dots, b_{t-1}, x_t, x_{t+1}, \dots) \succsim (b_0, \dots, b_{t-1}, y_t, y_{t+1}, \dots). \end{aligned}$$

Finally, we assume the monotone continuity axiom from Villegas (1964), Arrow (1966), and Chateauneuf et al. (2005) adapted to utility streams. This axiom states that the limit of the utility stream is not given a positive weight. Relaxing this axiom has been studied in Drugeon and Ha Huy (2022).

Axiom 6 (Monotone Continuity) For all $x, y, z \in \ell^\infty$, if $x \succ y$, then there exists $t \in \mathbb{N}$ such that

$$(x_0, \dots, x_{t-1}, z_t, z_{t+1}, \dots) \succ y \text{ and } x \succ (y_0, \dots, y_{t-1}, z_t, z_{t+1}, \dots).$$

These six axioms characterize the envy-guilt asymmetry.

Theorem 1 (Dynamically Consistent Intergenerational Welfare) \succsim satisfies Axioms 1-6 iff. there exist for each $t \in \mathbb{N}$, $\delta_t^+, \delta_t^- \in (0, 1)$ such that $\prod_{i=0}^\infty \max\{\delta_i^+, \delta_i^-\} = 0$, for each $t \in \mathbb{N}$ and $x \in \ell^\infty$, we have a recursive function

$$V_t(x_t, x_{t+1}, \dots) = x_t + \delta_t^+ \min \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\} + \delta_t^- \max \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\}$$

with $\limsup_{t \rightarrow \infty} |V_t(x_t, x_{t+1}, \dots)| < \infty$ and the recursive solution V_0 represents \succsim .

Additionally, $(\delta_t^+, \delta_t^-)_{t \in \mathbb{N}}$ are unique.

Here, the restrictions $\prod_{i=1}^\infty \max\{\delta_i^+, \delta_i^-\} = 0$ and $\limsup_{t \rightarrow \infty} |V_t(f_t, f_{t+1}, \dots)| < \infty$ capture Axiom 6 and that the recursive formulation has a convergent solution.

In this representation, each generation has envy-guilt asymmetry. Higher utility than future generations' utility is discounted differently than lower utility than future generations' utility. This result shows that the general dynamically consistent intergenerational welfare aggregation has a simple and tractable formulation only as envy-guilt asymmetry.

Remark In this theorem, the condition $\limsup_{t \rightarrow \infty} |V_t(f_t, f_{t+1}, \dots)| < \infty$ is equivalent to the following two conditions: monotonicity of each V_t , for all $x, y \in \ell^\infty$, if for all $t' \in \mathbb{N}$, $x_{t'} \geq y_{t'}$, then $V_t(x_t, x_{t+1}, \dots) \geq V_t(y_t, y_{t+1}, \dots)$, and normalization of each V_t , for all $\theta \in \mathbb{R}$, $V_t(\theta, \theta, \dots) = \theta$.

2.2 Stationarity

Next, we strengthen the history independence axiom with the stationarity axiom from Koopmans (1960).

Axiom 7 (Stationarity) Let $\theta \in \mathbb{R}$ and $x, y \in \ell^\infty$. Then

$$(x_0, x_1, \dots) \succsim (y_0, y_1, \dots) \iff (\theta, x_0, x_1, \dots) \succsim (\theta, y_0, y_1, \dots).$$

The stationarity axiom captures that the passage of time does not affect the preferences and so the discounting is the same at each generation. The next result shows that this gives concavity or convexity of preferences.

Theorem 2 (Stationary Intergenerational Welfare) \succsim satisfies Axioms 1-4 and 7 iff. there exist $\delta^1, \delta^2 \in (0, 1)$ with $\delta^1 \leq \delta^2$ such that there exists a recursive function V defined by for each $x \in \ell^\infty$,

$$V(x_0, x_1, \dots) = \min_{\delta \in [\delta^1, \delta^2]} (1 - \delta)x_0 + \delta V(x_1, x_2, \dots)$$

or for each $x \in \ell^\infty$,

$$V(x_0, x_1, \dots) = \max_{\delta \in [\delta^1, \delta^2]} (1 - \delta)x_0 + \delta V(x_1, x_2, \dots)$$

with $\limsup_{t \rightarrow \infty} |V(x_t, x_{t+1}, \dots)| < \infty$ such that V represents \succsim .

The connection to our previous theorem is that an alternative way to write the recursive formula is in the case of min operator

$$V(x_0, x_1, \dots) = x_0 + \delta^2 \min \{V(x_1, x_2, \dots) - x_0, 0\} + \delta^1 \max \{V(x_1, x_2, \dots) - x_0, 0\}$$

and similarly in the case of max operator.

This offers a simple and tractable characterization with only two parameters for the general stationary intergenerational welfare aggregation. This result generalizes the exponentially discounted utility and allows for uncertainty on the exact discount factor while maintaining its tractability and dynamic consistency. This model has been studied from a programming perspective in Drugeon et al. (2019).

Remark In this theorem, the condition $\limsup_{t \rightarrow \infty} |V(f_t, f_{t+1}, \dots)| < \infty$ is equivalent to the following monotonicity condition: for all $x, y \in \ell^\infty$, if for all $t \in \mathbb{N}$, $x_t \geq y_t$, then $V(x_0, x_1, \dots) \geq V(y_0, y_1, \dots)$.

3 Conclusion

When considering intergenerational choices, dynamic consistency is crucial for the credibility of the choice policy. Without dynamic consistency, future generations might reevaluate the choice policy since there is no intergenerational commitment to the policy. However, the previous literature (Zuber, 2011; Jackson and Yariv, 2015) has highlighted the tension between dynamic consistency and Pareto efficiency.

In this paper, we relaxed Pareto efficiency to apply for all generations. We showed that under dynamic consistency, these choice rules are characterized by envy-guilt asymmetry in discounting. This offers a tractable and simple model for intergenerational welfare that has been studied from a programming perspective in Drugeon et al. (2019).

Another alternative would be to relax the comparability of utilities, co-cardinality axiom, following Dong-Xuan and Bich (2024). Under this relaxation, it remains unknown if Pareto efficiency for a set of exponential discounters and dynamic consistency still leads to a dictatorial choice rule and what is the general dynamic consistent characterization. Additionally, it remains an open question on how the envy and guilt discount factors should be related to individuals' discount factors.

4 Proofs

Proof of Theorem 1. First, by Axioms 2 and 4, we have for all $x, y \in \ell^\infty$ with for all t , $x_t \geq y_t$, $x \succsim y$.

Second, by Axiom 3, we have for all for all $x, y \in \ell^\infty$, $\alpha \in (0, 1)$, $\theta \in \mathbb{R}$,

$$x \succsim y \iff \alpha x + (1 - \alpha)\theta \succsim \alpha y + (1 - \alpha)\theta.$$

Hence, by Mononen (2024, Theorem 1), there exist an affine $u : \mathbb{R} \rightarrow \mathbb{R}$ and for each $t \in \mathbb{N}$, $\delta_t^+, \delta_t^- \in (0, 1)$ such that $\prod_{t=1}^\infty \max\{\delta_t^+, \delta_t^-\} = 0$, for each $t \in \mathbb{N}$,

$$V_t(x_t, x_{t+1}, \dots) = x_t + \delta_t^+ \min \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\} + \delta_t^- \max \left\{ V_{t+1}(x_{t+1}, x_{t+2}, \dots) - x_t, 0 \right\}$$

and the recursive solution V_0 represents \succsim .

Since u is affine, there exists $a > 0, b \in \mathbb{R}$ such that for all $x \in \mathbb{R}$ $u(x) = ax + b$. Thus Theorem 1 follows from the uniqueness result in Mononen (2024, Theorem 1). \square

The proof of Theorem 2 follows symmetrically from Mononen (2024, Theorem 2).

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