

# Should Cities Diversify? City Risk and Industrial Policy

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February 2024

## Abstract

We explore the diversification of an urban economy where the labor specialization choices of its residents determine the city's exposure to sectoral shocks. The presence of demand-driven externalities introduces the possibility of city-wide coordination failures. Residents, when making their specialization choices, do not account for the costs of these coordination failures, and as a result, the equilibrium level of diversification is inefficient. The optimal policies that address these externalities depend on the city's economic condition, with prosperous urban economies deriving a greater benefit from fostering diversification. Thus, the paper rationalizes the widespread industrial policies that in some cases promote diversifying, while in others, specializing a city's economy.

*Keywords: City diversification, industrial policy, city risk, coordination failures.*

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# 1 Introduction

Should urban policymakers promote economic diversification? While the benefits of developing industry clusters are often discussed by both policymakers and academics, the potential risk-reduction benefits of industrial diversification have received little attention in the economics literature, despite being frequently discussed by policymakers.<sup>1</sup> For instance, the Office of the New York City Comptroller expressed concern about the overreliance of the City’s economy on the finance industry in 2017.<sup>2</sup> More recently, in a 2021 New York Times opinion article, former New York’s City Mayor Michael Bloomberg argued that “[W]hatever policies the next mayor pursues, the crucial idea is that putting a city back on its feet economically requires more than aiding existing businesses. It requires creating the conditions for new ones to open and expand, further diversifying the economy.” This paper studies the costs and benefits of policies that favor industrial diversification to assess when these policies improve welfare and when they do not.

To explore these issues, we model a city which is a small part of a system of cities that constitute the aggregate economy. Location plays a role in this economy for two reasons. The first is that workers cannot quickly respond to sector-specific shocks by moving to more favorable locations. The second is that in addition to traded goods, that are transported freely between cities, city residents produce and consume non-traded goods, such as restaurant meals, entertainment, education, and health services, which must be produced in the city where they live. Shocks to traded-good prices create uncertainty in the model. These sectoral shocks, which affect the city’s revenue from producing traded goods, are transmitted to non-traded-good prices, e.g., the price of real estate in a city increases when the prices of the traded goods it produces increases. Because of the transmissions of these shocks, city residents are exposed to risks that we assume cannot be hedged. Specifically, markets are incomplete in that city residents cannot trade claims on either traded goods or real estate with residents in

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<sup>1</sup>It is noteworthy that, [Duranton and Venables \(2021\)](#) and [Juhasz, Lane and Rodrika \(2023\)](#), two recent articles discussing the motivations for place-based and industrial policies, respectively, do not mention risk.

<sup>2</sup>See [https://comptroller.nyc.gov/wp-content/uploads/documents/NYC\\_Economy\\_Diversified.pdf](https://comptroller.nyc.gov/wp-content/uploads/documents/NYC_Economy_Diversified.pdf).

other cities.<sup>3</sup>

The set of traded-good industries in a city – a city’s industrial base – is determined by the labor choices of its residents, which in turn, determine a city’s exposure to sectoral shocks. We assume that each city has a comparative advantage in the production of one particular traded-good. There is thus an exogenous force towards specialization: since traded goods can be transported freely between cities, production may be more efficient if each city specializes in producing the traded good in which it has a comparative advantage. There is also a benefit to diversification, even when the city’s residents are risk neutral. The benefit to diversification arises because some of the non-traded goods, such as real estate, are in fixed supply or more generally, they have a low supply elasticity. Intuitively, if a city produces or acquires 10% more traded goods, the utility of a representative risk-neutral resident increases less than 10% because the consumption of non-traded-goods cannot be proportionally adjusted. This implies that the residents of a city receive higher utility on average when the city’s income from traded goods is less volatile, that is, city residents benefit from diversification.

This benefit of diversification does not by itself justify policy interventions that favor specific industries. Indeed, in our benchmark setting, where non-traded-goods are in fixed supply, the equilibrium industrial base is socially optimal. In this equilibrium, some city residents may be willing to work in the sector that is less productive on average because by doing so, they generate relatively more income when the price of real estate and other non-traded goods in the city are lower.

When we extend the model by allowing the supply of non-traded goods to adjust to changing market conditions, the equilibrium level of diversification may no longer be socially optimal. Specifically, we assume that non-traded goods are produced by entrepreneurial city residents who increase production by exerting costly effort when the demand for non-traded goods is sufficiently high. In this setting, demand-driven complementarities exist in the production of non-traded goods – the incentives of entrepreneurs to produce and consume non-traded goods increases when other entrepreneurs produce and consume more. These

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<sup>3</sup>Section 5.1 discusses the implications of financial developments which complete markets.

complementarities create the possibility of a coordination failure, wherein all non-traded-good entrepreneurs produce at inefficiently low levels.

As we show, cities fall into coordination failure states when the income from its traded-good sector is sufficiently low. Intuitively, the decrease in demand for non-traded goods, triggered by a decline in income from the traded-good sector, is amplified by an endogenous drop in the demand within the non-traded sector. To understand this, suppose the price of software produced in Seattle experiences a large decline. Such a decline can lead to the closure of some restaurants that programmers frequent, causing both restaurateurs and programmers to attend fewer movies. This, in turn, results in the closure of cinemas, further reducing the demand for restaurants, and so forth.

Because workers fail to internalize the effect that their specialization choices have on the possibility of such a city-wide coordination failure, the equilibrium industrial base is not socially optimal. The optimal policy addressing this externality depends on the city's economic condition. Specifically, a city with a profitable traded-good sector that supports a "vibrant" non-traded-good sector, characterized by high-effort entrepreneurs, may benefit from subsidizing diversification. This subsidy reduces the likelihood of a negative sectoral shock triggering a coordination failure, thereby preserving the vibrancy of the city's non-traded-good sector. Conversely, a city with a failing traded-good sector, already suffering from a coordination failure, derives fewer benefits from industrial diversification. In such cases, policy choices that encourage a more focused industrial base may be preferred. Intuitively, such a city can potentially benefit from promoting one traded-good sector that acts as a catalyst to activate its economy.<sup>4</sup>

While there is a number of literatures that address related issues, we believe we are the first to explicitly model the risk-reduction benefits of urban diversification. The urban economics literature typically stresses economies of scale and scope driving a wedge between equilibrium outcomes and social optima to justify policies that promote either specialization

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<sup>4</sup>For example, [Glaeser \(2005\)](#) accounts for Boston's historical periods of success by referring to the success of a main traded-good activity: over centuries, Boston has transformed itself from a shipping and sailing city to a manufacturing city, and more recently, to an information-based city.

or diversification.<sup>5</sup> For instance, externalities from clustering (see [Henderson \(1974\)](#) and [Abdel-Rahman and Fujita \(1990\)](#)) can create a rationale for subsidizing specialization, while externalities from economies of scope (see [Abdel-Rahman \(1990\)](#) and [Abdel-Rahman and Fujita \(1993\)](#)) can create a rationale subsidizing diversification. In this literature, however, city risk is notably absent, despite the fact that promoting diversification as a strategy to mitigate risk is a key objective in the industrial policies of numerous municipalities and regions, e.g., [Figueiredo, Honiden, and Schumann \(2018\)](#).

There is an urban planning and economic geography literature that describes higher risk as an unfortunate consequence of an increase in urban focus. Borrowing from the finance literature, this literature views industrial diversification as a portfolio choice, which trades off risk and return. (See [Conroy, 1974](#), for an early example of this approach.) The literature has also documented a relation between industrial diversity and economic resilience (see [Brown and Greenbaum, 2017](#), for a recent example). As we show, risk reduction, by itself, does not rationalize the need for industrial policy, but we provide micro-foundations for a market failure based on demand externalities that does.

While the role diversification plays in our model is new, it is related to what [Krugman \(1991\)](#) characterizes as the benefits of labor pooling. This is the idea that firms that are exposed to different shocks may benefit from locating close to each other since the laid off workers at a firm experiencing a negative shock may have opportunities to work at a neighboring firm that may have experienced a positive shock. Our model abstracts from the benefits of labor pooling, since we assume that workers do not change jobs. However, non-traded-good workers realize a benefit from what we would characterize as effective labor pooling. In particular, although restaurant workers do not change jobs, they benefit from being located in a more diversified economy that allows them to serve more meals to programmers when the demand for software is higher and more meals to auto workers when the demand for autos is higher.

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<sup>5</sup>For overviews of this extensive literature see, for example, [Fujita, Krugman and Venables \(1999\)](#), [Duranton and Puga \(2000\)](#), [Fujita and Thisse \(2002\)](#), [Henderson and Thisse \(2004\)](#), and [Duranton, Henderson and Strange \(2015\)](#).

Our model builds on literature examining the potential for economy-wide coordination failures.<sup>6</sup> The common feature in these papers is the existence of multiple self-fulfilling equilibria that can be Pareto ranked. To overcome this indeterminacy stemming from multiple equilibria, we follow [Carlsson and Van Damme \(1993\)](#), which shows that introducing dispersed information in games of strategic complementarities can lead to a unique equilibrium pinned down by the realization of an underlying economic fundamental—in our case, the city’s income from its traded-good sector.<sup>7</sup> Intuitively, a city suffers a coordination failure when the income from its traded-good sector drops, leading to insufficient demand for non-traded goods within the city.<sup>8</sup> We contribute to this literature by endogenizing the ex-ante diversification choices that determine the volatility of the underlying economic fundamental, which ultimately, determines the probability of a coordination failure.

While our focus is on the diversification of cities, our analysis is also related to papers that explore the importance of systematic sources of risk at more aggregate levels. For example, [Acemoglu and Zilibotti \(1997\)](#) studies the link between the diversification of countries and growth. They argue that because of indivisibilities and capital scarcity, the competitive equilibrium is inefficient, in that too few risky high-expected-return projects are undertaken. Specifically, the opening of an additional sector creates a positive pecuniary externality on other potential projects since it allows consumers to bear less risk when they buy securities. Our paper also generates a link between diversification and growth since the coordination failures in our model can affect economic growth. However, unlike [Acemoglu and Zilibotti \(1997\)](#), in which diversification always creates a positive pecuniary externality, in our case, the city can feature excessive specialization or excessive diversification,

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<sup>6</sup>See [Diamond \(1982\)](#), [Hart \(1982\)](#), [Weitzman \(1982\)](#), [Kiyotaki \(1988\)](#), and [Cooper and John \(1988\)](#) for seminal papers in the literature.

<sup>7</sup>See [Carlsson and Van Damme \(1993\)](#) for the seminal global-games paper, and [Chamley \(1999\)](#) and [Bebchuk and Goldstein \(2011\)](#) for applications to economies in which investment decisions exhibit a generic form of complementarity.

<sup>8</sup>The importance of the income generated by the traded-good sector for the demand for non-traded goods in our model is consistent with the documented local multiplier effect of attracting new firms to a city. For instance, [Moretti \(2010\)](#) finds that adding one skilled manufacturing job in the city creates 2.5 jobs in its non-traded-good sector.

depending on its economic condition. Our channel, which does not rely on risk aversion, is also distinctly different: individuals do not internalize that their combined specialization choices determine the city's aggregate risk, and hence the probability of the city's economy falling into an equilibrium with a depressed level of economy activity.

Finally, our paper relates to current policy debates regarding the need to promote supply chain resilience and trade diversification, e.g., [Grossman, Helpman and Lhuillier \(2023\)](#).<sup>9</sup> Specifically, the model can be repositioned to explore the costs and benefits of diversifying the suppliers to the city's industrial base. Supply chain disruptions would then generate uncertainty regarding the city's traded-good-sector income and impact the likelihood of city-wide coordination failures.

## 2 Model

We consider a city that is a small part of a system of cities that constitute the aggregate economy. City residents are characterized by the goods that they produce. A portion of the residents produce traded consumption goods, like computers or automobiles, that can be consumed within the city or transported to be sold and consumed by residents in other cities. The city residents that produce traded goods choose the particular traded-good sector in which to specialize their labor and after making that choice, devote an exogenous unit of labor to the activity. Another portion of the residents produce non-traded goods, like restaurant meals, that can only be consumed within the city. In contrast to workers that produce traded goods, workers that produce non-traded goods do not face a specialization decision. While a city can specialize in the production of a few traded goods and trade to consume other traded goods, all non-traded goods consumed in a city need to be produced by city residents. Finally, there is a portion of residents endowed with real estate that is consumed by local residents. Next we describe in detail each type of city residents and the sectors that they work in.

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<sup>9</sup>[Grossman, Helpman and Lhuillier \(2023\)](#) points out that "See [Helpman and Razin \(1978\)](#) for seminal work on the role of risk-aversion and uncertainty in the theory of international trade.

## 2.1 Traded-Good Sectors

There is a unit interval of traded goods indexed by  $x_t \in [0, 1]$ . These goods are produced and consumed across the economy. Since the city we consider is small with respect to the whole economy, we take the prices of traded goods,  $p_t(x_t)$  for  $x_t \in [0, 1]$ , as exogenous. Specifically, traded goods prices are independent and uniformly distributed on  $(0, \bar{p})$ , and their fluctuations are the sole source of uncertainty in the model. Since there is a continuum of traded-good sectors with i.i.d. prices, a basket with one unit of each traded good has no price uncertainty. We take this basket as the economy's numeraire, and therefore normalize the upper bound  $\bar{p}$  to be equal to 2.

The city has a comparative advantage in the production of traded good  $x_t = 1$ : one unit of labor in the city generates  $\alpha + \delta$  units of traded-good 1, with  $\alpha > 0$  and  $\delta > 0$ . By contrast, in each of the other traded-good sectors  $x_t \in X_N$ , one unit of labor generates  $\delta$  units of the traded good, where  $X_N \subset [0, 1]$  is the set of  $N$  traded-good sectors available for production in the city, in addition to traded-good sector 1.<sup>10</sup> The city's greater productivity in the production of traded-good 1 captures the idea that cities have unique characteristics which provide them with comparative advantages in the production of certain goods. These comparative advantages can originate from their locations (e.g., by a river), their natural resources (e.g., oil) or their access to specific types of knowledge and human capital (e.g., being next to a major university). Note also that the model's production technology rules out economies of scale at the industry level or economies of scope across industries that generate externalities that workers fail to internalize, as there is already an extensive literature exploring the importance of these externalities for industrial policy. Instead, our analysis will focus on externalities that work through the city's aggregate risk, that is, through the city's exposure to sectoral shocks.

The city is populated by a mass  $\bar{L}$  of workers with the skills to produce traded goods.<sup>11</sup>

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<sup>10</sup>We introduce the subset  $X_N$  where  $N$  can be an arbitrarily larger number for technical reasons: having a finite set of traded-good sectors from which workers can choose keeps their specialization problem well-defined.

<sup>11</sup>See [Section 5.2](#) for a discussion of how to endogenize  $\bar{L}$ , and more generally, for the role of migration in the context of the model.



These workers are endowed with one unit of labor, which they supply inelastically, and decide ex-ante (before the traded-good prices are realized) the particular traded-good sector  $x_t \in X_N \cup \{1\}$  in which to specialize their labor. We let  $L(x_t)$  be the mass of workers that choose to specialize in the production of traded good  $x_t$  and refer to  $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$  as the city's industrial base, that is, the portfolio of traded goods that the city produces.<sup>12</sup> The industrial base determines the city's income from the production of traded goods:

$$Y_t = (\delta + \alpha)L(1)p_t(1) + \sum_{x_t \in X_N} \delta L(x_t)p_t(x_t). \quad (1)$$

Note that  $Y_t$  depends on the realization of the traded-goods prices  $\{p_t(x_t)\}_{x_t \in X_N \cup \{1\}}$  and therefore, the city's industrial base governs the city's exposure to these sector-specific shocks. For example, if  $L(x_t) = \bar{L}$ , the city is fully specialized and only exposed to shocks to traded-good sector  $x_t$ .

## 2.2 Non-Traded-Good Sectors and Real Estate

In addition to the traded goods, a separate group of workers produce a unit interval of non-traded goods, indexed by  $x_{nt} \in [0, 1]$ . Unlike the traded-good sectors, the non-traded-good sectors need not differ across cities, e.g., restaurants in Cleveland and Boston can potentially be identical. However, workers can make choices that generate endogenous differences in the non-traded sectors across cities.

Each non-traded good sector is populated with one of two possible types workers. The first type always produces a fixed amount, while the second type is more driven and can potentially increase production. To simplify our analysis, we assume there are exactly two workers in each non-traded-good sector  $x_{nt}$ , one is constrained to produce  $\frac{q_0}{2} > 0$  units of the non-traded good and the other can also produce  $\frac{q_0}{2}$  units of the non-traded good, but has the option to produce an additional  $q_1 > 0$  units of the good by incurring a non-pecuniary fixed cost  $c$ . Hereafter, we refer to workers that can increase production by incurring a non-pecuniary fixed cost  $c$  as

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<sup>12</sup>While the number of traded-good sectors in the city is at most  $N + 1$ , the mass of workers specialized in each of the  $N + 1$  sectors is endogenously determined. Therefore, the city may end up producing fewer than  $N + 1$  different traded goods, that is, in equilibrium, some sectors in  $X_N$  can have  $L(x_t) = 0$ .

entrepreneurs, therefore, each non-traded-good sector has one worker and one entrepreneur.<sup>13</sup> The entrepreneurs' production decisions are made after the traded-good prices are realized, and therefore, after the city's income from the production of traded goods  $Y_t$  becomes known.<sup>14</sup>

While the demand for traded goods is determined at the economy-wide level, the demand for non-traded goods is entirely determined at the city level. Consequently, the prices of non-traded goods,  $\{p_{nt}(x_{nt})\}_{x_{nt} \in [0,1]}$ , are endogenously determined and will depend on the city's industrial base,  $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$ , the realized prices of the traded goods,  $\{p_t(x_t)\}_{x_t \in [0,1]}$ , and the entrepreneurs' production decisions.

Note that there are differences in our modelling of workers in the traded- and non-traded good sectors. This asymmetry is in part motivated by the need to keep the model tractable, but also captures fundamental differences between the two types of goods. First, we endogenize workers' specialization decisions across traded-good sectors, but assume that workers and entrepreneurs are uniformly distributed throughout the non-traded-good sectors. Underlying this assumption is the idea that a city can specialize in the production of a few traded goods and trade to consume the other traded goods, while all non-traded goods consumed in the city need to be produced locally. Second, non-traded-good entrepreneurs make endogenous production choices that affect the level of output, while traded-good workers do not.<sup>15</sup> As we will show, the fact that the demand for non-traded goods is entirely determined at the city level generates strategic complementarities in the production of non-traded-goods that can lead to multiple equilibria and city-wide coordination failures. These strategic complementarities, however, do not arise in the production of traded-goods since the demand for these goods (and therefore, their prices) are exogenous to the city.<sup>16</sup>

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<sup>13</sup>As it will become clear below, the only role in the model of the worker that produces a fixed amount  $\frac{q_0}{2}$  of no-traded-good  $x_{nt}$  is to make the production decision of the entrepreneur in that sector  $x_{nt}$  a meaningful one.

<sup>14</sup>Even though the production of traded goods is entirely determined by the ex-ante specialization choices, all production (for traded and for non-traded goods) takes places after the traded-good prices are realized.

<sup>15</sup>Traded-good workers make ex-ante (before prices are realized) specialization decisions, but do not make ex-post (after prices are realized) production decisions, that is, they devote their one unit of labor to produce at  $t = 1$  the traded-good in which they have specialized at  $t = 0$ .

<sup>16</sup>If traded-good-workers were to make ex-post (after prices are realized) production decisions that affect their level of output, while non-traded-good entrepreneurs did not, these ex-post production decisions would be strategic substitutes rather than complements, and hence, there would be no scope for coordination failures.

Finally, real estate is a special case of a non-traded good. In contrast to the other non-traded goods, the supply of real estate is fixed. Specifically, we assume there is a mass  $R$  of real estate owners who are each endowed with one unit of real estate. After the realization of the traded-good prices, real estate  $R$  is competitively traded within the city at a market-clearing price  $p_r$ .

### 2.3 Consumption and Timing

Each consumer  $i$  (i.e., workers in the traded-good sectors, workers and entrepreneurs in the non-traded-good sectors, and real estate owners) have a utility function

$$U_i = \exp \left[ \int_0^1 \gamma_t \ln c_{i,t}(x_t) dx_t + \int_0^1 \gamma_{nt} \ln c_{i,nt}(x_{nt}) dx_{nt} + \gamma_r \ln c_{i,r} \right] \quad (2)$$

where  $c_{i,t}(x_t)$  is the consumption of traded good  $x_t \in [0, 1]$  by consumer  $i$ ,  $c_{i,nt}(x_{nt})$  is the consumption of non-traded good  $x_{nt} \in [0, 1]$ , and  $c_{i,r}$  is the consumption of real estate. We assume that  $\gamma_t + \gamma_{nt} + \gamma_r = 1$ , so that the utility function is homogenous of degree one, and the indirect utility function is a linear function of the consumer's income.

Figure 1 below illustrates the timing of events. At  $t = 0$ , workers in the traded-good sectors choose how to specialize their labor. At  $t = 1$ , after the traded-good prices are realized, production, trading, and consumption takes place.

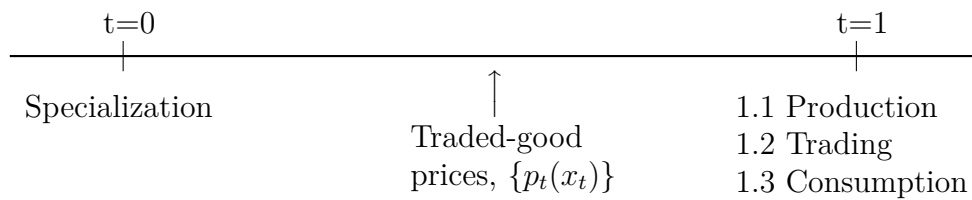


Figure 1. Timing of events.

## 3 Benchmark Case: No Coordination failures.

This section considers a benchmark case in which the only non-traded good is real estate (i.e.,  $\gamma_{nt} = 0$ ). This case allows us to study a city's diversification across traded-good sectors

without introducing the coordination failures in the production of non-traded goods sectors that we consider in [Section 4](#).

### 3.1 Trading and Consumption at $t = 1$

For a given city's industrial base  $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$  chosen at  $t = 0$  and traded-good prices,  $\{p_t(x_t)\}_{x_t \in [0,1]}$  realized at  $t = 1$ , consumer  $i$  with income  $y_i$  maximizes her utility in [eq. 2](#), which for  $\gamma_{nt} = 0$  writes

$$U_i = \exp \left[ \int_0^1 \gamma_t \ln c_{i,t}(x_t) dx_t + \gamma_r \ln c_{i,r} \right] \quad \text{with } \gamma_t + \gamma_r = 1, \quad (3)$$

subject to budget constraint

$$y_i = \int_0^1 p_t(x_t) c_{i,t}(x_t) dx_t + p_r c_{i,r}. \quad (4)$$

From this optimization problem and following standard derivations, consumer  $i$ 's indirect utility function writes

$$v(y_i) = \frac{\gamma_t^{\gamma_t} \gamma_r^{\gamma_r} \times y_i}{P_t^{\gamma_t} p_r^{\gamma_r}}, \quad (5)$$

where the geometric average  $P_t \equiv \exp \left[ \int_0^1 \ln p_t(x_t) dx_{nt} \right]$  is a constant as prices  $\{p_t(x_t)\}_{x_t \in [0,1]}$  are i.i.d. Note that consumers are risk-neutral since  $v(y_i)$  is linear in  $y_i$  for any given set of prices  $\{\{p_t(x_t)\}_{x_t \in [0,1]}, p_r\}$ .

Let  $Y$  be the city's income, that is, the sum of the income from the traded-good sectors,  $Y_t$  (see [eq. 1](#)) and from the real estate sector,  $Y_r = R \times p_r$ . The Cobb-Douglas utility function implies that the city's expenditure in the consumption of traded goods and real estate are  $\gamma_t Y$  and  $\gamma_r Y$ , respectively. Since the income in the real estate sector,  $Y_r = R \times p_r$ , must be equal to the city's expenditure in the consumption of real estate,  $\gamma_r (Y_t + Y_r)$ , it follows that

$$R \times p_r = \frac{\gamma_r}{\gamma_t} Y_t \Rightarrow p_r = \frac{\gamma_r}{\gamma_t} \frac{Y_t}{R}. \quad (6)$$

In words, the price of real estate at  $t = 1$  increases in the city's income from the production of traded goods,  $Y_t$ , and in the share of income spent on the consumption of real estate,  $\gamma_r$ , and decreases in the city's endowment of real estate,  $R$ , and in the share of income spent on

the consumption of traded goods,  $\gamma_t$ .

### 3.2 Equilibrium Industrial Base at $t = 0$

Consider now workers' decisions to specialize in the production of traded goods at  $t = 0$ . For each worker  $i$ , let  $l_i(x_t)$  be an indicator function defined on  $x_t \in X_N \cup \{1\}$  such that  $l_i(x'_t) = 1$  if the worker chooses to specialize in the production of traded good  $x'_t$ , and  $l_i(x'_t) = 0$ , if the worker chooses to specialize in the production of traded good  $x_t \neq x'_t$ . Therefore, the measure of workers that choose to specialize in the production of traded good  $x'_t$  is  $L_i(x'_t) = \int_0^{\bar{L}} l_i(x'_t) di$ .

For a given specialization choice  $l_i(x_t)$ , worker  $i$  obtains income

$$y_i = (\alpha + \delta) l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t) \quad (7)$$

and hence, from eq. 5, has indirect utility

$$v(y_i) = \gamma_t \frac{(\alpha + \delta) l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t)}{P_t^{\gamma_t} \left( \frac{Y_t}{R} \right)^{\gamma_r}}. \quad (8)$$

Note that the indirect utility in eq. 8 depends on  $Y_t$ , and therefore on the city's industrial base  $\{L(x_t)\}_{x_t \in X_N \cup \{1\}}$ . While the city's industrial base is the by-product of all the workers' specialization decisions, workers, being small with respect to the whole city, ignore the combined impact of their individual specialization decisions on the city's industrial base. More specifically, worker  $i$  solves the following optimization problem

$$\max_{\{l_i(x_t)\}_{x_t \in X_N \cup \{1\}}} \mathbb{E}[v(y_i)] \quad (9)$$

where  $l_i(x_t) \in \{0, 1\}$  and  $\sum_{x_t \in X_N \cup \{1\}} l_i(x_t) = 1$ , and the expectation is taken with respect to the traded-good prices,  $\{p_t(x_t)\}_{x_t \in [0,1]}$ . The next proposition characterizes the city's equilibrium industrial base and income from the production of traded goods.

**Proposition 1** *There exists  $\bar{\alpha}_N > 0$ , such that*

1. *if  $\alpha \geq \bar{\alpha}_N$ , the city fully specializes in sector 1:*

$$L^*(1) = \bar{L} \text{ and } Y_t = (\alpha + \delta) p_t(1) \bar{L}.$$

2. if  $\alpha < \bar{\alpha}_N$ , the city diversifies into sectors  $x_t \in X_N$ :

$$L^*(1) \in \left( \frac{\bar{L}}{N+1}, \bar{L} \right) \text{ and } Y_t = (\alpha + \delta) p_t(1) L^*(1) + \sum_{x_t \in X_N} \delta p_t(x_t) \frac{\bar{L} - L^*(1)}{N}.$$

Part (1) of [Proposition 1](#) is straightforward: if the productivity gap  $\alpha$  between traded-good sector 1 and the other traded-good sectors  $x_t \in X_N$  is large enough, all traded-good workers specialize in the production of traded-good 1. Part (2) shows that diversification can arise in equilibrium if the productivity gap  $\alpha$  is not too large, yet strictly positive. In that case, the equilibrium features a mass  $L^*(1) < \bar{L}$  of workers in traded-good sector 1 and an equal mass of workers  $L^*(x_t) = \frac{\bar{L} - L^*(1)}{N}$  in each other traded-good sector  $x_t \in X_N$ . In this equilibrium, each worker is indifferent between specializing in traded-good sector 1 or in any other traded-good sector  $x_t \in X_N$ ,

$$(\alpha + \delta) \mathbb{E}[v(p_t(1))] = \delta \mathbb{E}[v(p_t(x_t))], \quad (10)$$

despite sector 1 generating a higher expected income, i.e.,  $(\alpha + \delta) \mathbb{E}[p_t(1)] > \delta \mathbb{E}[p_t(x_t)]$ . Intuitively, the income from traded-good sector 1 is more correlated with the city's income from producing traded-goods,  $Y_t$ , than the income from any other traded-good sector. This is both because sector 1 is more productive and because a larger share of the workforce works in sector 1 than in any other sector. As a result, real estate prices are also more correlated with the income in traded-good sector 1 than with the income in other traded-good sectors. This makes traded-good sectors other than sector 1 attractive as workers are more likely to take advantage of low real estate prices when the city's income is low.

From [Proposition 1](#), if  $\alpha < \bar{\alpha}_N$ , the city is exposed to shocks to all traded-good sectors in  $X_N$ , but the labor devoted to the production of these traded-good sectors,  $\bar{L} - L^*(1)$ , is well-diversified, that is, an equal mass of workers  $L^*(x_t) = \frac{\bar{L} - L^*(1)}{N}$  choose to specialize in each traded-good sector  $x_t \in X_N$ . Therefore, as  $N$  increases, the city becomes less exposed to these other traded-good sectors  $x_t \in X_N$ , and, at the limit when  $N \rightarrow +\infty$ , the city becomes only exposed to shocks to traded-good sector 1.

**Corollary 1** *As  $N \rightarrow +\infty$ , there is an  $\bar{\alpha}_\infty > 0$ , such that  $Y_t = (\alpha + \delta)p_t(1)L^*(1) + \delta(\bar{L} - L^*(1))$  with  $L^*(1) = \bar{L}$  for  $\alpha \geq \bar{\alpha}_\infty$ , and  $L^*(1) \in (0, \bar{L})$  for  $\alpha < \bar{\alpha}_\infty$ .*

### 3.3 Welfare

We next examine the equilibrium industrial base in [Proposition 1](#) from the social point of view. Since the indirect utility function is linear in  $y_i$ , from [eq. 5](#) and [eq. 6](#), the sum of the utility of all the city's residents can be written as

$$v_s(Y) = \frac{(\gamma_t Y)^{\gamma_t} R^{1-\gamma_t}}{P_t^{\gamma_t}}. \quad (11)$$

where we have used the fact that  $Y_t = \gamma_t Y$ . While each consumer's indirect utility  $v(y_i)$  is linear in her income  $y_i$  (see [eq. 6](#)), the sum of the utility of all the city residents  $v_s(Y)$  is concave in the city's income  $Y$ , i.e.,  $v_s''(Y) < 0$ . At the city level, an increase in the city's income from the production of traded goods  $Y_t$  (and hence, in the city's consumption of traded goods) cannot be met with a proportional increase in the consumption of real estate, because real estate is in fixed supply. Therefore, because the supply of traded goods is perfectly elastic (the city is small with respect to the aggregate economy), while real estate is in fixed supply (or more generally, in less elastic supply), the city's social welfare function is concave with respect to the city's income.

Even though each consumer's indirect utility function  $v(y_i)$  is linear in  $y_i$ , workers internalize the social cost of specialization, that is, the concavity of the social indirect utility function  $v_s(Y)$ , through the positive correlation between the city's income and the price of real estate. This positive correlation decreases the marginal utility of a worker's income in those states in which the city's income is high. In equilibrium, unless  $\alpha > \bar{\alpha}_N$ , in which case we have a corner solution with full city specialization, workers are indifferent between specializing in traded-good sector 1 and specializing in any other traded-good sector. These equilibrium specialization choices are socially optimal, as the next proposition shows.

**Proposition 2** *The equilibrium city's industrial base in [Proposition 1](#) is socially optimal.*

While numerous studies have documented a relation between industrial diversification and

economic resilience (e.g., [Brown and Greenbaum, 2017](#)), [Proposition 2](#) illustrates that having a benefit associated to diversification does not imply the need for active industrial policy, that is, the equilibrium industrial base can be socially optimal. The next section, however, shows that, when the city produces non-traded as well as traded goods, individual choices lead to an equilibrium industrial base that is no longer socially optimal, even under risk-neutrality. Specifically, depending on the city’s economic conditions, the equilibrium industrial can feature excessive specialization or diversification.

## 4 City Risk and Coordination Failures

[Section 3](#) has one non-traded good (real estate) in fixed supply. This section introduces other non-traded goods whose supply can adjust to an increase in demand. We show that production decisions in these non-traded good sectors are strategic complements, which creates scope for coordination failures. We also show that shocks to traded-good sectors influence production decisions in non-traded good sectors, and therefore, also influence the likelihood of a coordination failure. Finally, we analyze how the potential for coordination failures affect the optimality of the equilibrium industrial base.

Concretely, we first examine the production and consumption decisions at  $t = 1$ , conditioned on the specialization decisions at  $t = 0$ . We then consider the  $t = 0$  equilibrium specialization decisions and assess their optimality.

### 4.1 Consumption at $t = 1$

The consumer’s problem is similar to the one in the benchmark case of [Section 3.1](#). For any given realization of the traded-good prices,  $\{p_t(x_t)\}_{x_t \in [0,1]}$ , consider consumption decisions at  $t = 1$ . Consumer  $i$  with income  $y_i$  maximizes her utility in [eq. 2](#) subject to the following budget constraint:

$$y_i = \int_0^1 p_t(x_t) c_{i,t}(x_t) dq_t + \int_0^1 p_{nt}(x_{nt}) c_{i,nt}(x_{nt}) dx_{nt} + p_r c_{i,r}. \quad (12)$$



From this optimization problem, consumer  $i$ 's indirect utility function can be expressed as

$$v(y_i) = \frac{\gamma_t^{\gamma_t} \gamma_{nt}^{\gamma_{nt}} \gamma_r^{\gamma_r} \times y_i}{P_t^{\gamma_t} \exp \left[ \gamma_{nt} \int_0^1 \ln p_{nt}(x_{nt}) dx_{nt} \right] p_r^{\gamma_r}}, \quad (13)$$

which, as in the benchmark case, is increasing and linear in  $y_i$ .

Let  $Y$  be the city's income, that is, the sum of the income from the production of traded goods  $Y_t$ , from the production and endowment of non-traded goods  $Y_{nt}$ , and from real estate  $Y_r$ . The Cobb-Douglas utility function implies that the city's expenditure in the consumption of traded goods, non-traded-foods, and real estate,  $E_t$ ,  $E_{nt}$  and  $E_r$ , respectively are

$$E_t = \gamma_t Y, \quad E_{nt} = \gamma_{nt} Y \quad \text{and} \quad E_r = \gamma_r Y. \quad (14)$$

Since the city's income from the production of traded goods must be equal to the city's consumption in traded goods, i.e.,  $Y_t = E_t$ , from [eq. 14](#), it follows that the market-clearing prices of non-traded good  $x_{nt}$  and real estate are

$$p_{nt}(x_{nt}) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})} \quad (15)$$

and

$$p_r = \frac{\gamma_r}{\gamma_t} \frac{Y_t}{R}. \quad (16)$$

where  $Q_{nt}(x_{nt})$  is the production of non-traded good  $x_{nt}$  in the city. While the prices of traded goods are taken as exogenous, the prices of non-traded goods and real estate are endogenously determined and depend on the city's income from the production of traded goods,  $Y_t$ , as well as by the production choices of the entrepreneurs supplying the non-traded goods. These production choices are considered in the next subsection.

## 4.2 Production at $t = 1$ .

At  $t = 1$ , the city's industrial base determines the production of traded goods, an industrial base which is the by-product of traded-good workers' specialization decisions at  $t = 0$ . Hence,  $L_i(x_t) = \int_0^{\bar{L}} l_i(x_t) di$  units of labor are devoted to the production of traded-good  $x_t \in X_N \cup \{1\}$  at  $t = 1$ , and  $Y_t$ , as expressed in [eq. 1](#), is the city's income from the production of traded

goods.

Regarding the production of non-traded-goods, there is one worker and one entrepreneur in each non-traded-good sector  $x_{nt}$ . The worker and the entrepreneur both produce  $\frac{q_0}{2} > 0$  units of the non-traded good, but the entrepreneur has the option to produce  $q_1 > 0$  additional units of the good by incurring a non-pecuniary fixed cost  $c$ . The total quantity produced is thus  $Q_{nt}(x_{nt}) = q_0 + q_1$  if the entrepreneur in non-traded-good sector  $x_{nt}$  produces  $q_1$  additional units of the good, and  $Q_{nt}(x_{nt}) = q_0$  if she does not.

The above specification contributes to the tractability of our model in a few of ways. First, the assumption that  $q_0 > 0$  guarantees that every non-traded good is available in the market, and therefore, consumed whatever the entrepreneurs' production decisions. This implies that the marginal utility of income is always positive, even when  $Q_{nt}(x_{nt}) = q_0$ . Second, the Cobb-Douglas utility function makes the elasticity of demand of each consumption good equal to 1, which implies that a single monopolist-entrepreneur would never incur a non-pecuniary fixed cost  $c$  to produce  $q_1$  additional units of the good. Therefore, having one worker and one entrepreneur for each non-traded good who compete to gain market share makes the entrepreneur's production decision a meaningful one. Finally, having only one worker and one entrepreneur, rather than two entrepreneurs, allows us to avoid solving a Cournot game (under asymmetric information in the case of the Global Games refinement) for each non-traded good market, and it just simplifies the analysis.

To determine the production choice of the entrepreneur in traded-good sector  $x_{nt}$ , from [eq. 15](#), the price of non-traded good  $x_{nt}$  is

$$p_{nt}(x_{nt}) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})}, \quad (17)$$

and then, the entrepreneur's income as a function of the total quantity produced of non-traded-good  $x_{nt}$  is

$$y(Q_{nt}(x_{nt})) = \frac{\gamma_{nt}}{\gamma_t} \frac{Y_t}{Q_{nt}(x_{nt})} \left( Q_{nt}(x_{nt}) - \frac{q_0}{2} \right). \quad (18)$$

Entrepreneur  $x_{nt}$  produces  $q_1$  additional units when the utility of the extra income from doing so provides sufficient compensation for the non-pecuniary fixed cost  $c$ , that is, when

$v[y(q_0 + q_1)] - v[y(q_0)] \geq c$ . From eq. 18, this condition can be expressed as

$$v \left[ \frac{\gamma_{nt} \frac{1}{2} q_0 + q_1}{\gamma_t \ q_0 + q_1} Y_t \right] - v \left[ \frac{\gamma_{nt} Y_t}{\gamma_t \ 2} \right] \geq c. \quad (19)$$

Using eq. 13 and eq. 16, the condition in eq. 19 writes

$$\frac{\gamma_t^{\gamma_t} \gamma_{nt}^{\gamma_{nt}} \gamma_r^{\gamma_r} \times \frac{\gamma_{nt} \ q_1 \ Y_t}{\gamma_t \ q_0 + q_1 \ 2}}{P_t^{\gamma_t} \exp \left[ \gamma_{nt} \int_0^1 \ln p_{nt}(x_{nt}) dx_{nt} \right] \left[ \frac{\gamma_r Y_t}{\gamma_t R} \right]^{\gamma_r}} \geq c, \quad (20)$$

where  $P_t \equiv \exp \left[ \int_0^1 \ln p_t(x_{nt}) dx_{nt} \right]$ . According to eq. 20, the production decision of entrepreneur in sector  $x_{nt}$  depends on the city's income from the traded-good sector  $Y_t$  (and hence, on the city's industrial base), and also on the prices of other non-traded goods in the city  $\int_0^1 \ln p_{nt}(x_{nt}) dx_{nt}$  (and hence, on the production decisions of entrepreneurs in all the other non-traded-good sectors  $x'_{nt} \in [0, 1] \setminus \{x_{nt}\}$ ).

Assume that all entrepreneurs  $x'_{nt} \in [0, 1] \setminus \{x_{nt}\}$  produce the same amount  $Q_{nt}$  of non-traded good  $x'_{nt}$ . Then, for all  $x'_{nt} \neq x_{nt}$ ,  $p_{nt}(x'_{nt}) = \frac{\gamma_{nt} Y_t}{\gamma_t Q_{nt}}$ , where  $Q_{nt} \in \{q_0, q_0 + q_1\}$ . The condition in eq. 20 can then be written as

$$\left[ \frac{Y_t}{P_t} \right]^{\gamma_t} \geq \frac{2c}{\gamma_{nt} Q_{nt}^{\gamma_{nt}} R^{\gamma_r}} \left( 1 + \frac{q_0}{q_1} \right). \quad (21)$$

According to eq. 21, an entrepreneur in sector  $x_{nt}$  is more likely to produce  $q_1$  additional units when entrepreneurs in other non-traded sectors also produce  $q_1$  additional units, that is, when  $Q_{nt} = q_0 + q_1$ . Intuitively, an increase in the supply of other non-traded goods in the city lowers their prices, which increases the amount of goods that entrepreneur in sector  $x_{nt}$  can consume if she decides to produce. This implies that entrepreneurs' production decisions are strategic complements. This strategic complementarity can lead to multiple equilibria in the production decisions at  $t = 1$ , as we show next.

Assume that all entrepreneurs  $x'_{nt} \in [0, 1] \setminus \{x_{nt}\}$  produce  $q_1$  additional units so that  $Q_{nt} = q_0 + q_1$ . From eq. 21, entrepreneur in sector  $x_{nt}$  also produces if

$$\left[ \frac{Y_t}{P_t} \right]^{\gamma_t} \geq \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}} \quad (22)$$

Alternatively, assume that all entrepreneurs  $x'_{nt} \in [0, 1] \setminus \{x_{nt}\}$  do not produce  $q_1$  additional

units so that  $Q_{nt} = q_0$ . From eq. 21, entrepreneur in sector  $x_{nt}$  does not produce either if

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} < \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}} \quad (23)$$

Combining eq. 22 and eq. 23, we obtain the following lemma.

**Lemma 1** *At  $t = 1$ , the production subgame among the entrepreneurs that produce non-traded goods is such that*

1. *If  $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} < \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}}$ , producing  $\frac{q_0}{2}$  is a strictly dominant strategy;*
2. *If  $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} > \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}}$ , producing  $\frac{q_0}{2} + q_1$  is a strictly dominant strategy.*
3. *If  $\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \in \left[\frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right]$ , an equilibrium where all entrepreneurs produce  $\frac{q_0}{2}$  coexists with an equilibrium where all entrepreneurs produce  $\frac{q_0}{2} + q_1$ ;*

Note that Lemma 1 establishes the existence of demand spillovers not only across non-traded goods, as discussed earlier, but also between traded and non-traded goods. Indeed, an equilibrium in which the city is vibrant, that is, in which the production of non-traded goods is high, i.e.,  $Q_{nt} = q_0 + q_1$ , is more likely when the income from the production of traded goods in the city,  $Y_t$ , is also high. Intuitively, when the income from traded goods is high, so is the demand for non-traded goods in the city, and hence, the price of non-traded-goods relative to the price of traded-goods. This increase in the relative price of non-traded-goods makes it more profitable for entrepreneurs to produce  $q_1$  additional units of any given non-traded good.

Lemma 1 also confirms the familiar intuition that strategic complementarities across non-traded goods can generate multiple equilibria when

$$\left[\frac{Y_t}{P_t}\right]^{\gamma_t} \in \left[\frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right]. \quad (24)$$

The multiplicity of equilibria arises because of the strategic complementarity in the production of non-traded goods: Since an entrepreneur is more likely to produce  $q_1$  additional units when entrepreneurs in other non-traded sectors also produce  $q_1$  additional

units, a self-fulfilling coordination failure can exist in which entrepreneurs do not produce  $q_1$  additional units because they expect other entrepreneurs to also not produce. In such case, the city is trapped in a low level of economic activity. The presence of multiple equilibria and the importance of aggregate demand is in line with the Keynesian narrative. For example, [Blanchard and Quah \(1989\)](#) use structural VARs to provide evidence in support of the idea that business cycles are driven by shifts in aggregate demand. The same idea is corroborated by recent work that exploits the regional variation in business cycles, such as [Mian and Sufi \(2014\)](#) and [Beraja, Hurst and Ospina \(2019\)](#). Low aggregate demand is also considered to be an important force behind the Great Recession and the slow recovery from it. (See, for instance, [Hall, 2011](#); [Eggertsson and Krugman, 2012](#); and [Mian, Rao and Sufi, 2013](#).)

### 4.3 Unique Equilibrium in Production at $t = 1$ .

The multiplicity of equilibria at the production stage  $t = 1$  captures the notion of strategic complementarities in the production of non-traded goods, but creates an indeterminacy that precludes the analysis of the specialization decisions at  $t = 0$ . To resolve this indeterminacy, we follow a global games approach in which the introduction of dispersed information breaks agents' ability to perfectly coordinate and delivers a unique equilibrium when agents' private information is sufficiently precise. A well-known property of global games is that equilibrium uniqueness carries over to the asymptotic case in which agents' private signals become infinitely precise and the information structure is arbitrarily close to common knowledge. This limit case highlights that equilibrium multiplicity is an artefact of the common knowledge assumption and will be our focus in the rest of the paper.

Formally, we enrich the production game by assuming that at  $t = 1$ , prior to making their production choices, the entrepreneur in each sector  $x_{nt} \in [0, 1]$  observes a private noisy signal of the city's income from the production of traded goods,  $s_{x_{nt}} = Y_t + \varepsilon_{x_{nt}}$ , where  $\varepsilon_{x_{nt}}$  is uniformly distributed in the interval  $[-\underline{\varepsilon}, \underline{\varepsilon}]$  and independent across entrepreneurs. So information is dispersed in the sense that each of these entrepreneurs observe a slightly

different signal of the same fundamental. We also make the following assumption:

**Assumption 1** *Upper dominance region*

$$\left[ \frac{2\delta\bar{L}}{P_t} \right]^{\gamma_t} > \frac{2c}{\gamma_{nt}R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}. \quad (25)$$

**Assumption 1** follows directly from Case 2 in **Lemma 1** by setting  $Y_t = 2\delta\bar{L}$ , and guarantees that, irrespective of workers' specialization decisions, there exist realizations of the traded-good income  $Y_t$  large enough for which producing  $\frac{q_0}{2} + q_1$  is a dominant strategy. Specifically, producing  $\frac{q_0}{2} + q_1$  becomes a dominant when the realized prices of the traded goods produced in the city are sufficiently high yet smaller than the upper bound  $\bar{p} = 2$ . Since these realized prices are positive but arbitrarily small, there also exist realizations of the traded-good income  $Y_t$  small enough that producing  $\frac{q_0}{2}$  is dominant strategy (Case 1 in **Lemma 1**). In other words, **Assumption 1** guarantees that the support of  $Y_t$  contains the entire region in which multiple equilibria exist in the original common-knowledge game. The existence of these upper and lower dominance regions is a standard requirement in global games, but the probability that  $Y_t$  falls in these regions (which depends on workers' specialization decisions at  $t = 0$ ) can be arbitrarily small.

The equilibrium derivation is a direct application of global games techniques and left to the proof of **Proposition 3** in the Appendix. In this derivation, we first show that for  $\underline{\varepsilon}$  small enough, the unique equilibrium strategy is for entrepreneur in sector  $x_{nt}$  to produce if and only if her signal  $s_{x_{nt}}$  is higher than a threshold  $\hat{s}$ . We then take the limit when  $\underline{\varepsilon}$  tends to 0 to recover an information structure arbitrarily close to common knowledge while retaining equilibrium uniqueness.

**Proposition 3** . *When  $\underline{\varepsilon} \rightarrow 0$ , (i) All entrepreneurs produce  $\frac{q_0}{2} + q_1$  units at  $t = 1$  if  $Y_t \geq Y_t^T$  and produce  $\frac{q_0}{2}$  units if  $Y_t < Y_t^T$  with*

$$\left( \frac{Y_t^T}{P_t} \right)^{\gamma_t} \equiv \frac{\ln(q_0 + q_1) - \ln q_0}{[q_0 + q_1]^{\gamma_{nt}} - q_0^{\gamma_{nt}}} \frac{q_0 + q_1}{q_1} \frac{2c}{\gamma_{nt}R^{\gamma_r}}; \quad (26)$$

*(ii) At the threshold  $Y_t^T$ , welfare is strictly higher if entrepreneurs produce  $\frac{q_0}{2} + q_1$  units than*

if they produce  $\frac{q_0}{2}$ ,

$$\left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} (q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} - c > \left(\frac{Y_t^T}{P_t}\right)^{\gamma_t} q_0^{\gamma_{nt}} R^{\gamma_r}. \quad (27)$$

The production threshold  $Y_t^T$  in [Proposition 3](#) belongs to the region in which multiple equilibria previously coexisted, i.e.,

$$\left[\frac{Y_t^T}{P_t}\right]^{\gamma_t} \in \left(\frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{(q_0 + q_1)^{\gamma_{nt}}}, \frac{2c}{\gamma_{nt} R^{\gamma_r}} \frac{1 + \frac{q_0}{q_1}}{q_0^{\gamma_{nt}}}\right). \quad (28)$$

Intuitively, dispersed information affects the structure of higher-order beliefs: when the entrepreneur in sector  $x_{nt}$  observes a lower signal  $s_{x_{nt}}$  of  $Y_t$ , it does not only imply that the marginal benefit to producing  $q_1$  additional units is lower, but also that other entrepreneurs are more likely to observe a low signal and therefore not produce. In turn, other entrepreneurs become less willing to produce  $q_1$  additional units when it is likely that entrepreneur in sector  $x_{nt}$  has observed a low signal and is not producing. This higher-order structure breaks equilibrium multiplicity and, importantly, equilibrium uniqueness is preserved when the model reverts to an information structure arbitrarily close to common knowledge, i.e., when  $\underline{\varepsilon} \rightarrow 0$ .

The unique equilibrium in [Proposition 3](#) has two important properties. First, entrepreneurs are more likely to produce  $q_1$  additional units when the city's income from the production of traded goods  $Y_t$  is higher, as in [Lemma 1](#), and therefore, this unique equilibrium is linked to the city's economic fundamentals. Second, the production threshold  $Y_t^T$  is in a region where it is strictly Pareto-improving that entrepreneurs produce these extra units. In other words, the equilibrium features coordination failures: there is a region below  $Y_t^T$  where welfare is lower than if all entrepreneurs jointly made the decision to produce more units.

Overall, [Proposition 3](#) underscores the importance of strategic uncertainty and higher-order beliefs in determining the city's level of economic activity. The importance of strategic uncertainty is consistent with a recent literature in macroeconomics that uses models of incomplete information to introduce coordination frictions in the economy, and which sheds light on the origin and dynamics of business cycles and economic crises.<sup>17</sup> For

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<sup>17</sup>See [Angeletos and Lian \(2016\)](#) for an overview of this literature.

instance, [Schaal and Taschereau-Dumouchel \(2019\)](#) show that a large transitory shock may push the economy into a quasi-permanent recession, helping explain the slow recovery and other salient features of the Great Recession. In the context of our model, [Proposition 3](#) shows that a shock to the city’s income from the production of traded goods  $Y_t$  can generate a sharp transition between low- and high-levels of economic activity. Indeed, there are numerous examples of cities whose economic fate has mirrored the fate of the sector in which the city was specialized in. For example, [Glaeser \(2005\)](#) accounts for Boston’s periods of success and decline over the last centuries by referring to the success and decline of its traded-good sector: “*Boston has reinvented itself three times: in the early 19th century as the provider of seafaring human capital for a far flung maritime trading and fishing empire, in the late 19th century as a factory town built on immigrant labor and Brahmin capital, and finally in the late 20th century as a center of the information economy.*” Importantly, there is also evidence of sectoral shocks having important local multiplier effects. For instance, [Moretti \(2010\)](#) finds that for each additional job in manufacturing in a given city, 1.6 jobs are created in the non-traded sector in the same city. This local multiplier is significantly larger for skilled jobs as they have higher earnings, and therefore, demand more local goods and services: One skilled job in the traded-good sector generates 2.5 jobs in the non-traded one.<sup>18</sup>

#### 4.4 Specialization at $t=0$ .

This section builds on the production equilibrium in [Proposition 3](#) to study traded-good workers’ specialization choices at  $t = 0$ . We focus our attention on a particular type of specialization equilibrium and prove that such an equilibrium exists. Specifically, we prove that there always exists an equilibrium industrial base that puts more weight on the favored traded-good sector –traded-good sector 1– than in another traded-good sector. We will then provide conditions under which a more or a less diversified industrial base improves welfare relative to this equilibrium allocation. In [Section 4.5](#) below, we will consider the possibility

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<sup>18</sup>See also [Foerster, Sarte and Watson \(2011\)](#) and [Di Giovanni, Levchenko and Mejean \(2014\)](#) for additional evidence of sectoral shocks having a large multiplying effect in rest of the economy.



of other equilibria that put less weight on the favored traded-good sector and assess their welfare.

As in [Section 3](#), let  $l_i(x_t)$  be an indicator function defined on  $x_t \in X_N \cup \{1\}$  such that  $l_i(x'_t) = 1$  if worker  $i$  specializes in the production of traded good  $x'_t$ , and  $l_i(x'_t) = 0$  otherwise. Therefore, the mass of workers that choose to specialize in the production of traded good  $x_t \in X_N \cup \{1\}$  is  $L_i(x_t) = \int_0^{\bar{L}} l_i(x_t) di$ . For a given specialization choice  $l_i(x_t)$ , worker  $i$  obtains income

$$y_i = (\alpha + \delta) l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t) \quad (29)$$

and from [eq. 13](#), his indirect utility function can be written

$$v(y_i) = \frac{\gamma_t Q_{nt}^{\gamma_{nr}} R^{\gamma_r} \times y_i}{P_t^{\gamma_t} Y_t^{\gamma_{nt} + \gamma_r}} \quad (30)$$

where, from [Proposition 3](#),  $Q_{nt} = q_0$  if  $Y_t < Y_t^T$  and  $Q_{nt} = q_0 + q_1$  if  $Y_t \geq Y_t^T$ . Therefore, at  $t = 0$ , worker  $i$  solves:

$$\max_{\{l_i(x_t)\}_{x_t \in X_N \cup \{1\}}} \mathbb{E}[v(y_i)]. \quad (31)$$

To simplify the exposition, in the rest of this section, we assume that  $N = 1$ , so that workers either specialize in traded-good sector 1 or in some other traded-good sector  $x_t \in X_1$ . ([Section 4.5](#) below considers the case in which  $N > 1$ .) The next proposition shows that there exists an equilibrium in which the city's industrial base has more weight in the production of the traded-good in which the city has a productivity advantage, i.e., traded-good 1.

**Proposition 4** *For  $\alpha > 0$ , there exists an equilibrium such that  $L(1)^* > L(x_t)^*$ .*

According to [Proposition 4](#), there always exists an equilibrium in which the mass of workers specialized in the production of traded-good 1 is greater than the mass of workers specialized in the production of traded-good  $x_t$ . Intuitively, at  $L(1) = L(x_t)$ , if  $\alpha = 0$ , a worker would be indifferent between specializing in traded-good sector 1 and  $x_t$ , and if  $\alpha > 0$ , a worker strictly prefers specializing in traded-good sector 1. However, as the next proposition shows, this does not imply that the city ends up fully specialized in traded-good sector 1.

**Proposition 5** *There is an  $\tilde{\alpha} > 0$  such that if  $\alpha \in (0, \tilde{\alpha})$ , there exists an equilibrium with  $L(1)^* > L(x_t)^* > 0$ .*

[Proposition 5](#) provides conditions for the existence of an interior equilibrium in which the city produces both traded-goods. In any interior equilibrium,  $L(1)^* > L(x_t)^* > 0$ , a worker must be indifferent between specializing in traded-good sectors 1 and  $x_t$ , that is

$$\mathbb{E}[v((\alpha + \delta)p_t(1))] = \mathbb{E}[v(\delta p_t(x_t))]. \quad (32)$$

where  $v(y_i)$  is defined as in [eq. 30](#). While the productivity advantage of traded-good sector 1 induces workers towards specializing in this sector, if this productivity advantage is not large enough (if  $\alpha < \tilde{\alpha}$ ), the only equilibrium with  $L(1)^* > L(x_t)^*$  is one in which the city does not fully specialize in traded-good sector 1, i.e.,  $L(1)^* < \bar{L}$ . In his specialization decision, a worker considers the expected income (i.e.,  $(\alpha + \delta)\mathbb{E}[p_t(1)]$  vs.  $\delta\mathbb{E}[p_t(x_t)]$ ) as well as the correlations between his income and the prices of real estate and of non-traded goods. These prices depend on the income from the production of traded-goods  $Y_t$ : In the case of real estate,  $Y_t$  determines the demand for real estate by the city residents, as we saw in [Section 3](#); In the case of non-traded goods,  $Y_t$  not only determines the demand for non-traded-goods, but also its supply  $Q_{nt}$ . As we see next, the effect of  $Y_t$  on  $Q_{nt}$  has welfare implications for the equilibrium industrial base.

Our concept of social optimum is *constrained* efficiency: The social planner chooses the industrial base at  $t = 0$ , but cannot avoid coordination failures at  $t = 1$  when  $Y_t < Y_t^T$ . From [eq. 30](#), the sum of the indirect utility of all the city's residents can be written as

$$v_s(Y) = \frac{Q_{nt}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c \mathbb{1}_{Y_t \geq Y_t^T}, \quad (33)$$

where we have used the fact that  $Y_t = \gamma_t Y$ , and  $\mathbb{1}_{Y_t \geq Y_t^T}$  is an indicator function that takes a value of 1 if  $Y_t \geq Y_t^T$ . The following proposition evaluates the welfare of any interior equilibrium industrial base, i.e.  $L(1)^* > L(x_t)^* > 0$ .

**Proposition 6** *Consider an interior equilibrium such that  $L(1)^* > L(x_t)^* > 0$ . If  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1)$  increases welfare, and if  $Y_t^T > \delta\bar{L}$ , a marginal increase*

in  $L(1)$  from  $L^*(1)$  increases welfare.

**Proposition 6** shows that the equilibrium industrial base is not socially optimal. To better understand the source of the inefficiency, consider the case in which  $Y_t^T < \delta\bar{L}$ . For  $p_t(x_t) \geq \frac{Y_t^T}{(\bar{L}-L(1))\delta}$ , the city's economy is in the high-activity equilibrium (i.e.,  $Q_{nt} = q_0 + q_1$ ) because, even if  $p_t(1) = 0$ ,  $Y_t \geq Y_t^T$ . Alternatively, for  $p_t(x_t) < \frac{Y_t^T}{(\bar{L}-L(1))\delta}$ , the city's economy is in the low-production equilibrium (i.e.,  $Q_{nt} = q_0$ ) unless the price  $p_t(1)$  is greater than  $\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}$ . Based on these considerations, the expectation of social welfare function in [eq. 33](#),  $\mathbb{E}[v_s(y_i)]$ , writes

$$\begin{aligned} & \frac{1}{4} \int_{\max\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}}^2 \left[ \int_0^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right] dp_t(x_t) \\ & + \frac{1}{4} \int_0^{\max\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} \left[ \int_{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t). \end{aligned} \quad (34)$$

The expression in [eq. 34](#) is a function of  $L(1)$  directly and through  $Y_t$ , as  $Y_t$  is a function of  $L(1)$  (see [eq. 1](#)). Therefore, the derivative of [eq. 34](#) with respect to  $L(1)$  has two components. One component is the derivative of [eq. 34](#) with respect to  $Y_t$  multiplied by the derivative of  $Y_t$  with respect to  $L(1)$ . This derivatives is proportional to

$$\mathbb{E}[v((\alpha + \delta)p_t(1))] - \mathbb{E}[v(\delta p_t(x_t))], \quad (35)$$

which is zero when  $L(1) = L^*(1)$  as, in equilibrium, workers are indifferent between specializing in traded-good sectors 1 and  $x_t$ . (See [eq. 32](#).) Therefore, workers' specialization choices internalize the welfare effect that works through  $Y_t$  for a given probability of a coordination failure (that is, the effect that works through the integrand in [eq. 34](#)) as they did in [Proposition 2](#). The second component is the direct derivative of [eq. 34](#) with respect to  $L(1)$ , which works through the integration boundaries  $\max\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}$  and  $\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}$ . The derivative with respect the integration boundary  $\max\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}$  is zero, and the derivative with respect the integration boundary  $\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}$  is negative when  $Y_t^T < \delta\bar{L}$ , which implies that a marginal decrease in  $L(1)$  from  $L^*(1)$  increases welfare.

Therefore, the social inefficiency of the equilibrium in [Proposition 6](#) is associated to the effect that  $L(1)$  has on the integration boundaries in [eq. 34](#), and, when  $Y_t^T < \delta\bar{L}$ , the sign of the inefficiency is one of excessive specialization.

Intuitively, workers do not internalize that their combined specialization choices determine the city's exposure to sectoral shocks and hence, the probability of the city's economy suffering a coordination failure, i.e.,  $\Pr(Y_t < Y_t^T)$ . Since the equilibrium industrial base is not socially optimal, an active industrial policy can increase welfare. The optimal industrial policy promotes a more diversified industrial base in some cities, and a more specialized one in others. If  $Y_t^T < \delta\bar{L}$ , the city's expected income from the production of traded goods  $Y_t$  is above threshold  $Y_t^T$ , and a decrease in  $L(1)$  from  $L^*(1)$ , which decreases the volatility of the city's income, reduces the probability of a coordination failure. In other words, when the city's productivity (as measured by  $\delta$ ) is high and the size of the traded-good sector is large (as measured by  $\bar{L}$ ) demand for non-traded goods is expected to be high and hence, the city is likely to be in the high-output equilibrium. In such case, there is a social benefit of increasing diversification: avoiding a city-wide coordination failure when there is a negative shock to traded-good sector 1. Alternatively, if  $Y_t^T > \delta\bar{L}$ , the city is likely to be in low-output equilibrium, and there is a social benefit of increasing specialization: escaping the low-output equilibrium when there is a positive shock to traded-good sector 1.

Our analysis is consistent with cities, when compared with countries, being highly specialized economies dependent on one or a few traded-good sectors. Moreover, it is also consistent with these traded-good sectors having important local multiplier effects as documented, for example, in [Moretti \(2010\)](#). In many instances, industrial policy has the objective of reducing the exposure to sectoral shocks by diversifying the city's industrial base. Indeed, there is a literature that emphasizes the importance of diversification for economic resilience. For instance, [Brown and Greenbaum \(2017\)](#) examines the influence of industrial diversity on unemployment rate stability in Ohio counties between 1977 and 2011, and finds that while more concentrated counties had lower unemployment rates when times were good, counties with more diverse industry structures fared better during times of local

employment shocks. Recent example of cities with policies aimed at diversifying their economies include Austin and New York among many others. While the objective of reducing the exposure to sectoral shocks through diversification has been a key driver of industrial policy, in some cases a different strategy has been followed. A Porter-type cluster strategy has been the guiding principle of industrial policy for many underdeveloped regions. (See [Porter, 1990](#).) Under this strategy, a city or region focuses its resources in a specific industry or in a number of related industries, the idea being that firms benefit from the activities of neighboring firms in the same and related industries, an idea that goes back to [Marshall \(1890\)](#). In the context of our model, this strategy entails increasing the city's exposure to sectoral shocks, something that according to our analysis is optimal for less productive regions (low  $\delta$ ) and for regions with scarce resources (low  $\bar{L}$ ). More recently, a smart specialization strategy (S3) has become central to industrial policy, notably for the less developed regions of the European Union.<sup>19</sup> While this strategy is different from a leading industry or a cluster-based strategy, it still requires identifying and investing in a limited number of areas that can be considered most promising given the capabilities of the region.

[Proposition 6](#) also implies that a city's diversification level need not be informative about the optimal industrial policy. Specifically, the degree of diversification depends on the productivity advantage of the favored traded-good sector,  $\alpha$ , while the optimal industrial policy depends on the size,  $\bar{L}$ , and productivity,  $\delta$ , of all traded-good sectors. In other words, if a city has a comparative advantage in one sector, the equilibrium industrial base will be less diversified, but the city may still suffer from excessive or insufficient diversification. Finally, note that this also implies that there is no a priori relation between volatility and expected output: A city with low  $\alpha$  will tend to have a more diversified industrial base, but if the city has a low (*high*)  $\delta$ , it will also have a (*high*) low expected outcome.

[Proposition 6](#) considers the case of an interior equilibrium,  $\bar{L} > L(1)^* > L(x_t)^* > 0$ , which we view as the leading case as cities produce more than one traded-good. For completeness,

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<sup>19</sup>See [https://www.interregeurope.eu/sites/default/files/inline/Smart\\_Specialisation\\_Strategy\\_\\_\\_S3\\_\\_\\_-Policy\\_Brief.pdf](https://www.interregeurope.eu/sites/default/files/inline/Smart_Specialisation_Strategy___S3___-Policy_Brief.pdf), and <https://s3platform.jrc.ec.europa.eu/lagging-regions>.

however, we next assess the welfare of a corner equilibrium,  $\bar{L} = L^*(1)$ .

**Proposition 7** *Consider a corner equilibrium, with  $L^*(1) = \bar{L}$ . If  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1)$  may increase or decrease welfare, and if  $Y_t^T > \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1)$  decreases welfare.*

As in an interior equilibrium, in a corner equilibrium, workers do not internalize the effect of their specialization choices on the probability of a coordination failure. However, unlike in an interior equilibrium, in a corner equilibrium, workers strictly prefer specializing in traded-good sector 1, i.e.,  $\mathbb{E}[v((\alpha + \delta)p_t(1))] > \mathbb{E}[v(\delta p_t(x_t))]$ , which implies that there is now an additional cost of reducing  $L^*(1)$  from  $\bar{L}$ . When  $Y_t^T < \delta\bar{L}$ , reducing  $L^*(1)$  from  $\bar{L}$  still increases welfare through reducing the probability of coordination failure, but it has to be now weighted against this additional cost. If the difference  $\mathbb{E}[v((\alpha + \delta)p_t(1))] - \mathbb{E}[v(\delta p_t(x_t))]$  is small, the effect of reducing  $L(1)$  on the probability of a coordination failure dominates, and hence, a decrease in  $L^*(1)$  from  $\bar{L}$  increases welfare, as it did in [Proposition 6](#). However, when this difference is large enough, a decrease in  $L^*(1)$  from  $\bar{L}$  decreases welfare. Alternatively, when  $Y_t^T > \delta\bar{L}$ , a decrease in  $L^*(1)$  from  $\bar{L}$  decreases welfare, not only because it increases the probability of a coordination failure (as it did in [Proposition 6](#)), but now also because workers prefer specializing in traded-good sector 1.

## 4.5 Specialization at $t = 0$ : Other Equilibria and $N > 1$ .

[Section 4.4](#) focuses on equilibria in which the mass of workers specialized in the production of traded-good 1 is greater than the mass of workers specialized in the production of traded-good  $x_t$  and assumes that  $N = 1$ . This section explores the possibility of other type of equilibria and studies the case in which  $N > 1$ .

[Proposition 4](#) guarantees the existence of an equilibrium industrial base that puts more weight in the production of traded-good 1. However, [Proposition 4](#) does not rule out the possibility of equilibria in which the city puts more weight in traded-good sector  $x_t$ , i.e.,  $L(x_t)^* > L(1)^*$ . While these equilibria are probably less interesting empirically –the city enjoys a productivity advantage in traded-good sector 1– we next study their properties.

In the absence of non-traded goods other than real estate, [Section 3](#) shows that there is a unique equilibrium industrial in which  $L^*(1) > L^*(x_t)$ , and that this equilibrium is socially optimal. Two forces shape this unique equilibrium. First, the productivity advantage of traded-good sector 1 induces workers towards specializing in sector 1. Second, the positive correlation between the income in traded-good sector 1 and  $Y_t$  – correlation that increases in  $L(1)$ – can induce some workers to specialize in other traded-good sectors to take advantage of the low real estate price when the city’s income  $Y_t$  is low, provided that  $\alpha$  is not too large. These two forces are also present in [Section 4.4](#), but the inclusion of non-traded goods and the possibility of a coordination failure generates an additional force: the correlation between  $Y_t$  and the prices of non-traded goods.

An increase in  $Y_t$  increases the demand for non-traded goods which, *ceteris paribus*, increases their prices, as it did for the price of real estate. However,  $Y_t$  also affects the prices of non-traded goods because it impacts their supply. The supply of non-traded goods depends on the resources available in the city to produce these goods (i.e., the entrepreneurs in the case of our model) and on the utilization of these resources (i.e., whether the city is in a low- or high-activity equilibrium). Around threshold  $Y_t^T$ , the production of non-traded goods rapidly increases as the city’s economy shifts from a low- to a high-production equilibrium. This rapid increase in the supply of traded goods provides workers with an incentive to coordinate their specialization choices, that is, it motivates workers to specialize in the same traded-good sector as other workers. To gain intuition, consider a case in which  $q_0 = 0$ , so there is no production of traded-goods when  $Y_t < Y_t^T$ , which makes the marginal utility of income zero in the low-production equilibrium. In such case, there is less incentives to diversify away from traded-good sector 1 because, when there is a negative shock to traded-good sector 1, the city is likely to be in the low-production equilibrium and hence, the marginal utility of income is likely to be zero. While the rapid increase in the supply of non-traded goods when the economy shifts from a low- to a high-production equilibrium generates strategic complementarities in workers’ specialization choices, within the low- and high-activity equilibria, production is fixed at  $q_0$  and  $q_0 + q_1$ , respectively. Again, to gain

intuition, assume now that  $q_1 = 0$ , so that non-traded goods are in fixed supply. Such case is essentially identical to the benchmark (real estate is just a non-traded in fixed supply) and, as discussed in [Section 3](#), when the supply of traded-goods is inelastic, workers benefit from diversification, that is, from receiving higher income when other workers do not. This effect generates strategic substitutabilities in workers' specialization choices. In summary, the resources available to produce non-traded goods limits their supply and tends to make workers' specialization choices strategic substitutes, while the rapid change in the supply of non-traded goods when the city's economy transitions between the low- and high-activity equilibria tends to make workers' specialization choices strategic complements. When this strategic complementarities are strong enough, multiple equilibria may arise, and in some of this equilibria, the mass of workers specialized in the production of traded-good  $x_t$  could be greater than the mass of workers specialized in the production of traded-good 1.

If there are equilibria in which the mass of workers specialized in the production of traded-good  $x_t$  is greater than the mass of workers specialized in the production of traded-good 1, this type of equilibria is again going to be socially inefficient. First, any equilibrium industrial base  $\{L^*(1), L^*(x_t)\}$  in which  $L^*(1) < L^*(x_t)$  is going to be dominated from the social point of view by an industrial base  $\{L'(1), L'(x_t)\}$  such that  $L'(1) = L^*(x_t) > L'(x_t) = L^*(x_t)$ . This inefficiency does not speak to the city's degree of specialization per se, but to the city specializing in the “wrong” (lower productivity) traded-sector  $x_t$ . Second, similarly to [Section 4.4](#), workers do not internalize that their combined specialization choices determine the city's income from the production of traded and hence, the probability of a coordination failure as the next proposition shows.

**Proposition 8** (i) *Consider an interior equilibrium, with  $0 < L^*(1) < L^*(x_t) < \bar{L}$ . If  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(x_t)$  from  $L^*(x_t)$  increases welfare, and if  $Y_t^T > \delta\bar{L}$ , a marginal increase in  $L(x_t)$  from  $L^*(x_t)$  increases welfare provided that  $\alpha$  is small enough.* (ii) *Consider a corner equilibrium, with  $L^*(x_t) = \bar{L}$ . If  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(x_t)$  from  $L^*(x_t)$  may increase or decrease welfare, and if  $Y_t^T > \delta\bar{L}$ , a marginal decrease in  $L(x)$  from  $L^*(1)$  decreases welfare provided that  $\alpha$  is small enough.*



The message of [Proposition 8](#) is similar to the message of [Proposition 6](#) and [Proposition 7](#). Combining the three propositions, it follows that for any interior equilibrium (i.e.,  $L^*(1) > 0$  and  $L^*(x_t) > 0$ ), when  $Y_t^T < \delta\bar{L}$ , an increase in diversification (i.e., a decrease in  $L(1)$  when  $L^*(1) > L(x_t)$  or a decrease in  $L(x_t)$  when  $L^*(x_t) > L^*(1)$ ) increases welfare. Alternatively when  $Y_t^T > \delta\bar{L}$ , an increase in specialization (i.e., an increase in  $L(1)$  when  $L^*(1) > L(x_t)$  or an increase in  $L(x_t)$  when  $L^*(x_t) > L(1)$  provided that  $\alpha$  is small enough) decreases welfare.<sup>20</sup> For any corner equilibrium, workers also do not internalize the probability that their specialization choices has of the probability of a coordination failure. However, as explained after [Proposition 7](#), this effect has to be weighted against the fact that in a corner equilibrium a workers prefer to specialize in traded-good sector 1 when  $L^*(1) = \bar{L}$  and in traded-good sector  $x_t$  when  $L^*(x_t) = \bar{L}$ . In summary, while equilibria in which  $L^*(1) > L^*(x_t)$  always exists and are probably more economically relevant than equilibria in which  $L^*(x_t) > L^*(1)$ , the welfare implications in terms of diversification of equilibria  $L^*(1) > L^*(x_t)$  carry to equilibria  $L^*(x_t) > L^*(1)$ .

Consider now the case with  $N > 1$ , so the set of traded-good sectors in which a worker can specialize,  $X_N \cup \{1\}$ , is greater than 2. The next proposition assesses the existence and welfare of a symmetric equilibrium industrial base (an equilibrium in which all sectors in  $X_N$  have the same size) when  $N \rightarrow +\infty$ . This symmetric equilibrium can be viewed as the combination of traded-good-sector 1 with  $L^*(1)$  workers and a composite traded-good sector with  $\bar{L} - L^*(1)$  workers, in which all traded-good sectors in  $X_N$  have equal weight.

**Proposition 9** *(i) There if an  $\tilde{\alpha} > 0$  such that if  $\alpha \in (0, \tilde{\alpha})$  and  $N \rightarrow +\infty$ , there exists an equilibrium with  $L(1)^* \in (0, \bar{L})$  workers specialize in the production of traded-good 1 and  $Y_t = (\alpha + \delta) p_t(1)L^*(1) + \delta(\bar{L} - L^*(1))$ ; (ii) In any such equilibrium, if  $0 < \frac{Y_t^T - \delta(\bar{L} - L^*(1))}{(\alpha + \delta)L^*(1)} < 2$ , a marginal decrease in  $L(1)$  from  $L^*(1)$  increases welfare when  $Y_t^T < \delta\bar{L}$ , and a marginal increase in  $L(1)$  from  $L^*(1)$  increases welfare when  $Y_t^T > \delta\bar{L}$ ; and (iii) In any such equilibrium, if  $\frac{Y_t^T - \delta(\bar{L} - L^*(1))}{(\alpha + \delta)L^*(1)} \notin (0, 2)$ , a marginal change in  $L(1)$  does not affect welfare*

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<sup>20</sup>For  $L^*(x_t) > L(1)$  and  $Y_t^T > \delta\bar{L}$ , the assumption that  $\alpha$  is not too large guarantees that marginal increase in  $L(x_t)$  does not reduce the probability of a coordination failure. Note however, that if  $\alpha$  is large an equilibrium with  $L^*(x_t) > L^*(1)$  is less likely to exist in the first place.

Note also that when  $N \rightarrow +\infty$ , city risk could be fully diversified, but provided that  $\alpha > 0$ , traded-good workers will choose not to do so. In fact, the message of [Proposition 9](#) is again similar to the messages in [Proposition 5](#) and [Proposition 6](#). An interior symmetric equilibrium exists provided that the productivity advantage of traded-good sector 1 is not too large. If in this interior equilibrium the city can shift between a high- and a low-activity equilibrium (i.e., between  $Q_{nt} = q_0 + q_1$  and  $Q_{nt} = q_0$ ) following a shock to traded-good sector 1, that is, if  $0 < \frac{Y_t^T - \delta(\bar{L} - L^*(1))}{(\alpha + \delta)L^*(1)} < 2$ , the equilibrium is not socially optimal because workers do not internalize the effect that their specialization choices have on the probability of the city ending up in the high- or low-activity equilibrium. More specifically, as in [Proposition 6](#), an increase in diversification increases welfare if  $Y_t^T < \delta\bar{L}$ , while an increase in specialization increases welfare if  $Y_t^T > \delta\bar{L}$ . In [Proposition 9](#) such an increase in diversification (*specialization*) can be achieved by shifting weight from traded-good sector 1 towards (*away*) the composite traded-good sector.

## 5 Discussion and Future Research

Up to this point we have made a number of assumptions that simplify our analysis. In this section we discuss two of these assumptions – specifically, the ruling out of cross-city hedging and city-to-city migration – and discuss possible extensions with the assumptions relaxed. As we discuss first, in our setting, markets that allow cross-city hedging can improve the allocation of resources, even though individuals are all risk neutral. Regarding migration, ex-ante migration (migration at  $t = 0$ , before workers specialized) does not materially affect our analysis, but allowing some ex-post migration (migration at  $t = 1$ , after the realization of the sectoral shocks) can in some situations reduce the possibility of a coordination failure and in others make coordination failures more likely. Finally we discuss the possibility of direct externalities in the consumption of non-traded goods, and how these externalities can reinforce the demand-driven complementarities that this paper studies.

## 5.1 Incomplete Markets

The agents in our model are assumed to be risk neutral, which simplifies our analysis, and perhaps more importantly, illustrates that diversification in our model plays a role that does not arise because of risk aversion, per se. Given our assumption of risk neutrality, the fact that we also preclude risk sharing across cities appears to be innocuous at first glance. However, because of the inelastic supply of real estate, individuals can in fact gain from hedging city level risk.<sup>21</sup> Indeed, if we extend our model to allow individuals to hedge city risk, we obtain a unique equilibrium in the benchmark case described in [Section 3](#) that has the property that all traded-good workers specialize in the production of traded-good 1, that is, in the traded good that can be most efficiently produced in the city. In this equilibrium, workers choose to hedge their city risk, and as a result, the city's (hedged) income from the production of traded-goods is  $E[Y_t] = (\alpha + \delta)\bar{L}$  with probability one. This unique equilibrium, which exhibits efficient production as well as constant consumption of traded goods, is socially optimal.

In the general case described in [Section 4](#), in which the city produces non-traded as well as traded goods, there can also be a socially optimal equilibrium in which all traded-good workers specialize in the production of traded-good 1. Specifically, if  $(\delta + \alpha)\bar{L} > Y_t^T$ , residents hedge their income (i.e., each worker exchanges  $(\delta + \alpha)p_t(1)$  for its expectation  $\delta + \alpha$ ), and the production of traded goods is  $Q_{nt} = q_o + q_1$  with probability one.<sup>22</sup> Overall, hedging across cities allows the city to benefit from the productivity advantage of a specialized industrial base, while avoiding the more volatile aggregate income  $Y_t$ .

There are, of course, a number of reasons why financial markets do not provide the kind of cross-city hedging that eliminates the benefits of a diversified industrial base. As the financial markets develop, however, these impediments to efficient risk sharing are reduced, which in theory, contributes to economic growth. It is noteworthy that our model suggests a novel channel through which financial market development promotes economic growth. Specifically,

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<sup>21</sup>As previously discussed, although individual indirect utility functions  $v(y_i)$  in [eq. 13](#) are linear in  $y_i$ , the social indirect utility function  $v_s(Y)$  in [eq. 33](#) is concave in the city's aggregate income  $Y$ , which is a consequence of the inelastic supply of real estate.

<sup>22</sup>If  $(\delta + \alpha)\bar{L} < Y_t^T$ , it may not be optimal to hedge city risk, as the city would end up in the low-activity equilibrium with probability one.

the model predicts that as financial markets develop, the overall economy benefits as industries migrate to locations in which they have a comparative advantage. This positive relation between financial development and growth is consistent with an extensive literature that studies the benefits of financial market development, see, e.g., [Greenwood and Jovanovic \(1990\)](#), [Obstfeld \(1994\)](#), and [Acemoglu and Zilibotti \(1997\)](#), and [Levine \(2005\)](#) for an overview of the literature. In our case, however, the benefits of hedging do not stem from risk aversion, but from the inelasticity of the supply of real estate and other non-traded goods.

## 5.2 The Effect of Migration

Another aspect of our analysis that merits discussion is our assumption that the labor force  $\bar{L}$  is exogenous and fixed. This is in contrast to existing models of systems of cities that consider migration between cities (see, for example, [Rauch, 1993](#)). Since our analysis holds for any  $\bar{L}$ , it also holds if we allow frictionless ex-ante migration. In particular, one can allow workers to freely migrate in the initial period so that they achieve the same reservation utilities in each location.

Our assumption that individuals cannot migrate ex-post (after the realization of a shock) is, however, not innocuous, and warrants further discussion. Intuitively, this assumption captures the enduring nature of a city's industrial base. Indeed, if we allow frictionless ex-post migration, cities would respond to a productivity shock by swiftly changing their industrial composition. For example, migration could allow Detroit to rapidly transform itself from an auto- to a software-manufacturing cluster following a negative shock to the auto industry.

While our model is designed to capture the numerous frictions that inhibit migration, e.g., individuals enjoy social networks in the locations where they have long lived, the types of shocks explored in this paper are likely to trigger at least some migration. If some workers move out of a city following a negative shock, this could amplify the decline in the demand for the city's non-traded goods, which could in turn, increase the probability of a coordination failure. Moreover, there might be strategic complementarities associated with migration choices. For example, the popular press has recently described what is referred to

as a “doom-loop” where office workers move out of central business districts because of increased crime and the deteriorating quality of restaurants and other services, and how this, in turn, triggers further deteriorations, accelerating the exit.<sup>23</sup> It should be noted that there can also be offsetting effects that arises because the price of real estate in a city declines when individuals leave the city, which can make the city less expensive for those that stay. In other words, a channel exists that make individual migration choices strategic substitutes that may at least partially offset those that make migration choices strategic complements. While the casual evidence seems to be more consistent with the complementarities, i.e., the "doom-loop" effect, this is a topic that warrants future research.

### 5.3 Urban Vibrancy

Our model assumes that each city exhibits a productivity advantage for one particular traded-good, and this productivity advantage is fully captured by the traded good workers. Our model, however, abstracts from productivity externalities, such as economies of scale at the industry level or economies of scope across industries. We do this to isolate the effect of externalities that arise directly from the level of city risk. That is, from the relation between the city’s exposure to sectoral shocks and the probability of the city being trapped in an equilibrium where the production of non-traded goods are inefficiently low.

One might characterize the coordination failure described in our model as a loss in “vibrancy” that arises when the production of non-traded goods in a city declines during a downturn. We believe that this notion of vibrancy can be further analyzed in an extension that captures the potential externalities that arise from the consumption of non-traded goods. For instance, externalities emanating from social interactions at restaurants, bars, entertainment venues, social clubs, etc. These vibrancy effects could amplify the link between traded-good shocks and the probability of a coordination failure in a couple of ways. First, to the extent that these social interactions facilitate the transmission of knowledge and ideas, they may directly affect the productivity of the city’s industrial base.

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<sup>23</sup><https://www.brookings.edu/articles/breaking-the-urban-doom-loop-the-future-of-downtowns-is-shared-prosperity/>

In addition, because these interaction benefits generate direct strategic complementarities in the consumption of non-traded goods, they can reinforce the demand-driven complementarities in the production of non-traded goods that this paper studies.

## 6 Conclusion

There is a substantial and varied literature that studies the tradeoffs between the productivity advantages of a specialized regional economy and the risk reduction benefits of diversification. For the most part, this literature focuses on the gains associated with specialization, e.g., knowledge spillovers that arise when firms co-locate with industry peers, and takes as given the inherent costs of having a less diversified economy. In this paper, we do the opposite. By assuming that regional economies are endowed with comparative advantages in specific industries, we consider a setting with an exogenous benefit of specialization. However, our modeling of the provision of non-traded goods provides micro-foundations for the benefits of regional diversification.

As we show, there are benefits of having a diversified industrial base even when the residents of a region are all risk neutral. These benefits are related to the idea of labor pooling, as introduced by [Krugman \(1991\)](#). In our case, instead of workers responding to exogenous shocks by shifting between jobs, the workers providing non-traded goods experience a shift in their clienteles. For example, restaurant workers serve more meals to auto workers when the auto market does well and to software programmers when the tech business does well. We have an analogous notion of land pooling that arises from the fact that workers consume more land when they are more prosperous, which plays the same role.

These diversification benefits do not imply that an activist industrial policy that taxes or subsidizes different industries necessarily improves welfare. In particular, when workers supply labor inelastically, cities optimally diversify without interventions. This, however, is not the case in a setting in which the entrepreneurs who provide non-traded goods have the flexibility to work somewhat less when the demand for their services decline. For example, a restaurant may cut back its hours during a downturn. As we show, a city in this setting can experience a

coordination failure when the demand for the traded goods produced in the city declines. The coordination failure arises because entrepreneurs providing non-traded goods cut production in bad times. This is partly due to less demand from the traded goods workers, but the effect is amplified because of reduced demand from other non-traded goods entrepreneurs who also scale back both their production and their consumption.

As discussed in detail in [Section 5](#), the model can be extended in a number of interesting directions. Since our focus is primarily on risk, the analysis of financial market developments and the hedging of city risk is clearly warranted. For instance, policy makers may consider subsidies for financial market development as a substitute for subsidies that attract diversifying industries. We also discussed the importance of thinking more carefully about migration and urban vibrancy, and how these factors can amplify the coordination failures that this paper has considered. Each of these topics are likely to be fruitful areas for future research.

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# Appendix

## Proof of Proposition 1

Let  $X_N$  be a subset of  $[0, 1)$  with  $N$  sectors.  $X_N \cup \{1\}$  defines the set of sectors available for work in the city. Worker's  $i$  indirect utility function (8) writes

$$v(y_i) = \gamma_t \frac{(\alpha + \delta) l_i(1) p_t(1) + \sum_{x_t \in X_N} \delta l_i(x_t) p_t(x_t)}{N_t^{\gamma_t} \left(\frac{Y_t}{R}\right)^{\gamma_r}}. \quad (\text{A.1})$$

In what follows, recall that workers are atomistic and therefore, each worker's individual sector choice does not affect  $Y_t$ .

**Claim 1** *In any equilibrium, all sectors in  $X_N$  have the same mass of workers.*

**Proof.** The proof is by contradiction. Suppose there exists  $\{x_t, x'_t\} \in X_N^2$  such that the mass of workers in  $x_t$  is strictly larger than the mass of workers in  $x'_t$ :  $L(x_t) > L(x'_t)$ . Let

$$\hat{Y}_t \equiv Y_t - L(x_t) \delta p(x_t) - L(x'_t) \delta p(x'_t) \quad (\text{A.2})$$

be the production of the city without sectors  $x_t$  and  $x'_t$ . From (A.1), a worker's net expected utility from working  $x'_t$  rather than in sector  $x_t$  is

$$\frac{\gamma_t R^{\gamma_r} \delta}{4 N_t^{\gamma_t}} \mathbb{E} \left[ \int_0^2 \int_0^2 \frac{p(x'_t) - p(x_t)}{(\hat{Y}_t + L(x_t) \delta p(x_t) + L(x'_t) \delta p(x'_t))^{\gamma_r}} dp(x'_t) dp(x_t) \right], \quad (\text{A.3})$$

where expectation is with respect to prices in sectors other than  $\{x_t, x'_t\}$ :  $\{p(\hat{x}_t)\}$ . Therefore, a worker strictly prefers working in  $x'_t$  than in  $x_t$  if

$$\int_0^2 \int_0^2 \frac{p(x'_t) - p(x_t)}{(\hat{Y}_t + L(x_t) \delta p(x_t) + L(x'_t) \delta p(x'_t))^{\gamma_r}} dp(x'_t) dp(x_t) > 0 \quad (\text{A.4})$$

$$\Leftrightarrow \int_0^2 \int_{p(x_t)}^2 \frac{p(x'_t) - p(x_t)}{(\hat{Y}_t + L(x_t) \delta p(x_t) + L(x'_t) \delta p(x'_t))^{\gamma_r}} dp(x'_t) dp(x_t) > \quad (\text{A.5})$$

$$\int_0^2 \int_{p(x'_t)}^2 \frac{p(x_t) - p(x'_t)}{(\hat{Y}_t + L(x_t) \delta p(x_t) + L(x'_t) \delta p(x'_t))^{\gamma_r}} dp(x_t) dp(x'_t) \quad (\text{A.6})$$

$$\Leftrightarrow \int_0^2 \int_0^{2-p(x_t)} \frac{z}{(\hat{Y}_t + (L(x_t) + L(x'_t)) \delta p(x_t) + L(x'_t) \delta z)^{\gamma_r}} dz dp(x_t) > \quad (\text{A.6})$$

$$\int_0^2 \int_0^{2-p(x'_t)} \frac{z}{(\hat{Y}_t + (L(x_t) + L(x'_t)) \delta p(x'_t) + L(x_t) \delta z)^{\gamma_r}} dz dp(x'_t),$$

(A.6) holds if  $L(x'_t) < L(x_t)$ . ■

**Claim 2** *There exists a unique  $\bar{\alpha}_N > 0$  such that  $L(1) = \bar{L}$  is an equilibrium if and only if  $\alpha \geq \bar{\alpha}_N$ .*

**Proof.** If  $\bar{L}$  workers work in traded-good sector 1, then  $Y_t = \bar{L}(\delta + \alpha)p(1)$ . Then from (A.1), the worker net utility from working in traded-good sector 1 rather than in  $x_t \neq 1$  has the sign of

$$\mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \delta p(x_t)}{(\bar{L}(\delta + \alpha)p(1))^{\gamma_r}} \right] = \frac{(\delta + \alpha)\mathbb{E}[p(1)^{1-\gamma_r}] - \delta\mathbb{E}[p(1)^{-\gamma_r}]}{(\bar{L}(\delta + \alpha))^{\gamma_r}}, \quad (\text{A.7})$$

using that  $p(x_t)$  is independent from  $p(1)$  and has mean 1. The concavity of  $p(1)^{1-\gamma_r}$  implies

$$\mathbb{E}[p(1)^{1-\gamma_r}] < (\mathbb{E}[p(x_t)])^{1-\gamma_r} = 1. \quad (\text{A.8})$$

The convexity of  $p(1)^{-\gamma_r}$  implies

$$\mathbb{E}[p(x_t)^{-\gamma_r}] > (\mathbb{E}[p(x_t)])^{-\gamma_r} = 1. \quad (\text{A.9})$$

It follows that if  $\alpha = 0$ , (A.7) is strictly negative. Then since (A.7) is strictly increasing in  $\alpha$  and tends to  $+\infty$  as  $\alpha$  tends to  $+\infty$ , there is a unique  $\bar{\alpha}_N > 0$  such that (A.7) equals zero. (A.7) is positive, i.e.,  $L(1) = \bar{L}$  is an equilibrium if and only if  $\alpha \geq \bar{\alpha}_N$ . ■

**Claim 3** *There exists an equilibrium such that  $L(1) < \bar{L}$  and for all  $x_t \in X_N$ ,  $L(x_t) = \frac{\bar{L}-L(1)}{N}$  if and only if  $\alpha < \bar{\alpha}_N$ .  $L(1)$  is then unique.*

**Proof.** In the candidate equilibrium, workers' net utility from working in traded-good sector 1 rather than in  $\hat{x}_t \in X_N$  has the sign of

$$\mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \delta p(\hat{x}_t)}{Y_t^{\gamma_r}} \right] \quad (\text{A.10})$$

where  $Y_t = (\delta + \alpha)L(1)p(1) + \frac{\bar{L}-L(1)}{N} \sum_{x_t \in X_N} \delta p(x_t)$ . If  $L(1) = 0$ , (A.10) becomes

$$\mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \delta p(\hat{x}_t)}{\left(\frac{\bar{L}}{N} \sum_{x_t \in X_N} \delta p(x_t)\right)^{\gamma_r}} \right] > \frac{\delta \mathbb{E}_{p(x_t) \neq p(\hat{x}_t)} \mathbb{E}_{p(\hat{x}_t)} \left[ (1 - p(\hat{x}_t)) \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right]}{\left(\delta \frac{\bar{L}}{N}\right)^{\gamma_r}}, \quad (\text{A.11})$$

as  $p(1)$  is independent from  $\{p(x_t)\}_{x_t \in X_N}$  and of mean 1. The LHS of (A.11) is strictly increasing in  $\alpha$  and  $\alpha > 0$ . Note that  $\mathbb{E}_{p(\hat{x}_t)} \left[ p(\hat{x}_t) \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right]$  is concave in  $p(\hat{x}_t)$  so

$$\mathbb{E}_{p(\hat{x}_t)} \left[ p(\hat{x}_t) \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right] < \left(1 + \sum_{x_t \in X_N \setminus \hat{x}_t} p(x_t)\right)^{-\gamma_r}, \quad (\text{A.12})$$

that  $\mathbb{E}_{p(\hat{x}_t)} \left[ \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right]$  is concave in  $p(\hat{x}_t)$  so

$$\mathbb{E}_{p(\hat{x}_t)} \left[ \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right] > \left(1 + \sum_{x_t \in X_N \setminus \hat{x}_t} p(x_t)\right)^{-\gamma_r}. \quad (\text{A.13})$$

It follows that  $\mathbb{E}_{p(\hat{x}_t)} \left[ (1 - p(\hat{x}_t)) \left(\sum_{x_t \in X_N} p(x_t)\right)^{-\gamma_r} \right] > 0$  and, therefore, that (A.10), i.e.,

workers' net utility from working in traded-good sector 1 rather than in  $\hat{x}_t$ , is strictly positive for  $L(1) = 0$ . We also know from the proof of [Claim 2](#) that if  $\alpha < \bar{\alpha}_N$ , [\(A.10\)](#) is strictly negative for  $L(1) = \bar{L}$ . Therefore if  $\alpha < \bar{\alpha}_N$ , there exists  $L(1) \in (0, \bar{L})$  such that [\(A.10\)](#) equals 0. Finally, the symmetry and independence of all traded-good sectors in  $X_N$  implies that if [\(A.10\)](#) equals 0 for some  $\hat{x}_t \in X_N$ , then it equals 0 for any  $x_t \in X_N$ . This shows existence of the equilibrium in [Claim 3](#) if  $\alpha < \bar{\alpha}_N$ .

For sufficiency and uniqueness, we use the equilibrium indifference condition that [\(A.10\)](#) equals 0 for all  $\hat{x}_t \in X_N$ . This implies a necessary equilibrium condition for  $L(1)$ :

$$\sum_{\hat{x}_t \in X_N} \frac{1}{N} \mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \delta p(\hat{x}_t)}{\left( (\delta + \alpha)L(1)p(1) + \frac{\bar{L} - L(1)}{N} \sum_{x_t \in X_N} \delta p(x_t) \right)^{\gamma_r}} \right] = 0 \quad (\text{A.14})$$

Then the derivative of the LHS of [\(A.14\)](#) with respect to  $L(1)$  is

$$-\gamma_r \mathbb{E} \left[ \frac{\left( (\delta + \alpha)p(1) - \frac{1}{N} \sum_{x_t \in X_N} \delta p(x_t) \right)^2}{\left( (\delta + \alpha)L(1)p(1) + \frac{\bar{L} - L(1)}{N} \sum_{x_t \in X_N} \delta p(x_t) \right)^{1+\gamma_r}} \right] < 0, \quad (\text{A.15})$$

which implies [\(A.14\)](#) has at most one solution. Furthermore, since we have just shown that the LHS of [\(A.14\)](#) is strictly positive for  $L(1) = 0$  and since [Claim 2](#) implies the LHS of [\(A.14\)](#) is positive for  $L(1) = \bar{L}$  if  $\alpha \geq \bar{\alpha}_N$  and strictly positive if  $\alpha > \bar{\alpha}_N$ , [\(A.14\)](#) has no solution in  $(0, \bar{L})$  if  $\alpha \geq \bar{\alpha}_N$ . This shows existence of the equilibrium in [Claim 3](#) only if  $\alpha < \bar{\alpha}_N$ . ■

To conclude, if  $\alpha \geq \bar{\alpha}$ , we have shown the existence of a (corner) equilibrium described in [Claim 2](#). If  $\alpha < \bar{\alpha}_N$ , we have shown that there is a unique (interior) equilibrium of the form described in [Claim 3](#). From [Claim 1](#), there is no other possible equilibrium. *QED*

## Proof of [Corollary 1](#)

$\lim_{N \rightarrow +\infty} L(x_t) = \frac{\bar{L} - L(1)}{N} = 0$ , which means that for all traded-good sectors  $x_t \in X_{+\infty}$ , there is a zero measure of workers devoted to the production of the traded good. Therefore, if there is a measure  $L(1)$  of workers devoted to the production of traded good 1 and a measure  $\bar{L} - L(1)$  of workers devoted to the production of traded goods other than traded-good 1 (with a zero measure devoted to the production of any one of these other traded goods), the city's income from the production of traded goods writes

$$Y_t = (\alpha + \delta)p_t(1)L(1) + \delta (\bar{L} - L(1)), \quad (\text{A.16})$$

an expression that makes use of the fact that  $\mathbb{E}[p_t(x_t)] = 1$  and that prices are i.i.d.. In the candidate equilibrium, workers' net utility from working in traded-good sector 1 rather than



in  $\hat{x}_t \in X_{+\infty}$  has the sign of

$$\mathbb{E} \left[ \frac{(\delta + \alpha)p_t(1) - \delta p(\hat{x}_t)}{Y_t^{\gamma_r}} \right] = \mathbb{E} \left[ \frac{(\delta + \alpha)p_t(1) - \delta}{[(\alpha + \delta)p_t(1)L(1) + \delta(\bar{L} - L(1))]^{\gamma_r}} \right] \quad (\text{A.17})$$

For  $L(1) = \bar{L}$ , (A.17) writes

$$\mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \delta}{[(\alpha + \delta)p_t(1)\bar{L}]^{\gamma_r}} \right] \quad (\text{A.18})$$

and following similar steps as in the proof of [Claim 2](#), it follows that there is a unique  $\bar{\alpha}_{+\infty} > 0$  such that (A.18) equals zero and that  $L(1) = \bar{L}$  is an equilibrium if and only if  $\alpha \geq \bar{\alpha}_{+\infty}$ .

For  $L(1) = 0$  and  $\alpha > 0$ , (A.17) is strictly positive. Following similar steps as in the proof of [Claim 3](#), it follows that there exists an equilibrium such that  $L(1) < \bar{L}$  if and only if  $\alpha < \bar{\alpha}_{+\infty}$ , and that  $L(1)$  is unique. *QED*

## Proof of [Proposition 2](#)

From (11), and since  $Y_t = \gamma_t Y$ , the social utility function writes

$$v_s = \frac{R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} \quad (\text{A.19})$$

Therefore, the social planner solves

$$\max_{\{L(x_t)\}_{x_t \in X_N \cup \{1\}}} \mathbb{E}(Y_t^{\gamma_t}) \quad (\text{A.20})$$

$$\text{s.t. } Y_t = (\delta + \alpha)L(1)p_t(1) + \sum_{x_t \in X_N} \delta L(x_t)p(x_t), \quad (\text{A.21})$$

$$\sum_{x_t \in X_N} L(x_t) + L(1) = \bar{L}, \quad (\text{A.22})$$

$$L(x_t) \geq 0 \text{ for } x_t \in X_N \cup \{1\}. \quad (\text{A.23})$$

First-order conditions with respect to  $L(1)$  and  $\{L(x_t)\}_{x_t \in X_N}$  write:

$$\gamma_t(\delta + \alpha)\mathbb{E} \left[ \frac{p_t(1)}{Y_t^{1-\gamma_t}} \right] - \mu + \lambda(1) = 0 \quad (\text{A.24})$$

$$\gamma_t \delta \mathbb{E} \left[ \frac{p_t(x_t)}{Y_t^{1-\gamma_t}} \right] - \mu + \lambda(x_t) = 0 \quad (\text{A.25})$$

where  $\mu$  and  $\lambda(x_t)$  are the non-negative Lagrange multipliers associated with the total labor constraint (A.22) and the non-negativity constraints (A.23).

**Claim 4** *At the social optimum, all sectors in  $X_n$  have the same amount of workers.*

**Proof.** By contradiction: suppose there exists  $\{x_t, x'_t\} \in X_n^2$  such that  $L(x_t) > L(x'_t)$ . This

implies  $L(x_t) > 0$ , therefore  $\lambda(x'_t) \geq \lambda(x_t) = 0$ . Thus (A.25) imply

$$\mathbb{E} \left[ \frac{p_t(x'_t) - p_t(x_t)}{Y_t^{\gamma_r}} \right] \leq 0. \quad (\text{A.26})$$

We have shown in the proof of [Claim 1](#) that if  $L(x_t) > L(x'_t)$ , this expression (which has the sign of (A.3)) is strictly positive. ■

[Claim 4](#) implies the social planner's problem reduces to

$$\max_{L(1) \in [0, \bar{L}]} \mathbb{E} \left[ \left( (\delta + \alpha)L(1)p_t(1) + \frac{\bar{L} - L(1)}{N} \sum_{x_t \in X_N} \delta p(x_t) \right)^{\gamma_t} \right] \quad (\text{A.27})$$

The first-order derivative of this objective function with respect to  $L(1)$  is

$$\gamma_t \mathbb{E} \left[ \frac{(\delta + \alpha)p(1) - \frac{1}{N} \sum_{x_t \in X_N} \delta p(x_t)}{\left( (\delta + \alpha)L(1)p_t(1) + \frac{\bar{L} - L(1)}{N} \sum_{x_t \in X_N} \delta p(x_t) \right)^{\gamma_r}} \right], \quad (\text{A.28})$$

which is, to the factor  $\gamma_t$ , the LHS of the equilibrium condition (A.14). Then the proof of [Claim 3](#) implies that the social planner's objective is strictly concave and that if  $\alpha < \bar{\alpha}_N$ , it admits an interior maximum  $L(1) \in (0, 1)$  given by the equilibrium condition (A.14). Similarly, if  $\alpha \geq \bar{\alpha}_N$ , (A.28) is strictly positive for  $L(1) \in (0, 1)$ , therefore the social optimum is  $L(1) = \bar{L}$ , and also coincides with equilibrium. *QED*

## Proof of [Lemma 1](#)

[Lemma 1](#) follows directly from (22) and (23).

## Proof of [Proposition 3](#)

The derivation of the unique equilibrium of the production subgame follows [Morris and Shin \(2003\)](#). Let  $\theta$  be the proportion of entrepreneurs that produce  $\frac{q_0}{2} + q_1$ , while  $1 - \theta$  produce  $\frac{q_0}{2}$ . Using (20), let

$$\Delta(\theta) \equiv \frac{\gamma_t^{\gamma_t} \gamma_{nt}^{\gamma_{nt}} \gamma_r^{\gamma_r} \times \frac{\gamma_{nt}}{\gamma_t} \frac{q_1}{q_0 + q_1} \frac{Y_t}{2}}{P_t^{\gamma_t} \left[ \left( \frac{1}{q_0 + q_1} \right)^\theta \left( \frac{1}{q_0} \right)^{1-\theta} \frac{\gamma_{nt}}{\gamma_t} Y_t \right]^{\gamma_{nt}} \left[ \frac{\gamma_r}{\gamma_t} \frac{Y_t}{R} \right]^{\gamma_r}} - c \quad (\text{A.29})$$

be the net payoff for an entrepreneur of producing  $\frac{q_0}{2} + q_1$  given that a proportion  $\theta$  of entrepreneurs produce  $\frac{q_0}{2} + q_1$ . Note that  $\Delta'(\cdot) > 0$ , i.e., entrepreneurs' decision to produce exhibit global strategic complementarities. Then the existence of upper and lower dominance regions (see [Assumption 1](#) and the discussion that follows) implies that if a unique equilibrium in threshold strategies exist, then it also is the unique equilibrium ([Morris and Shin \(2003\)](#)).

Assume that entrepreneur in sector  $x_{nt}$  produces  $\frac{q_0}{2} + q_1$  when her signal  $s_{x_{nt}}$  is above a

threshold  $s^T$ . Then, for a given  $Y_t$ , the mass of entrepreneurs that produce  $\frac{q_0}{2} + q_1$  is

$$\theta(Y_t) \equiv \begin{cases} 1 & \text{if } s^T < Y_t - \underline{\varepsilon} \\ \frac{Y_t + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}} & \text{if } s^T \in [Y_t - \underline{\varepsilon}, Y_t + \underline{\varepsilon}] \\ 0 & \text{if } s^T > Y_t + \underline{\varepsilon} \end{cases} . \quad (\text{A.30})$$

Let  $h(\cdot)$  be the probability density function (pdf) of  $Y_t$  (recall that  $h(\cdot)$  depends on workers' ex-ante specialization choices). For an entrepreneur who observes  $s_t$ , the pdf of the posterior distribution of  $Y_t$  is

$$\frac{\frac{1}{2\underline{\varepsilon}} h(Y_t)}{\int_{s_t - \underline{\varepsilon}}^{s_t + \underline{\varepsilon}} \frac{1}{2\underline{\varepsilon}} h(z) dz} = \frac{h(Y_t)}{\int_{s_t - \underline{\varepsilon}}^{s_t + \underline{\varepsilon}} h(z) dz} \quad (\text{A.31})$$

At the threshold  $s^T$ , entrepreneur  $x_{nt}$  must be indifferent between producing and not producing:

$$\int_{s^T - \underline{\varepsilon}}^{s^T + \underline{\varepsilon}} \Delta(\theta(Y_t)) \frac{h(Y_t)}{\int_{s_t - \underline{\varepsilon}}^{s_t + \underline{\varepsilon}} h(z) dz} dY_t = 0 \quad (\text{A.32})$$

The existence of upper- and lower-dominance region implies that that the LHS of (A.32) is strictly positive for  $s^T$  large enough and strictly negative for  $s^T$  low enough. It remains to study the monotonicity of the LHS of (A.32) for uniqueness.

For  $Y_t \in [s^T - \underline{\varepsilon}, s^T + \underline{\varepsilon}]$ ,  $\theta(Y_t) = \frac{Y_t + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}}$  and hence using (A.29), we can write (A.32) as

$$\int_{s^T - \underline{\varepsilon}}^{s^T + \underline{\varepsilon}} \left[ (q_0 + q_1) \frac{Y_t + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}} q_0^{1 - \frac{Y_t + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}}} \right]^{\gamma_{nt}} Y_t^{\gamma_t} \frac{h(Y_t)}{\int_{s^T - \underline{\varepsilon}}^{s^T + \underline{\varepsilon}} h(z) dz} dY_t = \frac{q_0 + q_1}{q_1} \frac{2cP_t^{\gamma_t}}{\gamma_{nt} R^{\gamma_r}} \quad (\text{A.33})$$

Changing variables,  $\theta = \frac{Y_t + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}}$  and  $v = \frac{z + \underline{\varepsilon} - s^T}{2\underline{\varepsilon}}$ , we can write (A.33) as

$$\begin{aligned} & \int_0^1 \left[ (q_0 + q_1)^\theta q_0^{1-\theta} \right]^{\gamma_{nt}} (\underline{\varepsilon}(2\theta - 1) + s^T)^{\gamma_t} \frac{h(\underline{\varepsilon}(2\theta - 1) + s^T)}{\int_0^1 h(\underline{\varepsilon}(2v - 1) + s^T) dv} d\theta \\ &= \frac{q_0 + q_1}{q_1} \frac{2cP_t^{\gamma_t}}{\gamma_{nt} R^{\gamma_r}} \end{aligned} \quad (\text{A.34})$$

Note that  $h$  is the density of a linear combination of uniform random variable, hence is differentiable almost everywhere. At any point of differentiability, the derivative of integrand of the LHS of (A.34) with respect to  $s^T$  has the sign of

$$\begin{aligned} & \gamma_t (\underline{\varepsilon}(2\theta - 1) + s^T)^{\gamma_t - 1} \frac{h(\underline{\varepsilon}(2\theta - 1) + s^T)}{\int_0^1 h(\underline{\varepsilon}(2v - 1) + s^T) dv} \\ &+ \frac{(\underline{\varepsilon}(2\theta - 1) + s^T)^{\gamma_t} \times}{\left( \int_0^1 h(\underline{\varepsilon}(2v - 1) + s^T) dv \right)^2} \\ & \frac{h'(\underline{\varepsilon}(2\theta - 1) + s^T) \int_0^1 h(\underline{\varepsilon}(2v - 1) + s^T) dv - h(\underline{\varepsilon}(2\theta - 1) + s^T) \int_0^1 h'(\underline{\varepsilon}(2v - 1) + s^T) dv}{\left( \int_0^1 h(\underline{\varepsilon}(2v - 1) + s^T) dv \right)^2} \end{aligned} \quad (\text{A.35})$$

which tends to  $\gamma_t(s^T)^{\gamma_t-1} > 0$  as  $\underline{\varepsilon}$  tends to 0. It follows that for  $\underline{\varepsilon}$  sufficiently small, (A.32) has a unique solution.

Taking the limit of (A.34) as  $\underline{\varepsilon} \rightarrow 0$ , we obtain

$$\int_0^1 (q_0 + q_1)^{\gamma_{nt}\theta} q_0^{\gamma_{nt}(1-\theta)} (Y_t^T)^{\gamma_t} d\theta = \frac{q_0 + q_1}{q_1} \frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}} \quad (\text{A.36})$$

where  $Y_t^T \equiv \lim_{\underline{\varepsilon} \rightarrow 0} s^T(\underline{\varepsilon})$ . (A.36) writes

$$\frac{(Y_t^T)^{\gamma_t}}{\gamma_{nt}} \left[ \frac{(q_0 + q_1)^{\gamma_{nt}\theta} q_0^{\gamma_{nt}(1-\theta)}}{\ln(q_0 + q_1) - \ln q_0} \right]_0^1 = \frac{q_0 + q_1}{q_1} \frac{2cP_t^{\gamma_t}}{\gamma_{nt}R^{\gamma_r}}, \quad (\text{A.37})$$

which in turn writes

$$\left( \frac{Y_t^T}{P_t} \right)^{\gamma_t} = \frac{\ln(q_0 + q_1) - \ln q_0}{(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}} \frac{q_0 + q_1}{q_1} \frac{2c}{R^{\gamma_r}}. \quad (\text{A.38})$$

Finally, to show that (27) holds, rewrite (A.38) as

$$c = \frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} \frac{1}{2} \frac{\ln(\frac{q_0}{q_1} + 1)}{\frac{q_0}{q_1} + 1} \quad (\text{A.39})$$

and substitute  $c$  in (27) to obtain after simplification

$$1 - \frac{1}{2} \left( \ln\left(\frac{q_1}{q_0} + 1\right) \left(\frac{q_0}{q_1} + 1\right) \right)^{-1} > 0, \quad (\text{A.40})$$

which holds because  $f(x) \equiv (\frac{1}{x} + 1)(x + 1) > 1$  for  $x > 0$  since  $\lim_{x \rightarrow 0} f(x) = 1$  and  $f'(x) > 0$  for  $x > 0$ . *QED*

## Proof of Proposition 4

A worker's net benefit from specializing in traded-good sector 1 rather than in traded-good sector  $x_t$  is

$$\mathbb{E}_{p_t(1), p_t(x_t)} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\delta + \alpha)p_t(1) - \delta p_t(x_t))}{(L(1)(\alpha + \delta)p_t(1) + (\bar{L} - L(1))\delta p_t(x_t))^{1-\gamma_t}} \right] \quad (\text{A.41})$$

Suppose (A.41) is strictly positive when evaluated at  $L(1) = \frac{\bar{L}}{2}$ . Then either (A.41) is always strictly positive for any  $L(1) > \frac{\bar{L}}{2}$  and then  $L(1) = \bar{L}$  is a (corner) equilibrium, or (A.41) crosses 0 for some  $L(1)$  strictly greater than  $\frac{\bar{L}}{2}$ , which is then an equilibrium.

To show Proposition 4, it is therefore sufficient to show (A.41) is strictly positive for  $L(1) = \frac{\bar{L}}{2}$ , which is equivalent to

$$\mathbb{E}_{p_t(1), p_t(x_t)} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\delta + \alpha)p_t(1) - \delta p_t(x_t))}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} \right] > 0 \quad (\text{A.42})$$

We consider three cases: (i)  $Y_t^T < \delta\bar{L}$ , (ii)  $\delta\bar{L} < Y_t^T < (\alpha + \delta)\bar{L}$ , and (iii)  $(\alpha + \delta)\bar{L} < Y_t^T <$

$2\delta\bar{L}$ . (Note: [Assumption 1](#) implies  $Y_t^T < 2\delta\bar{L}$ .)

**Case (i):**  $Y_t^T < \delta\bar{L}$

Given  $Y_t^T < \delta\bar{L}$  and  $L(1) = \frac{\bar{L}}{2}$ ,

- if  $p_t(x_t) \in [\frac{2Y_t^T}{L\delta}, 2)$ ,  $Y_t \geq Y_t^T$  for all  $p_t(1) \in (0, 2)$ ,
- if  $p_t(x_t) \in (0, \frac{2Y_t^T}{L\delta})$ ,  $Y_t \geq Y_t^T$  for  $p_t(1) \in \left(\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\bar{L}}{2}}, 2\right)$ .<sup>A1</sup>

Therefore, the LHS of [\(A.42\)](#) (the worker's net benefit from specializing in sector 1) has the sign of

$$\begin{aligned} & \int_{\frac{2Y_t^T}{L\delta}}^2 \left[ \int_0^2 (q_0 + q_1)^{\gamma_{nt}} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} dp_t(1) \right] dp_t(x_t) \\ & + \int_0^{\frac{2Y_t^T}{L\delta}} \left[ \int_{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\bar{L}}{2}}}^2 (q_0 + q_1)^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} dp_t(1) + \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\bar{L}}{2}}} q_0^{\gamma_{nt}} \frac{(\alpha+\delta)p_t(1) - \delta p_t(x_t)}{((\alpha+\delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} dp_t(1) \right] dp_t(x_t) \end{aligned} \quad (\text{A.43})$$

Note first that if  $\alpha = 0$ , then [\(A.43\)](#) is equal to 0: if both sectors have the same productivity and the same number of workers, then each worker is indifferent between working in sector 1 or  $x_t$ . Then to show [\(A.43\)](#) is strictly positive for  $\alpha > 0$ , it is sufficient to show that [\(A.43\)](#) is strictly increasing in  $\alpha$  for  $\alpha > 0$ , which is what we do next.

A marginal increase in  $\alpha$  affects both the integrands in [\(A.43\)](#) and the boundaries of the integrals in  $p_t(1)$ .<sup>A2</sup> For any  $p_t(1)$  and  $p_t(x_t)$ ,

$$\frac{\partial}{\partial \alpha} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1-\gamma_t}} = \frac{p_t(1)(\gamma_t(\alpha + \delta)p_t(1) + (2 - \gamma_t)\delta p_t(x_t))}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{2-\gamma_t}} > 0, \quad (\text{A.44})$$

that is, the effect on the integrands is strictly positive. The effect of a marginal increase in  $\alpha$

<sup>A1</sup>Note: For  $Y_t^T < \delta\bar{L}$  and  $L(1) = \frac{\bar{L}}{2}$ ,  $\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha+\delta)\frac{\bar{L}}{2}} \in (0, 2)$  for all  $p_t(x_t) \in (0, \frac{2Y_t^T}{L\delta})$ .

<sup>A2</sup>The boundaries of the integral in  $p_t(x_t)$  are independent from  $\alpha$  as  $Y_t^T$  is independent from  $\alpha$  (see [\(26\)](#)).

on the boundaries of the integral in  $p_t(1)$  has the sign of

$$\int_0^{\frac{2Y_t^T}{\bar{L}\delta}} (Y_t^T - \delta\bar{L}p_t(x_t)) \left( Y_t^T - \delta\frac{\bar{L}}{2}p_t(x_t) \right) dp_t(x_t) \quad (\text{A.45})$$

$$= \left[ (Y_t^T)^2 p_t(x_t) - \frac{3}{4} Y_t^T \delta\bar{L} (p_t(x_t))^2 + \frac{1}{6} \delta^2 \bar{L}^2 (p_t(x_t))^3 \right]_0^{\frac{2Y_t^T}{\bar{L}\delta}} \quad (\text{A.46})$$

$$= (Y_t^T)^2 \frac{2Y_t^T}{\bar{L}\delta} - 3 \frac{(Y_t^T)^3}{\bar{L}\delta} + \frac{4}{3} \frac{(Y_t^T)^3}{\bar{L}\delta} \quad (\text{A.47})$$

$$= \frac{(Y_t^T)^3}{\bar{L}\delta} \left[ 2 - 3 + \frac{4}{3} \right] > 0 \quad (\text{A.48})$$

To sum up, (A.43) equals 0 for  $\alpha = 0$ , is strictly increasing in  $\alpha$  and is therefore strictly positive for any  $\alpha > 0$ .

**Case (ii):**  $\delta\bar{L} < Y_t^T < (\alpha + \delta)\bar{L}$

For  $\delta\bar{L} < Y_t^T < (\alpha + \delta)\bar{L}$  and  $L(1) = \frac{\bar{L}}{2}$ , if  $p_t(x_t) \in (0, 2)$ ,  $Y_t < Y_t^T$  for  $p_t(1) \in \left( 0, \frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}} \right)$ .

Therefore the LHS of (A.42) has the sign of

$$\int_0^2 \left[ \int_{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}}^{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}} (q_0 + q_1)^{\gamma_{nt}} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1 - \gamma_t}} dp_t(1) + \int_0^{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}} q_0^{\gamma_{nt}} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1 - \gamma_t}} dp_t(1) \right] dp_t(x_t) \quad (\text{A.49})$$

Similarly to *Case (i)*, (A.49) is equal to 0 if  $\alpha = 0$  and the effect of a marginal increase in  $\alpha$  in (A.49) is strictly positive if the effect through the boundaries of the integral in  $p_t(1)$  is positive. This effect has the sign of

$$\int_{\frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}}^2 (Y_t^T - \delta\bar{L}p_t(x_t)) \left( Y_t^T - \delta\frac{\bar{L}}{2}p_t(x_t) \right) dp_t(x_t) \quad (\text{A.50})$$

$$= \left[ (Y_t^T)^2 p_t(x_t) - \frac{3}{4} Y_t^T \delta\bar{L} (p_t(x_t))^2 + \frac{1}{6} \delta^2 \bar{L}^2 (p_t(x_t))^3 \right]_0^2 \quad (\text{A.51})$$

$$= 2(Y_t^T)^2 - 3Y_t^T \delta\bar{L} + \frac{4}{3} \delta^2 \bar{L}^2 \quad (\text{A.52})$$

At  $Y_t^T = \delta\bar{L}$ , (A.52) is positive (i.e.,  $\delta^2 \bar{L}^2 (2 - 3 + \frac{4}{3}) > 0$ ). The derivative w.r.t.  $Y_t^T$  of (A.52) (i.e.,  $Y_t^T, 4Y_t^T - 3\delta\bar{L}$ ) is positive for  $Y_t^T > \delta\bar{L}$ , and therefore, (A.52) is positive for  $\delta\bar{L} < Y_t^T < (\alpha + \delta)\bar{L}$ .

**Case (iii):**  $(\alpha + \delta)\bar{L} < Y_t^T < 2\delta\bar{L}$ .

Given  $(\alpha + \delta)\bar{L} < Y_t^T < 2\delta\bar{L}$  and  $L(1) = \frac{\bar{L}}{2}$ ,

- if  $p_t(x_t) > \frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}$ ,  $Y_t < Y_t^T$  for  $p_t(1) \in \left(0, \frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}\right)$ ,
- if  $p_t(x_t) < \frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}$ ,  $Y_t < Y_t^T$  for  $p_t(1) \in (0, 2)$ .

Therefore, for  $Y_t^T > \bar{L}(\alpha + \delta)$ , the LHS of (A.42) has the sign of

$$\int_{\frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}}^2 \left[ \frac{\int_{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}}^2 (q_0 + q_1)^{\gamma_{nt}} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1 - \gamma_t}} dp_t(1) +}{\frac{Y_t^T - \frac{\bar{L}}{2}\delta p_t(x_t)}{(\alpha + \delta)\frac{\bar{L}}{2}}} \right] dp_t(x_t) \quad (\text{A.53})$$

$$+ \int_0^{\frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}} \left[ \int_0^2 q_0^{\gamma_{nt}} \frac{(\alpha + \delta)p_t(1) - \delta p_t(x_t)}{((\alpha + \delta)p_t(1) + \delta p_t(x_t))^{1 - \gamma_t}} dp_t(1) \right] dp_t(x_t)$$

Similarly to *Case (i)*, (A.53) is equal to 0 if  $\alpha = 0$ , and the effect of a marginal increase in  $\alpha$  in (A.53) is strictly positive if the effect though the boundaries of the integral in  $p_t(1)$  is positive.<sup>A3</sup> This effect has the sign of

$$\int_{\frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}}^2 (Y_t^T - \delta \bar{L} p_t(x_t)) \left( Y_t^T - \delta \frac{\bar{L}}{2} p_t(x_t) \right) dp_t(x_t) \quad (\text{A.54})$$

Equation (A.54) writes

$$\left[ (Y_t^T)^2 p_t(x_t) - \frac{3}{4} Y_t^T \delta \bar{L} (p_t(x_t))^2 + \frac{1}{6} \delta^2 \bar{L}^2 (p_t(x_t))^3 \right]_{\frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}}^2 \quad (\text{A.55})$$

$$= 2 (Y_t^T)^2 - 3 Y_t^T \delta \bar{L} + \frac{4}{3} \delta^2 \bar{L}^2 \quad (\text{A.56})$$

$$- \left[ 2 (Y_t^T)^2 \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta \bar{L}} - 3 Y_t^T \delta \bar{L} \left( \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta \bar{L}} \right)^2 + \frac{4}{3} \delta^2 \bar{L}^2 \left( \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta \bar{L}} \right)^3 \right]$$

---

<sup>A3</sup>Note that the boundaries of the integral in  $p_t(1)$  do depend on  $\alpha$ , unlike in case (i). However, at  $p_t(1) = \frac{2(Y_t^T - \bar{L}(\alpha + \delta))}{\bar{L}\delta}$ , the first and second terms between bracket in (A.53) are equal and therefore the marginal effect of  $\alpha$  through the boundaries of the integral in  $p_t(1)$  is 0.

The first derivative of (A.56) with respect to  $Y_t^T$  writes

$$4Y_t^T - 3\delta\bar{L} - \left[ 4Y_t^T \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} - 3\delta\bar{L} \left( \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} \right)^2 \right] \quad (\text{A.57})$$

$$- \left[ \frac{2(Y_t^T)^2}{\delta\bar{L}} - 6Y_t^T \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} + 4\delta\bar{L} \left( \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} \right)^2 \right]$$

$$= 4Y_t^T - 3\delta\bar{L} - \left[ \frac{2(Y_t^T)^2}{\delta\bar{L}} - 2Y_t^T \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} + \delta\bar{L} \left( \frac{Y_t^T - \bar{L}(\alpha + \delta)}{\delta\bar{L}} \right)^2 \right] \quad (\text{A.58})$$

$$= 2Y_t^T \frac{\delta - \alpha}{\delta} - 3\delta\bar{L} - \frac{[Y_t^T - \bar{L}(\alpha + \delta)]^2}{\delta\bar{L}} \quad (\text{A.59})$$

Note the following four facts:

1. (A.59) evaluated at  $Y_t = 2\bar{L}\delta$  is negative:

$$4\bar{L}\delta \frac{\delta - \alpha}{\delta} - 3\delta\bar{L} - \frac{[2\bar{L}\delta - \bar{L}(\alpha + \delta)]^2}{\delta\bar{L}} = -\bar{L}\alpha \left[ 2 + \frac{\alpha}{\delta} \right] < 0 \quad (\text{A.60})$$

2. (A.59) evaluated at  $Y_t = \bar{L}\delta$  is negative:

$$2\bar{L}\delta \frac{\delta - \alpha}{\delta} - 3\delta\bar{L} - \frac{[\bar{L}\delta - \bar{L}(\alpha + \delta)]^2}{\delta\bar{L}} = -\bar{L} \left[ 2\alpha + \delta + \frac{\alpha^2}{\delta} \right] < 0 \quad (\text{A.61})$$

3. The second derivative of (A.56) with respect to  $Y_t^T$  is positive:

$$\frac{2}{\delta\bar{L}} [2\delta\bar{L} - Y_t^T] > 0 \quad (\text{A.62})$$

4. (A.54) evaluated at  $Y_t^T = 2\delta\bar{L}$  is strictly positive since the integrand is then strictly positive for any  $p_t(x_t) \in (0, 2)$ .

Facts (1), (2) and (3) imply that (A.54) is strictly decreasing in  $Y_t^T$  for  $Y_t^T \in (\delta\bar{L}, 2\delta\bar{L})$ . Together with (4), this implies that (A.54) is strictly positive for  $Y_t^T \in (\delta\bar{L}, 2\delta\bar{L})$ . *QED*

## Proof of Proposition 5

Workers' net utility from specializing in traded-good sector 1 rather than in traded-good sector  $x_t$  has the sign of

$$\mathbb{E} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\delta + \alpha)p_t(1) - \delta p_t(x_t))}{Y_t^{1-\gamma_t}} \right] \quad (\text{A.63})$$

where  $Y_t = (\delta + \alpha)L(1)p_t(1) + \delta(\bar{L} - L(1))p_t(x_t)$ . If  $L(1) = \bar{L}$ , (A.63) becomes

$$\mathbb{E}_{p_t(1), p_t(x_t)} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\delta + \alpha)p_t(1) - \delta p_t(x_t))}{((\delta + \alpha)\bar{L}p_t(1))^{1-\gamma_t}} \right] \quad (\text{A.64})$$



For  $L(1) = \bar{L}$ ,  $Y_t$  and  $Q_{nt}^{\gamma_{nt}}$  do not depend on  $p_t(x_t)$ , and hence, (A.64) has the same sign as

$$\mathbb{E}_{p_t(1)} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\delta + \alpha)p_t(1) - \delta)}{p_t(1)^{1-\gamma_t}} \right] \quad (\text{A.65})$$

$$= \frac{1}{2} \int_{\frac{Y_t^T}{(\alpha+\delta)\bar{L}}}^2 (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \frac{(\delta + \alpha)p_t(1) - \delta}{p_t(1)^{1-\gamma_t}} dp_t(1) \quad (\text{A.66})$$

$$\begin{aligned} &+ \frac{1}{2} \int_0^{\frac{Y_t^T}{(\alpha+\delta)\bar{L}}} q_0^{\gamma_{nt}} \frac{(\delta + \alpha)p_t(1) - \delta}{p_t(1)^{1-\gamma_t}} dp_t(1) \\ &= \frac{1}{2} (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) 2^{\gamma_t} \left[ \frac{2(\delta + \alpha)}{\gamma_t + 1} - \frac{\delta}{\gamma_t} \right] \\ &\quad - \frac{1}{2} [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \left( \frac{Y_t^T}{(\alpha + \delta)\bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\bar{L}}}{\gamma_t + 1} - \frac{\delta}{\gamma_t} \right] \end{aligned} \quad (\text{A.67})$$

Notice that

$$\frac{\frac{Y_t^T}{\bar{L}}}{\gamma_t + 1} - \frac{\delta}{\gamma_t} = \frac{\gamma_t \frac{Y_t^T}{\bar{L}} - (\gamma_t + 1)\delta}{(\gamma_t + 1)\gamma_t} < \frac{\frac{Y_t^T}{\bar{L}} - 2\delta}{(\gamma_t + 1)} < 0 \quad (\text{A.68})$$

which means that (A.65) is increasing in  $\alpha$  and tends  $+\infty$  as  $\alpha \rightarrow +\infty$ .

Evaluated at  $\alpha = 0$ , equation (A.67) is negative:

$$= (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} \left[ \frac{2}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta \bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta \bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \quad (\text{A.69})$$

$$< (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta \bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta \bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta \bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \quad (\text{A.70})$$

$$= \left[ \frac{\frac{Y_t^T}{\delta \bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \left( (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta \bar{L}} \right)^{\gamma_t} \right) < 0, \quad (\text{A.71})$$

Therefore, (A.65) is increasing in  $\alpha$ , tends  $+\infty$  as  $\alpha \rightarrow +\infty$ , and is negative for  $\alpha = 0$ . This implies that there is a unique  $\alpha^* > 0$  such that  $L(1) = \bar{L}$  is an equilibrium if and only if  $\alpha > \alpha^*$ . From Proposition 4, we know that for  $\alpha > 0$  there always exists an equilibrium such that  $L(1) > \frac{\bar{L}}{2}$  and therefore, for  $\alpha \in (0, \alpha^*)$  there is an interior equilibrium such that  $L(1) > \frac{\bar{L}}{2} > L(x_t) > 0$ . *QED*

## Proof of Proposition 6

From eq. 33,  $\mathbb{E}[v_s(y_i)]$ , writes

$$\mathbb{E}[v_s(Y)] = \mathbb{E} \left[ \frac{Q_{nt}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c \mathbb{1}_{Y_t \geq Y_t^T} \right], \quad (\text{A.72})$$

Since

$$Y_t = L(1)(\alpha + \delta)p_t(1) + (\bar{L} - L(1))\delta p_t(x_t), \quad (\text{A.73})$$

the event  $Y_t \geq Y_t^T$  can be written as

$$p_t(1) \geq \frac{Y_t^T - (\bar{L} - L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)} \equiv p_t^T(p_t(x_t)). \quad (\text{A.74})$$

We first consider the case in which  $Y_t^T < \delta\bar{L}$  and then, the case in which  $Y_t^T > \delta\bar{L}$ .

**Case ( $Y_t^T < \delta\bar{L}$ ):** Note that

$$p_t^T(0) < 2 \Leftrightarrow L(1) > \frac{Y_t^T}{2(\alpha + \delta)} \quad (\text{A.75})$$

which holds if  $Y_t^T < \delta\bar{L}$  and  $L(1)^* > \frac{\bar{L}}{2}$ . This implies that even if  $p_t(x_t) = 0$ , there exists  $p_t(1)$  high enough such that  $Y_t > Y_t^T$ . On the other hand, for  $p_t(x_t)$  high enough, it could be that  $Y_t > Y_t^T$  for all  $p_t(1) \in (0, 2)$ . This is the case if

$$(\bar{L} - L(1))\delta p_t(x_t) > Y_t^T \Leftrightarrow p_t(x_t) > \frac{Y_t^T}{(\bar{L} - L(1))\delta} \quad (\text{A.76})$$

Therefore, (A.72) writes

$$\begin{aligned} & \frac{1}{4} \int_{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}}^2 \left[ \int_0^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right] dp_t(x_t) \\ & + \frac{1}{4} \int_0^{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} \left[ \int_{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t). \end{aligned} \quad (\text{A.77})$$

The derivative with respect to  $Y_t$  of the integrands in (A.77) is proportional to

$$\begin{aligned} & \frac{\gamma_t}{4} \int_{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}}^2 \left[ \int_0^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} [(\alpha + \delta)p_t(1) - \delta p_t(x_t)]}{N_t^{\gamma_t} Y_t^{1-\gamma_t}} dp_t(1) \right] dp_t(x_t) \\ & + \frac{\gamma_t}{4} \int_0^{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} \left[ \int_{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} [(\alpha+\delta)p_t(1) - \delta p_t(x_t)]}{N_t^{\gamma_t} Y_t^{\gamma_t}} dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} [(\alpha+\delta)p_t(1) - \delta p_t(x_t)]}{N_t^{\gamma_t} Y_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t) \\ & = \gamma_t (\mathbb{E}[v((\alpha + \delta)p_t(1))] - \mathbb{E}[v(\delta p_t(x_t))]) \end{aligned} \quad (\text{A.79})$$

where  $v(y_i)$  is defined as in eq. 30. At any interior equilibrium  $\mathbb{E}[v((\alpha + \delta)p_t(1))] = \mathbb{E}[v(\delta p_t(x_t))]$ , and therefore, (A.79) equals zero. Therefore, the effect not internalized by the workers in the specialization choices is the effect of  $L(1)$  on the boundaries of the integrals in (A.77). Consider first the effect on the integral with respect to  $p_t(x_t)$ . If  $\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\} = 2$  this effect is 0. If  $\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\} = \frac{Y_t^T}{(\bar{L}-L(1))\delta}$ , the marginal

effect of  $L(1)$  on the boundaries of the integrals with respect to  $p_t(x_t)$  in (A.77) is

$$\frac{1}{4} \frac{\partial \frac{Y_t^T}{(\alpha+\delta)L(1)}}{\partial L(1)} \left[ \begin{array}{c} \int_0^2 \left( \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c \right) dp_t(1) \\ - \int_0^2 \left( \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c \right) dp_t(1) \end{array} \right] = 0. \quad (\text{A.80})$$

Therefore, it follows that the sign of the only effect not internalized by individual workers is driven by the effect of  $L(1)$  on the boundaries of the integrals with respect to  $p_t(1)$  in (A.77). At the margin, this effect is

$$\frac{1}{4} \left[ \frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c \right] \int_0^{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} \frac{Y_t^T - \delta\bar{L}p_t(x_t)}{(\alpha + \delta)(L(1))^2} dp_t(x_t) \quad (\text{A.81})$$

From the definition of  $Y_t^T$  in Proposition 3

$$\frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \quad (\text{A.82})$$

so the marginal effect of  $L(1)$  that workers do not internalize has the same sign as

$$\int_0^{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} (Y_t^T - \delta\bar{L}p_t(x_t)) dp_t(2) \quad (\text{A.83})$$

$$= \left[ p_t(x_t)Y_t^T - \delta\bar{L}\frac{p_t(x_t)^2}{2} \right]_0^{\min\left\{2, \frac{Y_t^T}{(\bar{L}-L(1))\delta}\right\}} \quad (\text{A.84})$$

If  $Y_t^T < 2(\bar{L} - L(1))\delta$ , (A.84) writes  $2(Y_t^T - \delta\bar{L})$ , which is negative since we are considering the case in which  $Y_t^T < \delta\bar{L}$ . Alternatively,  $Y_t^T \geq 2(\bar{L} - L(1))\delta$ , (A.83) writes,

$$\left( \frac{Y_t^T}{(\bar{L} - L(1))\delta} \right)^2 \frac{\delta}{2} [\bar{L} - 2L(1)] < 0, \quad (\text{A.85})$$

since  $L^*(1) > \frac{\bar{L}}{2}$ . Therefore, if  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1)$  increases welfare.

**Case ( $Y_t^T > \delta\bar{L}$ ):**Note that

$$p_t^T(2) > 0 \Leftrightarrow L(1) > \bar{L} - \frac{Y_t^T}{2\delta} \quad (\text{A.86})$$

which holds if  $Y_t^T > \delta\bar{L}$  and  $L(1)^* > \frac{\bar{L}}{2}$ . This implies that even if  $p_t(x_t) = 2$ , there exists  $p_t(1)$  low enough that  $Y_t < Y_t^T$ . On the other hand, for  $p_t(x_t)$  low enough, it could be that  $Y_t < Y_t^T$  for all  $p_t(1) \in (0, 2)$ . This is the case if

$$L(1)(\alpha + \delta)2 + (\bar{L} - L(1))\delta p_t(x_t) < Y_t^T \Leftrightarrow p_t(x_t) < \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \quad (\text{A.87})$$

Therefore, (A.72) writes

$$\begin{aligned} & \frac{1}{4} \int_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta}\right\}}^2 \left[ \int_{\frac{Y_t^T - (\bar{L} - L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)}}^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c \, dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L} - L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} \, dp_t(1) \right] dp_t(x_t) \quad (\text{A.88}) \\ & + \frac{1}{4} \int_0^{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta}\right\}} \left[ \int_0^2 \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} \, dp_t(1) \right] dp_t(x_t) \end{aligned}$$

(Note: If  $Y_t^T < 2\bar{L}\delta$  then  $\frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} < 2$ .) Similarly to the previous **Case** ( $Y_t^T > \delta\bar{L}$ ), the sign of the effect not internalized is driven by the effect on the boundaries of the integrals with respect to  $L(1)$ . From the definition of  $Y_t^T$  in [Proposition 3](#)

$$\frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \quad (\text{A.89})$$

so the marginal effect of  $L(1)$  that workers do not internalize has the same sign a

$$\int_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta}\right\}}^2 (Y_t^T - \delta\bar{L}p_t(x_t)) \, dp_t(x_t) \quad (\text{A.90})$$

$$= \left[ p_t(x_t)Y_t^T - \delta\bar{L}\frac{p_t(x_t)^2}{2} \right]_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta}\right\}} \quad (\text{A.91})$$

If  $Y_t^T < L(1)(\alpha + \delta)2$ , (A.91) writes  $2(Y_t^T - \delta\bar{L})$  which is positive since we are now considering the case in which  $Y_t^T > \delta\bar{L}$ . Alternatively, if  $Y_t^T \geq L(1)(\alpha + \delta)2$ , (A.91) writes,

$$Y_t^T \left[ 2 - \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \right] - \frac{\delta\bar{L}}{2} \left[ 2^2 - \left( \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \right)^2 \right], \quad (\text{A.92})$$

whose sign is the same as the sign of

$$Y_t^T - \frac{\delta\bar{L}}{2} \left[ 2 + \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \right] \quad (\text{A.93})$$

$$= \frac{Y_t^T \left( \frac{\bar{L}}{2} - L(1) \right) + \bar{L}[\delta(2L(1) - \bar{L}) + L(1)\alpha]}{\bar{L} - L(1)} \quad (\text{A.94})$$

$$= \frac{Y_t^T \left( L(1) - \frac{\bar{L}}{2} \right) (2\bar{L}\delta - Y_t^T) + \bar{L}L(1)\alpha}{\bar{L} - L(1)} > 0 \quad (\text{A.95})$$

where the inequality follows from  $2\bar{L}\delta > Y_t^T$  and  $L(1) > \frac{\bar{L}}{2}$ . Therefore, if  $Y_t^T > \delta\bar{L}$ , a marginal increase in  $L(1)$  from  $L^*(1)$  increases welfare. *QED*

## Proof of Proposition 7

The welfare function writes as in (A.72). We first consider the case in which  $Y_t^T < \delta\bar{L}$  and then, the case in which  $Y_t^T > \delta\bar{L}$ .

**Case ( $Y_t^T < \delta\bar{L}$ ):** Following similar steps as the ones as in the **Case ( $Y_t^T < \delta\bar{L}$ )** in the proof of Proposition 6, for  $Y_t^T < \delta\bar{L}$ , the welfare function (A.72) writes as in (A.77). The derivative with respect to  $Y_t$  of the integrands in (A.77) is proportional to

$$\gamma_t (\mathbb{E} [v((\alpha + \delta)p_t(1))] - \mathbb{E} [v(\delta p_t(x_t))]) \quad (\text{A.96})$$

(see (A.79).) At a corner equilibrium  $L^*(1) = \bar{L}$ , (A.96) is zero for  $\alpha = \alpha^*$  and strictly positive for  $\alpha > \alpha^*$ . (See proof of Proposition 5.) From **Case ( $Y_t^T < \delta\bar{L}$ )** in the proof of Proposition 6, the derivative with respect to  $L(1)$  on the boundaries of the integrals in (A.77) is negative. Therefore, if  $Y_t^T < \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1) = \bar{L}$  increases or decreases welfare depending on whether in (A.77), the effect that works through the integrands or the that works through the boundaries of the integrals dominates.

**Case ( $Y_t^T > \delta\bar{L}$ ):** Following similar steps as the ones as in the **Case ( $Y_t^T > \delta\bar{L}$ )** in the proof of Proposition 6, for  $Y_t^T > \delta\bar{L}$ , the welfare function (A.72) writes as in (A.88). The derivative with respect to  $Y_t$  of the integrands in (A.88) is again proportional to

$$\gamma_t (\mathbb{E} [v((\alpha + \delta)p_t(1))] - \mathbb{E} [v(\delta p_t(x_t))]), \quad (\text{A.97})$$

and, at corner equilibrium  $L^*(1) = \bar{L}$ , (A.97) is zero for  $\alpha = \alpha^*$  and strictly positive for  $\alpha > \alpha^*$ . (See proof of Proposition 5.) From **Case ( $Y_t^T > \delta\bar{L}$ )** in the proof of Proposition 6, the derivative with respect to  $L(1)$  on the boundaries of the integrals in (A.77) is also positive. Therefore, if  $Y_t^T > \delta\bar{L}$ , a marginal decrease in  $L(1)$  from  $L^*(1) = \bar{L}$  decreases welfare. *QED*

## Proof of Proposition 8

From (33),  $\mathbb{E}[v_s(y_i)]$ , writes as in (A.72) and the event  $Y_t \geq Y_t^T$  can be written as in (A.74):

$$p_t(1) \geq \frac{Y_t^T - (\bar{L} - L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)} \equiv p_t^T(p_t(x_t)). \quad (\text{A.98})$$

In turn we consider (i)  $Y_t^T < \delta\bar{L}$  and  $0 < L^*(1) < L^*(x_t)$ ; (ii)  $Y_t^T > \delta\bar{L}$  and  $0 < L^*(1) < L^*(x_t)$ ; (iii)  $Y_t^T < \delta\bar{L}$  and  $L^*(x_t) = \bar{L}$ ; (iv)  $L^*(x_t) = \bar{L}$ .

**Case (i):**  $Y_t^T < \delta\bar{L}$  and  $0 < L^*(1) < L^*(x_t)$ :

$$p_t^T(x_t) < 0 \Leftrightarrow L(1) < \bar{L} - \frac{Y_t^T}{2\delta} \quad (\text{A.99})$$

which holds if  $Y_t^T < \delta\bar{L}$  and  $L(1)^* < \frac{\bar{L}}{2} < L^*(x_t)$ . This implies that for  $p_t(x_t)$  high enough

(positive but smaller than 2 given [Assumption 1](#)),  $Y_t > Y_t^T$ . However for  $p_t(x_t) = 0$ , there may exist  $p_t(1)$  high enough that  $Y_t > Y_t^T$ .

Therefore, [\(A.72\)](#) writes:

$$\begin{aligned} & \int_{\frac{Y_t^T}{(\bar{L}-L(1))\delta}}^2 \left[ \int_0^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right] dp_t(x_t) + \\ & + \int_{\frac{Y_t^T}{(\bar{L}-L(1))\delta}}^{\frac{Y_t^T}{(\bar{L}-L(1))\delta}} \left[ \int_{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}}^2 \frac{(q_0+q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha+\delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t) \\ & + \int_0^{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\bar{L}-L(1))\delta}\right\}} \left[ \int_0^2 \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t) \end{aligned} \quad (\text{A.100})$$

Similarly to the **Case** ( $Y_t^T > \delta\bar{L}$ ) in the Proof of [Proposition 6](#), the effect not internalized by workers'specialization decisions is the effect that  $L(1)$  has on the boundaries of the integrals in [\(A.100\)](#). From the definition of  $Y_t^T$  in [Proposition 3](#),

$$\frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \quad (\text{A.101})$$

and the marginal effect of  $L(1)$  that workers do not internalize has the same has the same sign as

$$\int_{\frac{Y_t^T}{(\bar{L}-L(1))\delta}}^{\frac{Y_t^T}{(\bar{L}-L(1))\delta}} \left[ Y_t^T - \delta\bar{L}p_t(x_t) \right] dp_t(x_t) \quad (\text{A.102})$$

$$= \left[ p_t(x_t)Y_t^T - \delta\bar{L}\frac{(p_t(x_t))^2}{2} \right]_{\min\left\{0, \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\bar{L}-L(1))\delta}\right\}}^{\frac{Y_t^T}{(\bar{L}-L(1))\delta}} \quad (\text{A.103})$$

If  $Y_t^T \leq (\alpha + \delta)L(1)$ , [\(A.103\)](#) has the sign of  $\bar{L} - 2L(1)$ , which is positive for  $L^*(x_t) > \frac{\bar{L}}{2}$ . Alternatively, if  $Y_t^T > (\alpha + \delta)L(1)$ , [\(A.103\)](#) has the sign of

$$Y_t^T - \frac{\delta\bar{L}}{2} \left[ \frac{Y_t^T}{(\bar{L}-L(1))\delta} + \frac{Y_t^T - L(1)(\alpha+\delta)2}{(\bar{L}-L(1))\delta} \right] \quad (\text{A.104})$$

$$= \frac{L(1)}{\bar{L}-L(1)} \left( (\alpha+\delta)\bar{L} - Y_t^T \right) \quad (\text{A.105})$$

which is positive when  $Y_t^T < \delta\bar{L}$ .

**Case (ii):**  $Y_t^T > \delta\bar{L}$  and  $0 < L^*(1) < L^*(x_t)$ :

$$p_t^T(0) > 2 \Leftrightarrow L(1) < \frac{Y_t^T}{2(\alpha+\delta)} \quad (\text{A.106})$$

which holds for some  $\alpha$  small enough, yet greater than zero, if  $Y_t^T > \delta\bar{L}$  and  $L(1)^* < \frac{\bar{L}}{2} < L^*(x_t)$ . Which implies that for  $\alpha$  and  $p_t(2)$  low enough,  $Y_t < Y_t^T$ . However, when  $Y_t^T > \delta\bar{L}$ , for  $p_t(x_t) = 2$ , there may exist  $p_t(1)$  low enough such that  $Y_t < Y_t^T$ .

Therefore, (A.72) writes:

$$\begin{aligned} & \int_{\min\left\{\frac{Y_t^T}{(\bar{L}-L(1))\delta}, 2\right\}}^2 \left[ \int_0^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right] dp_t(x_t) \\ & + \int_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\bar{L}-L(1))\delta}, 2\right\}} \left[ \int_{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)}}^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} - c dp_t(1) \right. \\ & \quad \left. + \int_0^{\frac{Y_t^T - (\bar{L}-L(1))\delta p_t(x_t)}{(\alpha + \delta)L(1)}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t) \\ & + \int_0^{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L}-L(1))\delta}\right\}} \left[ \int_0^2 \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \right] dp_t(x_t) \end{aligned} \quad (\text{A.107})$$

Similarly to the previous **Case** ( $Y_t^T > \delta\bar{L}$ ) in the Proof of [Proposition 6](#), the effect not internalized by workers'specialization decisions is the effect that  $L(1)$  has on the boundaries of the integrals in (A.107). From the definition of  $Y_t^T$  in [Proposition 3](#),

$$\frac{[(q_0 + q_1)^{\gamma_{nt}} - q_0^{\gamma_{nt}}] R^{\gamma_r} (Y_t^T)^{\gamma_t}}{N_t^{\gamma_t}} - c > 0, \quad (\text{A.108})$$

so the marginal effect of  $L(1)$  that workers do not internalize has the same sign as

$$\int_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\bar{L}-L(1))\delta}, 2\right\}} (Y_t^T - \delta\bar{L}p_t(2)) dp_t(x_t) \quad (\text{A.109})$$

$$= \left[ p_t(2)Y_t^T - \delta\bar{L}\frac{p_t(2)^2}{2} \right]_{\max\left\{0, \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L}-L(1))\delta}\right\}}^{\min\left\{\frac{Y_t^T}{(\bar{L}-L(1))\delta}, 2\right\}} \quad (\text{A.110})$$

If  $Y_t^T < L(1)(\alpha + \delta)2 < 2\delta(\bar{L} - L(1))$ , (A.110) has the sign of

$$\left( Y_t^T - \frac{\delta\bar{L}}{2} \frac{Y_t^T}{(\bar{L} - L(1))\delta} \right) = \frac{Y_t^T}{2(\bar{L} - L(1))} ((\bar{L} - 2L(1))) < 0. \quad (\text{A.111})$$

If  $L(1)(\alpha + \delta)2 < Y_t^T < 2\delta(\bar{L} - L(1))$ , (A.110) has the sign of

$$Y_t^T - \frac{\delta\bar{L}}{2} \left( \frac{Y_t^T}{(\bar{L} - L(1))\delta} + \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \right) \quad (\text{A.112})$$

$$= L(1) \frac{\bar{L}(\alpha + \delta) - Y_t^T}{\bar{L} - L(1)} \quad (\text{A.113})$$

which is negative for  $\alpha$  small when  $Y_t^T > \delta\bar{L}$ .

If  $L(1)(\alpha + \delta)2 < 2\delta(\bar{L} - L(1)) < Y_t^T$ , (A.110) has the sign of

$$Y_t^T - \frac{\delta\bar{L}}{2} \left( 2 + \frac{Y_t^T - L(1)(\alpha + \delta)2}{(\bar{L} - L(1))\delta} \right) \quad (\text{A.114})$$

$$= \frac{(2L(1) - \bar{L})(2\delta\bar{L} - Y_t^T) + 2\bar{L}L(1)\alpha}{2(\bar{L} - L(1))}, \quad (\text{A.115})$$

which is negative if for  $\alpha$  small, since  $2\bar{L}\delta > Y_t^T$  (??).

**Case (iii):**  $Y_t^T < \delta\bar{L}$  and  $L^*(x_t) = \bar{L}$ .

As in **Case (i)**, (A.72) writes like in (A.100). The proof of this case is similar to the proof of **Case (i)** in the Proof of **Proposition 7**: At a corner equilibrium  $L^*(x_t) = \bar{L}$ , the derivative of the integrand in (A.100) with respect to  $L(1)$  is non-positive and is strictly negative for  $\alpha$  small enough. From **Case (i)** in the Proof of **Proposition 8**, the derivative with respect to  $L(1)$  on the boundaries of the integrals in (A.100) is positive. Therefore, if  $Y_t^T < \delta\bar{L}$ , a marginal increase in  $L(1)$  from  $L^*(x_t) = \bar{L}$  increases or decreases welfare depending on whether in (A.100), the effect that works through the integrands or the that works through the boundaries of the integrals dominates.

**Case (iv):**  $Y_t^T > \delta\bar{L}$  and  $L^*(x_t) = \bar{L}$ .

As in **Case (ii)**, (A.72) writes like in (A.107). The proof of this case is similar to the proof of **Case (ii)** in the Proof of **Proposition 7**: At a corner equilibrium  $L^*(x_t) = \bar{L}$ , the derivative of the integrand in (A.107) with respect to  $L(1)$  is non-positive and is strictly negative for  $\alpha$  small enough. From **Case (ii)** in the Proof of **Proposition 8**, the derivative with respect to  $L(1)$  on the boundaries of the integrals in (A.100) is negative for  $\alpha$  small enough. Therefore, for  $\alpha$  small enough, if  $Y_t^T > \delta\bar{L}$ , a marginal increase in  $L(1)$  from  $L^*(x_t) = \bar{L}$  decreases welfare. *QED*

## Proof of Proposition 9

### Part (i)

In any interior equilibrium, a worker must be indifferent between specializing in traded-good sector 1 and in other traded-good sector.

$$\mathbb{E} \left[ \frac{Q_{nt}^{\gamma_{nt}} ((\alpha + \delta)p_t(1) - \delta p_t(x_t))}{Y_t^{1-\gamma_t}} \right] = 0. \quad (\text{A.116})$$

Note

$$\lim_{N \rightarrow +\infty} Y_t = (\alpha + \delta)L(1)p_t(1) + \delta(\bar{L} - L(1)) \quad (\text{A.117})$$



and hence when  $N \rightarrow +\infty$ , the LHS of (A.116) is

$$\begin{aligned} & \int_{\max\left\{\min\left\{\frac{Y_t^T - \delta(\bar{L} - L(1))}{(\alpha + \delta)L(1)}, 2\right\}, 0\right\}}^2 \frac{(q_1 + q_0)^{\gamma_{nt}} ((\alpha + \delta)p_t(1) - \delta)}{\left[(\alpha + \delta)L(1)p_t(1) + \delta(\bar{L} - L(1))\right]^{1-\gamma_t}} dp_t(1) \quad (\text{A.118}) \\ & + \int_0^{\max\left\{\min\left\{\frac{Y_t^T - \delta(\bar{L} - L(1))}{(\alpha + \delta)L(1)}, 2\right\}, 0\right\}} \frac{q_0^{\gamma_{nt}} ((\alpha + \delta)p_t(1) - \delta)}{\left[(\alpha + \delta)L(1)p_t(1) + \delta(\bar{L} - L(1))\right]^{1-\gamma_t}} dp_t(1), \end{aligned}$$

which, if  $L(1) = 0$ , is equal to <sup>A4</sup>

$$\frac{Q_{nt}^{\gamma_{nt}} \alpha}{(\delta\bar{L})^{1-\gamma_t}} > 0. \quad (\text{A.119})$$

It follows that for  $N$  large enough (A.118) is strictly greater than zero and therefore  $L^*(1) = 0$  cannot be an equilibrium.

For  $L(1) = \bar{L}$ ,  $Y_t$  and  $Q_{nt}$  do not depend on  $p_t(x_t)$ , and hence, (A.118) has the same sign as

$$\begin{aligned} & \int_{\frac{Y_t^T}{(\alpha + \delta)\bar{L}}}^2 (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \frac{(\delta + \alpha)p_t(1) - \delta}{p_t(1)^{1-\gamma_t}} dp_t(1) \quad (\text{A.120}) \\ & + \int_0^{\frac{Y_t^T}{(\alpha + \delta)\bar{L}}} q_0^{\gamma_{nt}} \frac{(\delta + \alpha)p_t(1) - \delta}{p_t(1)^{1-\gamma_t}} dp_t(1) \\ & = (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) 2^{\gamma_t} \left[ \frac{2(\delta + \alpha)}{\gamma_t + 1} - \frac{\delta}{\gamma_t} \right] \quad (\text{A.121}) \\ & - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \left( \frac{Y_t^T}{(\alpha + \delta)\bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\bar{L}}}{\gamma_t + 1} - \frac{\delta}{\gamma_t} \right] \end{aligned}$$

Notice that

$$\frac{\frac{Y_t^T}{\bar{L}}}{\gamma_t + 1} - \frac{\delta}{\gamma_t} = \frac{\gamma_t \frac{Y_t^T}{\bar{L}} - (\gamma_t + 1)\delta}{(\gamma_t + 1)\gamma_t} < \frac{\frac{Y_t^T}{\bar{L}} - 2\delta}{(\gamma_t + 1)} < 0 \quad (\text{A.122})$$

which means that (A.120) is increasing in  $\alpha$  and tends  $+\infty$  as  $\alpha \rightarrow +\infty$ .

Evaluated at  $\alpha = 0$ , (A.120) is equal to

$$\begin{aligned} & = (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} \left[ \frac{2}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta\bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta\bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \\ & < (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta\bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta\bar{L}} \right)^{\gamma_t} \left[ \frac{\frac{Y_t^T}{\delta\bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \\ & = \left[ \frac{\frac{Y_t^T}{\delta\bar{L}}}{\gamma_t + 1} - \frac{1}{\gamma_t} \right] \left( (q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) \delta 2^{\gamma_t} - [(q_0^{\gamma_{nt}} + q_1^{\gamma_{nt}}) - q_0^{\gamma_{nt}}] \delta \left( \frac{Y_t^T}{\delta\bar{L}} \right)^{\gamma_t} \right) < 0, \quad (\text{A.123}) \end{aligned}$$

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<sup>A4</sup>Recall  $Q_{nt} = q_1 \mathbb{1}_{Y_t^T \geq \delta\bar{L}} + q_0$  where  $\mathbb{1}$  is the indicator function.

Therefore, (A.120) is increasing in  $\alpha$ , tends  $+\infty$  as  $\alpha \rightarrow +\infty$ , and is negative for  $\alpha = 0$ . This implies that there exist  $\tilde{\alpha} > 0$  such that (A.120) is weakly positive if and only if  $\alpha \geq \tilde{\alpha}$ . If  $\tilde{\alpha} < \alpha$ , the RHS in (A.118) is negative and bounded away from 0 for  $L(1) = \bar{L}$  and positive and bounded away from 0 for  $L(1) = 0$ . It follows that for  $N$  large enough, there exists  $\tilde{\alpha}_N$  such that if  $\alpha < \tilde{\alpha}_N$ , (A.116) is strictly positive for  $L(1) = 0$  and strictly negative for  $L(1) = \bar{L}$ , in which case, any equilibrium of the specialization game  $L^*(1)$  is interior ( $0 < L^*(1) < \bar{L}$ ) and if  $\alpha \geq \tilde{\alpha}_N$ .

**Part (ii) and (iii).**

From (33),  $\mathbb{E}[v_s(y_i)]$ , writes

$$\mathbb{E}[v_s(Y)] = \mathbb{E} \left[ \frac{Q_{nt}^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{P_t^{\gamma_t}} - c \mathbb{1}_{Y_t \geq Y_t^T} \right] \quad (\text{A.124})$$

When  $N \rightarrow +\infty$ , (A.124) tends to

$$\begin{aligned} & \int_{\max \left\{ \min \left\{ \frac{Y_t^T - \delta(\bar{L} - L(1))}{(\alpha + \delta)L(1)}, 2 \right\}, 0 \right\}}^2 \frac{(q_0 + q_1)^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \\ & + \int_0^{\max \left\{ \min \left\{ \frac{Y_t^T - \delta(\bar{L} - L(1))}{(\alpha + \delta)L(1)}, 2 \right\}, 0 \right\}} \frac{q_0^{\gamma_{nt}} R^{\gamma_r} Y_t^{\gamma_t}}{N_t^{\gamma_t}} dp_t(1) \end{aligned} \quad (\text{A.125})$$

where  $Y_t = (\alpha + \delta)L(1)p_t(1) + \delta(\bar{L} - L(1))$ . For  $0 < \frac{Y_t^T - \delta(\bar{L} - L^*(1))}{(\alpha + \delta)L^*(1)} < 2$ , the derivative of (A.125) with respect to  $L(1)$  evaluated at the equilibrium  $L(1) = L^*(1)$  has the same sign as  $(Y_t^T - \delta\bar{L})$ . For  $\frac{Y_t^T - \delta(\bar{L} - L^*(1))}{(\alpha + \delta)L^*(1)} \notin (0, 2)$ , the derivative of (A.125) with respect to  $L(1)$  evaluated at the equilibrium  $L(1) = L^*(1)$  is zero. *QED*