Economic Development According to Chandler*

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Abstract

Chandler (1977) links the transition to modern economic growth in the United States to the rise of managerial capitalism. We show that a lack of skills in today's developing countries limits the pool of managers and white collar workers, preventing them from making the same transition. We use multiple data sources from around the world to show that larger and more productive firms systematically use white collar labor more intensively and that the aggregate supply of skills accounts for nearly all of the difference in the supply of white collar labor between developing and developed economies. Motivated by these facts, we develop a general equilibrium occupational choice model in the spirit of Lucas (1978). The key novel feature is that entrepreneurs can increase the returns to scale of their production process by delegating some of the firm's management functions to hired white collar workers. We show that countries with more aggregate skills have larger firms, more white collar workers, and less own account work, consistent with the data. We validate the causal role for skills using cross-regional evidence from a large, policy-driven expansion of colleges in Sweden.

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1 Introduction

In a seminal contribution, Chandler (1977) shows that the transition to managerial capitalism played a critical role in United States economic history.[1] The Industrial Revolution introduced a host of new, productive technologies that leveraged economies of scale and scope. Firms that adopted such technologies became large and encountered logistical challenges in terms of sourcing a constant supply of inputs, coordinating a high volume of production across establishments, and marketing and selling outputs. They solved these challenges by recruiting and training a hierarchy of white-collar workers, such as managers, accountants, purchasing agents, and clerks.

The main contribution of this paper is to show that a lack of sufficient skills in developing countries today prevents them from making the same transition. We start by documenting three motivating facts on the relationship between occupations, firm structure, and skills. First, we use cross-country data to show that there is a strong relationship between occupations and firm structure. Firms, particularly large firms, use white collar labor much more intensively that own account employment, consistent with Chandler's historical narrative.

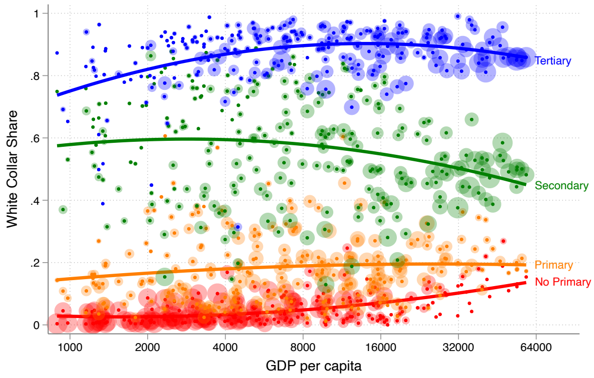
Second, we show that there is a close correspondence between the probability that a worker engages in white collar work and her skill level. Figure 1 provides one striking summary of this fact. It plots the average white collar employment share by country and education level (shown with different colors) against the country's GDP per capita. There are large differences in the white collar share across education levels, ranging from roughly 10 percent for workers without primary schooling to roughly 90 percent for workers with tertiary education. However, there are almost no cross-country differences in the white collar share after conditioning on education. Ninety percent of the lower share of white-collar workers in poor countries is accounted for by a lower aggregate supply of skills.

Third, we show that development alters the relationship between skills and firm structure. In developing countries less educated workers overwhelmingly engage in own account work. Among middle income countries these gaps shrink substantially, while there is essentially no difference in the own account employment rate by education level in developed economies.

These facts motivate us to develop a model that allows us to study the links between skills, occupational choices, and firm structure. The model features a continuum

[1] Chandler uses the term managerial capitalism to describe firms managed by teams or hierarchies of professional managers; this contrasts with personal capitalism, where the owner directly manages the firm. See also Chandler (1987) for a comparative history of the United States, Germany, and Great Britain.

FIGURE 1: SKILLS AND WHITE COLLAR WORKERS



of workers with heterogeneous skills. Workers make an occupational choice: they can become entrepreneurs, blue collar production workers, or white collar managers and professionals. Production workers and professionals supply their labor to firms in competitive markets. Similar to Lucas (1978), entrepreneurs hire production workers, pay them a wage, and receive firm profits as their income. The key novel twist is that entrepreneurs can also choose for a set of managerial functions whether to perform each function themselves or to delegate them to hired white collar workers.

We derive several results to help characterize the resulting equilibrium. First, we provide a useful representation result: the production function is equivalent to one where entrepreneurs choose directly the returns to scale. Intuitively, delegating a greater share of managerial functions allows the entrepreneurs to switch from using their own time, which is in fixed supply, to using the flexibly chosen time of professionals. We use this result to show that more skilled entrepreneurs hire more professional workers, consistent with our first motivating fact, which allows them to produce with larger (but still decreasing) returns to scale. Finally, given these results, we show that occupational choices satisfy a cutoff rule. Workers with low levels of skills become production workers or entrepreneurs that run small firms and perform all management functions themselves, which we interpret as own account workers in the spirit of Bassi *et al.* (2023). Workers with intermediate levels of skills become professionals. Finally, workers with the highest levels of skills become entrepreneurs of firms that hire both professional and production labor.

The model is useful as a laboratory to understand the implications of changes in skill endowments. We focus on a special case where entrepreneurs either delegate all managerial functions or none of them, which allows us to solve the model analytically. We compare two economies that are otherwise identical except that one has a distribution of skills that first order stochastically dominates the second. We show that a worker with the same skill level makes the same occupational choice in each of the two economies, consistent with our second motivating fact. The economy with more skills has more entrepreneurs and hence more demand for production labor. This demand pulls less educated blue collar workers out of own account work and into factories, consistent with our third motivating fact.

The full quantitative model relaxes the assumptions of the analytical model such that entrepreneurs can choose intermediate levels of delegation and returns to scale. It also extends the model by allowing for heterogeneous sectors, which capture the fact that returns to scale are more important than some sectors than others and allows the model to generate structural transformation through adoption of new technologies that leverage these returns to scale (Kuznets, 1973). Finally, we allow also for households' preferences to have flexible substitution and income effects (Comin *et al.*, 2021).

Our plan is to calibrate the model to fit cross-country and cross-sectional data on skills, occupational choices, and firm productivity. We want to use the model to study the effect on an economy of experiencing an increase in total factor productivity or skills alone, while holding the other fixed. The goal of this experiment is to shed light on the complementary importance of skills and the re-organization of production in growth. The combination of skills and re-organization of production will be more important in some sectors, leading to differential productivity growth by sector and structural transformation as in Ngai & Pissarides (2007). It will also have a direct effect on GDP per capita that we want to quantify. We also think that raising general total factor productivity while holding skills fixed might shed light on the experiences of developing countries that discover natural resources or generate outcomes similar to premature deindustrialization (Rodrik, 2016).

We support the main causal mechanism from skills to occupational choices and firm structure using a policy-induced expansion of colleges. This expansion involved a politically motivated decision to expand the access of college geographically by transforming teachers' colleges in a number of communities into universities. We show that cohorts that were newly graduated from high school in these local labor markets experienced a disproportionate increase in college attainment and that the overall labor market experienced a large increase in firm size.

Our work is most directly related to an earlier historical and development literature that took a wide-ranging perspective on the broad changes that accompanied economic development that linked together economic growth, technology adoption, and firm-restructuring (Kuznets, 1973; Chandler, 1977). We show that skills are also an important component of this story and bring to bear new microdata as well as quantitatively modeling to explore the complementary importance of these factors.

Our work also touches on several more recent literatures. Contemporaneous work by Gottlieb *et al.* (2023) also documents an important link between skills and occupational choices around the world. Our work touches on the recent literature that documents cross-country differences in firm structure in the form of self-employment rates, workers sorting, and firm size (Gollin, 2008; Bento & Restuccia, 2017; Porzio, 2017; Poschke, 2018; Bento & Restuccia, 2021). Our focus on the complementarity between white collar work and large firms relates also to the findings of Hjort *et al.* (2023) about the importance of management for large and leading firms. Finally, our findings relate to the large recent literature on structural transformation; Herrendorf *et al.* (2014) provide an able overview. We are particularly related to Buera *et al.* (2022) and Porzio *et al.* (2022), who also emphasize the relationship between skills and structural transformation; Duernecker & Herrendorf (2022), who show that structural transformation also involves a reallocation of labor across occupations; and Ding *et al.* (2022), who show that structural transformation happens within firms over time in the United States.

2 Motivating Evidence

We start by providing three facts that motivate our model and analysis. Drawing on Chandler's historical narrative, we start by showing that larger firms use white collar labor more intensively also for many countries across a wide range of development. We then provide novel evidence linking the aggregate supply of white collar labor back to the underlying aggregate supply of skills in an economy. These two facts suggest that skills may be a constraint on developing countries' ability to adopt large firms. Finally, we explore the relationship between education and firm size and show that development is associated with a strong shift away from self-employment, even for the least educated workers. This last fact suggests that the growth of large firms may draw workers away from self-employment and towards being production workers in firms. h

2.1 Fact 1: Larger Firms use White Collar Labor Intensively

We start with the relationship between firm structure and white collar employment shares. A core theme of Chandler's work is that white collar labor became particularly important for large firms as the United States developed. We examine the same patterns in cross-country data. In addition, given our interest in the full range of development, we also document differences in white collar employment between own account workers and firms.

To document differences by firm size we use a database of labor force surveys from

Donovan *et al.* (2023). The database covers people aged 16–65 living in urban areas of 49 countries with a wide range of income levels. The advantage of labor force surveys is that they frequently ask workers about how many employees work in their firm. The responses are coded into categories that vary across countries, but we can compare results for firms with ten or fewer versus eleven or more employees for most countries.

To compare differences between own account work and firms, we use microdata from Minnesota Population Center (2020), which collects and harmonizes censuses from around the world. Our sample includes all countries and years with available information on occupations and educational attainment, totaling to 233 cross-sections from 77 different countries. The data set spans six decades and covers most of the global income distribution.

Own account workers are non-employer self-employed, while employees are treated as working in firms. For both data sets we measure occupational choices using data classified according to the International Standardized Classification of Occupations (ISCO) scheme. The ISCO has undergone several revisions, but the codes are reasonably comparable at the 1-digit level for the two most recent revisions (1988 and 2008). We use data with either classification. We define white collar workers to include the 1-digit codes 1–4 (managers, professionals, technicians and associate professionals, and clerks) and blue collar workers to include the 1-digit codes 5–9 (service and sales, agriculture, crafts and trades, plant and machine operators, and elementary occupations).

FIGURE 2: WHITE COLLAR SHARE AND FIRM STRUCTURE

(a) Small vs Large Firms (b) Own Account vs Firms

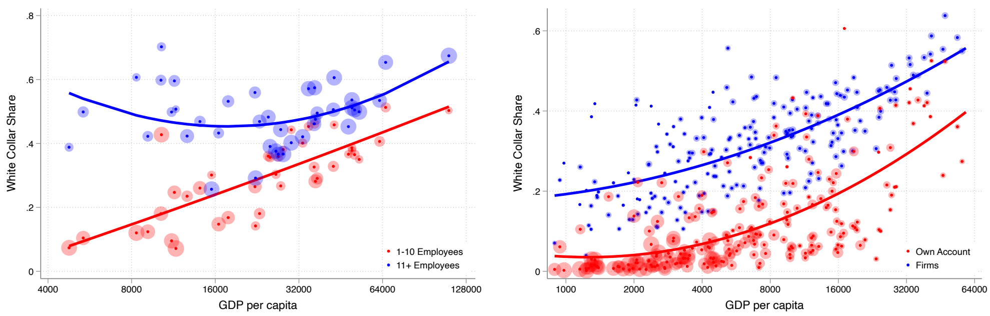


Figure 2 shows both sets of results. Figure 2a compares the white collar employment share for small versus large firms, while Figure 2b compares the white collar employment share for own account workers versus employees of firms. In each figure a dot corresponds to a country × year × firm type observation and the size of the dot captures the share of the country × year employment in that bin. There are two main findings. First, employees of firms are systematically more likely to be white collar workers. Second, larger firms use white collar labor more intensively than smaller firms. These findings motivate us to focus on non-homothetic production functions that endogenously generate a relationship between firm size and white collar labor intensity.

2.2 Fact 2: Skills Account for White Collar Labor Supply

We next turn to the relationship between skills and the occupational choices. Our main data set for this analysis is the international census data from Minnesota Population Center (2020). We measure skills using educational attainment in four broad bins: no school, primary, secondary, and tertiary education. We again measure white collar employment using occupational codes.

Figure 3 illustrates the general growth in white collar labor with development. We plot for each country in Minnesota Population Center (2020) the share of workers with white collar occupations against PPP GDP per capita from World Bank (2022). The share rises from roughly 10 percent in the poorest economies to 60 percent in the richest.

FIGURE 3: WHITE COLLAR WORK AND DEVELOPMENT

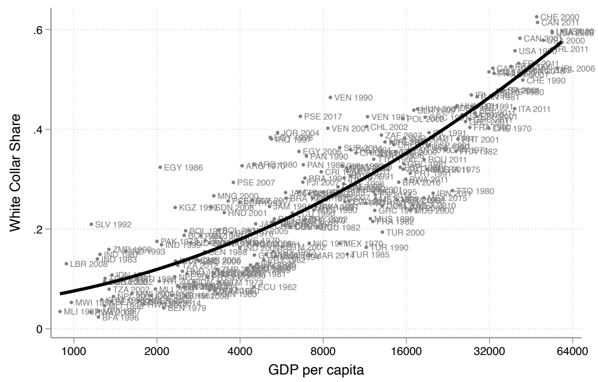
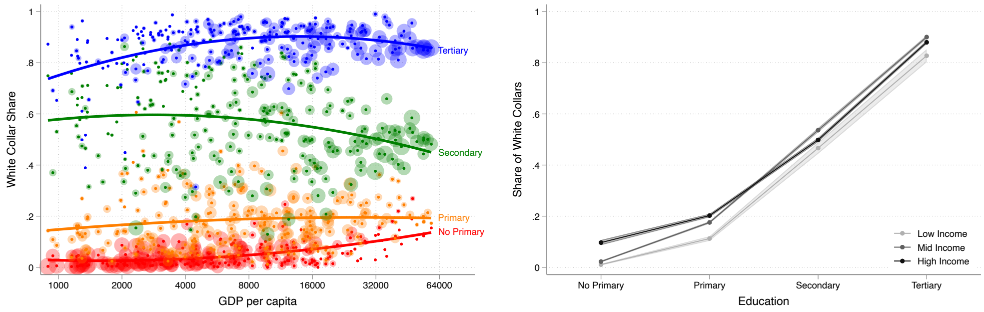


Figure 4a, which we also showed in the introduction, unpacks the relationship between the white collar share and development from Figure 3 across education levels. Each dot corresponds to a country $\times$ year $\times$ education group observation, and the bubbles around the dots are proportional to the size of each education group within each country. At all levels of development, highly educated workers are more likely to be white collars, and conditional on education there are relatively small differences between poor and rich countries. The aggregate pattern in Figure 3 is therefore to a large extent driven by rich countries being more abundant in highly educated workers (the size of the bubbles in Figure 4a).

Figure 4b visualizes the propensities of working in a white collar occupation by education, conditional on other observable characteristics. We pool countries in three

FIGURE 4: EDUCATION AND WHITE COLLAR OCCUPATIONS

(a) White Collar Share by Education (b) Estimated Propensities



income groups (according to the World Bank classification), and separately for each group we project a dummy identifying white collar workers on education, age, gender and country $\times$ year dummies. Consistently with Figure 4a, the propensities are strongly increasing with education, in a nearly identical fashion across countries with vastly different income levels.

A direct implication of these findings is that differences in the skill distribution account for almost all of the gaps in the white collar share between rich and poor countries. To appreciate this point, we estimate the semi-elasticity of the share of white collar workers with respect to development unconditionally and after conditioning on skills, and quantify the accounting contribution of skills as

$$\text{Accounting Share} = 1 - \frac{\text{Conditional Elasticity}}{\text{Unconditional Elasticity}}.$$

Table 1 shows the resulting unconditional semi-elasticity, conditional semi-elasticity, and accounting share. We find that variation in the aggregate supply of skills accounts for roughly 90 percent of the cross-country correlation between white collar employment share and development.[2]

TABLE 1: ACCOUNTING RESULTS

Unconditional Elasticity	Conditional Elasticity	Accounting Share
0.119	0.014	0.882
(0.001)	(0.001)	

Notes: The Table shows the results of the accounting exercises described in the text.

[2]In all these regressions we include a dummy for gender. The sample includes all employed individuals aged 25-65. All cross-sections are weighted equally.

This strong accounting relationship turns out to be extremely robust. We show in Appendix A that we find similarly large results if we study the cross-section or focus on the time series for countries we can track for long periods. It holds equally for both men and for women. We explore using alternative measures of skills such as childhood or adult test scores, motivated by the concern that licensure or credentialism may generate a mechanical relationship between educational attainment and occupational choices; the results do not change.

Finally, an important potential confounding factor is structural transformation, which shifts workers away from agriculture, the most blue collar labor-intensive sector. In the theory we lay out in the next section, structural transformation is in part a response to the accumulation of skills and re-organization of production. Nonetheless, we find it useful to explore the role of structural transformation versus skills in accounting for the correlation between white collar employment shares and development. To do so, we include sector fixed effects in the regressions used to estimate both the unconditional and conditional elasticity. We find sectoral composition accounts for a little more than half of the elasticity in this case, while skills account for the entirely of the remainder.

The strength and robustness of these results motivate us to model a strong link between a worker's skills and their occupational choices. However, Figure 4 has an even stronger implication: a worker's occupational choices is also nearly independent of the aggregate supply of skills in their country of work. This more surprising finding is an important goal for our analysis.

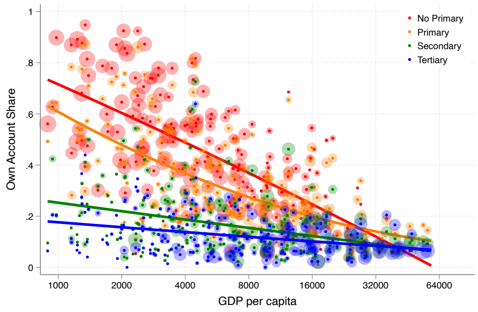
2.3 Fact 3: Development Pulls Unskilled Labor into Firms

Finally, we explore the relationship between education and firm structure. Again, we use two measures of firm structure: we compare own account workers versus employees in international census data; we compare employees of small versus large firms using labor force surveys.

The results are shown in Figure 5. There are two main findings. First, there is a strong correlation between education and firm structure. Less educated workers are systematically more likely to engage in own account work and less likely to work in large firms. This point follows naturally from our first two facts. The more surprising finding is that development is associated with a collapse of own account work even among least educated workers, such that for the very richest countries the rate of own account work is similarly low for all education levels.

FIGURE 5: EDUCATION AND FIRM STRUCTURE

(a) Own Account vs Firms



3 Model

The facts in Section 2 motivate us to turn now to a model that investigates the relationship between skills, occupational choices, and firm structure. We use this as a laboratory to understand these patterns and to investigate whether a lack of skills in developing countries may constrain their ability to develop large firms, adopt new technologies, and transition to modern economic growth.

All the proofs for the results in this and next Section are in Appendix B.

3.1 Environment

We model the long-run (static) equilibrium of an economy with one factor of production, labor, and many sectors.

Agents and Preferences. The economy is inhabited by a mass 1 of heterogeneous individuals. Each individual is endowed with a skill $z \sim \hat{G}(z|s)$, where $\hat{G}$ is a CDF which has an associated density function with strictly positive mass on the support $[0, \bar{z}]$ and s is the level of schooling.[3] We assume that schooling shift the distribution of skills so that if $s' > s$ then $\hat{G}(z'|s') < \hat{G}(z|s)$ for any $z' > z$. The distribution of skills in the country is given by

$$G(z) \equiv \int \hat{G}(z|s) d\Gamma(s) \tag{1}$$

where $\Gamma(s)$ is the schooling distribution.

Workers have log preferences over their income and are also endowed with a vector of idiosyncratic relative preferences for engaging in sector j – ε_j – that is drawn from a type-I extreme value distribution with shape parameter ν.

[3] In bringing the model to data, we are going to assume that we can observe the schooling of an individual, but not her skill level.

Choices of Sectors and Occupations. Each individual chooses in which sector j to enter, and within the sector, whether to start a firm (entrepreneurship), or work in firms as either a professional or a laborer.

The potential incomes of an individual of skill z in sector j are given by $\pi_j(z)$ as an entrepreneur, $w_b z^\lambda$ as a professional, and w_l as a laborer. As we describe below, $\pi_j(z)$ is the outcome of profit maximization, while $w_{j,b}$ and $w_{j,l}$ are the equilibrium wages per efficiency unit. $\lambda > 0$ is a parameter that modulates the extent to which skills are used more intensive by professionals than by laborers.

Within sector, each individual chooses the occupation that maximizes her income:

$$\underbrace{\phi_j(z)}_{\text{Income in Sector } j} = \max \left\{ \underbrace{\pi_j(z)}_{\text{Entrepreneur}}, \underbrace{w_b z^\lambda}_{\text{Professional}}, \underbrace{w_l}_{\text{Laborer}} \right\}. \tag{2}$$

The occupational choice yields functions $\omega_{\pi,j}(z), \omega_{p,j}(z), \omega_{\ell,j}(z)$ for the share of individuals of skill z, among those in sector j, that choose to be an entrepreneur, professional and laborer.

Given the properties of the extreme value distribution, the share of individuals of ability z in sector j is

$$\sigma_j(z) = \int \frac{\phi_j(z)^\nu}{\sum_{k \in J} \phi_k(z)^\nu} dG(z), \tag{3}$$

which implies that, defining $\overline{G}_j \equiv G_j(\overline{z})$ as the overall share of employment in sector j, the distribution of skills in sector j satisfies

$$G_j(z) = \frac{1}{\overline{G}_j} \int_0^z \sigma_j(z) dG(z). \tag{4}$$

Entrepreneur Problem and Production Function. The core element of our model is the production function, which embeds both a choice of *how* and *how much* to produce. Production entails to complete a series of tasks. One set of tasks concern production and requires laborers to be completed. Other tasks are managerial and may be either performed by the entrepreneur herself or through hired professionals. For example, an entrepreneur may decide to rely on word of mouth to sell her products, or she may hire marketing professionals; she may rely on personal connections to find workers or hire HR specialists; and so on. If the entrepreneur does not hire outside workers to perform a task i, she produces a fixed amount $\tilde{a}_{i,j}$ of task output, while $n_p(i)$ hired professionals produce $\frac{z}{A_j} n_p(i)$. All the tasks are aggregated through a Cobb-Douglas function.

Formally, each entrepreneur z in a sector j solves

$$\pi_j(z) = \max_{\{n_p(i)\}_{i\in[0,1]},n_\ell} p_j A_j \exp\left(\int_0^1 \log\left(\frac{\tilde{n}(i)}{\bar{\eta}_j - \gamma_{\ell,j}}\right)^{\bar{\eta}_j-\gamma_{\ell,j}} di\right)\left(\frac{n_l}{A_j\gamma_{\ell,j}}\right)^{\gamma_{\ell,j}} \tag{5}$$

$$- w_{p,j}\int_0^1 n_p(i) - w_{\ell,j}n_l$$

s.t.

$$\tilde{n}(i) = \max\left\{(\tilde{a}_{i,j}(\bar{\eta}-\gamma_l))^{\frac{1}{\bar{\eta}_j-\gamma_\ell}}, \frac{z}{A_j}n_p(i)\right\}$$

We can assume, without loss of generality, that the tasks i are ranked from 0 to 1 by their relative productivity $\frac{z}{\tilde{a}_{i,j}A_j}$. Under this assumption, Lemma 1 shows that the multi-dimensional problem (5) reduces to become the choice of which share (q) of tasks to *professionalize* – i.e. to hire professionals for – and how many professionals and laborers to hire.

LEMMA **1** (Equivalence Result). *The problem of the entrepreneur (5) is equivalent to the following simplified problem, where q is the share of task which are professionalized and n_p is the employment of white collar workers per each task:*

$$\pi_j(z) = \max_{q\in[0,1],n_p,n_l} A^{1-\eta(q)}\tilde{A}(q)^{1-q}\left[\left(\frac{qzn_p}{\alpha(q)\eta(q)}\right)^{\alpha(q)}\left(\frac{n_l}{(1-\alpha(q))\eta(q)}\right)^{1-\alpha(q)}\right]^{\eta(q)} \tag{6}$$

$$- qw_pn_p - w_ln_l$$

where

$$\begin{aligned}
\tilde{A}(q) &\equiv \exp\left(\frac{1}{1-q}\int_q^1 \log\tilde{a}(i)di\right)\\
\eta(q) &\equiv q\bar{\eta} + (1-q)\gamma_l\\
\alpha(q) &\equiv \frac{q(\bar{\eta}-\gamma_l)}{\eta(q)}
\end{aligned}$$

Lemma 1 also shows that the problem of the entrepreneur boils down to the maximization of a Cobb-Douglas production function in which the factor share of professionals ($\alpha(q)$) and the extent of decreasing returns to scale ($\eta(q)$) are endogenous objects, functions of the share of professionalized tasks q. This observation implies that we can interpret q as a choice of technology which encapsulates Chandler (1977) idea about economic development. By hiring white-collar professionals, entrepreneurs can make their business more scalable.

We define $y_j(z)$ to be the output produced by an entrepreneur z in sector j, which solves problem 5.

Closing the model. In order to close the model, we need to describe how the relative prices of each sectors are determined. We are going to postulate that the price in each sector satisfies the following log-linear relationship

$$\log p_j = -\frac{1}{\sigma}\log\frac{Y_j}{Y} + \varepsilon_j \log Y + \log \mathbb{P}_j \tag{7}$$

where σ is the elasticity of substitution across goods, ε_j is the sector-specific income elasticity of demand, $Y_j = \int y_j(z)dG_j(z)$ is the total output in sector j, and $Y = \sum_{j\in J} p_j Y_j$ is the total income in the economy.

The terms $\log \mathbb{P}_j$, which are kept constant across counterfactual, capture differences (up to a normalization) in the level of demand for each good.

While we simply postulate it, the price equation 7 can be derived from a standard non-homethetic CES preference system, as in (Comin *et al.*, 2021).[4]

3.2 Equilibrium

We define an equilibrium in our setting, which requires that all agents maximize and earnings for each occupation and in each sector are such that all the labor markets clear.

Definition of Competitive Equilibrium *The competitive equilibrium is given by: i. wages per efficiency unit for laborers and professionals in each sector j ($w_{p,j}$, $w_{\ell,j}$); ii. technology choice, number of hired professionals and laborers, and profits for each entrepreneur z in each sector j ($q_j(z)$, $n_{p,j}(z)$, $n_\ell, j(z)$, $\pi_j(z)$); iii. shares of individuals in each sector and occupation ($\sigma_j(z)$, $\omega_{\pi,j}(z)$, $\omega_{p,j}(z)$, $\omega_{\ell,j}(z)$); iv. distribution of talent z in each sector j ($G_j(z)$); v. sectoral prices ($\{p_j\}_{j\in J}$) such that:*

1. *entrepreneurs maximize firm profits solving (5);*

[4] An important remark is due. The non-homethetic CES as in (Comin *et al.*, 2021) does not satisfy the conditions for Gorman aggregation. As a result, we cannot derive their exact aggregate formulation starting from our framework with individual-level heterogeneity. In practice, for our purpose, we find it convenient to simply assume that the economy behaves as if there is an aggregate household with those preferences. This conflict implies that our model may be not the best suited to study welfare, which we do not intend to do.

2. $\omega_{\pi,j}(z)$, $\omega_{p,j}(z)$, $\omega_{\ell,j}(z)$ *satisfy the occupational choice* (2) *that is*

$$\begin{aligned} \omega_{\pi,j}(z) > 0 \quad &\textit{iff} \quad \phi_j(z) = \pi_j(z) \\ \omega_{p,j}(z) > 0 \quad &\textit{iff} \quad \phi_j(z) = w_{p,j} z^{\lambda} \\ \omega_{\ell,j}(z) > 0 \quad &\textit{iff} \quad \phi_j(z) = w_{\ell,j} \end{aligned}$$

3. *the markets for professionals and laborers clear in each sector*

$$\begin{aligned} \int q(z) n_p(z) \omega_\pi(z) dG_j(z) &= \int \omega_p(z) z^{\lambda} dG_j(z) \\ \int n_l(z) \omega_\pi(z) dG_j(z) &= \int \omega_l(z) dG_j(z) \end{aligned}$$

4. *the distributions of talent in each sector are consistent with individual choices* (3)*, and* (4)

5. *prices satisfy* (7).

3.3 Characterization

We now characterize a few key properties of the equilibrium. We begin by solving the problem of the firm in each sector, then turn to the occupational choice, and finally to the sectoral choice.

Before showing results, it is convenient to define a few objects of interest.

DEFINITION **1.** The *skill premium* ρ_j in sector j is the ratio of the wages per efficiency unit of professionals and laborers: $\rho_j \equiv \frac{w_{b,j}}{w_{\ell,j}}$.

DEFINITION **2.** The *skill bias* $\mathbb{A}_j$ in sector j is the inverse of productivity term evaluate at $q = 0$: $\mathbb{A}_j \equiv (A_j(0))^{-1}$.

Entrepreneurial Problem

We first consider the partial equilibrium profit maximization of an entrepreneur z in a given sector, who takes as given prices and wages. We drop the subscript j to ease notation.

Using the representation of Lemma 5 we can solve for the profit – $\tilde{\pi}(z;q)$ – as a function of the skill z and the (endogenous) technology choice q:

$$\tilde{\pi}(z;q) = A(1-\eta(q))\tilde{A}(q)^{1-q} \underbrace{\left[\left(\frac{z}{w_p}\right)^{\alpha(q)} \left(\frac{1}{w_l}\right)^{1-\alpha(q)} \right]^{\frac{\eta(q)}{1-\eta(q)}}}_{\equiv \tilde{y}} \tag{8}$$

Equation (8) shows the key ingredients in the model. The profit function follows a very familiar expression, with two departures: i. the factor share of professionals ($\alpha(q)$) and the decreasing returns to scale ($\eta(q)$) are a function of technology choice q; ii. there is an endogenous productivity term $\tilde{A}(q)^{1-q}$ which itself depends on q. Given how we construct the problem $\tilde{A}(q)$ is (weakly) increasing in q, however, the overall productivity $\left(\tilde{A}(q)\right)^{1-q}$ decreases as a function of q (at least for high enough q). Intuitively, as more tasks are professionalized, the weight of the entrepreneurial task-productivity $\tilde{A}(q)$ in production shrinks.

Taking the first order conditions of $\tilde{\pi}(z;q)$ with respect to q yields an intuitive equation for the share of tasks that are professionalized.

LEMMA **2** (Technology Choice). *The share of tasks of the entrepreneurial problem (6) satisfies*

- *if $z \leqslant \hat{z}$, then q is equal to 0*
- *if $z > \hat{z}$, then q is bigger than 0, is increasing in z, and satisfies*

$$\begin{aligned} \underbrace{\frac{\eta'(q)}{\eta(q)} \log \tilde{y}}_{\textit{Returns to Scale}} =& \underbrace{\alpha'(q)\eta(q) \log\left(\frac{w_p}{z w_\ell}\right)}_{\textit{Cost of Skill-Intensity}} + \underbrace{\log \tilde{A}(q)}_{\textit{Cost of Professionalization}} \\ & - \underbrace{\eta(q)(\log \tilde{A}(q) - \log \tilde{a}(q))}_{\textit{Gain from "Specialization"}} + \underbrace{\eta'(q)}_{\textit{Decrease in Entr. Share}} . \end{aligned} \tag{9}$$

Equation (9) summarizes the main trade-offs at play. A higher share of professionalized tasks q leads to larger returns to scale $\frac{\eta'(q)}{\eta(q)} > 0$ and the benefit of this on profits are modulated by the scale of production – i.e. by $\tilde{y}$. The cost of increasing q is driven by four components. First, it increases the share of white collar workers in production ($\alpha'(q) > 0$), which is costly as long as the effective price of white collar labor is higher than the one of blue collar labor – i.e. if $\log \frac{w_p}{z} > \log w_l$. Second, it has a direct cost ($\log \tilde{A}(q)$) since a higher q implies that a larger share of the managerial tasks are done by hired professional work: when q increases an input which is effectively free (the managerial tasks performed by entrepreneur) gets a smaller weight in production[5]. Third, this direct cost can be partially compensated by an increase of $\log \tilde{A}(q)$ due to a "specialization" effect. As the entrepreneur professionalizes more tasks, she picks the ones with lower $\log \tilde{a}(q)$, thus increasing the overall productivity term (as long as there is heterogeneity in $\log \tilde{a}(q)$). Fourth, a higher q reduces the share of entrepreneurial input in production, hence the share of total output that is distributed as profits – i.e. $\eta'(q) > 0$.

Higher skilled entrepreneurs have both higher benefits from increasing the returns to scale (since $\tilde{y}$ is higher) and lower cost (since $\log \frac{w_p}{A(q)z}$ is smaller).

Finally, we notice that, given the definition of $\tilde{y}$, the technology choice is not affected by the neutral TFP term A.

Lemma 2 shows that the marginal cost of increasing technology may declining with q (since $\alpha'(q)$ is declining) and that the marginal benefit may be increasing (since $\log \tilde{y}$ is increasing in q). As a result, in general we expect that $q(\hat{z}) > 0$: the lowest skilled entrepreneur that decides to professionalize some tasks, does so for more than just a tiny few of them. This property of the model, which we assume to hold (see Assumption 1), is very convenient since it endogenously generates duality.[6] Some entrepreneurs only rely on blue collar workers, face steep decreasing returns, and have in equilibrium small firm sizes. Others hire hierarchies of professionals, achieving less decreasing returns to scale and thus choosing in equilibrium to operate firms of larger sizes.

ASSUMPTION **1.** We assume that the distribution of task specific productivity $\{\tilde{a}(z)\}_{\in[0,1]}$ is such that $q(\hat{z}) > 0$.

Figure 6a illustrates the economics of choosing q for entrepreneurs of different skill levels. The grey lines represent the relationship between profits and entrepreneurial quality z for different values of q – i.e. plots $\tilde{\pi}(z, q)$ as a function of z. As q increases, the profit functions become less concave and eventually turn convex in z. This reflects that a higher q reduces the degree of diminishing returns, disproportionately benefiting high-z entrepreneurs. The blue line, which is the upper envelope of the grey curves, represents the resulting profits of entrepreneurs at each level of z, which take into account the optimal technology choice. The ability to choose $q > 0$ raises the equilibrium profits of high-quality entrepreneurs while having no effect on the profits of low-quality entrepreneurs, who optimally choose $q = 0$.

Figure 6a also demonstrates the emergence of duality in the choice of q. For instance, curve $\pi(z, 0.25)$ shows that $q = 0.25$ is not an optimal choice for any level of z.

[5] A natural interpretation of this first term is that is a fixed cost of setting up departments of white collar professionals.

[6] We are still in the process of proving features of the primitives so that this property is satisfied.

The reason is that entrepreneurs with quality below $\hat{z}$ are better off setting $q = 0$, while those with quality above $\hat{z}$ prefer setting $q = 0.5$. This holds true for any q less than 0.25, implying that once you go above $q = 0$, you will go at least to a value $q > 0.25$.

The *endogenous duality* in the choice of q manifests itself in the firm-size distribution, as shown in Figure 6b. Low-z entrepreneurs operate identical, small-scale firms. However, when entrepreneurs cross the threshold $\hat{z}$, there is a discrete jump in employment and the organization of production.

For further increases in z beyond this threshold, firms grow progressively bigger and more skill-intensive, as can be noted by the steeper slope of $n_p(z)$. The speed of this process is characterized in the following lemma.

LEMMA **3** (Firm scale)**.** *The relationship between the (optimal) output produced by an entrepreneur and her ability z satisfies*

$$
\begin{aligned}
& y = y_0 && \textit{if } z \leqslant \hat{z} \\
& \frac{d \log y}{d \log z} = \frac{\alpha(q(z))}{\frac{1}{\eta(q(z))} - 1 - \frac{\partial \log \eta(q(z))}{\partial \log y}} && z > \hat{z}
\end{aligned}
$$

where $\frac{\partial q}{\partial z}, \frac{\partial \eta}{\partial z} > 0$ *for* $z > \hat{z}$.

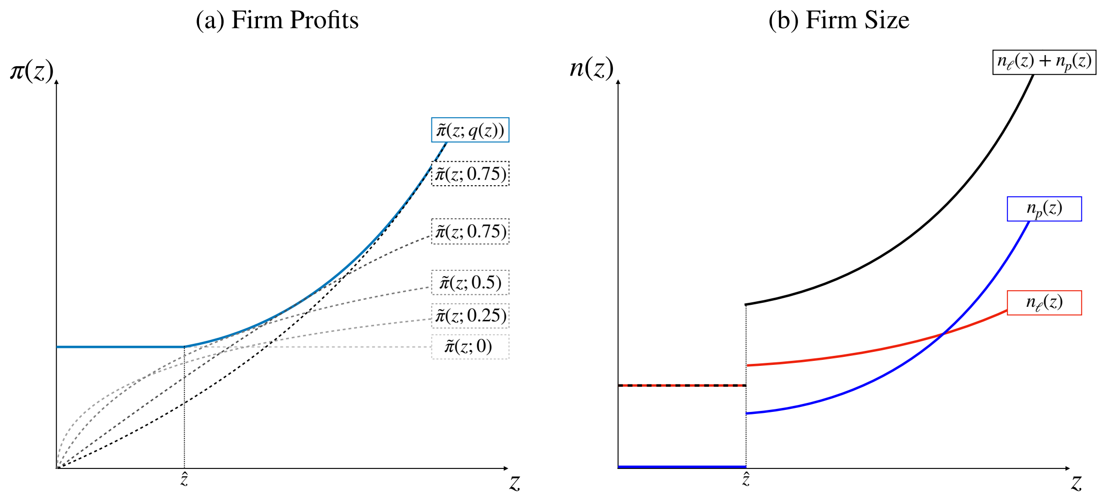
The lemma shows that the sensitivity of size to z depends on three components. First, the share $\alpha(q(z))$ which is the direct effect of z on productivity (given that z only applies to professional labor). Second, the local degree of decreasing returns $1/\eta(q) - 1$, with growth being faster when production is closer to constant returns to scale. Third, an additional effect stemming from endogenous organizational change, represented by $\frac{\partial \log \eta(q(z))}{\partial \log y}$: more skilled entrepreneurs choose a technology more conducive to have larger firms.

The lemma shows that endogenous re-organization amplifies the sensitivity of firm size to entrepreneurial skill at the top of the ability distribution. High-z entrepreneurs endogenously choose a production function that is closer to constant returns, allowing them to scale more aggressively to exploit their high productivity. This feature distinguishes our model from a standard Hopenhayn model, where η would be constant across firms, and in which there would be no effect of organizational change on the relationship between entrepreneurial ability and firm size.

Occupational Choice and Equilibrium within Sector

Next, we describe the equilibrium occupational choice and the wages and profits that support it. We are describing the equilibrium that must hold within each sector, given

FIGURE 6: TECHNOLOGY CHOICE AND ENDOGENOUS DUALITY



the endogenous prices and distribution of talent. For this reason, we still drop the subscript j in the analysis.

To connect the model with the empirical analysis is convenient to define what white and blue collar workers are in our model. The *endogenous duality* feature explained in the previous section is convenient towards this goal since it divides entrepreneur into two groups which are effectively disjointed.

DEFINITION **3.** *Blue collar workers* are laborers and traditional entrepreneurs – i.e. those that do not professionalize any tasks – : $\omega_{BC}(z) \equiv \omega_\ell(z) + \omega_\pi(z)\mathbb{I}_{z\leqslant\hat{z}}$.

DEFINITION **4.** *White collar workers* are professionals and modern entrepreneurs – i.e. those that professionalize tasks – : $\omega_{WC}(z) \equiv \omega_p(z) + \omega_\pi(z)\mathbb{I}_{z>\hat{z}}$.

For the rest of this section, we work under a convenient assumption which guarantees that the equilibrium displays talent-segmentation by occupation. We are going to relax this assumption for the quantification.

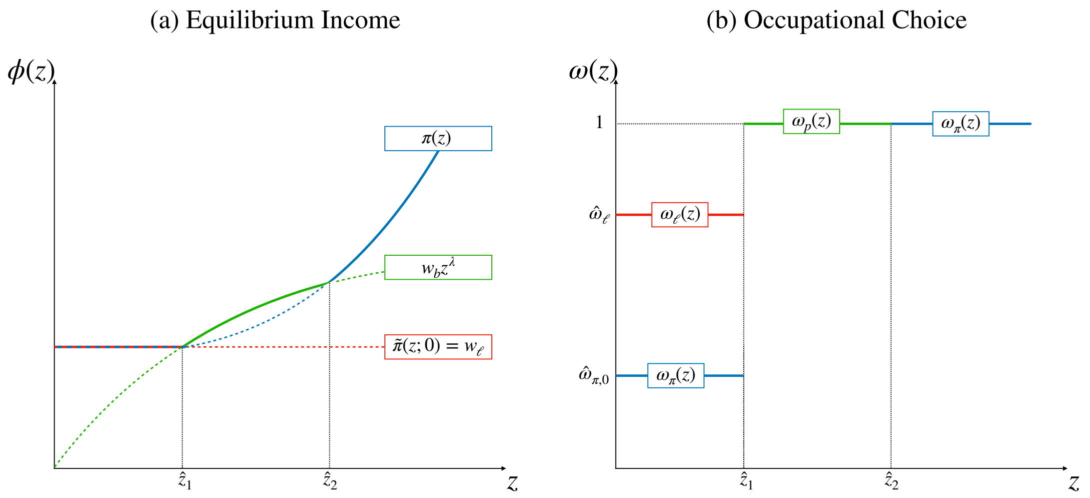
ASSUMPTION **2.** The skill sensitivity λ of professionals satisfies: $\lambda \leqslant w_p^{-1}\hat{z}^{1-\lambda}\pi'(\hat{z})$.

LEMMA **4** (Occupational Choice)**.** *If assumptions 1 and 2 are satisfied, the equilibrium yields the following properties*

1. *there exists two cutoffs $\hat{z}_1$ and $\hat{z}_2$ such all individuals with $z \leqslant \hat{z}_1$ are blue-collar workers, all those with $z \in (\hat{z}_1, \hat{z}_2)$ are professionals, all those with $z \geqslant \hat{z}_2$ are white-collar entrepreneurs;*

2. *the equilibrium wages satisfy $w_\ell = \tilde{\pi}(z, 0)$, $w_\ell = w_p\hat{z}_1^\lambda$, $w_p\hat{z}_2^\lambda = \pi(\hat{z}_2)$;*

FIGURE 7: INDIVIDUAL INCOME AND OCCUPATIONAL CHOICE



3. *the share of laborers among the individuals less skilled than $\hat{z}_1$ clears the labor market:*

$$\underbrace{\hat{\omega}_\ell G(z)}_{\text{Supply of Laborers}} = \underbrace{n_{\ell,0}\hat{\omega}_{\pi,0}}_{\text{Demand from BC Entr.}} + \underbrace{\int_{\hat{z}_w}^{\overline{z}} n_\ell(x)dG(x)}_{\text{Demand from WC Entr.}} \tag{10}$$

Lemma 4 is visualized in Figure 7. The left panel shows how the equilibrium income $\phi(z)$ is determined. It shows the earnings that an individual would make, given equilibrium prices, as a function of individual skill z. The red line is the wage for laborers, which is identical to the profit of blue-collar entrepreneurs (in equilibrium). The green line is the wage of professionals, which is increasing in z, with elasticity modulated by λ. Finally, the blue line shows the profit of entrepreneurs (both blue and white collars), which take into account the optimal choice of technology q. Individuals choose the occupation which maximize their earnings, as shown in equation 2.

The right panel shows the resulting occupational choice. Importantly, the low skilled individuals are indifferent between being a laborers or a traditional, blue-collar, entrepreneur. The reason is that both occupations are not skill-intensive and thus low-skilled entrepreneurs have a comparative advantage there. As shown in Lemma 4, the share of laborers is thus purely determined by market clearing. As a result, an increase in the mass of white-collar entrepreneurs would decrease, in equilibrium, the blue-collar ones by pulling low-skilled workers into firms. This mechanism is consistent with the evidence of Sub-Section 2.3, as we further discuss and formalize in the next Section.

4 Bringing the Model to Data: Skills for Scale

In this section, we show that a limit version of our model is able to account for the empirical patterns documented in Section 2. Increasing the supply of skills leads to a re-organization of the economy that closely match, both qualitatively and quantitatively, the empirical patterns documented.

4.1 A Convenient Limit Case

We consider a single sector economy, hence drop any subscript j from the analysis, and make two assumptions to simplify the analysis.

ASSUMPTION **3.** Each task has identical productivity if performed by the entrepreneur: $\tilde{a}(i) = \tilde{a}$.

ASSUMPTION **4.** The income of professional is as skill-intensive as the profit of an entrepreneur with $q = 1$: $\lambda = \frac{\bar{\eta}-\gamma_\ell}{1-\bar{\eta}}$.

Assumption 3 implies that task-productivity term simplifies to $\tilde{A}(q) = \tilde{a}$ and the problem (8) reads as

$$\max_{q\in[0,1],n_p,n_l} A^{1-q\bar{\eta}-(1-q)\gamma_\ell}\tilde{a}^{1-q}\left(\frac{zn_p}{\bar{\eta}-\gamma_\ell}\right)^{q(\bar{\eta}-\gamma_\ell)}\left(\frac{n_l}{\gamma_\ell}\right)^{\gamma_\ell} - qw_pn_p - w_ln_l \tag{11}$$

The production function thus simplifies to a Cobb-Douglas, which has one fixed input $\tilde{a}$, whose weight is decreasing in q. Problem 11 is convex in q implying that entrepreneurs would only choose values of $q \in \{0, 1\}$

Assumption 4 further simplifies the analysis by guaranteeing that, in equilibrium, the most skilled individuals are indifferent between being a professional or an entrepreneur with $q = 1$.

Under these assumptions the equilibrium can be fully characterized analytically.

LEMMA **5** (Limit Case)**.** *Under assumptions 3 and 4 the equilibrium satisfy the following properties:*

1. *there is a cutoff type $\hat{z}$, with*

$$\hat{z} \equiv \left(\frac{w_p}{w_l}\right)\left(\frac{1-\gamma_\ell}{1-\bar{\eta}}\right)^{\frac{\bar{\eta}(1-\bar{\eta})}{\bar{\eta}-\gamma_\ell}} w_l^{-\frac{\bar{\eta}}{1-\gamma_\ell}}\tilde{a}^{\frac{\bar{\eta}(1-\bar{\eta})}{\bar{\eta}-\gamma_\ell}}$$

such that all individuals $z \leqslant \hat{z}$ are indifferent between being laborers or own-account entrepreneurs, while all those with $z > \hat{z}$ are indifferent between being professionals or entrepreneur with $q = 1$;

2. *wages and profits satisfy*

$$\begin{aligned} \pi(z \leqslant \hat{z}) &= w_l &\propto \tilde{a}^{\frac{1}{1-\gamma_l}} \\ \pi(z > \hat{z}) &= w_p z^{\lambda} &\propto z \end{aligned}$$

3. *the skill premium does not depend on skill supply, but only on skill bias.*

4.2 Skill Supply and Organization of Production

We now use the simple model to study the aggregate effects of an increase in the supply of skills, as captured by upward shift in distribution of schooling $\Gamma(s)$. This allows us to show how the predictions of the model are in line with the cross-country facts documented in Section 2.

Skills and White Collar Occupations. In the model, we think about modern entrepreneurs with $q = 1$ and professionals as capturing white collar occupations. As shown in Lemma 5, workers with $z > \hat{z}$ become white collars, so that the white collar share conditional on schooling is given by

$$\Omega_{WC}(s) = 1 - G(\hat{z}|s)$$

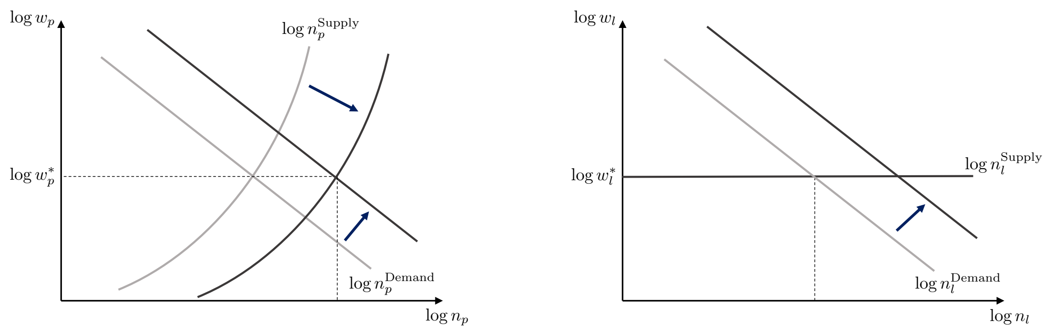
that is increasing in s. Given that $\hat{z}$ only depends on technological parameters, $\Omega_{WC}(s)$ is independent of $\Gamma(s)$. In other words, a shift in the schooling distribution affects the white collar share only through composition effects, but does not impact the white collar share conditional on schooling. This is in line with the evidence in Section 2: countries with vastly different skill supplies have similar white collar propensities by education, and higher aggregate white collar shares are almost entirely accounted for by differences in the educational composition of the workforce.

The key driver of this result is that the skill premium is invariant to changes in the skill supply. Figure 8 illustrates the equilibrium in the markets for professional efficiency units and laborers. An increase in the skill supply leads to both an increase in the supply of professional efficiency units, as more educated workers are more likely to be above the skill threshold $\hat{z}$, as well as in their demand, due to the entry of highly skilled entrepreneurs that want to leverage economies of scale by hiring professionals.

In this simple version of the model these two effects perfectly compensate, so that wages and skill premium are unaffected (Figure 8a). The rise of high-skill entrepreneurs also leads to a higher demand for laborers (Figure 8b). Given the outside option of being a traditional entrepreneur, the supply of laborers is perfectly elastic at the equilibrium w_l, which is therefore unaffected by the increase in demand. An increase in the demand for blue-collar workers thus simply bring low skilled individuals from being own-account entrepreneurs to becoming laborers in firms.

FIGURE 8: AN INCREASE IN SKILL SUPPLY

(a) Professionals (b) Laborers



Skills and Traditional Entrepreneurship. Consider now the traditional entrepreneurs with $q = 0$. Denoting their share conditional on schooling as $\Omega_T(s)$, we can write

$$\Omega_T(s) = g(\mathbb{Z}_{WC})\,[1 - \Omega_{WC}(s)]$$

where $\mathbb{Z}_{WC} = \int \omega_{WC}(z) z dG(z)$ is the total supply of white collar efficiency units and $g(.)$ is a strictly decreasing function. Given that $\Omega_{WC}(s)$ is increasing in s, individuals with higher schooling are less likely to be traditional entrepreneurs. Moreover, a higher overall supply of skills lowers $\Omega_T(s)$ for all levels of schooling through the $\mathbb{Z}_{WC}$ term. Intuitively, the increase in the quantity and skill of entrepreneurs and professional leads to larger firms and hence an increase in the demand for laborers, which pulls labor away from traditional entrepreneurship.

These results are qualitatively in line with the relationship between education and own account work, which we interpret as a proxy of traditional entrepreneurship. As shown in Section 2.3, highly educated workers are generally less likely to be in own account, especially so in poorer countries. Conditional on education, the own account share declines with development, and more so for lower education groups. Thus, the own account share varies across countries both as a function of differences in the educational distribution and due to rich countries having a lower own account share by education. Through the lens of our model, the latter gap can be rationalized by a higher supply of skills in rich countries that endogenously pulls labor from traditional entrepreneurship into modern firms.

Skills and Firm Size. The average firm size is given by

$$\underbrace{\frac{\int_{\hat{z}}^{\overline{z}} \omega_\pi(x) dG(x)}{\int_0^{\overline{z}} \omega_\pi(x) dG(x)}}_{\substack{\text{Share of}\\ \text{Modern Firms}\\ \downarrow \\ \text{Increasing in } \mathbb{Z}^{WC}}} \times \underbrace{\int_{\hat{z}}^{\overline{z}} (n_\ell(x) + n_p(x))\omega_\pi(x) dG(x)}_{\substack{\text{Avg Size of}\\ \text{Modern Firms}\\ \downarrow \\ \text{Increasing in } z^{WC}}} + \underbrace{\frac{\int_0^{\hat{z}} \omega_\pi(x) dG(x)}{\int_0^{\overline{z}} \omega_\pi(x) dG(x)}}_{\substack{\text{Share of}\\ \text{Traditional Firms}\\ \downarrow \\ \text{Decreasing in } \mathbb{Z}^{WC}}} \times n_{\ell,0}$$

where $\mathbb{Z}^{WC}$ and z^{WC} are respectively the total and the average of the efficiency units supplied by white collar individuals. A higher supply of skills increases firm size through two channels. First, it leads to a higher prevalence of modern firms, that in equilibrium are larger than traditional firms. Second, it also causes an increase in the average size of modern firms, due to the increase in the average skill of entrepreneurs and professionals.

5 Calibration and Quantitative Results

We calibrate the model to fit cross-country and cross-sectional data on skills, occupational choices, and firm productivity. We use the calibrated model to study the effect on an economy of experiencing an increase in total factor productivity or skills alone, while holding the other fixed. The goal of this experiment is to shed light on the complementary importance of skills and the re-organization of production in growth. The combination of skills and re-organization of production will be more important in some sectors, leading to differential productivity growth by sector and structural transformation as in Ngai & Pissarides (2007). It will also have a direct effect on GDP per capita that we want to quantify.

5.1 Calibration

We calibrate our model using three sets of targets. First, some parameters, such as the preferences, are set using external references. Second, we calibrate the model so that it is consistent with our motivating facts. This disciplines the distribution of skills by country; the link from skills to occupational choice; the share of workers in own account versus large firms; and the white collar intensity of production.

Finally, our model makes rich predictions about the distribution of outcomes for entrepreneurs with different skill levels, including their firm's size, hiring patterns, and productivity. To help discipline the parameters that govern these predictions, we turn to matched employer-employee data, which allows us to study the distribution of firm characteristics and link each firm to the workers they employ.

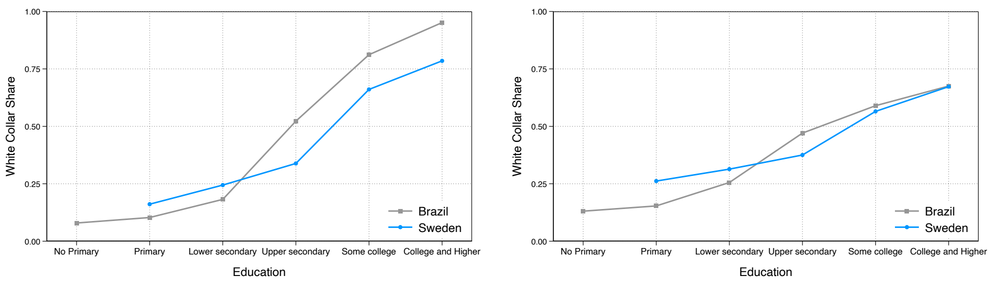
5.2 Calibration: Matched Employer-Employee Data

We use matched employer-employee data from Brazil (RAIS) and Sweden (JOBB). These data provide detailed information on the role of firms in two countries at different stages of development and structural transformation. The data cover the universe of formal firms and workers employed at those firms in each country. For Sweden the number of informal firms is generally considered to be small; for Brazil we are missing the smallest and least productive firms.

Figure 9 starts by showing that Brazil and Sweden are little different than the rest of the world, in the sense that more educated individuals are much more likely to be white collar (panel (a)). In particular, while only 10–20 percent of uneducated workers choose white collar occupations, 90 percent of college-educated workers do. Panel (b) plots the average white collar share of a worker's co-workers. This figure shows that highly educated workers are sorted into firms that hire high shares of white collar workers; in the model, these are firms operated by highly skilled entrepreneurs.

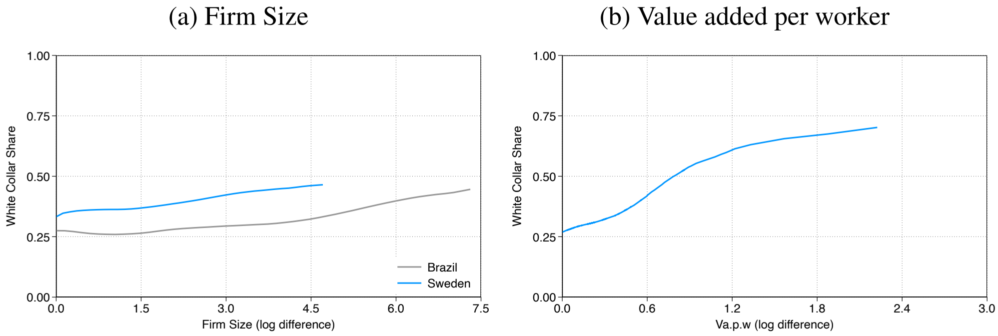
FIGURE 9: WHITE COLLAR WORKERS AND WHITE COLLAR FIRMS

(a) Raw share (b) Accounted for by sorting into firms



To delve into these findings further, Figure 10 links the share of white collar workers to two firm-level outcomes: average size and average labor productivity. According to panel (a), larger firms employ a greater share of white collar workers, in both Brazil and Sweden. Conditional on size, firms in Sweden employ a larger white collar share than their Brazilian counterparts. Panel (b) uses the firm financial data from Sweden to show that more (labor) productive firms employ a larger share of white collar workers. Data limitations prevent the equivalent analysis in Brazil.

FIGURE 10: THE SHARE WHITE-COLLAR AND FIRM OUTCOMES



We calibrate the parameters of the production function so that the model is consistent with these patterns given the distribution of skills.

5.3 Quantitative Results

[TBD]

5.4 Validating the Mechanism

Our theory relies on a particular causal mechanism: raising skills changes optimal occupational choices and firm structure. This is consistent with the results from a growing literature that studies the effect of policy-induced, exogenous expansions of education, typically through building schools or universities. Many of these papers show that expansions of schooling lead to a reallocation of labor across sectors, generally away from agriculture and towards manufacturing or skill-intensive services (Porzio *et al.*, 2022; Coelli *et al.*, 2023; Nimier-David, 2023; Russell *et al.*, forthcoming; Cox, 2023). These findings offer indirect support of our model. Cox (2023) shows that building colleges in Brazil led to an increase in the number of large firms in the local area, directly supporting our main mechanism.

We add to this literature by using spatial variation in a large expansion of universities across Swedish commuting zones (CZs) to provide additional evidence. We use again

Swedish matched employer-employee data available between 1990 and 2018 that cover the universe of individuals aged 16 to 70 who are legally residing in Sweden, as well as the universe of firms, including incorporated and unincorporated businesses. The data record standard demographic characteristics of individuals, including gender, age and education level, as well as sector and location. Firm and establishment identifiers allow us to construct firm and establishment size.

The national statistical agency of Sweden, *Statistics Sweden*, divides Sweden into 69 CZs based on commuting flows. The number of CZs gradually declined over time thanks to improving infrastructure that made it feasible to commute longer distances. We consistently use the 2018 delineation.

We restrict attention to individuals who are 25 to 64 years old, and drop those working in the public sector. As a baseline, we focus on those who remain in the same CZ as they were born in order to limit the impact of migration on our results (but results are robust to not doing so).

We construct the share of employment in CZ i in year t with a college degree, college_{it}. We also record log average firm size in CZ i in year t, size_{it} (unweighted by employment).[7]

Skill Supply and the Organization of Production: A First Look. Figure 11 offers a first look at the data. Specifically, panel (a) plots log average firm size against the fraction of the workforce with a college degree across Sweden's 69 CZs, pooling all years of data. We normalize the former relative to the CZ with the lowest average firm size. Firms are larger in local labor markets with a more educated workforce. Panel (b) repeats the analysis in long run changes. Specifically, it plots within-CZ changes in log average firm size between 1990–2004 and 2005–2018 against the change in the fraction of the workforce with a college degree over the same time frame. CZs that experienced a relatively larger increase in educational attainment saw a disproportionate rise in average firm size.

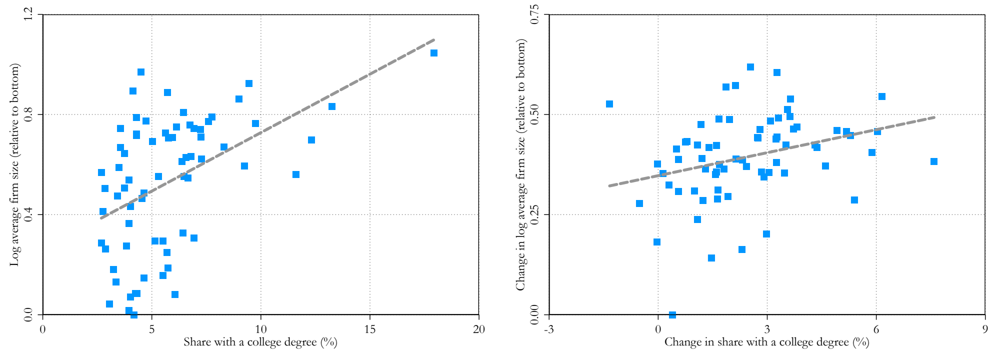
While the patterns in Figure 11 are consistent with the predictions of our theory, there are clearly many alternative explanations for this correlation. For instance, a local economic boom may incentivize college attendance and raise firm size. Hence, we proceed to look for exogenous variation in the local supply of skills.

Exogenous variation in local skill supply. Starting in the mid-1960s, Sweden embarked on a dramatic expansion of its higher education system. Specifically, the central

[7] Detailed firm financial data are available, but only since the late 1990s. Since we require a long panel to exploit within-location variation as we do below, we do not use firm financial information.

FIGURE 11: SKILL SUPPLY AND AVERAGE FIRM SIZE

(a) Levels 1990–2018 (b) Changes 1990–2004 to 2005–2018



government constructed new colleges in various parts of Sweden, which in most cases had at most a school for teachers prior to the construction of these new colleges. While the decision of where to locate these colleges was not random, it was largely driven by political considerations, including a desire to provide more equal access to college across the country. In total, 23 of Sweden's 37 colleges were established over the next decades, with a particularly rapid expansion during the second half of the 1970s and first half of the 1980s.

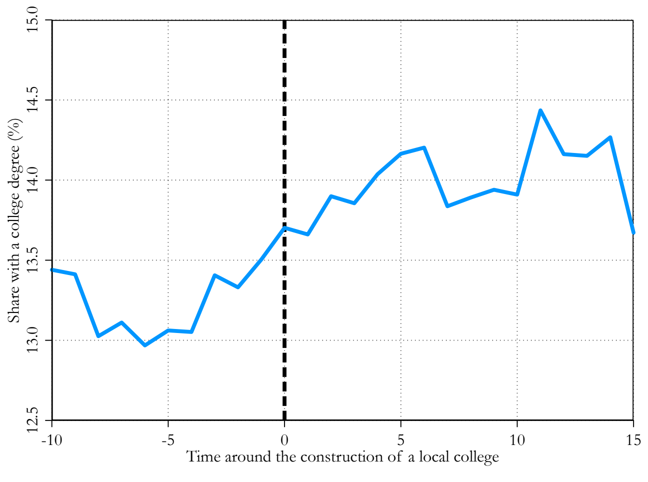
We exploit this rapid build out as a source of exogenous variation in college access. To that end, we collect data on the founding dates of Sweden's 37 colleges. Subsequently, we construct a set of indicators for whether a new college opened in CZ i up to 15 years prior to and up to 10 years after a cohort c turned 25, $\{b^{\tau}_{ci}\}^{15}_{\tau=-10}$. We take the opening of a new college at age 25 as a proxy for whether a cohort gained additional access to college. It understates access if some 25 year olds return to school once a local college becomes available, while it overstates access if some 25 year olds find it too late to return to school when a local college becomes available.

To provide a first look at whether the establishment of a college is associated with an increase in educational attainment for the cohorts that benefited from additional college access, we project the fraction of a cohort c in CZs i of age a that has at least a college degree, $\widehat{\text{college}}_{cia}$, on the leads and lags indicating access to college, controlling for CZ-age and cohort-age fixed effects

$$\widehat{\text{college}}_{cia} = \sum_{\tau=-10}^{15} \beta^{\tau} b^{\tau}_{ci} + \xi_{ia} + \phi_{ca} + \varepsilon_{cia} \tag{12}$$

Figure 12 plots the estimated coefficients $\{\beta^{\tau}\}^{15}_{\tau=-10}$. Relative to other cohorts of the same age in other CZs, a treated cohort's college attainment rises after the construction of a local college, from an average of just over 13 percent to a little over 14 percent.[8] The construction of a college appears to raise skill supply among cohorts that turned 25 a few years before the new college opens. Possible interpretations is that some individuals opt to go back to school once a local college becomes available. The estimated effect of additional college access on cohort college attainment is strongly statistically significant after the opening of a college, but in general not prior to the opening of a local college (clustering standard errors at the CZs level).

FIGURE 12: COHORT SKILL SUPPLY AROUND THE OPENING OF A COLLEGE



Skill Supply and the Organization of Production: A Detailed Look. To formalize the discussion above, we first compute the average number of colleges available to the cohorts active in CZ i in year t, i.e. the cohorts aged 25–64 in year t

$$\text{colleges}_{it} = \sum_{c \in \mathcal{C}_t} \omega_{cit} \widehat{\text{colleges}}_{ci}$$

where we measure the number of colleges available to cohort c in CZ i, $\widehat{\text{colleges}}_{ci}$, as the number of colleges in CZ i in the year cohort c turned 25. Subsequently, we project log average firm size on the share of college graduates across 69 CZs and 29 years, controlling for CZ (ξ_i) and year (ϕ_t) fixed effects

$$\text{size}_{it} = \beta \text{college}_{it} + \xi_i + \phi_t + \varepsilon_{it} \tag{13}$$

[8] The level of college attainment is significantly higher among recent cohorts, at over 30 percent. The overall level is so low because we include cohorts born as early as 1926, which have much lower levels of educational attainment.

Alternatively, we instrument for the share with a college degree in CZ i in year t with the average number of colleges available to the cohorts in CZ i in year t

$$\text{college}_{it} \;=\; \alpha \text{colleges}_{it} \;+\; \psi_i \;+\; \zeta_t \;+\; \nu_{it}$$

We cluster standard errors at the CZ level.

Table 2 presents our OLS and IV estimates of the impact of workforce skill on average firm size. An increase in the share of the workforce with a college degree is associated with a strongly statistically significant rise in average firm size. Moreover, the opening of a new college is a strong predictor of subsequent changes in the fraction of the local workforce with a college degree. When used as an instrument for local skill supply, the effect of an increase in workforce skills on average firm size is even larger (although the difference relative to the OLS estimate is not statistically significant). In terms of magnitudes, these estimates imply that going from no college graduates to all college graduates is associated with 6–15 fold increase in average firm size.

TABLE 2: LOCAL SKILL SUPPLY AND FIRM SIZE

	OLS	IV
Share college, β	1.861	2.745
	(0.612)	(1.118)
First stage, α		0.057
		(0.010)
F-test		33.886
N	2,001	2,001
Clusters	69	69

Notes: The Table shows the results from regression (13) estimated either by OLS or 2SLS using the construction of colleges as instrument.

6 Conclusion

Chandler (1977) shows that the transition to managerial capitalism played a critical role in United States economic history. We argue that a lack of sufficient skills in developing countries today prevents them from making the same transition. Specifically, we make three contributions. First, we provide new data linking skills to occupational choices and firm structure. We show that larger firms use white collar labor more intensively;

that skills account for almost all of the gap in white collar labor supply between developing and developed economies; and that development pulls unskilled labor out of own account work and into firms.

Second, we develop an occupational choice model in the spirit of Lucas (1978), but with the key novel twist that entrepreneurs can also choose for a set of managerial functions whether to perform each function themselves or to delegate them to hired white collar workers. This effectively allows sufficiently productive entrepreneurs to use hired professionals to scale up production, consistent with the historical narrative of Chandler. We provide analytical results showing that the economy is consistent with all three of our motivating facts.

Finally, our work in progress is to calibrate this model to cross-country evidence as well as richer cross-sectional evidence derived from matched employer-employee data from Sweden and Brazil. We plan to use the model to study the effect on an economy of experiencing an increase in total factor productivity or skills alone, while holding the other fixed. The goal of this experiment is to shed light on the complementary importance of skills and the re-organization of production in growth.

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A Data Appendix

This Appendix provides further results related to our second motivating fact from Section 2.2, which is that differences in the skill distribution account for almost all of the gaps in the white collar share between rich and poor countries. Table A.1 summarizes some of the key relationships in terms of how much of the correlation between white collar employment share and development can be accounted for through skills.

TABLE A.1: ACCOUNTING RESULTS: ROBUSTNESS

	Unconditional Elasticity	Conditional Elasticity	Accounting Share
(1) Baseline	0.119	0.014	0.882
	(0.001)	(0.001)	
(2) Sector FE	0.053	-0.001	1.001
	(0.001)	(0.001)	
(3) Literacy Score	0.133	0.008	0.939
	(0.002)	(0.002)	

Notes: The Table shows the results of the accounting exercises described in the text. Rows 1-2 use data from IPUMS International, while Row 3 uses data from PIAAC and STEP.

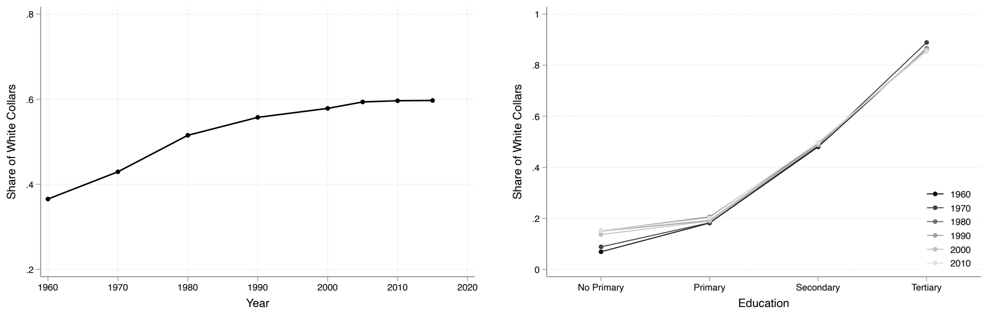
A.1 Time Series Results

The analysis in Section 2 combines the cross-sectional and time series variation by pooling all available surveys. This Appendix illustrates the results when focusing on the time series alone. Figure A.1 starts by focusing on the United States, the country with the longest available time series. Panel (a) shows that the white collar has increased by more than 20 percentage points between 1960 and 2015. Panel (b) shows that the white collar propensity by education is remarkably constant across decades, implying that virtually all the aggregate increase in Panel (a) can be accounted for by changes in the educational composition over time.

Figure A.2 displays the white collar propensities from all countries in the sample, group by income level, across three time periods. While the propensities by education are very stable in rich countries, for low and middle income countries we see a decline in the white collar share of primary and secondary educated workers, in particular between 1970 and 2010, a period of rapid educational expansion in the developing world. One possible reason for this might be a decline in education quality resulting from the rapid increase in enrolment, consistently with Le Nestour *et al.* (2023). Nevertheless,

FIGURE A.1: WHITE COLLAR OCCUPATIONS OVER TIME - UNITED STATES

(a) White Collar Share Over Time (b) Propensities by Decade

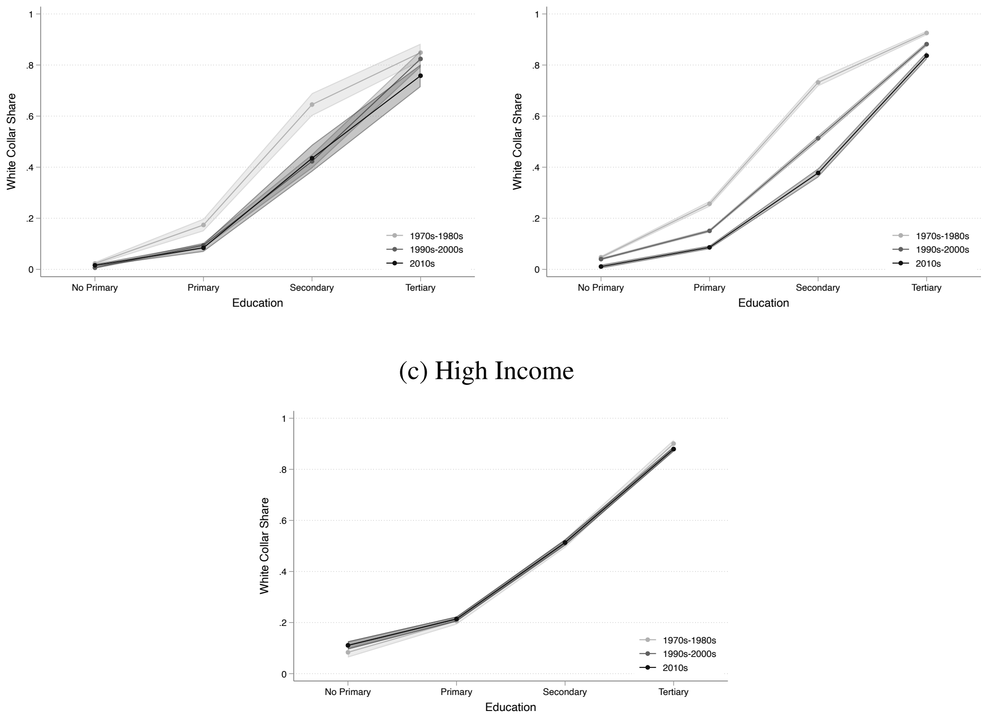


differences across education groups remain large in all periods, and changes in the education composition can account for most of the variation in the white collar share over time.

FIGURE A.2: WHITE COLLAR PROPENSITIES OVER TIME - ALL COUNTRIES

(a) Low Income (b) Mid Income

(c) High Income

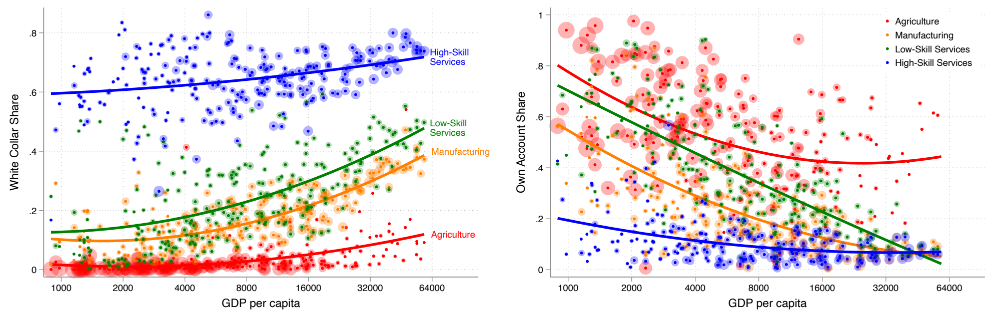


A.2 The Role of Sectors

A natural question is whether the association between the white collar and own account shares and economic development simply reflect structural change across sectors. As shown in Figure A.3, sectors do play an important role. Agriculture has few white collars and many own account workers in most countries, and the reallocation of labor away from this sector into manufacturing and services contributes to the aggregate patterns in Figure 3. However, the organizational structure also changes within sectors, in particular for manufacturing and low-skill services.

FIGURE A.3: THE ROLE OF SECTORS

(a) White Collar (b) Own Account



A.3 Alternative Measures of Skills

This section investigates the relationship between white collar employment shares and skills for several alternative measures of skills.

A.3.1 Adult Test Scores

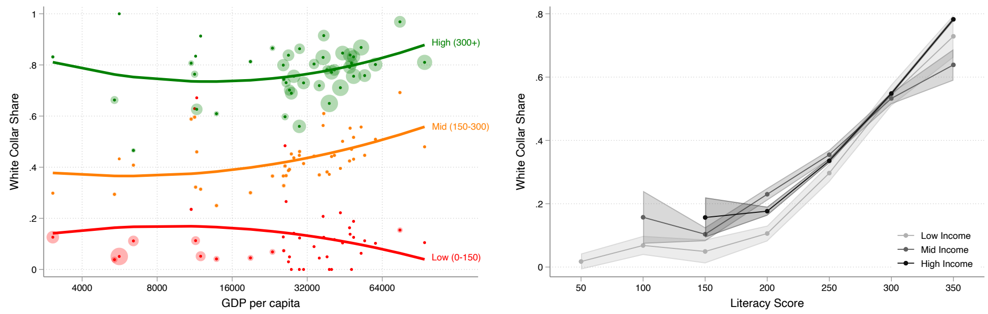
In addition to educational attainment, we can study trends in white collar employment shares as a function of adult test scores for a large number of countries around the world. For this analysis we use data from the OECD PIAAC Survey of Adult Skills and the World Bank STEP Skills Measurement Program. The OECD PIAAC surveyed roughly 5,000 adults age 15–65 in more than 40 countries. Its tests measure skills in literacy, numeracy, and problem solving. The World Bank STEP program builds on and expands the scope of PIAAC by surveying 2,000–4,000 adults age 16-65 in 12 poorer countries/regions. They measure literacy and socioemotional skills. We combine the two datasets and focus on literacy skills, which are measured in both, as done elsewhere in the literature (Caunedo *et al.*, 2023). Our final sample includes 43 countries, spanning the income distribution between Kenya and Norway.

Figure A.4 repeats Figure 4 using adult literacy scores (a direct measure of skills) in place of education. The same patterns apply: workers with higher test scores are much more likely to engage in white collar work; cross-country differences in white collar employment shares conditional on skills are small. Figure A.4b shows again that the propensities are strongly increasing with adult test scores, in a nearly identical fashion across countries with vastly different income levels. Row (3) of Table 1 shows that these results again imply that skills account for most of the correlation between white collar employment shares and development.

FIGURE A.4: LITERACY AND WHITE COLLAR OCCUPATIONS

(a) White Collar Share by Literacy Score

(b) Estimated Propensities



A.3.2 Childhood Test Scores

The key advantage of adult test scores is that they measure the skills workers have (rather than how long they sat in a classroom). However, they are plausibly endogenous, in the sense that workers' skills may in part be caused by practicing and using those skills more in the course of performing their occupation. As an alternative approach, we also explore the relationship between occupational choices and childhood test scores.

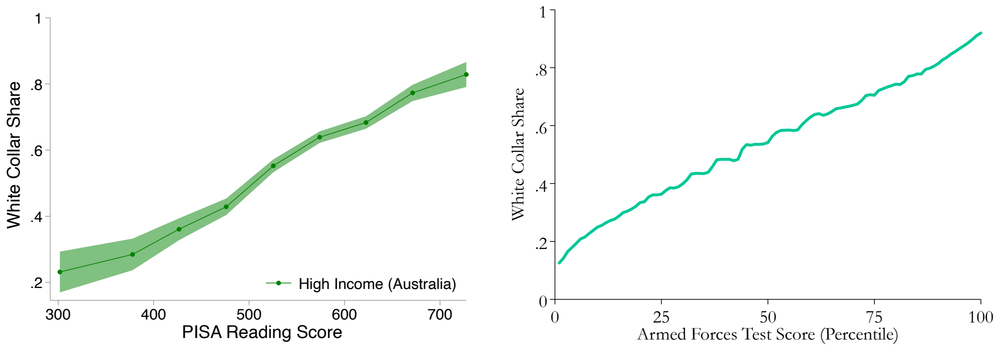
We source this data from two sources. First, we combine Programme for International Student Assessment (PISA) and Longitudinal Survey of Australian Youths (LSAY) data. The former measures literacy and mathematics proficiency of 15-year olds in countries around the world. The latter builds on the PISA in Australia. It tracks test-takers into early adulthood, as late as age 25, and hence allows us to link the test scores of Australian students with their subsequent occupational choices. This dataset has the advantage that it is directly linked to PISA. However, the sample size is relative small; after pooling waves we have test scores and occupational choices for 12,000 Australians. Given that Australians score relatively well on the PISA exam, this implies that we have a small sample of students with low test scores in terms of the global distribution. To help address this final concern, we turn to the Swedish microdata. We measure childhood skills using scores from the military conscription test given to all men at age 18. This allows us to link test scores to occupational choice for all Swedish man, providing a much larger sample of millions of men.

Figure A.5a plots the propensity of being a white collar as a function of the PISA score in Australia. The relationship is strongly increasing. Figure A.5b replicates the same analysis on the Swedish data. Once again, workers with higher skills at age 18 are more likely to subsequently work in a white collar occupation.

FIGURE A.5: WHITE COLLAR OCCUPATIONS AND ADULT SKILLS

(a) Childhood Test Scores (Australia)

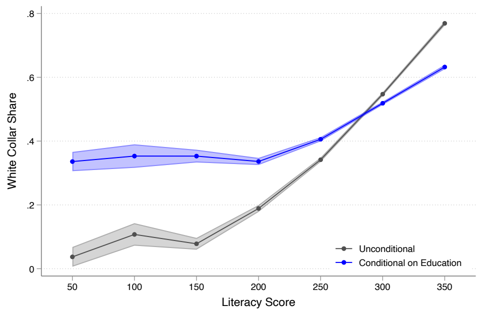
(b) Armed Forces Score (Sweden)



A.3.3 Literacy Conditional on Education

Using the PIAAC-STEP data this Appendix examines the relationship between occupational choices and literacy conditional on education. Figure A.6 plots the estimated white collar propensity by literacy score (pooling all countries), with and without education dummies. While a marginal increase in literacy does not matter much at the bottom of a distribution, going from a score of 200 (around the 10th percentile of the global distribution) to a score of 350 (around the 99th percentile of the global distribution) keeping education constant increases the white collar share by about 30 percentage points. This corroborates the point that the sorting into white collar jobs reflects skills and not just educational credentials.

FIGURE A.6: LITERACY CONDITIONAL ON EDUCATION



B Model Appendix

Proof of Lemma 1. The cost function for producing output y is

$$c[y, w_l, w_p] = \min_{n_l, \{n_{p,i}\}} w_l n_l + \int_0^1 n_p(i) w_p di$$

subject to

$$A \exp\left[\int_0^1 \log\left(\frac{\tilde{n}(i)}{\bar{\eta} - \gamma_l}\right) di\right]^{\bar{\eta}-\gamma_l} \left[\frac{z n_l}{A\gamma_1}\right]^{\gamma_l} \geqslant y$$

$$\tilde{n}(i) \leqslant \max\{\tilde{a}(i)(\bar{\eta} - \gamma_l)^{1/(\bar{\eta}-\gamma_l)}, \frac{z}{A} n_p(i)\}$$

$$n_p(i) \geqslant 0$$

First order conditions

$$
\begin{aligned}
w_l &= \frac{\lambda_y \gamma_l}{y n_l} \\
\frac{\bar{\gamma}_p}{\tilde{n}(i)} \frac{\lambda_y}{y} &= \lambda_{\tilde{n}}(i) \\
w_p &\geqslant \lambda_{\tilde{n}}(i) \frac{\partial \max\{\tilde{a}(i)(\bar{\eta} - \gamma_l)^{1/(\bar{\eta}-\gamma_l)}, \frac{z}{A} n_p(i)\}}{\partial z} && = \text{if } n_p(i) > 0
\end{aligned}
$$

We can further simplify to

$$
w_p \geqslant \begin{cases} 0 & \text{if } \frac{z}{A} n_p(i) < \tilde{a}(i)(\bar{\eta} - \gamma_l)^{1/(\bar{\eta}-\gamma_l)} \\ \frac{z}{A} \lambda_{\tilde{n}}(i) & \text{if } \frac{z}{A} n_p(i) \geqslant \tilde{a}(i)(\bar{\eta} - \gamma_l)^{1/(\bar{\eta}-\gamma_l)} \end{cases}
$$

with equality if $n_p(i) > 0$. Note that if $n_p(i) > 0$, then $n_p(i) \geqslant \frac{1}{z}\tilde{a}(i)$. Now, assume that $\tilde{a}(i)$ is differentiable and strictly increasing in i. Then, there exist a unique cutoff $q \in [0, 1]$ such that $n_p(i) > 0$ for all $i \leqslant q$ and $n_p(i) = 0$ for all $i > q$. To establish this, we show that $n_p(i) > 0$ that implies that $n_p(i') > 0$ for all $i' < i$ and that $n_p(i) = 0$ implies $n_p(i') = 0$ for all $i' > i$. Let's start with case 1. Suppose that $n_p(i) > 0$. Now, $n_p(i) > 0$ implies $n_p(i) \geqslant \frac{1}{z} \frac{\tilde{a}(i)}{a(i)}$ and thus $\tilde{n}(i) \geqslant \tilde{a}(i)$, meaning that $\lambda_n(\tilde{i}) \leqslant \frac{\bar{\gamma}_p}{\tilde{a}(i)} \frac{1}{y}$. Now, suppose that there exists $i' < i$ such that $n_p(i') = 0$. Then, we have

$$
\tilde{\lambda}_n(i') = \frac{\bar{\gamma}_p}{\tilde{a}(i')} \frac{1}{y}.
$$

However, this means that we have

$$
\begin{aligned}
\frac{z}{A} \lambda_{\tilde{n}}(i') a(i') &= \frac{z}{A} \frac{1}{\tilde{a}(i')} \bar{\gamma}_p \frac{1}{y} \\
&> \frac{z}{A} \frac{1}{\tilde{a}(i)} \bar{\gamma}_p \frac{1}{y} \\
&\geqslant \frac{z}{A} \frac{1}{\left(\frac{\bar{\gamma}_p}{\lambda_{\tilde{n}}(i)} \frac{1}{y} \right)} \bar{\gamma}_p \frac{1}{y} \\
&= \frac{z}{A} \lambda_{\tilde{n}}(i) \\
&= w_p
\end{aligned}
$$

which is a contradiction.

Conversely, suppse that $n_p(i) = 0$. Then we have

$$
\lambda_{\tilde{n}}(i) = \frac{\bar{\gamma}_p}{\tilde{a}(i)} \frac{1}{y}
$$

Suppose now that we have some $i' > i$ with $n_p(i) > 0$. Then we have at least output $\tilde{a}(i)$, so we have

$$\tilde{\lambda}_n(i) \leqslant \frac{\bar{\gamma}_p}{\tilde{a}(i)}\frac{1}{y}.$$

Now, we get

$$\begin{aligned}
\frac{z}{A}\lambda_{\tilde{n}}(i')a(i') &\leqslant \frac{z}{A}\frac{\bar{\gamma}_p}{\tilde{a}(i')}\frac{1}{y}a(i') \\
&< \frac{z}{A}\bar{\gamma}_p\frac{1}{y}\frac{1}{\tilde{a}(i)} \\
&= \frac{z}{A}\bar{\gamma}_p\frac{1}{y}\frac{1}{\frac{\bar{\gamma}_p}{\lambda_{\tilde{n}}(i)}\frac{1}{y}} \\
&= \frac{z}{A}\lambda_{\tilde{n}}(i) \\
&\leqslant w_p
\end{aligned}$$

Contradicting that $\frac{z}{A}\lambda_n(i') = w_p$ when $n_p(i') > 0$.

Thus, we can define the problem in terms of choosing a cutoff q and then choosing optimally given the cutoff. Formally, defining a problem P_q with that property, we first note that $V[P_q] \leqslant V[P]$ for all q since the choice set is restricted. However, given an optimal solution, we know that there exists q^* so that the solution has the right form. Hence $V[P_{q^*}] = V[P]$, and

$$\min_q V[P_q] = V[P]$$

gives you the same minimum cost as the full problem. We obtain the problem

$$V_q = \min_{n_l > 0, n_p(i) > 0} w_l n_l + \int_0^q n_p(i) w_p$$

subject to

$$\begin{aligned}
A\left(\frac{n_l}{A\gamma_1}\right)^{\gamma_1}\exp\left[\left(\int_0^q \log\left[\frac{z}{A}\frac{n_p(i)}{\bar{\eta}-\gamma_l}\right]di + \int_q^1 \log\tilde{a}(i)di\right)\right]^{\bar{\eta}-\gamma_l} &\geqslant y \\
n_p(i) &> 0 \quad \forall i \leqslant \bar{\eta} \\
n_l &> 0.
\end{aligned}$$

Since profit maximization implies cost minimization, this means that a profit maximization problem that operates under this constraint will find the same optimum, which is the lemma.