Capital Deepening, Technology Choice, and Labor Share Dynamics^{*}

Juin-Jen Chang, Academia Sinica[†]
T. Terry Cheung, Academia Sinica[‡]
Han Yang, Academia Sinica[§]

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Abstract: We analyze the labor income share decline amidst capital accumulation and technological shifts, noting three trends: capital prices have fallen and capital per worker has increased; the elasticity of substitution between capital and labor is below one, countering expectations that more or cheaper capital diminishes labor share; and a significant labor share reduction is observed post-1990s. We propose that increasing capital stock encourages adoption of capital-intensive technologies, reducing labor share on the extensive margin, regardless of elasticity levels. This explains the U.S. labor share dynamics in 1980-2010 based on technology adoption intensity.

JEL codes: E25, E22, J31, J33

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 $^{^{*}\}mathrm{TBA}$

 $^{^{\}dagger} jj chang @econ.sinica.edu.tw$

[‡]terrycheung@econ.sinica.edu.tw

[§]hanyang@econ.sinica.edu.tw

1 Introduction

Keynes (1939) noted the surprising stability of labor's share of national income as one of the most established facts in economic statistics. However, Karabarbounis and Neiman (2014) argue that the global labor share has declined by approximately 5 percentage points since 1975. In the U.S. data, we observe that the labor share has declined from around 70% of GDP to below 65% as shown in Figure 1. This has sparked considerable research into the reasons and implications of this trend (see Grossman and Oberfield, 2022, for review). Karabarbounis and Neiman (2014) suggest that this decline can be understood within the context of an aggregate production function, which incorporates the substitutability of capital and labor. According to their analysis, technological advancements that lower the cost of capital lead to an increase in capital accumulation. As capital and labor are substitutes in production, this accumulation subsequently reduces the demand for labor and decreases the labor income share.

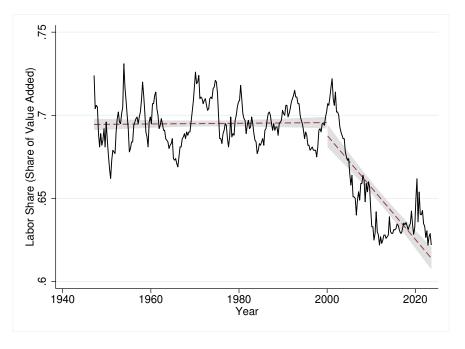


Figure 1: Labor Share and Trend Before and After 2000. Note: Calculated as compensation divided by gross value added less taxes on production and imports. Source: Bureau of Economic Analysis.

This explanation faces three significant challenges. Firstly, the narrative presumes capital and labor are substitutes with an elasticity of substitution (EOS) greater than one. Contrary to this, most empirical research points to an EOS of less than one (Chirinko and Mallick, 2017; Oberfield and Raval, 2021; Gechert et al., 2022). Secondly, the theory suggests that technological advancements should decrease capital costs, leading to an increase in capital demand. However, if capital and labor are complements, the increase in capital and the reduction in capital price should increase the labor share. Thirdly, even if capital prices, capital per worker, and the EOS favor a declining labor share, the actual trend only begins its pronounced downturn after the 1990s, as pointed out by Grossman et al. (2017). Before this period, there is a slight upward (yet insignificant) trend as shown in Figure 1.¹

This paper enhances the capital accumulation narrative by incorporating a technological adoption framework, addressing the challenges previously outlined. It argues that differences in capital intensities across technologies can lead to greater adoption of capital-intensive technologies as capital stock increases, aligning with the Rybczynski Theorem. While a rise in capital stock generally increases the labor share of the economy, the adoption of technologies that are less labor-intensive tends to lower the labor income share, even when the EOS within each technology type remains below one. Thus, due to this dual effect of increased capital stock and technological adoption favoring capital intensity, the overall EOS may actually exceed one.²

Labor share is influenced by factor prices and capital intensity across two dimensions. On the intensive margin, changes in factor input prices can affect total costs, often resulting in a higher relative price of labor as capital becomes more prevalent. This usually leads to a higher labor share when the EOS between capital and labor is below one. On the extensive margin, firms accumulating more capital are inclined to adopt more capital-intensive technologies, which decreases the overall labor share, a trend that continues even if the EOS for each technology is below one. This offers a novel insight into how capital accumulation affects labor share.

The paper highlights the critical role of technology availability in this context. Without alternative technological options, an increase in capital per worker tends to increase the labor share when EOS is below one. However, the availability of alternative technologies means that more capital per worker can lead to the selection of more capital-intensive technologies. In this analysis, computers are crucial, symbolizing the presence of alternative technological options. The limited use of computers prior to the 1980s and their widespread adoption thereafter, as noted by Burstein,

¹The observed trend in labor share remains consistent across different cutoff years. For instance, selecting 1975 as the cutoff year, following Karabarbounis and Neiman (2014), highlights a more pronounced upward trend compared to using 2000. Conversely, choosing 1995 as the cutoff year, which aligns with the period when computers became prominent (refer to Figure 5), the labor share trend appears flat before the cutoff year. Regardless of the specific cutoff year, the labor share typically shows an initial period of slight increase or stability, followed by a subsequent decline.

²Oberfield and Raval (2021) provides both theoretical and empirical support for the idea that micro- and macrolevel elasticities can differ.

Morales and Vogel (2019), is associated with a marked decline in labor share, demonstrating the significant impact of technology adoption on the economy.

This paper contributes to the extensive body of literature exploring the decline in labor share, a topic that has garnered significant attention due to its implications for economic inequality and productivity. Various studies have proposed alternative explanations for this trend, including trade liberalization (Elsby, Hobijn and Şahin, 2013), demographic shifts (Glover and Short, 2023), the increasing market power of firms (De Loecker, Eeckhout and Unger, 2020; Barkai, 2020), the rise of "superstar" firms (Autor et al., 2020; Kehrig and Vincent, 2021), and the reduction in public sector enterprises (Bridgman and Greenaway-McGrevy, 2022). The insights from Karabarbounis and Neiman (2014) and Piketty and Zucman (2014) regarding investment goods and capital accumulation are of particular relevance. The study by Boldrin and Zhu (2021) also stands out for its examination of how technological heterogeneity influences the relationship between product market concentration and labor share, highlighting a significant decline in labor share within the highly concentrated manufacturing sector – a topic further explored by Kehrig and Vincent (2021).

Unlike these studies, our research studies the interplay between technological adoption and capital accumulation and its effects on labor share. We argue that this relationship leads to a decline in labor share, even when the EOS within each technology type remains below one. Our analysis aims to capture the full dynamics of labor share fluctuations, accounting for the stable trend observed before the 1990s and the subsequent decline after the 2000s.

Our paper enhances the capital accumulation narrative surrounding the declining labor share by introducing an additional, realistic mechanism: technological choice.³ This approach diverges from traditional analyses that primarily focus on automation or labor-augmenting technological changes. The novelty of our paper lies in establishing a mechanism where, at the micro level, the EOS is less than one, aligning with much of the empirical evidence (see, for example, Chirinko and Mallick, 2017; Oberfield and Raval, 2021; Gechert et al., 2022). At the macro level, however, the implied aggregate EOS can exceed one, as required to explain the declining labor share.⁴ Additionally, our study underscores the role of computers in influencing the labor share. It suggests a stable labor share pattern before widespread computer adoption, followed by a declining trend as personal computers became prevalent (Aum and Shin, 2020).

 $^{^{3}}$ Farrokhi and Pellegrina (2023) use a similar discrete choice model as we have. However, their focus is to study the effect of globalization on technological choice in the agricultrual sector.

⁴Houthakker (1955) famously demonstrated that micro and macro elasticities can differ significantly: an economy of Leontief micro units can have a Cobb–Douglas aggregate production function.

2 Empirical Fact

2.1 Elasticity of Substitution – Meta Analysis

In classical capital-deepening theory, technological advancements in capital production are posited to lead to increased capital accumulation and a consequent decline in the labor share of income, but this occurs only if the elasticity of substitution (EOS) between capital and labor exceeds 1. However, a substantial body of evidence, particularly at the micro-level – including data specific to firms and industries – suggests that the EOS typically falls below 1.

In the analysis that follows, we conduct a meta-analysis using data from Gechert et al. (2022).⁵ The principal finding from Gechert et al. (2022) is that the EOS between capital and labor does not exceed 1, indicating that the majority of studies report an EOS below 1. The consideration of publication bias further reduces the estimated value of EOS.

The time series of the labor share exhibits distinct trends. Following the approach of Karabarbounis and Neiman (2014), we use the year 1980 as a cutoff to study if EOS varies across time.⁶ Our findings indicate that the majority of EOS estimates are below 1, as illustrated in Figure 2. Although there has been evidence on the time-varying EOS, most data in Gechert et al. (2022) agree with the fact that EOS before and after the cutoff are well below 1, even without correcting the publication bias.

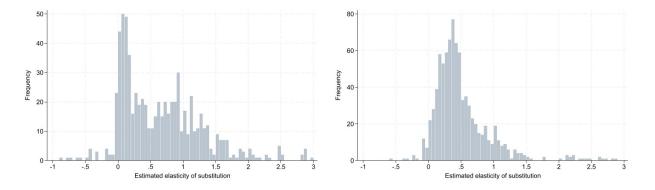


Figure 2: EOS Estimates. Left: 1950-1980. Right: After 1980. Source: Gechert et al. (2022).

 $^{{}^{5}}$ We adhere as closely as possible to their methodology and select the midpoint year of the data period for each study to maximize representativeness.

 $^{^{6}}$ Following the approach of Grossman et al. (2017) and use the year 2000 as a cutoff will result in too few observations for the second period.

2.2 Capital Price and Capital Stock

The significance of capital accumulation in quantitative terms largely depends on the extent to which technological advancements in the production of capital goods reduce capital costs, thereby fostering further capital accumulation. The increase in the capital accumulation per worker is well known in the literature and we use the data from Penn World Table (Feenstra, Inklaar and Timmer, 2015) to illustrate the trend in Figure 3 (Left Panel).

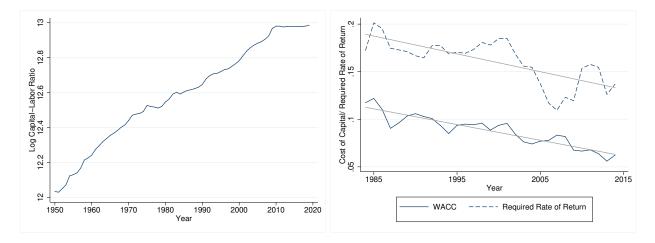


Figure 3: Log Capital-Labor Ratio and Price of Capital. Source: Feenstra, Inklaar and Timmer (2015) and Barkai (2020).

To empirically assess the reduction in capital costs, more work is needed. Within an aggregate model, prices of capital and investment are considered equivalent, both influencing the rental rate. Yet, in practical scenarios, different assets contribute variably to capital, investment, and rental costs. To address this complexity, we utilize the formula developed by Hall and Jorgenson (1967) and recently applied by Barkai (2020) for analyzing the required return on a specific capital type s:

$$R_s = \left[\left(\frac{D}{D+E} i^D (1-\tau) + \frac{E}{D+E} i^E \right) - \mathbb{E}(\pi_s) + \delta_s \right] \frac{1-z_s \tau}{1-\tau}$$

Subsequently, the required rate of return on aggregated capital R can be determined as:

$$E = \sum_{s} R_{s} P_{s}^{K} K_{s} = \underbrace{\sum_{s} \frac{P_{s}^{K} K_{s}}{\sum_{j} P_{j}^{K} K_{j}}}_{R} \times \underbrace{\sum_{s} P_{s}^{K} K_{s}}_{P^{K} K}$$

Here, E is the aggregate capital costs are the sum of the asset-specific capital costs. The term R represents the weighted average of the asset-specific required rates of return, with the weight

for asset s being proportional to its capital stock's nominal value. The term $P^{K}K$ indicates the nominal value of the total capital stock. The result is shown in Figure 3 (Right Panel), indicating the downward trend of capital cost.

2.3 The Trend Computer Usage and Relationship with Labor Share

Computers began entering workplaces in the 1980s but only became widely popular after the late 1990s. As shown in Figure 4 (Left Panel), the adoption of computers, in terms of both overall quantity and as a proportion of total capital stock, significantly increased in mid-1990. Paradoxically, a decrease in the rate of computer adoption post-2010 corresponds with a stabilization of the previously declining labor share.

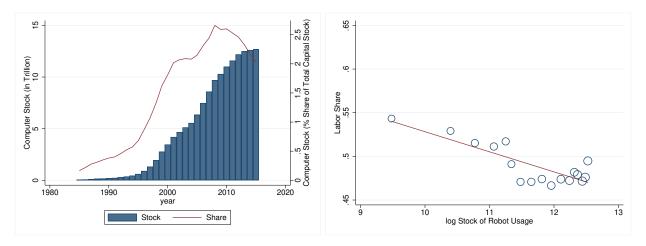


Figure 4: Left: Computer Stock and Share of Total Capital Stock 1985-2015. Right: Robot Usage and Labor Share in Manufacturing Sector. Source: Caunedo, Jaume and Keller (2023) and BLS.

To illustrate the partial effect of adopting computers on the labor share, we utilize data from the International Federation of Robotics (IFR) on the stock of robots in operation. The use of robots, which encompasses computers and software, effectively demonstrates the impact of robot utilization on labor share.⁷ Due to data limitations, our analysis is concentrated on the manufacturing sector, which has experienced one of the largest declines in labor share, as identified by Boldrin and Zhu (2021). Figure 4 (Right Panel) shows a clear negative correlation between robot usage and labor share in the manufacturing sector across various years.

⁷Refer to Aum (2017) for an analysis of the effects of computer and software utilization on job polarization.

3 Model

In this model, we consider a closed economy consisting of a single sector, where final goods is produced by a continuum of varieties, denoted as ν , ranging from 0 to 1. These varieties can be produced using either high capital-intensive (T_H) or low capital-intensive (T_L) technologies. The decision of technology adoption resting with firms. There are 2 inputs: labor which is fixed, denoted as L, and capital, denoted as K.⁸ Capital accumulation is endogenous, and its supply being determined by the household saving based on various economic interactions and factors. All markets are competitive. Finally, the model adopts a steady-state analysis to understand the long-term implications of different technological choices in production.

3.1 Production

There are $v \in [0, 1]$ varieties. All the varieties are combined using a CES technology in the overall production process M to give the quantity produced q^M , which can be expressed as

$$q^{M} = \left(\int_{0}^{1} y^{g}(v)^{\frac{\eta-1}{\eta}} dv\right)^{\frac{\eta}{\eta-1}} \tag{1}$$

In the production process, firms have the option to select a technology, denoted as g, from the available set \mathbb{G} , for producing each varieties v. The technology choices include capital-intensive "high" technology (T_H) , and labor-intensive "low" technology (T_L) . Both technologies utilize labor and capital as inputs. The output $y^g(v)$ for each variety v using technology g is defined by the equation:

$$y^{g}(v) = a^{g}(v) \left[(\alpha^{g})^{1/\rho^{g}} L^{g}(v)^{\frac{\rho^{g}-1}{\rho^{g}}} + (1-\alpha^{g})^{1/\rho^{g}} K^{g}(v)^{\frac{\rho^{g}-1}{\rho^{g}}} \right]^{\frac{\rho^{g}}{\rho^{g}-1}}, \quad g \in \{T_{H}, T_{L}\}$$
(2)

Here, $a^g(v)$ represents the productivity level for firm producing variety v, and follows a Fréchet distribution with $\Pr(x < a) = \exp\left(-A^g a^{-\theta}\right)$. The term α^g indicates the labor intensity of technology g, with $\alpha^{T_L} > \alpha^{T_H}$. $L^g(v)$ and $K^g(v)$ are the labor and capital used for producing variety v in technology g, respectively. Lastly, $\rho^g\left(=\frac{\partial \ln(K^g/L^g)}{\partial \ln(w/r)}\right)$ refers to the elasticity of substitution between labor L and capital K in technology g.

⁸We keep labor fixed so that it is flexible to incorporate skilled and unskilled labors in our definition. Our robustness check indicates this assumption is innocuous.

Cost minimization determines the unit bundle cost for technology g, given by the equation:

$$c^{g} = \left[\alpha^{g} w^{1-\rho^{g}} + (1-\alpha^{g}) r^{1-\rho^{g}}\right]^{\frac{1}{1-\rho^{g}}}$$
(3)

where w represents the labor wage, and r is the rate of return on capital. This formula provides an aggregated cost measure for employing technology g, factoring in the costs of labor and capital. Additionally, the model calculates the expenditure shares on labor and capital within technology g. The expenditure share on labor, denoted as e_L^g , and capital, denoted as e_K^g , for technology g are given by

Labor:
$$e_L^g = \alpha^g \left(\frac{w}{c^g}\right)^{1-\rho^g}$$
 (4)

Capital:
$$e_K^g = (1 - \alpha^g) \left(\frac{r}{c^g}\right)^{1 - \rho^g}$$
 (5)

These equations reflect how labor and capital costs are proportionally allocated within each technology, based on their respective intensities (α^g) and the overall cost of employing the technology. Furthermore, the model also accounts for the marginal cost associated with producing a specific variety v using technology g. The marginal cost, $c^g(v)$, is determined by

$$c^{g}(v) = \frac{c^{g}}{a^{g}(v)} \tag{6}$$

Firms select the most cost-effective technology for each variety v. The cost for a specific firm is determined by choosing the cheapest available technology, which is represented as:

$$c(\upsilon) = \min_{g \in \mathbb{G}} \left\{ c^g(\upsilon) \right\} = \min_{g \in \mathbb{G}} \left\{ \frac{c^g}{a^g(\upsilon)} \right\}$$
(7)

As the productivity a^g follows Fréchet distribution with shape parameter θ , the probability of selecting a technology g is given by:

$$\tau^g = \frac{A^g(c^g)^{-\theta}}{\sum_{g' \in \mathbb{G}} A^{g'}(c^{g'})^{-\theta}}$$
(8)

This probability depends on the relative cost efficiency of technology g compared to other tech-

nologies in set \mathbb{G} . The unit cost of the overall production process M is given as

$$P^{M} = \Gamma \left(\frac{\theta - 1 + \eta}{\theta}\right)^{\frac{1}{1 - \eta}} \left[\sum_{g \in \mathbb{G}} A^{g} (c^{g})^{-\theta}\right]^{-\frac{1}{\theta}}$$
(9)

where $\Gamma(\cdot)$ represents the gamma function and η is the elasticity of substitution between the varieties v. This equation incorporates the cost efficiencies of all technologies and the characteristics of the production function.

The production of the final consumption good involves using the production process M. The output of the final good, denoted as Y, is expressed as:

$$Y = Aq^M \tag{10}$$

where A represents the TFP in the production of final goods, and q^M is the quantity produced in the production process. The price of the final goods is given by:

$$P = \frac{P^M}{A} \tag{11}$$

3.2 Household

The household faces an intertemporal optimization problem, aiming to maximize its utility over an infinite time horizon, by making dynamic saving and consumption decisions. The labor supply decision of the household is assumed to be inelastic to as to make the steady state Euler Equation (15) independent of wage and labor decision. The household problem is

$$\max\sum_{t=0}^{\infty} \beta^t u(C_t) \tag{12}$$

s.t.
$$\frac{r_t K_t + w_t L_t}{P_t} = C_t + I_t$$
 (13)

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{14}$$

In the steady state, when the labor is assumed inelastic, the FOCs simplify to:

$$1 = \beta \times \left[\frac{r}{P} + (1 - \delta)\right] \tag{15}$$

$$I = \delta K \tag{16}$$

3.3 Equilibrium

The total output in the economy, Y, is produced using CRS production function and hence is defined by the sum of wages and capital returns, as given by the equation:

$$Y = wL + rK \tag{17}$$

where wL represents the total wage bill, and rK signifies the total return on capital. The factor market clearing conditions ensure that the total compensation in the economy is distributed across different technologies. These equations imply that the wage bill and capital returns are allocated across technologies in proportion to their expenditure shares and adoption rates. These conditions are specified for labor and capital as follows:

Labor Bill:
$$wL = \sum_{g \in \mathbb{G}} e_L^g \tau^g Y$$
 (18)

Capital Bill:
$$rK = \sum_{g \in \mathbb{G}} e_K^g \tau^g Y$$
 (19)

The aggregate labor and capital shares in the economy are calculated as the weighted average of the respective shares across all technologies $g \in \mathbb{G}$, using their adoption shares as weights. The aggregate labor share, s^L , and the aggregate capital share, s^K , are determined by:

Aggregate Labor Share:
$$s^L = \sum_{g \in \mathbb{G}} e^g_L \tau^g$$
 (20)

Aggregate Capital Share:
$$s^K = \sum_{g \in \mathbb{G}} e^g_K \tau^g$$
 (21)

These shares represent the overall distribution of income between labor and capital in the economy, reflecting the average impact of different technologies.

An equilibrium in this context is defined as a set of values w, r, K that satisfy all the equilibrium conditions mentioned above. This equilibrium state ensures that the factor markets for labor and capital are cleared, and the distribution of income across different technologies is balanced. Thus, from equations (17)-(19), the equilibrium aggregate labor share can be rewritten as

$$s^{L} = \frac{wL}{Y} = \frac{wL}{wL + rK} = \frac{1}{1 + \frac{rK}{wL}}$$
 (22)

where

$$\frac{rK}{wL} = \frac{e_K^{T_H} \cdot \tau^{T_H} + e_K^{T_L} \cdot \tau^{T_L}}{e_L^{T_H} \cdot \tau^{T_H} + e_L^{T_L} \cdot \tau^{T_L}}$$
(23)

Following Dekle, Eaton and Kortum (2007), we solve the model in relative changes. In this analysis, let x represent a baseline economic variable, and $\hat{x} = x'/x$, where x' is the counterfactual outcome or the outcome from the new equilibrium.

The change in the factor shares can be expressed as

Change in Labor Share:
$$\hat{s}^L = \frac{\sum_{g \in \mathbb{G}} (\hat{e}^g_L e^g_L) \times (\hat{\tau}^g \tau^g)}{s^L}$$
 (24)

where the respective changes in expenditure shares, \hat{e}^{g} , and in probability of selecting a technology $g, \hat{\tau}^{g}$, can be expressed as the changes in wage, rent and capital stock, $\{\hat{w}, \hat{r}, \hat{K}\}$:

$$\hat{e}_L^g = \left(\frac{\hat{w}}{\hat{c}^g}\right)^{1-\rho^g}; \quad \hat{e}_K^g = \left(\frac{\hat{r}}{\hat{c}^g}\right)^{1-\rho^g} \tag{25}$$

where $\hat{c}^g = \left[e_L^g(\hat{w})^{1-\rho^g} + e_K^g(\hat{r})^{1-\rho^g} \right]^{\frac{1}{1-\rho^g}}$, and

$$\hat{\tau}^{g} = \frac{\hat{A}^{g}(\hat{c}^{g})^{-\theta}}{\sum_{g' \in \{T_{H}, T_{L}\}} \tau_{i}^{g'} \hat{A}^{g'}(\hat{c}^{g'})^{-\theta}}$$
(26)

3.4 Discussion of Mechanism

Capital deepening impacts the labor share along two dimensions: the intensive margin effect and the extensive margin effect. Both affect the aggregate labor share through the relative return of capital to labor $\frac{rK}{wL}$, as shown in equation (22).

The intensive margin effect echoes the conventional one, as in Karabarbounis and Neiman (2014), whereby capital accumulation changes the relative price of factor inputs and hence the labor share. To recover the conventional effect, we shut down the extensive margin effect by restraining firms from the technology choice (imposing only one type of technology for firms). Thus, from equations (1), (2), and (10), we have the aggregate production function

$$Y = A \left[(\alpha^g)^{\frac{1}{\rho^g}} \cdot L^{\frac{\rho^g - 1}{\rho^g}} + (1 - \alpha^g)^{\frac{1}{\rho^g}} \cdot K^{\frac{\rho^g - 1}{\rho^g}} \right]^{\frac{\rho^g}{\rho^g - 1}}$$

Accordingly, we can use the first-order conditions (r = MPK and W = MPL) to obtain the ratios

of capital and labor bills as follows

$$\frac{rK}{wL} = \frac{e_K^g}{e_L^g} = \left(\frac{K}{L}\right)^{1-\frac{1}{\rho^g}} \cdot \left(\frac{\alpha}{1-\alpha}\right)^{-\frac{1}{\rho^g}} \tag{27}$$

Note that, because there is only one type of technology, the relative return of capital to labor in equation (23) is simply equal to the relative expenditure of capital to labor $\left(\frac{rK}{wL} = \frac{e_K^g}{e_L^g}\right)$, i.e., $\tau^g \equiv 1$. Differentiating equation (27) with respect to the ratio of capital to labor immediately yields

$$\frac{\partial \left(\frac{rK}{wL}\right)}{\partial \left(\frac{K}{L}\right)} = \left(1 - \frac{1}{\rho^g}\right) \left(\frac{\alpha K}{(1 - \alpha)L}\right)^{-\frac{1}{\rho^g}} < 0$$
(28)

given that capital and labor are gross complements ($\rho^g < 1$). This is intuitive since an increase in capital K, on the one hand, raises the capital-labor ratio $\frac{K}{L}$ (recalling that the aggregate labor force L = 1) and, on the other hand, lowers the relative price of capital to labor $\frac{r}{w}$. If capital and labor are gross complements, the decrease in $\frac{r}{w}$ is larger than the increases in $\frac{K}{L}$, lowing the relative return of capital to labor $\frac{rK}{wL}$. Thus, capital deepening increases, rather than decreases, the labor share s^L when $\rho^g < 1$.

The labor share decline resulting from capital deepening is not theoretically feasible if capital and labor are gross complements. Although the theoretical prediction requires the labor-capital substitution elasticity to be greater than one, most empirical studies find estimates of the laborcapital substitution elasticity to be less than one (Grossman et al., 2017). Particularly, our previous Section 2.1 and that in Gechert et al. (2022) show that the EOS estimates typically falls below 1.

With an endogenous technology choice, the extensive margin effect indicates that differences in capital intensities across technologies can lead to greater adoption of capital-intensive technologies as capital stock increases, aligning with the Rybczynski Theorem. To shed light on such an effect, we assume that $\tau^{T_H} = (1+\theta)\tau^{T_L}$, where θ measures the additional increase in the adoption of high technology, and rearrange equation (23) as

$$\frac{rK}{wL} = \frac{\frac{\tau^{T_L}}{e_K^{T_H}} \left(e_K^{T_H} + e_K^{T_L} \right) + \theta}{\frac{\tau^{T_L}}{e_K^{T_H}} \left(e_L^{T_H} + e_L^{T_L} \right) + \theta \cdot \frac{e_L^{T_H}}{e_K^{T_H}}}$$
(29)

Note that if high-technology production is more capital-intensive than low-technology production (i.e., the labor intensity $\alpha^{T_H} < \alpha^{T_L}$) and if $\alpha^{T_H} < 1/2$, then $\frac{e_L^{T_H}}{e_K^{T_H}} < 1$ are true under the production function with constant returns to scale and complementarity between capital and labor ($\rho^g < 1$).

When capital deepening increases the capital stock, there is an increased adoption of higher capitalintensive technology (i.e., high-technology production), leading to an increase in the probability of adopting high technology θ , Thus, we can see that in contradiction to the conventional intensive margin effect, a higher θ increases the relative return of capital to labor $\frac{rK}{wL}$ (see equation (29) with $\frac{e_L^{TH}}{e_K^{TH}} < 1$), resulting in a lower labor share (see equation (22)). The extensive margin effect represents a novel channel in this model that can decrease the labor share in general equilibrium, even if the EOS is less than one.

Overall, on the intensive margin, a rise in capital stock always increases the labor share of the economy when capital and labor are complements, but, on the extensive margin, the adoption of technologies that are less labor-intensive tends to lower the labor income share, even when the EOS within each technology type remains below one. Thus, due to this dual effect of increased capital stock and technological adoption favoring capital intensity, the labor share can increase, remain constant or decrease with capital deepening. Hence, the overall aggregate EOS in the whole economy may actually exceed one (see our numerical analysis in Section 4) even if the individual firm's EOS is less than 1.⁹ In our analysis, the extensive margin effect helps us reconcile the discrepancy in labor shares not only between theory and practice, but also between micro-level and macro-level estimates for factor substitution,

4 Quantitative Results

4.1 Parameterization

We solve the model using relative changes, following the approach of Dekle, Eaton and Kortum (2007). This reduces the number of parameters requiring calibration, as some are eliminated in the computation of relative terms. Table 1 summarizes these parameters and their respective values. The methodology for determining these values is discussed subsequently.

Our model includes a total of six time-invariant parameters $\{\rho^g, \alpha_L^{T_L}, \alpha_L^{T_H}, \tau_0^{T_H}, \tau_0^{T_L}, \theta\}$ and three time-variant parameters $\{L_t, A_t^{T_L}, A_t^{T_H}\}$ that need to be determined.¹⁰ We establish the values for $\{\rho^g, \alpha_L^{T_H}, \theta\}$ and $\{L_t, A_t\}$ using existing literature and empirical data. Then, we calibrate $\{\alpha_L^{T_L}, \tau_0^{T_H}\}$ to align with the initial and final labor share observed in 1980 and 2010. All the data

 $^{^{9}}$ This also echos the finding in Oberfield and Raval (2021) that the aggregate EOS is larger than that of the micro-level EOS.

¹⁰Throughout the baseline analysis, we assume that the TFP of the final goods production is identical to 1, i.e. $A \equiv 1$

Parameter	Value	Definition	Source
Time-Invariant Parameters			
Production Parameters			
$ ho^g$	0.30	Elasticity of Sub. btw K and L	Gechert et al. (2022)
$lpha_L^{T_L} lpha_L^{T_H}$	0.87	Labor Share of T_L Tech	Decline in Labor Share
$lpha_L^{T_H}$	0.10	Labor Share of T_H Tech	U.S. Auto-Mobile Industry
Adoption Parameters			
$\frac{\tau_0^{T_H}}{\tau_0^{T_L}}$	0.44	Initial Adoption Share of the T_H Tech	Initial Labor Share
$ au_0^{T_L}$	0.56	Initial Adoption Share of the T_L Tech	$1 - \tau_0^{Lo}$
Distribution Parameters			
heta	4	Shape Parameter	Bernard et al. (2003)
Time-Varying Parameters			
L_t	Varies	Labor Supply	Penn World Table
$A_t^{T_L}$	Varies	Productivity Progress in T_L Tech	Penn World Table
$A_t^{\check{T}_H}$	Varies	Productivity Progress in T_H Tech	BEA

Table 1: Model parameters in the baseline model.

used in the exercise are from the U.S.

Production Parameters $\{\rho^g, \alpha_L^{T_H}, \alpha_L^{T_L}\}$. Gechert et al. (2022) performed a meta-analysis revealing that the elasticity of substitution (EOS) between capital and labor varies between 0.3 and 0.9, with 0.5 emerging as the median value across studies. They argue that, when adjusting for publication bias, the EOS should be considered as 0.3. Based on these findings, our baseline analysis assumes an EOS of 0.3, and we plan to explore the sensitivity of this assumption across a range of $\rho^g \in [0.2, 0.9]$ in Section 5.3.¹¹

We calculate the labor intensity, denoted as $\alpha_L^{T_H}$, for industries with high capital intensity by examining the labor share within the sector reporting the highest robot density, according to the International Federation of Robotics (IFR). The objective is to assess labor intensity in an industry characterized by full automation. The motor vehicle manufacturing industry is identified for this purpose. Given the variability of labor share over time, we take time average, at the value roughly equals to 0.1. Consequently, we adopt this minimum as our labor intensity value, setting $\alpha_L^{T_H} = 0.10$. Subsequently, we calibrate the value of $\alpha_L^{T_L} = 0.87$ to align with the overall decline in labor share in the Penn World Table.

Adoption Parameters $\{\tau_0^{T_H}, \tau_0^{T_L}\}$. The adoption share parameters, $\tau_0^{T_H}$ and $\tau_0^{T_L}$, are designed to sum to unity, as expressed by the equation $\tau_0^{T_H} + \tau_0^{T_L} = 1$. By adjusting the initial values of

¹¹A significant consideration in our analysis is the assumption that the EOS is consistent across different technologies, although it is conceivable that technologies with higher capital intensity might substitute labor more efficiently, thereby warranting a higher ρ^g value. However, due to data constraints, this hypothesis cannot be directly tested. As such, in Section 5.3, we will investigate the implications of varying ρ^g values to address this potential variability.

 $\alpha_L^{T_H}, \alpha_L^{T_L}$, we aim to align the initial adoption share with the labor share of 0.62 reported in 1980, according to the Penn World Table. This adjustment results in an implied value of $\tau_0^{T_H} = 0.44$, which appears high in comparison to empirical data, such as the computer stock's proportion of the total capital stock depicted in Figure 4. Yet, referencing the findings of Burstein, Morales and Vogel (2019), the adoption rate during that period should not be underestimated.¹² To deduce the adoption share for 1980, we employ linear extrapolation based on the trends observed between 1984 and 1993 as reported by Burstein, Morales and Vogel (2019). This method suggests an adoption rate of 0.24, lower than our calculated value but significantly above zero.

Distribution Parameter $\{\rho^g\}$. We follow the value used in Bernard et al. (2003) and set the shape parameters of the Fréchet distribution at 4.

Time-Variant Parameters $\{L_t, A_t^{T_H}, A_t^{T_L}\}$. To identify three time-varying parameters, we employ the Penn World Table (Feenstra, Inklaar and Timmer, 2015) to determine the number of workers, L_t , in the U.S., as illustrated in the left panel of Figure 5. For the TFP progress associated with the low-tech (T_L) technology, we use the overall TFP growth in the U.S. as a proxy. Meanwhile, TFP growth in the computer-producing sector, as reported by the Bureau of Economic Analysis (BEA), serves as a proxy for TFP progress in the high-tech (T_H) technology. It is important to note, as Aum, Lee and Shin (2018) argue, that the adoption of computers is primarily driven by technological advancements within the computer-producing sector. The trends for these three parameters over time are depicted in Figure 5.

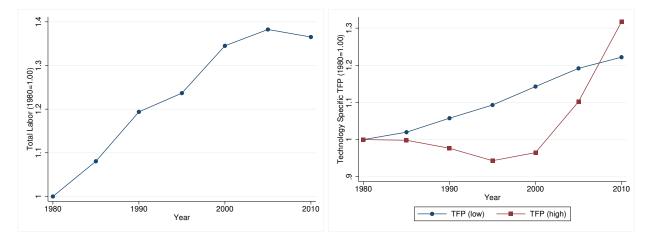


Figure 5: Computer Stock and Share of Total Capital Stock 1985-2015. Source: Feenstra, Inklaar and Timmer (2015) and BEA Industry Account.

¹²Data on computer usage from Burstein, Morales and Vogel (2019) is derived from analysis of the CPS October Supplement surveys from 1984, 1989, 1993, 1997, and 2003, which collected information on respondents' direct use of computers at work.

4.2 Model Fit and Discussion

To validate our model, we compare its predictions with untargeted moments, with a particular focus on the implied economy-wide EOS. While we assume an EOS of capital and labor, ρ^g , of 0.3 across all technologies, the aggregate EOS may differ due to distribution effects from the use of various technologies within the economy, as discussed by Oberfield and Raval (2021). Consequently, the aggregate EOS is determined by the equation:

$$EOS = -\frac{d\ln(K/L)}{d\ln(r/w)} = -\frac{\hat{K} - \hat{L}}{\hat{r} - \hat{w}},$$

where the implied value of the aggregate EOS in our model for 2010 is 3.2. This is compared to an empirical value of 3.0, derived from data post-1990s in Gechert et al. (2022).

The second aspect of our validation focuses on assessing whether our model accurately replicates the observed increase and then decline in labor share from 1980 to 2010. As depicted in Figure 6 (Left Panel), the model successfully captures the trend of an increasing labor share from 1980 to 2000, followed by a decrease from 2000 to 2010. This trend is primarily attributed to the varying rates of adoption of high-type technology in the economy. As detailed in Section 3.4, two main forces drive the dynamics of the labor share:

$$\hat{s}^L = \frac{\sum_{g \in \mathbb{G}} (\hat{e}^g_L e^g_L) \times (\hat{\tau}^g \tau^g)}{s^L},$$

where the first force is the intensive channel, influenced by the change in factor price, $\hat{e}_L^g e_L^g$. With ρ_{KL} being less than 1, this intensive channel contributes to an increase in the labor share. The second force is the extensive channel, which affects the labor share through the adoption of technology associated with a lower labor share, $\hat{\tau}^g \tau^g$, tending to decrease the labor share. However, as shown in Figure 6 (Right Panel), the model suggests a slight decline in the adoption of computers from 1980 to 2000, resulting in a weakened extensive channel and, consequently, an increase in the labor share during this period.

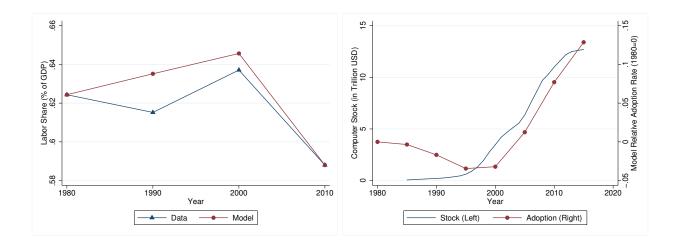


Figure 6: Labor Share Dynamics: Model Versus Data

5 Discussion

5.1 Positive and Increasing Profit

The baseline model assumes firms do not generate profits. Nevertheless, recent studies have shown that the most productive firms often achieve higher profits and a lower labor share (Barkai, 2020; De Loecker, Eeckhout and Unger, 2020; Autor et al., 2020), contributing to a decline in the overall labor share (Kehrig and Vincent, 2021). In light of these findings, we adjust our model to incorporate a production function that includes entrepreneurial skill $M^g(v)$, utilized by technology g for task v, with the entrepreneurs' input share interpreted as profit share. The production function is thus:

$$y^{g}(v) = a^{g}(v) \left(\left[\alpha_{L}^{g} \right]^{1/\rho^{g}} L^{g}(v) + \left[\alpha_{K}^{g} \right]^{1/\rho^{g}} K^{g}(v) + \left[\alpha_{M}^{g} \right]^{1/\rho^{g}} M^{g}(v) \right)^{\frac{\rho^{g}}{\rho^{g} - 1}}$$

This formulation, while not exactly fitting into Lucas' span-of-control framework (Lucas, 1978), shares a similar principle that a portion of the output is allocated to entrepreneurs. In this revised production function, we must determine six parameters: $\{\alpha_f^g\}_{f\in\{L,K,M\}}^{g\in\{T_H,T_L\}}$. Following the baseline calibration, we maintain labor shares $(\alpha_L^{T_L}, \alpha_L^{T_H}) = (0.87, 0.10)$. Owing to data limitations, we adopt a fixed rule for dividing inputs between capital and entrepreneurial skill across all technologies $g \in \mathbb{G}$, setting $(\alpha_K^g, \alpha_M^g) = (\kappa \times [1 - \alpha_L^g], [1 - \kappa] \times [1 - \alpha_L^g])$, effectively reducing the number of parameters to just κ . We select $\kappa = 0.81$ to align the profit share of GDP in the 1980s with the 7.6% figure reported by Dobbs et al. (2015).

The model is successful in predicting the profit share of GDP in the 2010s, estimating a profit share of 9.8% in 2010, which matches the figure for 2013 reported by Dobbs et al. (2015). However,

the model's explanatory power regarding the declining labor share has diminished. It now accounts for approximately 20% of the labor share decline (see Figure 7, left panel). This is largely because adding another factor reduces the aggregate EOS between capital and labor, attenuating the impact of increased capital stock on labor observed in previous cases.

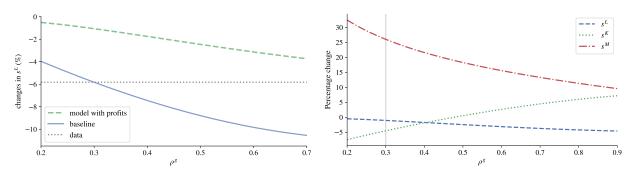


Figure 7: Changes in Labor Share for Different Values of ρ^g in Model with Positive Profit.

Furthermore, as shown in Figure 7 (Right Panel), both labor share and capital share decline. However, the profit share, representing entrepreneurs' factor share, increases, as evidenced by the positive percentage change between 1980 and 2010. This finding aligns with the evidence provided by Barkai (2020).

5.2 Sensitivity of Elasticity of Substitution

In our baseline analysis, we follow Gechert et al. (2022) by adopting their lowest estimate for the elasticity of substitution (EOS) in our model. A critique regarding our methodology concerns the uniform EOS assumed for both high (T_H) and low (T_L) technology sectors. This critique is warranted given that the exact EOS values are challenging to pinpoint from the available data. Nevertheless, evidence suggests that the EOS for T_H technology could be higher than that for T_L technology, as Aum, Lee and Shin (2018) estimate the EOS between computers and occupations to consistently exceed 1.

To address these concerns and explore the impact of varying EOS values, we model different ρ^{g} values ranging from 0.2 to 0.9, all below the threshold of 1. As illustrated in Figure 8, every evaluated ρ^{g} value results in a predicted reduction in the labor share, as evidenced by the negative percentage change in s^{L} . Furthermore, the model indicates that an increase in the EOS corresponds with a more pronounced decline in the labor share, thereby reinforcing the underlying mechanism of our model.

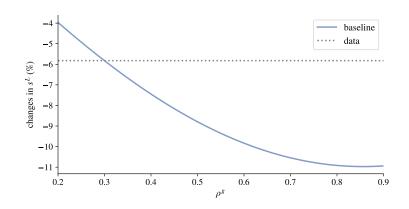


Figure 8: Changes in Labor Share for different values of ρ^g

5.3 The Importance of Factor Intensity

In our baseline analysis, we link the labor intensity of high capital-intensive technology to the 4-digit industry reported by the IFR with the highest robot density. For low capital-intensive technology, we calibrate it to reflect the actual labor share accurately. This section explores alternative factor share intensities to assess our mechanism's robustness across different parameterizations of $\left(\alpha_L^{T_L}, \alpha_L^{T_H}\right)$. We find that a significant difference between $\alpha_L^{T_L}$ and $\alpha_L^{T_H}$ is essential for our mechanism to hold.

Consider a hypothetical scenario where $\left(\alpha_L^{T_L}, \alpha_L^{T_H}\right) = (0.5, 0.4)$, with the adoption share adjusted to match the initial labor share. We also incorporate time-varying parameters as depicted in Figure 5. Results, shown in Figure 9, unexpectedly indicate an increase in labor share across all examined values of ρ^g , contradicting the expected decline.

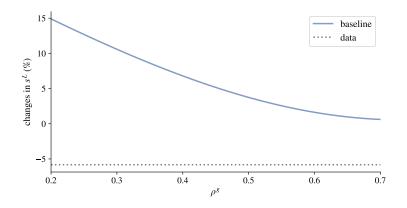


Figure 9: Changes in Labor Share for different values of ρ^g with $\left(\alpha_L^{T_L}, \alpha_L^{T_H}\right) = (0.5, 0.4)$

This result is intuitive. Recall our formula for labor share change: $\hat{s}^L s^L = \sum_{g \in \mathbb{G}} (\hat{e}_L^g e_L^g) \times (\hat{\tau}^g \tau^g)$. Here, the extensive margin $\hat{\tau}^g \tau^g$, which influences the labor share, is dependent on the difference in labor shares between technologies. Therefore, if the difference in labor shares between the two technologies is minimal, the extensive margin's negative impact on the labor share will also be diminished, leading to a counterfactual increase in labor share.

5.4 Endogenous Labor Choice

To accommodate endogenous labor supply and keep the baseline conditions intact, we separate the labor and capital supply problems and assume that labor works hand to mouth. The capital supply problem is the same as the one in Section 3.2. The utility function of workers in follows Greenwood, Hercowitz and Huffman (1988) utility function. Workers' optimization problem follows

$$u(C,L) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{L^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

s.t. $PC = wL$,

S

where ν is the Frisch elasticity of of labor supply. Workers supply labor according to optimality condition $C^{\sigma}L^{1/\nu} = \frac{w}{P}$. Since workers are hand-to-mouth, we can rewrite FOC as $L = \left(\frac{w}{P}\right)^{\frac{1-\sigma}{\sigma+1/\nu}}$.

All other equilibrium conditions remain the same as in the baseline case. In the quantitative exercise below, we further pick $\sigma = 2$ and $\nu = 1$. The result is reported in Figure 10.

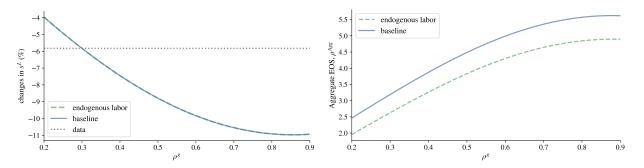


Figure 10: Changes in Labor Share for Different Values of ρ^{g} in Model with Endogenous Labor Supply.

In the scenario with endogenous labor supply, capital adjustments ensure that the optimal capital-labor ratio in equilibrium remains constant. Consequently, the change in labor share mirrors that of the baseline case and so there is no difference between the labor share changes in model with exogenous and endogenous labor supply. However, the aggregate elasticity of substitution (EOS) differs due to the labor's endogenous response to wage changes, as illustrated by the equation:

$$EOS = -\frac{d\ln(K/L)}{d\ln(r/w)} = -\frac{\hat{K} - \hat{L}}{\hat{r} - \hat{w}},$$

where the change in labor (\hat{L}) is now nonzero. This endogenous response results in a reduced magnitude of the EOS for the same wage and capital rental rate changes.

6 Conclusion

Our study presents a groundbreaking analysis of the declining labor share, attributing it to the intricate interplay between technological adoption and capital accumulation. This approach distinguishes itself from existing literature by focusing on the internal dynamics of economic systems rather than external factors like globalization or demographic changes. We propose that the labor share's decline is driven by the adoption of capital-intensive technologies, a process that affects the labor share negatively even when the elasticity of substitution (EOS) within each technology type remains below one. This process unfolds through two distinct mechanisms: the extensive and intensive channels.

The extensive channel relates to how firms, in response to an increase in capital, shift towards more capital-intensive technologies, thereby reducing the labor share. This shift is a strategic move to optimize production efficiency by leveraging technological advancements, exemplified by the widespread incorporation of computers into business operations since the 1980s. The intensive channel, on the other hand, focuses on the direct impact of changes in factor prices on the cost structure, which can lead to an increased labor share when capital becomes relatively more expensive, assuming the EOS between capital and labor is below one. However, the rising prevalence of technology that favors capital intensity has tipped the balance, leading to an overall decline in labor share despite the intensive channel's potential to counteract this trend.

Our analysis underscores the significance of technology availability, particularly the role of computers, in economic outcomes. The diffusion of personal computing technology marks a critical juncture in labor share dynamics, aligning with the observed shift from stability to decline in labor's income share. This correlation highlights the transformative impact of technological adoption on labor markets and income distribution.

By weaving the concepts of technological choice, capital accumulation, and the dual channels of economic adjustment into our narrative, we provide a nuanced framework for understanding the labor share's decline. This model not only offers insights into the mechanics of income distribution but also sheds light on the broader economic implications of technological progress and capital deepening.

In sum, our contribution lies in elucidating a dual mechanism—comprising extensive and intensive channels—that explains the declining labor share in the context of technological adoption and capital accumulation. This analysis not only enriches the discourse on economic inequality but also offers valuable perspectives for policymakers grappling with the challenges of technological change and its impact on the labor market.

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