Strategic Use of Product Delays to Shape Word-of-Mouth Communication*

Alexei Parakhonyak† and Nick Vikander‡

January 2024

Abstract

This paper investigates the advantages a seller can gain by strategically creating product scarcity to manipulate consumer word-of-mouth communication. The seller offers a product of uncertain quality and sets a service speed that determines whether opinion leaders are immediately served or delayed when attempting to purchase the product. Opinion leaders subsequently share their experiences with other consumers, influencing these consumers' beliefs about product quality and their purchase decisions. We show that delaying opinion leaders can significantly impact consumer learning by altering both the content and level of word-of-mouth communication. Specifically, the content effect alone can incentivize the seller to delay opinion leaders, except in niche markets where private information is highly accurate. In settings where information about purchased products spreads more easily than information about delays, the level effect limits the potential for suppressing service speed, particularly in markets with high expected product quality and many opinion leaders.

Keywords: word of mouth, product delays, scarcity, capacity constraints

JEL codes: D83, L15

---

*We thank Philipp Brunner, Julia Salmi, Egor Starkov, Peter Norman Sørensen, Greg Taylor, as well as participants of Oligo Workshop 2023, EARIE 2023 and seminar audiences in Copenhagen, Oxford and from the Nordic Theory Group for their comments.

†University of Oxford, Department of Economics and Lincoln College. E-mail: aleksei.parakhonyak@economics.ox.ac.uk. Address: Lincoln College, Turl Street, OX1 3DR, Oxford, UK.

‡University of Copenhagen, Department of Economics. E-mail: nick.vikander@econ.ku.dk. Address: Øster Farimagsgade 5, Building 26, 1353 K, Copenhagen, Denmark.

We gratefully acknowledge funding from the Independent Research Fund Denmark (grant no. 8019-00075B).

1 Introduction

This paper considers how a seller may profitably shape consumer 'buzz' about its product by strategically creating product scarcity. The idea is that initial scarcity limits the number of opinion leaders who can make early purchases. As such, it can influence both *what* leaders can communicate about the product and *how much* they communicate to other consumers in the market. We explore how creating initial product scarcity can either help or hurt the seller by affecting subsequent sales, via this word-of-mouth channel.

Consumers can communicate in many ways, such as face-to-face conversations, social media posts, online discussion groups, and product reviews.[1] This word-of-mouth communication can in particular be a valuable tool to help consumers learn about product quality. For example, when considering purchasing a mobile phone, individuals may seek out information from acquaintances who have ordered or received the same model. Additionally, online product reviews can provide insights from other consumers that aid in the decision-making process. By leveraging these sources, individuals can effectively learn from the experiences of others before making a purchase decision.[2]

Given the power of word of mouth, it is no wonder that firms may try to strategically influence consumer communication in a variety of ways. Firms can use so-called 'buzz marketing', where digital advertising about a product aims to help get people talking (Mohr, 2017). They may in particular try to shape 'pre-release buzz' by influencing how consumers talk before receiving the product (Xiong and Bharadwaj, 2014). Firms may also strategically use pricing to increase product awareness, as in frequent zero-price sales for smartphone apps, to attract new consumers who then talk to others (Ajorlou et al., 2018). They may even go so far as to directly manipulate consumer communication, by posting fake product reviews on platforms such as TripAdvisor (Mayzlin et al., 2014), Yelp (Luca and Zervas, 2016), and Amazon (He et al., 2022).

[1]An established literature has shown how word-of-mouth communication can affect many consumer decisions, including television viewing (Godes and Mayzlin, 2004), book purchases (Chevalier and Mayzlin, 2006), movie viewing (Liu, 2006; Duan et al., 2008), even adoption of microfinance (Banerjee et al., 2013), and much more.

[2]In this spirit, Nam et al. (2010) consider word-of-mouth learning about product quality for a 'video on demand' service, where signal quality is uncertain ex ante and varies with geographical location, and where 86% of surveyed household reported discussing the service with someone they knew. They conclude that word of mouth is associated with faster (slower) local adoption of the service when quality is high (low). Relatedly, Neelamegham and Jain (1999) emphasize the importance of word-of-mouth communication for products, such as movies, with ex ante quality uncertainty, and distinguish between positive and negative word of mouth.

Our paper considers a novel way a seller may strategically influence word of mouth to shape consumer beliefs about product quality: by using scarcity to delay the purchases of opinion leaders. Delays and stocks outs are common in practice. Prominent examples that have received widespread attention include shortages for the Nintendo Switch in 2020[3], Tesla in 2021[4] and the Sony PlayStation 5 over a similar period.[5] While delays can certainly arise for many reasons, such as unexpected production problems, our focus is on how delay can allow a seller to shape consumer buzz.

We show that delaying opinion leaders can sometimes be profitable, precisely because of its impact on word-of-mouth communication. We also describe how the profitability of delay depends on factors such as ex ante beliefs about quality, the precision of consumers' private information, the relative number of opinion leaders in the market, and the extent to which leaders talk if their purchases are delayed.

More specifically, we present a two-period model, where a seller first faces a cohort of opinion leaders and then a cohort of followers who can potentially talk to the leaders. All parties are initially unsure whether product quality is high or low and each consumer receives a boundedly informative private signal. The seller chooses a service speed which influences how quickly leaders can be served, after which leaders arrive and simultaneously choose whether to place orders. All demand from leaders up to the service speed is immediately served while all other leaders who place orders are delayed. Each of these leaders can then tell some followers about their experience: whether they were delayed or served, and in the latter case, their utility from buying. Followers then simultaneously choose whether to buy. Those that buy receive the product, as do leaders who were delayed.[6]

We first consider a baseline case, where leaders talk to the same number of followers regardless of whether they are immediately served or delayed. Delaying leaders then affects only the informational content of word-of-mouth communication. Rather than telling followers hard information about product quality, delayed leaders can only convey

[3]See https://www.forbes.com/sites/davidthier/2020/04/19/the-real-reason-nintendo-switch-is-out-of-stock-everywhere/?sh=4e08af9d5694, accessed on January 05 2024.

[4]See https://www.cnbc.com/2021/08/18/months-long-delivery-delays-confound-would-be-tesla-owners.html, accessed on January 05 2024.

[5]See https://www.npr.org/2023/01/05/1147157065/sony-playstation-5-shortage-over, accessed on January 05 2024.

[6]Assuming delayed leaders also eventually receive the product allows us to isolate the informational impact of delay on profits. Reducing service speed then does not directly cost sales but only affects word of mouth. As such, in the absence of word of mouth or quality uncertainty, the seller would be indifferent between immediately serving or delaying leaders.

soft information about their intention to buy and the fact that they could not get the product immediately. It is via this *content effect* that a lower service speed reduces learning.

We show that delaying leaders may be strictly profitable. That is, for certain parameter values, there exists a pure strategy equilibrium where all leaders are delayed. Perhaps surprisingly, the seller will hide information via delay whenever product quality is likely high, as then meeting a delayed leader is sufficiently good news to induce purchases. The seller may also use delay when product quality is likely low, but only if signals are not too accurate. The first case corresponds to 'mass markets' (see Ivanov (2009)), i.e. markets in which all consumers would be willing to buy based solely on the prior. The second case corresponds to 'niche markets', where only consumers who obtain positive information about quality, either via private signals or word of mouth, will purchase the product.

In contrast, no pure strategy equilibrium exists in which leaders are immediately served. Followers who expect a high service speed will infer that demand is high upon unexpectedly meeting a delayed leader, which pushes the seller to instead set a low service speed to induce delay. By the same token, it is never strictly profitable in equilibrium to immediately serve leaders rather than delay them. Nonetheless, if private signals are strong enough, then in niche markets there exists a mixed strategy equilibrium where the seller serves all leaders almost surely, and where followers who meet delayed leaders randomize between always buying and acting on their private signal.[7]

We then consider a more general case, where immediately-served leaders talk to more followers than leaders who are delayed.[8] A low service speed then reduces learning both by the *content effect* from the baseline, as well as a *level effect*, since fewer followers then hear from leaders. Intuitively, the less that delayed leaders talk, the stronger the level effect will be when the seller induces delay.

The level effect affects expected sales via two channels. First, if expected quality is high enough, the seller may prefer consumers to act on hard information learned via word of mouth rather than private signals. Second, when the number of leaders is large relative to the number of followers, not meeting a leader may provide particularly bad

[7]Our baseline results are robust to changes in the amount of word of mouth that leaders engage in, and in the number of leaders relative to followers. These results are also unchanged if we assume that word of mouth not only transmits utility information but also spreads product awareness, i.e. if we assume that only followers who hear from leaders are able to buy.

[8]Arguably, consumers are more likely to share information about the product they actually have, than about the fact that they are waiting to receive it. Such information sharing, e.g. can take the form of a product review.

news about product quality, resulting in unconnected followers refusing to buy regardless of their private signal. Thus, the level effect makes it less attractive for the seller to use delay compared to the baseline, in particular in situations with many leaders and where expected quality is high. In such markets, the level effect leads to the existence of a pure strategy equilibrium in which all leaders are served. Nonetheless, the seller still finds it profitable to delay leaders under alternative market conditions.

The second channel of the level effect also implies that follower beliefs about the seller's service speed may be self-reinforcing, resulting in equilibrium multiplicity. This means that both a pure strategy equilibrium with delay and a pure strategy equilibrium where all leaders are immediately served can exist for the same parameter values. Intuitively, followers who do not hear from leaders will be more pessimistic if they expect a high service speed, as serving leaders increases the correlation between the level of word of mouth and product quality. This, in turn, pushes the seller to set a high service speed so that more followers hear from leaders. Our analysis suggests that equilibrium multiplicity is more of an issue in markets with a moderate number of opinion leaders. In this case, the level effect is strong enough to offset incentives to delay all leaders for some, but not for all, follower beliefs.

Our paper's main contribution is to show how a seller can profitably use product scarcity to influence word-of-mouth communication. As such, it adds to the literature on seller strategic behavior and word of mouth.[9] Our approach of explicitly modelling consumer communication differs from work that directly assumes local interaction effects in consumers' payoff functions (see, e.g., Galeotti and Goyal (2009), Galeotti et al. (2020)). Our focus on quality uncertainly also stands in contrast to much of the literature, which has instead focused on how word-of-mouth communication can inform consumers about product existence, and explored possible interactions with pricing decision (Campbell (2015), Ajorlou et al. (2018)), information release (Campbell et al., 2017), and both pricing and advertising (Campbell, 2013).[10]

Relatively few papers consider how word-of-mouth communication may help consumers learn about product quality. Godes (2017) assume quality is endogenous and

[9]The broader literature on word-of-mouth communication considers how agents can learn from hearing about outcomes from earlier agents in different settings. See, e.g., Ellison and Fudenberg (1995), Bala and Goyal (1998), Banerjee and Fudenberg (2004). The focus has often been on whether agents will converge on the same action in the long run and whether this action is efficient.

[10]Galeotti (2010) instead assumes consumers know about product existence but communicate about prices, and shows how this affects equilibrium price dispersion.

focuses on firm investment incentives. They show that more word of mouth that transmits utility information is associated with higher equilibrium investment in quality. Campbell et al. (2020) assume consumers engage in costly search to learn product quality from earlier buyers, focusing on how the network structure of communication affects the distribution of quality in the market via entry and exit. In contrast, we explore how the seller's strategic choice of scarcity can affect consumer learning about quality via word of mouth.

Our focus on consumer word of mouth and product quality also has some relation to the literature on customer reviews. A major focus there has been on how firms may directly manipulate consumers. In terms of theory, Mayzlin (2006) shows that firm fake reviews can make consumer communication less persuasive, by making it less credible. Relatedly, Smirnov and Starkov (2022) show that a firm's ability to censor bad reviews can affect the informational content of the bad reviews that do end up appearing in equilibrium.[11] We also look at how a seller may benefit from strategically reducing the informational content of consumer communication, but via product scarcity rather than deception, and without being privately informed about product quality.

By looking at a seller's incentive to delay opinion leaders, our paper also contributes to work on optimal product launch strategies. There the distinction is often made between a sequential 'waterfall' strategy to promote learning, and a simultaneous 'sprinkler' strategy to restrict it (see, e.g. Sgroi (2002); Aoyagi (2010); Liu and Schiraldi (2012); Bhalla (2013); Parakhonyak and Vikander (2019)). Consumers there just observe each others choices, whereas we allow consumers to potentially learn each others' utility information via word of mouth. Another difference is that serving consumers simultaneously in this literature essentially shuts down any information transmission. This contrasts with our paper, as delayed leaders still talk to followers, which in turn contributes to making delay more attractive for the seller.

Our paper also contributes to the literature on firms' strategic use of product scarcity to influence consumer behavior. Previous work on scarcity has looked at discouraging consumer strategic delay (DeGraba (1995); Nocke and Peitz (2007); Möller and Watanabe (2010)) as well as facilitating price discrimination (Wilson (1988); Bulow and Roberts (1989); Ferguson (1994); Loertscher and Muir (2022)). A few papers in this literature share our focus on how scarcity can affect consumer learning about product quality, but via different mechanisms. Consumers learn either via firm signaling (Stock and Balachander,

[11] Hauser (2023) look at censorship but without explicitly considering consumer communication. There, censorship reduces the arrival rate of potential bad news, which can affect firm incentives to invest in quality.

2005), observational learning from realised sales (Parakhonyak and Vikander, 2023) or a combination of the two (Debo et al., 2012). In contrast, we look at how scarcity affects how consumers learn via both the content and the level of word-of-mouth communication.

Finally, follower learning via word of mouth causes a demand rotation in the spirit of Johnson and Myatt (2006), by effectively making valuations more dispersed. Delaying leaders limits this increase in dispersion by rotating demand in the opposite direction. As such, delay brings the posterior beliefs of followers closer to the prior. Johnson and Myatt (2006) show that firms tend to benefit from extremes: increasing dispersion when charging a high price (niche market strategy), but decreasing dispersion when charging a low price (mass market strategy). As the seller does not price strategically in our model, the distinction between niche and mass market is exogenous and lies in whether the value of consumers' outside option exceeds their prior beliefs about quality.

A fundamental difference with Johnson and Myatt (2006) is that consumer beliefs about the seller's chosen strategy crucially affect the way that word of mouth, and delaying leaders, rotates demand. Their applications focus mainly on mean-preserving demand rotations corresponding to Bayesian persuasion mechanisms (Kamenica and Gentzkow, 2011), where Bayes plausibility dictates that the expected posterior belief is equal to the prior. This is also true in our model *on the equilibrium path*. Off-path, however, due to unobservable service speed (i.e., lack of commitment power in Bayesian persuasion terms), the expected posterior need not equal to the prior, since follower inference about product quality will depend on the service speed they expect. As a result, delaying leaders can sometimes even allow the seller to increase follower valuations in the sense of first-order stochastic dominance.

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 contains all main results of the paper, with the baseline case, corresponding to an equal amount of word of mouth for served and delayed leaders, discussed in section 3.1, and the general case discussed in 3.2. Section 4 considers an extension where followers only become aware of the product if they hear about it through word of mouth. Section 5 concludes.

2 Setting

Suppose there is a product of unknown quality and two possible states of the world, $\Omega = \{G, B\}$. In state G, quality is good and each consumer who buys obtains $u_G = 1$. In state B, quality is bad and each consumer who buys obtains $u_B = 0$. A consumer who does not buy gets reservation utility $r \in (0, 1)$. The actual state is known neither to the seller nor to consumers. Prior beliefs of all players are that $P(G) \equiv \beta$ and $P(B) = 1 - \beta$, where $\beta \in (0, 1)$.

There are two cohorts of potential buyers in the market. We refer to the first cohort as opinion leaders, or simply leaders, and the second cohort as followers. The number of leaders is N whereas the number of followers is nN, with $n, N \in \mathbb{N}$.

Each potential buyer has unit demand and learns about quality based on a noisy private signal, $s \in \{g, b\}$, where $P(g|G) = P(b|B) \equiv \alpha \in (1/2, 1)$. By $\alpha < 1$, signals are *boundedly informative* about the state. Additionally, followers can potentially learn by hearing from leaders, as described more precisely below. We focus on situations where $P(G|s = g) > r > P(G|s = b)$. This means that in the absence of further information, it is optimal for a potential buyer to follow their own private signal.

The timing of the game is as follows. At $t = -1$, the seller sets a service speed $K \in \mathbb{Z}^+$, with $K \leq N$. This speed will influence how quickly leaders can receive the product. Then, at $t = 0$, nature chooses product quality, $\omega \in \{G, B\}$. The rest of the game consists of three stages.

In stage $t = 1$, the leaders enter the market, each receives their private signal, and they simultaneously decide whether to order the product. If the total quantity ordered is less than K, then all leaders who ordered receive the product immediately. If instead total orders exceed K, then K randomly chosen leaders receive the product immediately, whereas the remaining $N - K$ leaders do not. We will say that the former group of leaders are served immediately and that the latter group are delayed.

In stage $t = 2$, the followers enter the market, each receives their private signal, and each may also learn from a leader via word-of-mouth communication. That is, each follower connects with at most one leader who ordered the product and hears about that leader's experience in stage 1: whether they were delayed or served immediately, and in the latter case, their utility from receiving the product.[12] Each leader who is served,

[12]We could have assumed that the leader's utility is transmitted with noise, e.g. that with a small probability, $\tilde{u} = 1 - u$ is reported instead of u. Doing so would not qualitatively change our results.

and each leader who is delayed, connects with m_1 and m_0 followers, respectively. We assume $m_1 \geq m_0$ to capture the idea that leaders who immediately receive the product, and discover its quality, plausibly should not communicate less than leaders who have yet to receive the product due to delays.[13] Since each follower is connected to at most one leader, we impose $m_1 \leq n$.

Finally, in stage $t = 3$, followers simultaneously decide whether to buy. All followers who want to buy the product receive it, as do all leaders who were previously delayed.

We normalize the per consumer profit of the seller to 1. The seller sets service speed K so as to maximize expected profits, given the subsequent behavior of consumers.[14] Each consumer makes the purchase decision that is optimal, given their beliefs about product quality. These beliefs about quality follow from Bayes' rule and the other players' equilibrium strategies, whenever possible.

We will need to characterise beliefs when the seller is expected to immediately serve all leaders in stage 1, but a follower nonetheless meets a delayed leader in stage 2. Our approach is to allow for seller 'trembles', where the implemented service speed, with small probability, can differ slightly from the seller's profit-maximizing choice of K, in order to pin down these out-of-equilibrium beliefs. Specifically, if the seller chooses $K \geq 1$, then we assume that the implemented service speed is $K - 1$ with probability ϵ and K with probability $1 - \epsilon$, where $\epsilon > 0$ can be arbitrarily small.

Our analysis will focus on the limiting case of large markets, $N \to \infty$. We therefore normalize both service speed and profits by the total market size, where the seller sets service speed $k \equiv K/N$ to maximizes expected profits per consumer. This service speed $k \in [0, 1]$ should be thought of as the limiting case of the profit-maximizing normalized service speed K/N in a finite market, as N become large.

There is no discounting between stages.[15] Moreover, we assume that followers observe neither the seller's choice of service speed K nor the total number of leaders who are immediately served. As a result, the only information available to followers is what they hear from leaders via word-of-mouth communication.

[13]We could also allow for each leader who did not order to connect with say $m' > 0$ followers and communicate their own private signal. Doing so will not qualitatively change our results, as long as such leaders do not communicate more broadly than leaders who ordered: $m' \leq m_0 \leq m_1$.

[14]While we focus on service speed, rather than pricing, the consumer reservation utility r can also be interpreted as reflecting the product's price if consumers have an outside option of zero.

[15]Alternatively, the second period sales can be interpreted as a net present value of future sales.

3 Analysis

Since leaders follow their private signals, the probabilities that j leaders order the product in the good and bad states are, respectively,

$$Q_G(j) = \binom{N}{j}\alpha^j(1-\alpha)^{N-j}, \quad Q_B(j) = \binom{N}{j}(1-\alpha)^j\alpha^{N-j}. \tag{1}$$

Followers who meet a served leader will learn the state by hearing the leader's utility, and therefore only buy if the state is good. The decision of other followers may depend on what they infer from meeting a delayed leader or remaining unconnected, as well as their own private signal.

As a first step to write out the seller profit function, we will assume a follower buys with probability γ_ω if they meet a delayed leader, and buys with probability δ_ω if they remain unconnected. These probabilities may depend on the state $\omega \in \{G, B\}$, since they take into account both possible realizations of the follower's private signal $s \in \{g, b\}$, and followers with different signals may take different decisions. We later derive what values γ_ω and δ_ω should take in equilibrium, given follower beliefs.

Denote the expected number of leaders served in state ω as $S_\omega(K)$, and the expected number of leaders delayed as $D_\omega(K)$, where

$$S_\omega(K) = \sum_{j=0}^{N} \min\{j, K\} Q_\omega(j), \quad D_\omega(K) = \sum_{j=0}^{N} \max\{j-K, 0\} Q_\omega(j).$$

Profits from setting service speed $K \geq 1$, normalized by market size N, are then

$$\pi(K) = (1-\varepsilon)\tilde{\pi}(K) + \varepsilon\tilde{\pi}(K-1)$$

where

$$\begin{aligned}\tilde{\pi}(K) =& \frac{\beta}{N}\left[\sum_{j=0}^{N} jQ_G(j) + m_1 S_G(K) + m_0\gamma_G D_G(K) + \delta_G\left(nN - m_1 S_G(K) - m_0 D_G(K)\right)\right] \\ &+ \frac{1-\beta}{N}\left[\sum_{j=0}^{N} jQ_B(j) + 0 + m_0\gamma_B D_B(K) + \delta_B\left(nN - m_1 S_B(K) - m_0 D_B(K)\right)\right],\end{aligned} \tag{2}$$

and we define profits from setting service speed $K = 0$ as $\pi(0) = \tilde{\pi}(0)$. In each square bracket in expression (2), the first term refers to the number of leaders who order the product and therefore receive it either in stage 1 (if served immediately) or in stage 3 (if delayed); the second term refers to followers who meet immediately-served leaders and therefore learn the state; the third term refers to receivers who meet delayed leaders; and the fourth term refers to followers who remain unconnected.

Before stating our first result, we introduce the notion of a demand rotation from Johnson and Myatt (2006) to our setting. Fix some consumer belief about the service speed and suppose that leaders follow their private signals. Let $F_K(\mu)$ denote the distribution function for follower willingness to pay (belief about the state) when the seller sets service speed K. Note that K and consumers beliefs about the service speed need not coincide. The expected demand from followers, given ex ante uncertainty about the state and signal realizations and given outside option r, is then $nN(1-F_K(r))$. Then we make the following definition.

Definition 1. *Consider $K \in [0, N]$ and $K' \in [0, N]$. Then we say that setting service speed K' rather than K induces a demand rotation if there is μ such that $F_{K'}(\mu) \neq F_K(\mu)$, and there exists $\mu^* \in [0, 1]$ such that either (i) $F_{K'}(\mu) \geq F_K(\mu)$ for all $\mu < \mu^*$ and $F_{K'}(\mu) \leq F_K(\mu)$ for all $\mu > \mu^*$, or (ii) $F_{K'}(\mu) \leq F_K(\mu)$ for all $\mu < \mu^*$ and $F_{K'}(\mu) \geq F_K(\mu)$ for all $\mu > \mu^*$.*

Intuitively, a demand rotation corresponds to an unambiguous increase, or decrease, of the dispersion of follower valuations (at least weakly). Case (i), where dispersion increases, constitutes an anti-clockwise demand rotation, whereas Case (ii), where dispersion decreases, constitutes a clockwise demand rotation. The effect of a demand rotation on follower valuations is not necessarily mean preserving. In particular, setting $\mu^* = 1$ shows that the definition also encompasses a decrease or increase of follower valuations in the sense of first-order stochastic dominance. We then have the following result.

Lemma 1. *For any market size N and $\epsilon > 0$, and any follower beliefs about the service speed, the seller maximizes expected profits by setting service speed $K = 0$ or $K = N$, or by randomizing between these two values. Moreover, for any $K \in [0, N]$ and $K' \in [0, N]$, changing service speed from K to K' induces a demand rotation.*

Lemma 1 implies in particular that any equilibrium will involve the seller either delaying all leaders, immediately serving all leaders, or possibly randomizing between the two. Intuitively, a small increase in service speed can help the seller when the state is good and hurt when the state is bad, as more immediately-served leaders may then reveal the state to followers, but only if demand is sufficiently high (e.g if demand is very low then nobody is delayed even at a low service speed). Demand tends to be higher in the good state. It follows that whenever a small increase in service speed helps the seller in expectation, then a larger increase will help even more, by revealing information for those demand realizations that are even more indicative of the good state.

An increase in the seller's chosen service speed induces a demand rotation because it allows the followers to learn more from leaders. This learning, in turn, increases the dispersion of follower valuations, seen from an ex-ante perspective. Intuitively, immediately-served leaders engage in (at least weakly) more word of mouth than leaders who are delayed, and they also reveal hard information about the state when speaking to followers. Since a high service speed implies more leaders are immediately served, it will therefore increase follower learning through both these channels.

Lemma 1 shows that the optimal service speed always either maximizes or minimizes the dispersion of follower valuations. Which of these two the seller prefers may depend both on ex ante beliefs about quality and the precision of consumers' private signals. It may also depend on follower beliefs about the service speed, which is an issue that we return to in our equilibrium analysis.[16]

We now turn to follower beliefs about the state, in order to derive the purchase probabilities γ_G, γ_B, δ_G, and δ_B in the expression for seller profits.

Meeting a delayed leader provides good news, since demand tends to be higher when the state is good. A follower with a good private signal will clearly want to buy upon receiving this good news. The question is whether this good news is enough to convince a follower to buy after receiving a bad private signal.

Suppose that the seller sets service speed $K = N$ with probability q and sets $K = 0$ with probability $(1 - q)$. The expected number of delayed leaders in state ω is then

$$D_\omega(q) = (1 - q) \sum_{j=0}^{N} jQ_\omega(j) + q\varepsilon Q_\omega(N).$$

In particular, if the seller sets $K = 0$ for sure then all consumers are delayed. If the seller sets $K = N$ then one follower is delayed with probability ε, which is the probability

[16]Relatedly, whether a given demand rotation is mean increasing, mean decreasing, or mean preserving, may depend on beliefs about the service speed, as these beliefs affect followers' inference when meeting a delayed leader or when remaining unconnected.

that service speed $N-1$ is implemented due to a tremble. Using Bayes' rule, the belief of a follower with $s=b$ who met a delayed leader is therefore

$$\mu(q,N) \equiv \frac{\beta(1-\alpha)D_G(q)}{\beta(1-\alpha)D_G(q)+(1-\beta)\alpha D_B(q)}. \tag{3}$$

Properties of these beliefs are described in the following Lemma.

Lemma 2. *In any equilibrium, the belief of a follower with a bad private signal who met a delayed leader satisfies the following properties:*

1. $\lim_{N\to\infty}\lim_{\varepsilon\to 0}\mu(1,N)=1$.

2. *For any fixed* $q<1$, $\lim_{N\to\infty}\lim_{\varepsilon\to 0}\mu(q,N)=\beta$.

3. *For any* $r>\beta$ *there exists* $\overline{N}(r)$ *such that for any* $N>\overline{N}(r)$ *there exists a unique* $q^*(N)$ *which solves* $\mu(q^*(N),N)=r$. *Moreover,* $\lim_{N\to\infty}\lim_{\varepsilon\to 0}q^*(N)=1$.

Followers will be more optimistic after meeting a delayed leader if they expect a high service speed. The first part of Lemma 2 states that if followers expect the seller to immediately serve all leaders, then followers will infer that the state is good, since delay could only result from the combination of high demand and a tremble.[17] The second part states that delaying leaders with any fixed probability, including playing a pure strategy $K=0$, would lead followers with bad private signals who meet delayed leaders to revert to their prior beliefs, because the good private signal of the delayed leader effectively cancels the bad signal of the follower. Finally, the third part implies that there is a candidate equilibrium in which both the seller and followers may randomize, but as markets grow large, the seller's strategy converges to serving all leaders with probability one. We conclude that followers with $s=b$ who met a delayed leader are only willing to buy if they believe that high service speed is sufficiently likely (equals or approaches 1) or if their prior exceeds r.

We now turn our attention to followers who did not meet any leaders, either served or delayed. Not meeting a leader is bad news, because only leaders who place orders engage in word of mouth, and demand is higher when the state is good. Therefore, followers with bad private signals who remain unconnected prefer not to buy. Now consider an

[17] This result in fact holds for any market size N, not just in the limit.

unconnected follower with a good private signal. Bayes' Rule implies that their beliefs are given by

$$\nu(q, N) = \frac{\beta\alpha U_G(q, N)}{\beta\alpha U_G(q, N) + (1-\beta)(1-\alpha)U_B(q, N)},$$

where U_ω is the expected number of unconnected followers in state ω, given by

$$U_\omega(q, N) = (1-q)\left(nN - m_0 \sum_{j=0}^{N} jQ_\omega(j)\right) + q\left(nN - m_1 \sum_{j=0}^{N} jQ_\omega(j) - \varepsilon(m_0 - m_1)Q_\omega(N)\right).$$

Thus, unconnected followers with good private signals will buy as long as $\nu(q, N) \geq r$. Solving for $\lim_{N\to\infty} \lim_{\varepsilon\to 0} \nu(q, N) = r$ allows us to formulate the following Lemma.

Lemma 3. *In a large market, a follower with a good private signal who did not meet a leader will buy the product if and only if $\beta \geq \overline{\beta}_q(r, \alpha)$ where*

$$\overline{\beta}_q(r, \alpha) \equiv \frac{(1-\alpha)\big(n - [(1-q)m_0 + qm_1](1-\alpha)\big)r}{(1-\alpha)\big(n - [(1-q)m_0 + qm_1](1-\alpha)\big)r + \alpha\big(n - [(1-q)m_0 + qm_1]\alpha\big)(1-r)} \tag{4}$$

Having described follower beliefs and their implications for purchase behavior, we now consider the seller's equilibrium choice of service speed. To do so, we focus on large markets. Taking the limit $N \to \infty$ in expression (2) yields the seller's normalized large-market profit function:

$$\begin{aligned}\pi(k) =& \beta\,[\alpha + \min\{\alpha, k\}m_1 + \max\{\alpha - k, 0\}m_0\gamma_G + \\ & (n - \min\{\alpha, k\}m_1 - \max\{\alpha - k, 0\}m_0)\delta_G] + \\ & (1-\beta)\,[1 - \alpha + \max\{1 - \alpha - k, 0\}m_0\gamma_B + \\ & (n - \min\{1-\alpha, k\}m_1 - \max\{1 - \alpha - k, 0\}m_0)\delta_B]\,,\end{aligned} \tag{5}$$

where γ_ω is the purchase probability of a follower who met a delayed leader (consistent with Lemma 2), and δ_ω is the purchase probability of a follower after remaining unconnected (consistent with Lemma 3), conditional on the state being ω.[18]

Notice that although the limit profit function $\pi(k)$ is constant on $k \equiv \frac{K}{N} \in (\alpha, 1]$, setting service speed $K \in [1, N-1]$ is never optimal for any finite market size N.

[18]Expression (5) also gives expected seller profits per consumer, taking into account the possibility of trembles. Given that the market is large we obtain $\lim_{N\to\infty} \tilde{\pi}(K) = \lim_{N\to\infty} \tilde{\pi}(K-1) = \lim_{N\to\infty} \pi(K)$.

Taking the limit of finite markets, we either have that $\lim_{N\to\infty} \frac{1}{N} \arg\max \pi(K) = 1$ or $\lim_{N\to\infty} \frac{1}{N} \arg\max \pi(K) = 0$, from Lemma 1. Thus, we will use $k = 1$ as the profit maximizer of expression (5) over $(\alpha, 1]$ in what follows.

3.1 Baseline case: service speed affects informational content

We first consider the baseline case where all leaders who place orders talk to the same number of followers, regardless of whether they themselves are immediately served or delayed: $m_1 = m_0 = m$. The seller's choice of service speed therefore does not affect the total level of word of mouth but only its informational content.

From Lemma 3, the critical value of β for an unconnected follower to follow their private signal then simplifies to

$$\overline{\beta}_q(r,\alpha)\bigg|_{m_1=m_0=m} = \frac{(1-\alpha)(n-m(1-\alpha))r}{(1-\alpha)(n-m(1-\alpha))r + \alpha(n-m\alpha)(1-r)} \equiv \overline{\beta}(r,\alpha). \tag{6}$$

Note that $\overline{\beta}(r,\alpha)$ now does not depend on q and that $\overline{\beta}(r,\alpha) < r$ for all values of $\alpha \in (1/2, 1)$.

Consider a candidate equilibrium with $k = 0$. From Lemma 2, the posterior belief of a follower with a bad private signal who met a delayed leader is just β. Therefore,

$$\gamma_G = \begin{cases} \alpha, & \beta < r \\ 1, & \beta \geq r \end{cases}, \quad \gamma_B = \begin{cases} 1-\alpha, & \beta < r \\ 1, & \beta \geq r \end{cases} \tag{7}$$

From Lemma 3 and equation (6) we obtain the purchase probabilities of unconnected followers

$$\delta_G = \begin{cases} 0, & \beta < \overline{\beta}(r,\alpha) \\ \alpha, & \beta \geq \overline{\beta}(r,\alpha) \end{cases}, \quad \delta_B = \begin{cases} 0, & \beta < \overline{\beta}(r,\alpha) \\ 1-\alpha, & \beta \geq \overline{\beta}(r,\alpha). \end{cases} \tag{8}$$

In what follows, we will use the following terminology, inspired by Ivanov (2009).

Definition 2. *We call the market mass if $\beta \geq r$ and niche otherwise. We say that private signals are strong if $\beta > 1 - \alpha$ and that private signals are weak otherwise.*

We can now directly substitute the purchase probabilities from (7) and (8) into the expression for seller profits. In a mass market, i.e. for $\beta \geq r$, expression (5) reduces to

$$\begin{aligned}\pi(k) =& \beta\left[\alpha(m+1) + (n-\alpha m)\alpha\right] + \\ & (1-\beta)\left[(1-\alpha) + \max\{1-\alpha-k,0\}m + (n-(1-\alpha)m)(1-\alpha)\right],\end{aligned}$$

which is decreasing in k. Thus, $k=0$ is an equilibrium if the market is mass.

If the market is niche with $\beta \in [\overline{\beta}(r,\alpha), r)$, the profit function reduces to

$$\begin{aligned}\pi(k) =& \beta\left[\alpha + \min\{\alpha,k\}m + \max\{\alpha-k,0\}m\alpha + (n-\alpha m)\alpha\right] + \\ & (1-\beta)\left[1-\alpha + \max\{1-\alpha-k,0\}m(1-\alpha) + (n-(1-\alpha)m)(1-\alpha)\right].\end{aligned}$$

Now consider a deviation from $k=0$ to $k=1$.[19] This deviation is profitable if and only if

$$\beta\alpha^2 m + (1-\beta)(1-\alpha)^2 m < \beta\alpha m,$$

which simplifies to $\beta > 1-\alpha$. Thus, $k=0$ is an equilibrium for $\beta \in [\overline{\beta}(r,\alpha), r)$ as long as $\beta \leq 1-\alpha$, i.e. if private signals are weak.

Finally, for $\beta < \overline{\beta}(r,\alpha)$, the profit function reduces to

$$\pi(k) = \beta\left[\alpha + \min\{\alpha,k\}m + \max\{\alpha-k,0\}m\alpha\right] + (1-\beta)\left[1-\alpha + \max\{1-\alpha-k,0\}m(1-\alpha)\right],$$

which again is increasing in k if an only if $\beta > 1-\alpha$. Taken together, we can conclude that $k=0$ in an equilibrium in mass markets, as well as in niche markets with weak private signals.

Intuitively, reducing service speed affects seller profits by influencing the informational content of consumer word of mouth. Delayed leaders cannot transmit precise information to followers about the state; instead they just transmit coarse information by revealing that they received good private signals but could not get the product immediately. As such, delaying leaders maximizes the number of followers who receive positive information via word of mouth, but also limits how convincing this information will be.

It pays off to restrict service speed in a mass market (i.e. $\beta \geq r$) since meeting delayed leaders is then convincing enough to induce followers to buy regardless of the followers' private signals. In contrast, in a niche market (i.e. $\beta < r$), followers who meet delayed

[19] We can restrict attention to this deviation by Lemma 1.

leaders just act on their own signal. Delaying leaders is then a double-edged sword: hiding hard information via delay helps the seller if the state turns out to be bad but hurts the seller if the state turns out to good. When private signals are strong, more (fewer) leaders want to buy in the good (bad) state, so reducing service speed tends to hide hard information whose revelation would help the seller.

We can also interpret this result in terms of a (anti-clockwise) demand rotation. More specifically, deviating from $k = 0$ to any $k \in (0, 1)$ induces a mean-reducing spread of follower valuations. Followers will make incorrect inference about the state when meeting a delayed leader off the equilibrium path; they underestimate the good news that meeting a delayed leader suggests about the state, because they underestimate the service speed. However, the spread induced by a deviation to $k = 1$ is mean preserving, since no leaders are delayed. The optimal deviation for the seller therefore just increases the dispersion of follower valuations.

Whether more dispersion helps or hurts the seller depends on parameter values. In a mass market, both followers with good and bad private signals will buy after meeting a delayed leader. The seller therefore has no reason to further spread their valuations by deviating to $k = 1$. In a niche market, the seller benefits from spreading the valuations of followers with bad signals (pushing some above r), but not from those with good signals (pushing some below r). Whether a deviation to $k = 1$ pays off depends on the size of the former group in the good state, relative to the latter group in the bad state, which depends on whether private signals are strong or weak.

We now rule out a pure strategy equilibrium with $k = 1$. In such a candidate equilibrium, the purchase probabilities of unconnected followers, δ_G and δ_B, are still given by (8). The posterior belief of followers who meet delayed leaders equals to 1, from Lemma 1, so these followers will buy regardless of their private signal: $\gamma_G = \gamma_B = 1$.

For $\beta \geq \overline{\beta}(r, \alpha)$, substituting these purchase probabilities into expression (5) for seller profits gives

$$\begin{aligned}\pi(k) =& \beta\left[\alpha(m+1) + (n - \alpha m)\alpha\right] + \\ & (1-\beta)\left[(1-\alpha) + \max\{1-\alpha-k, 0\}m + (n-(1-\alpha)m)(1-\alpha)\right].\end{aligned}$$

Doing the same for $\beta < \overline{\beta}(r, \alpha)$ gives

$$\pi(k) = \beta \left[\alpha(m+1)\right] + (1-\beta)\left[(1-\alpha) + \max\{1-\alpha-k, 0\}m\right].$$

Both these profit expressions are decreasing in k, so setting $k = 1$ cannot be optimal.

Deviating to $k < 1$ when followers expect $k = 1$ induces a particular kind of demand rotation: follower valuations increase in the sense of first-order stochastic dominance. If followers expect a high service speed, then meeting a delayed leader in a large market provides compelling evidence of high demand and the good state. This evidence will induce followers to buy regardless of the prior and their own private signal. That is, reducing the service speed implies that word of mouth pools the beliefs of connected followers at 1, rather than splitting their beliefs between 0 and 1. Thus, the purchase behavior of these followers pushes the seller to deviate to a low service speed, to delay as many leaders as possible. Note the key distinction between followers' beliefs about the service speed, which determines how followers react when meeting a delayed leader, and the seller's actual choice of service speed, which determines how many followers in fact meet delayed leaders.

Whereas no pure strategy equilibrium exists in niche markets with strong private signals, $1-\alpha < \beta < r$, we now derive a mixed strategy equilibrium in which the seller randomises between $k = 0$ and $k = 1$, and in which followers who meet delayed leaders randomize between buying and not buying.

Lemma 2 showed that for any finite market size N large enough, there exists a mixed strategy of the seller $q^*(N)$ that leaves followers who meet delayed leaders indifferent. We now verify that whenever $1-\alpha < \beta < r$, there exists a mixed strategy for these followers that leaves the seller is indifferent between setting $k = 0$ and $k = 1$. From (5), the seller indifference condition is[20]

$$\begin{aligned}\pi(0) = &\beta[\alpha + \alpha m\gamma_G + (n-\alpha m)\delta_G] + (1-\beta)[1-\alpha+(1-\alpha)m\gamma_B + (n-(1-\alpha)m)\delta_B] \\ &= \beta[\alpha + \alpha m + (n-\alpha m)\delta_G] + (1-\beta)[1-\alpha + (n-(1-\alpha)m\delta_B] = \pi(1),\end{aligned}$$

[20]We work directly with profit expressions for the limiting case of large markets. It is easy to show that for any finite N, profit expressions for both $\pi(K=0)$ and $\pi(K=N)$ are continuous in γ_ω and hence there exists a unique pair $1-\alpha < \gamma_B < \alpha < \gamma_G$ that solves $\pi(K=0) = \pi(K=1)$. Thus, the mixed strategy equilibrium corresponding to the limit of this solution exists and is unique.

which simplifies to

$$\beta\alpha\gamma_G + (1-\beta)(1-\alpha)\gamma_B = \beta\alpha.$$

Thus, the expected sales to followers who meet delayed leaders when $k = 0$, in both states, must equal the expected sales to followers who learn the state is good when $k = 1$.

Recall that γ_G and γ_B denote the ex ante purchase probabilities of followers who meet delayed leaders, depending on the state. These followers who receive good private signals will buy; meeting a delayed leader provides good news, so their posterior beliefs exceed the value of the outside option, $P(G|g) > r$. If these unconnected followers who receive bad private signals buy with probability $\chi \in [0,1]$, then $\gamma_G = \alpha + (1-\alpha)\chi$ and $\gamma_B = (1-\alpha) + \alpha\chi$. Substituting into the seller indifference condition yields

$$\beta\alpha[\alpha + (1-\alpha)\chi] + (1-\beta)(1-\alpha)[(1-\alpha) + \alpha\chi] = \beta\alpha.$$

The left-hand side of this condition is increasing in χ and equals $\beta\alpha^2 + (1-\beta)(1-\alpha)^2$ when $\chi = 0$ and $\beta\alpha + (1-\beta)(1-\alpha) > \beta\alpha$ when $\chi = 1$. Thus, whenever $\beta\alpha^2 + (1-\beta)(1-\alpha)^2 < \beta\alpha$, or equivalently $\beta > 1-\alpha$, there exists a value $\chi^* \in (0,1)$ that leaves the seller indifferent. If followers with bad signals who meet delayed leaders then buy with probability χ^*, then the seller is willing to mix between $k = 0$ and $k = 1$.

It therefore follows that a mixed strategy equilibrium exists for $1-\alpha < \beta < r$. Since $q^*(N) \to 1$ by Lemma 2, we can conclude that the seller sets a service speed $k = 1$ almost surely, whereas followers who meet delayed leader randomize with a probability strictly greater than 0 and strictly less than 1.

The difference between this mixed strategy equilibrium, which exists for $1-\alpha < \beta < r$, and a pure strategy equilibrium with $k = 1$, which never exists in the baseline, relates to whether followers ascribe delay to a low service speed or to a tremble. In the mixed strategy equilibrium, followers who meet delayed leaders understand that either the seller set $k = 0$, in which case meeting a delayed leader is relatively likely; or the seller set $k = 1$, in which case meeting a delayed leader is very unlikely (due to a combination of high demand and a tremble). In the limit $N \to \infty$, the probability of meeting a delayed leader when $k = 1$ becomes vanishingly small, so followers update beliefs with positive weight on $k = 0$ even if the seller's mixed strategy calls on it to serve all leaders almost surely. In contrast, in a candidate pure strategy equilibrium with $k = 1$, followers update beliefs with full weight on $k = 1$ and interpret delay as the result of an unlikely tremble.

We can summarize the results of this section in the following Proposition.

Proposition 1. *Suppose that $m_1 = m_0$, so that immediately-served leaders and delayed leaders engage in the same amount of word of mouth. Then:*

1. *In niche markets with strong private signals a unique equilibrium exists and is in mixed strategies, where the seller almost surely sets $k = 1$ immediately serves all leaders.*
2. *If either markets are mass or private signals are weak, then there exists a unique equilibrium in pure strategies, where the seller sets $k = 0$ and delays all leaders.*

We can represent these results in the following figure, for parameter values $r = 0.2$, $m = 3$, $n = 4$. Signal precision α is depicted on the horizontal axis and the prior β on the vertical axis. The dashed line $\beta = r$ separates the mass and niche market cases, and the solid line $\beta = 1 - \alpha$ separates markets with strong and weak private signals.

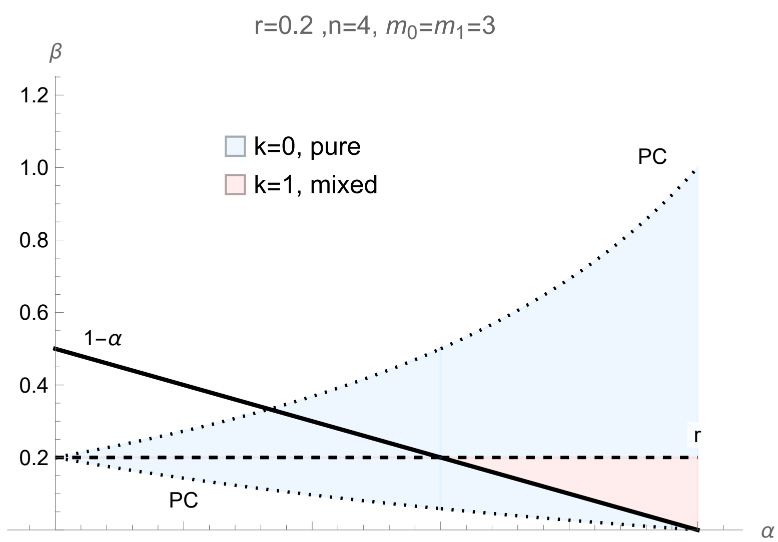


Figure 1: Optimal Service Speed

The region between the two dotted curves in Figure 1 shows where leaders follow their own signals. The light blue part of this region shows where there is a pure strategy equilibrium with $k = 0$, whereas the light red triangle shows where there is a mixed strategy equilibrium where the seller sets $k = 1$ with probability 1. Delay is inevitable in mass markets, but happens in niche markets only if private signals are imprecise, in line with Proposition 1.

Notice that it is the unobservability of the seller's service speed that drives the existence of a mixed-strategy-equilibrium region. If k were observable, then follower inference about the state, after meeting a delayed leader, would always be correct, both and off the equilibrium path. An increase in service speed would then always induce a mean-preserving spread of follower valuations, and the seller would just set $k = 1$ in the light-red region of Figure 1.

We conclude this section by noting that the seller's equilibrium choice of service speed does not depend on the number of leaders relative to followers in the market (the parameter n) or on the amount that leaders talk (the parameter m). That is, the fact that leaders engage in word of mouth matters for seller strategic behavior, but the exact number of followers who learn via word of mouth does not. An important reason is that service speed does not affect the total level of word mouth when all leaders who place orders communicate to the same extent, as assumed in the baseline. As the number of connected followers does not depend on the service speed, the choice of this speed is also independent of the amount of word of mouth.

3.2 General Case: service speed affects both the informational content and the level of word of mouth

We now consider a more a general case with $m_1 \geq m_0$, so where immediately-served leaders may talk to more followers than leaders who are delayed. As a result, the seller's chosen service speed will now affect the overall level of word-of-mouth communication, as well as its informational content. Specifically, if $m_1 > m_0$, then reducing service speed will reduce the number of followers who hear from leaders. We will refer to this impact of delay as the *level effect*.

We proceed along similar lines as in the baseline analysis, looking for both pure strategy equilibria with $k = 0$ and $k = 1$, as well as mixed strategy equilibria where the seller randomizes between the two service speeds. By Lemma 1, we can again restrict attention to these candidate equilibria, as well as possible deviations to $k = 0$ and $k = 1$.

Throughout the analysis we try to make use of the intuition from the baseline and also highlight the differences implied by the level effect. We are interested, in particular, in the seller's incentive to delay leaders in niche and mass markets, as well as in how the number of leaders and the amount that they talk influences the seller's incentive to delay.

Before proceeding, recall that $\overline{\beta}_0(r, \alpha)$ denotes the lowest value of the prior β for which unmatched followers with good private signals buy when they expect $k = 0$, and $\overline{\beta}_1(r,\alpha)$ denotes the corresponding threshold when they expect $k = 1$. Expression (4) implies that $\overline{\beta}_1(r,\alpha) > \overline{\beta}_0(r,\alpha)$ whenever $m_0 < m_1$, and also that both $\overline{\beta}_1(r,\alpha)$ and $\overline{\beta}_0(r,\alpha)$ are decreasing in n, the number of followers relative to leaders.[21]

Intuitively, the level effect's impact on unconnected followers' inference is driven by the size of the unconnected population, which in turn depends on the state and the service speed. For example, the number of unconnected followers is $N(n - m_1\alpha)$ in the good state and $N(n-m_1(1-\alpha))$ in the bad state if the seller sets $k = 1$, with corresponding expressions involving m_0 if the seller sets $k = 0$. The ratio of the number of unconnected followers in the good state, relative to the bad state, is therefore larger when the seller serves all leaders than when it delays them. This difference, however, decreases as n becomes large, i.e. when there are only few leaders relative to followers. Thus, for given values of m_0 and m_1, it is convenient to characterize the level effect's impact on unconnected followers through the number of leaders in the market, as we do in the following definition.

Definition 3. *For a niche market, we say that there are a* **low** *number of leaders if $\beta \geq \overline{\beta}_1(r,\alpha)$, a* **moderate** *number of leaders if $\overline{\beta}_0(r,\alpha) \leq \beta \leq \overline{\beta}_1(r,\alpha)$, and a* **high** *number of leaders if $\beta < \overline{\beta}_0(r,\alpha)$.*

First consider a candidate equilibrium with $k = 0$. As in the baseline, followers who meet delayed leaders may either all buy, or follow their private signals, depending on the value of the prior. The corresponding purchase probabilities γ_G and γ_B are therefore still given by equation (7), and the purchase probabilities δ_G and δ_B, for an unconnected follower, correspond to (8) but with cutoff $\overline{\beta}_0(r,\alpha)$. Thus, we have

$$\gamma_G = \begin{cases} \alpha, & \beta < r \\ 1, & \beta \geq r \end{cases}, \quad \gamma_B = \begin{cases} 1-\alpha, & \beta < r \\ 1, & \beta \geq r \end{cases}$$

$$\delta_G = \begin{cases} 0, & \beta < \overline{\beta}_0(r,\alpha) \\ \alpha, & \beta \geq \overline{\beta}_0(r,\alpha) \end{cases}, \quad \delta_B = \begin{cases} 0, & \beta < \overline{\beta}_0(r,\alpha) \\ 1-\alpha, & \beta \geq \overline{\beta}_0(r,\alpha) \end{cases}$$

It is straightforward to verify that $n > m_0$ implies that $\overline{\beta}_0(r,\alpha) < r$, so we can sequentially consider three cases: when $\beta \geq r$, when $\beta \in [\overline{\beta}_0(r,\alpha), r)$ and when $\beta < \overline{\beta}_0(r,\alpha)$.

[21] Note that $\lim_{n\to\infty} \overline{\beta}_q(r,\alpha) = \frac{(1-\alpha)r}{(1-\alpha)r+\alpha(1-r)}$, and so the condition $\beta \geq \overline{\beta}_q(r,\alpha)$ is equivalent to the participation constraint (i.e. leaders will follow a bad private signal). Meeting no-one does not constitute bad news in a setting with so few leaders, relative to followers. Moreover, $\lim_{n\to m_i} \overline{\beta}_i(r,\alpha) = r$, were $n \geq m_i$, $i = 0, 1$. Meeting no-one effectively cancels the positive private signal when the number of leaders is high.

Consider a mass market, i.e. $\beta \geq r$, which implies $\gamma_G = \gamma_B = 1$, $\delta_G = \alpha$, $\delta_B = 1 - \alpha$. Plugging these purchase probabilities into expression (5) for profits, then evaluating at $k = 0$, gives

$$\pi(0) = \beta \left[\alpha(m_0 + 1) + (n - \alpha m_0)\alpha\right] + (1 - \beta)\left[(1 - \alpha)(m_0 + 1) + (n - (1 - \alpha)m_0)(1 - \alpha)\right].$$

Doing the same but evaluating at $k = 1$ yields deviation profits

$$\pi(1) = \beta \left[\alpha(m_1 + 1) + (n - \alpha m_1)\alpha\right] + (1 - \beta)\left[1 - \alpha + (n - (1 - \alpha)m_1)(1 - \alpha)\right].$$

A direct comparison shows that we can rule out a profitable deviation to $k = 1$ when

$$\beta \leq \hat{\beta}(\alpha) \equiv 1 - \frac{m_1 - m_0}{m_1}\alpha. \tag{9}$$

If immediately-served leaders talk strictly more than delayed leaders, $m_1 > m_0$, then an equilibrium with $k = 0$ can exist in a mass market, but only provided that the good state is not sufficiently likely. The seller faces the following trade-off, which arises due to the level effect. Delaying all leaders helps the seller if the state turns out to be bad, as followers who meet delayed leaders do not learn the state and instead are all induced to buy. In contrast, delaying all leaders hurts if the state turns out to be good. Fewer followers then hear from leaders and buy regardless of their private signal, and more followers end up unconnected, compared to if the seller set a high service speed.

Now consider a niche market with either a low or moderate number of leaders, i.e. $\beta \in [\overline{\beta}_0(r, \alpha), r)$, which implies $\gamma_G = \delta_G = \alpha$, $\gamma_B = \delta_B = 1 - \alpha$. That is, followers who do not meet immediately-served leaders just follow their private signals. Profits from setting $k = 0$ are then

$$\pi(0) = \beta\left[\alpha + \alpha^2 m_0 + (n - \alpha m_0)\alpha\right] + (1 - \beta)\left[1 - \alpha + (1 - \alpha)^2 m_0 + (n - (1 - \alpha)m_0)(1 - \alpha)\right].$$

whereas a deviation to $k = 1$ yields

$$\pi(1) = \beta\left[\alpha + \alpha m_1 + (n - \alpha m_1)\alpha\right] + (1 - \beta)\left[1 - \alpha + (n - (1 - \alpha)m_1)(1 - \alpha)\right].$$

Thus, there is no profitable deviation when $\beta \leq 1 - \alpha$, i.e. when private signals are weak.

Finally, consider a niche market with a high number of leaders, i.e $\beta < \overline{\beta}_0(r, \alpha)$, which implies $\gamma_G = \alpha,\ \gamma_B = 1-\alpha$, and $\delta_G = \delta_B = 0$. Profits from setting $k = 0$ are then

$$\pi(0) = \beta\left[\alpha + \alpha^2 m_0\right] + (1-\beta)\left[1-\alpha+(1-\alpha)^2 m_0\right],$$

while profits from a deviation to $k = 1$ are

$$\pi(1) = \beta\left[\alpha + \alpha m_1\right] + (1-\beta)(1-\alpha).$$

It follows that we can rule out any profitable deviation whenever

$$\beta \leq \frac{(1-\alpha)^2 m_0}{(1-\alpha)m_0 + (m_1 - m_0)\alpha}. \tag{10}$$

We denote

$$\tilde{\beta}(\alpha) \equiv \frac{(1-\alpha)m_0}{(1-\alpha)m_0 + (m_1 - m_0)\alpha}.$$

Note that $\tilde{\beta}(\alpha)$ depends on $m_1 - m_0$, the difference between the levels of word of mouth between served and delayed leaders. Thus, the values of $\tilde{\beta}(\alpha)$ and $\hat{\beta}(\alpha)$ both reflect the strength of the level effect. Moreover, $\hat{\beta}(\alpha) > \tilde{\beta}(\alpha)$ for $m_1 > m_0$, and $\hat{\beta}(\alpha) = \tilde{\beta}(\alpha) = 1$ when $m_1 = m_0$. This leads us to introduce the following definition.

Definition 4. *We say that the level effect is* **strong** *if* $\beta > \hat{\beta}(\alpha)$, **substantial** *if* $\beta > \tilde{\beta}(\alpha)$[22], **modest** *if* $\beta \in [(1-\alpha)\tilde{\beta}(\alpha), \tilde{\beta}(\alpha)]$ *and* **weak** *otherwise.*

Notice, from Definition 2, that strong private signals are necessary for the level effect to be strong, since $\hat{\beta}(\alpha) > 1-\alpha$. Similarly, weak private signals are necessary for the level effect to be weak, since $(1-\alpha)\tilde{\beta}(\alpha) < 1-\alpha$.

Using this notation, we sum up all these cases to get the following result.

Lemma 4. *An equilibrium with $k = 0$ exists if and only if*

1. *a mass market has a level effect that is not strong*
2. *a niche market has at most a moderate number of leaders, and weak private signals*
3. *a niche market has a high number of leaders, and a weak level effect*

[22]Note that a strong level effect will also be substantial.

The level effect of restricting service speed reduces the range of parameter values for which the seller sets $k = 0$. Comparing case 1 from Lemma 4 to case 2 from Proposition 1 shows that the seller no longer always delays leaders in a mass market. Instead, the seller only does so if the level effect is not strong. Moreover, comparing case 3 from Lemma 4 shows that in niche markets with a high number of leaders, the seller does not always set $k = 0$ when there are a weak private signals: the level effect must be weak as well.

The level effect's impact on seller incentives is also reflected by what demand rotation is induced by an increase in service speed. In the baseline, a deviation from $k = 0$ to $k = 1$ resulted in a mean-preserving spread of follower valuations. The level effect implies that this deviation now results in a mean-increasing spread, making the deviation more attractive. Intuitively, followers who remain unconnected make incorrect inference following the deviation; they overestimate the probability of being unconnected but underestimate the bad news that being unconnected suggests about the state. As a result, average follower posterior beliefs will exceed the prior.

Now we consider a candidate equilibrium with $k = 1$. As in the baseline, followers who meet immediately-served leaders will act based on hard information about the state, whereas followers who meet delayed leaders may either all buy, or follow their private signals, depending on the value of the prior. Unconnected followers will act on their privates signals if $\beta \geq \overline{\beta}_1(r, \alpha)$, with $\overline{\beta}_1(r, \alpha)$ given by (4) evaluated at $q = 1$, and otherwise all refuse to buy. Recall that $\overline{\beta}_1(r, \alpha) > \overline{\beta}_0(r, \alpha)$ when $m_1 > m_0$, as unconnected followers become more pessimistic (and hence will refuse to buy for a larger range of β) if they expect a higher service speed.

As in the baseline, meeting a delayed leader when followers expect $k = 1$ is interpreted as a result of high demand and a tremble, which provides decisively good news. Therefore, $\gamma_G = \gamma_B = 1$ for all β. The behaviour of unconnected followers may vary with parameters.

We start our analysis with the case where the number of leaders is low, $\beta > \overline{\beta}_1(r, \alpha)$. The purchase probabilities are then $\delta_G = \alpha$ and $\delta_B = 1 - \alpha$, so setting $k = 1$ yields profits

$$\pi(1) = \beta\left[\alpha + \alpha m_1 + (n - \alpha m_1)\alpha\right] + (1 - \beta)\left[1 - \alpha + (n - (1 - \alpha)m_1)(1 - \alpha)\right].$$

A deviation to $k = 0$ gives

$$\pi(0) = \beta\left[\alpha + \alpha m_0 + (n - \alpha m_0)\alpha\right] + (1 - \beta)\left[1 - \alpha + (1 - \alpha)m_0 + (n - (1 - \alpha)m_0)(1 - \alpha)\right].$$

Therefore setting $k = 1$ is optimal whenever inequality (9) is reversed, i.e. when the level effect is strong.

Now consider the case when the number of leaders is moderate or high, i.e. $\beta \leq \overline{\beta}_1(r, \alpha)$, so that purchase probabilities are $\gamma_\omega = 1$, $\delta_\omega = 0$. Followers who remain unconnected never buy, whereas followers who meet delayed leaders (off the equilibrium path) again always buy.

Profits from setting $k = 1$ are then

$$\pi(1) = \beta \left[\alpha + \alpha m_1\right] + (1 - \beta)(1 - \alpha),$$

whereas deviating to $k = 0$ gives

$$\pi(0) = \beta \left[\alpha + \alpha m_0\right] + (1 - \beta) \left[1 - \alpha + (1 - \alpha) m_0\right].$$

It follows that the deviation is unprofitable for

$$\beta \geq \frac{(1 - \alpha) m_0}{(1 - \alpha) m_0 + \alpha(m_1 - m_0)} = \tilde{\beta}(\alpha),$$

i.e., when the level effect is substantial.

These results can be gathered and restated as the following Lemma.

Lemma 5. *An equilibrium with $k = 1$ exists if and only if*

1. *a market has a strong level effect*

2. *a niche market has at least a moderate number of leaders and a substantial level effect.*

Lemma 5 shows that a pure strategy equilibrium in which the seller immediately serves all leaders will sometimes exist, unlike in the baseline. Specifically, this equilibrium will exist if the level effect is strong enough. A deviation to $k = 1$ no longer increases follower valuations in the sense of first-order stochastic dominance, but instead results in either a mean-increasing or mean-reducing contraction of follower valuations.

When immediately-served leaders talk more than leaders who are delayed, setting a high service speed allows the seller to maximize the number of followers who learn via word of mouth, and these followers will all buy if the state turns out to be good. The less delayed leaders talk, i.e. the lower the value of m_0, the larger is the parameter region for which $k = 1$ constitutes an equilibrium. Moreover, an equilibrium with $k = 1$ is easier to sustain when there are at least a moderate number of leaders: $\beta < \overline{\beta}_1(r, \alpha)$, so that unconnected followers never buy. Intuitively, the seller is less tempted to deviate to $k = 0$ when unconnected followers refuse to buy, because delay then means that more followers end up unconnected.

We now turn to mixed strategy equilibria. From Lemma 1, in any such equilibrium, the seller must randomize between $k = 1$ and $k = 0$. We start by looking for an equilibrium where a follower who meets a delayed leader and gets $s = b$ randomizes between buying and not buying. From Lemma 2, such an equilibrium requires $\beta < r$ and $q^*(N) \to 1$ as $N \to \infty$. That is, in large markets, the seller must set $q^* = 1$.

The seller's indifference condition can be written as

$$\begin{aligned}\pi(0) \equiv &\beta[\alpha + \alpha m_0 \gamma_G + (n - \alpha m_0)\delta_G] + (1-\beta)[1 - \alpha + (1-\alpha)m_0\gamma_B + (n - (1-\alpha)m_0)\delta_B] \\ &= \beta[\alpha + \alpha m_1 + (n - \alpha m_1)\delta_G] + (1-\beta)[1 - \alpha + (n - (1-\alpha)m_1\delta_B] = \pi(1).\end{aligned}$$

Suppose that followers who meet a delayed leader but get $s = b$ buy with probability $\chi \in [0, 1]$. This implies $\gamma_G = \alpha + (1-\alpha)\chi$ and $\gamma_B = (1-\alpha) + \alpha\chi$. Since in this candidate equilibrium the seller sets $k = 1$ almost surely, for unconnected followers we obtain

$$\delta_G = \begin{cases} 0, & \beta < \overline{\beta}_1(r, \alpha) \\ \alpha, & \beta \geq \overline{\beta}_1(r, \alpha) \end{cases}, \quad \delta_B = \begin{cases} 0, & \beta < \overline{\beta}_1(r, \alpha) \\ 1-\alpha, & \beta \geq \overline{\beta}_1(r, \alpha) \end{cases}$$

Recall that $\overline{\beta}_1(r, \alpha) < r$. If $\beta \in [\overline{\beta}_1(r, \alpha), r]$, i.e. in a niche market with a low number of leaders, the seller's indifference condition reduces to

$$\begin{aligned}\pi(0, \chi) = &\beta[\alpha + \alpha m_0(\alpha + (1-\alpha)\chi) + (n - \alpha m_0)\alpha] + \\ &(1-\beta)[1 - \alpha + (1-\alpha)m_0(1 - \alpha + \alpha\chi) + (n - (1-\alpha)m_0)(1-\alpha)] \\ = &\beta[\alpha + \alpha m_1 + (n - \alpha m_1)\alpha] + (1-\beta)[1 - \alpha + (n - (1-\alpha)m_1)(1-\alpha) \\ = &\pi(1).\end{aligned}$$

Profits $\pi(0, \chi)$ are increasing in χ, so there will exist a value of $\chi \in (0, 1)$ that makes the seller indifferent, $\pi(0, \chi) = \pi(1)$ if $\pi(0, \chi = 0) < \pi(1) < \pi(0, \chi = 1)$. It is straightforward to verify that this condition is equivalent to

$$1 - \alpha < \beta < \hat{\beta}(\alpha),$$

i.e. private signals are strong, but the level effect is not.

If instead $\beta < \overline{\beta}_1(r, \alpha)$, so there are at least a moderate number of leaders, the seller's indifference conditions reduces to

$$\begin{aligned}\pi(0, \chi) =& \beta[\alpha + \alpha m_0(\alpha + (1-\alpha)\chi)] + (1-\beta)[1-\alpha + (1-\alpha)m_0(1-\alpha+\alpha\chi)] \\ =& \beta(\alpha + \alpha m_1) + (1-\beta)(1-\alpha) \\ =& \pi(1)\end{aligned}$$

As above, there exists a value of $\chi \in (0, 1)$ that makes the seller indifferent if $\pi(0, \chi = 0) < \pi(1) < \pi(0, \chi = 1)$. This condition is now equivalent to

$$(1-\alpha)\tilde{\beta}(\alpha) < \beta < \tilde{\beta}(\alpha),$$

i.e. the level effect is modest. We can summarize these results in the following lemma, which we structure in a slightly different way to ease the comparison with our baseline results.

Lemma 6. *A mixed strategy equilibrium in which the seller almost surely sets $k = 1$ and immediately serves all leaders, and where followers with bad signals who meet delayed leaders randomize, exists if and only if*

1. *a niche market has strong private signals and either (i) a low number of leaders and a not-strong level effect, or (ii) at least a moderate number of leaders and a modest level effect*
2. *a niche market has weak private signals, at least a moderate number of leaders and a modest level effect*

Case 1 of Lemma 6 corresponds to case 1 of Proposition 1 from the baseline, but with extra conditions : if the number of leaders is low then the level effect must not be strong, and if the number of leaders is not low then the level effect must be modest. If these conditions are violated, then the seller will instead play a pure strategy with $k = 1$, due to the level effect (Lemma 5). Case 2 deals with a parameter region where the seller played a pure strategy with $k = 0$ in the baseline. Now, the level effect pushes the seller to set $k = 1$ almost surely. Note, that cases 1.ii and 2 of Lemma 6 imply that the seller sets $k = 1$ in a niche markets with at least a moderate number of leaders and a modest level effect irrespective of the strength of private signals.

We now look for a mixed strategy equilibrium where a follower who remains unconnected and receives a good private signal randomizes.[23] An unconnected follower will be indifferent if $\overline{\beta}_q(r, \alpha) = \beta$, by Lemma 3. Since $\overline{\beta}_q(r, \alpha)$ is increasing in q, the necessary condition for the equilibrium to exist is $\overline{\beta}_0(r, \alpha) \leq \beta \leq \overline{\beta}_1(r, \alpha)$, i.e. there must be a moderate number of leaders. This means that an unconnected follower with a bad signal would buy if they think all leaders were delayed, but would not buy if they think all leaders were immediately served. Moreover, for any interior $q \in (0, 1)$, a follower who meets a delayed leader and receives a bad signal retains their prior beliefs, by Lemma 2.

Suppose that an unconnected follower with $s = g$ buys with probability ξ. Then seller profits from setting $k = 0$ are

$$\pi(0, \xi) = \beta[\alpha + \alpha^2 m_0 + (n - m_0\alpha)\alpha\xi] + (1-\beta)[1 - \alpha + (1-\alpha)^2 m_0 + (n - m_0(1-\alpha))(1-\alpha)\xi],$$

whereas profits from setting $k = 1$ are

$$\pi(1, \xi) = \beta[\alpha + \alpha m_1 + (n - m_1\alpha)\alpha\xi] + (1-\beta)[1 - \alpha + (n - m_1(1-\alpha))(1-\alpha)\xi].$$

Since $\frac{\partial\pi(0,\xi)}{\partial\xi} > \frac{\partial\pi(1,\xi)}{\partial\xi} > 0$, by $m_0 < m_1$, there exists a unique ξ that satisfies the seller indifference condition, $\pi(0, \xi) = \pi(1, \xi)$, whenever $\pi(0, \xi = 0) < \pi(1, \xi = 0)$ and $\pi(0, \xi = 1) > \pi(1, \xi = 1)$. The first inequality is equivalent to $\beta > (1-\alpha)\tilde{\beta}(\alpha)$ and the second is equivalent to $\beta < 1 - \alpha$, which implies the following.

Lemma 7. *A mixed strategy equilibrium in which the seller randomizes between $k = 0$ and $k = 1$ with $q \in (0, 1)$, where unconnected followers with good signals randomize, exists if and only if the market is niche with a moderate number of leaders, the level effect is at least modest, and private signals are weak.*

The mixed strategy equilibrium from Lemma 7, when it exists, will not be unique, by Lemmata 4 - 6. That is, in any niche market with weak private signals and a moderate number of leaders, there exists a pure strategy equilibrium with $k = 0$. There also exists either a pure strategy equilibrium with $k = 1$ if the level effect is substantial, or a mixed strategy equilibrium where the seller sets $k = 1$ almost surely if the level effect is modest.[24]

[23] A follower who remains unconnected and receives a bad private signal will always have a strict incentive not to buy, because their posterior is less than the value of their outside option, $P(G|s = b) < r$.

[24] Notice, that if the mixed strategy equilibrium from Lemma 7 does not exist, then it follows from Lemmata 4 - 6 that the equilibrium which does exist is unique.

Having derived all pure and mixed strategy equilibria for the general case, we now combine Lemmata 4 - 7 to formulate an overall result.

Proposition 2. *1. In any market with a strong level effect, or in niche markets with a substantial level effect and at least a moderate number of leaders, there is a pure strategy equilibrium, in which the seller sets $k = 1$.*

2. *In niche markets with either (i) a not-strong level effect, strong private signals and a low number of leaders; or (ii) a modest level effect and at least a moderate number of leaders, there is a mixed strategy equilibrium in which the seller sets $k = 1$ almost surely.*

3. *In niche markets with an at-least modest level effect, weak private signals, and a moderate number of leaders, there is a mixed strategy equilibrium in which the seller sets $k = 1$ with probability $q \in (0, 1)$.*

4. *In mass markets with a not-strong level effect, or in niche markets with either (i) weak private signals and at most a moderate number of leaders, or (ii) a weak level effect and a high number of leaders, there is a pure strategy equilibrium in which the seller sets $k = 0$.*

We now return to the question of whether the delay is more attractive in niche or mass markets, as well as how the number of leaders in the market, and the amount that leaders talk, affects the seller's incentive to delay. We take each of these issues in turn.

First, Proposition 2 shows that in a mass market, an equilibrium with $k = 0$ exists as long as the level effect is not strong. In contrast, this equilibrium only exists under stricter conditions in niche markets, e.g., weak private signals (which is also implied by a weak level effect). This feature of Proposition 2 broadly echoes our result from the baseline case, where the seller always used delay in mass markets, but only did so in niche markets with weak signals.

Second, consider a situation with very few leaders, or equivalently, one with very many followers relative to leaders ($n \to \infty$). Then unconnected followers all act on their private signals, for all values of the prior and signal precision that satisfy the participation constraint, regardless of their beliefs about the seller's chosen service speed. That is,

$$\lim_{n\to\infty} \overline{\beta}_0(r, \alpha) = \lim_{n\to\infty} \overline{\beta}_1(r, \alpha) = \frac{(1-\alpha)r}{(1-\alpha)r + \alpha(1-r)},$$

where it then follows directly that $\beta \geq \lim_{n\to\infty} \overline{\beta}_0(r, \alpha)$ and $\beta \geq \lim_{n\to\infty} \overline{\beta}_1(r, \alpha)$ are equivalent to the participation constraint, $P(g|G) = \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\beta)} \geq r$.

Then cases 1, 2 and 4 of Proposition 2 are relevant, and we can reformulate this result in terms of the following corollary:

Corollary 1. *Suppose there are very few leaders relative to followers, $n \to \infty$, so all followers act on their private signals. Then we have the following:*

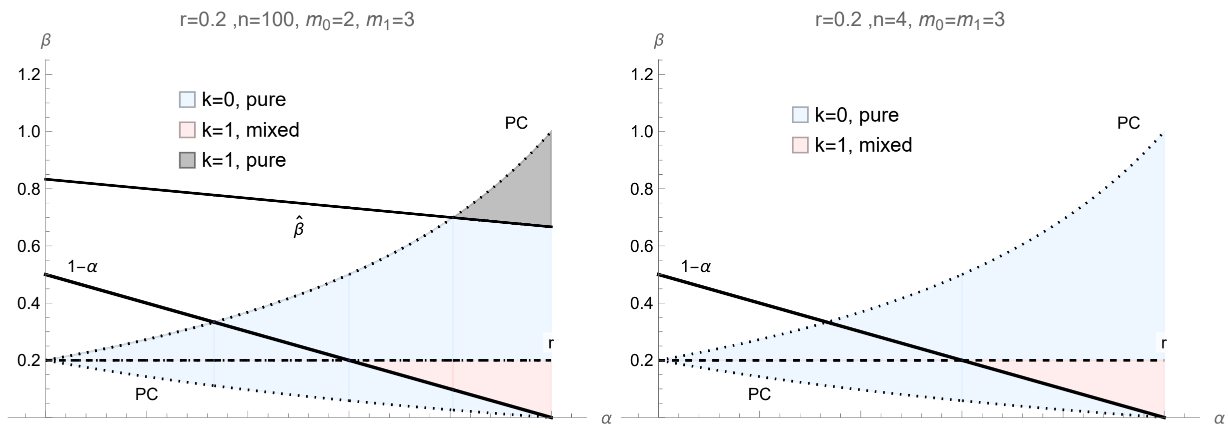
1. *If the level effect is strong, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 1$ immediately serves all leaders.*
2. *If the market is niche, private signals are strong, but the level effect is not strong, then a unique equilibrium exists and is in mixed strategies, where the seller almost surely sets $k = 1$ and immediately serves all leaders.*
3. *If the market is mass and the level effect is not strong, or if private signals are weak, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 0$ and delays all leaders.*

Corollary 1 shows that in a niche market with very few leaders, the seller will set $k = 0$ whenever private signals are weak, $\beta < 1 - \alpha$. In contrast, Proposition 2 shows the seller will only set $k = 0$ in a niche market with many leaders if an additional condition holds, i.e. the level effect must be weak as well. This illustrates how the presence of many leaders in the market reduces the seller's incentive to delay.

Notice that Corollary 1 is very similar to Proposition 1 from the baseline. For ease of comparison, Figure 2 depicts both scenarios. The only difference is that a pure strategy equilibrium with $k = 1$ now exists whenever $\beta \geq \hat{\beta}(\alpha)$, i.e. whenever the level effect is strong. The smaller the value of m_0, the less delayed leaders talk, and hence the less tempted the seller is to delay them. As such, the parameter region with the pure strategy equilibrium $k = 1$ grows larger as m_0 decreases, and the level effect has a larger impact. In situations where all unconnected followers act according to their private signals, this is the only way that seller behavior in the general case differs from that in the baseline.

Finally, consider a situation where delayed leaders never talk, $m_0 = 0$. The cutoff values are then $\tilde{\beta}(\alpha) = 0$; $\overline{\beta}_0(r, \alpha) = \frac{(1-\alpha)r}{(1-\alpha)r+\alpha(1-r)}$, which is equivalent to the participation constraint; and $\hat{\beta}(\alpha) = 1 - \alpha$. Therefore, the relevant considerations are whether private signals are strong or weak (implying a strong or substantial level effect respectively), and whether the number of leaders is low or moderate.

Hence, only cases 1, 3, and 4 are relevant in Proposition 2, which gives this corollary:



(a) Large n, Corollary 1 (b) Baseline Case

Figure 2: Level effect for large n

Corollary 2. *Suppose that delayed leaders do not engage in word of mouth, $m_0 = 0$. Then we have the following:*

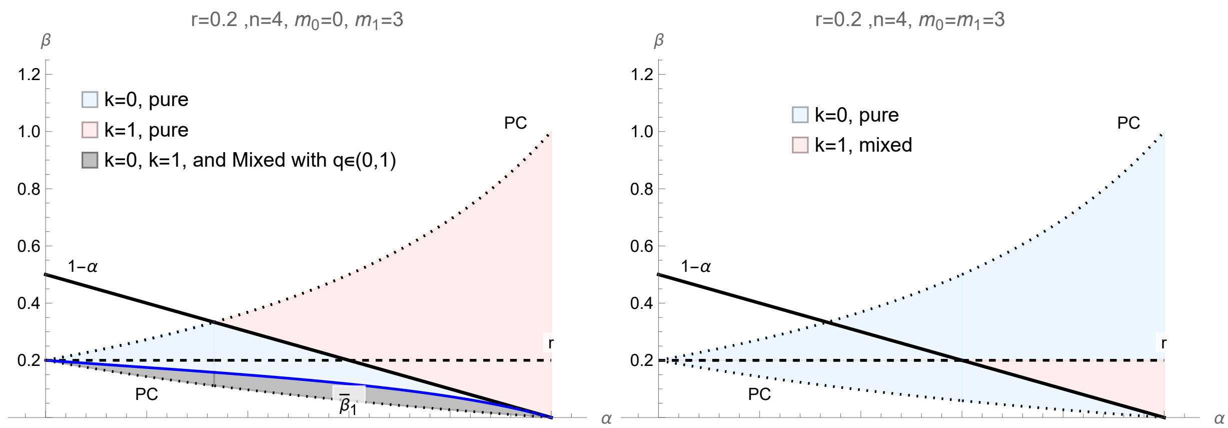
1. *If private signals are strong, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 1$ immediately serves all leaders.*

2. *If private signals are weak and the number of leaders is low, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 0$ and delays all leaders.*

3. *If private signals are weak and the number of leaders is at least moderate, then the following equilibria exist: a pure strategy equilibrium with $k = 0$, a pure strategy equilibrium with $k = 1$, and a mixed strategy equilibrium in which $q \in (0, 1)$.*

Setting $m_0 = 0$ maximizes the strength of the effect that described after Corollary 1. As delayed leaders talk less and less, the region where $k = 0$ constitutes an equilibrium becomes smaller. In particular, when $m_0 = 0$, the seller never sets $k = 0$ when private signals are strong, $\beta > 1 - \alpha$, as shown in Figure 3. The takeaway is that delay tends to be more (less) attractive to the seller when delayed leaders communicate to a large (small) extent.

As m_0 drops, unconnected followers also become less and less pessimistic about the state, but only if they believe the seller has engaged in delay. Thus, unlike in the baseline, there is a region where the behavior of unconnected follower depends on their beliefs about the seller's chosen service speed. This is what occurs when the number of leaders is moderate, $\overline{\beta}_0(r, \alpha) < \beta < \overline{\beta}_1(r, \alpha)$. The function $\overline{\beta}_0(r, \alpha)$ is decreasing in m_0, and coincides



(a) Delayed leaders do not talk, m_0

(b) Baseline Case

Figure 3: Level effect when delayed leaders do not talk

with the participation constraint when $m_0 = 0$, so that the relevant constraint becomes $\beta < \overline{\beta}_1(r, \alpha)$, the blue line in the Figure.

Equilibrium multiplicity can arise because the unconnected followers' optimal behavior depends on their beliefs about the service speed in a way that can be self reinforcing. These followers will all refuse to buy if they expect a high service speed, which pushes the seller to increase service speed to minimize the number of followers who are unconnected. Similarly, these followers will follow their signal if they expect a low service speed, which makes setting a low service speed more attractive.

4 Word of mouth and product awareness

How would seller strategic behavior change if word-of-mouth communication was not just necessary to transmit information about product quality, but also to learn about product existence? We explore this issue by assuming now that followers are simply unable to buy if they remain unconnected, $\delta_G = \delta_B = 0$. This assumption could be reasonable for situations where the seller launches a product that leaders are aware of, e.g., by virtue of their high level of interest in new products in the market, but where followers only become aware if they hear from leaders. Many of the works surveyed in the Introduction assume that word of mouth plays precisely this role of spreading product awareness. The question is whether the seller will still sometimes delay leaders in equilibrium, even though delay implies more followers will end up unconnected.

In what follows, we assume followers who meet delayed leaders still receive a private signal with accuracy α, so the purchase probabilities γ_G and γ_B coincide with those in the general analysis. The interpretation is that accessing noisy information about product quality may in principle be straightforward, but only for consumers who first become aware of the product.[25]

We start our analysis with the case of mass markets, i.e. $\beta \geq r$, so followers who meet delayed leaders always buy, regardless of the service speed they expect. Profits from setting $k = 0$ are

$$\pi(0) = \beta\left[\alpha(m_0 + 1)\right] + (1 - \beta)(1 - \alpha)(m_0 + 1),$$

whereas profits from setting $k = 1$ are

$$\pi(1) = \beta\left[\alpha(m_1 + 1)\right] + (1 - \beta)(1 - \alpha).$$

Thus, we have $\pi(0) \geq \pi(1)$ if and only if

$$\beta \leq \frac{m_0(1 - \alpha)}{m_0(1 - \alpha) + (m_1 - m_0)\alpha} = \tilde{\beta}(\alpha).$$

Now consider niche markets, $\beta < r$. Since unconnected followers do not buy, regardless of the candidate equilibrium in question, we can use our earlier results from Proposition 2, involving a high number of leaders: $\beta < \beta_0(r, \alpha)$, so unconnected followers never found it optimal to buy. Taken together, we have the following.

Proposition 3. *Suppose that unconnected followers are unable to buy. Then we have the following:*

1. *If the level effect is substantial, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 1$ immediately serves all leaders.*

2. *If the market is niche and the level effect is modest, then a unique equilibrium exists and is in mixed strategies, then the seller almost surely sets $k = 1$ and immediately serves all leaders.*

3. *If the market is mass and the level effect is not substantial, or if the market is niche and the level effect is weak, then a unique equilibrium exists and is in pure strategies, where the seller sets $k = 0$ delays all leaders.*

[25] At the end of the section we discuss how the absence of any follower private signals would affect the results.

For the interpretation of Proposition 3, we start by focusing on two special cases. If $m_0 = m_1$, so immediately-served and delayed leaders talk to the same number of followers, then the level effect plays no role, and seller behavior is just like in the baseline. That is, $\tilde{\beta}(\alpha) = 1$, so the level effect is never substantial, and a modest (respectively, weak) level effect is equivalent to strong (respectively, weak) private signals. Case 1 of Proposition 3 then does not apply, and cases 2 and 3 are equivalent to Proposition 1. The total number of unconnected followers is independent of service speed when $m_0 = m_1$, and so the seller's strategic behavior does not depend on whether unconnected followers are able to buy.

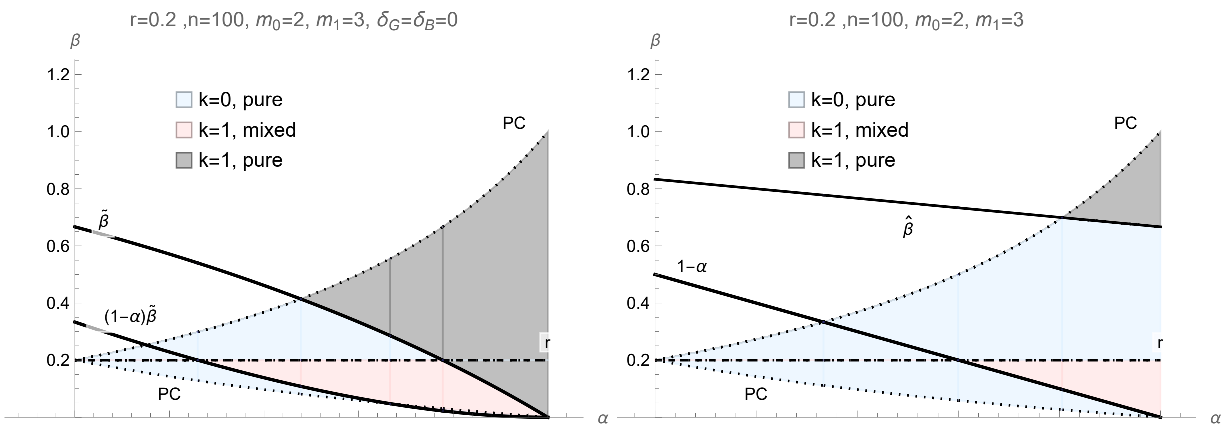
If instead $m_0 = 0$, so delayed leaders don't talk to any followers, then the level effect is always substantial, and the seller will never engage in delay. That is, $\tilde{\beta}(\alpha) = 0$ and so the seller always sets $k = 1$. Clearly, it does not pay off to delay leaders if they never talk to followers, who therefore never buy.

For intermediate values of $m_0 < m_1$, the seller may still delay leaders in equilibrium, but for a smaller set of parameter values than in the general case. To see this, consider the case of mass markets, $\beta \geq r$. An equilibrium with $k = 0$ will exist according to Proposition 3 when the level effect is not substantial, whereas it will exist according to Proposition 2 as long as the level effect is not strong (which is a weaker condition). Intuitively, assuming that unconnected followers cannot buy reduces profits to the greatest extent when the seller delays leaders, since then more followers end up unconnected. This makes it less attractive for the seller to use delay.

These ideas are illustrated in Figure 4. Panel (a) depicts the seller's equilibrium behavior for particular parameter values, assuming that unconnected followers cannot buy. For the sake of comparison, Panel (b) depicts seller behavior from Corollary 1 in the general analysis, where unconnected followers always act on their private signals, and where setting $k = 0$ is therefore more attractive.

Finally, we explore what would happen if the only way for followers to learn about the product was through talking to leaders, and if followers did not receive any private signals. Followers would then behave the same way in niche markets as in mass markets. In particular, all followers who meet delayed leaders would buy, since meeting a delayed leader is just as convincing as receiving a good signal.

It follows that Case 2 of Proposition 3, where the seller set $k = 1$ almost surely, would no longer apply. In that case, followers who met a delayed leader sometimes followed their private signals. Now, in the absence of private signals, these followers would be



(a) Unconnected do not buy

(b) Unconnected may buy, Large n

Figure 4: Unconnected followers cannot buy

convinced by word of mouth and buy. This makes setting $k = 0$ relatively more attractive, and the red region in panel (a) of Figure 4 would become blue. Even though the behavior of connected followers is independent of parameter values in the absence of private signals, the expected number of connected followers willing to buy is not, and the seller maximizes this number by setting $k = 1$ whenever $\beta > \tilde{\beta}(\alpha)$ and by setting $k = 0$ otherwise.

This implies that the comparison with Corollary 1 is not as clear-cut as before. If quality is ex ante relatively high, corresponding to a substantial level effect, then the inability of unconnected followers to buy would push the seller towards setting $k = 1$. However, the opposite is true if quality is ex ante relatively low, corresponding to a modest level effect, where setting $k = 0$ would now become more attractive.

5 Conclusions

In this paper, we explored how a seller can strategically use product delay to influence consumer learning about the quality of its product through word of mouth. The effect of service speed on learning can be decomposed into the content effect and the level effect.

The content effect influences the behavior of consumers who encounter either served or delayed leaders. In mass markets, meeting a delayed leader is sufficiently good news for these followers to buy. The seller then prefers to conceal information about quality by delaying, rather than revealing quality to leaders who are served. In niche markets, meeting a delayed leader may not significantly impact followers' behavior. In this case the seller prefers to hide information by delaying leaders if signals are inaccurate, and otherwise to serve all leaders almost surely. Broadly, if the seller mainly cares about influencing those followers who encounter leaders, and if the total amount of word of mouth depends little on how many leaders are immediately served, then it is often optimal for the seller to delay.

The level effect plays a significant role in determining how many consumers receive information from leaders, and it also affects the behavior of consumers who do not hear from leaders at all. This effect arises in situations where immediately-served leaders engage in more word-of-mouth communication than leaders who are delayed. Service delay then reduces the likelihood that a consumer will hear from a leader, prompting the seller to increase service speed when product quality is high. The level effect may also push the seller to set a high service speed in markets with a large number of leaders, because consumers who expect this service speed will perceive not hearing from leaders as particularly negative news. Moreover, if word-of-mouth communication is crucial for spreading product awareness, then the level effect is amplified. Nonetheless, the seller may still use delay to influence consumer word-of-mouth communication.

In this paper, we made an assumption that the minimal service speed equals to 0, so it is possible for the seller to delay all leaders. Alternatively, we could assume that the seller must serve a minimum number $\underline{K}$ of leaders when launching a new product. For completeness, we note that if $\lim_{N\to\infty} \underline{K}/N = 0$, then all our results will immediately go through. Otherwise, parts 1 and 3 of Lemma 2 still apply, but in part 2 the limit belief, which we can denote as $\overline{\mu}$, may be smaller than β. In this case, the relevant purchase probabilities from (7) depend on $\overline{\mu} < r$ rather than $\beta < r$, and the rest of the analysis follows the same steps as in the paper.

Finally, we note that explicitly introducing discounting into our model would mechanically reduce the seller's payoff when setting $k = 0$ to a larger extent than when $k = 1$, but would not affect the mechanism driving delay. That is, applying a discount factor $\delta < 1$ to the sales of delayed leaders and followers, in contrast to immediately-served leaders, would increase the seller's incentive to serve leaders immediately. However, the informational motive for delay which we focus on would still be present, and our qualitative results would be unchanged.

6 Appendix: Proofs

Proof of Lemma 1. Let us again denote the number of leaders served in state ω as $S_\omega(K)$, and the number of leaders delayed as $D_\omega(K)$, where

$$S_\omega(K) = \sum_{j=0}^{N} \min\{j, K\} Q_\omega(j), \quad D_\omega(K) = \sum_{j=0}^{N} \max\{j - K, 0\} Q_\omega(j).$$

For notational simplicity we denote $S_\omega(-1) = 0$ and $D_\omega(-1) = D_\omega(0) = \sum_{j=0}^{N} jQ_\omega(j)$, i.e. when the seller sets $K = 0$, a downward tremble is not possible and all consumers are delayed.

Let γ_ω be the probability that a consumer who met a delayed leader buys the product, and let δ_ω be the probability that a consumer who did not meet anyone buys the product. Profits from implementing service speed K are then

$$\begin{aligned}
\pi(K) =& \frac{\beta}{N}\Bigg[\sum_{j=0}^{N} jQ_G(j) + (1-\varepsilon)\left(m_1 S_G(K) + m_0\gamma_G D_G(K) + \delta_G\left(nN - m_1 S_G(K) - m_0 D_G(K)\right)\right) \\
&+ \varepsilon\left(m_1 S_G(K-1) + m_0\gamma_G D_G(K-1) + \delta_G\left(nN - m_1 S_G(K-1) - m_0 D_G(K-1)\right)\right)\Bigg] \\
&+ \frac{1-\beta}{N}\Bigg[\sum_{j=0}^{N} jQ_B(j) + (1-\varepsilon)\left(m_0\gamma_B D_B(K) + \delta_B\left(nN - m_1 S_B(K) - m_0 D_B(K)\right)\right) \\
&+ \varepsilon\left(m_0\gamma_B D_B(K-1) + \delta_B\left(nN - m_1 S_B(K-1) - m_0 D_B(K-1)\right)\right)\Bigg].
\end{aligned}$$

We now show that the profit-maximizing service speed K cannot take on any value $1 \leq K \leq N-1$. Consider the difference $\pi(K+1) - \pi(K)$. Note that

$$S_\omega(K+1) - S_\omega(K) = \sum_{j=0}^{N} \left(\min\{j, K+1\} - \min\{j, K\}\right) Q_\omega(j) = \sum_{j=K+1}^{N} Q_\omega(j),$$

and

$$D_\omega(K+1) - D_\omega(K) = \sum_{j=0}^{N} (\max\{j-K-1,0\} - \max\{j-K,0\})\, Q_\omega(j) = -\sum_{j=K+1}^{N} Q_\omega(j).$$

which implies

$$\begin{aligned}\Delta(K) \equiv& \pi(K+1) - \pi(K) = \\ & \frac{\beta}{N}[m_1(1-\delta_G) - m_0(\gamma_G - \delta_G)]\left(\sum_{j=K+1}^{N} Q_G(j) + \varepsilon Q_G(K)\right) \\ & \frac{1-\beta}{N}[-m_1\delta_B - m_0(\gamma_B - \delta_B)]\left(\sum_{j=K+1}^{N} Q_B(j) + \varepsilon Q_B(K)\right),\end{aligned}$$

where the expression in the second set of square brackets is non-positive, by $m_1 \geq 0$. Thus, we have that

$$\operatorname{sign}\Delta(K) = \operatorname{sign}\left(\frac{\beta[m_1(1-\delta_G) - m_0(\gamma_G - \delta_G)]}{(1-\beta)([m_1\delta_B + m_0(\gamma_B - \delta_B)])}\frac{\sum_{j=K+1}^{N} Q_G(j) + \varepsilon Q_G(K)}{\sum_{j=K+1}^{N} Q_B(j) + \varepsilon Q_B(K)} - 1\right).$$

Note, that if $\Delta(K_0) > 0$ holds for some K_0 it must be the case that $m_1(1-\delta_G) - m_0(\gamma_G - \delta_G) > 0$. As such, $\Delta(K_0) > 0$ must also hold for all $K > K_0$, since $Q_G(K)/Q_B(K)$ is increasing in K. Therefore, $\pi(K)$ attains its maximum either at $K = 0$ or at $K = N$.

We now consider a change in service speed from $K \in [0, N)$ to $K' > K$ and show that it induces a demand rotation. The case of $K' < K$ follows essentially the same steps, and hence is omitted.

Follower willingness to pay is just equal to their ex post belief about product quality. A follower who meets an immediately-served leader and learns that the state is good holds belief $\mu = 1$. Similarly, a follower who meets an immediately-served leader and learns that the state is bad holds belief $\mu = 0$. Denote the belief of a follower who meets a delayed leaders and receives a good (bad) signal by μ^+ $(\mu^-) \in (0,1)$. Denote the belief of a follower who remains unconnected and receives a good (bad) signal by ν^+ $(\nu^-) \in (0,1)$.

For ease of exposition, we will assume $\nu^- < \nu^+ < \mu^- < \mu^+$. The proof goes through a parallel argument, regardless of the ordering of these beliefs, and in particular if $\nu^+ > \mu^-$.

Given service speed K, the expected number of followers with beliefs $\mu = 0$ is

$$(1-\beta)m_1 S_B(K) \equiv X_0(K).$$

The corresponding expected number with beliefs ν^- is

$$\beta(1-\alpha)[N - m_1 S_G(K) - m_0 D_G(K)] + (1-\beta)\alpha[N - m_1 S_B(K) - m_0 D_B(K)] \equiv X_1(K);$$

with beliefs ν^+ is

$$\beta\alpha[N - m_1 S_G(K) - m_0 D_G(K)] + (1-\beta)(1-\alpha)[N - m_1 S_B(K) - m_0 D_B(K)] \equiv X_2(K);$$

with beliefs μ^- is

$$\beta(1-\alpha)m_0 D_G(K) + (1-\beta)\alpha m_0 D_B(K) \equiv X_3(K);$$

with beliefs μ^+ is

$$\beta\alpha m_0 D_G(K) + (1-\beta)(1-\alpha)m_0 D_B(K) \equiv X_4(K);$$

and with beliefs $\mu = 1$ is

$$\beta m_1 S_G(K).$$

The distribution function $F_K(\mu)$ for follower valuations can therefore be written as follows:

$$F_K(\mu) = \begin{cases} X_0(K)/N, & \text{if } \mu \in [0, \nu^-) \\ \sum_{i=0}^{1} X_i(K)/N, & \text{if } \mu \in [\nu^-, \nu^+) \\ \sum_{i=0}^{2} X_i(K)/N, & \text{if } \mu \in [\nu^+, \mu^-) \\ \sum_{i=0}^{3} X_i(K)/N, & \text{if } \mu \in [\mu^-, \mu^+) \\ \sum_{i=0}^{4} X_i(K)/N, & \text{if } \mu \in [\mu^+, 1) \\ 1 & \text{if } \mu = 1 \end{cases} \tag{11}$$

where in particular, $\sum_{i=0}^{4} X_i(K)/N = 1 - \beta m_1 S_G(K)/N$.

Note that $K' > K$ implies $S_\omega(K') > S_\omega(K)$ and $D_\omega(K') < D_\omega(K)$, for $\omega \in \{G, B\}$. This in turn implies $X_0(K') > X_0(K)$ and $\sum_{i=0}^4 X_i(K') < \sum_{i=0}^4 X_i(K)$, by $X_0(K) = N(1-\beta)m_1 S_B(K)$ and $\sum_{i=0}^4 X_i(K) = N - \beta m_1 S_G(K)$. As such, using expression (11), we can conclude that the distribution function $F(K')$ crosses $F(K)$ at least once from above.

Let $\mu^* = \inf\{\mu : F_{K'}(\mu) < F_K(\mu)\}$, where $\mu^* \in [\nu^-, \mu^+]$. We know that $F_{K'}(\mu) \geq F_K(\mu)$ holds for all $\mu < \mu^*$. We also know that $F_{K'}(\mu) < F_K(\mu)$ holds for all $\mu \in [\mu^+, 1)$. It remains to show that $F_{K'}(\mu) \leq F_K(\mu)$ holds for all $\mu > \mu^*$. We proceed by contradiction.

Suppose that there exists some $\mu^{**} > \mu^*$ for which $F_{K'}(\mu^{**}) > F_K(\mu^{**})$. Notice that $\mu^{**} < \mu^+$, since $F_{K'}(\mu) < F_K(\mu)$ for $\mu \in [\mu^+, 1)$.

By expression (11), a necessary condition for $F_{K'}(\mu^*) < F_K(\mu^*)$ and $F_{K'}(\mu^{**}) > F_K(\mu^{**})$ is as follows: $X_j(K') > X_j(K)$ must hold for at least some $j \in \{1, 2, 3\}$.

However, from above, we have that $m_1 \geq m_0$ implies that $X_j(K') \leq X_j(K)$ for all $j \in \{1, 2, 3\}$, which generates a contradiction. We therefore conclude that $F_{K'}(\mu) \leq F_K(\mu)$ holds for all $\mu > \mu^*$, and hence increasing service speed from K to K' induces a demand rotation.

□

Proof of Lemma 2. Part 1. Note that

$$D_\omega(q) = (1-q)\sum_{j=0}^{N} jQ_\omega(j) + q\varepsilon Q_\omega(N),$$

$$\begin{aligned}\mu(1, N) &= \frac{\beta(1-\alpha)D_G(1)}{\beta(1-\alpha)D_G(1) + (1-\beta)\alpha D_B(1)} \\ &= \frac{\beta(1-\alpha)\varepsilon Q_G(N)}{\beta(1-\alpha)\varepsilon Q_G(N) + (1-\beta)\alpha\varepsilon Q_B(N)} = \frac{1}{1 + \frac{(1-\beta)(1-\alpha)}{\beta\alpha}\frac{Q_G(N)}{Q_B(N)}}.\end{aligned}$$

Now, due to $\lim_{N\to\infty} Q_G(N)/Q_B(N) = 0$ we get that $\lim_{N\to\infty} \mu(1, N) = 1$.

Part 2. For any fixed q, we get

$$\mu(q, N) = \frac{\beta(1-\alpha)\left((1-q)\sum_{j=0}^{N} jQ_G(j) + q\varepsilon Q_G(N)\right)}{\beta(1-\alpha)\left((1-q)\sum_{j=0}^{N} jQ_G(j) + q\varepsilon Q_G(N)\right) + (1-\beta)\alpha\left((1-q)\sum_{j=0}^{N} jQ_B(j) + q\varepsilon Q_B(N)\right)}.$$

Thus, for $q < 1$ we get

$$\lim_{\varepsilon \to 0} \mu(q, N) = \frac{\beta(1-\alpha)\sum_{j=0}^{N} jQ_G(j)}{\beta(1-\alpha)\sum_{j=0}^{N} jQ_G(j) + (1-\beta)\alpha\sum_{j=0}^{N} jQ_B(j)},$$

and is independent of q. Using $\sum_{j=0}^{N} jQ_G(j) = \alpha N$ and $\sum_{j=0}^{N} jQ_G(j) = (1-\alpha)N$, we get that

$$\lim_{\varepsilon \to 0} \mu(q, N) = \frac{\beta(1-\alpha)\alpha}{\beta(1-\alpha)\alpha + (1-\beta)\alpha(1-\alpha)} = \beta.$$

Part 3. We can rewrite the follower belief in the form

$$\mu(q, N) = \frac{\beta(1-\alpha)}{\beta(1-\alpha) + (1-\beta)\alpha\frac{(1-q)\sum_{j=0}^{N} jQ_B(j) + q\varepsilon Q_B(N)}{(1-q)\sum_{j=0}^{N} jQ_G(j) + q\varepsilon Q_G(N)}}.$$

Note that

$$\frac{\partial}{\partial q}\left(\frac{(1-q)\sum_{j=0}^{N} jQ_B(j) + q\epsilon Q_B(N)}{(1-q)\sum_{j=0}^{N} jQ_G(j) + q\epsilon Q_G(N)}\right) = \frac{Q_B(N)\sum_{j=0}^{N} jQ_G(j) - Q_G(N)\sum_{j=0}^{N} jQ_B(j)}{\left[(1-q)\sum_{j=0}^{N} jQ_G(j) + q\epsilon Q_G(N)\right]^2}\varepsilon < 0,$$

and therefore $\mu(q, N)$ is increasing in q. Moreover, $\mu(0, N) = \beta$ and $\lim_{N\to\infty} \mu(1, N) = 1$. Thus, for any r, there exists $\overline{N}(r)$ such that $\mu(1, N) > r$ holds for all $N > \overline{N}(r)$. As such, for any $r \in (\beta, 1)$, and any $N > \overline{N}(r)$, there exists a unique $q^*(N)$ such that $\mu(q^*(N), N) = r$.

Now we prove that $\lim_{N\to\infty} q^*(N) = 1$. To do so, we proceed by contradiction. Suppose that $q^*(N) < \overline{q}$ for all N. Then, using monotonicity of the belief in q, we get

$$\lim_{N\to\infty} \mu(q^*(N), N) < \lim_{N\to\infty} \mu(\overline{q}, N) = \beta < r$$

, which is not possible as $\mu(q, N)$ is continuous in q. □

References

Ajorlou, A., Jadbabaie, A., and Kakhbod, A. (2018). Dynamic pricing in social networks: The word-of-mouth effect. *Management Science*, 64(2):971–979.

Aoyagi, M. (2010). Optimal sales schemes against interdependent buyers. *American Economic Journal: Microeconomics*, 2(1):150–182.

Bala, V. and Goyal, S. (1998). Learning from neighbours. *The Review of Economic Studies*, 65(3):595–621.

Banerjee, A., Chandrasekhar, A. G., Duflo, E., and Jackson, M. O. (2013). The diffusion of microfinance. *Science*, 341(6144):1236498.

Banerjee, A. and Fudenberg, D. (2004). Word-of-mouth learning. *Games and Economic Behavior*, 46(1):1–22.

Bhalla, M. (2013). Waterfall versus sprinkler product launch strategy: Influencing the herd. *The Journal of Industrial Economics*, 61(1):138–165.

Bulow, J. and Roberts, J. (1989). The simple economics of optimal auctions. *Journal of Political Economy*, 97(5):1060–1090.

Campbell, A. (2013). Word-of-mouth communication and percolation in social networks. *American Economic Review*, 103(6):2466–2498.

Campbell, A. (2015). Word of mouth model of sales. *Economics Letters*, 133:45–50.

Campbell, A., Leister, C. M., and Zenou, Y. (2020). Word-of-mouth communication and search. *The RAND Journal of Economics*, 51(3):676–712.

Campbell, A., Mayzlin, D., and Shin, J. (2017). Managing buzz. *The RAND Journal of Economics*, 48(1):203–229.

Chevalier, J. A. and Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research*, 43(3):345–354.

Debo, L. G., Parlour, C., and Rajan, U. (2012). Signaling quality via queues. *Management Science*, 58(5):876–891.

DeGraba, P. (1995). Buying frenzies and seller-induced excess demand. *The RAND Journal of Economics*, pages 331–342.

Duan, W., Gu, B., and Whinston, A. B. (2008). The dynamics of online word-of-mouth and product sales—an empirical investigation of the movie industry. *Journal of Retailing*, 84(2):233–242.

Ellison, G. and Fudenberg, D. (1995). Word-of-mouth communication and social learning. *The Quarterly Journal of Economics*, 110(1):93–125.

Ferguson, D. G. (1994). Shortages, segmentation, and self-selection. *Canadian Journal of Economics*, pages 183–197.

Galeotti, A. (2010). Talking, searching, and pricing. *International Economic Review*, 51(4):1159–1174.

Galeotti, A., Golub, B., and Goyal, S. (2020). Targeting interventions in networks. *Econometrica*, 88(6):2445–2471.

Galeotti, A. and Goyal, S. (2009). Influencing the influencers: a theory of strategic diffusion. *The RAND Journal of Economics*, 40(3):509–532.

Godes, D. (2017). Product policy in markets with word-of-mouth communication. *Management Science*, 63(1):267–278.

Godes, D. and Mayzlin, D. (2004). Using online conversations to study word-of-mouth communication. *Marketing Science*, 23(4):545–560.

Hauser, D. N. (2023). Censorship and reputation. *American Economic Journal: Microeconomics*, 15(1):497–528.

He, S., Hollenbeck, B., and Proserpio, D. (2022). The market for fake reviews. *Marketing Science*, 41(5):896–921.

Ivanov, M. (2009). Niche market or mass market? *Economics letters*, 105(3):217–220.

Johnson, J. P. and Myatt, D. P. (2006). On the simple economics of advertising, marketing, and product design. *American Economic Review*, 96(3):756–784.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.

Liu, T. and Schiraldi, P. (2012). New product launch: herd seeking or herd preventing? *Economic Theory*, 51:627–648.

Liu, Y. (2006). Word of mouth for movies: Its dynamics and impact on box office revenue. *Journal of Marketing*, 70(3):74–89.

Loertscher, S. and Muir, E. V. (2022). Monopoly pricing, optimal randomization, and resale. *Journal of Political Economy*, 130(3):566–635.

Luca, M. and Zervas, G. (2016). Fake it till you make it: Reputation, competition, and yelp review fraud. *Management Science*, 62(12):3412–3427.

Mayzlin, D. (2006). Promotional chat on the internet. *Marketing science*, 25(2):155–163.

Mayzlin, D., Dover, Y., and Chevalier, J. (2014). Promotional reviews: An empirical investigation of online review manipulation. *American Economic Review*, 104(8):2421–2455.

Mohr, I. (2017). Managing buzz marketing in the digital age. *Journal of Marketing Development and Competitiveness*, 11(2):10–16.

Möller, M. and Watanabe, M. (2010). Advance purchase discounts versus clearance sales. *The Economic Journal*, 120(547):1125–1148.

Nam, S., Manchanda, P., and Chintagunta, P. K. (2010). The effect of signal quality and contiguous word of mouth on customer acquisition for a video-on-demand service. *Marketing Science*, 29(4):690–700.

Neelamegham, R. and Jain, D. (1999). Consumer choice process for experience goods: An econometric model and analysis. *Journal of marketing research*, 36(3):373–386.

Nocke, V. and Peitz, M. (2007). A theory of clearance sales. *The Economic Journal*, 117(522):964–990.

Parakhonyak, A. and Vikander, N. (2019). Optimal sales schemes for network goods. *Management Science*, 65(2):819–841.

Parakhonyak, A. and Vikander, N. (2023). Information design through scarcity and social learning. *Journal of Economic Theory*, 207:105586.

Sgroi, D. (2002). Optimizing information in the herd: Guinea pigs, profits, and welfare. *Games and Economic Behavior*, 39(1):137–166.

Smirnov, A. and Starkov, E. (2022). Bad news turned good: reversal under censorship. *American Economic Journal: Microeconomics*, 14(2):506–560.

Stock, A. and Balachander, S. (2005). The making of a "hot product": A signaling explanation of marketers' scarcity strategy. *Management science*, 51(8):1181–1192.

Wilson, C. A. (1988). On the optimal pricing policy of a monopolist. *Journal of Political Economy*, 96(1):164–176.

Xiong, G. and Bharadwaj, S. (2014). Prerelease buzz evolution patterns and new product performance. *Marketing Science*, 33(3):401–421.