The role of central bank digital currency in an increasingly digital economy*

Benjamin Hemingway †

February 2024

Abstract

Current indications from major central banks are that should they introduce a central bank digital currency (CBDC) it would be unremunerated. Motivated by the increasing trend in online sales in the UK, this paper provides a theoretical framework where an unremunerated CBDC leads to aggregate welfare gains. A cash-credit model with two sectors is developed where absent CBDC, physical retailers have a competitive advantage relative to online firms as their ability to accept cash as a means of payment lowers their banking fees, leading to inefficiently low entry of online firms. Introducing a CBDC removes this inefficiency and leads to higher welfare.

Keywords: Cashless, Credit, Digital Currency, Money

JEL codes: E41, E42, E58

---

*The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of England or its committees.

†Bank of England. benjamin.hemingway@bankofengland.co.uk

1 Introduction

The introduction of a retail central bank digital currency (CBDC) is currently being considered by the major central banks. Current proposals by the Bank of England and the ECB have suggested that should they decide to issue a CBDC, it would be unremunerated. This limits a lot of the financial stability concerns of issuing a CBDC, but also limits the potential benefits. An obvious question is if CBDC offers the same rate of return as cash, how does it differ from cash, and what is the motivation for introducing a CBDC?

One motivation for introducing a CBDC is to address the declining use of cash. Figure 1 shows that the number of transactions in the UK using cash has fallen significantly in the last 10 years. This decline in the number of cash transactions has been mirrored by the increase in debit card and, to a lesser extent, credit card transactions. As cash is currently the only form of central bank money directly used as a means of payment by households, there is concern that cash will no longer be able to play the role of monetary anchor should this trend towards a cashless society continue. A related trend is that an increasing number of retail transactions are taking place online, where cash cannot be used as a means of payment at all. Figure 2 shows the share of internet sales as a percentage of total retail sales in the UK between January 2008 and July 2023. The chart captures the rapid increase in the share of online transactions from under 5% to over 25%. Excluding a spike in the proportion of retail sales during the COVID-19 pandemic period, the series shows a steady upward trend. Despite these obvious concerns, it is not immediately clear whether either of these trends necessitates the issuance of a retail CBDC as a digital fiat money.

This paper builds a theoretical framework that is able to model the increase in online retail sales and suggests a mechanism through which the issuance of a CBDC would lead to aggregate welfare gains in many situations. In cases where CBDC does not improve aggregate welfare, the model finds that the introduction of a CBDC would lower consumer prices and increase consumer welfare at the cost of lower bank profitability.

The model developed in this paper is a cash-credit monetary model based on the framework developed by Lagos and Wright (2005) and Rocheteau and Wright (2005). Specifically, it builds on the recent paper by Lagos and Zhang (2022) who showed that the moneyless limit differs from a nonmonetary model, as money acts as a constraint on the market power of bankers by acting as an outside option. This paper adopts this mechanism but extends the model in two key ways. First, I assume that there are two distinct firm types, digital and physical, which differ in the means of payment they are able to accept. Money is usable only in physical, face-to-face transactions and not in digital transactions. Second, I allow for endogenous firm entry into these two sectors, which is made possible by introducing search frictions and solving for the competitive search equilibrium. Allowing for endogenous firm entry allows the model to study different possible channels for the increasing trend in online sales identified by Figure 2 and the different welfare implications they have for the introduction of a CBDC. For a detailed survey of the competitive search equilibrium used in this paper, see Wright et al. (2021).

In the absence of a CBDC, the model suggests that the entry of online retailers is inefficiently low. This stems from physical retailers having a competitive advantage: their ability to accept cash, which constrains the market power of banks and lowers the intermediation fees that physical retailers are required to pay to banks. As online retailers are unable to accept cash as a means of payment, they face larger intermediation fees from banks than physical retailers. Introducing a CBDC levels the playing field in the sense that online retailers have the same outside option as physical retailers, leading to efficient entry by both online retailers and physical retailers.

There is extensive literature that discusses the welfare implications of introducing a CBDC, although much of this focusses on a remunerated rather than an unremunerated CBDC. For example, Barrdear and Kumhof (2022) find that a countercyclical remuneration rate rule for CBDC can contribute to stabilising the business cycle, while Bordo (2021) suggests that a remunerated CBDC can strengthen the transmission of monetary policy. This paper emphasises a possible benefit of an unremunerated CBDC, which can be seen as complementary to papers that highlight the potential risks of introducing a CBDC such as Fernández-Villaverde et al. (2021) that consider the increased risk of bank runs, especially in cases where the CBDC is remunerated.

This paper is closest to recent work by Williamson (2022), Chiu et al. (2023) and Keister and Sanches (2023) who study the impact of a CBDC using money-search models that have distinct types of sellers that can accept different methods of payment. Williamson (2022) studies various implementations of CBDC and shows how an interest-bearing CBDC can increase welfare by competing with private means of payment. Keister and Sanches (2023) consider CBDC's substitutability with other means of payment as a design choice. Specifically, they distinguish between a cash-like CBDC that only competes with cash based payments, a deposit-like CBDC which competes with bank deposits, and a universal CBDC which competes with bank deposits and cash payments. A key modelling difference in this paper is that I introduce search frictions and allow for endogenous entry of the different seller types.

This is not the first paper to consider the impact of a declining trend in cash use. Jiang and Shao (2020) not only document the decline in cash usage, but also suggest an answer to the 'cash paradox', that is why cash usage has not declined as much as might be expected. A paper closer in spirit to this one is Chiu et al. (2023), who study the impact

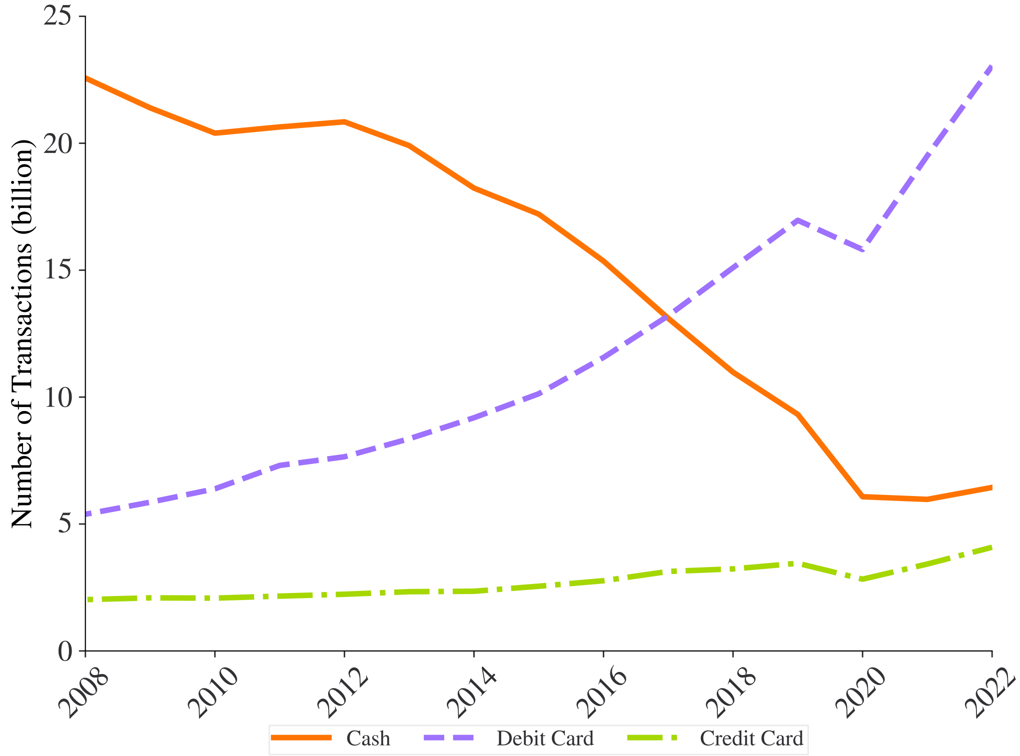

Figure 1: Total Number of UK Payments (in billions) by Method of Payment.
Source: UK Finance.

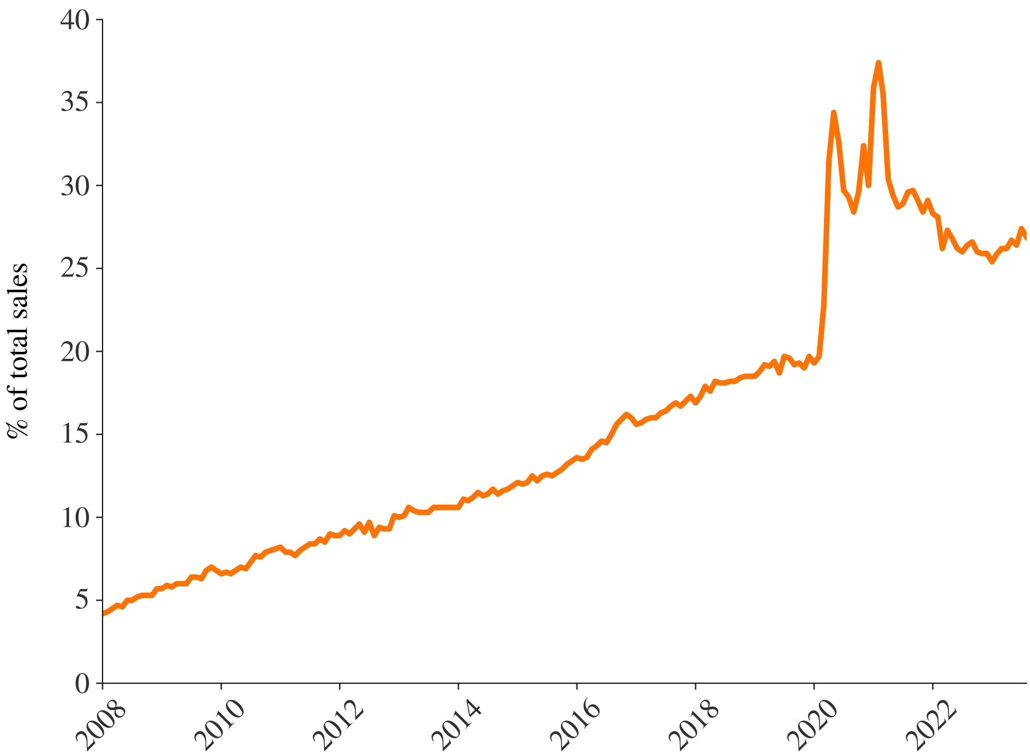

Figure 2: Value of Internet Sales as Percentage of Total Value of UK Retail Sales (seasonally adjusted).
Source: UK ONS, Retail Sales Index.

of an exogenous shift toward sellers that do not accept cash. As I allow for endogenous entry of firms, this paper is able to show that the welfare implications of introducing a CBDC may depend on the underlying factors driving the declining trend in cash usage.

The remainder of this paper is organised as follows. Section 2 presents the model. Section 3 describes the equilibrium and discusses the impact of introducing a CBDC. Section 4 calibrates the model to UK data and studies the effect of introducing a CBDC. Section 5 concludes.

2 The Model

In this section, I build a cash-credit model based on the framework of Lagos and Zhang (2022) with two innovations. First, the model features two distinct types of retailer, a physical retailer and a digital retailer. The retailers are distinguished in the means of payment they are able to accept. Physical retailers are able to accept cash as a means of payment, whereas digital retailers cannot. Second, consumers are subject to search frictions and choose between physical and digital retailers in a directed search setting along the lines of Rocheteau and Wright (2005).

2.1 Environment

Time is discrete, lasts forever, and is represented by a sequence indexed by $t \in \mathbb{T} \equiv \{0, 1, \ldots\}$. There are three types of agents, denoted by $i \in \{c, f, b\}$: a measure 1 of consumers, c, a measure 1 of bankers b, and firms, f. Firms are divided into two permanent types: physical retailers and digital retailers, denoted by p and d, respectively. There is a measure 1 of firms of type $j \in \{p, d\}$.

The discount factor from the current period to the next is $\beta \in (0, 1)$. Each period is divided into two stages: first, a decentralised frictional trading stage (DM) and second, a centralised frictionless settlement stage (CM). There are two non-storable goods: y in the DM and x in the CM.

At the beginning of period t there is a quantity M_t of money. Money is an intrinsically useless financial asset issued by the monetary authority. Money is perfectly divisible, and agents can hold any nonnegative amount. In the baseline model, money is physical and can only be used as a medium of exchange by physical retailers. In this sense, money can be thought of as being banknotes that cannot be used in transactions with online retailers. Later, I consider the possibility that the monetary authority issues digital cash, such as an unremunerated central bank digital currency (CBDC), that can be used by both digital retailers and physical retailers. The initial money stock, $M_0 \in \mathbb{R}_{++}$, is taken as given and distributed uniformly among consumers. The monetary authority is assumed to constantly adjust the money supply through lump-sum transfers or taxes to consumers in the CM stage of every period so that the law of motion of the money supply is $M_{t+1} = \mu M_t$ with $\mu \in \mathbb{R}_{++}$.

In the first stage, consumers obtain utility from consuming y of the DM good which can only be produced by firms. The utility consumers get from consumption in the DM is $u(y)$, with $u'(0) = \infty$, $u' > 0$, $u'' < 0$ and $u(0) = 0$. Firm j's marginal cost of producing y , which may depend on their type, is denoted by $\kappa_j > 0$.

In the second stage, agents of all types consume the CM good, x, and are able to supply labour, h, which can be used to produce good x through a linear production technology. All agents obtain utility $v(x)$ from consuming x of the CM good, with $v'(0) = \infty$, $v' > 0$, $v'' < 0$ and there exists $x^* > 0$ such that $v'(x^*) = 1$.

As is common in money-search models, money has a meaningful role as a medium of exchange because consumers are unable to commit in the DM and firms cannot enforce consumers' promises. Bankers are endowed with the ability to enforce and commit and thus are able to play the role of financial intermediaries between consumers and producers. Specifically, as in Lagos and Zhang (2022), consumers issue bonds through bankers in the first stage in order to purchase goods in the DM, where each bond is a claim to one unit of the CM good.

In the second stage, all agents can trade the CM good and money in a spot Walrasian market. In the first stage, there are two markets: a goods market where money and bonds are exchanged for the DM good and a bond market where money and bonds are traded.

The bond market in the first stage is organised as follows. All bankers have access to the bond market where they can trade bonds and money competitively. All consumers are able to access the bond market through a randomly assigned banker. Consumers are assumed to make a take-it-or-leave-it offer to the banker. Only a random subset of firms are able to access the bond market. With probability $\alpha \in [0, 1]$, a firm matches bilaterally with a banker. If a firm and a banker make contact, they negotiate the quantity of bonds and money that the banker sells on behalf of the firm in the competitive bond market, as well as an intermediation fee paid to the banker. The intermediation fee is expressed in terms of the CM good and paid in the second stage. The terms of trade between a firm and banker are determined by Nash bargaining, where the firm has bargaining power $\theta \in [0, 1]$.

The goods market in the first stage is organised according to the competitive search equilibrium, as described in Rocheteau and Wright (2005). There is a price posting mechanism where the terms of trade are publicly announced. As in Moen (1997), this can be interpreted as arising from competition between market makers. Trade takes place bilaterally between firms and consumers, and each submarket is subject to trading frictions that occur due to a search externality. Formally, denote the ratio of firms to consumers in submarket s as $n_s = n_{f,s}/n_{c,s}$. where $n_{f,s}$ is the mass of firms that enter submarket s and $n_{c,s}$ is the mass of consumers. Consumers can choose to enter any submarket they choose at no cost, while firms must pay an entry cost $\eta \geq 0$ to enter a submarket.

The probability that a consumer who searches in submarket s matches with a firm is $\delta(n_s)$. As trade occurs bilaterally, the probability that a firm matches with a consumer is $\delta(n_s)/n_s$. It is assumed that $\delta'(n) > 0$, $\delta''(n) < 0$, $\delta(n) \leq \min\{1, n\}$, $\delta(0) = 0$, $\delta'(0) = 1$, and $\delta(\infty) = 1$. The market maker posts the terms of trade before the firm knows if it has access to banking services. Each submarket s consists of the following $\omega_s = (y_s, p_s, n_s)$. Where n_s is the ratio of firms to consumers in the submarket and y_s and p_s are the quantity and nominal price traded conditional on the ability of the firm and consumer to trade. In the physical sector, matched consumers and firms are always able to trade. Physical retailers with access to the credit market are able to trade using credit, while unbanked physical retailers trade using money. In the baseline model, digital retailers are unable to trade using money, and thus matched consumers and firms in the digital sector are only able to trade if the firms obtain access to credit.

The instantaneous utility of a consumer at date t is

$$U_{c,t} = u(y_t) + v(x_t) - h_t \tag{1}$$

and a consumer's lifetime utility is $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{c,t}$.

Similarly, the instantaneous utility of a type $j \in \{p, d\}$ firm at date t is

$$U_{f,j,t} = -\kappa_j y_t + v(x_t) - h_t \tag{2}$$

and the firm's lifetime utility is $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{f,j,t}$.

Finally, the instantaneous utility of a banker at date t is

$$U_{b,j,t} = v(x_t) - h_t \tag{3}$$

and the banker's lifetime utility is $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{b,t}$.

2.2 Efficient Allocation

A useful benchmark to consider is the efficient allocation. Consider a social planner that chooses n_j the ratio of firm type $j \in \{p, d\}$ to consumers in each submarket, as well as an allocation $\Omega = \left\{ (y_{j,t})_{j \in \{p,d\}}, (x_{i,t}, h_{i,t})_{i \in \{c,f,b\}} \right\}_{t=0}^{\infty}$. The planner maximises the equally weighted utility of all agents. Thus, let

$$\mathcal{W}_t = \sum_{j \in \{p,d\}} \left(\frac{n_{f,j}}{n_j} \delta(n_j) \left[u(y_{c,j,t}) - \kappa_j y_{f,j,t} \right] - \eta n_{f,j} \right) + \sum_{i \in \{c,f,b\}} \left(v(x_{i,t}) - h_{i,t} \right). \quad (4)$$

where $n_{f,j}$ is the mass of firms of type j that attempt to match with consumers, and $n_j = n_{f,j}/n_{c,j}$ is the ratio of firms of type j to the number of consumers who want to purchase from firms of type j. The planner aims to maximise $\sum_{t=0}^{\infty} \beta^t \mathcal{W}_t$ subject to several feasibility constraints. First, the allocation in the DM goods market must be feasible $y_{c,j,t} \leq y_{f,j,t}$. Similarly, the allocation in the CM goods market must also be feasible, $\sum_{i \in \{c,f,b\}} x_{i,t} \leq \sum_{j \in \{c,f,b\}} h_{i,t}$. Next, the mass of firms wishing to trade must be feasible, and thus $n_{f,j} \leq 1$ must hold for all $j \in \{p, d\}$. Finally, since consumers must type with at most one type of firm, $\sum_j n_{c,j} \leq 1$. Thus, an efficient allocation is an allocation Ω that maximises $\sum_{t=0}^{\infty} \beta^t \mathcal{W}_t$ subject to the feasibility constraints.

The efficient allocation consists of $y^*_{f,j,t} = y^*_{c,j,t} = y^*_{j,t}$ where $y^*_{j,t} = u'^{-1}(\kappa_j)$ and $x^*_{i,t} = x^*_t$ for all $i \in \{c, f, b\}$. Turning to the efficient submarket composition, the first order condition from the planner's problem yields

$$\delta'(n_j) S^*_j \geq \eta, \quad (5)$$

where $S^*_j \equiv u\left(y^*_{j,t}\right) - \kappa_j y^*_{j,t}$ denotes the surplus of trading with a firm in sector $j \in \{p, d\}$. Equation (5) holds with strict equality if $n_{f,j} < 1$. As $\delta'\prime(n_j) < 0$, for entry into sector j to be profitable for at least one firm, the entry cost must not be too high. Formally $\eta < \delta'(0) S^*_j$.

An efficient allocation where both digital and physical firms enter requires that all consumers attempt to trade, $\sum_j n_{c,j} = 1$, and that the following condition holds

$$(\delta(n_d) - n_d \delta'(n_d)) S^*_d = (\delta(n_p) - n_p \delta'(n_p)) S^*_p. \quad (6)$$

Equation (6) highlights the trade-off between sectors. A sector with a higher surplus will be offset in equilibrium by a lower n and thus more congestion.

2.3 Settlement Stage

Consider the utility of an agent of type $i \in \{c, f, b\}$ who enters the second stage with money holdings a_t^M and bond holdings a_t^B. Their value function can be expressed as

$$\begin{aligned} W_{i,t}\left(a_t^M, a_t^B\right) &= \max_{\left(x_t, h_t, a_{t+1}^M\right) \in \mathbb{R}_+^3} \left[v\left(x_t\right) - h_t + \beta V_{i,t+1}\left(a_{t+1}^M\right)\right], \\ \text{s.t.} \qquad p_{2,t} x_t + a_{t+1}^M &\leq p_{2,t}\left(h_t + a_t^B\right) + a_t^M + T_t^M \mathbb{I}_{\{i=c\}}, \end{aligned} \tag{7}$$

where $T_t^M \in \mathbb{R}$ is the time t lump-sum monetary injection to an individual consumer, $p_{2,t}$ is the nominal price of the CM good.

As a welfare maximising agent will ensure their budget constraint binds, by substituting out h_t using the budget constraint equation (7) can be written as

$$W_{i,t}\left(a_t^M, a_t^B\right) = \frac{a_t^M}{p_{2,t}} + a_t^B + \bar{W}_{i,t}, \tag{8}$$

where

$$\bar{W}_{i.t} \equiv \frac{T_t^M}{p_{2,t}} \mathbb{I}_{i=c} + v\left(x^*\right) - x^* + \max_{a_{t+1}^M \in \mathbb{R}_+} \left[\beta V_{i,t+1}\left(a_{t+1}^M\right) - \frac{a_{t+1}^M}{p_{2,t}}\right]. \tag{9}$$

The first-order condition with respect to the money demand of an agent of type $i \in \{c, f, b\}$ yields the following Euler equation

$$\beta \frac{\partial V_{i,t+1}}{\partial a_{i,t+1}^M} \frac{1}{p_{2,t+1}} \leq \frac{1}{p_{2.t}}, \quad \text{with “ = ” if } a_{i,t+1}^M > 0 \text{ for } i \in \{c, f, b\}. \tag{10}$$

2.4 Individual Portfolio Choice and Bargaining

Consider the portfolio problem of a banker at the end of the first stage of period t. Let a_t^M be their money holdings and a_t^B be their existing holdings of bonds, which they could have obtained by trading with other agents earlier in period t. The banker is able to re-enter the bond market in order to rebalance their portfolio and solves the following problem in order to maximise their expected discounted payoff

$$\begin{aligned} \hat{W}_{b,t}\left(a_t^M, a_t^B\right) &= \max_{\left(\bar{a}_t^M, \bar{a}_t^B\right) \in \mathbb{R}_+ \times \mathbb{R}} W_{b,t}\left(\bar{a}_t^M, \bar{a}_t^B\right), \\ \text{s.t.} \qquad \bar{a}_t^M + q_t \bar{a}_t^B &\leq a_t^M + q_t a_t^B. \end{aligned} \tag{11}$$

Let $\bar{a}_t^M = \bar{a}_{b,t}^M\left(a_t^M, a_t^B\right)$ and $\bar{a}_t^B = \bar{a}_{b,t}^B\left(a_t^M, a_t^B\right)$ denote the solution to the maximisation problem in equation (11).

A consumer who enters the beginning of period t with a quantity of money $a_t^M \geq 0$ and incurs a nominal expenditure of $\chi_t \geq 0$ on DM goods following the trading stage. Thus, a consumer that purchases y_t goods at a price p_t will have expenditure of $\chi_t = p_t y_t$, while a consumer that is not matched with a firm in the DM will have an expenditure of $\chi_t = 0$. All consumers have access to a banker and thus are able to rebalance their portfolio of money and bonds following the trading stage and do so in order to maximise

$$\begin{aligned} &\max_{\left(\bar{a}_t^M, \bar{a}_t^B\right) \in \mathbb{R}_+ \times \mathbb{R}} && W_{c,t}\left(\bar{a}_t^M, \bar{a}_t^B\right) \\ &\text{s.t.} && \bar{a}_t^M + \chi_t + q_t \bar{a}_t^B \leq a_t^M \end{aligned} \tag{12}$$

Let $\bar{a}_t^M = \bar{a}_{c,t}^M\left(\chi_t, a_t^M\right)$ and $\bar{a}_t^B = \bar{a}_{c,t}^B\left(\chi_t, a_t^M\right)$ denote the solution to the maximisation problem in equation (12).

A firm of type j that does not match with a banker is unable to trade in the bond market. The only asset an unbanked firm can hold money between the two stages of period t is money. Denote $\tilde{\chi}_{j,t}$ as the revenue that the firm receives from the sale of DM goods. In the baseline model, only physical firms are able to accept money in exchange for DM goods they sell and so

$$\tilde{\chi}_{j,t} = \begin{cases} p_{p,t} y_{p,t}, & \text{if } j = p \\ 0, & \text{if } j = d \end{cases} \tag{13}$$

The continuation value of such a firm that enters the beginning of period t with a quantity of money $a_t^M \geq 0$ and sells DM goods in exchange for $\chi_t \geq 0$ units of money is

$$W_{f,j,t}\left(\tilde{a}_{f,j,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right), 0\right) \tag{14}$$

where $\tilde{a}_{f,j,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right) = a_t^M + \tilde{\chi}_{j,t}$.

Finally, consider a firm of type j that enters period t with money holdings $a_{j,t}^M$ and matches with a banker. The firm must simultaneously choose its optimal portfolio of money and bonds to bring into the second stage of period t and bargain over the intermediation fee paid to the banker, $k_{j,t}$. The firm and banker engage in Nash bargaining, where the firm's bargaining power is θ and solve the following problem

$$\begin{aligned} &\max_{\left(k_{j,t}, \bar{a}_{j,t}^M, \bar{a}_{j,t}^B\right) \in \mathbb{R}_+^2 \times \mathbb{R}} && \left[W_{f,j,t}\left(\bar{a}_{j,t}^M, \bar{a}_{j,t}^B\right) - W_{f,j,t}\left(\tilde{a}_{f,j,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right), 0\right)\right]^\theta k_{j,t}^{1-\theta}, \\ &\text{s.t.} && \bar{a}_{j,t}^M + q_t\left(\bar{a}_{j,t}^B - k_{j,t}\right) \leq a_t^M + p_{j,t} y_{j,t}, \\ &&& W_{f,j,t}\left(\tilde{a}_{f,j,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right), 0\right) \leq W_{f,j,t}\left(\bar{a}_{j,t}^M, \bar{a}_{j,t}^G\right). \end{aligned} \tag{15}$$

Let $k_{j,t} = k_{j,t}\left(\omega_{j,t}, a_t^M\right)$, $\bar{a}_{j,t}^M = \bar{a}_{f,j,t}^M\left(\omega_{j,t}, a_t^M\right)$, $\bar{a}_{j,t}^B = \bar{a}_{f,j,t}^B\left(\omega_{j,t}, a_t^M\right)$ be the solution to the maximisation in equation (15).

2.5 Trading Problem

The consumer's value function can be written as

$$\begin{aligned} V_{c,t}\left(a_t^m\right) = &\max_{\omega_j}\Big\{\delta\left(n_j\right)\alpha\left[u\left(y_{j,t}\right) - p_{j,t}y_{j,t} + W_{c,t}\left(\bar{a}_{c,t}^M\left(p_{j,t}y_{j,t}, a_t^M\right), \bar{a}_{c,t}^B\left(p_{j,t}y_{j,t}, a_t^M\right)\right)\right] \\ &+ \delta\left(n_j\right)\left(1-\alpha\right)\left[1\left(j=p\right)\left(u\left(y_{j,t}\right) - p_{j,t}y_{j,t}\right) + W_{c,t}\left(\bar{a}_{c,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right), \bar{a}_{c,t}^B\left(\tilde{\chi}_{j,t}, a_t^M\right)\right)\right] \\ &+ \left(1-\delta\left(n_j\right)\right)W_{c,t}\left(\bar{a}_{c,t}^M\left(0, a_t^M\right), \bar{a}_{c,t}^B\left(0, a_t^M\right)\right)\Big\} \end{aligned} \tag{16}$$

where $1\left(j=p\right)$ is an indicator function which equals 1 if $j=p$.

The firm's value function can be written as

$$\begin{aligned} V_{f,j,t}\left(a_t^m\right) = &\max_{\omega_j}\left\{\frac{\delta\left(n_j\right)}{n_j}\alpha\left[-\kappa_j y_{j,t} + W_{f,j,t}\left(\bar{a}_{f,t}^M\left(p_{j,t}y_{j,t}, a_t^M\right), \bar{a}_{f,t}^B\left(p_{j,t}y_{j,t}, a_t^M\right)\right)\right]\right. \\ &+ \frac{\delta\left(n_j\right)}{n_j}\left(1-\alpha\right)\left[-1\left(j=p\right)\kappa_j y_{j,t} + W_{f,j,t}\left(\bar{a}_{f,j,t}^M\left(\tilde{\chi}_{j,t}, a_t^M\right), 0\right)\right] \\ &\left.+ \left(1 - \frac{\delta\left(n_j\right)}{n_j}\right)W_{f,j,t}\left(a_t^M, 0\right)\right\} \end{aligned} \tag{17}$$

The banker's value function can be written as

$$\begin{aligned} V_{b,t}\left(a_t^M\right) = &\alpha\sum_j n_{c,j,t}\delta\left(n_j\right)\int W_{b,t}\Big(\bar{a}_{b,t}^M\left(a_t^M, k_{j,t}\left(\omega_{j,t}, a_{f,j,t}^M\right)\right), \bar{a}_{b,t}^B\left(a_t^M, k_{j,t}\left(\omega_{j,t}, a_{f,j,t}^M\right)\right)\Big)dF_{j,t}\left(a_{f,j,t}^M\right) \\ &+ \left(1-\alpha\right)W_t^B\Big(\bar{a}_{b,t}^M\left(a_t^M, 0\right), \bar{a}_{b,t}^B\left(a_t^M, 0\right)\Big) \end{aligned} \tag{18}$$

where $F_{j,t}$ is the beginning-of-period t cumulative distribution function of money holdings across firms of type j and $n_{c,j,t}$ are the mass of consumers that enter submarket j and $n_{c,j,t}\delta\left(n_j\right)$ is the number of trades, and hence the number of active firms, in submarket j.

3 Equilibrium

First, in order to simplify notation, denote the relative price of the DM good in submarket j in terms of the CM good as

$$\varphi_{j,t} \equiv \frac{p_{j,t}}{p_{2,t}}. \tag{19}$$

Similarly, denote the interest rate on bonds as i_t which is defined as follows

$$i_t \equiv \frac{p_{2,t}}{q_t} - 1. \tag{20}$$

If $i_t > 0$ then agents will strictly prefer to hold bonds instead of money. Thus, the only agents who hold money are unbanked firms that are unable to access the bond market.

Using equation (8) the value function for firms that meet with both a firm and a banker can be rewritten as

$$W_{f,j,t}\left(\bar{a}^M_{j,t}, \bar{a}^B_{j,t}\right) = \left[-\kappa_j y_{j,t} - \frac{1}{p_{2,t}} i_t \bar{a}^M_{j,t} + (1+i_t)\left(\frac{1}{p_2, t} a^M_t + \bar{\varphi}_{j,t} y_{j,t}\right) - k_{j,t} + \bar{W}_{f,j,t}\right] \tag{21}$$

Firms and bankers bargain over the total surplus generated by allowing firms to access the bond market. Defining this surplus as $\Delta W_{f,j,t} \equiv W_{f,j,t}\left(\bar{a}^M_{j,t}, \bar{a}^B_{j,t}\right) - W_{f,j,t}\left(\tilde{a}^M_{j,t}, 0\right)$, it can be expressed as

$$\Delta W_{f,j,t} = -\kappa_j y_{j,t}\left(1 - 1\left(j = p\right)\right) + \frac{1}{p_{2,t}} i_t \left(a^M_t - \bar{a}^M_{j,t}\right) + \left(1 - 1\left(j = p\right) + i_t\right)\varphi_{j,t} y_{j,t} - k_{j,t} \tag{22}$$

Taking the first order condition of the Nash bargaining problem and rearranging yields the following

$$k_{j,t} = (1-\theta)\,\Delta W_{f,j,t}, \tag{23}$$

where the equilibrium intermediation fee paid to bankers is set so that bankers receive a fraction $1-\theta$ of the total available surplus.

Next, turn to the design of the submarkets by market makers who maximise the expected utility of consumers subject to a participation constraint for firms and feasibility constraints. Using equation (8), the expected utility of consumers that choose to enter submarket j can be written

$$\begin{aligned} V_{c,j,t} &= \alpha\delta\left(n_j\right)\left(u\left(y_{j,t}\right) - (1+i_t)\varphi_{j,t} y_{j,t}\right) \\ &+ (1-\alpha)\,\delta\left(n_j\right) 1\left(j = p\right)\left(u\left(y_{j,t}\right) - (1+i_t)\varphi_{j,t} y_{j,t}\right) \\ &+ (1+i_t)\frac{1}{p_{2,t}} a^M_{j,t} + \bar{W}_{c,t}. \end{aligned} \tag{24}$$

Similarly, substituting in the solution to the firm's Nash bargaining problem, the expected utility of firms that enter submarket j can be written

$$\begin{aligned} V_{f,j,t} &= \alpha\theta\frac{\delta\left(n_j\right)}{n_j}\left((1+i_t)\varphi_{j,t} - \kappa_j\right) y_{j,t} \\ &+ (1-\alpha\theta)\frac{\delta\left(n_j\right)}{n_j} 1\left(j = p\right)\left(\varphi_{j,t} - \kappa_j\right) y_{j,t} \\ &+ (1+\alpha\theta i_t)\frac{1}{p_{2,t}} a^M_{j,t} + \bar{W}_{f,t} - \eta_t. \end{aligned} \tag{25}$$

A submarket with a posted $(\bar{y}_{j,t}, \bar{\varphi}_{j,t}, \bar{y}_{j,t}, \bar{\varphi}_{j,t})$ will attract a measure n_j of firms of type j per consumer, where n_j satisfies

$$\frac{\delta(n_j)}{n_j}\Big(\alpha\theta\left(\left(1+i_t\right)\varphi_{j,t}-\kappa_j\right)y_{j,t}+1\left(j=p\right)\left(1-\alpha\theta\right)\left(\varphi_{j,t}-\kappa_j\right)y_{j,t}\Big)=\pi_{j,t}+\eta. \tag{26}$$

The term $\pi_{j,t} \geq 0$ is the expected profit of a j type seller in the DM. The market maker posts ω_j in order to maximise equation (24) subject to the firm participation constraint, equation (26). Furthermore, the prices posted by the submarket must also ensure that the consumer obtains nonnegative utility from trading with firms regardless of whether the firm they meet is able to access the bond market or not. Similarly, ω_j must yield a non-negative surplus from the firm's access to credit, $\Delta W_{f,j,t} \geq 0$. In the case where this latter constraint were violated, the firm would voluntarily remain unbanked, producing an outcome equivalent to the case where $\Delta W_{f,j,t} = 0$.

In equilibrium, the market will segment into at most two submarkets where each submarket will feature entry by only one type of firm. Consumers are free to choose which of the submarkets to enter and thus, for a positive mass of consumers to enter both submarkets in equilibrium, consumers must be indifferent between the two submarkets, $V_{c,p,t} = V_{c,d,t}$.

3.1 Equilibrium without CBDC

In this section, I characterise the equilibrium without CBDC.

First, the equilibrium output for digital firms is a function of the marginal cost and is equal to the optimal output level, as described above.

$$y_{d,t} = u'^{-1}\left(\kappa_d\right). \tag{27}$$

The output level of physical firms is given by the following equation

$$y_{p,t} = u'^{-1}\left(\left(\frac{1+i_t}{1+\alpha\theta i_t}\right)\kappa_p\right), \tag{28}$$

where output is below the optimal level whenever $\alpha\theta < 1$.

$$\delta'\left(n_p\right)\Gamma_{p,t} = \left(\frac{1+i_t}{1+\alpha\theta i_t}\right)\left(\pi_{p,t}+\eta_t\right), \tag{29}$$

and

$$\delta'\left(n_d\right)\Gamma_{d,t} = \frac{1}{\alpha\theta}\left(\pi_{d,t}+\eta_t\right). \tag{30}$$

where $\Gamma_{j,t} \equiv u(y_{j,t}) - y_{j,t}u'(y_{j,t})$ is the surplus available from trade between a consumer and a type j firm given an output level of $y_{j,t}$.

A requirement for firms of type j to enter their submarket is that they at least cover the entry cost and make non-negative profits, and so $\pi_{j,t} \geq 0$. In the case where firms make positive profits, all firms of type j enter and $n_{f,j,t} = N_{f,j,t}$, while if banks make zero profits, free-entry holds and $n_{f,j,t} < N_{f,j,t}$.

As with the efficient allocation, if the surplus available $\Gamma_{j,t}$ is small enough, it may not be possible for the firms to cover their entry costs. In particular, for physical firms to enter the market, it must be the case that

$$\delta'(0)\,\Gamma_{p,t} > \left(\frac{1+i_t}{1+\alpha\theta i_t}\right)\eta_t, \tag{31}$$

and for digital firms to enter the market it must be the case that

$$\delta'(0)\,\Gamma_{d,t} > \frac{1}{\alpha\theta}\eta_t. \tag{32}$$

Solving the market maker's problem with respect to the optimal firm to seller ratio, in the physical sector yields

$$V_{c,p,t} = \left(\delta(n_p) - n_p\delta'(n_p)\right)\Gamma_{p,t} \tag{33}$$

similarly, in the digital sector

$$V_{c,j,t} = \alpha\left(\delta(n_d) - n_d\delta'(n_d)\right)\Gamma_{d,t}. \tag{34}$$

For consumers to be indifferent between the two submarkets, it must be the case that $V_{c,p,t} = V_{c,d,t}$. In this case, n_p and n_d must be such that

$$\alpha\left(\delta(n_d) - \delta'(n_d)\,n_d\right)\Gamma_{d,t} = \left(\delta(n_p) - \delta'(n_p)\,n_p\right)\Gamma_{p,t} \tag{35}$$

Consumers trade off the probability of trading with a firm with the benefit of trading. In addition to the matching friction, absent CBDC, digital firms are only able to transact with physical firms if they have access to the credit market; thus, in the case where the gains from trade are the same across sectors, consumers would require a higher probability of trade ($n_d > n_p$) in order to be indifferent between the two sectors.

When these equations are compared to optimal allocation, the equilibrium without CBDC will generally be only efficient if $\alpha\theta = 1$ and where all firms have access to credit and drive the profits of the credit sector to zero. Here, all trades are made using credit. In cases where $\alpha\theta < 1$, frictions in the credit market result in an inefficiently high number of consumers attempting to trade with physical firms and an inefficiently low number of consumers attempting to trade with digital firms. Money has a dual role to play in this case; first, it provides an alternative form of payment in cases where firms do not have access to credit which facilities a greater number of transactions, and second, it improves the bargaining position of firms in the credit market which allows for more firms to enter the physical retail market.

3.2 Introduction of a CBDC

Consider now the case where the monetary authority decides to introduce a digital form of money, for example, a CBDC, that can be used as a means of payment by digital firms. This relaxes the constraint on digital firms that is set out in equation (13) and instead this would become $\Gamma_{j,t} = p_{j,t}y_{j,t}$. The problem for digital firms becomes identical to the problem for physical firms. As such, introducing a CBDC will result in the economy reaching the efficient allocation.

As discussed above, in the case where $\alpha\theta = 1$ the economy achieves the efficient allocation without the need for money, digital or otherwise. In cases where $\alpha\theta < 1$ and there are frictions in the credit market, the introduction of CBDC will only lead to the efficient outcome if $i_t = 0$, essentially in the presence of frictions in the credit market, efficiency requires implementation of the Friedman rule.

4 Numerical analysis

As discussed in the previous section, the welfare implications of introducing a CBDC depend at least partly on what is driving the trend towards digital payment methods and the increase in online retail sales. This section provides a numerical analysis that emphasises this point.

4.1 Baseline Calibration

The model is parameterised as follows. The utility function of consumers in the DM is

$$u\left(y_t\right) = \frac{1}{1-\sigma}\left[\left(y_t + \epsilon\right)^{1-\sigma} - \epsilon^{1-\sigma}\right], \tag{36}$$

where $\sigma > 0$ and the parameter ϵ is set to 0.001. This ensures $u(0) = 0$ and $u(\cdot)$ is close to CRRA. This parameter has little impact quantitatively and is common in the money-search literature. The utility of agents in the CM is assumed to have the following functional form:

$$v(x) = B\ln(x), \tag{37}$$

which implies that $x^* = B$. The matching function specified is analogous to the one used in Rocheteau and Wright (2009) and is

$$\delta(n_j) = \frac{n_j}{1+n_j}. \tag{38}$$

The model is calibrated to UK annual data. The credit availability parameter, α, is calibrated to the proportion of cash transactions relative to the total number of cash and card transactions. Data are obtained from the UK Finance Payment Markets Survey, with around 76% of transactions in 2007 made with cash. The marginal cost of physical firms, κ_p, is normalised to 1, while the marginal cost of digital firms, κ_d, is calibrated to the proportion of internet sales as a percentage of the total value of retail sales. Data are obtained from the ONS Retail Sales Index, where a fraction 3.4% of retail sales in 2007 were made via the internet. The firm's bargaining power, θ, is calibrated to card handling costs as a fraction of the turnover from card transactions. Data are obtained from the British Retail Consortium Payments Survey. In contrast to the other parameters where the 2007 value is used in the baseline calibration, data is available only from 2011, where card handling costs stood at 0. 53% of the turnover from card transactions. This is unlikely to significantly affect the calibration as card handling costs are relatively stable over time. Finally, the fixed cost of the firm, η, is calibrated so that the ratio of fixed costs to revenue in the model matches the ratio of consumption of fixed capital by private non-financial firms to their revenue in 2007.

The parameters (σ, B) are calibrated analogously to those of Lucas (2000) and Lagos and Wright (2005) where the parameters are set to match the relationship between the nominal interest rate i and the demand for money $L \equiv M/PY$ using a longer time series of data.

In the model, money is held only in order for firms in the physical market without access to credit to transact. Equilibrium money demand is then

$$M_t = (1-\alpha)\, n_{c,p}\delta(n_p)\, p_{p,t}y_{p,t} \tag{39}$$

Nominal output in the decentralised market is $\alpha n_{c,d}\delta(n_d)\, p_{d,t}y_{d,t} + n_{c,p}\delta(n_p)\, p_{p,t}y_{p,t}$ and nominal output in the centralised market is $4p_{2,t}B$. Thus the model implied function for

Table 1: Baseline Calibration

Parameters	Notation	Value	Calibration Target
Credit availability	α	0.232	Fraction of cash transactions 2007
Digital firm cost	κ_d	0.376	Fraction of online transactions 2007
Firm bargaining power	θ	0.887	Card handling costs 2011
Firm fixed cost	η	0.052	PNFC Consumption of fixed capital / revenue 2007
Curvature of DM Consumption	σ	0.563	CIC/GDP demand function 1982-2023
Coefficient of CM Consumption	B	4.889	CIC/GDP demand function 1982-2023

L is given by

$$L = \frac{(1-\alpha)\, n_{c,p}\delta\,(n_p)\,\varphi_{p,t}y_{p,t}}{\alpha n_{c,d}\delta\,(n_d)\,\varphi_{d,t}y_{d,t} + n_{c,p}\delta\,(n_p)\,\varphi_{p,t}y_{p,t} + 4B} \tag{40}$$

The series for i is the spot yield on UK one-year Gilts and PY is UK nominal GDP. I deviate from Lucas (2000) and Lagos and Wright (2005) by letting M be the amount of currency in circulation (CIC) as opposed to M1. The reasoning behind this is that money in the model is best interpreted narrowly as cash, as opposed to M1. However, calibrating the model to M1 instead of CIC does not significantly alter the results.

Table 1 summarises the parameter values in the baseline calibration along with the corresponding calibration target. Figure 3 shows the model-predicted curve for CIC/GDP as a function of the nominal rate compared to the UK data between 1982 and 2023.

4.2 Effect of introducing a CBDC

I now consider the effect of the introduction of a CBDC into the model. The introduction of a CBDC allows digital firms to transact in the absence of access to the payments market. This has two effects: first, the number of possible transactions in the digital sector increases as transactions are no longer conditional on firms receiving access to the payments market, and second, digital firms have a better bargaining position with payment operators due to the existence of an outside option for transactions. The effect on welfare and key data moments of the introduction of a CBDC is presented in Table 2. The introduction of CBDC increases the proportion of sales in the digital sector and reduces the number of cash transactions that occur on average. The proportion of Internet sales following the introduction of CBDC is close to optimal. The model also predicts a significant increase in welfare of around 130%. Putting this welfare increase in context, the model abstracts away from any welfare costs associated with the introduction of CBDC,

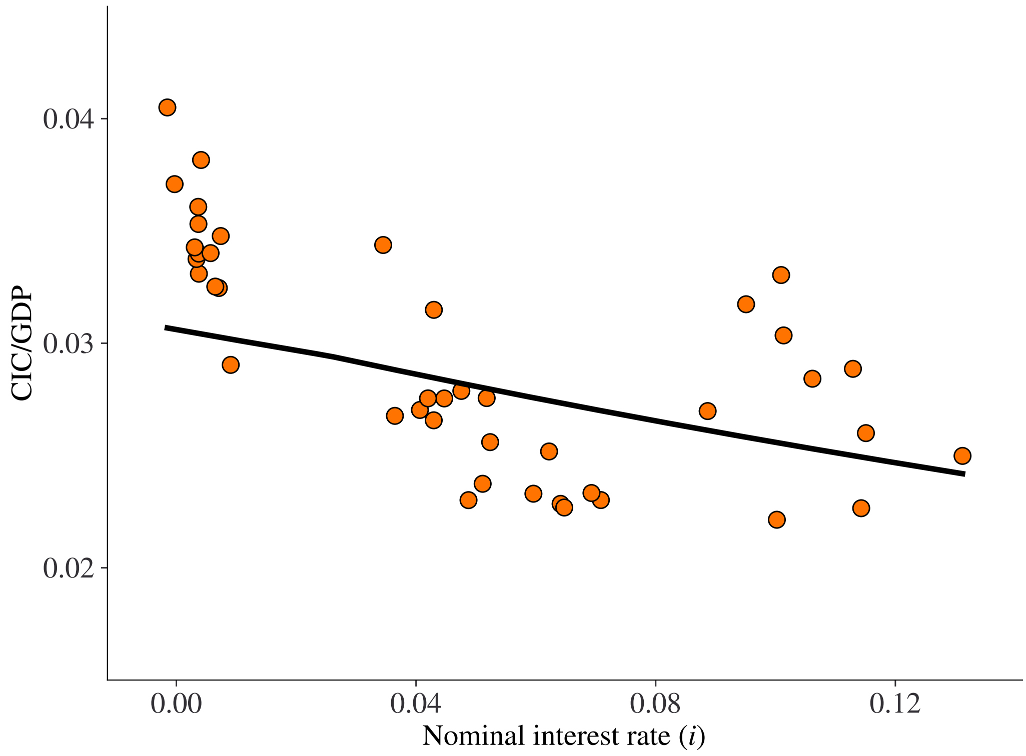

Figure 3: Model and Data (1982-2023)

such as issues surrounding financial stability and bank disintermediation. However, it is worth considering what generates this welfare benefit.

In the baseline calibration, the proportion of Internet sales is relatively low, while the number of cash transactions is relatively high. The model captures this through low credit availability ($\alpha = 0.203$) and with digital firms facing lower production costs compared to physical firms ($\kappa_d = 0.705$). Given that digital firms are more efficient than physical firms in this calibration, it would be optimal for digital firms to have a much larger share of sales revenue. This results in a large welfare loss that stems from digital firms without access to credit not being able to transact with consumers. The introduction of a CBDC increases the ability of digital firms to transact with consumers, and thus the proportion

	Data	Baseline Model	Model with CBDC	Optimal Allocation
Internet Sales	0.034	0.034	0.924	0.914
Cash Transactions	0.763	0.763	0.396	0.0
Card Handling Costs	0.0053	0.0053	0.0043	0.0
CFC / Revenue	0.0712	0.0712	0.1630	0.0476
Welfare (baseline=100)		100	230	237

Table 2: Effect of CBDC introduction on Moments and Welfare

of sales made by digital firms is close to the optimal level.

To better understand what generates the welfare gains from introducing a CBDC in the baseline calibration, Figure 4 compares the welfare of the model without CBDC to both the model with CBDC and the optimal allocation while altering the values of key parameters of the model. Other parameters are fixed at the baseline calibration values. Figure 4 highlights the sensitivity of welfare gains to the parameterisation. In particular, the welfare gains are largest at low values of α and low values of κ_d. As $\alpha \to 1$ the model without CBDC approaches the optimal allocation and the welfare gains from introducing a CBDC disappear. It should be noted that in the baseline calibration, interest rates are relatively low ($i = 0.0476$) and θ is close to 1, introducing a CBDC leads to an equilibrium close to optimal.

4.3 Capturing the rise of the digital economy

In the baseline calibration, the model was matched to 2007 data where the share of Internet sales was relatively low and the share of cash transactions relatively high. As illustrated in Figures 2 and 1, the years following this featured a rapid increase in both the proportion of Internet retail sales and a decrease in the share of cash transactions in the UK.

The model suggests two possible explanations for the observed trends. First, a decrease in κ_d relative to κ_p would result in digital firms facing lower marginal production costs and hence are more productive than physical firms. This could occur due to technological advances being made in the economy that benefit digital firms to a greater extent than physical stores. This increase in relative productivity would lead to a higher proportion of consumers entering the digital submarket and lead to digital firms receiving a larger share of total trades. Second, an increase in credit availability, α, would result in digital firms having a higher probability to transact with consumers, and as a consequence consumers switch from physical stores to digital stores. As previously discussed, a decrease in κ_d would increase the welfare gains from the introduction of a CBDC while an increase in α would decrease the welfare gains from introducing a CBDC.

To address which of these channels dominates, I now extend the baseline calibration by allowing the parameters $(\alpha_t, \kappa_{d,t}, \theta_t, \eta_t)$ to vary over the years 2007-2022 while keeping the remaining parameters fixed at their values in the baseline calibration. The parameters α_t, $\kappa_{d,t}$ and η_t are calibrated to the time series of the proportion of cash transactions, the proportion of Internet sales, and the consumption of fixed capital ratio, respectively. Due to the lack of available data, the parameter θ_t is calibrated so that the card handling costs remain at the baseline calibration level.

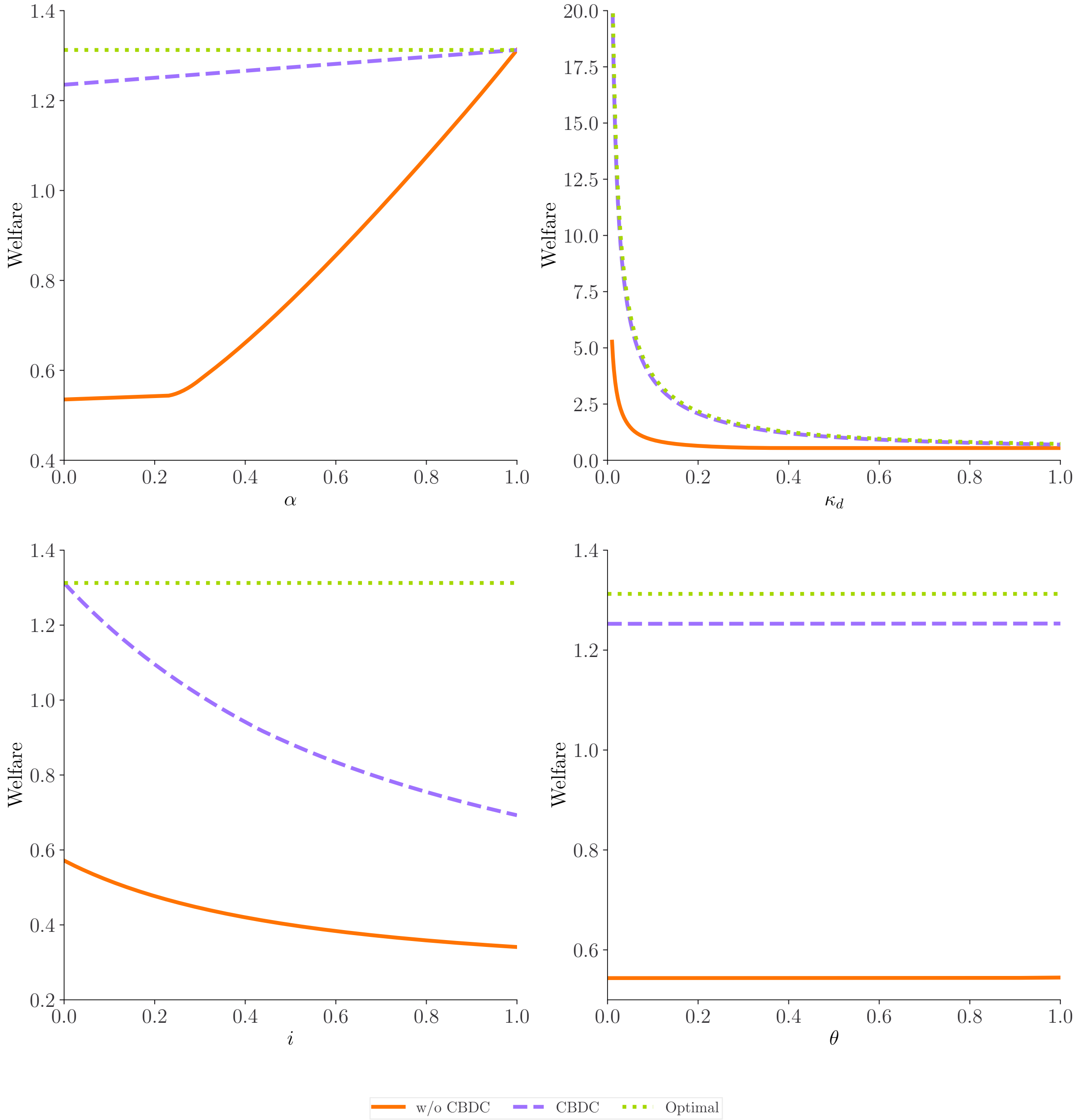

Figure 4: Welfare Sensitivity to Parameters

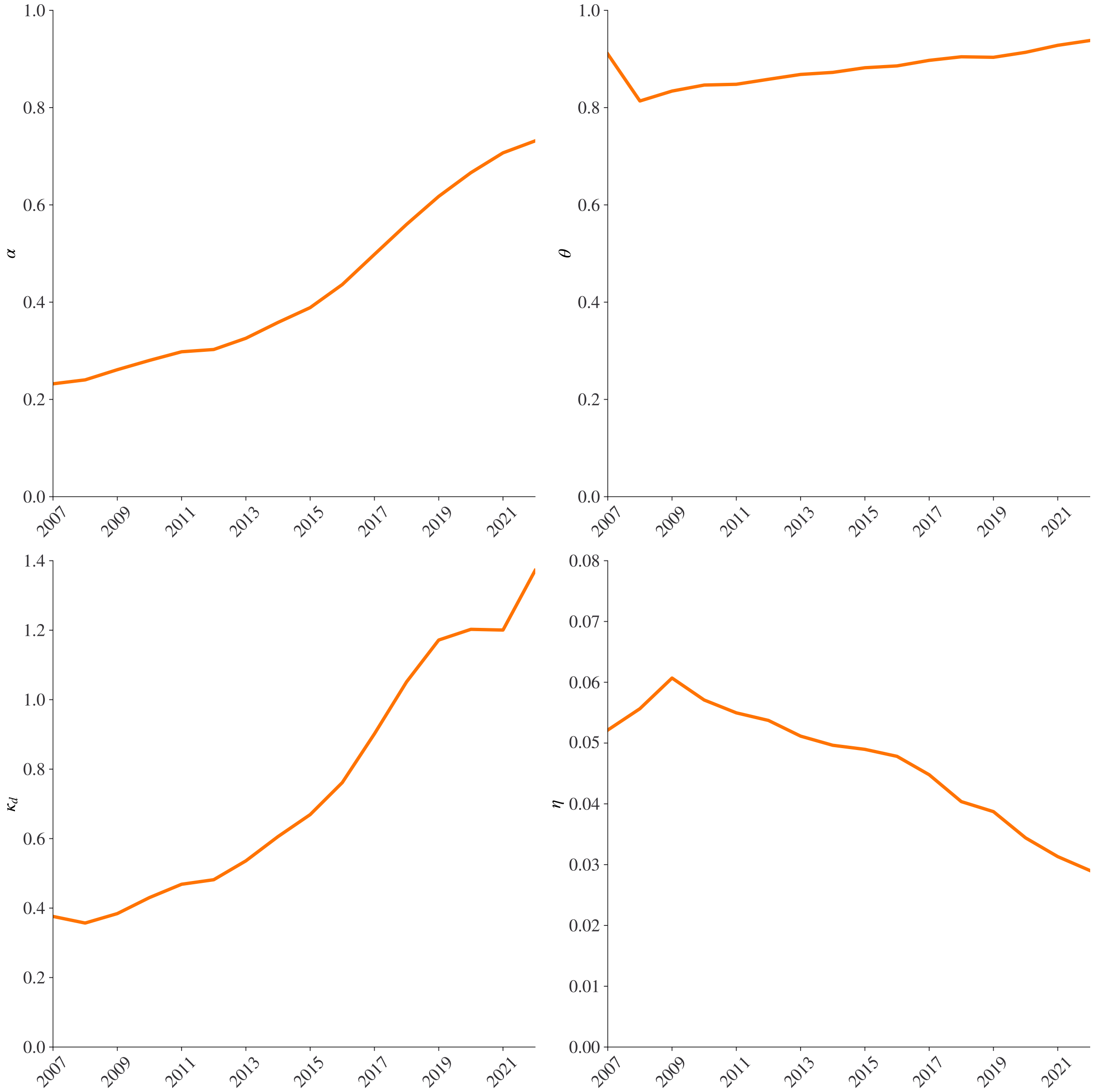

Figure 5: Parameter Calibration (2007-2022)

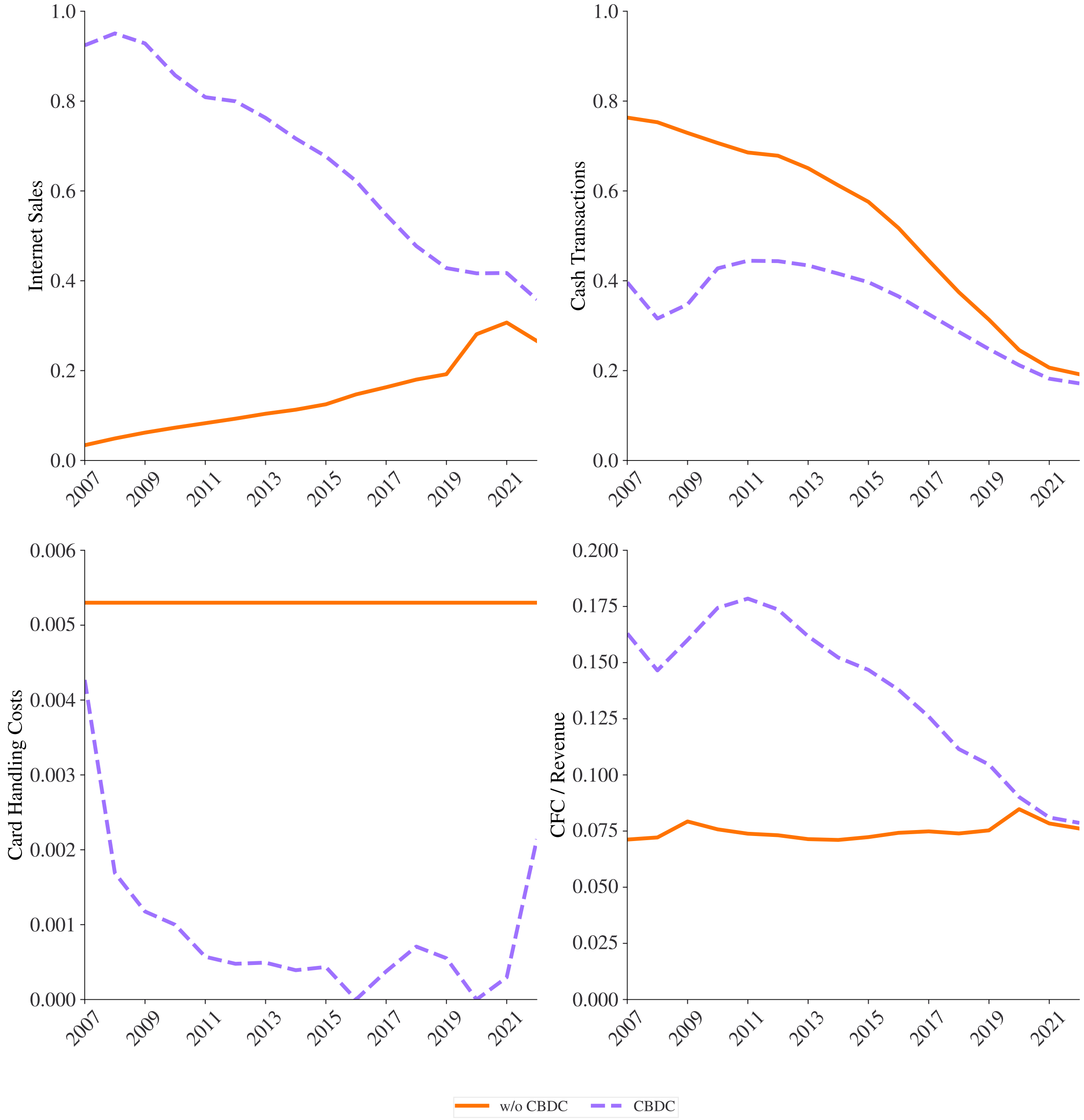

Figure 6: Calibrated Moments and Counterfactual Moments (2007-2022)

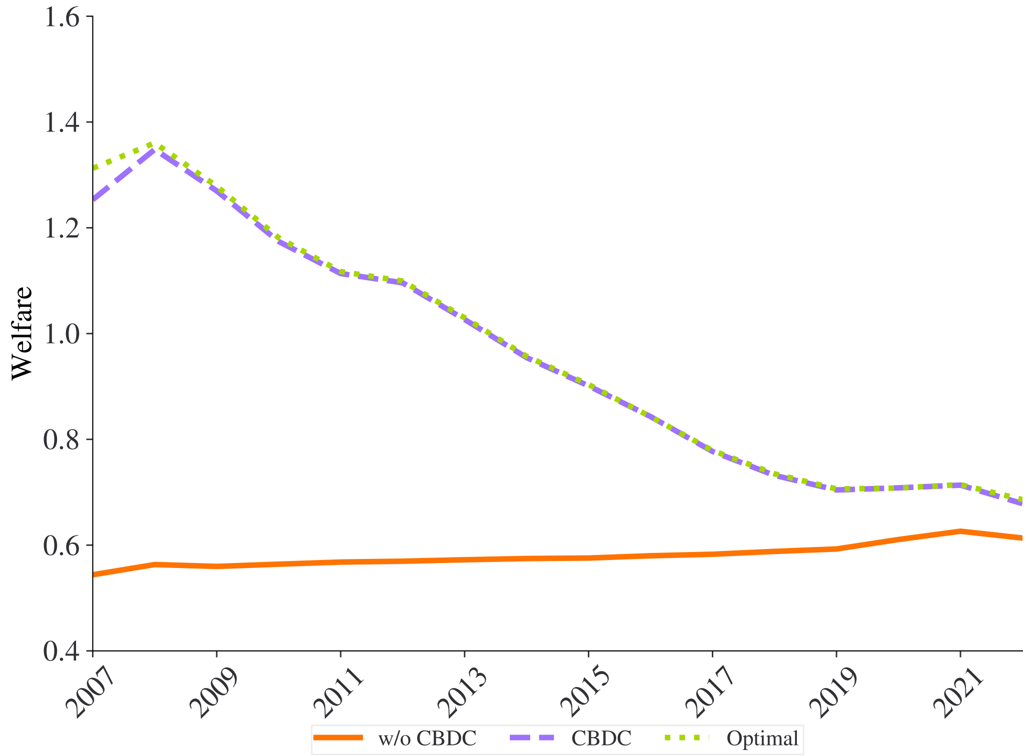

Figure 7: Welfare

Figure 5 shows the series of calibrated parameters. The model suggests that the increase in the share of Internet sales and the decrease in cash use are driven by an increase in α, greater availability of credit, as opposed to lower marginal costs of digital firms, κ_d. In fact, the model suggests that the marginal costs of digital firms increased over the period 2007-2022.

Figure 6 shows the calibrated moments for the benchmark model and the model with CBDC. The benchmark model matches the increase in Internet sales and the decline in cash transactions found in the data. Cash transactions in the CBDC model capture payments in the physical sector made using money which, given that the model does not distinguish between the two, could consist of both cash payments and CBDC payments. As the model suggests κ_d increases over the period 2007-2022, the gap between the benchmark model and the model with CBDC narrows for both Internet sales and cash transactions.

The implications of this calibration are that the welfare gains of introducing a CBDC fall over the period 2007-2022. Figure 7 shows that welfare in the benchmark model increases over time, while welfare in the model with CBDC decreases over time. While the welfare gains of introducing a CBDC were 130% in 2007, this falls to 10.6% in 2022. This result is driven by the increase in both α and κ_d over the time period. However, welfare in the model with CBDC is very close to optimal. This is because nominal interest rates are close to zero for most of the period.

As discussed previously, since the model abstracts from possible negative implications of introducing a CBDC, the level of welfare gains of introducing a CBDC to the model should only be interpreted as measuring the possible benefit of offering an alternative means of payment for digital transactions. Given the benefit of introducing a CBDC benefits firms in the digital sector, one might think that the welfare gains would be greater when there are a higher number of Internet transactions and fewer transactions made using cash. The results presented here suggest that this may not necessarily be the case. Instead, it is important to gain an understanding of what is driving the shift toward Internet sales and away from cash transactions.

5 Conclusion

Current proposals for the introduction of a CBDC, such as those made by the Bank of England and the ECB, have coalesced around an unremunerated CBDC. An obvious question then is if it offers the same rate of return, how would a CBDC be distinct from existing forms of money? This paper proposes a simple mechanism through which an unremunerated CBDC may improve welfare by lowering the market power banks have in negotiating fees with digital firms that cannot use cash as an alternative means of payment.

Although the introduction of a CBDC has the potential to improve welfare, the model suggests that the size of any welfare increase depends on the degree to which the inability to trade with physical money deters the entry of online retailers. Given the trend for an increasing proportion of retail transactions to be done online and the increasing use of digital payment methods, it is important to understand the underlying drivers of these changes. When calibrated to the UK data, the model suggests that while there are still significant welfare gains from the introduction of a CBDC, the benefits have actually fallen over the period 2007 to 2022 as the proportion of online transactions has increased and cash usage has fallen.

Finally, while the model presented in this document abstracts from many of the design choices central banks are currently facing, the basic mechanism of the model relies on a CBDC as offering a credible alternative to existing digital payment methods. Thus, should a CBDC reuse existing payment infrastructure without providing additional competition, the welfare benefits set out in this paper would be overstated.

References

Barrdear, John and Michael Kumhof (2022). "The macroeconomics of central bank digital currencies". *Journal of Economic Dynamics and Control* 142.104148.

Bordo, Michael D (2021). *Central Bank Digital Currency in Historical Perspective: Another Crossroad in Monetary History.* Working Paper 29171. National Bureau of Economic Research.

Chiu, Jonathan, Seyed Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu (2023). "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment". *Journal of Political Economy* 131.5, pp. 1213–1248.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig (2021). "Central Bank Digital Currency: Central Banking for All?" *Review of Economic Dynamics.*

Jiang, Janet Hua and Enchuan Shao (2020). "The cash paradox". *Review of Economic Dynamics* 36, pp. 177–197.

Keister, Todd and Daniel Sanches (2023). "Should Central Banks Issue Digital Currency?" *The Review of Economic Studies* 90.1, pp. 404–431.

Lagos, Ricardo and Randall Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis". *Journal of Political Economy* 113.3, pp. 463–484.

Lagos, Ricardo and Shengxing Zhang (2022). "The Limits of *ONETARY ECONOMICS*: On Money as a Constraint on Market Power". *Econometrica* 90.3, pp. 1177–1204.

Lucas, Robert E. Jr. (2000). "Inflation and Welfare". *Econometrica* 68.2, pp. 247–274.

Moen, Espen R (1997). "Competitive Search Equilibrium". *Journal of Political Economy* 105.2, pp. 385–411.

Rocheteau, Guillaume and Randall Wright (2005). "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium". *Econometrica* 73.1, pp. 175–202.

Rocheteau, Guillaume and Randall Wright (2009). “Inflation and Welfare in Models with Trading Frictions”. In: *Monetary Policy in Low-Inflation Economies*. Ed. by David E. Altig and Ed Nosal. 1st ed. Cambridge University Press, pp. 89–116.

Williamson, Stephen (2022). “Central Bank Digital Currency: Welfare and Policy Implications”. *Journal of Political Economy* 130.11, pp. 2829–2861.

Wright, Randall, Philipp Kircher, Benoît Julien, and Veronica Guerrieri (2021). “Directed Search and Competitive Search Equilibrium: A Guided Tour”. *Journal of Economic Literature* 59.1, pp. 90–148.