Life-cycle forces make monetary policy wealth-centric*

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Abstract

This paper focuses on the implications of life-cycle forces for monetary policy. We show that life-cycle preoccupations place financial wealth at the center of consumption/saving decisions, leading aggregate demand to depend upon the link between interest rates, asset demand, valuation effects, and the division of wealth between active and retired households. When interest rate changes are quite temporary, the impact of life-cycle preoccupations on the monetary transmission process is modest. But when interest rate changes are more persistent, we show that life-cycle preoccupations have important implications for the monetary transmission mechanism, as well as for the appropriate response to financial shocks. In particular, we show that "low for long" or "high for long" policies may have very muted (or even perverse) effects in terms of influencing the economy, and that this dampening can be compounded by Quantitative Easing. Furthermore, we show why it can be desirable to prop up asset markets in response to adverse financial shocks. Life-cycle preoccupations thereby offer a new perspective on several aspects of monetary interventions.

JEL-classification: E21, E43, E44, E52, G51.

Key words: monetary policy, unconventional monetary policy, monetary transmission mechanism, retirement savings.

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1 Introduction

Recent times have brought substantial research efforts directed at understanding the monetary transmission mechanism. While the canonical New Keynesian model emphasizes the role of intertemporal substitution, models have since been enriched to include financial frictions, informational frictions, liquidity-constrained consumers, and more. All of these have been shown to matter for monetary policy. We aim to add to this literature by examining how the monetary transmission process is affected by life-cycle forces (which establish an interesting link between interest rates and households' desire to hold assets) and how monetary policy needs to be conducted to favor price stability.

The paper builds on the sticky-price setup of New Keynesian models, but instead of infinitely-lived consumers, we introduce agents that transit from an active phase of life (in which they work and save), to a retirement phase in which they live off their accumulated savings and the associated rate of return. Following Beaudry et al. (2023) we build on a household structure close to Yaari (1965), Blanchard (1985) and – especially – Gertler (1999).[1] The introduction of life-cycle saving motives is shown to change optimal consumption/saving decisions, yielding a central role to financial wealth. We show that it is not directly measured wealth that affects consumption decisions, but wealth corrected for the expected future path of interest rates. When rates are viewed as "low for long", this tends to increase wealth holdings through valuation effects. However, whether this boosts consumption will depend on how such lower interest rates simultaneously affect asset demand, as households may now want to save *more*, to compensate for the lower flow return per unit of asset held. This implies that if wealth levels are high only because of low interest rates, the propensity to consume out of this wealth may be very low.

In our framework, the elasticity of intertemporal substitution (EIS) plays an important role in the monetary transmission process. When the EIS is greater than one, introducing life-cycle forces is shown to have only modest effects on the monetary transmission mechanism. However, when the EIS is less than one – which seems to be the more relevant case (see Yogo (2004) and Best et al. (2020), who report the EIS to be around 0.1) – life-cycle preoccupations substantially change how interest rates affect aggregate demand. The reason is that when the EIS is less than one, persistently low interest rates can *increase* asset demand by more than supply, lowering demand for goods.

The interaction of life-cycle forces with monetary policy will be shown to depend

[1] Gertler's (1999) framework has also been used to analyze issues related to monetary policy by, among others, Sterk and Tenreyro (2016, focusing on a redistribution channel of monetary policy transmission when prices are fully flexible) and Galí (2021, analyzing the conduct of monetary policy in the presence of bubble-driven fluctuations).

crucially on the persistence of interest rate changes. For temporary changes, life-cycle forces can almost be ignored as these will neither have big valuation effects nor big effects on asset demand. Intertemporal substitution will then likely be the dominant force, implying that temporary rate cuts unambiguously stimulate the economy. In contrast, for persistent rate changes, life-cycle preoccupations can substantially alter the monetary transmission mechanism. Persistent rate changes have a large impact on the value of assets, but are accompanied by a countervailing effect on asset demand (with lower rates boosting asset demand, to compensate for the lower flow return per unit of asset held). This can cause "low for long" or "high for long" policies to have very muted effects, or even opposite effects, relative to temporary changes.[2] Accordingly, in the presence of life-cycle forces, a central bank faced with a persistent fall in aggregate demand could find it very challenging – possibly counter-productive – to stabilize the economy by signalling that interest rates will be "low for long". In this context, Rajan (2013) already worried that the post-GFC situation of persistently-low interest rates might not be expansionary because "savers put more money aside as interest rates fall in order to meet the savings they think they will need when they retire".

More generally, our framework implies that interest rate changes will often need to have a sufficiently strong impact on asset prices to stimulate aggregate demand. This makes transmission through financial markets central to monetary policy. Interestingly, models used for policy analysis at the Fed and ECB often focus on the asset price channel of monetary policy, instead of the more traditional intertemporal substitution effect (Boivin et al., 2010: 379). Our paper provides a micro-founded basis for such a view. Through textual analysis of FOMC statements, Cieslak and Vissing-Jorgensen (2021) provide further support for the notion that Fed policy makers pay explicit attention to asset prices (or "financial conditions"), and this seems to have increased since the 1990s. They also present evidence that the FOMC members view the associated wealth effect on consumption as an important driving force of the economy.[3] The existence of such a spending effect has further been documented by studies like Chodorow-Reich et al. (2021)

[2]This is consistent with the findings of Uribe (2022), who documents empirically that only transitory rate changes give rise to the "conventional" effects (with rate hikes being contractionary, and vice versa). When looking at permanent rate changes, he actually finds rate hikes (cuts) to be expansionary (contractionary) for both output and inflation. While he rationalizes his findings through a neo-Fisherian lens, our model offers an alternative explanation.

[3]They for example quote Bill Dudley (then-President of the New York Fed) as saying "We care about financial conditions not for themselves, but instead for how they can affect economic activity and ultimately our ability to achieve [...] maximum employment and price stability. [...] A rise in equity prices can boost household wealth, which is one factor that underpins consumer spending". Earlier VAR-based work by Rigobon and Sack (2003) and Bjørnland and Leitemo (2009) also found that the Federal Reserve tends to tighten (ease) in response gains (losses) in stock markets.

and Di Maggio et al. (2020) who report a marginal propensity to consume (MPC) out of financial wealth of around 3 to 20 percent (depending on the percentile of the wealth distribution, with poorer households having a higher MPC).

When it comes to mapping the saving behavior central to our model back to reality, one can either think of this savings response being directly implemented by households or indirectly by pension funds. Pension funds, who act on behalf of households, employ scores of asset-liability management specialists to solve the type of problem that rests at the core of our model. The effects of interest rate changes on the asset demand by pension funds is commonly discussed. For example, back in 2019, Dutch pension fund ABP (among the five largest in the world) issued a statement conveying how persistently low rates increase the current price of future consumption:[4]

- "*Pensions are becoming increasingly expensive* based on the current scheme. With the current pension ambition *and the expectation that interest rates will remain low for a long time*, higher premiums will be needed from 2021 onwards to finance pensions. The expectation is that the premium will increase." (italics added)

Here, the premium increases are analogous to higher savings in our model.

We also show that the extent to which interest rate changes affect aggregate demand is influenced by the composition of the central bank's balance sheet, which is in turn affected by Quantitative Easing (QE). QE can be viewed as an asset swap, with the central bank financing purchases of longer-term bonds by issuing overnight reserves. By taking out duration from financial markets, QE reduces the responsiveness of private wealth to changes in interest rates – making it more likely that the dominant effect of interest rate changes will be a "dissonant" effect, whereby many households – both active and retired – feel richer as rates increase, lowering their demand to hold assets, thus boosting the economy. This implies that QE may have weakened the conventional working of monetary policy. Concerns related to this aspect of our model have recently come to the fore. As noted in Bloomberg (2023):

- "UK households are on aggregate about £10 billion ($12.7 billion) a year better off as a result of a jump in interest rates (...) At current interest rates, savers collectively are earning £24 billion more a year than in November 2021 (...) Respondents to GfK's June consumer confidence barometer said their personal finance situation had improved sharply last month, despite the surge in mortgage rates (...) The data suggests interest rates may not be as effective a monetary policy tool as they were in 2008."

[4]See www.abp.nl/content/dam/abp/nl/documents/persbericht%20premie-indexatie%202020.pdf. A year later, ABP made a similar statement (see www.abp.nl/content/dam/abp/nl/documents/persbericht-premie-2021.pdf).

Holm et al. (2021) also document such an interest-income effect in the Norwegian administrative data.

Our framework furthermore provides a perspective for why monetary authorities have often been seen to cut rates aggressively, propping up the value of financial assets, following adverse financial shocks. Such actions have become known as the “Greenspan put” (or “Fed put” more generally), referring to interventions first observed under the Chairmanship of Alan Greenspan at the US Federal Reserve after the 1987 stock market crash.[5] If a central bank wants to maintain price stability, we show that, in our model (and in contrast to the New Keynesian model), a Greenspan put can be an optimal response to financial shocks.

Related literature. Our paper relates to several contributions to the monetary transmission literature. Firstly, our paper links to earlier papers that have enriched the standard New Keynesian model with additional transmission mechanisms. A prominent recent example of this is the “TANK/HANK” literature, extending the standard model with liquidity-constrained “hand-to-mouth” consumers. This makes transmission run less through intertemporal substitution and more via general equilibrium effects (Kaplan et al., 2018). In this sense, our work complements that of Auclert (2019) who analyzes the impact of transitory interest rate changes – showing how the unhedged interest rate exposure (distinguishing only between net assets that pay “today” versus “in the future”) is a sufficient statistic with respect to the first-order response of consumption to shocks. When considering persistent rate changes, the exact timing of cash flows starts to matter; just separating “today” from “the future” no longer suffices. In this context, Greenwald et al. (2023) develop a life-cycle model to understand how the observed decline in real rates has affected wealth inequality, also documenting how lower rates contract consumption possibilities for “the young” who have not yet accumulated many financial assets with positive duration, but have a long consumption stream to finance going forward. When rates fall, they show that the current price of future consumption rises – which makes them worse off. Fagereng et al. (2023) work out the associated welfare implications, pointing out how increases in asset valuations driven by reductions in discount rates benefit

[5] Once inflation started accelerating over 2021-22 the “Fed put” quickly transformed into a “Fed call”, the idea being that the Fed needed to *reduce* asset prices in order to bring inflation back to target. As observed by Englander (2022): “the Fed may push back against equity market gains until it is comfortable that disinflation is a lock – in other words, [there is] a Fed call”. This view was echoed by the FOMC minutes associated with the December 2022 policy meeting, which stated: “Participants noted that, because monetary policy worked importantly through financial markets, an unwarranted easing in financial conditions, especially if driven by a misperception by the public of the Committee’s reaction function, would complicate the Committee’s effort to restore price stability.”

prospective asset sellers not just any asset holder.

Our work furthermore relates to papers which have pointed out that certain aspects of lower rates may have a contractionary impact. Bilbiie (2008) features "inverted aggregate demand logic" stemming from limited asset market participation: when interest rate cuts increase marginal costs, this can lead to situations where profits flowing to asset holders *decrease*, leading to a contraction. In Mian et al. (2021) monetary stimulus promotes debt accumulation, which – while being stimulative in the short run – ultimately starts forming a drag on the economy (as savers have lower MPCs in their model). Abadi et al. (2023) and Cavallino and Sandri (2023) also present frameworks in which rate cuts can be contractionary, due to an adverse impact on banking sector profitability or capital flows, respectively. In contrast, our model often favors responses carrying the conventional signs (with rate cuts still being expansionary for many calibrations), but it implies that these responses can be very muted due to countervailing effects on asset demand. This implies that very large swings in interest rates may be needed to achieve a given effect on the economy. This may create a challenging environment from a financial stability point of view, but this is not a dimension we explore in this paper.

Finally, by placing asset prices central to the monetary transmission process, our paper builds on classical contributions like Pigou (1943), Patinkin (1948), and Keynes (1936). The latter noted that "there are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 percent (...) Perhaps the most important influence (...) depends on the effect of these changes on the appreciation or depreciation in the price of securities." More recently, Caramp and Silva (2021) show how wealth effects enter the monetary transmission mechanism in a HANK model featuring rare disasters, while Caballero and Simsek (2020, 2022, 2023) obtain a similar result in a "risk-centric" model: they decompose the demand side of the economy, normally represented by an Euler-type equation, into an output-asset price relation (capturing the notion that higher asset prices boost demand through a wealth effect) and a risk balance condition (which prices assets).

Outline of this paper. First, the next section will present evidence suggesting that it is important to control for asset demand effects when analyzing the impact of household wealth holdings on consumption. Section 3 will then present a model which has this characteristic, after which we will discuss the implications for the conduct of monetary policy in Section 4. Finally, Section 5 concludes.

2 Motivating evidence

A key implication of life-cycle forces is that households' consumption decisions will be influenced by their need/desire to put aside wealth for retirement. This force can be seen as creating a target for desired wealth holdings. The value of such target will depend on many factors including longevity, income, intertemporal substitution, and time preferences. But, very importantly, such a target level will also likely depend on the expected path for interest rates.

In the presence of life-cycle forces, it is not wealth *per se* that should drive consumption but a notion of "excess wealth", that is, the difference between the market value of wealth held by households and their targeted wealth holdings. For example, when interest rates are reduced, this tends to increase measured wealth holdings through valuation effects. However, it is not immediately clear whether this will boost consumption, as desired wealth holdings may well increase simultaneously. This is especially relevant if a reduction in interest rates is viewed as persistent, since this reduces the flow value of wealth – thereby possibly creating an increased desire to accumulate assets to compensate for the lower flow return per unit held.[6] Without controlling for interest rate effects on asset demand, the link between consumption and wealth may therefore be very weak, possibly even negative, since measured wealth can be poorly correlated with excess wealth. In contrast, once one controls for possible interest rate effects on demand for assets, consumption and excess wealth should exhibit strong positive co-movement – as people will want to spend their asset holdings in excess of their desired (targeted) levels.

In this context, Figure 1a shows the relationship between the natural log of detrended U.S. real consumption per capita ($\ln C_t$) and the natural log of detrended beginning-of-period real per-capita U.S. wealth holdings ($\ln W_{t-1}$) over 1982Q1-2019Q4.[7] Consumption and wealth are made stationary by linear detrending using the average growth rate of U.S. real GDP per capita over our sample (a quarterly rate of 0.4%). The correlation between these two series is very weak, actually slightly negative: $corr(\ln C_t, \ln W_{t-1}) = -0.064$. On the face of it, this suggests that wealth holdings are unlikely to be playing a major role in driving consumption fluctuations. However, an alternative interpretation is that this correlation is low because we are not controlling for potential asset demand effects.

[6] Indeed, Norwegian data suggest that capital gains are unlikely to be spent if stemming from lower rates of interest (Fagereng et al., 2021).

[7] All data are at the quarterly frequency and available from FRED for 1982Q1-2022Q3. The consumption series has code PCE; the wealth series has code TABSHNO. Price deflation is done using the CPI (CPIAUCSL), while per-capita amounts are calculated by dividing by POPTHM.

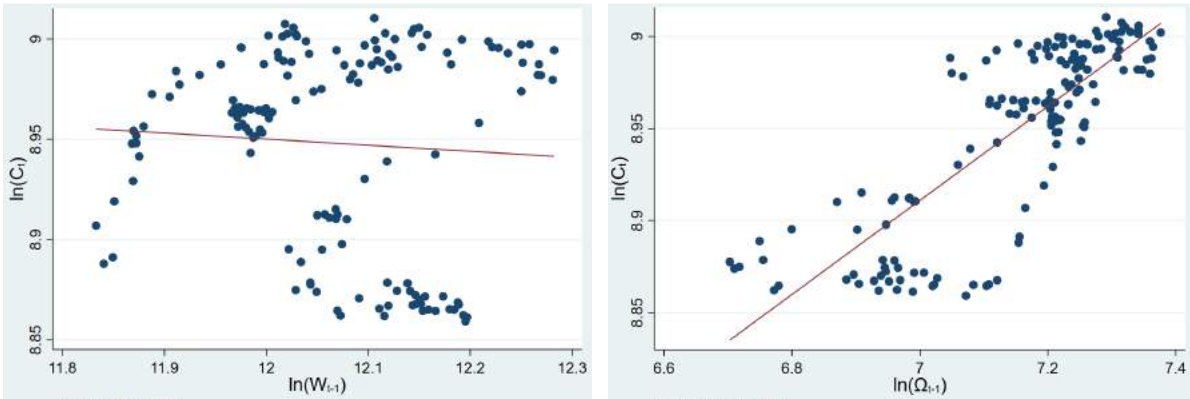

Figure 1: Scatter plot illustrating the correlation between detrended U.S. real consumption levels and detrended real wealth holdings, without and with adjusting for the level of interest rates via (1). Quarterly data from 1982Q1-2019Q4.

Figure 1b presents an alternative plot of the relationship between consumption and wealth, but now we adjust wealth by multiplying it with an interest rate factor $\mathcal{A}_t(r_t)$ to control for asset demand effects in the manner consistent with the theory we will present. In this adjustment factor, r_t is the real rate of return on long term bonds.[8] We take $\mathcal{A}_t(\cdot)$ to have the functional form prescribed by the life-cycle model developed in Section 3. As shown in Beaudry et al. (2023), this form is:

$$\mathcal{A}_t(r_t) = (\rho + \delta_2 + (\sigma - 1)r_t)\,(\rho + \delta_1 + \sigma g - r_t)^{1/\sigma}\,, \tag{1}$$

where ρ is the pure rate of time preference, δ_1 (δ_2) governs the average length of the household's working life (retirement), g the average growth rate of the economy, and σ the coefficient of relative risk aversion. We take standard values for these parameters,[9] and calculate our adjusted wealth metric as $\Omega_t \equiv \mathcal{A}_t(r_t)W_t$. Using this measure of "excess wealth"[10] we now observe a tight positive correlation with consumption: $corr(\ln C_t, \ln \Omega_{t-1}) = +0.825$. This stands in stark contrast to the -0.064 correlation we obtained *without* carrying out this simple rate-adjustment via $\mathcal{A}_t(r_t)$. Figure 1 illustrates

[8] This real rate is taken as the ex-ante 10-year real rate, available from FRED via code REAINTRATREARAT10Y.

[9] In particular, we use: $\sigma = 3$, $\rho = 0.005$, $\delta_1 = 0.025$ (average working life of 40 years), $\delta_2 = 0.05$ (average retirement period of 20 years), $g = 0.016$ (average real GDP growth rate observed in the U.S. economy over our sample period).

[10] As shown in Beaudry et al. (2023), households in our life-cycle setup have a target level of wealth that is a fraction of long run income, with this fraction depending on interest rates. We can express the target level of wealth as $\mathcal{A}_t(r_t)^{-1}Y_t$, where $\mathcal{A}_t(r_t)^{-1}$ is the relevant fraction and Y_t is the long-run income level. The ratio of observed (non-detrended) wealth $\overline{W}_t$ to targeted wealth – which is our measure of "excess wealth" – can then be expressed as $\mathcal{A}_t(r_t)\overline{W}_t/Y_t$, which corresponds to growth-detrended observed wealth $W_t \equiv \overline{W}_t/Y_t$ adjusted by the factor $\mathcal{A}_t(r_t)$.

this contrast graphically, where the improved fit of the regression line is also of note (the R-squared rises from 0.004 to 0.680).[11]

Recall that the adjustment factor $\mathcal{A}_t(r_t)$ is used to capture potential effects of changes in expected returns, as captured by long-term interest rates, on households' desire to hold assets. For this reason, it is insightful to look at the shape of the implied demand factors, that is, look at the shape of $\mathcal{A}_t(r_t)$, which is done in Figure 2. Under the parameterization considered in footnote 9, these demand factors take on a C-shape – with asset demand being an increasing function of r_t at high interest rates, but decreasing in r_t at low interest rates. As we will discuss in the theory section, the lower arm of the C-shape is driven by lower rates giving working households a desire to enter retirement with a larger stock of assets, to compensate for their lower flow return.[12]

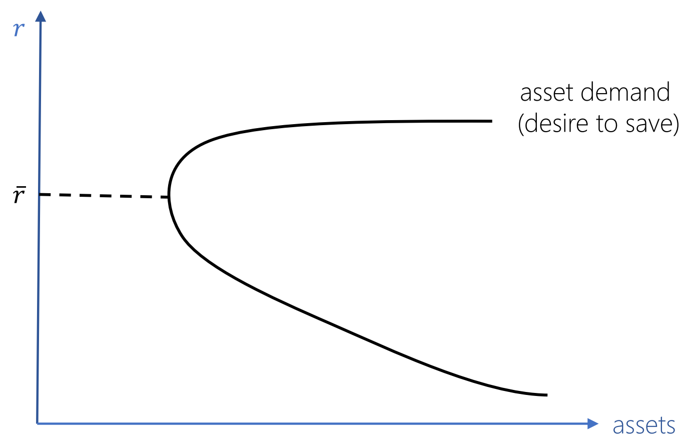

Figure 2: Illustration of equation (1), implying a C-shaped asset demand curve (considering permanent changes in r). Below the cut-off rate $\bar{r}$ reductions in r *increase* the desire of households to save (as they then need to enter retirement with a larger stock of assets to compensate for the lower flow return).

[11]A similar result emerges when taking a less theoretical approach, instead performing the adjustment by first regressing $\ln C_t$ on $\ln W_{t-1}$, the ex ante 10-year real rate (r_t) and its square (r_t^2, to allow for the effect coming from rates to be non-linear). In that case, the correlation between the adjusted wealth series and our consumption series is similarly high (+0.858, vs +0.825 for our theoretically-consistent adjusted wealth series).

[12]Interestingly, the purely empirical approach described in footnote 11 gives rise to a similar C-shape. The results from that regression moreover suggest that the U.S. economy has mostly operated on the *lower* arm of the C-shape over our sample period. To see this, note that if one fails to control for r_t, a regression of $\ln C_t$ on $\ln W_{t-1}$ leads to a downward bias for the coefficient on wealth if $\mathcal{A}_t(r_t)$ is mainly increasing in r_t, while leading to an upward bias if $\mathcal{A}_t(r_t)$ is mainly decreasing in r_t. Since regressing $\ln C_t$ on $\ln W_{t-1}$ gives rise to a *negative* coefficient on wealth (equal to -0.03), while standard logic and studies like Chodorow-Reich et al. (2021) suggest this coefficient should be positive, this is pointing to a downward bias – implying that $\mathcal{A}_t(r_t)$ is mainly increasing in r_t. That, in turn, implies that the target level of wealth $\mathcal{A}_t(r_t)^{-1}Y_t$ goes *up* as rates fall; recall footnote 10. Indeed, when adding r_t and r_t^2 as controls to the regression, the coeffiicent on $\ln W_{t-1}$ rises to 0.14.

When it comes to the possibility that asset demand may be C-shaped, with low interest rates increasing asset demand, a look at the raw data on savings provides a complementary perspective on the same theme. Figure 3 plots monthly observations of the saving rate (calculated in percent of disposable income) alongside the 10-year ex ante real rate.[13]

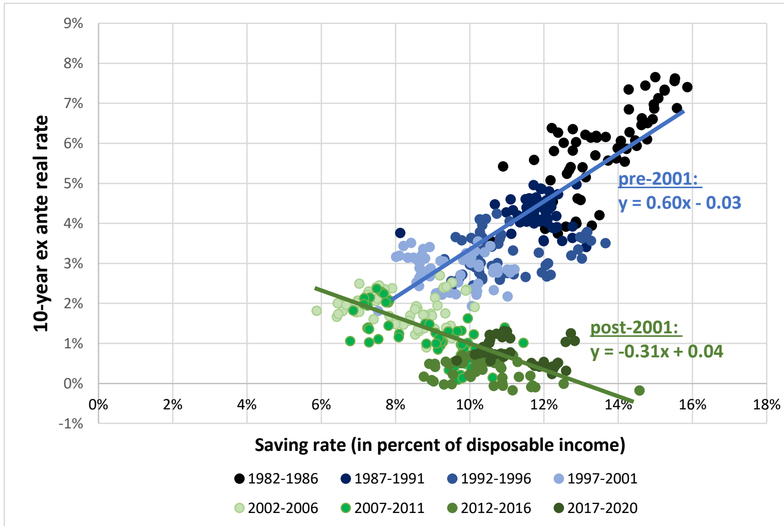

Figure 3: Scatter plot of U.S. savings rate vs. the 10-year ex ante real rate. Monthly data from January 1982-February 2020.

As Figure 3 shows, the early (pre-2001) part of the sample displays a strong positive relationship between the real rate and saving rate. But in the late (post-2001) sample, which is characterized by lower rates on average, the relationship flips sign and reductions in the 10-year real rate are associated with a *higher* saving rate. This is consistent with the notion that the lower rates pushed up households' desire to hold assets by more than the accompanying valuation gain. While these are just raw data with various possible explanations,[14] it is exactly the type of pattern one would expect to arise if asset demand is a C-shaped function of interest rates. Going beyond raw data, Nabar (2011) reports similar findings for China: controlling for various factors (like income growth and volatility), he finds that Chinese households' desire to save went up by more in provinces that saw a bigger fall in real rates. Further supporting this idea, recent papers (Van den End et al., 2020; Felici et al., 2023; Ahmed et al., forthcoming) have observed that the

[13]All data are taken from FRED. In particular we use the 10-year ex ante real rate ("REAINTRATREARAT10Y"), personal consumption expenditures ("PCE"), and disposable personal income ("DSPI"). Savings are then calculated as "DSPI−PCE".

[14]For example: it could be the case the early part of the sample was dominated by asset supply shocks (tracing out a conventional, positively-sloped asset demand function) while the later part of the sample was dominated by asset demand shocks (tracing out the negatively-sloped asset supply function).

standard substitution effect appears especially weak when interest rates are low.

To summarize, this section has presented data patterns to support two points. First, that consumption is strongly correlated with a notion of "excess wealth", that is, wealth corrected for potential effects of interest rates on asset demand. Second, that the adjustment factor for such demand effects takes a C-shaped form, suggesting that the demand for wealth may be increasing at both high and low interest rates. This C-shaped pattern was further supported by savings patterns.

We now turn to a presenting a life-cycle model which gives rise to such an asset demand structure.

3 A life-cycle model for monetary policy

This section describes our model, which can be seen as a discrete-time and stochastic version of the model in Beaudry et al. (2023), extended to feature both short- and long-term debt and embedded in a New Keynesian setup. In particular, firms are assumed to be monopolistically competitive while facing price adjustment costs – giving rise to a standard New Keynesian Phillips Curve. One could refer to this type of model as a "FLANK", for Finitely-Lived Agent New Keynesian model.

Environment. There is a measure one of households, subject to a life-cycle dynamic that follows Gertler (1999). Each household starts life in a work state and transits out with Poisson probability δ_1. At this transition, the household retires with probability z (i.e., "it survives the retirement shock") while the household instantly dies with complementary probability $(1 - z)$; households falling in this latter category can be seen as dying during their working life, so allowing for $z < 1$ enables us to match overall life expectancy (in addition to life expectancy upon reaching retirement). If the household survives to enter the retirement stage, it faces a per-period Poisson probability of dying equal to $\delta_2 \geq \delta_1$. Deceased households are immediately replaced by new, working households, implying that the fraction of working households is constant at $\vartheta = \delta_2/(z\delta_1 + \delta_2)$.

Working households supply labor and own all firms operating in the economy. Each household owns and manages a good-producing firm and a financial firm. Upon retirement, both are liquidated and replaced by new ones owned by a new working household.

Retired Households. The household structure of the model is best understood backwards. In the retirement state, a household only derives income from its financial wealth a_t^j. Retired households invest their financial wealth in risk-free deposits issued by domestic banks that pay the gross nominal interest rate i_t^d. The problem of a retired household j (characterized by CRRA-preferences with risk aversion parameter σ) reads:

$$V_t^r\left(\tilde{a}_t^j\right) = \max_{c_t^j}\left\{\frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[(1-\delta_2)\,V_{t+1}^r\left(\tilde{a}_{t+1}^j\right)\right]\right\},$$
$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}\left(\tilde{a}_t^j - c_t^j\right), \tag{2}$$

where c_t^j is consumption, $r_{t+1} \equiv i_t^d/\pi_{t+1}$ denotes the gross real rate of return of deposits and $\tilde{a}_t^j \equiv r_t a_{t-1}^j$ is the beginning-of-period t assets held by household j, such that the real rate of return r_t gets to work on whatever is left after period-$(t-1)$ consumption has been financed, i.e. on $a_{t-1}^j = \tilde{a}_{t-1}^j - c_{t-1}^j$. This yields:

$$\left(c_t^j\right)^{-\sigma} = \beta\left(1-\delta_2\right)\mathbb{E}_t\left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j}r_{t+1}\right]. \tag{3}$$

At the same time, the envelope theorem implies that:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(c_t^j\right)^{-\sigma}. \tag{4}$$

If we combine this with our guess that $V_t^r\left(\tilde{a}_t^j\right) \equiv \frac{\left(\tilde{a}_t^j\right)^{1-\sigma}}{1-\sigma}\Gamma_t$, with Γ_t conjectured to be a function of the future path of r_t and independent of $\tilde{a}_t^j$, this gives:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(\tilde{a}_t^j\right)^{-\sigma}\Gamma_t. \tag{5}$$

By combining (4) and (5) we obtain that:

$$\left(c_t^j\right)^{-\sigma} = \left(\tilde{a}_t^j\right)^{-\sigma}\Gamma_t \Leftrightarrow c_t^j = \tilde{a}_t^j\Gamma_t^{-\frac{1}{\sigma}}, \tag{6}$$

which we can plug into (2) to yield:

$$\tilde{a}_{t+1}^j = r_{t+1}\tilde{a}_t^j\left[1 - \Gamma_t^{-\frac{1}{\sigma}}\right]. \tag{7}$$

Finally, plugging (5), (6), and (7) into (3) gives a non-linear difference equation for Γ_t:

$$\left(\Gamma_t^{\frac{1}{\sigma}} - 1\right)^{\sigma} = \left(1-\delta_2\right)\beta\mathbb{E}_t\left[\left(r_{t+1}\right)^{1-\sigma}\Gamma_{t+1}\right]. \tag{8}$$

This verifies our guess that Γ_t is independent of $\tilde{a}_t^j$, confirming that it is only a function of future expected rates of return.

Writing utility of retired agents as $V^r\left(\tilde{a}_t^j, \Gamma_t\right) = (1-\sigma)^{-1}(\tilde{a}_t^j)^{1-\sigma}\Gamma_t$ illustrates that it depends both on the stock of assets with which the household enters retirement at date t ($\tilde{a}_t^j$) as well as on the entire future path of interest rates working over that stock (captured by Γ_t). For a given level of assets $\tilde{a}_t^j$, retired households are better off when rates are expected to be high (as this offers them a possible superior stream of interest revenues).

The linearity of (6) allows us to construct a representative retired household. Let $c_t^r \equiv \int_{\mathbf{R}_r} c_t^j dj / (1-\vartheta)$ be the consumption of the representative retired agent and define $a_t^r \equiv \int_{\mathbf{R}_r} a_t^j dj / (1-\vartheta)$ as its financial wealth, where $\mathbf{R}_r$ denotes the set of households in the retired state. Then:

$$c_t^r = a_t^r \left(\Gamma_t^{\frac{1}{\sigma}} - 1\right)^{-1},$$

which shows how consumption of retirees is driven by their wealth holdings (a_t^r) adjusted for the expected path of interest rates (as captured by the term $\left(\Gamma_t^{\frac{1}{\sigma}} - 1\right)^{-1}$). Finally, a_t^r evolves as:

$$a_{t+1}^r = r_{t+1}\left(1 - \Gamma_{t+1}^{-\frac{1}{\sigma}}\right)\left[(1-\delta_2)\, a_t^r + \delta_2 a_t^w \left(1 + x_{t+1}^w\right)\right],$$

with a_t^w being the financial wealth introduced by the retiring cohort of workers and x_{t+1}^w representing the net excess return on its portfolio, as defined below.

Working Households. Next, consider a working household. It receives a real wage w_t for any labor input ℓ_t it provides, plus dividends from the good-producing and financial firms it owns, and transfers from/to the government. A working household also faces a $\delta_1 z_s$ probability of moving into retirement next period. With probability $\delta_1(1-z_s)$ the household ceases to work but dies – thus leaving the model immediately. Upon retirement, a household liquidates its firms and lives off its financial wealth until death, as described above. Consequently, a working household's problem can be written as:

$$V_t^w\left(\tilde{a}_t^j\right) = \max_{c_t^j, a_t^j, \tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} - \chi \frac{(\ell_t)^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_t\left[(1-\delta_1)\, V_t^w\left(\tilde{a}_{t+1}^j\right) + \delta_1 z_s V_t^r\left(\tilde{a}_{t+1}^j\right)\right] \right\},$$

$$\begin{aligned} s.t.\ \tilde{a}_{t+1}^j &= r_{t+1} a_t^j + d_{t+1}^j, \\ a_t^j &= \tilde{a}_t^j - c_t^j + \ell_t w_t + \tau_t, \end{aligned}$$

where d_{t+1}^j are dividends received from financial firms, ℓ_t is labor, $z_s = z + \varpi$ represents the household's *subjective* probability of surviving the retirement shock (which they over-estimate by $\varpi > 0$, for reasons explained below), w_t denotes the real wage and τ_t is a lump-sum income component. It is given by $\tau_t = \tau_t^y + \tau_t^g - \tau_t^\phi$ where the first term represents dividends received from good-producing firms and τ_t^g denotes transfer received from the government. Finally, τ_t^ϕ is a tax designed to equalize the assets of new and existing working households. The optimality conditions give rise to the following Euler equation:

$$\left(c_t^j\right)^{-\sigma} = \beta \left\{ (1-\delta_1)\,\mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} r_{t+1} \right] + z_s \delta_1 \mathbb{E}_t \left[\left(\tilde{a}_{t+1}^j\right)^{-\sigma} \Gamma_{t+1} r_{t+1} \right] \right\}, \tag{9}$$

and the labor supply schedule:

$$\chi c_t^j \left(\ell_t^j\right)^\varphi = w_t.$$

Note how the Euler equation for working households (9) features two terms on the RHS: the first term is familiar from the standard models without retirement and implies that a lower interest rate, *ceteris paribus*, works to decrease the household's desire to save; this is the standard force of intertemporal substitution. The second term on the RHS of (9), however, stems from the introduction of the prospect of retirement and shows how consumption is again driven by wealth ($\tilde{a}_{t+1}^j$) adjusted for the expected path of interest rates (as captured by $\Gamma_{t+1} r_{t+1}$).

At this point, it can be helpful to highlight an important implication of equation (9), namely that its steady state version can be thought of as defining a target wealth-to-consumption of ratio for active households. The steady state of $\tilde{a}^j/c^j$ is given by:

$$\frac{\tilde{a}^j}{c^j} = \left(\frac{\beta z_s \delta_1 \Gamma r}{1 - \beta(1-\delta_1) r} \right)^{1/\sigma} = \left(\frac{\beta z_s \delta_1}{\left[r^{-1} - \beta(1-\delta_1)\right] \left[1 - \left(\left((1-\delta_2)\beta r^{1-\sigma}\right)^{\frac{1}{\sigma}} \right)^{\sigma} \right]} \right)^{1/\sigma}, \tag{10}$$

where the we have used the steady-state version of (8) to express Γ as a function of a constant r. A key result from Beaudry et al. (2023), which carries over to our setup, is that when $\sigma > 1$ (implying an EIS < 1), equation (10) implies that the target asset-to-consumption ratio of working households becomes "C-shaped" in r as illustrated in Figure 2. It follows that there exists a critical level of r (call it $\bar{r}$) below which reductions in the interest rate *increase* households' desire to hold assets. The reason is that when interest rates are low and are expected to remain low, retirement considerations can outweigh the standard intertemporal substitution channel. In particular, when $\sigma > 1$ and $r_t < \bar{r}$, the marginal value of owning an asset goes up as the associated flow income (determined by the interest rate) falls – as one now needs to possess more assets to finance a given level of consumption in retirement. This echoes the 2019 statement issued by Dutch pension fund ABP (quoted in the Introduction), noting that "pensions are becoming increasingly expensive" due to "the expectation that interest rates will remain low for a long time".

Since the assets of new and existing working households are equalized at each point in time, working households are homogeneous. Let c_t^w denote the consumption of the representative working household and a_t^w its end-of-period financial wealth. Then c_t^w solves:

$$(c_t^w)^{-\sigma} = \beta\left\{(1-\delta_1)\,\mathbb{E}_t\left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1}\right] + z_s\delta_1\,(a_t^w)^{-\sigma}\,\mathbb{E}_t\left[\left(1+x_{t+1}^w\right)^{-\sigma}\Gamma_{t+1}\,(r_{t+1})^{1-\sigma}\right]\right\},$$

where a_t^w– once we incorporate the transfers τ_t^ϕ needed to keep wealth constant across working cohorts – evolves as:

$$a_{t+1}^w = (1-z\delta_1)\,a_t^w r_{t+1}\left(1+x_{t+1}^w\right) + z\delta_1 a_t^r r_{t+1} - c_{t+1}^w + \ell_{t+1}w_{t+1} + \tau_t^y + \tau_t^g,$$

and $x_{t+1}^w = \frac{1}{a_t^w}\frac{d_{t+1}}{r_{t+1}}$ is the working household's net excess return on its portfolio of assets, which includes both deposits and dividends from banks.

Good-producing firms. Each working household $j \in \mathbf{R}_w$ owns and manages a firm that produces a differentiated good using the technology $y_t^j = A\ell_t^j$. Firms are monopolistically competitive and set prices subject to a quadratic adjustment cost *a la* Rotemberg (1982). Let P_t^j be the price chosen by firm j at time t and $\pi_t^j \equiv P_t^j/P_{t-1}^j$ be its growth rate. Then, the firm pays the following adjustment cost $\Theta\left(\pi_t^j\right) = y_t^j\frac{\theta}{2}\left(\pi_t^j-\bar{\pi}\right)^2$, where $\bar{\pi}$ is the inflation target of the central bank, and θ governs the cost of adjusting prices. The resulting Phillips curve takes the standard form (which, to a first-order approximation, has the same reduced form as the one resulting from Calvo-pricing; see Roberts (1995)):

$$(\pi_t-\bar{\pi})\,\pi_t = \kappa\,(mc_t-1) + \mathbb{E}_t\left[\Lambda_{t,t+1}\,(\pi_{t+1}-\bar{\pi})\,\pi_{t+1}\frac{y_{t+1}}{y_t}\right],$$

where $\kappa \equiv (\epsilon-1)/\theta$ represents the slope of the Phillips curve and ϵ is the elasticity of substitution across product varieties,[15] $y_t = \int_{\mathbf{R}_w} y_t^j dj$ denotes aggregate output (since there is no price dispersion all firms produce the same amount of output), while $\Lambda_{t,t+1}$ is the stochastic discount factor of the representative working household:

$$\Lambda_{t,t+1} = \beta\frac{(1-\delta_1)\left(c_{t+1}^w\right)^{-\sigma} + z_s\delta_1\,(a_t^w)^{-\sigma}\left(1+x_{t+1}^w\right)^{-\sigma}\Gamma_{t+1}r_{t+1}^{-\sigma}}{(c_t^w)^{-\sigma}}.$$

[15]Households consume a CES aggregate of all varieties: $c_t^j = \left[\int_{\mathbf{R}_w} c_t^j\,(j)^{\frac{\epsilon-1}{\epsilon}}\,dj\right]^{\frac{\epsilon}{\epsilon-1}}$. Notice that the Phillips curve already assumed that the government implements a constant labor subsidy to undo the steady-state markup $\epsilon/(\epsilon-1)$. This subsidy is financed through lump-sum taxes levied on firms.

This captures the familiar notion that households place more weight on the future when their marginal utility is high, but it features the additional forces stemming from retirement preoccupations. In particular, households now place more weight on the future when they hold fewer assets a_t^w or when the interest rate path is lower (the former aspect of this is captured by $r_{t+1}^{-\sigma}$, while Γ_{t+1} captures the future rate path).

The real marginal cost of production is $mc_t = (1 - \tau_t^w)\, w_t/A$, where τ_t^w is a wage subsidy financed through lump-sum taxes levied directly on good-producing firms. The real dividend generated by each firm is $\tau_t^y = \frac{y_t}{\vartheta}\left[1 - \frac{\theta}{2}(\pi_t - \bar{\pi})^2\right] - \ell_t w_t$ while their liquidation value is always zero because they have no physical capital.[16]

Financial firms. The model also features a continuum of identical financial firms, each owned and managed by a working household. Financial firm collect deposits from households and invest in short- and long-term government bonds. Short-term government bonds are one-period assets whose nominal return, i_t, is set by the central bank. Following Woodford (2001), we model long-term bonds as real perpetuities with coupons that decay geometrically at rate ρ. This implies that a bond issued in period t pays $(1-\rho)^h$ units of consumption $h+1$ periods later. Setting $\rho = 1$ reduces this bond to a one-period instrument, while $\rho = 0$ represents an infinitely lived consol (so the bond's duration is decreasing in ρ). The return on this long-term bond is:

$$r_{t+1}^b = \frac{1 + (1-\rho)\, q_{t+1}}{q_t},$$

where q_t is the price of the long-term bond.

For simplicity, we assume that financial firms operate with no capital. The balance sheet of intermediary $j \in \mathbf{R}_w$ is

$$q_t b_t^j + s_t^j = a_t^j,$$

where a_t^j are deposits collected from retired and working households,[17] while b_t^j and s_t^j represent intermediary j's holding of long- and short-term bonds, respectively. The return on its asset portfolio is given by:

$$d_{t+1}^j = s_t^j \left(\frac{i_t}{\pi_{t+1}} - r_{t+1}\right) + q_t b_t^j \left(r_{t+1}^b - r_{t+1}\right).$$

[16] Alternatively, we could assume that retiring households sell their good-producing firms to new households. This would strenghten the "asset valuation channel" (described later) as a rate cut would then not only increase bond prices, but also stock prices.

[17] For the moral hazard problem described below to arise, we must assume that each household cannot deposit in its own financial intermediary.

Notice that, since the intermediary operates without capital, d^j_{t+1} equals both the dividend paid to household j and the liquidation value of the financial intermediary at time $t+1$.

To capture the role of limited financial risk-bearing capacity, we assume that financial firms are subject to a moral hazard problem which might limit their ability to raise deposits. In each period, after taking positions but before shocks are realized, the manager of the financial firm can divert a fraction of its long-term assets. If it diverts the funds, the firm is unwound and depositors recover a portion $1-\mu_t\left|\bar{b}^j_t\right|$ of their deposits b^j_t, where $\bar{b}^j_t \equiv b^j_t/b_t$ represents bank j's holding of long-term government bonds relative to the rest of the market. The variable $\mu_t \geq 0$ captures the severity of the financial friction that affects the economy. An increase in μ_t is akin to a negative financial shock, as it implies a more severe moral hazard problem between the financial intermediary and its creditors. Our functional assumption regarding the diversion of funds is not only a convenient specification for tractability, but it also captures the idea that (relatively) bigger balance sheets lead to more complex positions that are more difficult and costly for creditors to unwind.

Due to this moral hazard problem, depositors are willing to lend to the financial firm if and only if the following incentive compatibility constraint holds:

$$V^j_{f,t} \geq \mu_t \bar{b}^j_t q_t b^j_t,$$

where $V^j_{f,t}$ is the value of financial intermediary j and solves the following constrained maximization problem:

$$V^j_{f,t} = \max_{s^j_t, b^j_t} \mathbb{E}_t\left[\Lambda^j_{t,t+1} d^j_{t+1}\right] \quad \text{s.t.} \quad V^j_{f,t} \geq q_t b^j_t \mu_t \bar{b}^j_t.$$

Since the value of the financial intermediary is linear in b^j_t, while the right-hand side of the constraint is convex in b^j_t, the constraint always binds. Hence, the first order conditions with respect to b^j_t give rise to the following pricing equation:

$$\mathbb{E}_t\left[\Lambda^j_{t,t+1}\left(r^b_{t+1} - r_{t+1}\right)\right] = \mu_t \bar{b}^j_t \tag{11}$$

where $b_t = \int b^j_t dj$ denotes the aggregate financial sector holding of long-term government bonds. Since all intermediaries are identical, integrating (11) across js yields:

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\left(r^b_{t+1} - r_{t+1}\right)\right] = \mu_t \tag{12}$$

Finally, the first order condition with respect to s^j (the intermediaries' holdings of short-term bonds) yields $i_t^d = i_t$, which implies $r_{t+1} = i_t/\pi_{t+1}$.

Government. The budget constraint of the government reads:

$$s^g + q_t b^g = q_{t-1} b^g r_t^b + s^g r_t - \vartheta \tau_t^g,$$

where s^g and b^g are the supply of short- and long-term government bonds, respectively, which we assume to be constant for now. This implies that tax policy must satisfy:

$$\phi \tau_t^g = b^g \left(1 - \rho q_t\right) + s^g \left(r_t - 1\right).$$

The central bank conducts monetary policy according to the following Taylor rule:

$$i_t = r\bar{\pi} \left(\frac{\mathbb{E}_t \left[\pi_{t+1}\right]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t}, \tag{13}$$

where $\phi > 0$ governs the responsiveness of the central bank to expected deviations from the inflation target ($\bar{\pi}$), r is the steady-state real interest rate, and ε_t is a monetary policy shock.

Market clearing . Market clearing requires that:

$$\vartheta c_t^w + (1 - \vartheta) c_t^r = y_t \left[1 - \frac{\theta}{2} \left(\pi_t - \bar{\pi}\right)^2 \right],$$
$$\vartheta a_t^w + (1 - \vartheta) a_t^r = s^g + q_t b^g,$$

where it should be recalled that ϑ is the share of working households, while we used $\int s_t^j dj = s^g$ and $\int b_t^j dj = b^g$.

Exogenous Processes. We allow the model to be hit by two types of shocks: first, a standard monetary policy shock "ε_t" to the Taylor rule (13) and, second, a shock to the severity of the financial friction "μ_t" in (12) . In that regard, we model $\mu_t = \mu + \zeta_t$. The exogenous variables ε_t and ζ_t are assumed to follow AR(1) processes:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \sigma_\varepsilon v_t,$$
$$\zeta_t = \rho_\zeta \zeta_{t-1} + \sigma_\zeta u_t,$$

with the innovations "v" and "u" following a standard-normal distribution (σ_ε and σ_ζ scale the shocks' standard deviations).

We furthermore assume that the inflation target is zero ($\bar{\pi} = 1$). The equilibrium and steady state equations of our full model can be found in Appendix A.

4 Model properties: analytical and quantitative

In order to highlight how life-cycle forces affect monetary policy, we proceed in two steps. We first simplify our model to derive a set of analytical results that help clarify the main mechanisms at play. Then we return to the more general setup and examine implications quantitatively. In particular, the quantitative model will validate the usefulness of the simplified model by showing that its main insights are maintained in the more general setup despite the simplifying assumptions being quite strong.

4.1 Simplifying the model

The presence of life-cycle forces has the effect of both influencing the consumption-saving decisions of active households, and gives rise to a retirement state where asset income is important. A priori, the relative importance of each of these elements for understanding monetary policy transmission is unclear. The role of the retirement state is intuitively more straightforward as a change in interest rates will have a direct effect on the consumption of any retiree who lives off interest income. However, as our analysis will illustrate, the effect of retirement preoccupations on the behavior of active households may be equally (if not more) important. To show why this may be the case, this section simplifies our model to focus solely on the role of retirement preoccupations in influencing the behavior of active households – thus abstracting from actual retirees. To achieve this, we assume that $z = 0$ and $z_s = \varpi$. This simplification implies that no one actually survives the retirement shock, even though all households think they need retirement savings. Retirement savings can be interpreted as "prudent" in this simplified setup, stemming from the fact that households over-estimate their true probability of surviving the retirement shock by a degree $\varpi > 0$. This creates a "prudent perpetual youth" (PPY) structure that enables us to abstract from the asset demand stemming from retired agents and obtain closed-form solutions capturing the behavior of active households. To further simplify, we set $\delta_2 = 0$ so that active households expect to live off the flow income from their assets in retirement indefinitely. Finally, we set ϖ such that $r = 1/\beta$.[18] This will allow us to perform simple comparative statics with respect to δ_1 ($\delta_1 \in [0, 1]$) , where variations in δ_1 can be seen as determining the salience of retirement preoccupations for active households.

This set of assumptions simplifies the log-linearized equilibrium of the model to (using

[18]This implies that $\varpi = \left(\frac{1-\beta}{\beta} \frac{s^g + b^g \left(\frac{1}{\beta} - 1 + \rho\right)^{-1}}{y} \right)^{\sigma}$.

hats to denote a variable's log-deviation from steady state):

$$\hat{y}_t = (1-\delta_1)\left[\mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right] + \delta_1\left[\eta\left(\hat{q}_t + \zeta_t\right) + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{\Gamma}_{t+1}\right] \quad (14)$$

$$\hat{\Gamma}_t = \beta\left[\mathbb{E}_t\hat{\Gamma}_{t+1} - (\sigma-1)\,\mathbb{E}_t\hat{r}_{t+1}\right] \quad (15)$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa\,(1+\varphi)\,\hat{y}_t \quad (16)$$

$$\hat{q}_t = -\hat{r}_{t+1} + \beta\,(1-\rho)\,\mathbb{E}_t\hat{q}_{t+1} - \zeta_t \quad (17)$$

$$\mathbb{E}_t\hat{r}_{t+1} = \phi\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon_t, \quad (18)$$

where $\eta \equiv qb/(s+qb) \in [0,1]$ denotes the steady-state share of long-term assets over total assets. The parameter η becomes a measure of how sensitive the value of financial assets in the economy are to interest rates.

From (14) one can see how the Euler equation incorporates both the standard force of intertemporal substitution, as captured by the first term on the RHS, and a second term which captures the wealth-related factor associated with retirement preoccupations. As the perceived probability of entering the retirement state (δ_1) goes up, the weight on the wealth-related factor increases relative to the role of intertemporal substitution. In this sense, life-cycle forces can be seen as placing wealth at the center of consumption decisions and the monetary transmission mechanism.

From (14), one can see that the wealth-related factor consist of two distinct parts: a direct wealth effect (in blue) and an effect stemming from asset demand (in red). Let's discuss these in turn, starting with the former. As (17) shows, a higher real rate depresses the price q of the long-term bond contemporaneously. Via the blue term in equation (14) this exerts a negative effect on $\hat{y}_t$. We will call this the "asset valuation channel". It works as a pure wealth effect, with rate hikes weighing on economic activity.

But at the same time, the red terms in (14) indicate that if $\sigma > 1$ a higher real rate also exerts a countervailing force *increasing* $\hat{y}_t$. The reason is that, for a fixed value of assets, a higher interest rate implies that these assets will deliver a greater flow return to the owning household. This greater flow return lowers the need to hold as many assets for retirement when $\sigma > 1$, thus lowering asset demand, thereby stimulating demand for goods. To the extent that the increase in the interest rate is expected to persist, equation (15) – which summarizes the expected path of future interest rates – shows that this gets captured through a lower $\mathbb{E}_t\hat{\Gamma}_{t+1}$, giving this channel a boost. We call this red term in (14) the "asset flow channel", but one should keep in mind that it stems from interest rates affecting asset demand.

As illustrated in Figure 2 (which holds asset valuation constant and has $\sigma > 1$), the non-linear version of this setup gives rise to a cut-off rate of interest $\bar{r}$ where the intertemporal substitution channel exactly cancels out against the asset flow channel. In the region where $r > \bar{r}$, intertemporal substitution dominates the asset flow channel, while the asset flow effect is dominant if $r < \bar{r}$. In the latter case, the effects of persistent interest rate changes only carry the conventional signs if the asset valuation effect dominates the sum of the substitution and asset flow effects. This is the sense in which the asset valuation effect can become essential to the monetary transmission mechanism: without it, monetary policy operates in the "wrong" direction (with rate hikes being expansionary to output). So when the asset market equilibrium is located on the lower arm of the C-shape, a strong asset valuation effect becomes *necessary* for rate hikes to obtain their standard contractionary effect. This contrasts with the upper arm of the C-shape, where intertemporal substitution dominates the countervailing asset flow effect – implying that rate hikes are contractionary even when asset prices are held constant. For equilibria located on the upper arm of the C-shape, any asset valuation effects are thus a mere bonus to the monetary transmission process: nice to have, but strictly not necessary.

In the linearized version of the model, for fixed parameter values, the economy cannot switch between being on the upper versus lower arm of the C-shape representing asset demand in Figure 2. Instead, the economy will be operating on either the upper *or* the lower arm. If $EIS > 1$ ($\sigma < 1$), there is no lower arm. Given our simplifying assumptions,[19] when $EIS < 1$ ($\sigma > 1$), the economy will be operating on the lower arm if retirement preoccupations are sufficiently strong, that is, when $\delta_1 \geq (1-\beta)/(\sigma-\beta) \equiv \bar{\delta_1}$; for $\delta_1 < \bar{\delta_1}$, the economy operates on the upper arm.

Equipped with this PPY-version of the model, we are now able to establish some analytical results.

4.2 Analytical results under the prudent perpetual youth structure

Equilibrium determinacy. Recall that monetary policy is governed by the parameter ϕ, which expresses the degree to which expected real interest rates are increased in response to expected inflation. A policy with $\phi = 0$ corresponds to a constant real interest rate policy. The Taylor principle would suggest that ϕ may need to be strictly greater than zero. However in our setup, as expressed in Proposition 1, the model always

[19] To further simplify the model, we will treat long term bonds as perpetuities ($\rho = 0$)

maintains determinacy if $\phi = 0$. More generally, determinacy appears to require that $\underline{\phi} \leq \phi \leq \bar{\phi}$ where $\underline{\phi} < 0$ and $\bar{\phi} > 0$. While we are not able to provide an explicit expression for $\bar{\phi}$ for this version of our model, we can derive explicit expressions if we focus on the special case of a static Phillips Curve, that is, when $\hat{\pi}_t = \kappa(1+\varphi)\hat{y}_t$. For that case, it can be shown that determinacy requires $\underline{\phi} \leq \phi \leq \bar{\phi}$ where $-\infty < \underline{\phi} < 0$, and if $\delta_1 > (1-\beta)/[(1-\eta)\sigma - \beta]$, then $0 < \bar{\phi} < \infty$.[20] We will also show numerically that, in the calibrated version of the full model, ϕ will generally need to satisfy both an upper and a lower bound with $\bar{\phi} > 0$ and $\underline{\phi} < 0$. This implies that monetary policy cannot react too aggressively or too dovishly to inflation if it wants to maintain determinacy.

Proposition 1. *With $\theta > 0$ (sticky prices), a constant real rate policy ($\phi = 0$) always delivers determinacy.*

Proofs of all propositions are recorded in Appendix B. In light of Proposition 1, the rest of this section will derive results under $\phi = 0$ as to ensure determinacy.

Effect of Monetary policy shocks. When $\phi = 0$, equilibrium dynamics in the presence of monetary policy shocks "ε" are described by:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & -\frac{\delta_1}{\sigma} & 0 & \delta_1\eta\beta \\ 0 & \beta & 0 & 0 \\ \kappa(1+\varphi) & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{y}_{t+1} \\ \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} - \delta_1(1-\eta) \\ \beta(\sigma-1) \\ 0 \\ 1 \end{bmatrix} \varepsilon_t,$$

The impact responses of output and inflation to a monetary policy shock are then determinate and given by:

$$\hat{y}_0 = -\frac{1-\rho_\varepsilon}{\sigma} \frac{1-\delta_1\frac{\sigma(1-\eta)-\rho_\varepsilon\beta}{1-\rho_\varepsilon\beta}}{1-\rho_\varepsilon(1-\delta_1)} \bar{\varepsilon}, \tag{19}$$

$$\hat{\pi}_0 = \kappa\frac{1+\varphi}{1-\rho_\varepsilon\beta}\hat{y}_0, \tag{20}$$

where $\bar{\varepsilon} \equiv \sum_{t=0}^{\infty} \rho_\varepsilon^t v_0 = v_0/(1-\rho_\varepsilon)$ is the overall monetary policy impulse. Since the PPY-version of our model features no state variables, we have that $\hat{y}_t = \rho_\varepsilon^t \hat{y}_0$ and $\hat{\pi}_t = \rho_\varepsilon^t \hat{\pi}_0$ – implying that the propositions we are about to present continue to apply to longer

[20] To be more precise, in this case, the value of $\bar{\phi}$ is given by $\frac{\delta_1\sigma}{\kappa(1+\varphi)}\left[\frac{(1-\eta)\sigma-\beta}{1-\beta}\delta_1 - 1\right]^{-1} < \infty$.

horizons $t > 0$.

Equations (19) and (20) carry several interesting implications that are captured in Propositions 2-5. First, Proposition 2 indicates that that the sensitivity of output to the monetary policy shock is decreasing in δ_1, the strength of retirement preoccupations. By the structure of the Phillips curve (20), the same applies to the potency of monetary policy over inflation.

Proposition 2. *The ability of a surprise interest rate cut $\bar{\varepsilon} < 0$ (hike, $\bar{\varepsilon} > 0$) to boost (contract) output and inflation is decreasing in retirement preoccupations δ_1.*

An increase in retirement preoccupations, as captured by δ_1, affects the monetary transmission mechanism in several ways. First, it reduces the strength of intertemporal motives in determining savings. Second, it introduces the financial channel, which consists of the asset valuation channel as well as the asset flow channel. Proposition 2 shows that the net effect of these changes is to decrease the potency of monetary policy.

Proposition 3 clarifies how the interest-rate sensitivity of government bonds held by the public, as captured by the average bond duration η, affects the monetary transmission mechanism.

Proposition 3. *With $\delta_1 > 0$, the ability of an interest rate cut, $\bar{\varepsilon} < 0$ (hike, $\bar{\varepsilon} > 0$), to boost (contract) output and inflation is increasing in the duration of assets held by the public (η).*

Proposition 3 is relevant when thinking about the potential role of QE in affecting the monetary transmission mechanism.[21] In particular, QE acts like an asset swap, with the central bank replacing high-duration assets (longer-term government bonds) with overnight central bank reserves carrying zero duration. Accordingly, QE can thus be seen as the central bank lowering η, which renders conventional monetary policy (changes in the interest rate) less potent. This suggests that in a post-QE (lower η) world, central banks may need to move the interest rate by more to bring about a given effect on output and prices. The main reason that η matters for monetary policy is that it governs the strength of the asset valuation channel. Recall that when the economy is operating on the lower arm of the C-shape asset demand, then the asset valuation channel is the key mechanism by which a decrease in interest rates boosts economic activity.

From equation (19), we can also infer that a monetary easing has the potential to be contractionary (and vice versa). The condition for this is given in Proposition 4.

[21] At this stage it is important to recall that our Blanchard-Yaari-Gertler setup implies that Ricardian Equivalence does not hold; because of this breakdown, the maturity structure of assets held by the public starts to matter. For $\delta_1 = 0$, Ricardian Equivalence holds and η no longer matters for (19) and (20).

Proposition 4. *If $\eta < (\sigma - 1)/\sigma$, then there exists a $\delta_1^* \in (\bar{\delta}_1, 1)$ such that an interest rate cut (hike) becomes contractionary (expansionary) for all $\delta_1 > \delta_1^*$.*

To understand Proposition 4, it is important to know whether the asset market equilibrium is located on the upper branch of the C-shape ($\delta_1 < \bar{\delta}_1 \equiv \frac{1-\beta}{\sigma-\beta}$) or on the lower one ($\delta_1 > \bar{\delta}_1$). In this regard, it can be shown that $\delta_1^* = \frac{1-\rho_\varepsilon \beta}{\sigma(1-\eta)-\rho_\varepsilon \beta} \in [\bar{\delta}_1, 1]$, meaning that the critical value for δ_1, beyond which the effects of changes in interest rates become "perverse", is always located on the lower branch of the C-shape.

The intuition for why monetary easings can lead to output contractions reflects the fact that when asset market equilibrium is located on the lower branch of the C-shape illustrated in Figure 2, the "asset flow effect" implies that a lower interest rate has a contractionary effect on the demand for goods. In this configuration, only when the (contractionary) flow effect is accompanied by a sufficiently strong "asset valuation effect" does a monetary easing boost output. This again explains why η is important in Proposition 4, as a lower η implies a weaker "asset valuation effect", thus increasing the odds of a monetary easing being contractionary.

Another interesting feature of our model is that the impact of monetary policy depends not only on the total size of the monetary impulse ($\bar{\varepsilon}$), but also on its distribution over time. Hence our model has implications for interest rate smoothing and the power of forward guidance. In the conventional New Keynesian model ($\delta_1 = 0$) we have that $\hat{y}_0 = -\bar{\varepsilon}/\sigma$ and the impact of monetary policy on output depends only on the total size of the impulse, $\bar{\varepsilon} \equiv v_0/(1-\rho_\varepsilon)$, where v_0 can be varied alongside ρ_ε to keep $\bar{\varepsilon}$ constant. But in the presence of retirement preoccupations ($\delta_1 > 0$), we find that the shape of the interest rate path starts to matter for its effectiveness. This is best seen by taking the limit of (19) for $\beta \nearrow 1$. In that case we get:

$$\hat{y}_0 = -\left[\frac{1}{\sigma} - \frac{\delta_1 (1-\eta)}{1-\rho_\varepsilon (1-\delta_1)}\right] \bar{\varepsilon},$$

which highlights how the response of output to a monetary impulse is decreasing in its persistence ρ_ε.

Proposition 5. *With $\delta_1 > 0$, the ability of a surprise interest rate cut $\bar{\varepsilon} < 0$ (hike, $\bar{\varepsilon} > 0$) to boost (contract) output is decreasing in its persistence ρ_ε.*

Proposition 5 implies that larger, less persistent interest rate changes are more effective than smaller, more persistent changes – suggesting that monetary policy is less potent when the central bank implements monetary policy with a greater degree of interest rate smoothing. The reason is that more persistent rate changes weaken motives to substitute intertemporally, since a persistent move does less to alter the relative price between today and tomorrow.

Finally, our setup can also speak to the power of forward guidance. In particular, when taking the limit as $\beta \nearrow 1$, the time-0 impact on output of an anticipated one-time surprise change in the interest rate at time $T > 0$ (pre-announced at time-0) is given by:

$$\hat{y}_0 = \hat{y}_T + \left[1 - (1 - \delta_1)^T\right](1 - \delta_1)(1 - \eta)\,\varepsilon_{T;0}. \tag{21}$$

The right-hand side of (21) captures the effect of forward guidance – here modelled as a future shock to the interest rate at time T that is pre-announced at time 0. This gives rise to:

Proposition 6. *When $\eta < 1$, the effect that pre-announced monetary policy shocks have on current output is decreasing in retirement preoccupations, δ_1, and the announcement horizon, T.*

Our model also implies that forward guidance can be counterproductive, in the sense of giving rise to an impact effect on output ($\hat{y}_0$) that is different in sign to the effect it ends up having when the shock actually materializes ($\hat{y}_T$). The latter is given by:

$$\hat{y}_T = \left[-\frac{1}{\sigma} + \delta_1 (1 - \eta)\right]\varepsilon_{T;0}. \tag{22}$$

Comparing (21) and (22) tells us that such flip is more likely to arise when (a) retirement preoccupations are more important (higher δ_1), (b) assets held by households are of shorter duration (lower η), or (c) the longer the anticipation period (higher T). This delivers at least two interesting insights.

First, (b) points to the existence of an (unpleasant) interaction between two "unconventional" monetary policies: upon hitting the ELB, many central banks embarked on a strategy which combined forward guidance with QE – hoping that the combination of these policies would stimulate the economy. But taking the aforementioned view that QE can be seen as the central bank lowering η (the interest-rate sensitivity of assets held by the public), our model suggests that this has the side effect of making forward guidance less powerful.[22]

Second, (c) stands in sharp contrast to the power of forward guidance in the standard New Keynesian model. As for example noted by McKay et al. (2016), the standard

[22] The model of Caballero and Farhi (2018) shares the implication that this form of QE and forward guidance are not particularly powerful. In their framework this is however driven by the existence of a "safe asset shortage", which they obtain in a framework featuring risk-neutral financiers that interact with consumers who are infinitely risk averse.

model implies that forward guidance is implausibly powerful – more so the longer the announcement lag T, which adds to the puzzle. Our model can allow for the power of forward guidance to be *decreasing* in T, which many deem more plausible.

Financial shocks. In the previous sub-section we examined the impact of retirement preoccupations on the transmission of monetary policy shocks. In this section we want to examine how retirement preoccupations change how monetary policy needs to be conducted in order to replicate the flex-price equilibrium and thereby maintain price stability and full employment (defined as the level of employment achieved under flexible prices). We will focus on the needed response of monetary policy to financial shocks, υ_t. We focus on financial shocks since they can require different responses depending on whether or not the economy is functioning on the lower branch of the C-shaped asset demand curve.[23]

As can be seen by equation (17), the direct effect of a financial shock is to decrease the value of financial assets held by households. In the absence of a change in interest rates, this will therefore put downward pressure on consumption if $\delta_1 > 0$.

We can analyze how monetary authorities should respond to such shocks if they aim to reproduce the flex-price outcome. In particular, we ask whether it could ever be desirable to fully offset such shocks (or even over-compensate by producing an asset price boom). As we shall show, if $\delta_1 = 0$ (i.e., in a representative agent setup) or if the financial shock is transitory, there would be no need to offset such a shock. In contrast, when $\delta_1 > 0$ (retirement preoccupations are present) and σ^{-1} (the EIS) is sufficiently low, we will show that when financial shocks are sufficiently persistent, fully insulating (or even over-insulating) wealth from such shocks becomes necessary to reproduce the flex-price outcome. In other words, our model gives a justification to the "Greenspan put" – a type of monetary response aimed at insulating asset values from financial shocks.

To examine this question, let us consider the dynamic system driving assets prices when goods prices are fully flexible. This is given by:

$$\begin{bmatrix} \hat{\Gamma}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} \beta\frac{1-\delta_1+\delta_1\sigma\eta}{1-\sigma\delta_1(1-\eta)} & -\frac{\eta\delta_1\sigma\beta^2(\sigma-1)}{1-\sigma\delta_1(1-\eta)} \\ \frac{\delta_1}{1-\sigma\delta_1(1-\eta)} & \frac{\beta(1-\sigma\delta_1)}{1-\sigma\delta_1(1-\eta)} \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda\zeta_t \end{bmatrix},$$

where ζ_t is the financial shock and $\lambda \equiv (1+\mu)^{-1}$ is the direct effect of the shock on the asset price q_t. This system can be solved forward with the response of the asset price to

[23]For this reason we do not expand on fluctuations in asset prices driven by demand shocks: the needed response of monetary policy is not qualitatively affected by whether the economy is on the upper or lower arm of the C-shape. In both cases, it is necessary to cut rates following negative demand shocks if monetary authorities want to maintain full employment.

a shock ζ_t being given by:[24]

$$\hat{q}_t = -\left\{1 - \frac{\eta\sigma\beta\rho_\zeta\delta_1}{\delta_1\left[\beta\rho_\zeta - (1-\eta)\,\sigma\right] + 1 - \beta\rho_\zeta}\right\}\frac{\lambda}{1-\beta\rho_\zeta}\zeta_t. \tag{23}$$

For $\delta_1 = 0$ this expression reduces to:

$$\hat{q}_t = -\frac{\lambda}{1-\beta\rho_\zeta}\zeta_t, \tag{24}$$

which implies that the asset price adjusts "fully" on impact (in the sense that the right-hand side of (24) is equal to the present-discounted value of the shock, accounting for its persistence ρ_ζ). Hence, if $\delta_1 = 0$, the flex-price equilibrium does not require any adjustment of real interest rates, with the full effect of the financial shock being passed through to asset prices and, hence, wealth. In particular: if $\zeta_t > 0$, which represents an *adverse* financial shock, $\hat{q}_t < 0$ (meaning that the asset price falls). So under $\delta_1 = 0$, there is no role for a Greenspan put. Even if prices are sticky, monetary authorities need not react to the financial shock since this is not necessary to reproduce the flex-price equilibrium. Keeping interest rates unchanged in this situation will not put pressure on inflation as employment is held at its natural level. The reason, ultimately, lies in the fact that neither household wealth holdings nor asset prices affect consumption when $\delta_1 = 0$ (see equation (14)). Similarly, if the financial shock is temporary ($\rho_\zeta = 0$), then $\hat{q}_t = -\lambda\zeta_t$, which again implies that monetary authorities need not react in order to reproduce the flex-price outcome.

However, when introducing retirement preoccupations by setting $\delta_1 > 0$, asset prices start affecting consumption (through household wealth holdings; again recall equation (14)) and monetary authorities will generally need to adjust real interest rates if they want to maintain full employment and inflation at target in the face of financial shocks. The question is: could the required monetary policy response involve a full Greenspan put, or perhaps even more? This brings us to Proposition 7:

Proposition 7. *If $\delta_1 < \bar{\delta}_1$, a "Greenspan put" (here defined as the central bank ensuring that asset prices don't fall following a negative financial shock) is never required to reproduce the flex-price equilibrium. If $\delta_1 > \bar{\delta}_1$ and $\eta > (1-\beta/\sigma)(1-\bar{\delta}_1/\delta_1)$, then a*

[24] For the forward solution to be the unique stationary solution we need $\eta > \frac{\sigma-\beta}{\delta_1\sigma}\left(\delta_1 - \frac{1-\beta}{\sigma-\beta}\right)$ or $\eta < \frac{\sigma+\beta}{\delta_1\sigma}\left(\delta_1 - \frac{1+\beta}{\sigma+\beta}\right)$.

Greenspan put is required to reproduce the flex-price outcome whenever:

$$\rho_\zeta \geq \frac{1}{\beta}\frac{1-\delta_1\sigma+\delta_1\eta\sigma}{1-\delta_1+\delta_1\eta\sigma}.$$

Proposition 7 implies that a (full) Greenspan put is never needed if the economy is operating on the upper arm of the C-shaped asset demand curve (i.e., if $\delta_1 < \bar{\delta_1}$). But if the economy is operating on the lower arm of the C-shape ($\delta_1 > \bar{\delta_1}$), and the financial shock is sufficiently persistent, then the central banks needs to adjust interest rates sufficiently to ensure that asset prices don't fall when faced with a negative shock. In fact, if ρ_ζ is strictly greater than $\frac{1}{\beta}(1-\delta_1\sigma+\delta_1\eta\sigma)/(1-\delta_1+\delta_1\eta\sigma)$, the central bank actually needs to *boost* asset prices in order to maintain price stability/full employment (since $\hat{q}_t$ now has the same sign as ζ_t, where it should be kept in mind that a positive shock to ζ_t implies a negative financial shock that, if left unaddressed, lowers asset prices).

4.3 Quantitative results for the full model

Our analytical results so far have been obtained under the "prudent perpetual youth" assumption which allows us to abstract from retired agents. In this section we will solve the full model numerically, featuring both workers and retirees. As we will see, the findings we have just established analytically continue to hold qualitatively in the full model.

4.3.1 Calibration and nature of the exercise

We calibrate the model at a quarterly frequency using values commonly found in the literature (Table 1). We set $\sigma = 3$, which implies an elasticity of intertemporal substitution of 0.33, and $\varphi = 0.33$, which gives a labor supply elasticity of 3. As in Gertler (1999), we calibrate δ_1 and δ_2 such that the average working life is 45 years and the average life-expectancy in retirement is 10 years. This implies $\delta_1 = 0.0056$ and $\delta_2 = 0.025$. We set $z = 0.9975$ such that 18% of the population is in the retirement state, a number in line with the historical average old-age dependency ratio for the US.[25] For good-producing firms, we assume a steady-state mark-up of 20%, implying an elasticity of substitution between varieties (ϵ) of 6. We set the price adjustment cost parameter θ such that the slope of the Phillips curve is 0.085, as in Galí (2008). We choose $\chi = 1$ such that steady-state labour supply is equal to 1 and normalize productivity to $A = 1$. We set $\beta = 0.995$

[25] The old-age dependency ratio is defined as the number of individuals aged 65 and over per 100 people of working age defined as those at ages 20 to 64.

and choose the steady-state value of assets such that the steady-state real interest rate is $r = 1/\beta$. This implies an annual real interest rate of 2% and allows us to maintain the same calibration when solving the model for $\delta_1 = 0$.[26]

Table 1: calibration (quarterly frequency)

Parameter	Description	Value
β	discount factor	0.995
σ^{-1}	elasticity of intertemporal substitution	0.33
δ_1	Poisson probability of retiring	0.0056
δ_2	Poisson probability of dying (for retirees)	0.025
$1 - z$	Poisson probability of dying (for workers)	0.0025
ϵ	substitution elasticity between goods varieties	6
φ	inverse Frisch elasticity of labor supply	3
θ	adjustment cost for prices	78.43
η	share of long-term bonds	0.7
$1 + \phi$	Taylor coefficient on inflation	1
ρ	decay of coupons for long-term bond	0.025
χ	disutility from supplying labor	1
A	steady-state level of technology	1
ρ_ε	AR(1)-coefficient of the monetary policy shock	0.84
μ	steady-state share of assets which can be diverted	0

We set $\rho = 0.025$, which implies a long-term bond maturity of around 8.4 years. While this yields an average maturity for government debt which is higher than in the US data (around 5 years) it better captures the interest-rate sensitivity of the financial assets held by US households – which also includes equities.[27] We calibrate the amount of long-term bonds such that their steady-state ratio over total assets (η) is 0.7. This matches the 30-percent share of currency and deposits over assets held by US households. We furthermore set $\mu = 0$, implying that the financial friction is barely binding in steady state. Finally, we set the persistence parameter of the monetary shock (ρ_ε) to 0.84, implying a half-life of one year, and work with a Taylor coefficient on inflation ($1 + \phi$) of 1: this maximizes

[26] For any $\delta_1 > 0$ it is not necessary that the steady-state real interest rate equals the inverse of the time-discount factor β.

[27] Van Binsbergen (2021) estimates the duration of the S&P 500 at around 20 years. Another option, that we leave for future work, is to extend the model to also allow for equity holdings. Since bonds are pension funds' preferred mode of investment (OECD, 2021: 213), we keep the model simple by only allowing for this possibility.

cross-comparability between IRFs by ensuring that the endogenous response of monetary policy is not too different across responses.

To understand how retirement preoccupations affect the economy and monetary policy, we solve the model for various probabilities of retiring, δ_1, around its calibrated value. When varying δ_1, we keep the steady state of the model constant. To do so we choose ϖ (the degree by which households overestimate the probability of them surviving the retirement shock) such that the steady-state interest rate remains $r = 1/\beta$. This implies that the steady-state interest rate is independent from δ_1 and allows us to solve the model for $\delta_1 = 0$, which makes it coincide with the canonical New Keynesian model.

4.3.2 Findings

Armed with the above calibration, we can solve the model and distil various insights from our numerical results. As we will see, they by-and-large confirm our analytical findings established under the "prudent perpetual youth" assumption. When retirees are fully accounted for, an increase in δ_1 not only raises the retirement preoccupations of workers, but it also increases the share of aggregate demand coming from retirees. Both factors tend to increase the weight of the "asset flow effect" in the transmission mechanism of monetary policy, which tends to work in the "dissonant" direction (with higher rates stimulating aggregate demand).

First consider Figure 4, which offers a numerical counterpart to Proposition 1 – investigating the determinacy properties of our model. The figure shows that, as retirement becomes more important (i.e., as δ_1 rises), the region of determinacy shrinks. As explained before, the introduction of retirement preoccupations introduces an "asset flow effect", which (in isolation) implies that higher interest rates *boost* the economy. The presence of this effect not only lowers the potency of monetary policy, but also implies that the central bank needs to walk a finer line in order to render the equilibrium determinate. As δ_1 rises (making the asset market equilibrium slide down the C-shape), there is the sudden emergence of an *upper*-bound for the Taylor coefficient on inflation ϕ, meaning that the central bank cannot respond too aggressively to inflation. This implies that full inflation stabilization, which normally emerges as $\phi \to \infty$, may be infeasible. The reason is that if the central bank is too aggressive in responding to inflation, it may end up validating certain initial beliefs on future interest income being high – which can lead to self-fulfilling equilibria.

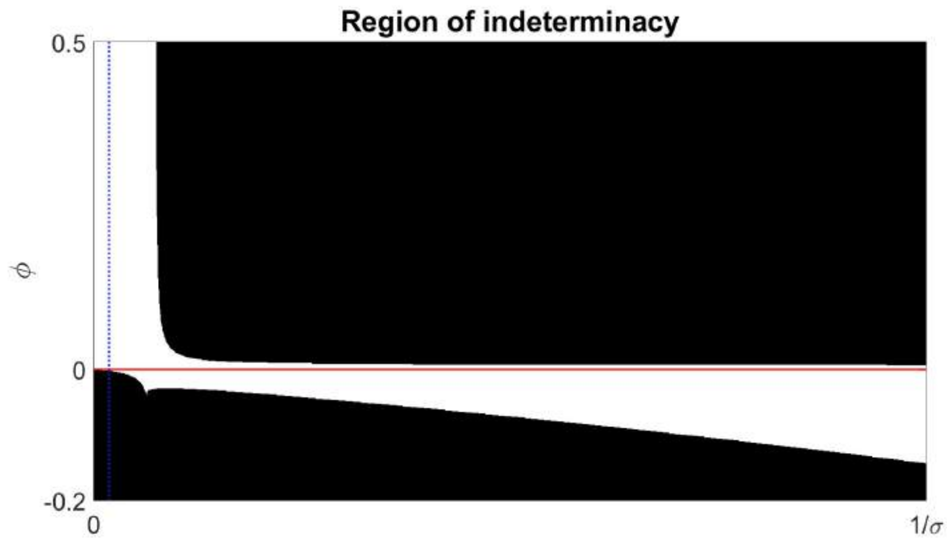

Figure 4: Region of determinacy (in white) for different weights placed on the retirement motive, δ_1.

Next consider Figure 5, which offers a numerical counterpart to Proposition 2 – investigating the potency of conventional monetary policy for different values of δ_1. The figure shows a "fan" of impulse responses to an expansionary monetary policy shock, all generated by different values for δ_1 (with retirement motives becoming more important as the IRFs get bluer). The greenest IRF shows the responses for the standard New Keynesian model with $\delta_1 = 0$. The dashed line represents the impulse response associated with our baseline calibration, as described in Section 4.2.1. The nature of this exercise illustrates that the behaviour of the model is "smooth" when it comes to whether the asset market equilibrium is located on the upper- or lower branch of the C-shape. In particular, the muted effects implied by Proposition 2 start emerging the moment we move from the standard New Keynesian model with $\delta_1 = 0$, to any calibration with $\delta_1 > 0$ (no matter how small).

As one can see from the figure, the potency of conventional monetary policy to affect output and prices decreases as δ_1 rises. The reason is twofold. There are the forces we discussed around Proposition 2, which relates to the saving behavior of active households and its link to expected retirement needs. However, there is now an additional effect due to the presence of retirees. As interest rates are lowered, retirees' wealth decreases and this forms an extra drag on the economy. Interestingly, the potency of monetary policy declines faster for inflation than for output. Under our baseline calibration, an expansionary monetary policy shock boosts output temporarily but lowers inflation persistently. This suggests that inflation is more forward-looking than output (i.e., future activity has a greater impact on current inflation than on current output, bringing the ultimate decline in inflation forward to the present).

Figure 6 relates to Proposition 3, as it investigates how the potency of monetary policy varies with the duration of assets held by the public. To produce this figure, we fixed δ_1 at our baseline calibration ($\delta_1 = 0.0056$) whilst varying η, which is the share of assets that is long-term in nature (i.e., carrying positive duration). As this figure shows, and in line with Proposition 3, monetary policy loses potency with respect to output and inflation as the maturity structure of public sector liabilities shortens (bluer IRFs correspond to lower values of η, i.e., households' asset holdings being less interest rate-sensitive). The reason is that lower duration in households' asset holdings implies a weaker "valuation channel". Recall that a sufficiently strong valuation channel is needed for the transmission mechanism to work in the "conventional" direction when the economy is operating on the lower arm of the C-shape. This points to the possibility that past rounds of QE could lower the potency of conventional monetary policy.

Finally, Figure 7 offers a numerical counterpart to Proposition 6 – analyzing the impact of forward guidance. Here, "forward guidance" is modelled as a preannounced cut in the monetary policy rate in 10 quarters' time. The standard New Keynesian model is known to feature a "forward guidance puzzle", with its effects being implausibly large because of the strength of the intertemporal substitution effect (Del Negro et al., 2013; McKay et al., 2016). In our setup, the importance of intertemporal substitution diminishes as δ_1 increases and the asset market equilibrium slides down the C-shape. Accordingly, the power of forward guidance can be seen to fall in Figure 7, with IRFs become bluer as the importance of retirement preoccupations "δ_1" increases. As the figure shows, and in line with our analytical result in Proposition 6, an "expansionary" forward-guidance shock can even *contract* output and *lower* inflation if retirement preoccupations are sufficiently strong. The reason for this again derives from the forces we discussed in our simplified model, reinforced in our general model by the behavior of the retirees, who can be seen to consume less as (future) interest rates fall.

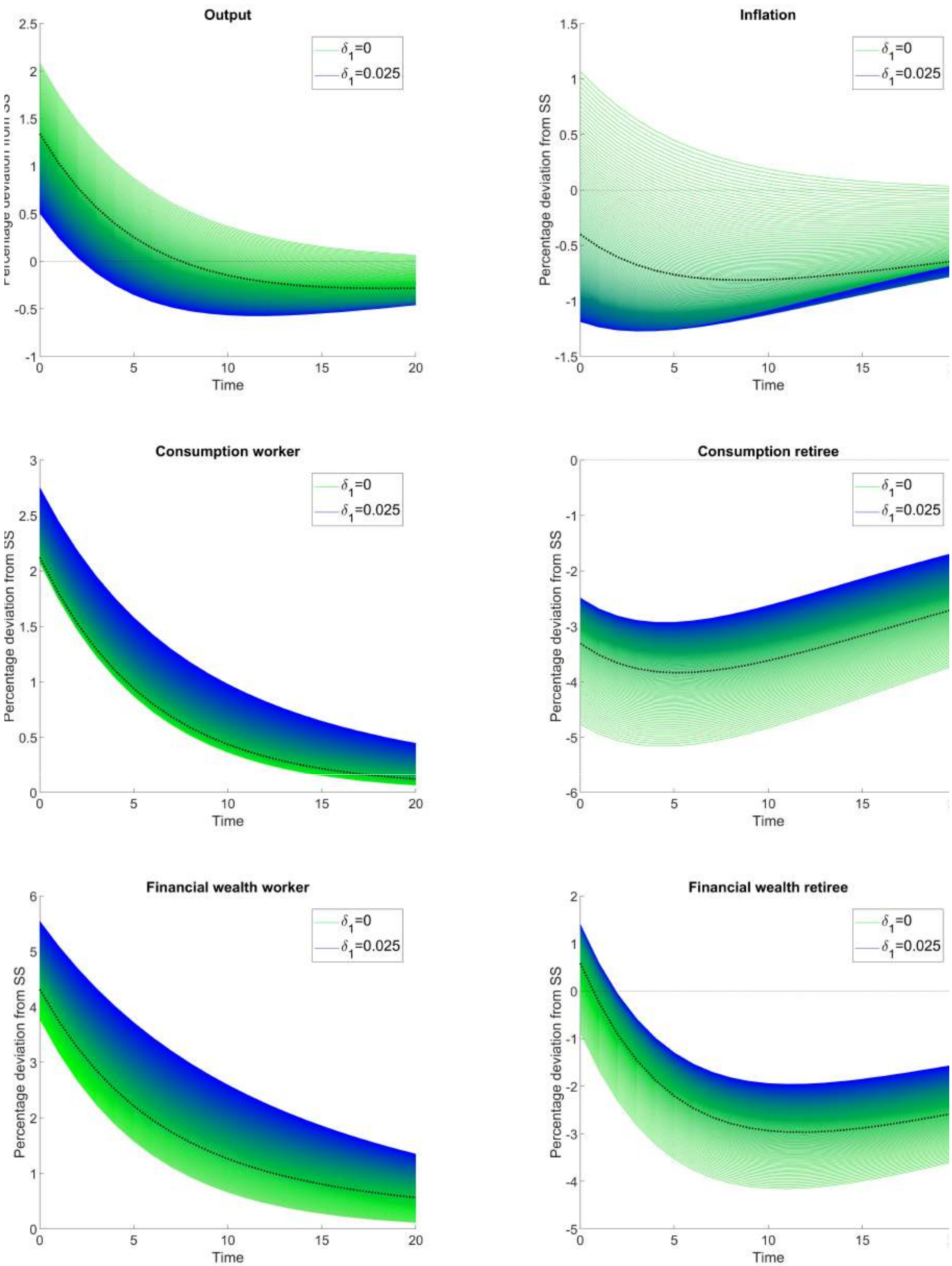

Figure 5: IRFs to an expansionary monetary policy shock for different weights placed on the retirement motive, δ_1.

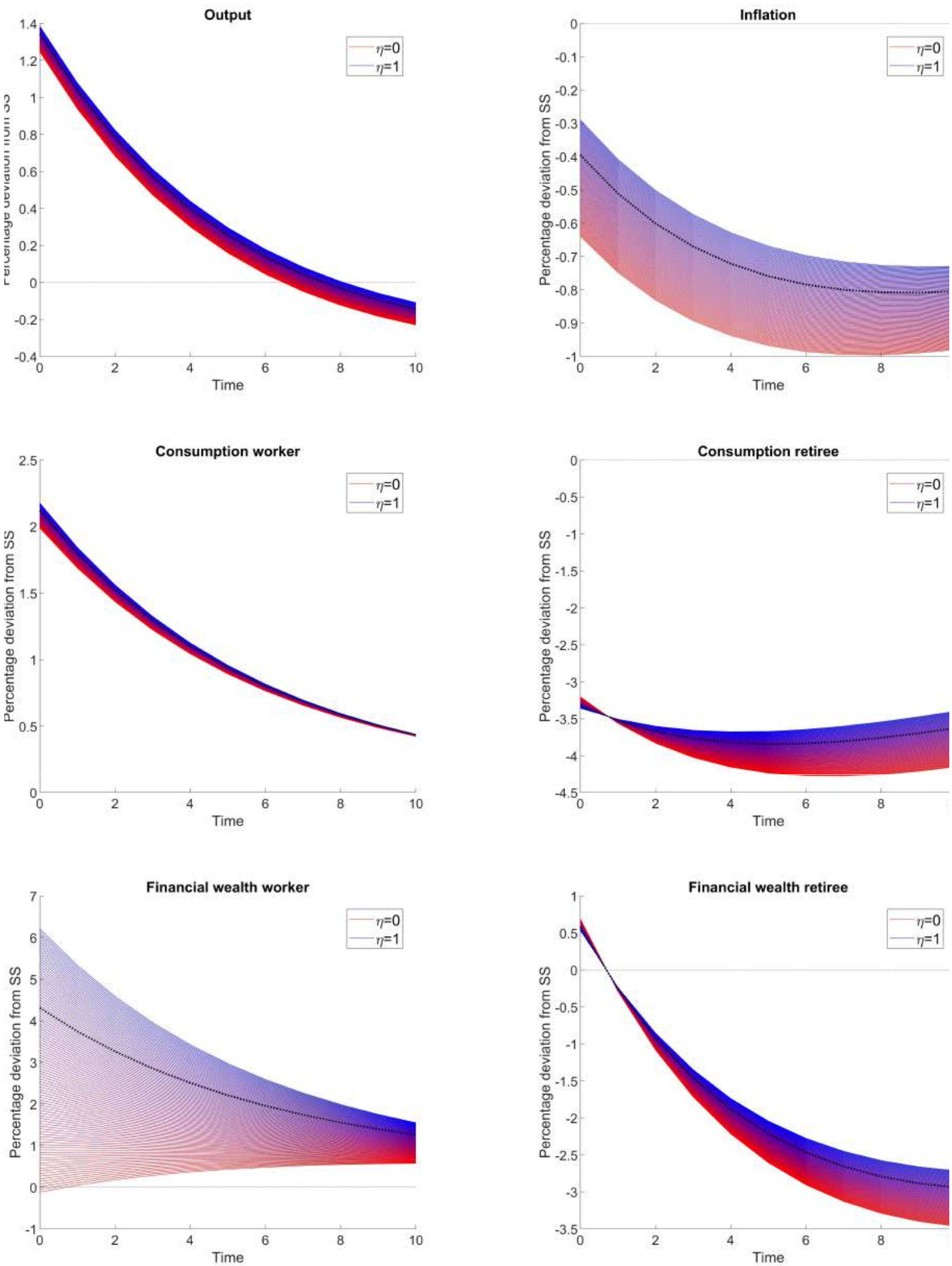

Figure 6: IRFs to an expansionary monetary policy shock for different shares of assets being long term, η.

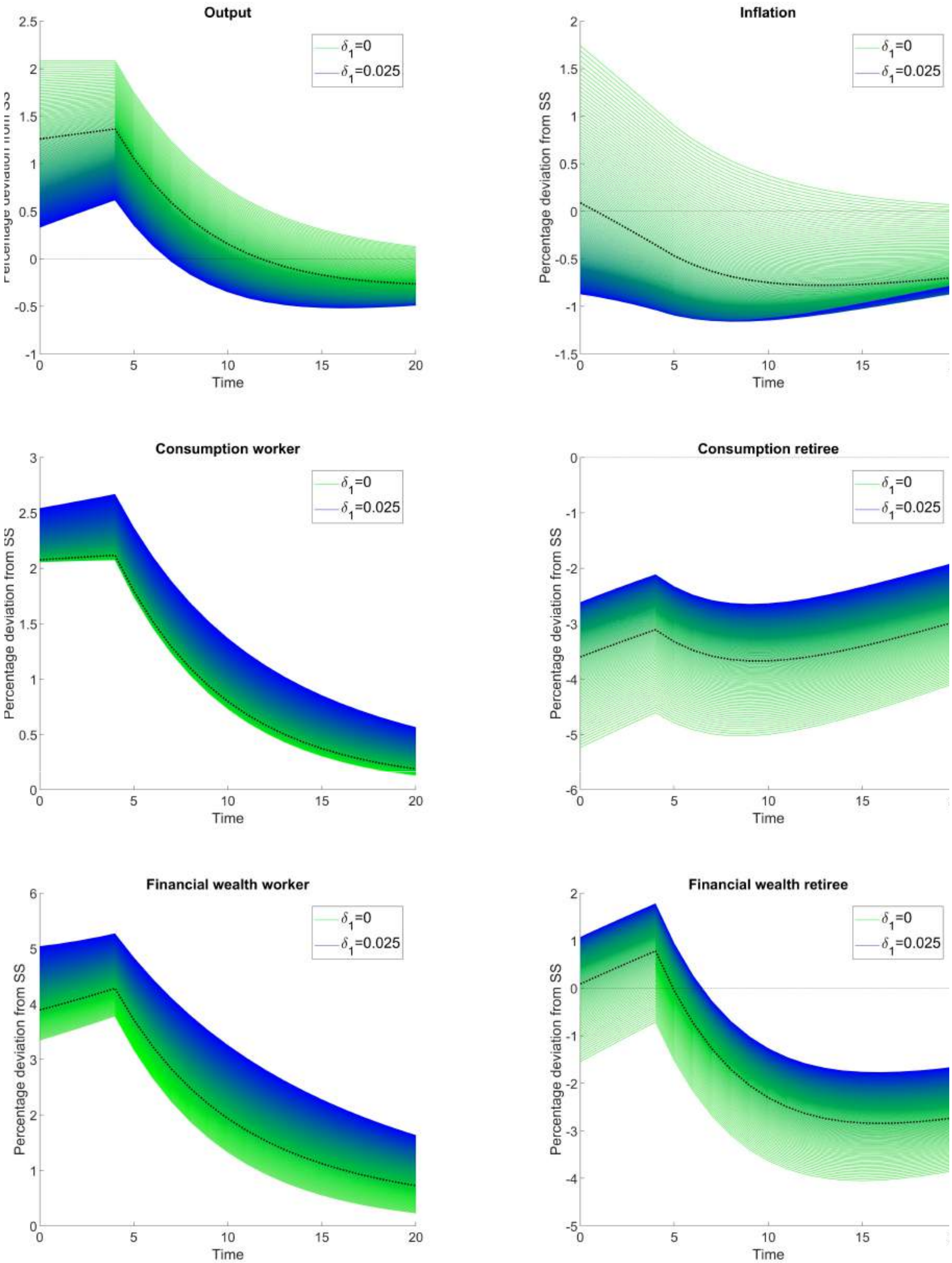

Figure 7: IRFs to a forward-guidance shock for different weights placed on the retirement motive, δ_1.

5 Conclusion

In this paper we have proposed a tractable extension of a standard New Keynesian DSGE model to include life-cycle forces (leading to a "FLANK model", a Finitely-Lived Agent New Keynesian model). Our framework allows for active households that save for their retirement – and retirees that live off their assets and associated interest income. This makes interest rates affect demand for goods not only through intertemporal substitution (which is central to the monetary transmission process in the standard New Keynesian model), but also by affecting asset demand. In particular, we have shown how life-cycle forces fundamentally alter the Euler equation for active households, giving rise to a "C-shaped" asset demand function – featuring a cut-off level in the interest rate below which further rate cuts *increase* households' desire to hold assets (which can come at the expense of demand for goods). We used the model to examine issues related to monetary policy. We show that our model pushes the monetary transmission mechanism away from intertemporal substitution (for which empirical evidence is mixed at best), towards transmission via a financial channel – placing household wealth holdings center stage. While an earlier tradition of "monetarist" models focused squarely on liquid wealth holdings (often for reasons related to tractability; Duca and Muellbauer (2013)), our model yields a tractable way of handling a richer wealth structure – demonstrating that it is wealth *adjusted for the level of interest rates* which drives consumption.

Our model shows that interest rates not only influence asset valuations, but can also affect asset demand. This makes our financial channel consist of two parts, pulling in opposite directions. First, by affecting the price of assets with positive duration, interest rate changes bring about an "asset valuation effect", which works in the conventional direction (rate hikes being contractionary, and vice versa). In addition, there is an "asset flow effect", stemming from the impact that interest rates have on asset demand. Taken in isolation, this effect implies that rate hikes are *expansionary*, as higher rates imply a greater per-period income flow per unit of assets held, reducing asset demand – as fewer assets suffice to finance post-retirement consumption needs. When the asset market equilibrium is located on the lower arm of the C-shape, the "dissonant" asset flow effect dominates intertemporal substitution. There, a strong asset price effect becomes *necessary* for rate hikes to obtain their standard contractionary impact. In contrast, on the upper arm of the C-shape any asset valuation effects are a mere bonus to the monetary transmission process: intertemporal substitution dominates the asset flow effect, implying rate hikes (cuts) are contractionary (expansionary) *even when asset prices are held constant.*

We find that the introduction of life-cycle forces has important implications for the conduct of monetary policy, irrespective of whether the asset market equilibrium rests on the upper- or lower arm of the C-shape. First, in order to produce a determinate equilibrium, the central bank may get to face an *upper* bound on its reaction coefficient to inflation-deviations from target (in addition to the conventional lower bound known from the Taylor principle). This upper bound implies that full inflation stabilization nay not be feasible, as responding too aggressively to inflation rising above target may cause indeterminacy. Second, the countervailing "asset flow effect" that comes with the introduction of retirement preoccupations renders conventional monetary policy – changing the interest rate – less potent. This holds particularly true when households' wealth holdings are of lower duration (as that weakens the "asset valuation channel"), implying that conventional monetary policy is less powerful in a post-QE world. Third, our model resolves the so-called "forward guidance puzzle": by pushing the monetary transmission mechanism away from intertemporal substitution, towards our financial channel, forward guidance becomes less potent. It can even become contractionary in the short run. Interestingly, forward guidance is also less potent when the assets held by households are of shorter duration – pointing to an unpleasant interaction between forward guidance and QE. While these two policies have typically been used in conjunction over the past ELB decade, our findings suggest that QE may actually weaken the potency of forward guidance – implying that simultaneously pursuing both policies could be somewhat counterproductive. Finally, when located on the lower branch of the C-shaped asset demand curve, optimal policy considerations may require the central bank to prop up asset markets in response to adverse financial shocks; when the asset market equilibrium lies on the upper branch of the C-shape, such a "Greenspan put" is never optimal.

By offering a tractable framework combining life-cycle forces and monetary policy, our work opens several avenues for future work. Our finding that conventional monetary policy may be less potent when retirement preoccupations are more prevalent suggests that central banks may need to move the interest rate *by more* to achieve a given effect on output and prices in an aging society. This may have adverse consequences for financial stability. We do not model these interactions in the present paper, but such an extension would be warranted.

Second, while our FLANK model is already heterogenous-agent in nature (distinguishing between workers and retirees), it could be interesting to incorporate other dimensions of heterogeneity. A natural candidate would involve allowing for heterogeneity in the marginal propensity to consume (MPC) out of wealth for both active and retired households. Empirical studies document that this object varies strongly across the wealth distribution, with richer households being characterized by lower MPCs (Di Maggio et al., 2020; Chodorow-Reich et al., 2021). In that case, our model's logic suggests that greater inequality (a smaller fraction of households owning a bigger share of the asset supply) can weaken the monetary transmission mechanism – as this now has the "asset valuation effect" at its core. When consumption demand of asset holders is not very sensitive to valuation effects, as would be the case when most assets are held by low-MPC households, this channel loses potency.

Third, to keep the analysis clean, our model intentionally abstracts from various other transmission mechanisms, such as cash flow-related channels (e.g. operating via mortgage debt), mechanisms running through investment, as well as effects on labor supply. Augmenting our model with such channels could be a natural next step, but we expect that the core lesson from our present analysis will survive: relative to any baseline model – abstracting from the life-cycle preoccupations central to our paper – incorporation of these forces will alter the monetary transmission mechanism in ways that we have described.

When it comes to adding realism, countries typically do not exclusively rely on fully-funded pension arrangements – instead providing retirees with some basic retirement income financed through a pay-as-you-go (PAYG) system (financed by taxing working individuals). The generosity of such schemes however tends to be limited (for example: 2023 US Social Security payments were about \$1,782 per month[28]), leaving an important role for the saving dynamics central to our paper – a role that would only increase in importance if one were to explicitly model bequest motives (in contrast to savings, a PAYG pension cannot be bequeathed to one's offspring). What our model also makes clear, is that the importance of life-cycle forces to the monetary transmission mechanism is greater in countries where PAYG pensions are less important. As demographic forces (increasing old age dependence ratios) are currently putting PAYG systems under pressure (OECD, 2021), our paper suggests that the importance of life-cycle forces to monetary policy makers may increase over time.

Finally, our theoretical model can serve as a guide to empirical researchers in formulating the correct econometric specification when trying to estimate the MPC out of wealth. In particular, our model suggests that it is important to control – in a non-linear way – for the accompanying *level* of interest rates. If wealth levels are high because of low discount rates, the MPC to consume out of this wealth is likely to be relatively low, as households would want to hold on to their stock of assets to compensate for the lower

[28] See https://www.cbpp.org/sites/default/files/atoms/files/8-8-16socsec.pdf. Most young, working Americans are moreover pessimistic about their future Social Security benefits (Turner and Rajnes, 2021), increasing the importance of their own saving efforts.

flow return (the low r). This suggests that the MPC to consume out of wealth not only varies with wealth holdings (with richer households having a lower MPC) but also with the prevailing level of long-term interest rates (with the propensity to consume out of wealth being lower when rates are lower).

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Appendix

A Equilibrium and steady state

The equilibrium of the model is described by the following equations:

$$
\begin{aligned}
y_t &= \frac{\vartheta c_t^w + (1-\vartheta)\, c_t^r}{1-\frac{\theta}{2}(\pi_t-\bar{\pi})^2} \\
c_t^r &= \left(\Gamma_t^{\frac{1}{\sigma}} - 1\right)^{-1} a_t^r \\
(c_t^w)^{-\sigma} &= \beta(1-\delta_1)\,\mathbb{E}_t\left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1}\right] + \beta z_s \delta_1 (a_t^w)^{-\sigma}\,\mathbb{E}_t\left[\left(1+x_{t+1}^w\right)^{-\sigma}\Gamma_{t+1}(r_{t+1})^{1-\sigma}\right] \\
\left(\Gamma_t^{\frac{1}{\sigma}} - 1\right)^{\sigma} &= (1-\delta_2)\,\beta\mathbb{E}_t\left[(r_{t+1})^{1-\sigma}\,\Gamma_{t+1}\right] \\
(\pi_t-\bar{\pi})\,\pi_t &= \kappa\left[\chi\left(\frac{y_t}{\vartheta A}\right)^{1+\varphi} - 1\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}(\pi_{t+1}-\bar{\pi})\,\pi_{t+1}\frac{y_{t+1}}{y_t}\right] \\
\Lambda_{t,t+1} &= \beta\frac{(1-\delta_1)\left(c_{t+1}^w\right)^{-\sigma} + z_s\delta_1 (a_t^w)^{-\sigma}\left(1+x_{t+1}^w\right)^{-\sigma}\Gamma_{t+1}(r_{t+1})^{-\sigma}}{(c_t^w)^{-\sigma}} \\
s^g + q_t b^g &= \vartheta a_t^w + (1-\vartheta)\, a_t^r \\
a_{t+1}^r &= r_{t+1}\left(1-\Gamma_{t+1}^{-\frac{1}{\sigma}}\right)\left[(1-\delta_2)\, a_t^r + \delta_2 a_t^w\left(1+x_{t+1}^w\right)\right] \\
q_t &= \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{1+(1-\rho)\, q_{t+1}}{1+\mu_t}\right] \\
x_{t+1} &= \frac{b^g}{\vartheta a_t^w}\left[\frac{1+(1-\rho)\, q_{t+1}}{r_{t+1}} - q_t\right] \\
r_{t+1} &= r\frac{\bar{\pi}}{\pi_{t+1}}\left(\frac{\mathbb{E}_t[\pi_{t+1}]}{\bar{\pi}}\right)^{1+\phi} e^{\varepsilon_t} \\
\mu_t &= \mu + \zeta_t \\
\varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + \sigma_\varepsilon v_t \\
\zeta_t &= \rho_\zeta \zeta_{t-1} + \sigma_\zeta u_t
\end{aligned}
$$

Assuming that the inflation target is zero ($\bar{\pi} = 1$) and $\mu = 0$, the steady state real interest rate r solves:

$$
\frac{y}{r - [(1-\delta_2)\,\beta r]^{\frac{1}{\sigma}}}\,\frac{1 + z\delta_1\frac{[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}{1-[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}(1-\delta_2)}}{\left[\frac{z_s\delta_1\beta r}{1-(1-\delta_1)\beta r}\right]^{-\frac{1}{\sigma}} + \frac{z\delta_1}{1-(1-\delta_2)[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}} = s^g + \frac{b^g}{r-1+\rho}
$$

The left-hand side of this equation represents the steady-state demand for savings, while the right-hand side captures the steady-state value of the assets supplied to the economy. Steady states for the other variables are given by:

$$\Gamma = \left\{1 - \left[(1-\delta_2)\,\beta r^{1-\sigma}\right]^{\frac{1}{\sigma}}\right\}^{-\sigma}$$

$$y = A\frac{\delta_2}{z\delta_1+\delta_2}\left(\frac{1}{\chi}\right)^{\frac{1}{1+\varphi}}$$

$$\Lambda = \frac{1}{r}$$

$$q = \frac{1}{r-1+\rho}$$

$$a^r = \alpha\frac{s^g+qb^g}{1-\vartheta};\; a^w = (1-\alpha)\,\frac{s^g+qb^g}{\vartheta}$$

$$c^r = \gamma\frac{y}{1-\vartheta};\; c^w = (1-\gamma)\,\frac{y}{\vartheta},$$

and $x = 0$. α and γ denote the retirees' share of assets held and output consumed, respectively. These shares are given by:

$$\alpha = \frac{z\delta_1}{\left[(1-\delta_2)\,\beta r\right]^{-\frac{1}{\sigma}} - 1 + \delta_2 + z\delta_1},$$

$$\gamma = z\delta_1\frac{r\left[(1-\delta_2)\,\beta r\right]^{-\frac{1}{\sigma}} - 1}{\left[(1-\delta_2)\,\beta r\right]^{-\frac{1}{\sigma}} - 1 + \delta_2 + z\delta_1}\frac{s^g+qb^g}{y}.$$

B Proofs of Propositions

B.1 Proof of Proposition 1

When $\phi = 0$, the equilibrium dynamics are captured by:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & -\frac{\delta_1}{\sigma} & 0 & \delta_1\eta\beta \\ 0 & \beta & 0 & 0 \\ \kappa(1+\varphi) & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{y}_{t+1} \\ \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix}$$

The four eigenvalues of this system are defined by the characteristic equation $(\beta-\lambda)^3(1-\delta_1-\lambda) = 0$, where the λs are the eigenvalues. Since all four eigenvalues are less than 1 (given $\beta < 1$ and $\delta_1 < 1$), this system has a unique stable (determinate) solution.

B.2 Proof of Propositions 2, 3 and 5

Note that one can rewrite equation (19) as $\hat{y}_0 = -\frac{1}{\sigma}\Psi\bar{\varepsilon}$, for $\Psi \equiv \frac{(1-\rho_\varepsilon)\left[1-\delta_1\frac{\sigma(1-\eta)-\rho_\varepsilon\beta}{1-\rho_\varepsilon\beta}\right]}{1-\rho_\varepsilon(1-\delta_1)}$. The content of these three propositions follows directly from taking the first derivate of Ψ with respect to δ_1, η, and ρ_ε, respectively. This yields:

$$\begin{aligned}
\frac{\partial\Psi}{\partial\delta_1} &= -\frac{(1-\beta)\rho_\varepsilon + (1-\rho_\varepsilon)\sigma\,(1-\eta)}{(1-\rho_\varepsilon\beta)\,[1-\rho_\varepsilon\,(1-\delta_1)]^2}\,(1-\rho_\varepsilon) < 0 \\
\frac{\partial\Psi}{\partial\eta} &= \frac{\delta_1\sigma\,(1-\rho_\varepsilon)}{(1-\rho_\varepsilon\beta)\,[1-\rho_\varepsilon\,(1-\delta_1)]} > 0 \\
\left.\frac{\partial\Psi}{\partial\rho_\varepsilon}\right|_{\beta=1} &= -\frac{(1-\delta_1)\delta_1(1-\eta)}{(1-(1-\delta_1)\rho_\varepsilon)^2} < 0
\end{aligned}$$

B.3 Proof of Proposition 4

Note from equation (19) that if $\delta_1 > \frac{1-\rho_\varepsilon\beta}{\sigma(1-\eta)-\rho_\varepsilon\beta} \equiv \delta_1^*$, we have that $1-\delta_1\frac{\sigma(1-\eta)-\rho_\varepsilon\beta}{1-\rho_\varepsilon\beta} < 0$, flipping the normal negative relationship between the monetary policy shock $\bar{\varepsilon}$ and output $\hat{y}_0$. For the critical retirement probability δ_1^* to be less than one (i.e., for it to be a probability), we need $\eta < (\sigma-1)/\sigma$.

B.4 Proof of Proposition 6

Proposition 6 follows directly from taking the first derivate of $\varUpsilon \equiv \left[1-(1-\delta_1)^T\right](1-\delta_1)(1-\eta)$ from equation (21) with respect to δ_1 and T:

$$\begin{aligned}
\frac{\partial\varUpsilon}{\partial\delta_1} &= -\left(\left[1-(1-\delta_1)^T\right] + (1-\delta_1)\left[1-T\,(1-\delta_1)^{T-1}\right]\right)(1-\eta) < 0, \\
\frac{\partial\varUpsilon}{\partial T} &= -\,(1-\delta_1)^{T+1}\ln(1-\delta_1)\,(1-\eta) < 0,
\end{aligned}$$

implying that the time-0 impact of a pre-announced monetary policy shock is decreasing in both δ_1 and T.

B.5 Proof of Proposition 7

Defining $\Omega \equiv 1 - \frac{\eta\sigma\beta\rho_\zeta\delta_1}{\delta_1\left[\beta\rho_\zeta-(1-\eta)\sigma\right]+1-\beta\rho_\zeta}$ one can rewrite equation (23), which gives the response of $\hat{q}_t$ to a financial shock under fully flexible prices, as $\hat{q}_t = -\Omega\frac{\lambda}{1-\beta\rho_\zeta}\zeta_t$. The conditions in

Proposition 7 yield $\Omega \leq 0$, meaning that – in order to replicate the flex-price equilibrium – the asset price $\hat{q}_t$ needs to weakly *increase* following an adverse financial shock ($\zeta_t > 0$).