Fiscal policy and human capital in the race against the machine*

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Abstract

We analyze the policy trade-offs facing fiscal policy in a dynamic growth model with automation, education choice, and human capital formation. Although beneficial for economic growth, automation contributes to wage inequality. While redistributive taxation mitigates inequality at the cost of lower economic growth, productivity-enhancing education spending boosts production, eventually increasing inequality. The composition of taxation (labor vs robot tax) financing government spending on transfers and education is key in determining the effects of fiscal policy on economic growth and inequality, as the robot tax is relatively more redistributive than the labor tax. We calibrate our model to the US economy and determine the welfare-maximizing tax policy. Optimality requires an initial reduction in the robot tax to foster automation and boost economic growth, followed by its gradual increase as redistributive motives become more important. Additionally, we explore education subsidies and identify the conditions leading to welfare improvements.

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1 Introduction

Technological advancement is recognized to be a principal driver of economic growth. However, in the last decades, the increased efficiency of automated technology – that is, the automated operation of production tasks through the use of robots, software technologies, artificial intelligence, etc. – has also significantly contributed to rising inequality due to the skill-biased nature of this form of technological progress (cf. e.g. Autor, 2019). Indeed, high-skilled workers often stand to benefit from an increase in the productivity (or a reduction in the price) of automated technologies as the tasks performed by them are complementary to machines. By contrast, low-skilled workers are more likely to be substituted by machines. Automation thus induces a pattern of rising skill premia along with stagnating, or even falling, wages for less-educated workers.

Importantly, however, skill premia and the resulting inequality across different types of workers reflect not only technological developments that determine the demand for skills, but also their supply, which is crucially shaped by education (cf. e.g. Goldin and Katz, 2010). In consequence, policy options in the 'race against the machine' must be assessed in terms of their impact on both the demand and the supply of skills.

Starting from this basic insight, this paper examines the trade-offs facing fiscal policy in the context of a dynamic general equilibrium model in which both technological progress and human capital formation are endogenous. Technological progress is skill-biased and facilitated by R&D which creates patents for automation capital. Human capital formation entails not only an extensive margin via the education choices (basic versus higher education) individuals take at the beginning of their working life, but also an intensive margin via the amount of resources devoted to the different stages of education. To capture this effect along the intensive margin, we consider a *hierarchical education system* with a sequential process for basic (primary and secondary) and higher (college/tertiary) education. Publicly funded education spending thus augments workers' human capital in two stages. Whereas all workers benefit from spending on basic education, only those enrolled in higher education get the additional benefits from education spending at the tertiary level.

We embed these mechanisms for technological progress and human capital formation into an overlapping generations model, which successfully captures the secular trend of sustained productivity growth alongside a rising college share, an increasing skill premium and a declining labor share. R&D-driven growth thus induces automation and is accompanied by increasing inequality between high-skilled and low-skilled workers. Fiscal policy – formalized initially in terms of variations in tax policy for a given configuration of public spending under a balanced budget – can affect these dynamics via two channels: *the redistribution and the human capital channel.* To the extent that part of the tax revenue is used to transfer resources from richer households (high-skilled workers) to poorer households (low-skilled workers), the redistribution channel implies reduced inequality. But to the extent that some tax revenue is used to fund education spending, the human capital channel works in the opposite direction since the hierarchical education system benefits proportionally more the high-skilled workers. Contrasting a labor tax (i.e., a linear tax on wage income) and a robot tax (i.e., an ad-valorem tax on machines), we find that the magnitude of these effects crucially depends on the way transfers and education spending are financed. The redistribution channel through the robot tax is, indeed, stronger than the redistribution channel when government spending is financed through the labor tax. This is because the robot tax has a direct negative effect on high-skilled workers as complements to machines, while the labor tax affects high- and low-skilled workers proportionally.

Since the two channels work in opposite directions, the net effect of taxation on inequality is generally ambiguous. We therefore calibrate the general equilibrium model – and in particular the breakdown of fiscal policy – to US data and observe that, following an increase in the labor tax relative to the calibrated status quo, both economic growth and inequality increase. By contrast, an increase in the robot tax has opposite effects on economic growth and inequality. In other words, given the structure of the US economy and the allocation of fiscal revenue for transfer payments and education funding, we find that, for the labor tax, the human capital channel dominates the redistribution channel; for the robot tax, the dominance is reversed.

While one-dimensional tax policy interventions (changes in either the labor or the robot tax) entail a fundamental trade-off between economic growth and inequality, we identify conditions for coordinated two-dimensional tax policy packages (combined changes in both taxes) to achieve both higher growth and lower inequality. For instance, we observe that a joint increase in the robot tax (which reduces inequality through redistribution) and in the labor tax (which increases growth via the human capital channel) can break the growth-inequality trade-off. This result highlights the importance of accounting for endogenous human capital accumulation along the intensive margin: In a version of the model in which we ignore the human capital-enhancing effect of education spending, a joint increase of both taxes would always lead to a reduction in production.

Taking this insight to a normative setting in which the optimal tax policy over time is characterized for a utilitarian welfare function, we find that the government should initially reduce the robot tax significantly and compensate for the loss in revenues with a higher labor tax. Subsequently, the government should progressively increase the robot tax and reduce the labor tax. This dynamic pattern of taxation initially provides incentives for increased R&D and automation. As machine productivity increases and the skill premium widens, the government then finds it optimal to increase the robot tax and reduce the labor tax to contain inequality.

A decomposition exercise disentangles the determination of the optimal tax policy via technological progress, education choices and human capital formation along the intensive margin, respectively. When the latter is missing so that the human capital channel is not operating, it becomes optimal to finance government spending exclusively via robot taxes: Absent a motive for education spending, the only task left for fiscal policy is to redistribute, which can be achieved more effectively via the robot tax and becomes increasingly important as automation proceeds.

As our approach highlights the interaction between the revenue (taxation) and the spending side of fiscal policy, we also study the role of education subsidies – formalized as tax-funded transfer payments to individuals undertaking higher education. When per-capita education spending is kept constant for both basic and higher education so that the intensive margin of human capital formation remains unchanged, there is no scope for education subsidies to increase aggregate welfare. However, welfare gains are attainable when the per-capita spending on basic education adjusts in response to the policy. It is thus crucial that the subsidy policy's extensive margin effects via education choice are accompanied by intensive margin effects via augmented human capital formation. When education subsidies can be targeted specifically to individuals who would not have undertaken higher education without the subsidy, the welfare effects can be further improved. Importantly, however, for all these education policies beneficial welfare effects can only be obtained under financing via the labor tax, whereas funding via the robot tax always has adverse welfare consequences. Accordingly, despite the hierarchical nature of the education system, the labor tax robustly emerges as the preferred instrument to finance education subsidies.

Finally, we consider a model extension in which individuals can privately invest in their higher education. Variations in public education spending then induce substitution effects that crowd out private contributions and result in a diminished magnitude of the human capital channel. As the importance of this mechanism hinges on the underlying funding mix for higher education, the implications for optimal fiscal policy now depend on the specific environment: For a 'European setting' where higher education is largely financed by the government, the optimal financing for the government's redistribution and education policy involves a positive robot tax. But for a 'US setting' where higher education is mostly privately funded, the optimal robot tax is zero. This difference is again rooted in the interplay between the human capital channel and the redistribution channel, but is also amenable to an intuitive normative interpretation. When high-skilled workers do not sufficiently contribute to their education, it is optimal to achieve redistribution by taxing them indirectly through a positive robot tax. When they instead contribute sufficiently, the indirect taxation through the robot tax becomes suboptimal as redistribution can be achieved more efficiently via the labor tax.

Related Literature Our paper contributes to several strands of the literature. Most closely related to our work is the paper by Prettner and Strulik (2020), which is also the specific point of departure for our model. Relative to other studies of automation-driven growth like Acemoglu and Restrepo (2018) or Hémous and Olsen (2022), we thus share the emphasis on the household side of the economy while technology is kept relatively simple.[1] Like Prettner and Strulik (2020), we examine inequality and redistribution in the context of an economy in which both technology and education are endogenous. However, we conceptualize education not just in terms of an extensive margin choice that generates skilled versus unskilled workers, but also in terms of the intensive margin effects of public education spending on the effective human capital commanded by these groups. Public policy therefore has interacting effects via the redistribution and the human capital channel, which entails important positive and normative implications for the scope, design and composition of desirable tax policies.

More broadly, our paper also contributes to a growing body of research investigating the joint dynamics of growth and inequality, driven by automation and human capital. Concerning innovation and automation, Krusell et al. (2000) identify capital-skill complementarity as a key mechanism generating inequality. Similarly, Brynjolfsson and McAfee (2011), Frey and Osborne (2017), Graetz and Michaels (2018), Acemoglu and Restrepo (2020) highlight the negative consequences of automation for wages and employment, particularly for workers with lower education. Common to these papers is that they analyze the impact of automation technologies on inequality along the skill distribution assuming an exogenous supply of high- and low-skilled workers. On the other hand, Goldin and Katz (2010), Acemoglu et al. (2012) and Goldin et al. (2020) emphasize the role of (higher) education in the context of increased adoption of automated technologies. Our model incorporates this 'race between education and technology' by considering endogenous education choices in the face of a widening skill premium under automation. But we extend the analysis by also allowing public policies to affect human capital formation via the intensive margin and provide a detailed analysis of fiscal policy.

This connects our work to papers analyzing the effects of tax and education policies on human capital accumulation, growth and inequality. For example, Guvenen et al. (2013) study the role of taxation for human capital formation via a generic accumulation equation. Considering an occupational choice model where school quality is endogenous to public education spending, Artige and Cavenaile (2023) find that there is a theoretically ambiguous relationship between public education and inequality and hence also between growth and inequality. In a similar vein, our paper follows the literature on public education finance (Restuccia and Urrutia, 2004; Blankenau, 2005; Arcalean and Schiopu, 2010) and models a hierarchical education system in which basic education spending affects all workers, while college education spending affects only high-skill workers. Differently from these papers, however, we examine the relevance of such an education system for educational

[1] Both Acemoglu and Restrepo (2018) and Hémous and Olsen (2022) do not consider human capital accumulation along the extensive or the intensive margin. On the production side, they allow for a richer structure where final output is produced by a variety of tasks or intermediate inputs, and where endogenous growth comes from new tasks or intermediate inputs generated by R&D. By contrast, we follow Prettner and Strulik (2020) and assume that R&D creates patents for automation capital, consistent with the evidence in Mann and Püttmann (2023).

sorting and inequality in a setting with endogenous technological progress from R&D and adoption of automation.

Considering the implications for desirable tax and education policies, Krueger and Ludwig (2013, 2016) characterize the optimal mix of progressive income taxes and education subsidies in an overlapping generations model with endogenous human capital under incomplete financial markets. They find that the welfare-maximizing fiscal policy features a progressive labor income tax code combined with a sizable subsidy for college education. This highlights the complementarity between the redistributive role of progressive taxation and appropriate education subsidies to offset the tax-induced labor supply distortions along the intensive and extensive margin (cf. Bénabou, 2002). In addition to education transfers, our work emphasizes the importance also of spending which augments the quality (intensive margin) of education and thus materializes via the human capital channel. Moreover, we obtain detailed results for the desirable breakdown of funding via the labor versus the robot tax in an environment with endogenous technological change.

Finally, given the fundamental importance of the equity-efficiency trade-off for the appropriate design of fiscal policy, our work is also related to the public finance literature on optimal taxation following Mirrlees (1971) and Diamond and Mirrlees (1971). In particular, we refer to the recent literature on robot and capital taxation (Slavik and Yazici, 2014; Jacobs and Thuemmel, 2020; Guerreiro et al., 2022; Thuemmel, 2022) in which production efficiency is a crucial policy objective.[2] Similar to Guerreiro et al. (2022), we show that the optimal robot tax can be different from zero. However, while in their paper this is driven by the presence of older workers in the labor market who are constrained by their initial education choices, in our model the positive robot tax reflects that its redistributive role is accompanied by its growth-enhancing effect through the human capital channel. Moreover, in contrast to their paper which finds that the optimal robot tax is zero in the long-run, our model suggests a progressive increase in the robot tax over time as an efficient way to reduce growing wage inequality. To put these findings into perspective, it is important to recall that our approach differs from the classical optimal taxation approach where the consequences of

[2] In this literature, information frictions constrain the government's tax-and-transfer system not to discriminate across unobservable types. In the particular two type-case of 'skilled' versus 'unskilled' workers considered in Stiglitz (1982), production efficiency (and thus a zero robot tax) is optimal when worker types are exogenous and they enter production as perfect substitutes. With imperfect substitutability across types, this is no longer the case, provided labor supply distortions along the intensive margin are the relevant concern. But adding an extensive margin via skill acquisition tends to restore the optimality of production efficiency, that is, the absence of robot taxes (cf. Guerreiro et al., 2022). By contrast, in our economy, worker types are observable so that the tax-and-transfer system can condition on skills in a discriminatory way. Imperfect substitutability in production and endogenous skill acquisition, though, are shared features in both environments. In addition, in our economy workers' wages are endogenous not only due to interaction in production, but also due to the human capital generated via the government's tax-financed education policy. This justifies deviations from production efficiency via positive robot taxes. Over time, this motive actually becomes stronger because of the skill-biased nature of the technological progress which is endogenously generated in our model.

tax policy are generally studied under either exogenous spending needs or lump-sum redistribution. By contrast, our model provides an integrated assessment of the government's tax and spending policy, whereby our baseline takes the composition of spending for transfers versus education as given and focuses on the optimal mix of labor and robot taxes.

The rest of this paper is structured as follows. Section 2 sets up our model, before Section 3 details the interaction between the redistribution channel and the human capital channel in a partial equilibrium environment with given supply of skills. Section 4 presents our calibration strategy and the general equilibrium dynamics of the calibrated economy. Sections 5 and 6 provide a detailed analysis of tax and education policies, respectively. Section 7 extends the model to account for privately funded education spending and Section 8 concludes.

2 Model

Similar to Prettner and Strulik (2020), we consider an overlapping generations economy in which individuals live for two periods. Having completed basic (i.e., primary and secondary) education, individuals enter the economy as young adults with a unit endowment of time. Then, they decide whether or not to spend a certain (fixed) fraction of their time studying to obtain higher (i.e., tertiary/college) education. If they decide against higher education, individuals allocate their full unit time endowment between leisure and labor as low-skilled workers. If they spend time in higher education, they allocate their remaining time between leisure and labor as high-skilled workers. On one hand, higher education is associated with an opportunity cost in terms of reduced marketable time; on the other hand, it augments individuals' human capital (intensive margin of human capital) and endows them with skills that differentiate them from unskilled workers (extensive margin of human capital). Both types of workers use their labor market income for consumption and savings for their second period of life when they are retired and simply consume the return to their savings. After the second period, individuals die with certainty. Time t evolves discretely, each period corresponds to one generation. The population size is constant in every period and equal to N.

Individuals differ in their ability to complete a college degree, which affects education decisions via the disutility (of effort) while acquiring higher education. In consequence, each generation is partitioned into two groups: those who opt into higher education and become high-skilled workers, and those who do not and become low-skilled workers. The two groups are distinct both in their human capital and the way they interact with machines in production. Low-skilled workers are employed in the final production sector and can be substituted by machines. By contrast, high-skilled workers are employed either in the final production sector as complements to machines, or in the R&D sector for developing the blueprints for machines that are used in the final production sector.

The government raises taxes through a labor tax (linear tax on wage income) and a robot tax (ad-valorem tax on the use of machines in the final production sector) and uses the revenue to finance expenditure on education and transfers. Education spending is allocated to basic education and higher education. While basic education spending affects the human capital of both low-skilled and high-skilled workers, higher education spending directly affects only the human capital of high-skilled workers. Transfers are differentiated across worker types allowing for progressivity in the model.

2.1 Households

Individuals obtain utility from consumption and leisure, and disutility from completing higher education. Lifetime utility of an agent of type $j \in \{H, L\}$ is given by:

$$\mathcal{U}_{j,t} = \log(c_{j,t}) + \beta \log(\bar{R}s_{j,t}) + \gamma \log(z_{j,t}) - \mathbb{1}_{[j=H]}\ v(a), \tag{1}$$

where $c_{j,t}$ is the consumption of the young agent in period t; $\beta \in [0, 1]$ is the discount factor; $\bar{R}s_{j,t}$ is the consumption of the old agent in period $t + 1$ (consisting of savings $s_{j,t}$ and gross interest payments $\bar{R}$); $\gamma > 0$ is a preference weight; and $z_{j,t}$ is leisure. Following Prettner and Strulik (2020), we make the simplifying assumption of a small open economy such that the interest rate $\bar{R}$ is determined on the world capital market and, therefore, exogenously given. The last term in (1) is the effort cost from higher education. Individuals are heterogeneous in terms of their innate ability a. Individuals with a higher ability suffer lower effort costs from completing higher education. In particular, we assume:

$$v(a) = \begin{cases} \psi_1 \cdot \log(\frac{\psi_2}{a-\underline{a}}), & \text{if} \quad a \geqslant \underline{a} \\ +\infty, & \text{if} \quad a < \underline{a}, \end{cases} \tag{2}$$

where $\psi_1 > 0$ and $\psi_2 > 0$ determine the level and the slope of the effort cost, and $\underline{a} > 0$ captures the idea that not all agents are able to obtain a college degree.

Given their ability level a, individuals maximize their lifetime utility by choosing consumption, savings, and leisure in period t subject to the following budget constraint:

$$(1 - \tau_{W,t})(1 - \eta_j - z_{j,t})w_{j,t} + \hat{T}_{j,t} = c_{j,t} + s_{j,t}, \tag{3}$$

where $\tau_{W,t}$ represents the linear tax rate on wage income; η_j is the time spent to acquire higher education, which is equal to zero for agents who do not go to college ($\eta_L = 0$) and equal to $\eta > 0$ for agents obtaining a college degree ($\eta_H = \eta$); $w_{j,t}$ is the type-specific wage; and $\hat{T}_{j,t}$ is the per-capita transfer to an agent of type j.[3] As old individuals do not work, they only consume their savings.

[3]Given a total volume of transfers $T_{j,t}$ to young individuals of type j, the per-capita payments are computed as $\hat{T}_{L,t} \equiv T_{L,t}/L_t$ and $\hat{T}_{H,t} \equiv T_{H,t}/H_t$, where L_t and H_t denote the mass of low- and high-skilled workers, respectively. Since per-capita transfers are not necessarily uniform across types, even under linear labor taxation, the system can be progressive (or regressive).

From the household utility maximization problem, we obtain:

$$c_{j,t} = \left(\frac{1}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t}+\hat{T}_{j,t}\right), \tag{4}$$

$$s_{j,t} = \left(\frac{\beta}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t}+\hat{T}_{j,t}\right), \tag{5}$$

$$z_{j,t} = \left(\frac{\gamma}{(1+\beta+\gamma)(1-\tau_{W,t})w_{j,t}}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t}+\hat{T}_{j,t}\right). \tag{6}$$

Given the ability level a, an individual decides to go to college to acquire higher education if $\mathcal{U}_{H,t}(a) \geqslant \mathcal{U}_{L,t}(a)$. Hence, there exists a threshold level a_t^* such that if $a \geqslant a_t^*$ the individual attends college, and if $a < a_t^*$ the individual does not. From the indifference condition, $\mathcal{U}_{H,t}(a) = \mathcal{U}_{L,t}(a)$, we obtain:

$$a_t^* = \psi_2 \left(\frac{c_{H,t}}{c_{L,t}}\right)^{-\frac{1+\beta+\gamma}{\psi_1}} \left(\frac{w_{H,t}}{w_{L,t}}\right)^{\frac{\gamma}{\psi_1}} + \underline{a}. \tag{7}$$

Assuming that individual ability a is distributed according to a cumulative distribution function $\mathcal{F}$, this implies that the mass of low-skilled workers is given by $L_t = \mathcal{F}(a_t^*)N$, and the mass of high-skilled workers by $H_t = (1-\mathcal{F}(a_t^*))N$, where $H_t = H_{Y,t} + H_{A,t}$, i.e., high-skilled workers are either employed in the final production sector or the R&D sector.

2.2 Final production sector

Aggregate output is produced according to the following production function:

$$Y_t = \left(h_{H,t}\tilde{H}_{Y,t}\right)^{1-\alpha}\left(\left(h_{L,t}\tilde{L}_t\right)^{\alpha} + \sum_{i=1}^{A_t}(x_{i,t})^{\alpha}\right), \tag{8}$$

where $\tilde{H}_{Y,t} \equiv (1 - z_{H,t} - \eta)H_{Y,t}$ is high-skilled labor employed in the final goods sector; $\tilde{L}_t \equiv (1 - z_{L,t})L_t$ is low-skilled labor; $h_{j,t}$ is the human capital of an agent of type j at time t; $x_{i,t}$ are machines of type i; $\alpha \in (0,1)$ is the elasticity of output with respect to (effective) labor that can be easily automated; and A_t represents the technological frontier. Let $p_{i,t}$ denote the price of a machine of type i and $\tau_{R,t}$ the ad-valorem tax on the use of machines in the final production sector, i.e., the robot tax. The profit maximization problem faced by competitive firms in the final production sector is:

$$\max_{\left\{\tilde{H}_{Y,t},\tilde{L}_t,\{x_{i,t}\}_{i=1}^{A_t}\right\}} Y_t - w_{H,t}\tilde{H}_{Y,t} - w_{L,t}\tilde{L}_t - (1+\tau_{R,t})\sum_{i=1}^{A_t} p_{i,t}x_{i,t}, \tag{9}$$

from which factor prices are obtained as:

$$w_{H,t} = (1-\alpha)\left(h_{H,t}\tilde{H}_{Y,t}\right)^{-\alpha} h_{H,t}\left((h_{L,t}\tilde{L}_t)^{\alpha} + \sum_{i=1}^{A_t}(x_{i,t})^{\alpha}\right), \tag{10}$$

$$w_{L,t} = \alpha\left(\frac{h_{H,t}\tilde{H}_{Y,t}}{h_{L,t}\tilde{L}_t}\right)^{1-\alpha} h_{L,t}, \tag{11}$$

$$(1+\tau_{R,t})p_{i,t} = \alpha\left(\frac{h_{H,t}\tilde{H}_{Y,t}}{x_{i,t}}\right)^{1-\alpha}. \tag{12}$$

The effect of technological progress (in the form of an increase in A_t, i.e., the variety of machines used in the production process measuring the technological frontier) is different across skill groups. On the one hand, technological progress is quasi labor-augmenting in the sense that it increases the productivity of high-skilled labor in the final production sector (see 10). On the other hand, productivity of low-skilled individuals is unaffected by technological progress (see 11), leading to a decline of the relative importance of low-skilled labor in the final production sector when technological progress occurs.[4]

2.3 R&D sector

The R&D sector produces the blueprints for new machines by employing only high-skilled labor. Similar to Romer (1990) and Jones (1995, 2022), we consider the following process for expanding the technological frontier via R&D:

$$A_t - A_{t-1} = \bar{\delta}_t h_{H,t}\tilde{H}_{A,t}, \tag{13}$$

where $\tilde{H}_{A,t} \equiv (1 - z_{H,t} - \eta)H_{A,t}$ represents high-skilled labor employed in the R&D sector; $\bar{\delta}_t \equiv \delta\frac{(A_{t-1})^{\lambda_1}}{\left(h_{H,t}\tilde{H}_{A,t}\right)^{1-\lambda_2}}$ is a measure of the productivity in the R&D sector capturing intertemporal knowledge spillovers (measured by $\lambda_1 \in (0,1]$) and congestion externalities (measured by $(1-\lambda_2)$ with $\lambda_2 \in [0,1]$), and δ is a scaling parameter. R&D firms' profits are given by the revenues generated by selling patents net of labor costs, $p_{A,t}\bar{\delta}_t h_{H,t}\tilde{H}_{A,t} - w_{A,t}\tilde{H}_{A,t}$, where $p_{A,t}$ denotes the price of blueprints and $w_{A,t}$ is the wage rate in the R&D sector. Optimality requires that $w_{A,t} = p_{A,t}\bar{\delta}_t h_{H,t}$. Patent protection is assumed to last for one model period.[5]

2.4 Intermediate goods sector

The intermediate goods sector rents capital to produce machines. We consider a linear technology, $x_{i,t} = k_{i,t}$, where $k_{i,t}$ is the amount of capital used by the intermediate firms producing machines

[4]This is in line with increasing high-skilled wages and stagnating low-skilled wages observed in the data in the last decades (Autor, 2019).

[5]This assumption is quite reasonable as the standard patent duration in the US is 20 years (US Patent and Trademark Office, 2023), i.e., similar to the length of one model period.

of type i, and assume that physical capital depreciates fully within one model period. Firms in the intermediate goods sector either produce the latest vintage (denoted by n) or the older vintage machines (denoted by m). Producers of older vintage machines do not need to acquire patents and operate under perfect competition. Free entry implies zero profits, i.e., $\pi_{m,t} = 0$. Producers of the latest vintage machines use patents from the R&D sector as input, which endows them with a certain degree of market power. Free entry implies that the profits, $\pi_{n,t}$, for the producers of the latest vintage machines must be equal to the patent costs, i.e., $\pi_{n,t} = p_{A,t}$. The profit maximization problem faced by the latest vintage machine producers is given by:

$$\max_{x_{n,t}} p_{n,t}(x_{n,t})x_{n,t} - \bar{R}x_{n,t} \tag{14}$$

subject to (12). Optimality requires:

$$\frac{\partial p_{n,t}(x_{n,t})}{\partial x_{n,t}} \frac{x_{n,t}}{p_{n,t}} + 1 = \frac{\bar{R}}{p_{n,t}} \quad \Longleftrightarrow \quad p_{n,t} = \frac{\bar{R}}{\alpha} \quad \Longleftrightarrow \quad \pi_{n,t} = \frac{1-\alpha}{\alpha}\bar{R}x_{n,t}, \tag{15}$$

from which the supply of machines of the latest vintage is obtained as:

$$x_{n,t} = \left(\frac{\alpha^2}{\bar{R}(1+\tau_{R,t})} \right)^{\frac{1}{1-\alpha}} h_{H,t}\tilde{H}_{Y,t}. \tag{16}$$

Older vintage machine producers, instead, face the following problem:

$$\max_{x_{m,t}} p_{m,t}x_{m,t} - \bar{R}x_{m,t}, \tag{17}$$

from which we obtain the optimality condition $p_{m,t} = \bar{R}$ and the supply of machines of older vintage:

$$x_{m,t} = \left(\frac{\alpha}{\bar{R}(1+\tau_{R,t})} \right)^{\frac{1}{1-\alpha}} h_{H,t}\tilde{H}_{Y,t}. \tag{18}$$

Combining (16) and (18), we obtain $x_{m,t} = \alpha^{\frac{1}{\alpha-1}} x_{n,t}$. Finally, aggregating over all vintages, we can rewrite the final goods production function as:

$$Y_t = \left(h_{H,t}\tilde{H}_{Y,t} \right)^{1-\alpha} \left(\left(h_{L,t}\tilde{L}_t \right)^{\alpha} + \tilde{A}_t \left(x_t \right)^{\alpha} \right), \tag{19}$$

where $x_t \equiv x_{n,t}$ and $\tilde{A}_t \equiv \left(\alpha^{\frac{\alpha}{\alpha-1}} - 1 \right) A_{t-1} + A_t$.[6]

2.5 Human capital

Human capital formation takes place via education, the effectiveness of which is determined by the amount and composition of public education spending, E_t. We assume a hierarchical public education system with a sequential process for basic and higher education following the literature

[6]Indeed, $\sum_{i=1}^{A_t} (x_{i,t})^{\alpha} = A_{t-1} (x_{m,t})^{\alpha} + (A_t - A_{t-1}) (x_{n,t})^{\alpha} = \left(\left(\alpha^{\frac{\alpha}{\alpha-1}} - 1 \right) A_{t-1} + A_t \right) (x_t)^{\alpha} = \tilde{A}_t (x_t)^{\alpha}$.

on public education finance (Restuccia and Urrutia, 2004; Blankenau, 2005; Arcalean and Schiopu, 2010). Accordingly, basic human capital, $h_{B,t}$, is determined only by public spending on basic education, $E_{B,t}$, while the human capital of high-skilled workers, $h_{H,t}$, depends on both the spending on higher education, $E_{H,t}$, and the human capital previously acquired through basic education, $h_{B,t}$. Total public education spending is the sum of the expenditures across the two tiers, i.e., $E_t = E_{B,t} + E_{H,t}$.

Basic human capital, $h_{B,t}$, and human capital of the low-skilled workers, $h_{L,t}$, coincide:

$$h_{L,t} = h_{B,t} = B \cdot \left(\hat{E}_{B,t}\right)^{\mu_B}, \tag{20}$$

where $\hat{E}_{B,t} \equiv E_{B,t}/N$ is the per-capita level of public education spending for basic education. Building on the human capital acquired from basic education, those individuals selecting into higher education acquire additional skills, which (i) differentiate them from unskilled workers and (ii) raise their human capital. Specifically, we assume:

$$h_{H,t} = B_H \cdot (h_{B,t})^{1-\mu_H} \cdot \left(\hat{E}_{H,t}\right)^{\mu_H}, \tag{21}$$

where $\hat{E}_{H,t} \equiv E_{H,t}/H_t$ is the per-capita level of public education spending for higher education. The parameters $B > 0$ and $B_H > 0$ are productivity parameters, while $\mu_B \in (0,1)$ and $\mu_H \in (0,1)$ govern the elasticity of human capital formation to public spending inputs at the basic and higher level, respectively. Notice that the above specification of human capital formation entails a role for public education spending via both the extensive margin (by drawing a larger number of individuals into higher education) and the intensive margin (by making education at both tiers more productive).

2.6 Fiscal policy

The government raises revenues by taxing labor and robots. Total tax revenues, $\mathcal{G}_t$, are given by the sum of labor, $\mathcal{G}_{W,t}$, and robot tax revenues, $\mathcal{G}_{R,t}$, defined as:

$$\mathcal{G}_{W,t} = \tau_{W,t}\left(w_{H,t}\tilde{H}_{Y,t} + w_{A,t}\tilde{H}_{A,t} + w_{L,t}\tilde{L}_t\right), \tag{22}$$

$$\mathcal{G}_{R,t} = \tau_{R,t}\sum_{i=1}^{A_t} p_{i,t}x_{i,t} = \tau_{R,t}\hat{A}_t\bar{R}x_t, \tag{23}$$

where $\hat{A}_t \equiv \alpha^{\frac{1}{\alpha-1}}A_{t-1} + \alpha^{-1}(A_t - A_{t-1})$.[7]

On the spending side, tax revenues are used to finance public education, E_t, and transfers, T_t, such that the budget is balanced, i.e., $\mathcal{G}_t = E_t + T_t$. We define $\phi_t \in (0,1]$ as the share of total public spending allocated to total education, and $\phi_{B,t} \in (0,1)$ as the share of total education spending allocated to basic education. This implies that we can rewrite the total public education spending, the total transfer spending, the basic education spending, and the higher education spending as:

[7] Indeed, $\sum^{A_t} p_{i,t}x_{i,t} = A_{t-1}\bar{R}x_{m,t} + (A_t - A_{t-1})\frac{\bar{R}}{\alpha}x_{n,t} = (\alpha^{\frac{1}{\alpha-1}}A_{t-1} + \alpha^{-1}(A_t - A_{t-1}))\bar{R}x_t = \hat{A}_t\bar{R}x_t$.

$E_t = \phi_t \mathcal{G}_t$; $T_t = (1-\phi_t)\mathcal{G}_t$; $E_{B,t} = \phi_{B,t} E_t$; and $E_{H,t} = (1-\phi_{B,t})E_t$, respectively. We further define the share of transfers to low-skilled individuals as $\omega_t \in [0,1]$ and rewrite the total transfers to low- and high-skilled agents as $T_{L,t} = \omega_t T_t$ and $T_{H,t} = (1-\omega_t)T_t$, respectively.

2.7 Competitive equilibrium

For a given balanced-budget fiscal policy $\{\tau_W, \tau_R, \phi, \phi_B, \omega\}_t$, the competitive equilibrium is such that (i) the household maximizes her utility; (ii) firms in the intermediate, final, and R&D sectors maximize their profits; (iii) the markets for low- and high-skilled labor, and the markets for machines, final goods, and patents clear; (iv) the population constraint holds, i.e., $N = H_t + L_t$, and (v) the no-arbitrage condition in the high-skilled labor market holds, i.e., $w_{H,t} = w_{A,t}$.[8]

3 Partial equilibrium analysis

Before turning to the model analysis for a calibrated environment, this section examines the main channels through which a change in taxation affects inequality. Key to our results is the fact that taxation generates fiscal revenues, which are used not only for transfer payments but also for public education. In consequence, tax policy affects inequality both via its redistributive effects and its effects on human capital formation. To highlight these effects, we proceed with a partial equilibrium analysis, keeping the aggregate supply of skills constant both along the education choice and the individual labor supply, i.e., we keep the extensive margin of human capital formation fixed. This is achieved by fixing the ability threshold a^* for selecting into higher education and considering the case of inelastic individual labor supply ($\gamma = 0$).[9] As a measure of inequality, we consider the consumption ratio between high- and low-skilled workers.[10] Our inequality measure is, therefore, defined as:

$$\frac{c_H}{c_L} = \frac{(1-\tau_W)(1-\eta)w_H + \hat{T}_H}{(1-\tau_W)w_L + \hat{T}_L}. \tag{24}$$

As hinted above, taxation affects the consumption ratio through two main channels: the *redistribution channel* (RE) and the *(intensive margin) human capital channel* (HC). The redistribution channel accounts for the direct effect of taxes on disposable income and transfers, while the human

[8]The equilibrium conditions are detailed in Appendix Section A.

[9]For the rest of this section, we drop time subscripts as they are not necessary for the analysis. Proofs for the theoretical results are available in Appendix Section B.

[10]The consumption ratio for young and old agents is the same due to the constant saving rate under log utility.

capital channel accounts for the effect of education spending on wages through human capital:

$$\frac{d\ c_H/c_L}{d\ \tau_W} = \underbrace{\frac{\partial c_H/c_L}{\partial \tau_W} + \sum_{j\in H,L} \frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \tau_W}}_{\frac{d\ c_H/c_L}{d\ \tau_W}\Big|_{\text{RE}}} + \underbrace{\sum_{j\in H,L}\left(\frac{\partial c_H/c_L}{\partial w_H}\frac{\partial w_H}{\partial h_j} + \frac{\partial c_H/c_L}{\partial w_L}\frac{\partial w_L}{\partial h_j}\right)\frac{\partial h_j}{\partial \tau_W}}_{\frac{d\ c_H/c_L}{d\ \tau_W}\Big|_{\text{HC}}}, \tag{25}$$

$$\frac{d\ c_H/c_L}{d\ \tau_R} = \underbrace{\frac{\partial c_H/c_L}{\partial w_H}\frac{\partial w_H}{\partial \tau_R} + \sum_{j\in H,L} \frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \tau_R}}_{\frac{d\ c_H/c_L}{d\ \tau_R}\Big|_{\text{RE}}} + \underbrace{\sum_{j\in H,L}\left(\frac{\partial c_H/c_L}{\partial w_H}\frac{\partial w_H}{\partial h_j} + \frac{\partial c_H/c_L}{\partial w_L}\frac{\partial w_L}{\partial h_j}\right)\frac{\partial h_j}{\partial \tau_R}}_{\frac{d\ c_H/c_L}{d\ \tau_R}\Big|_{\text{HC}}} \tag{26}$$

Redistribution channel Within the redistribution channel, we can identify distinct mechanisms through which taxation affects consumption inequality. Considering a marginal increase in the labor tax, τ_W, we can distinguish three effects:

$$\frac{d\ c_H/c_L}{d\ \tau_W}\Big|_{\text{RE}} = \underbrace{\frac{\partial\ c_H/c_L}{\partial\ \tau_W}}_{\text{RE}_W(1)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \tau_W}}_{\text{RE}_W(2)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\sum_{j'\in H,L}\frac{\partial \mathcal{G}}{\partial w_{j'}}\sum_{j''\in H,L}\frac{\partial w_{j'}}{\partial h_{j''}}\frac{\partial h_{j''}}{\partial \tau_W}}_{\text{RE}_W(3)}, \tag{27}$$

where $\text{RE}_W(1)$ is the direct negative effect on *disposable income*; $\text{RE}_W(2)$, the *tax rate* effect, positively affecting transfers due to the higher tax rate; and $\text{RE}_W(3)$, the *tax base* effect that increases transfers through the positive effect on wages driven by the higher education spending.[11] We can show that:

$$\frac{d\ c_H/c_L}{d\ \tau_W}\Big|_{\text{RE}} < 0 \qquad \Longleftrightarrow \qquad \omega > \frac{w_L L}{w_L L + w_H(1-\eta)H} \equiv \tilde{\omega}. \tag{28}$$

Accordingly, the redistribution channel works to reduce consumption inequality if the share of transfers given to low-skilled workers is sufficiently high, namely exceeding their relative share of labor income. Intuitively, this means that if the low-skilled workers get more transfers than their contribution to the government budget through labor taxes, then their relative consumption increases.

Consider now a marginal change in the robot tax, τ_R. As for the labor tax, we can distinguish three effects: the effect on high-skilled workers' wages and consumption arising due to their *complementarity* in production to machines, $\text{RE}_R(1)$; the *tax rate* effect, $\text{RE}_R(2)$; and the *tax base* effect, $\text{RE}_R(3)$, which unfold in analogy to the case of the labor tax:

$$\frac{d\ c_H/c_L}{d\ \tau_R}\Big|_{\text{RE}} = \underbrace{\frac{\partial\ c_H/c_L}{\partial\ w_H}\frac{\partial\ w_H}{\partial\ \tau_R}}_{\text{RE}_R(1)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \tau_R}}_{\text{RE}_R(2)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\sum_{j'\in H,L}\frac{\partial \mathcal{G}}{\partial w_{j'}}\sum_{j''\in H,L}\frac{\partial w_{j'}}{\partial h_{j''}}\frac{\partial h_{j''}}{\partial \tau_R}}_{\text{RE}_R(3)}. \tag{29}$$

[11] As $\text{RE}_W(3)$ rests on the interaction of human capital and transfers, it could in principle also be subsumed under the human capital channel. Abstracting from the intensive margin of human capital formation, $\text{RE}_W(3)$ disappears; condition (28) holds identically also in this case.

As for the labor tax, $\text{RE}_R(2)$ and $\text{RE}_R(3)$ are negative if low-skilled workers obtain proportionally more transfers relative to their labor income share than the high-skilled workers, i.e., if $\omega > \tilde{\omega}$. However, differently from the labor tax, $\text{RE}_R(1)$ is always negative since the robot tax affects disposable income not uniformly across the two skill groups, but instead has a direct negative effect only on the wages of the high-skilled workers and no direct effect on the wages of the low-skilled workers. This implies that $\omega > \tilde{\omega}$ is a sufficient (but not necessary) condition for the redistribution channel to reduce inequality through a change in the robot tax. Since this condition is sufficient for the redistribution effect to reduce consumption inequality through τ_R, while it is a necessary condition for τ_W, the robot tax is more redistributive. Intuitively, since the robot tax only directly affects the wages of high-skilled workers, while the labor tax has a symmetric direct effect on the wages of low- and high-skilled workers, the robot tax has a stronger redistributive effect. We can summarize these results with the following proposition.

Proposition 3.1. *An increase in taxation reduces consumption inequality, c_H/c_L, through the redistribution channel (RE) if $\omega > \tilde{\omega}$. This is a necessary condition for the linear labor tax τ_W and a sufficient condition for the ad-valorem robot tax τ_R.*

Human capital channel To highlight the human capital channel, we abstract from transfers by assuming $\phi = 1$.[12] The consumption ratio then simplifies to:

$$\frac{c_H}{c_L} = (1-\eta)\frac{w_H}{w_L}. \tag{30}$$

Within the human capital channel, considering a marginal increase of either labor or robot taxes, we can distinguish two mechanisms: the direct human capital effect on wages via increased *education spending*, $\text{HC}_g(1)$; and the additional human capital effect on wages via increased *R&D activity* and machine intensity $\tilde{A}$, $\text{HC}_g(2)$. Specifically, for $g \in \{W, R\}$, we have:

$$\left.\frac{d\ w_H/w_L}{d\ \tau_g}\right|_{\text{HC}} = \underbrace{\frac{\partial w_H/w_L}{\partial h_H/h_L}\frac{\partial h_H/h_L}{\partial \tau_g}}_{\text{HC}_g(1)} + \underbrace{\frac{\partial w_H/w_L}{\partial \tilde{A}}\frac{\partial \tilde{A}}{\partial h_H}\frac{\partial h_H}{\partial \tau_g}}_{\text{HC}_g(2)}. \tag{31}$$

We find that an increase in taxes (labor or robot) always increases the human capital ratio, h_H/h_L. This result is driven by the assumption of a hierarchical education system. Indeed, while an increase in the spending for basic education benefits the human capital of both types of workers, an increase in spending in higher education only benefits the human capital of high-skilled workers. Therefore, an increase in aggregate education spending financed by increased taxation leads to an increase in both basic and college education spending, which benefits mostly the high-skilled workers. This implies that $\text{HC}_g(1)$ is always positive. Also, the second term, $\text{HC}_g(2)$, is always positive as an increase in the human capital of high-skilled workers – via the R&D process – leads to higher

[12] This is without loss of generality as the effects via transfers are already captured within the redistribution channel.

machine intensity, $\tilde{A}$, and thus – via their complementarity in production – to higher wages of high-skilled workers. This implies that higher taxes (labor or robot) unambiguously increase the wage ratio w_H/w_L and hence consumption inequality through the human capital channel. The following proposition summarizes these findings.

Proposition 3.2. *An increase in taxation via the linear labor tax τ_W or the ad-valorem robot tax τ_R unambiguously increases consumption inequality, c_H/c_L, through the human capital channel (HC).*

Since the redistribution channel reduces consumption inequality in the empirically relevant situation in which low-skilled workers receive proportionally more transfers, while the human capital channel always increases consumption inequality, the overall effect of taxation is ambiguous. We therefore proceed with a quantitative exercise for a calibrated environment to determine the net effect of a change in tax policy on inequality. To simultaneously analyze the effect on production growth, we also take into account the endogenous household response in terms of educational choices (extensive margin of human capital formation) and individual labor supply. That is, we allow for adjustments in the ability threshold a^* and calibrate $\gamma > 0$. Although these general equilibrium adjustments modify the precise nature of the redistribution and the human capital channels, their sign is preserved.

4 Calibration and model dynamics

4.1 Calibration

We calibrate our model to the US economy of the year 2020 and assume that each model period corresponds to 25 years. The model has 19 parameters, eleven of which are set externally and eight internally calibrated. On top of these, we must specify fiscal policy $\{\tau_W, \tau_R, \phi, \phi_B, \omega_t(\rho)\}$ assuming time-invariant tax rates and public education spending shares, and allowing the share of transfers to low-skilled agents to vary to maintain a constant progressivity level, ρ, of the tax-and-transfer system.

External calibration We set $\beta = 0.55$, corresponding to an annual discount factor of about 0.98. The parameter $\gamma = 1.44$ is set to match the average US gross saving rate of 0.184 from 1996-2020 (US Bureau of Economic Analysis, 2023). The elasticity of output with respect to effective human labor that can easily be automated is set in line with Prettner and Strulik (2020), proposing a value of $\alpha = 0.80$.[13] Following Prettner and Strulik (2020), we assume that a) innate learning

[13]This value is justified based on information from the International Federation of Robotics indicating a 60 percent decline in the quality-adjusted price of robots between 1993 and 2005 together with an increase in the stock of robots by about 300 percent over the same period. From (12), the implied price elasticity of robot demand of 5 then pins

abilities are normally distributed with a mean $\mu_a = 100$ and standard deviation $\sigma_a = 15$ mimicking the empirically observed IQ distribution, and b) only half of the population has the potential to obtain higher education, i.e., $\underline{a} = 100$. Assuming that individuals have a working-age period of approximately 44 years between leaving high school (at age 19) and retiring (at age 63) and that the time spent in higher education amounts to five years, we set the fraction of time that high-skilled workers need to spend in higher education, η, to 0.11. The size of one generation is normalized to $N = 1000$. The intertemporal knowledge spillover parameter ($\lambda_1 = 0.67$) and the congestion externality parameter ($\lambda_2 = 0.44$) are set in line with the estimates for knowledge production in a class of semi-endogenous growth models obtained by Coe and Helpman (1995), Bottazzi and Peri (2007) and McMorrow and Röger (2009).[14] The interest rate factor $\bar{R} = 2.32$ is set to match the average (annual) real interest rate for the US from 1996-2020 of 3.43 percent (World Bank, 2023).

Internal calibration The remaining parameters are internally calibrated by minimizing the quadratic distance between model moments and empirically observed targets. The R&D productivity parameter δ, the disutility parameters for educational effort (ψ_1 and ψ_2) and the parameters governing the effectiveness of the education system (μ_B, μ_H, B and B_H) are set to fit the following seven targets: human capital level of the high-skilled individuals in year 2020 normalized to unity; share of college graduates of 34.7 percent in the year 2020 (US Census Bureau, 2023b); college wage premium of 1.86 in 2020 (US Census Bureau, 2023a);[15] average annual TFP growth rate for 1996-2020 of approximately 0.91 percent (OECD, 2022a); employment share in the R&D sector of around 1 percent in year 2020 (OECD, 2023c); average elasticity of low-skilled wages with respect to per-capita spending on basic education of 0.54 (Jackson et al., 2015);[16] and average elasticity of college education with respect to its price of 1.2 (Dynarski, 2003).[17] Finally, we ex-post set A_0, the level parameter representing the technological frontier of the economy in the initial model period, to 87.3 to minimize the loss from the internal calibration routine.

down $\alpha = 0.8$.

[14]Compared to Prettner and Strulik (2020), these values imply less pronounced endogenous growth effects from knowledge production. Instead, our model assigns part of these effects to productivity-enhancing education.

[15]The college wage premium is calculated as the ratio of the mean annual earnings of the total population with a Bachelor's degree relative to high school graduates.

[16]In detail, Jackson et al. (2015) find that a 10 percent increase in per-pupil spending in each year for all 12 years of public school leads to about 7 percent higher wages. Since this effect captures a mix of low-skilled and high-skilled wages, we use the college share and the college premium reported above to infer the effect on low-skilled individuals only. This results in an elasticity of $0.7/(0.653 + 0.347 \times 1.86) = 0.54$.

[17]Dynarski (2003) studies the effects of financial aid on college enrollment and estimates the elasticity of college *attendance* with respect to overall schooling costs (comprising both the direct cost of tuition and the opportunity cost of foregone earnings) at about 1.5. In order to arrive at the relevant elasticity measure for college *completion*, thus taking into account the effect of drop-outs, we exploit information on the additional years of education generated by financial aid, which are also reported by Dynarski (2003). The downward adjustment results in an elasticity of college completion of about 1.2.

External		Internal		Policy	
Parameter	Value	Parameter	Value	Parameter	Value
β	0.55	δ	0.584	τ_W	0.28
γ	1.44	ψ_1	0.479	τ_R	0.05
α	0.80	ψ_2	17.09	ϕ	0.27
μ_a	100	μ_B	0.354	ϕ_B	0.78
σ_a	15	μ_H	0.223	ρ	0.18
$\underline{a}$	100	B	1.720		
η	0.11	B_H	6.236		
N	1000	A_0	87.3		
λ_1	0.67				
λ_2	0.44				
$\bar{R}$	2.32				

Table 1: Calibration. Parameters for the baseline model. See text for details.

Policy parameters The baseline configuration of fiscal policy $\{\tau_W, \tau_R, \phi, \phi_B, \omega_t(\rho)\}$ is determined as follows. The average labor income tax rate ($\tau_W = 0.28$) is taken from OECD (2022b) and the average robot tax ($\tau_R = 0.05$) is set in line with Acemoglu et al. (2020).[18] The share of government spending on education used for higher (college) education amounts to 0.89 percent of GDP in the US for the year 2019, whereas the share spent on basic (primary and secondary) education is 3.22 percent for the same year (OECD, 2023b). Therefore, total government spending on education amounts to 4.11 percent of GDP, with a share of basic education spending of total government spending on education of $\phi_B = 0.78$. We observe in the data that total social spending relative to GDP net of public pension payments makes up a share of 11.17 percent in 2019 (OECD, 2023d,a). Summing up both parts of the government budget and calculating the share of government spending on education within total government spending then leads to $\phi = 0.27$. The share of total lump-sum transfer payments to low-skilled individuals, ω_t, is specified as time-varying and set such that the implied progressivity of the model's tax-and-transfer system is constant over time and in line with $\rho = 0.18$, the value estimated for the US by Heathcote et al. (2017).[19]

[18] Acemoglu et al. (2020) estimate time paths of effective tax rates on different types of capital (structures, software and equipment) computed from effective tax rates on capital and depreciation allowances for C-corporations, S-corporations and other pass-through businesses, and the differential taxation of capital financed with debt and equity. We use the 2018 value for the estimated tax rate on software and equipment of around 5 percent. The estimated tax rate on structures is slightly higher (around 7 percent). Therefore, our parameter choice is located at the lower end of the spectrum for the capital tax in the US in 2018.

[19] In detail, we calculate the discrete elasticity of post-government ($\tilde{y}_{i,t}$) to pre-government ($y_{i,t}$) earnings; this elasticity is then evaluated at the low-skilled income level, with ω_t adjusting such that the following condition is fulfilled in each period,

$$\frac{\Delta \tilde{y}_{i,t}}{\Delta y_{i,t}} \frac{y_{i,t}}{\tilde{y}_{i,t}} = \frac{(1-\tau_{W,t})[w_{H,t}\tilde{h}_t - w_{L,t}\tilde{l}_t] + [\hat{T}_{H,t} - \hat{T}_{L,t}]}{[w_{H,t}\tilde{h}_t - w_{L,t}\tilde{l}_t]} \frac{w_{L,t}\tilde{l}_t}{(1-\tau_{W,t})w_{L,t}\tilde{l}_t + \hat{T}_{L,t}} = 1 - \rho = 0.82, \tag{32}$$

Target	Data	Model
high-skilled human capital (normalization)	1.00	1.00
college share (%)	34.7	34.7
R&D employment share (%)	1.00	1.69
college wage premium	1.86	1.86
elasticity of college attendance wrt. its overall price	1.20	1.20
elasticity of low-skilled wages wrt. per-capita spending on lower education	0.54	0.54
TFP growth (annual, %)	0.91	0.91

Table 2: Calibration. Goodness of fit of the baseline model. See text for details.

4.2 Model validation

The parameters used for the initial calibration are summarized in Table 1. Table 2 reports the fit of the calibrated baseline model to the respective moments in the data. Six out of seven moments are matched exactly. The only exception is the employment share in the R&D sector, which the model predicts at a value of 1.69 percent for the year 2020, relative to a value of 1 percent in the data. As the model abstracts from the use of capital in the R&D sector, this is a natural outcome of the model economy, as the importance of human labor is overstated by assumption.

Figure 1 further validates the model by comparing the inferred dynamics of the baseline calibration (blue solid lines) with actual observations for the US spanning the period from 1970 to 2020 (red dotted lines). The model predicts a smooth positive trend in average TFP growth over time, which broadly aligns with real-world observations. The model can also capture the substantial increase in the skill premium documented in the US over the past 50 years. Concurrently, we also observe a rise in the proportion of the population pursuing higher education. While the model does not precisely replicate the level of the R&D employment share due to the absence of capital in this sector, it successfully mirrors the overall growth trajectory of the average R&D employment share over time. The same pattern is observed for the R&D expenditure share, where the model reproduces the average growth rate rather than the exact level. Moreover, the model replicates the shrinking labor share observed in the US.

4.3 Model dynamics

Figure 2 illustrates the general equilibrium dynamics for the calibrated economy, which displays endogenous growth and endogenous skill acquisition under the baseline fiscal policy $\{\tau_W, \tau_R, \phi, \phi_B, \omega_t(\rho)\}$. We observe that the R&D-induced productivity growth of machines leads to an exponential increase in TFP, production, and automation (i.e., the use of machines in the final production sector) over time. Due to workers' different degrees of complementarity with machines, the increased produc-

where $\hat{T}_{L,t} = \omega_t \cdot \frac{T_t}{L_t}$, $\hat{T}_{H,t} = (1-\omega_t) \cdot \frac{T_t}{H_t}$, and $\tilde{h}_t$ and $\tilde{l}_t$ denote the effective individual labor supply of high-skilled and low-skilled workers, respectively.

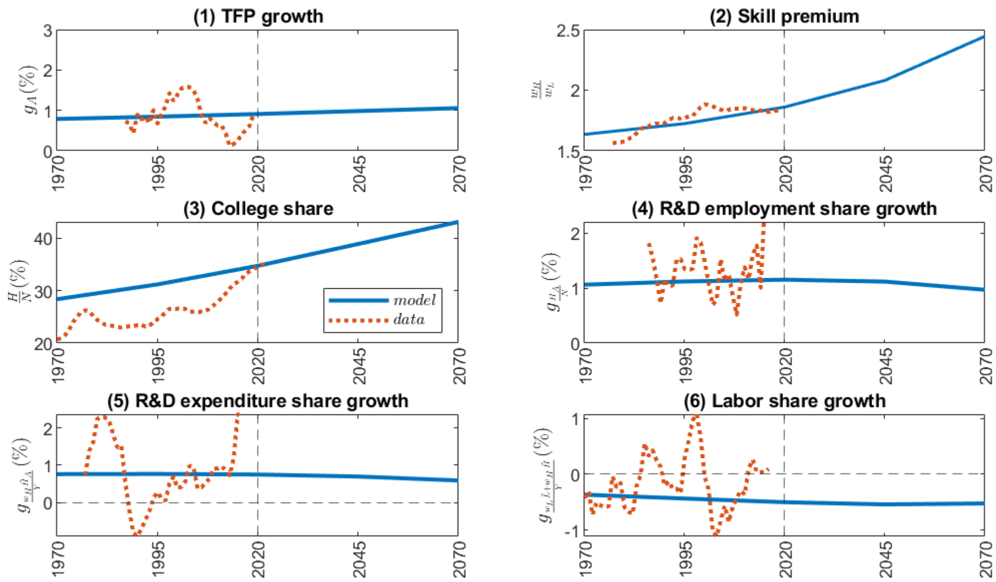

Figure 1: Model validation. Real world data for the US (red dotted lines) are represented in five-year moving averages. Labor share data are taken from FRED (2023), and data on the R&D expenditure share from NSF (2023). See text for sources for the rest of the data. Model growth rates are converted to yearly values.

tivity of machines disproportionally benefits the high-skilled workers, exponentially increasing the skill premium measured via the pre-tax ratio of high- to low-skilled wages, w_H/w_L. Although the tax-and-transfer system is progressive, the consumption ratio c_H/c_L (equivalent to the after-tax and transfer income ratio) follows a similar pattern. Throughout, post-government income inequality is slightly smaller than its pre-government value as transfers are used to redistribute resources from high- to low-skilled individuals.

The increase in the skill premium creates stronger incentives to undertake education (extensive margin of human capital formation) leading to a reduction in the number of low-skilled workers. As the share of low-skilled workers declines, the share of transfers to low-skilled agents, ω_t, declines as well to maintain the progressivity level constant. Vice versa, the number of high-skilled workers increases in both the final production and R&D sectors.

Changes in the composition of the type of workers are also accompanied by adjustments in the individual labor supply affecting the aggregate supply of labor, $\tilde{N}$. While the individual labor supply of high-skilled workers, $\tilde{h} \equiv 1 - \eta - z_H$, exhibits a modest decline over time, the individual labor supply of low-skilled workers, $\tilde{l} \equiv 1 - z_L$, decreases more substantially as the increase in the per-capita transfers, driven by economic growth, mostly affect low-skilled workers. Although the individual supply of high-skilled labor slightly declines, aggregate high-skilled labor supply $\tilde{H}$ rises, driven by the higher number of high-skilled individuals over time. The opposite is true for the aggregate labor supply of low-skilled individuals, $\tilde{L}$, driven by a decline in both individual labor

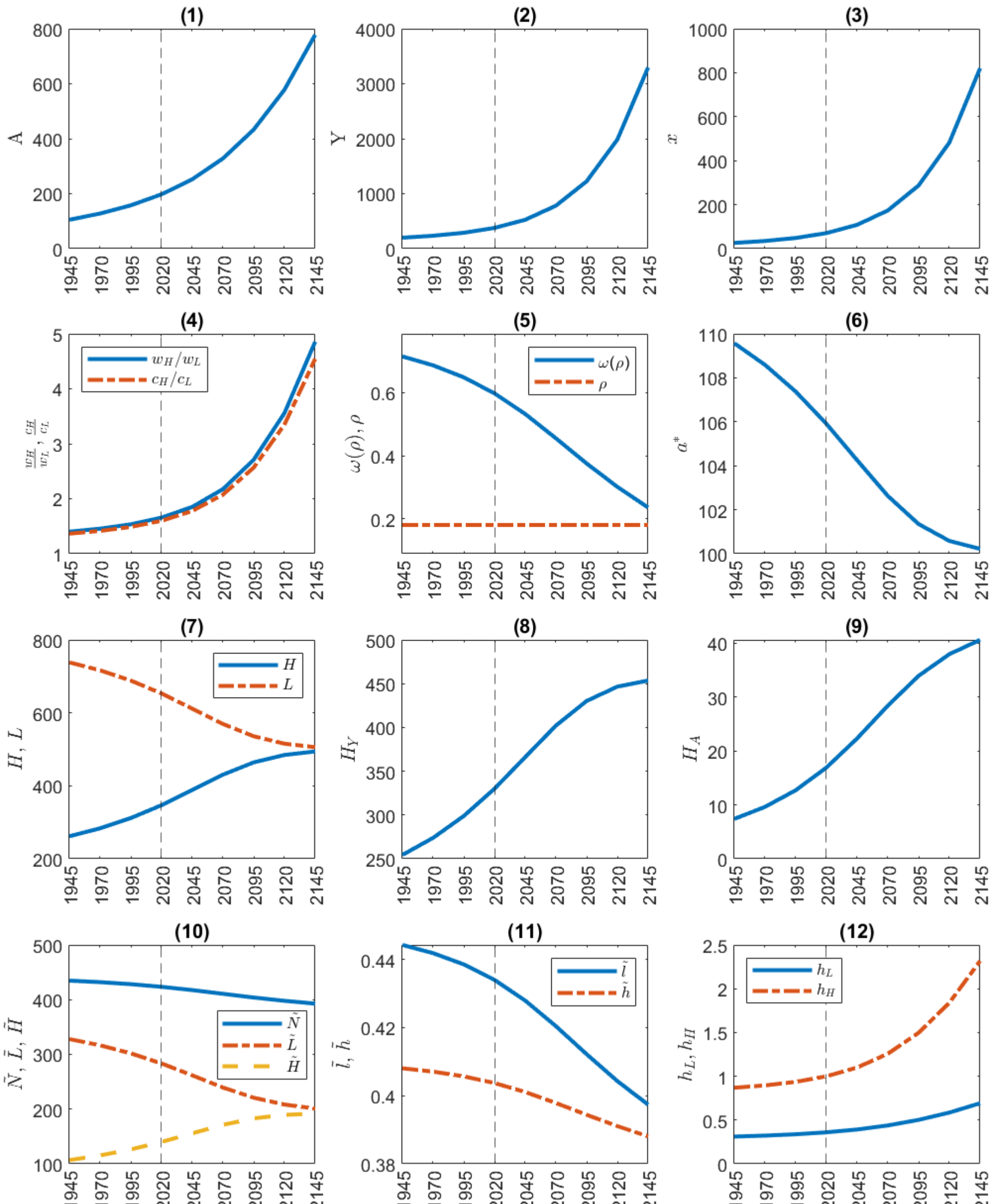

Figure 2: Model dynamics. Calibrated to the US economy in 2020. See text for details.

supply and the number of low-skilled workers in the economy. Summing the effects across the two skill groups, we observe a mild contraction in aggregate labor supply.

Finally, human capital increases over time for both low- and high-skilled individuals (intensive margin of human capital formation). Indeed, under the baseline fiscal policy, education spending is a constant fraction of GDP and, as production expands over time, the spending on both basic and higher education increases, leading to an increase in human capital. The human capital gradient h_H/h_L rises over time, confirming the argument from the partial equilibrium analysis that higher education spending under a hierarchical education system creates higher inequality as it disproportionately benefits high-skilled individuals. There exists, therefore, a fundamental trade-off between higher economic growth and lower inequality.

5 Tax policy

In view of the above trade-off, we proceed by examining the implications of different tax policies for production growth and inequality within our general equilibrium framework. We, initially, consider the effect of one-dimensional tax policy interventions (an exogenous change in either the labor or the robot tax) on production growth and inequality. We, then, consider coordinated two-dimensional tax policy packages (a combined change in both taxes) highlighting the scope for tax policies that can reduce inequality without harming production growth. We, finally, characterize the dynamic optimal tax policy maximizing aggregate welfare and provide a decomposition exercise to examine the underlying determinants of the optimal tax structure.

5.1 Exogenous tax policy

Figure 3 shows the effect of one-dimensional tax policy interventions (a change in either the labor or the robot tax) on inequality – measured alternatively in terms of the consumption ratio (high-skilled relative to low-skilled) or the Gini coefficient – and production growth.[20] We observe that increasing the labor tax compared to the initially calibrated situation leads to a rise in inequality (panel 1a), while increasing the robot tax has the opposite effect (panel 2a). For the labor tax, the human capital channel thus dominates the redistribution channel, whereas the reverse is true for the robot tax. This general equilibrium result is in line with the theoretical proposition from Section 3 showing that the robot tax is the more redistributive tax. The increase in either the labor or the robot tax, however, entails a fundamental trade-off between production growth and equality. We, indeed, observe that the increase in the labor tax increases both inequality (panel 1a) and production growth (panel 1b), while the increase in the robot tax reduces both (panels 2a and 2b).

[20] In addition to inequality at the individual level, the Gini coefficient also reflects composition effects across the two skill groups.

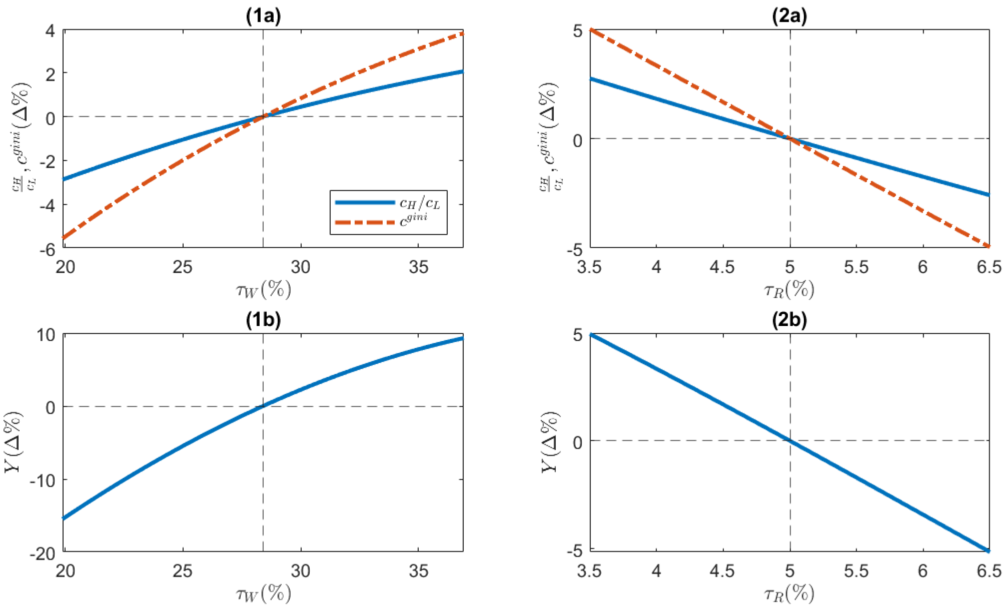

Figure 3: One-dimensional tax policy interventions. Percentage deviations of inequality (first row) and production (second row) in 2045 from the baseline situation for a change in either the labor tax (first column) or the robot tax (second column).

Figure 4 considers coordinated two-dimensional tax policy packages (changes in both the labor and robot tax). Panel 1 shows the effect on production growth and inequality of different tax reforms in the year 2045 relative to the status quo (grey dot), highlighting four different regions: two trade-off regions, in which production and inequality both increase (yellow) or reduce (orange); the benign region, in which production increases and inequality reduces (green); and finally the region with unwelcome consequences along both dimensions (red). As seen, there exists a combination of higher labor and robot taxes that leads to both higher production and lower inequality, thus breaking the production growth-equality trade-off entailed in the one-dimensional policy reforms.[21]

Panel 2, instead, shows the effect of a coordinated two-dimensional tax reform for a counterfactual economy in which we abstract from the intensive margin of human capital formation keeping the extensive margin of human capital formation (endogenous skill choice) active as in Prettner and Strulik (2020). In particular, we exogenously fix the time paths of both low- and high-skilled human capital before the tax adjustment, thus eliminating the endogenous response of human capital at the intensive margin to changes in fiscal policy. For this model without human capital formation at the intensive margin, we observe that a joint increase in both taxes would necessarily lead to lower production growth and inequality as taxation is purely redistributive and does not entail a

[21] For a moderate increase in both the labor and the robot tax (green region), the positive effect of a higher labor tax on production growth is stronger than the negative effect of the higher robot tax, and the positive effect of the higher labor tax on inequality is more than compensated by the reduction in inequality due to the increase in the robot tax.

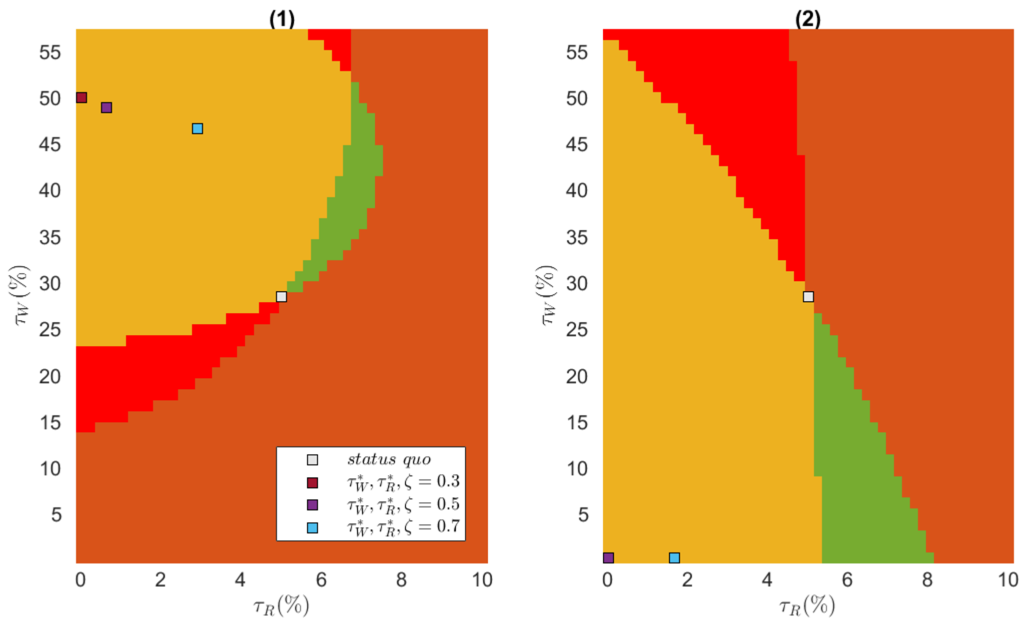

Figure 4: Coordinated two-dimensional tax policy packages (change in labor and robot tax). Panel 1: economy with intensive margin human capital formation. Panel 2: counterfactual without intensive margin human capital formation. Grey dot: status quo calibrated to the US economy for 2045; in green: benign region (higher production, lower inequality); in red: unwelcome region (lower production, higher inequality); in yellow and orange: trade-off regions (higher production, higher inequality – in yellow; lower production, lower inequality – in orange). The red-burgundy, purple, and light blue dots, respectively, represent the welfare-maximizing labor and robot tax for different welfare weights of the social planner, $\zeta \in \{0.3, 0.5, 0.7\}$.

productivity-enhancing effect through the intensive margin of human capital formation.

5.2 Optimal tax policy

As a simultaneous increase in both labor and robot taxes can lead to an increase in production and a reduction in inequality when we consider the intensive margin of human capital formation, we can now think about the welfare-optimizing tax policy within this framework. We consider the following welfare function:

$$\Omega_t = \zeta \cdot \underbrace{\mathcal{F}(a_t^*) \cdot N}_{=L_t} \cdot \mathcal{U}_{L,t} + (1-\zeta) \cdot \underbrace{(1-\mathcal{F}(a_t^*)) \cdot N}_{=H_t} \cdot \mathcal{U}_{H,t}, \tag{33}$$

where $\zeta \in [0,1]$ is the weight that the social planner places on the welfare of the low-skilled workers implicitly measuring the planner's preference for equality relative to production growth.[22] We focus on the optimal combination of labor and robot taxes while keeping the progressivity level of the tax-and-transfer system constant.[23]

Figure 4, panel 1 shows that for the utilitarian welfare weight, i.e., $\zeta = 0.5$ (purple dot), the welfare-optimizing tax policy implies increasing the labor tax and lowering the robot tax (though not to zero) relative to the situation in 2045; this policy leads to higher production at the cost of higher inequality. A similar pattern also applies under different levels of ζ, namely $\zeta = 0.3$ (red-burgundy dot) and $\zeta = 0.7$ (light blue dot).[24]

Figure 5 complements this analysis by detailing the dynamics of the optimal tax system over time. Initially, the planner should substantially increase the labor tax (panel 1) and reduce the robot tax (panel 2) relative to the baseline situation in 2020. This policy fosters economic growth by boosting human capital formation at both the intensive and extensive margins and increasing the incentives to invest in R&D which boosts machines' productivity. In later periods, the government should then progressively increase the robot tax and reduce the labor tax to mitigate the increasing wage gap. This result crucially depends on the intensive margin of human capital formation. In a model without this intensive margin (Prettner and Strulik, 2020), indeed, we observe that, although

[22]To ease the comparison with the results in Prettner and Strulik (2020), we follow their metric for welfare optimality. Different from the standard Ramsey problem, the welfare function in (33) is static in the sense that it neglects the dynamics of A_t. It can be rationalized, however, as emerging from a political economy framework where policy is determined such as to maximize the lifetime utility of the generations currently alive. As the model abstracts from a pension system and the interest rate is exogenous, the utility of the old generation is actually invariant to policy. Hence, the optimal tax policy can be determined by considering only the young generation as in (33).

[23]As a robustness check, we analyze the optimal tax policy under varying degrees of progressivity. An increase in the progressivity level makes the tax-and-transfer system more redistributive, calling for a slight reduction in the robot tax for future periods (see Online Appendix Section D).

[24]Notice that the competitive equilibrium delivers too little R&D relative to a planner solution which internalizes the effects of monopoly markups, intertemporal knowledge spillovers and congestion externalities. So there is no built-in efficiency rationale favoring a positive robot tax.

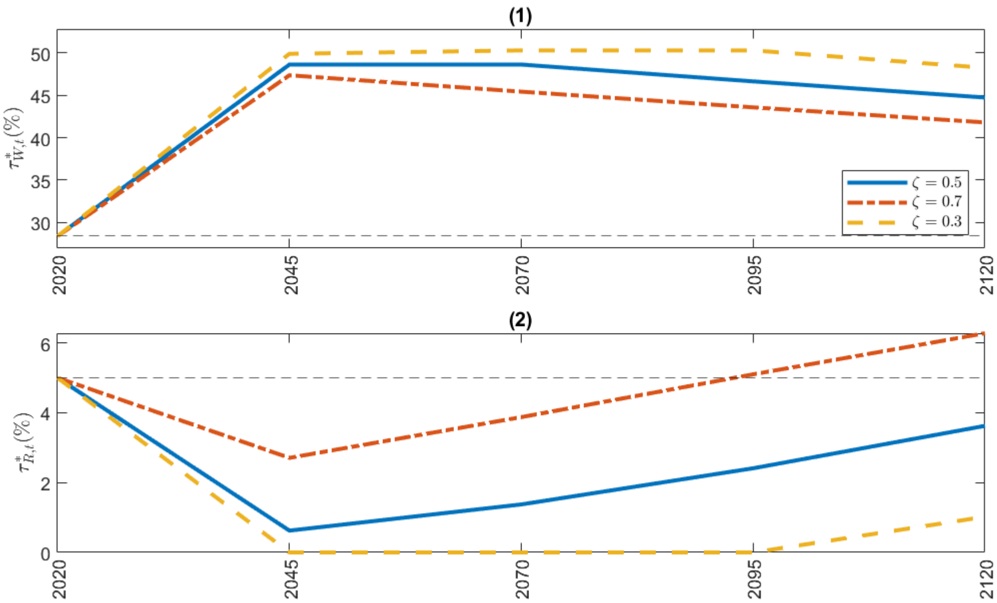

Figure 5: Dynamic optimal tax policy. Panels 1 and 2 show the welfare-optimizing labor tax and robot tax, respectively, dependent on the welfare weight, $\zeta \in \{0.3, 0.5, 0.7\}$. Values for the year 2045 correspond to the respective dots in Figure 4, panel 1.

the optimal robot tax may in principle be positive depending on the welfare weights, it is generally zero or very close to zero due to its adverse growth effects. In our model in which we consider the intensive margin of human capital formation, instead, the optimal robot tax becomes significantly positive following the initial policy change.[25] The underlying reason is that the revenue from the robot tax can be used to fund (i) productive education spending and (ii) redistributive transfers. Compared to the labor tax, the robot tax is the more effective tool for redistribution (cf. Section 3). Accordingly, as the planner's preference for equality increases from $\zeta = 0.5$ to $\zeta = 0.7$, the pattern of the optimal tax policy shifts away from the labor tax towards the robot tax.

Decomposition Figure 6 presents a decomposition analysis of the relevant margins – technological progress, extensive margin of human capital (education decision), and intensive margin of human capital formation – determining the optimal tax policy. Panels 1a and 1b describe the optimal tax policy when technological progress, A, is not affected by the change in policy (red dash-dotted lines), compared to the baseline model in which all the variables are endogenously determined (blue solid lines).[26] The profile of optimal taxation under an exogenous time path of

[25]The tax rates displayed in Figure 5 correspond to a revenue share coming from the robot tax of around 9% in 2045, rising to 28% in the long run.

[26]The time path of technological progress (A) is exogenous and equals the time path that would have prevailed under the constant calibrated policy ($\tau_W = 0.28$, $\tau_R = 0.05$). The same strategy also applies to our experiments when abstracting from the extensive margin of human capital (i.e., we assume exogenous skill choice), and the intensive

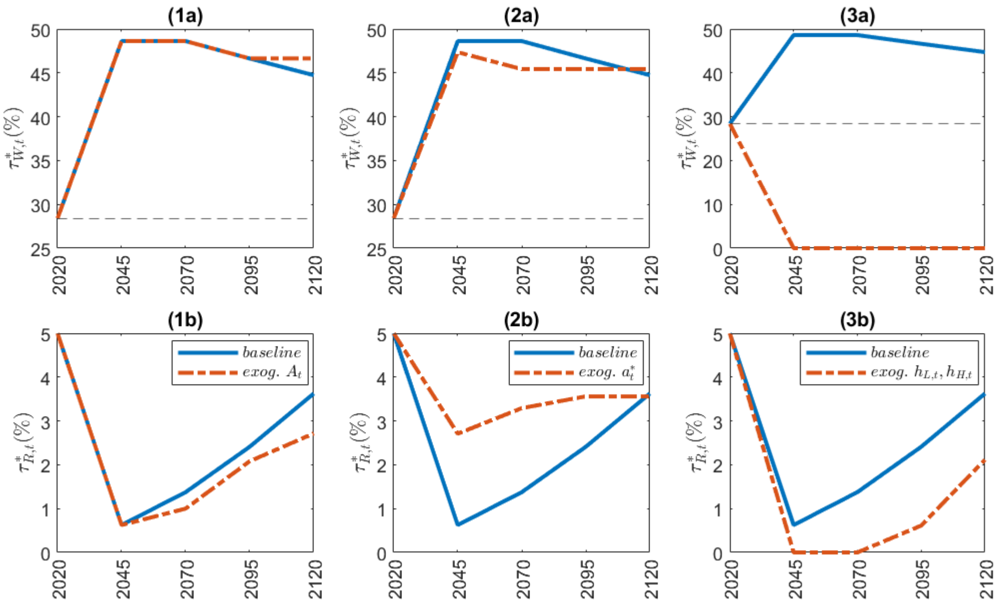

Figure 6: Decomposition of the optimal tax policy. Blue solid lines represent the baseline model, i.e., full adjustment of technological progress, extensive margin of human capital adjustment (share of low- and high-skilled workers), and intensive margin of human capital adjustment. Red dash-dotted lines in panels 1a and 1b represent the model in which the time path of technological progress is exogenous; red dash-dotted lines in panels 2a and 2b represent the model in which the time path of the extensive margin of human capital is exogenous; red dash-dotted lines in panels 3a and 3b represent the model in which the time path of the intensive margin of human capital is exogenous.

technological progress differs from the optimal baseline policy and entails a lower rate for the robot tax and a higher rate for the labor tax. This is because the initial drop of the robot tax in the baseline model induces a higher rate of technological progress (higher A) which, in turn, increases inequality. By contrast, under exogenous technology, this mechanism is shut down, implying a reduced need to redistribute and hence a lower robot tax.

Panels 2a and 2b examine the case in which the time path of the number of high- and low-skilled workers, L and H, is exogenously determined (by fixing the time path of a^*) and does not react to changes in taxation (red dash-dotted lines), compared to the baseline model. As seen, the welfare-optimal robot tax is higher and the welfare-optimal labor tax is lower under exogenous skill choice. In this case, since education decisions do not respond to the initial drop in the robot tax, the number of high-skilled workers is lower than in the baseline model with endogenous skill choice. The corresponding higher number of low-skilled workers thus calls for increased redistribution, which is achieved via a higher robot tax.

Panels 3a and 3b again illustrate the different redistributive nature of the labor and the robot tax. The red dash-dotted lines depict optimal taxes when the path for human capital, h_L and h_H, is exogenously determined, compared to the baseline model with endogenous adjustment. When the

margin of human capital formation.

intensive margin of human capital formation is not operating, it is efficient to finance government spending using exclusively robot taxes. The reason is that there is no motive for education spending when human capital accumulation is exogenous. The only role of the government is thus to redistribute, and since the robot tax achieves this more effectively, the government exclusively relies on this instrument. Notice, however, that also the robot tax is optimally equal to zero for some time (here in 2045 and 2070), before it is phased in at increasingly positive values. This pattern emerges because, initially (i.e., for a relatively low state of automation, A), both taxes are optimally used only for education spending (and hence equal to zero when there is no intensive margin of human capital formation). Later, as technological progress continues and inequality raises, the optimal policy relies on taxation also for redistribution and employs the more effective tool to this end.[27]

To summarize, the decomposition analysis highlights the role of endogenous technological progress, skill choice, and intensive margin human capital formation as determinants of the optimal tax policy. While endogenous technological progress leads to a higher optimal robot tax, endogenous education decisions (extensive margin of human capital) induce adjustments in the opposite direction. Finally, the intensive margin of human capital formation is critically needed to justify a positive labor tax as part of the optimal fiscal policy; moreover, the optimal robot tax is always higher than in the counterfactual setup in which the intensive margin of human capital formation is not operating.

6 Education subsidies

In this section, we examine different education policies with respect to their impact on production growth, inequality, and welfare. Specifically, we consider education subsidies, defined as additional transfers to high-skilled individuals financed through either an increase in labor or the robot tax.[28] We begin with the case of *non-targeted education subsidies* (*NT-ES*) in which the additional transfers are provided to all individuals who acquire higher education, irrespective of whether they would have acquired higher education also in the absence of transfers. Given this setup, we first assume that per-capita spending for both basic and higher education is kept constant.[29] This allows us

[27]In detail, the optimal robot tax is equal to zero in 2045 and 2070 and becomes positive only in later periods when the redistributive motive materializes due to the significant increase in inequality. Expressed in terms of the underlying optimal tax rates in the baseline model, the redistribution motive accounts for 0% of the robot tax in 2045 and 2070, 25.4% in 2095, and 58.3% in 2120.

[28]See Online Appendix Section E for the detailed specification of the model version with education subsidies. We also considered on-the-job training policies which seek to increase the human capital of low-skilled workers. As long as these measures do not render low-skilled workers more complementary to machines, they are largely ineffective. The reason is intuitive: On-the-job training would make low-skilled workers more productive, but also more expensive to hire. Hence, the implications for optimal tax policy are very minor relative to the baseline in Figure 5.

[29]Formally, this is achieved by fixing ω at its baseline value and letting ϕ and ϕ_B adjust to accommodate the additional transfer payments to high-skilled individuals and the induced changes in skill choice, respectively.

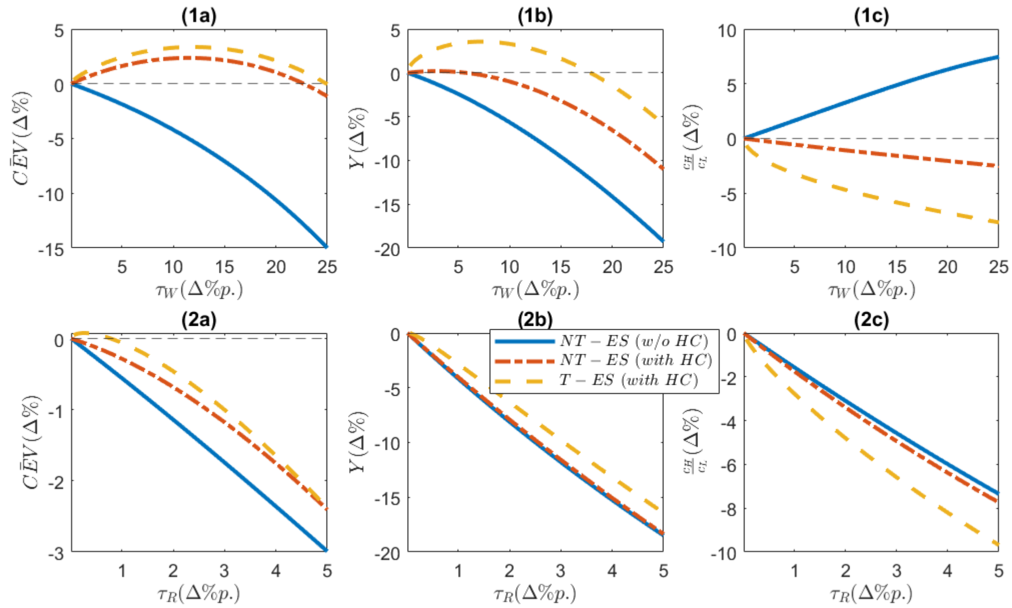

Figure 7: Education subsidies. Percentage deviations of aggregate welfare (measured as the average consumption equivalent variation, first column), production (second column), and inequality (third column) in 2045 from the baseline situation under education subsidies financed either with an increase in the labor tax (first row) or the robot tax (second row). Blue solid lines denote non-targeted education subsidies under constant per-capita spending on both basic and higher education, *NT-ES (w/o HC)*; red dash-dotted lines denote non-targeted education subsidies with adjustment of the per-capita spending on basic education, *NT-ES (with HC)*; yellow dashed lines denote targeted education subsidies with adjustment of the per-capita spending on basic education, *T-ES (with HC)*. Results are qualitatively similar when considering different levels of ζ (see Figure 11 and Table 6 in Appendix Section E).

to keep constant the level of human capital acquired through basic and higher education, thus highlighting the effect of the subsidies when the human capital channel is shut down (*w/o HC*). Alternatively, we maintain the assumption that the level of per-capita spending for higher education is kept constant, but allow adjustments in the per-capita spending for basic education.[30] Accordingly, the tax-and-transfer policy now has a direct effect on human capital formation at the basic level, and – via the hierarchical education system – an indirect one at the higher level (*with HC*). Finally, we analyze the effect of *targeted education subsidies* (*T-ES*), i.e., additional transfers paid only to the marginal individuals who would not have acquired higher education in the absence of transfers, whereby we again allow for adjustments in the per-capita spending for basic education (*with HC*).

Figure 7 shows the effect of the different education policies on welfare in terms of average consumption-equivalent variation ($C\bar{E}V$), production growth, and inequality, contrasting the financing of the subsidies through an increase in the labor tax (panels 1a-c) or an increase in the robot tax (panels 2a-c).

[30]For this scenario, ω and ϕ_B remain fixed at their baseline values, while ϕ and $\hat{E}_B$ adjust.

Labor tax-financed education subsidies When the funding for untargeted education subsidies comes from increases in the labor tax and the human capital channel is controlled for, the transfer payments have unambiguously negative welfare effects for any increase in tax rate (blue solid line, panel 1a). These adverse consequences arise due to (i) reduced production growth, caused by the distortion of labor supply under increased taxation and more generous transfer payments, and (ii) increased inequality, caused by the regressive nature of transfers to high-skilled workers (blue solid lines, panels 1b-c).

The red dash-dotted lines examine the effect of non-targeted subsidies when the human capital channel is active, i.e., part of the additional revenue from the increase in the labor tax is allocated to basic education spending. The joint effect of education subsidies, which encourage skill acquisition, and increased spending on the basic tier of the hierarchical education system is to augment the human capital of both low- and high-skilled workers. Compared to the case in which the human capital channel is not active, we therefore observe higher labor productivity and higher production growth (red dash-dotted line, panel 1b). Interestingly, we also observe reduced inequality (red dash-dotted line, panel 1c). This effect materializes despite the fact that the human capital channel itself actually works to accentuate inequality (cf. Proposition 3.2 and panel 1a of Figure 3). But the underlying fiscal system assumed in the experiment at hand builds on a fixed share ω of (non-educational) transfers going to low-skilled workers and hence responds to the increased number of high-skilled workers with a substantially increased progressivity of (non-educational) transfer payments that ultimately leads to lower inequality. Taken together, these effects on production growth and inequality give rise to an increase in welfare, with a maximum effect for an increase in the labor tax of 11.4 percentage points (red dash-dotted line, panel 1a).[31]

Although education subsidies financed by the labor tax can lead to an increase in aggregate welfare when the human capital channel is active, an untargeted policy is sub-optimal since it also rewards inframarginal individuals who would have acquired higher education also in the absence of additional transfers. Education subsidies targeted at ex-ante unskilled individuals who can actually be induced to take up higher education, indeed, lead to lower inequality and higher production growth and welfare for any change in the labor tax (panels 1a-c, yellow dashed lines). For the targeted policy, the increase in welfare reaches its peak for an increase in the labor tax of 11.9 percentage points, which is similar in terms of magnitude to the non-targeted education subsidy case.[32]

[31] The increase in the labor tax by 11.4 percentage points is welfare-maximizing, but still involves the trade-off between production and equality, as it leads to a decline in production by 1.5 percent, combined with a decline in inequality by 1.2 percent.

[32] Targeted education subsidies facilitate higher welfare gains and are also able to break the production-equality trade-off. An increase of the labor tax by 11.9 percent leads to an increase in production by 2.8 percent, combined with a decline in inequality by 5.1 percent.

Robot tax-financed education subsidies When the funding for additional transfers comes from increases in the robot tax, all considered policy scenarios generally entail negative welfare effects, where the induced reduction in inequality is not sufficient to compensate for the lost production growth. Higher robot taxes, indeed, have strong negative implications on production growth as they reduce the incentives for automation which is the driver of economic growth. Only under targeted education subsidies a marginal increase in the robot tax can induce a slightly positive effect on welfare. However, the effect appears negligible (at least for the considered welfare weight of $\zeta = 0.5$).[33] At any rate, and despite the hierarchical nature of the education system, the labor tax robustly emerges as the preferred alternative for the financing of education subsidies, whereas raising the robot tax generally has adverse welfare consequences.

7 Private education spending

In the US, private spending for college education accounts for an important fraction of total spending.[34] Under such co-funding for higher education from substitutable sources, changes in tax-funded public input can have important repercussions on the private spending component. To take this channel into account, we augment our model with an endogenous decision on how much individuals want to privately invest in higher education. We, therefore, rewrite the individual budget constraint of agent $j = \{L, H\}$ as:

$$(1 - \tau_{W,t})(1 - \eta_j - z_{j,t})w_{j,t} + \hat{T}_{j,t} - \mathbb{1}_{[j=H]}\, \theta_t = c_{j,t} + s_{j,t}, \tag{34}$$

where θ_t represents private spending in higher education, which equals zero if the individual does not acquire higher education ($j = L$) and is otherwise optimally chosen when the individual undertakes higher education ($j = H$).[35] The substitutability of private and public spending on higher education is captured via the process of human capital formation of high-skilled workers:

$$h_{H,t} = B_H \cdot (h_{B,t})^{1-\mu_H} \cdot \left(\epsilon \cdot (\theta_t)^{\frac{\nu-1}{\nu}} + (1-\epsilon) \cdot \left(\hat{E}_{H,t}\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1} \cdot \mu_H}, \tag{35}$$

where $\nu \in (0, \infty)$ governs the elasticity of substitution between private and public spending and $\epsilon \in (0, 1)$ is the associated share parameter of the CES aggregate.[36]

Calibration Integrating private education spending on higher education into the model necessitates modifications to the calibration strategy. The external parameters as well as the parameter

[33] Results for different levels of ζ are presented in Figure 11 and Table 6 in Online Appendix Section E.

[34] In 2019, private spending on college education accounted for 64.3 percent of the total (private and public) expenditure on higher education in the US (OECD, 2023e).

[35] See Online Appendix Section F for details.

[36] When $\epsilon = 0$ and $\nu \to \infty$, (35) collapses back to our baseline specification in (21) without private spending on higher education.

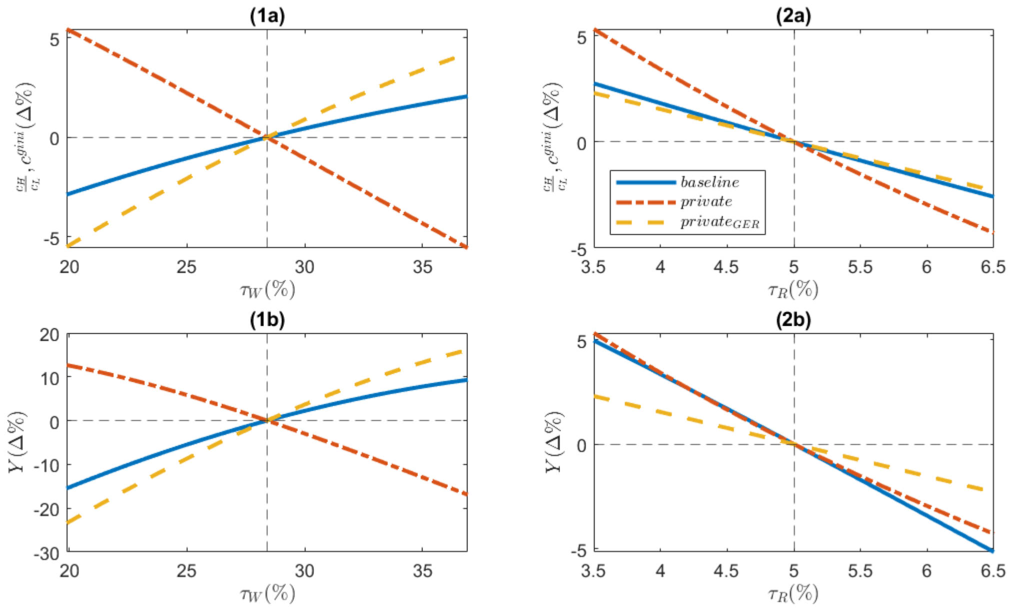

Figure 8: One-dimensional tax policy interventions with private education spending. Percentage deviations of inequality (first row) and production (second row) in 2045 from the baseline situation for a variation in either the labor tax (first column) or the robot tax (second column). Blue solid lines: baseline model without private education spending; red dash-dotted lines: private spending model calibrated to the US; yellow dashed lines: private spending model for the German counterfactual.

A_0, which governs the technological frontier in the initial model period, remain unchanged at their baseline values. From (35), there are then two additional parameters to calibrate in the extended model, ϵ and ν. Given the relatively constant private spending share on higher education over time, we set the elasticity parameter ν to unity. The value of ϵ is then determined via internal calibration, whereby we target the average ratio of private to public higher education spending observed in the US for 2000-2019. At a value of 1.54, this ratio is significantly higher than what is observed in most European countries (OECD, 2023e).[37] To assess the implications of the different funding composition for higher education (public versus private), we, therefore, complement the US calibration with a counterfactual scenario based on a private-to-public ratio for higher education spending set in line with German data.[38]

Exogenous tax policy Including private spending for college education has important implications for the effects of tax policies. In particular, while the effects on growth and inequality remain similar when considering changes in the robot tax, results are qualitatively different when considering a change in the labor tax. Indeed, in the model with private spending, an increase in the labor tax now reduces both inequality and production growth (Figure 8, panels 1a and 1b,

[37] Italy, France, and Germany have average values for 2000-2019 of 0.43, 0.24, and 0.17, respectively.

[38] The calibration of the private spending model and its counterfactual version are detailed in Appendix Section C.

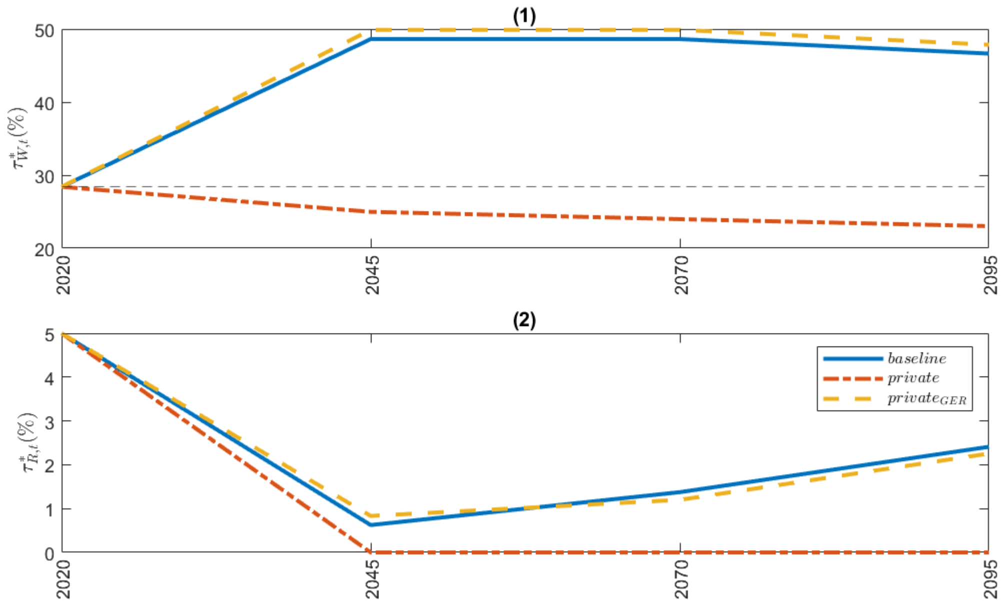

Figure 9: Dynamic optimal tax policy with private education spending. Panels 1 and 2 show the welfare-optimizing labor tax and robot tax, respectively. Blue solid lines: baseline model without private education spending; red dash-dotted lines: private spending model calibrated to the US; yellow dashed lines: private spending model for the German counterfactual.

red dash-dotted lines). Hence, the findings from the baseline model (blue solid lines), where the (intensive margin) human capital channel dominates the redistribution channel for the labor tax, are overturned. This is because private and public college education spending are now substitutes so that additional public spending on higher education financed through the labor tax crowds out private spending.[39] This diminishes the magnitude of the human capital channel and leads to a reduction in inequality and production growth, i.e., the redistribution channel now dominates the human capital channel. However, when considering the counterfactual scenario based on a reduced role for private spending as in Germany (yellow dashed lines), the original result from the baseline model (dominant human capital channel) is reinstated for variations in the labor tax.

Optimal tax policy In analogy to the optimal tax analysis from Figure 5, panels 1 and 2 in Figure 9 show the optimal tax policy for a welfare weight $\zeta = 0.5$ when private education spending is included in the model. We again contrast alternative scenarios in which the importance of private spending corresponds to the US (red dash-dotted lines) or German (yellow dashed lines) system of funding higher education. When college education is predominantly publicly funded, the pattern of optimal labor and robot taxes remains almost unchanged relative the baseline model without private spending (blue solid lines). When college education spending is mostly privately financed,

[39]See Figure 12 in Online Appendix Section F for details illustrating the importance of the underlying crowding out mechanism.

instead, the optimal tax policy significantly differs from the baseline. In particular, the labor tax immediately declines from its starting rate at $\tau_W = 0.28$, while the robot tax drops to zero.

The general reduction in the level of taxation can be accounted for by the reduced need for public education spending when a significant contribution comes from private funds. The specific dynamics of optimal tax rates over time again reflect the interaction of their effects on human capital and redistribution. As discussed above, the weaker human capital channel implies that both taxes are predominantly redistributive when private spending on higher education is sufficiently important. Hence, it becomes optimal to rely exclusively on the labor tax since this is the less distortionary source of revenue. Moreover, as the economy grows over time, private contributions towards higher education rise and can increasingly substitute for public spending. In consequence, optimal labor taxes are decreasing over time.

As a final point, notice that the comparison of the alternative model versions under the utilitarian welfare function with $\zeta = 0.5$ suggests a pattern of optimal robot taxes that are positive whenever the human capital channel effects of the labor tax dominate its redistributive effects. Accordingly, there is a normative case for the robot tax when higher education is mostly publicly funded. High-skilled workers then do not pay for their education and it becomes optimal to tax them indirectly through a positive robot tax. By contrast, when there are sufficiently important private contributions to college education, this indirect taxation is no longer justified. However, this clear taxonomy is complicated by the welfare weight that the social planner attaches to low-skilled workers. Indeed, for a sufficiently high weight on the low-skilled workers, it becomes eventually optimal to have a positive robot tax, which increases over time along with the dynamics of the state of automation.[40]

8 Conclusion

In this paper, we highlight the role of tax policy and education spending for economic growth and inequality in a dynamic growth model with automation, endogenous education choice, and endogenous human capital. Although beneficial for economic growth, automation contributes to wage inequality by replacing low-skilled workers. While direct redistribution mitigates inequality at the cost of lower economic growth, education spending boosts production growth and increases inequality as it favors the human capital accumulation of high-skilled workers.

Higher government spending that increases both redistributive transfers and education spending, therefore, has an ambiguous effect on economic growth and inequality. We show that the tax composition (labor versus robot tax) financing government spending is key to determining the effect on production and inequality as the robot tax is relatively more redistributive than the labor tax. In particular, we observe that, in a model accounting for the intensive margin of human capital

[40] See e.g. the case of $\zeta = 0.7$ depicted in Figure 13 in Online Appendix Section F.

formation, a combined increase in labor and robot taxes can reduce inequality without harming production growth. We determine the welfare-maximizing tax policy over time, showing that the robot tax should be initially reduced relative to the status quo to boost economic growth and then progressively increased to mitigate the increase in inequality.

We further show that education subsidies in the form of higher transfers to agents undertaking higher education can lead to an increase in welfare if two conditions are satisfied: (i) per-capita spending on basic education spending is allowed to adjust to the policy (so that the human capital channel is active), and (ii) the increase in transfers is financed through an increase in the labor tax. In particular, we observe that the increase in welfare is maximized by an increase in the labor tax of around 11 percentage points.

Extending the model to encompass also private spending on higher education, we finally show that the desirability of robot taxes crucially depends on the extent of private contributions. For European countries in which the higher education system is mostly financed through public funds, the optimal robot tax is significantly positive; but in countries like the US which rely heavily on private spending, the robot tax is optimally zero. In contrast to education subsidies, which should be financed via the labor tax, there is thus an important role for the robot tax in financing quality-enhancing spending on higher education.

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Appendix

A Model

Equilibrium conditions Type-specific aggregate labor supply can be calculated as the type-specific individual labor supply multiplied with the mass of type-specific agents:

$$\tilde{H}_{Y,t} = (1 - \eta - z_{H,t})\, H_{Y,t}, \tag{36}$$

$$\tilde{H}_{A,t} = (1 - \eta - z_{H,t})\, H_{A,t}, \tag{37}$$

and

$$\tilde{L}_t = (1 - z_{L,t})\, L_t. \tag{38}$$

Aggregate high-skilled labor supply is defined as the sum of aggregate high-skilled labor supplied in the final production and R&D sector

$$\tilde{H}_t = \tilde{H}_{Y,t} + \tilde{H}_{A,t}, \tag{39}$$

and aggregate labor supply in the economy as

$$\tilde{N}_t = \tilde{H}_t + \tilde{L}_t. \tag{40}$$

The total number of high-skilled individuals is the sum of high-skilled individuals working in the final production sector and the researchers (the high-skilled individuals working in the R&D sector):

$$H_t = H_{Y,t} + H_{A,t}. \tag{41}$$

Overall, the number of high- and low-skilled individuals needs to sum up to (the constant population size) N in all model periods:

$$N = H_t + L_t. \tag{42}$$

To close the model economy, we impose a no-arbitrage condition on high-skilled wage rates in both final production and R&D sector:

$$w_{H,t} = w_{A,t}. \tag{43}$$

B Partial equilibrium analysis

Proof: Redistribution channel Consider a change in the labor tax, τ_W:

$$\left.\frac{d\ c_H/c_L}{d\ \tau_W}\right|_{\text{RE}} = \underbrace{\frac{\partial\ c_H/c_L}{\partial\ \tau_W}}_{\text{RE}_W(1)} + \underbrace{\sum_{j \in H,L} \frac{\partial c_H/c_L}{\partial \hat{T}_j} \frac{\partial \hat{T}_j}{\partial \mathcal{G}} \frac{\partial \mathcal{G}}{\partial \tau_W}}_{\text{RE}_W(2)} + \underbrace{\sum_{j \in H,L} \frac{\partial c_H/c_L}{\partial \hat{T}_j} \frac{\partial \hat{T}_j}{\partial \mathcal{G}} \sum_{j' \in H,L} \frac{\partial \mathcal{G}}{\partial w_{j'}} \sum_{j'' \in H,L} \frac{\partial w_{j'}}{\partial h_{j''}} \frac{\partial h_{j''}}{\partial \tau_W}}_{\text{RE}_W(3)} \tag{44}$$

where:

$$\mathrm{RE}_W(1) = \frac{-(1-\eta)w_H\left[(1-\tau_W)w_L + \hat{T}_L\right] + w_L\left[(1-\tau_W)(1-\eta)w_H + \hat{T}_H\right]}{\left[(1-\tau_W)w_L + \hat{T}_L\right]^2}, \tag{45}$$

which is positive if $\omega < \frac{w_L L}{(1-\eta)w_H H + w_L L} \equiv \tilde{\omega}$;

$$\mathrm{RE}_W(2) = \frac{\partial c_H/c_L}{\partial \hat{T}_H}\frac{\partial \hat{T}_H}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \tau_W} + \frac{\partial c_H/c_L}{\partial \hat{T}_L}\frac{\partial \hat{T}_L}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \tau_W} \tag{46}$$

where $\frac{\partial \mathcal{G}}{\partial \tau_W}$ is always positive. This implies that $\mathrm{RE}_W(2) > 0$ if:

$$\frac{\partial c_H/c_L}{\partial \hat{T}_H}\frac{\partial \hat{T}_H}{\partial \mathcal{G}} + \frac{\partial c_H/c_L}{\partial \hat{T}_L}\frac{\partial \hat{T}_L}{\partial \mathcal{G}} > 0 \tag{47}$$

where:

$$\frac{\partial c_H/c_L}{\partial \hat{T}_H}\frac{\partial \hat{T}_H}{\partial \mathcal{G}} = \frac{1}{(1-\tau_W)w_L + \hat{T}_L} \cdot \frac{(1-\omega)(1-\phi)}{H} \tag{48}$$

and

$$\frac{\partial c_H/c_L}{\partial \hat{T}_L}\frac{\partial \hat{T}_L}{\partial \mathcal{G}} = -\frac{(1-\tau_W)(1-\eta)w_H + \hat{T}_H}{[(1-\tau_W)w_L + \hat{T}_L]^2} \cdot \frac{\omega(1-\phi)}{L}. \tag{49}$$

Substituting $\hat{T}_H = \frac{(1-\omega)(1-\phi)}{H}\mathcal{G}$ and $\hat{T}_L = \frac{\omega(1-\phi)}{L}\mathcal{G}$, we obtain that $\mathrm{RE}_W(2) > 0$ if $\omega < \tilde{\omega}$;

$$\mathrm{RE}_W(3) = \left(\frac{\partial c_H/c_L}{\partial \hat{T}_H}\frac{\partial \hat{T}_H}{\partial \mathcal{G}} + \frac{\partial c_H/c_L}{\partial \hat{T}_L}\frac{\partial \hat{T}_L}{\partial \mathcal{G}}\right)\underbrace{\sum_{j'\in H,L}\frac{\partial \mathcal{G}}{\partial w_{j'}}\sum_{j''\in H,L}\frac{\partial w_{j'}}{\partial h_{j''}}\frac{\partial h_{j''}}{\partial \tau_W}}_{D}. \tag{50}$$

Since $D > 0$, $\mathrm{RE}_W(3) > 0$ if $\frac{\partial c_H/c_L}{\partial \hat{T}_H}\frac{\partial \hat{T}_H}{\partial \mathcal{G}} + \frac{\partial c_H/c_L}{\partial \hat{T}_L}\frac{\partial \hat{T}_L}{\partial \mathcal{G}} > 0$ which is the same condition that implies $\mathrm{RE}_W(2) > 0$. Therefore, $\mathrm{RE}_W(3) > 0$ if $\omega < \tilde{\omega}$. From the above, we obtain that: $\left.\frac{d\ c_H/c_L}{d\ \tau_W}\right|_{\mathrm{RE}} < 0$ iff $\omega > \tilde{\omega}$.

Consider a change in the robot tax (τ_R):

$$\left.\frac{d\ c_H/c_L}{d\ \tau_R}\right|_{\mathrm{RE}} = \underbrace{\frac{\partial\ c_H/c_L}{\partial\ w_H}\frac{\partial\ w_H}{\partial\ \tau_R}}_{\mathrm{RE}_R(1)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \tau_R}}_{\mathrm{RE}_R(2)} + \underbrace{\sum_{j\in H,L}\frac{\partial c_H/c_L}{\partial \hat{T}_j}\frac{\partial \hat{T}_j}{\partial \mathcal{G}}\sum_{j'\in H,L}\frac{\partial \mathcal{G}}{\partial w_{j'}}\sum_{j''\in H,L}\frac{\partial w_{j'}}{\partial h_{j''}}\frac{\partial h_{j''}}{\partial \tau_R}}_{\mathrm{RE}_R(3)}. \tag{51}$$

Following the same steps as for the labor tax, we find that $\mathrm{RE}_R(2)$ and $\mathrm{RE}_R(3)$ are negative if $\omega > \tilde{\omega}$. $\mathrm{RE}_R(1)$ is instead always negative. This implies that a sufficient condition for $\left.\frac{d\ c_H/c_L}{d\ \tau_R}\right|_{\mathrm{RE}}$ to be negative is $\omega > \tilde{\omega}$.

Proof: Human capital channel Consider a change in either the labor or the robot tax, i.e., $g \in \{W, R\}$:

$$\left.\frac{d\ w_H/w_L}{d\ \tau_g}\right|_{\text{HC}} = \underbrace{\frac{\partial w_H/w_L}{\partial h_H/h_L}\frac{\partial h_H/h_L}{\partial \tau_g}}_{\text{HC}_g(1)} + \underbrace{\frac{\partial w_H/w_L}{\partial \tilde{A}}\frac{\partial \tilde{A}}{\partial h_H}\frac{\partial h_H}{\partial \tau_g}}_{\text{HC}_g(2)}. \tag{52}$$

We can write the wage and human capital ratios as:

$$\frac{w_H}{w_L} = \frac{1-\alpha}{\alpha}\frac{L}{H}\left[1 + \tilde{A}\left(\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{h_H H}{h_L L}\right)^{\alpha}\right], \tag{53}$$

$$\frac{h_H}{h_L} = \frac{B_H}{B^{\mu_H}}\left(\frac{\phi_B \cdot \phi}{N}\right)^{-\mu_B\mu_H}\left(\frac{(1-\phi_B)\phi}{H}\right)^{\mu_H}\mathcal{G}^{\mu_H(1-\mu_B)} \tag{54}$$

Since $\frac{\partial \mathcal{G}}{\partial \tau_g} > 0$, then $\frac{\partial h_H/h_L}{\partial \tau_g} > 0$. Therefore, since also $\frac{\partial w_H/w_L}{\partial h_H/h_L} > 0$, $\text{HC}_g(1) > 0$. Moreover, since all the three terms in the second term of (52) are positive, $\text{HC}_g(2)$ and $\left.\frac{d\ w_H/w_L}{d\ \tau_g}\right|_{\text{HC}}$ are positive.

C Calibration

Table 3 presents parameters for the baseline and the private education spending model, both for the US calibration and the German counterfactual. The goodness of fit of the baseline model and both private spending exercises is given in Table 4. External and policy parameters are kept at their baseline values. Internal parameters are re-calibrated by minimizing the squared difference between model moments and data. The inclusion of private spending on higher education requires the internal calibration of two additional parameters (ν and ϵ). We assume a unitary elasticity of substitution between private and public education spending on higher education, i.e. (35) simplifies to $h_{H,t} = B_H \cdot (h_{B,t})^{1-\mu_H} \cdot \left((\theta_t)^{\epsilon} \cdot \left(\hat{E}_{H,t}\right)^{1-\epsilon}\right)^{\mu_H}$. The share parameter ϵ adjusts such that the model replicates the observed ratio of private to public education spending on higher education in the US of 1.54 (or Germany of 0.17 for the German counterfactual). A better fit of the model implied college share to its real world counterpart for both, the US calibration and the German counterfactual, would require an adjustment of the technological frontier in the initial model period (A_0). We decided to keep the initial technology level from our baseline calibration to remain comparability of the technology levels across all versions.

External*		Internal				Policy*	
Parameter	Value	Parameter	Baseline	Private	$Private_{GER}$	Parameter	Value
β	0.55	δ	0.584	0.599	0.570	τ_W	0.284
γ	1.44	ψ_1	0.479	0.497	0.403	τ_R	0.050
α	0.80	ψ_2	17.09	16.31	18.99	ϕ	0.269
μ_a	100	μ_B	0.354	0.338	0.357	ϕ_B	0.783
σ_a	15	μ_H	0.223	0.514	0.481	ρ	0.181
$\underline{a}$	100	B	1.720	1.628	1.669		
η	0.11	B_H	6.236	14.02	18.05		
N	1000	A_0	87.3	87.3	87.3		
λ_1	0.67	ϵ	-	0.921	0.099		
λ_2	0.44	ν	-	1	1		
$\bar{R}$	2.32						

Table 3: Calibration parameters. *Parameters identical across all model versions.

Target	Data	Baseline	Private	$Private_{GER}$
high-skilled human capital (normalization)	1.00	1.00	1.00	1.00
college share (%)	34.7	34.7	31.2	35.6
R&D employment share (%)	1.00	1.69	0.90	1.20
college wage premium	1.86	1.86	1.86	1.86
elast. of college attendance wrt. its overall price	1.20	1.20	1.20	1.20
elast. of low-sk. wages wrt. per-cap. spend. on low. educ.	0.54	0.54	0.58	0.55
TFP growth (annual, %)	0.91	0.91	0.91	0.91
ratio of private to public educ. spending on higher educ.	1.54/0.17	-	1.54	0.17

Table 4: Goodness of fit across all model versions.

Online-Appendix

D Tax policy analysis

t	2045	2070	2095	2120	2045	2070	2095	2120	2045	2070	2095	2120
	$\zeta = 0.5$				$\zeta = 0.3$				$\zeta = 0.7$			
$\tau^*_{W,t}(\%)$	48.6	48.6	46.6	44.7	49.9	50.3	50.3	48.2	47.4	45.4	43.6	41.8
$\tau^*_{R,t}(\%)$	0.6	1.4	2.4	3.6	0	0	0	1.0	2.7	3.9	5.1	6.3
$C\bar{E}V_t(\%)$	8.8	11.3	13.5	15.1	9.9	13.6	16.8	19.2	8.3	9.5	10.4	11.1
$\Delta Y_t(\%)$	37.2	42.6	44.0	42.6	41.0	52.5	62.5	65.1	24.5	24.5	21.9	18.3
$\Delta \frac{c_{H,t}}{c_{L,t}}(\%)$	14.8	21.1	25.1	25.9	16.9	27.0	38.1	42.2	9.0	10.8	11.0	9.5

Table 5: Dynamic optimal tax policy. Optimal tax policy mix over time and its effects on production and inequality (relative to the baseline model) dependent on welfare weights in the welfare function, $\zeta \in \{0.3, 0.5, 0.7\}$.

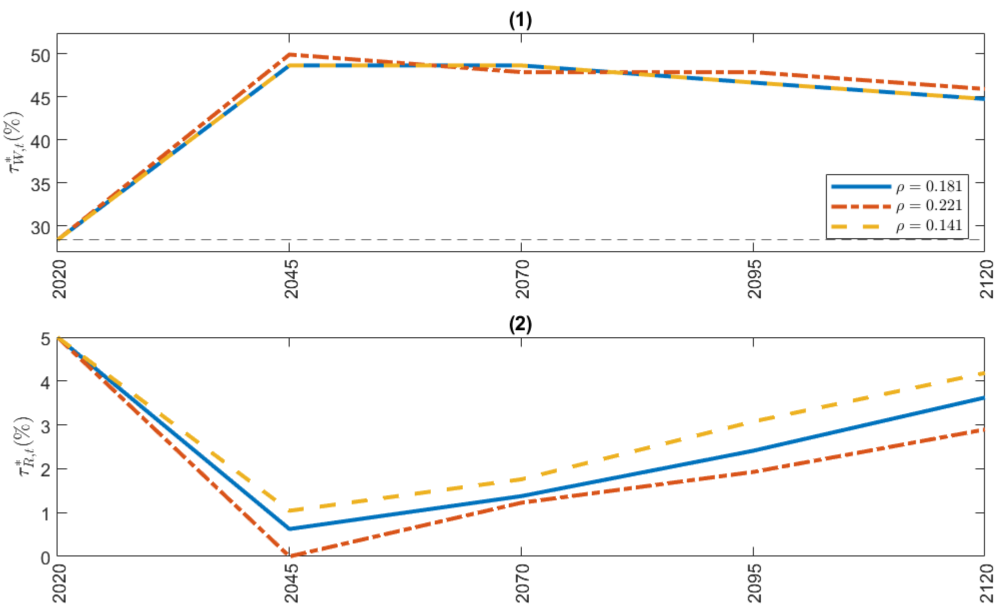

Figure 10: Dynamic optimal tax policy for different progressivity levels of the tax-and-transfer system, $\rho \in \{0.181, 0.221, 0.141\}$ and $\zeta = 0.5$. Panels 1 and 2 show the dynamic optimal labor tax and robot tax respectively. Compared to the baseline with $\rho = 0.181$ (blue solid line), higher progressivity requires less redistribution through the robot tax (red dash-dotted line), whereas lower progressivity calls for stronger redistribution through the robot tax (yellow dashed line).

E Education policies

We allow the government to increase either of its taxes to raise additional tax revenues that can be used for education policies. These education policies can either be education subsidies paid to all young high-skilled individuals in the economy or targeted education subsidies paid only to young individuals close to the ability threshold (marginal individuals, M_t). (Targeted) education subsidies realize as additional per-capita transfer payments to high-skilled (marginal) individuals, $T^{add.}_{H,t}(T^{add.}_{M,t})$. The education policy experiments work as follows: we fix per-capita spending on higher education and the transfer share determining the amount of total (per-capita) transfers paid to low-skilled individuals to their respective time paths from the baseline model ($\hat{E}_{H,t} = \bar{\hat{E}}_{H,t}$ and $\omega_t = \bar{\omega}_t,$). We then calculate aggregate higher education spending that is required to maintain this per-capita level. The government can increase one of the two tax rates (either $\tau_{W,t}$ or $\tau_{R,t}$). Additional tax revenues are then used to finance (targeted) education subsidies.

E.1 Education subsidies

In this education policy version, additional tax revenues from higher labor or robot taxation are used as additional per-capita transfer payments to high-killed individuals $\hat{T}^{add}_{H,t}$, calculated as

$$\hat{T}^{add}_{H,t} = \frac{T^{add.}_{H,t}}{H_t} = \frac{(1-\phi_{B,t}) \cdot E_t - \bar{\hat{E}}_{H,t} \cdot H_t}{H_t}, \tag{55}$$

providing additional incentives for low-skilled individuals to become high-skilled and leading to an adjustment in the young-age budget constraint of high-skilled individuals, such that

$$(1-\tau_{W,t})(1-\eta - z_{H,t})w_{H,t} + \hat{T}_{H,t} + \hat{T}^{add}_{H,t} = c_{H,t} + s_{H,t}. \tag{56}$$

As a consequence, high-skilled individual's optimal consumption, savings and leisure decisions are then of the following form

$$c_{H,t} = \left(\frac{1}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta)w_{H,t} + \hat{T}_{H,t} + \hat{T}^{add}_{H,t}\right), \tag{57}$$

$$s_{H,t} = \left(\frac{\beta}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta)w_{H,t} + \hat{T}_{H,t} + \hat{T}^{add}_{H,t}\right), \tag{58}$$

$$z_{H,t} = \left(\frac{\gamma}{(1+\beta+\gamma)(1-\tau_{W,t})w_{H,t}}\right)\left((1-\tau_{W,t})(1-\eta)w_{H,t} + \hat{T}_{H,t} + \hat{T}^{add}_{H,t}\right). \tag{59}$$

For the specific version of this education policy in which we fix the human capital channel, we assume in addition that per-capita spending on basic education is also fixed at its baseline model time path ($\hat{E}_{B,t} = \bar{\hat{E}}_{B,t}$). Additional per-capita transfer payments to high-skilled individuals are then calculated as

$$\hat{T}^{add}_{H,t} = \frac{T^{add.}_{H,t}}{H_t} = \frac{E_t - \bar{\hat{E}}_{H,t} \cdot H_t - \bar{\hat{E}}_{B,t} \cdot N}{H_t}. \tag{60}$$

E.2 Targeted education subsidies

In this education policy version, additional tax revenues from higher labor or robot taxation are used as additional per-capita transfer payments $\hat{T}^{add.}_{M,t}$, but explicitly targeted to marginal individuals (M_t, individuals close to the ability threshold). This idea creates a three-type economy of high-skilled (H_t), low-skilled (L_t) and marginal individuals (M_t). Marginal individuals are high-skilled but receive additional transfer payments from the government, calculated as

$$\hat{T}^{add}_{M,t} = \frac{T^{add.}_{M,t}}{M_t} = \frac{(1-\phi_{B,t}) \cdot E_t - \bar{\hat{E}}_{H,t} \cdot (H_t + M_t)}{M_t}. \tag{61}$$

Households Lifetime utility for an agent of type $j \in \{H, M, L\}$ becomes

$$\mathcal{U}_{j,t} = \log(c_{j,t}) + \beta \log(\bar{R} s_{j,t}) + \gamma \log(z_{j,t}) - \mathbb{1}_{[j=H,M]}\; v(a), \tag{62}$$

and the respective budget constraint changes to

$$(1-\tau_{W,t})(1-\eta_j - z_{j,t})w_{j,t} + \hat{T}_{j,t} + \mathbb{1}_{[j=M]}\; \hat{T}^{add}_{j,t} = c_{j,t} + s_{j,t}. \tag{63}$$

Marginal agents have to spend the same amount of time in higher education as (non-marginal) high-skilled individuals ($\eta_H = \eta_M = \eta$). Optimal decisions follow as

$$c_{j,t} = \left(\frac{1}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} + \mathbb{1}_{[j=M]}\; \hat{T}^{add}_{j,t}\right), \tag{64}$$

$$s_{j,t} = \left(\frac{\beta}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} + \mathbb{1}_{[j=M]}\; \hat{T}^{add}_{j,t}\right), \tag{65}$$

$$z_{j,t} = \left(\frac{\gamma}{(1+\beta+\gamma)(1-\tau_{W,t})w_{j,t}}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} + \mathbb{1}_{[j=M]}\; \hat{T}^{add}_{j,t}\right). \tag{66}$$

Instead of ending up with one ability threshold

$$a^*_t = \psi_2 \left(\frac{c_{H,t}}{c_{L,t}}\right)^{-\frac{1+\beta+\gamma}{\psi_1}} \left(\frac{w_{H,t}}{w_{L,t}}\right)^{\frac{\gamma}{\psi_1}} + \underline{a}. \tag{67}$$

from $\mathcal{U}_{H,t}(a) \geqslant \mathcal{U}_{L,t}(a)$, we get an additional threshold level

$$a^{**}_t = \psi_2 \left(\frac{c_{M,t}}{c_{L,t}}\right)^{-\frac{1+\beta+\gamma}{\psi_1}} \left(\frac{w_{M,t}}{w_{L,t}}\right)^{\frac{\gamma}{\psi_1}} + \underline{a}. \tag{68}$$

from $\mathcal{U}_{M,t}(a) \geqslant \mathcal{U}_{L,t}(a)$. As marginal individuals are high-skilled individuals, both receive the same wage rate ($w_{H,t} = w_{M,t}$). General per-capita transfers for marignal and non-marginal high-skilled individuals are equivalent ($\hat{T}_{M,t} = \hat{T}_{H,t}$). As both high-skilled types (marginal and non-marginal) also receive the same wage rate, it holds true that $a^{**}_t \leqslant a^*_t$, as long as $\hat{T}^{add}_{M,t} \geqslant 0$ (which is true by assumption). The mass of (non-marginal) high-skilled individuals is therefore given by $H_t = (1-\mathcal{F}(a^*_t)) \cdot N$, the mass of marginal individuals by $M_t = (\mathcal{F}(a^*_t) - \mathcal{F}(a^{**}_t)) \cdot N$ and the mass of low-skilled individuals by $L_t = \mathcal{F}(a^{**}_t) \cdot N$.

Final production sector Aggregate output is produced as

$$Y_t = \left(h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}\right)^{1-\alpha}\left(\left(h_{L,t}\tilde{L}_t\right)^{\alpha} + \sum_{i=1}^{A_t}(x_{i,t})^{\alpha}\right), \tag{69}$$

inducing the following maximization problem

$$\max_{\left\{\tilde{H}_{Y,t},\tilde{M}_{Y,t},\tilde{L}_t,\{x_{i,t}\}_{i=1}^{A_t}\right\}} Y_t - w_{H,t}\tilde{H}_{Y,t} - w_{M,t}\tilde{M}_{Y,t} - w_{L,t}\tilde{L}_t - (1+\tau_{R,t})\sum_{i=1}^{A_t} p_{i,t}x_{i,t}. \tag{70}$$

Factor prices are obtained as

$$w_{H,t} = (1-\alpha)\left(h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}\right)^{-\alpha} h_{H,t}\left((h_{L,t}\tilde{L}_t)^{\alpha} + \sum_{i=1}^{A_t}(x_{i,t})^{\alpha}\right), \tag{71}$$

$$w_{M,t} = (1-\alpha)\left(h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}\right)^{-\alpha} h_{M,t}\left((h_{L,t}\tilde{L}_t)^{\alpha} + \sum_{i=1}^{A_t}(x_{i,t})^{\alpha}\right), \tag{72}$$

$$w_{L,t} = \alpha\left(\frac{h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}}{h_{L,t}\tilde{L}_t}\right)^{1-\alpha} h_{L,t}, \tag{73}$$

$$(1+\tau_{R,t})p_{i,t} = \alpha\left(\frac{h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}}{x_{i,t}}\right)^{1-\alpha}, \tag{74}$$

from which we find that $w_{H,t} = w_{M,t}$.

R&D sector Blueprints for new machines are produced as

$$A_t - A_{t-1} = \bar{\delta}_t\left(h_{H,t}\tilde{H}_{A,t} + h_{M,t}\tilde{M}_{A,t}\right) \tag{75}$$

with

$$\bar{\delta}_t = \delta \cdot \frac{(A_{t-1})^{\lambda_1}}{(h_{H,t}\tilde{H}_{A,t} + h_{M,t}\tilde{M}_{A,t})^{1-\lambda_2}}. \tag{76}$$

R&D profits are given as

$$\max_{\left\{\tilde{H}_{A,t},\tilde{M}_{A,t}\right\}} p_{A,t}\bar{\delta}_t\left(h_{H,t}\tilde{H}_{A,t} + h_{M,t}\tilde{M}_{A,t}\right) - w_{HA,t}\tilde{H}_{A,t} - w_{MA,t}\tilde{M}_{A,t}. \tag{77}$$

Optimality requires

$$w_{HA,t} = p_{A,t}\bar{\delta}_t h_{H,t}, \tag{78}$$

and

$$w_{MA,t} = p_{A,t}\bar{\delta}_t h_{M,t}. \tag{79}$$

Intermediate goods sector The supply of machines of the latest vintage is obtained as

$$x_t \equiv x_{n,t} = \left(\frac{\alpha^2}{\bar{R}(1+\tau_{R,t})} \right)^{\frac{1}{1-\alpha}} \cdot (h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}), \tag{80}$$

and the supply of machines of older vintage is given by

$$x_{m,t} = \left(\frac{\alpha}{\bar{R}(1+\tau_{R,t})} \right)^{\frac{1}{1-\alpha}} \cdot (h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t}). \tag{81}$$

It holds that

$$x_{m,t} = \alpha^{\frac{1}{\alpha-1}} x_{n,t} \tag{82}$$

and the final production function can be rewritten as

$$Y_t = \left(h_{H,t}\tilde{H}_{Y,t} + h_{M,t}\tilde{M}_{Y,t} \right)^{1-\alpha} \left(\left(h_{L,t}\tilde{L}_t \right)^{\alpha} + \tilde{A}_t (x_t)^{\alpha} \right), \tag{83}$$

with

$$\tilde{A}_t \equiv \left(\alpha^{\frac{\alpha}{\alpha-1}} - 1 \right) A_{t-1} + A_t. \tag{84}$$

Human capital Total public spending is still the sum of the expenditures across basic and higher education spending

$$E_t = E_{B,t} + E_{H,t}. \tag{85}$$

Basic (and low-skilled) human capital is given as

$$h_{B,t} = h_{L,t} = B \cdot \left(\hat{E}_{B,t} \right)^{\mu_B}, \tag{86}$$

with

$$\hat{E}_{B,t} \equiv \frac{E_{B,t}}{N}. \tag{87}$$

High-skilled human capital follows (21) with

$$\hat{E}_{H,t} \equiv \frac{E_{H,t}}{H_t + M_t}. \tag{88}$$

Human capital of marginal individuals is equal to high-skilled human capital

$$h_{M,t} = h_{H,t}. \tag{89}$$

Fiscal policy The government still runs a balanced budget

$$\mathcal{G}_t = \mathcal{G}_{W,t} + \mathcal{G}_{R,t}, \tag{90}$$

with revenues from taxing labor income and machines as

$$\mathcal{G}_{W,t} = \tau_{W,t} \cdot (w_{H,t}\tilde{H}_{Y,t} + w_{M,t}\tilde{M}_{Y,t} + w_{HA,t}\tilde{H}_{A,t} + w_{MA,t}\tilde{M}_{A,t} + w_{L,t}\tilde{L}_t), \tag{91}$$

$$\mathcal{G}_{R,t} = \tau_{R,t} \cdot \sum_{i=1}^{A_t} p_{i,t} x_{i,t} = \tau_{R,t} \hat{A}_t \bar{R} x_t, \tag{92}$$

with

$$\hat{A}_t \equiv \alpha^{\frac{1}{\alpha-1}} A_{t-1} + \alpha^{-1}(A_t - A_{t-1}). \tag{93}$$

Government budget balance implies

$$\mathcal{G}_t = E_t + T_t, \tag{94}$$

with

$$E_t = \phi_t \cdot \mathcal{G}_t \tag{95}$$

and

$$T_t = (1 - \phi_t) \cdot \mathcal{G}_t. \tag{96}$$

Public spending on basic education is given as

$$E_{B,t} = \phi_{B,t} \cdot E_t, \tag{97}$$

and public spending on higher education as

$$E_{H,t} = (1 - \phi_{B,t}) \cdot E_t. \tag{98}$$

The share of total transfers to low-skilled individuals is given as

$$T_{L,t} = \omega_t \cdot T_t \tag{99}$$

and to high-skilled (and marginal) individuals as

$$T_{H,t} = (1 - \omega_t) \cdot T_t, \tag{100}$$

leading to general per-capita transfer payments of

$$\hat{T}_{L,t} = \frac{T_{L,t}}{L_t} \tag{101}$$

and

$$\hat{T}_{H,t} = \hat{T}_{M,t} = \frac{T_{H,t}}{(H_t + M_t)} \tag{102}$$

Market clearing and equilibrium conditions The population constraint

$$N = H_t + M_t + L_t \tag{103}$$

holds, with the number of high-skilled individuals as

$$H_t = H_{Y,t} + H_{A,t}, \tag{104}$$

and the number of marginal individuals as

$$M_t = M_{Y,t} + M_{A,t}. \tag{105}$$

We impose two no-arbitrage conditions on wages in form of

$$w_{H,t} = w_{HA,t} \tag{106}$$

and

$$w_{M,t} = w_{MA,t}. \tag{107}$$

Type-specific aggregate labor supply is given as

$$\tilde{H}_{Y,t} = (1 - \eta - z_{H,t}) \cdot H_{Y,t}, \tag{108}$$

$$\tilde{H}_{A,t} = (1 - \eta - z_{H,t}) \cdot H_{A,t}, \tag{109}$$

$$\tilde{M}_{Y,t} = (1 - \eta - z_{M,t}) \cdot M_{Y,t}, \tag{110}$$

$$\tilde{M}_{A,t} = (1 - \eta - z_{M,t}) \cdot M_{A,t}, \tag{111}$$

and

$$\tilde{L}_t = (1 - z_{L,t}) \cdot L_t. \tag{112}$$

Type-specific individual labor supply follows

$$\tilde{h}_t = 1 - \eta - z_{H,t}, \tag{113}$$

$$\tilde{m}_t = 1 - \eta - z_{M,t}, \tag{114}$$

and

$$\tilde{l}_t = 1 - z_{L,t}. \tag{115}$$

Aggregate high-skilled labor supply is given as

$$\tilde{H}_t = \tilde{H}_{Y,t} + \tilde{H}_{A,t}, \tag{116}$$

aggregate marginal labor supply as

$$\tilde{M}_t = \tilde{M}_{Y,t} + \tilde{M}_{A,t} \tag{117}$$

and aggregate labor supply follows

$$\tilde{N}_t = \tilde{H}_t + \tilde{M}_t + \tilde{L}_t. \tag{118}$$

Welfare function The modified welfare function for the three-type economy is of the following form

$$\Omega_t = \zeta \cdot \underbrace{\mathcal{F}(a_t^{**}) \cdot N}_{=L_t} \cdot \mathcal{U}_{L,t} + (1 - \zeta) \cdot \left(\underbrace{(\mathcal{F}(a_t^{*}) - \mathcal{F}(a_t^{**})) \cdot N}_{=M_t} \cdot \mathcal{U}_{M,t} + \underbrace{(1 - \mathcal{F}(a_t^{*})) \cdot N}_{=H_t} \cdot \mathcal{U}_{H,t} \right). \tag{119}$$

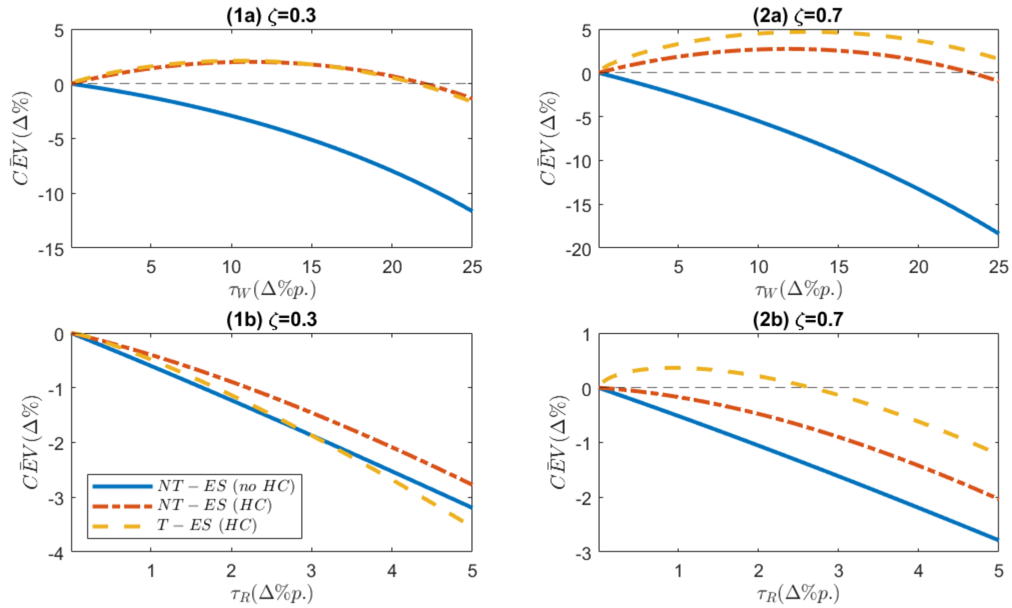

Figure 11: Education subsidies. Percentage deviations of aggregate welfare (measured as the average consumption equivalent variation) in 2045 from the baseline situation for a tax increase in either the linear income tax rate (first row) or the robot tax (second row) for a lower weight on low-skilled individuals in the aggregate welfare function ($\zeta = 0.3$, first column) and a higher weight on low-skilled individuals in the aggregate welfare function ($\zeta = 0.7$, second column) for education subsidies without intensive margin human capital formation (blue solid line), education subsidies (red dash-dotted line) and targeted education subsidies (yellow dashed line).

Education policy	τ_g^*	$\Omega\uparrow$	$\Delta\tau_g(\%p.)$	$C\bar{E}V(\%)$	$\Delta Y(\%)$	$\Delta\frac{c_H}{c_L}(\%)$
$\zeta = 0.5$						
Education subsidies	τ_W	✓	11.4	2.4	-1.5	-1.2
	τ_R	✗				
Targ. education subsidies	τ_W	✓	11.9	3.4	2.8	-5.1
	τ_R	✓	0.3	0.1	-0.5	-0.9
$\zeta = 0.3$						
Education subsidies	τ_W	✓	11.1	2.0	-1.4	-1.2
	τ_R	✗				
Targ. education subsidies	τ_W	✓	10.6	2.1	3.2	-4.8
	τ_R	✗				
$\zeta = 0.7$						
Education subsidies	τ_W	✓	11.9	2.7	-1.7	-1.3
	τ_R	✗				
Targ. education subsidies	τ_W	✓	13.1	4.7	2.4	-5.4
	τ_R	✓	1.0	0.4	-2.7	-2.7

Table 6: Education subsidies. Welfare optimal tax increases for 2045 for different education subsidies and their effects on production and inequality. Tax adjustments (specified as percentage-point increases) refer to the initial calibration for the labor income tax rate of 28.4% and the robot tax of 5%. ($^*g = \{W, R\}$)

F Private education spending

Household optimality FOC:

$$1 = (1 - \tau_{W,t})(1 - \eta - z_{H,t})\frac{\partial w_{H,t}}{\partial \theta_t}, \tag{120}$$

where:

$$\frac{\partial w_{H,t}}{\partial \theta_t} = (1 - \alpha)^2 \frac{\left((h_{L,t}\tilde{L}_t)^\alpha + \tilde{A}_t x_t^\alpha\right)}{(h_{H,t}\tilde{H}_{Y,t})^\alpha} \cdot \frac{\partial h_{H,t}}{\partial \theta_t}, \tag{121}$$

and

$$\frac{\partial h_{H,t}}{\partial \theta_t} = B_H \cdot (h_{B,t})^{1-\mu_H} \cdot \mu_H \cdot \epsilon \cdot (\theta_t)^{-\frac{1}{\nu}} \cdot \left(\epsilon \cdot (\theta_t)^{\frac{\nu-1}{\nu}} + (1-\epsilon) \cdot \left(\hat{E}_{H,t}\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1} \cdot \mu_H - 1}. \tag{122}$$

(4), (5) and (6), therefore, become:

$$c_{j,t} = \left(\frac{1}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} - \mathbb{1}_{[j=H]}\ \theta_t\right), \tag{123}$$

$$s_{j,t} = \left(\frac{\beta}{1+\beta+\gamma}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} - \mathbb{1}_{[j=H]}\ \theta_t\right), \tag{124}$$

and

$$z_{j,t} = \left(\frac{\gamma}{(1+\beta+\gamma)(1-\tau_{W,t})w_{j,t}}\right)\left((1-\tau_{W,t})(1-\eta_j)w_{j,t} + \hat{T}_{j,t} - \mathbb{1}_{[j=H]}\ \theta_t\right). \tag{125}$$

From solving the indifference condition ($\mathcal{U}_{H,t} = \mathcal{U}_{L,t}$), we obtain the same functional form as in (68).

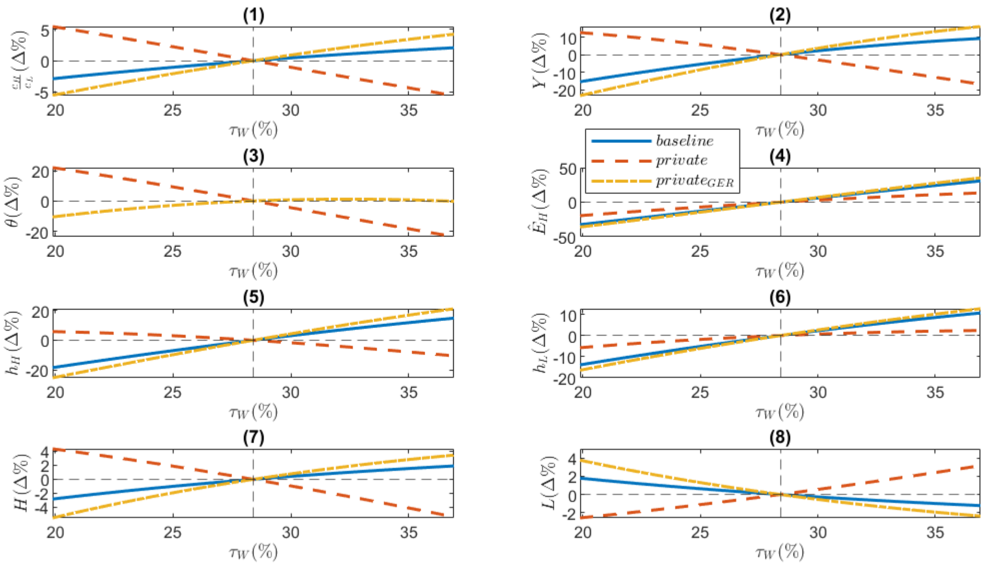

Figure 12: One-dimensional tax policy interventions with private education spending. Percentage deviations of different model variables in 2045 from the baseline situation for a variation in the labor tax. Blue solid lines: baseline model without private education spending; red dash-dotted lines: private spending model calibrated to the US; yellow dashed lines: private spending model for the German counterfactual.

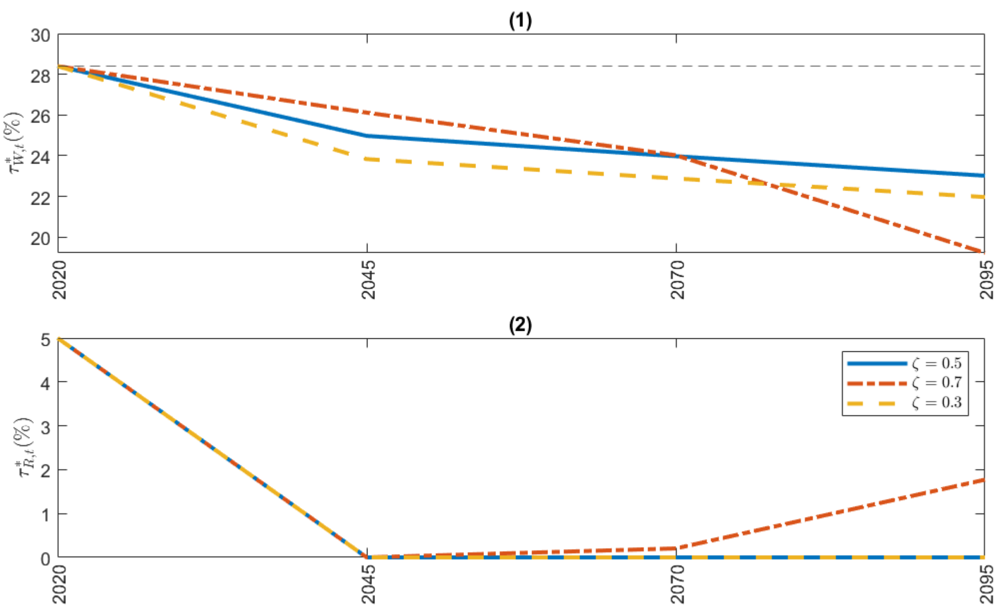

Figure 13: Dynamic optimal tax policy with private education spending. Panels 1 and 2 show the dynamic optimal labor tax and robot tax, respectively, for the private education model calibrated to the US and different levels of the welfare weight, $\zeta \in \{0.3, 0.5, 0.7\}$.