The Clean Transition in a Network Economy*

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Abstract

Governments increasingly provide sector-level subsidies to support the adoption of clean technologies. I study how the structure and endogeneity of the production network mediate the effect of such policies on technology choices, total emissions, and aggregate output. I develop an analytically tractable model featuring multiple sectors, heterogeneous firms, endogenous technology choice, and climate damages. I provide a general characterisation of the impact of a sector-specific subsidy increase on aggregate output. This effect can be decomposed into damage-related and non-damage components: the former is the direct output effect of the change in emissions, while the latter measures the output effect of the change in technology use and resource allocation given the level of damages. The existing network structure plays a crucial role in both components and policymakers should exploit this structure to identify interventions with the largest welfare improvements. Finally, I discuss several examples where well-meaning policies can, perversely, increase total emissions and reduce welfare.

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1 Introduction

In pursuit of net zero emissions, many large economies have adopted spending-based measures to encourage the adoption and deployment of technologies that directly use little or no fossil fuels (henceforth clean technologies). For example, the US Inflation Reduction Act, signed into law in 2022, provides tax credits for electric vehicles and a wide range of renewable energy systems such as solar, wind, geothermal, and energy storage[1]. At the same time, economies face significant fiscal constraints due to elevated debt levels and weak growth prospects (IMF, 2023). Given these pressures, how should governments target support for clean technologies across different sectors?

In this paper, I take an endogenous network perspective to answer this question. To motivate this approach, it is useful to discuss a simplified example. Consider the steel industry, which accounts for 7% of global carbon dioxide emissions (IEA, 2020). Traditionally, steel is produced by combining coal, iron ore and limestone in a blast furnace, releasing a significant amount of greenhouse gases as a byproduct. An alternative production technology involves using hydrogen instead of coal, producing far fewer emissions at the point of production. In this setting, how would a subsidy for hydrogen-based steel affect total emissions? First of all, the subsidy may encourage firms in the steel sector to change their production technology by switching from the traditional production method to the hydrogen-based method. This, in turn, would change the structure of intersectoral linkages by increasing the steel sector's demand for hydrogen and reducing its demand for coal. While this leads to a decrease in the sector's direct emissions, the total change in emissions depends on the nature of hydrogen production. If the hydrogen is extracted from natural gas, the most common production technique today, then the increase in hydrogen demand would lead to an increase in indirect emissions and, potentially, total emissions. If the hydrogen is produced via electrolysis[2] and, in turn, electricity is produced from renewables, then the increase in hydrogen demand would lead to no change in indirect emissions and, thus, a decrease in total emissions. This example highlights how the structure of an economy's production network is not only shaped by technology subsidies, but also mediates their effects.

With this intuition in mind, I develop a novel and analytically tractable model that nests an endogenous production network inside a simple integrated assessment model. To the best of my knowledge, my paper is the first to bring an endogenous network framework to study climate change mitigation. The model is static and considers the global economy as a single entity. There is a representative household, multiple sectors, including fossil fuel sectors, and a single factor of production, labour. Within each sector, there are a continuum of heterogeneous firms that produce firm-specific varieties that are imperfect substitutes. These firms make an endogenous choice whether to produce with a dirty technology or a clean technology. Both

[1] https://www.epa.gov/green-power-markets/summary-inflation-reduction-act-provisions-related-renewable-energy

[2] Electrolysis involves the passing of an electrical current through water to extract hydrogen and oxygen

technologies combine labour with intermediate inputs, but differ in their productivity and the set of inputs used. Specifically, the dirty technology's inputs include fossil fuels while the clean technology's does not. As firms are heterogeneous, both the clean and dirty technologies may be active in a sector. Firms are subject to an exogenous set of sector- and technology-specific subsidy rates. Finally, on the climate side, the use of fossil fuels generates emissions. For simplicity, I model the negative effect of these emissions (henceforth, climate damages or, simply, damages) as a homogeneous reduction in labour productivity across the economy. Within this setup, I study the effects of marginal changes in sector-specific subsidies for clean technologies on technology choices, emissions, and aggregate output.

My findings are threefold. First, I show that aggregate TFP can be decomposed into two parts: damage-related efficiency, which accounts for the direct damage from emissions to productivity, and non-damage efficiency, which accounts for technical and allocative efficiency given the level of damages.

Second, I show that the structure and endogeneity of the production network determine the effect of marginal changes in sector-specific clean technology subsidies on total emissions. The intuition behind this result is as follows. The subsidy induces a reduction in marginal cost in the target sector that propagates downstream along existing input-output linkages. Sectors with the largest direct and indirect exposure to the target sector, given the existing production network, experience the largest decrease in their marginal cost. The subsidy and resulting change in sectoral prices affect the relative competitiveness of the clean and dirty technologies across sectors, inducing firms to change their technology choices. The direction of this effect (i.e., whether it induces an increase or decrease in the clean share) depends on the direct and indirect reliance of each sector-specific technology on the target sector, while the size of the effect depends on the mass of firms at the margin of switching between the two technologies. The changes in technology (and input) choices then propagate upstream affecting the total demand for fossil fuels and, thus, the level of emissions. To illustrate this, I discuss several stylised examples where an increase in sector-specific clean technology subsidies can, perversely, increase total emissions depending on the structure of the network.

Third, and finally, I show that the change in aggregate output depends on the change in emissions as well as the change in non-damage efficiency. The latter captures how changes in subsidies interact with existing subsidies. I discuss several examples where an increase in a clean technology subsidy generates a double dividend i.e., a reduction in emissions and an improvement in non-damage efficiency. This analysis suggests that a policymaker interested in improving welfare should exploit the network structure to find interventions with the largest improvement in aggregate output, taking into account the changes in both damage-related and non-damage efficiency.

1.1 Literature Review

This paper relates to three areas of research. First, my work relates to the literature on climate change mitigation. Broadly, my model extends the tradition of Integrated Assessment Models (see Nordhaus (2013) for a survey) by developing a general equilibrium model of the economy and the carbon cycle. The two most related papers in this literature are Cavalcanti et al. (2022) and King et al. (2019), both of which consider the impact of carbon taxation in a production network. Cavalcanti et al. study the aggregate and distributional impacts of a carbon tax while King et al. study the impact of marginal changes to sector-specific carbon taxes. My work differs from these papers in three ways. First, these papers consider exogeneous networks while my model features endogeneous technology choice and, thus, an endogeneous production network. Second, these papers do not directly consider the negative impact of emissions, while I explicitly model the damages from emissions. Finally, I study heterogeneous firms rather than representative firms.

My work also relates to research on endogenous production networks. While early research in production networks took the structure of input-output linkages as given (e.g., Acemoglu et al. (2012), Acemoglu et al. (2015), Baqaee and Farhi (2019), Carvalho et al. (2021))[3], more recent studies consider the formation of intersectoral linkages and how this process influences macroeconomic outcomes. For example, Lim (2018) and Huneeus (2020) explore how frictions to the formation and modification of customer-supplier relationships affect the propagation of shocks. The closest paper in this literature to my work is Acemoglu and Azar (2020). The authors consider a multi-sector economy with endogeneous choice over the set of intermediate inputs used in production. They then study how changes in productivity or distortions affect the structure of production, specifically the density of the network. My paper differs along several dimensions. First, I study a continuum of heterogeneous firms in each sector rather than representative firms. This allows for multiple technologies to co-exist in my framework. Second, I include a climate module to track emissions and damages. Third, I study the impact of sector- and technology-specific subsidies while Acemoglu and Azar consider only sector-level distortions. Further, I focus on emissions and aggregate output rather than the density of the production network.

Finally, my work extends the literature on distortions in production networks. Bigio and La'O (2020) characterise how frictions compounded over the network, driving prices away from marginal cost and the economy away from productive efficiency. Similarly, Liu (2019) describes how the distortionary effects of market imperfections compound along backward demand linkages, leading to the largest distortions in upstream sectors. Baqaee and Farhi (2020) decompose changes in output into changes in technical and allocative efficiency. This allows the authors to trace out how distortions mediate the impact of productivity shocks.

[3] See survey by Carvalho and Tahbaz-Salehi (2019) for further discussion

1.2 Outline

The remainder of the paper is structured as follows. In Section 2, I describe the model. In Section 3, I characterise the equilibrium of the model. Section 4 contains my key results, including a general characterisation of the effect of a marginal increase in a sector-specific technology subsidy on technology choices, emissions and aggregate output. Section 5 concludes and discusses avenues for future research.

2 Model

In this section, I describe my model, which combines elements of integrated assessment models and endogenous production network models. I begin with an overview of the key features before explaining each element in more detail. At the end of the section, I define the equilibrium concepts to study this economy.

2.1 Overview

I model the global economy as a single entity composed of multiple intermediate sectors, indexed $i \in \mathcal{N}$, and a final good sector. A subset of intermediate sectors, $\mathcal{N}_F \subset \mathcal{N}$, are fossil fuel sectors. Within each sector, there is a continuum of firms that can choose to produce their firm-specific variety with a dirty technology $\theta = d$, which combines labour with intermediate inputs including some fossil fuel inputs, or a clean technology $\theta = c$, which combines labour with a different set of intermediate inputs. Importantly, clean technologies do not use any fossil fuels.

The use of fossil fuels generates emissions, which are a global negative externality. For simplicity and tractability, I model the negative effect of emissions as a reduction in labour productivity across the economy. To focus on the role of the production network and technology choice in shaping aggregate outcomes, I model the consumption side of the economy with a representative household. There are an exogenous set of sector- and technology- specific subsidies, σ_i^θ, which will ultimately influence technology choices, emissions and aggregate output. There is a lump sum tax levied on the household to balance the government's budget. In the following subsections, I discuss each element of the model.

2.2 Climate

In my model, the use of fossil fuels generates emissions, which impose a negative externality on the economy. I assume the quantity of emissions generated by the use of one unit of fossil fuel $i \in N_F$ is exogenous, homogeneous across sector of use, and denoted by ϵ_i. Thus, the total level of emissions E can be expressed as

$$E = \sum_{i \in \mathcal{N}_F} \epsilon_i Y_i \tag{1}$$

where Y_i is the output of fossil fuel sector i. This specification allows for heterogeneity in emissions content across different fossil fuels, such as coal, oil and natural gas.

I model the negative effect of emissions as a reduction in labour productivity. This can be interpreted as the negative impact of high temperature and extreme weather events. Specifically, the proportion of labour productivity lost D is defined as

$$D = \delta(E), \tag{2}$$

where $\delta(E)$ is continuous, differentiable, and increasing in emissions E. I normalise $\delta(0) = 0$ and assume damages approach 100% as emissions rise, $\lim_{E\to\infty} \delta(E) = 1$.

2.3 Production

2.3.1 Firms and Technologies

Each sector is composed of a unit mass of firms, indexed $v \in [0, 1]$. Each firm produces a firm-specific variety that is imperfectly substitutable. For example, consider the steel industry where each firm may produce different qualities of steel or the automotive sector where firms produce different styles of vehicles. To focus on the role of the network, I assume markets are perfectly competitive and model firms as profit-maximising price-takers.

Each firm can produce with a sector-specific dirty technology or a sector-specific clean technology, denoted by $\theta = d$ and $\theta = c$, respectively. These technologies combine labour and intermediate inputs in production but, importantly, differ in the set of intermediate inputs used. Specifically, the dirty technology uses some fossil fuel inputs, while the clean technology does not.

I now provide some examples to illustrate this setup. In the hydrogen industry, the dirty technology could refer to the process of natural gas reformation, which combines natural gas with high-temperature steam to produce hydrogen, while the clean technology could refer to the process of electrolysis, where an electric current splits water into hydrogen and oxygen. In the case of cement production, the dirty technology could refer to the process of using coal to heat a kiln with limestone and gypsum, while the clean technology could refer to a production process using hydrogen or biofuels to power the kiln. Finally, in the steel sector, the dirty technology can denote the traditional method of using coal to power the blast furnace, while the clean technology could refer to the use of hydrogen instead of coal to power the process.

For analytical tractability, I assume both technologies are Cobb-Douglas with constant returns to scale. Formally, the production function for variety v in sector i using technology $\theta \in \{c, d\}$ is

$$y_i^\theta(v) = \frac{1}{A_i^\theta} q_i^\theta(v) \Big((1 - D) l_i^\theta(v)\Big)^{a_i^\theta} \prod_{j\in\mathcal{N}} \Big(x_{ij}^\theta(v)\Big)^{m_{ij}^\theta} \tag{3}$$

where A_i^θ is a normalising contant[4], $q_i^\theta(v)$ is a Hicks-neutral productivity shifter, D is the proportion of labour productivity lost due to the negative effects of emissions, $l_i^\theta(v)$ is the quantity of labour, $x_{ij}^\theta(v)$ is the quantity of good j, $a_i^\theta \in (0, 1]$ is the labour share, $m_{ij}^\theta \in [0, 1)$ is the intermediate input share of good j, and $a_i^\theta + \sum_j m_{ij}^\theta = 1 \, \forall \, i, \theta$. Within each sector, the labour and intermediate input shares differ across technologies, reflecting differences in the inputs required for different production processes.

2.3.2 Technology Choice

Given profit maximisation, the representative firm in each variety will choose the technology with a lower marginal cost. Given that production is subject to sector- and technology-specific subsidies, denoted by σ_i^θ, the marginal cost $\kappa_i^\theta(v)$ for variety v in sector i using technology θ is given by

$$\kappa_i^\theta(v) \equiv \frac{1 - \sigma_i^\theta}{q_i^\theta(v)} \left(\frac{W}{1 - D} \right)^{a_i^\theta} \prod_{j \in \mathcal{N}} P_j^{m_{ij}^\theta} \tag{4}$$

where W is the wage and P_j is the price of good j. From this expression, we see that increases in the wage, damages, or sectoral prices raise marginal cost, while increases in the productivity shifter or subsidy rate reduce marginal cost.

Let $\gamma_i(v) \equiv q_i^d(v)/q_i^c(v)$ denote the exogenous technology gap between the dirty and clean technologies for variety iv. We can then express a firm's technology choice, denoted by $\theta_i^*(v)$, as a threshold function

$$\theta_i^*(v) = \begin{cases} c \text{ if } \log \gamma_i(v) < \log \Gamma_i \\ d \text{ otherwise} \end{cases} \tag{5}$$

where

$$\begin{aligned} \log \Gamma_i \equiv & (\alpha_i^d - \alpha_i^c)\big(\log W - \log(1 - D)\big) \\ & + \sum_j (m_{ij}^d - m_{ij}^c) \log P_j \\ & + \big(\log(1 - \sigma_i^d) - \log(1 - \sigma_i^c)\big) \end{aligned} \tag{6}$$

denotes the competitiveness threshold in sector i. Therefore, as the competitiveness threshold rises, more firms will choose to produce with the clean technology. This could occur if there is an increase in the price of some good j that is relatively more important to the dirty technology, $m_{ij}^d > m_{ij}^c$. Alternatively, this could occur if there is an increase in the subsidy for the clean technology, σ_i^c.

Let $G_i(X) \equiv \int_0^1 \mathbb{1}(\log \gamma_i(v) < X) \, dv$ denote the exogenous cumulative distribution function

[4] The constant is defined as $A_i^\theta \equiv \left(a_i^\theta\right)^{a_i^\theta} \prod_{j \in \mathcal{N}} \left(m_{ij}^\theta\right)^{m_{ij}^\theta}$. This simplifies the subsequent algebra, but does not affect any of my results.

of the technology gaps in sector i. Then the share of sector i using the clean technology is given by $G_i(\log \Gamma_i)$. For brevity, and with an abuse of notation, I will drop the $\log \Gamma_i$ term and simply use G_i to refer to the clean technology share in sector i.

I now re-write this expression in a format that will be more convenient and insightful to work with. Let $\overline{P}_i \equiv (1-D)P_i/W$ denote the normalised price of good i. The competitiveness threshold can, thus, be re-expressed as

$$\log \Gamma_i \equiv \sum_j (m_{ij}^d - m_{ij}^c) \log \overline{P}_j + \big(\log(1-\sigma_i^d) - \log(1-\sigma_i^c)\big). \tag{7}$$

This reveals that the absolute level of the wage or damages have no impact on technology choice after we account for normalised prices. Later, I show that equilibrium normalised prices do not depend on the wage or damage. Intuitively, we get this result as labour is the only factor of production. This means all inputs are embodied labour. If a variety is produced via a technology with a low labour input share, α_i^θ, then it is exposed to the wage via the embodied labour via its high intermediate input share.

2.3.3 Sectoral Goods

In each sector, the unit mass of varieties are aggregated into a composite good that is sold to other sectors as an intermediate input in production. I model this aggregation with a price-taking sectoral good producer using the following Cobb-Douglas production function

$$\log Y_i = \int_0^1 \log x_i(v)\, dv \tag{8}$$

where Y_i denotes the output of good i, and $x_i(v)$ is the quantity of variety iv.

2.3.4 Final Good

There is a competitive final good sector populated by a representative firm with the following constant returns to scale Cobb-Douglas production function[5]

$$Y = \frac{\prod_{i\in\mathcal{N}} X_i^{b_i}}{\prod_{i\in\mathcal{N}} b_i^{b_i}} \tag{9}$$

where Y is the output of the final good, X_i is the quantity of sectoral good i, b_i denotes the share of sectoral good i, and $\sum_i b_i = 1$.

[5]The denominator $\prod_{i\in\mathcal{N}} b_i^{b_i}$ simplifies the algebra, but does not change any results

2.4 Consumption

As the focus of this paper is the role of the network in mediating policy interventions, I simplify the consumption side of my model. There is a representative household endowed with L units of labour, which are supplied inelastically. Preferences are given by

$$U = \log C \tag{10}$$

where C is consumption of the final good. Setting the final good as the numeraire, the household budget constraint is given by

$$C \leq WL - T \tag{11}$$

where W is the wage and T is the lump sum tax to balance the government's budget. Note that the household does not earn any profits as markets are perfectly competitive throughout the model.

2.5 Equilibrium Concept

I end this section by defining the equilibrium concept I will use to study this economy.

Definition 1 (Equilibrium)**.** *An equilibrium is a set of prices, quantities, technologies choices, emissions and damages such that:*

- *(Household) The household chooses consumption C to maximise utility* (10) *subject to its budget constraint* (11)
- *(Final Good) The final good producer chooses quantities $\{X_i\}_{i\in\mathcal{N}}$ to maximise profit given sectoral prices $\{P_i\}$*
- *(Sectoral Goods) The sectoral good producers choose quantities $x_i(v)$ to maximise profit given variety prices $p_i(v)$*
- *(Varieties) Firms choose a technology and quantities to minimise cost given the wage W, sectoral good prices $\{P_i\}$, and damages D*
- *(Emissions) Emissions E satisfies* (1)
- *(Damages) Damages D satisfies* (2)
- *(Market clearing) The quantities satisfy market clearing for varieties, sectoral goods, the final good, and labour*

$$x_i(v) = y_i(v) \tag{12}$$

$$X_i + \sum_{j\in N} \int_0^1 x_{ji}(v)\, dv = Y_i \tag{13}$$

$$C = Y \tag{14}$$

$$\sum_{i \in N} \int_0^1 l_i(v)\, dv = L \tag{15}$$

3 Equilibrium Characterisation

In this section, I characterise the equilibrium of my model. The section is organised as follows. First, I introduce some useful notation that aggregates firm-level decisions to sector- and aggregate-level. This includes notation to describe the equilibrium production network, following Baqaee and Farhi (2020). After this, I establish the existence and uniqueness of equilibrium. There are several challenges to establish this result, which I discuss in the subsection. Following this, I characterise the equilibrium relationships between prices, quantities, and emissions in terms of the equilibrium values of other endogenous objects. I finish the section with a decomposition of aggregate TFP (and output) into damage-related and non-damage components. Taken together, these results provide the foundation for me to study the impact of subsidy changes in Section 4.

3.1 Useful Notation

I now introduce some notation to aggregate firm-level decisions and quantities to sector-level. First, let $\log Q_i \equiv \int_0^1 \log q_i(v)\, dv$ denote the average Hicks-neutral productivity shifter of firms in sector i. Note that while the productivity shifters for each firm and technology are exogenous, the average productivity shifter in a sector is an endogenous variable that depends on the technology choices of firms. Next, let $\log(1 - S_i) \equiv \int_0^1 \log(1 - \sigma_i(v))\, dv$ denote the average markdown of firms in sector i. Similarly to sectoral productivity, average markdowns are an endogenous quantity that depend on the exogenous subsidy rates, given by σ_i^d and σ_i^c, and the endogenous technology choice of firms.

Following Baqaee and Farhi (2020), I introduce some notation to describe the economy's equilibrium production network. First, let the $N \times N$ matrix $\tilde{\boldsymbol{M}}^{\theta} \equiv [\tilde{m}_{ij}^{\theta}]$ denote the intermediate input shares matrix for technology θ. Next, let $\tilde{\boldsymbol{\Omega}}_{N \times N}$ denote the economy's cost-based input-output matrix, where $\tilde{\Omega}_{ij} = \int_0^1 m_{ij}^{\theta_i^*(v)}\, dv$. I define the economy's cost-based Leontief inverse matrix as $\tilde{\boldsymbol{\Psi}}$ where

$$\tilde{\boldsymbol{\Psi}} = (\mathbf{I} - \tilde{\boldsymbol{\Omega}})^{-1}. \tag{16}$$

Intuitively, each element $\tilde{\Psi}_i j$ denotes the importance of sector j as a supplier to sector i given the existing production network. Finally, let the $N \times 1$ column vector $\tilde{\boldsymbol{\lambda}}$ denote the economy's cost-based Domar weights, where

$$\tilde{\boldsymbol{\lambda}}' \equiv \mathbf{b}'\tilde{\boldsymbol{\Psi}} \equiv \mathbf{b}'(\mathbf{I} - \tilde{\boldsymbol{\Omega}})^{-1}. \tag{17}$$

This object measures the direct and indirect importance of each sector to the production of the final good.

I now define a similar set of revenue-based objects. These broadly describe the direct and indirect expenditure of a given sector on intermediate inputs from other sectors as a share of the former's revenue. I use the $N \times N$ matrix $\boldsymbol{\Omega}$ to denote the economy's revenue-based input-output matrix, where each element is defined as

$$\Omega_{ij} \equiv \int_0^1 \frac{m_{ij}^{\theta_i^*(v)}}{1 - \sigma_i^{\theta_i^*(v)}}\, dv. \tag{18}$$

This aggregates the unit of firms' expenditure on good j to find the share of sector i's revenue spent directly on good j. I define the economy's revenue-based Leontief inverse matrix as $\boldsymbol{\Psi}$ where

$$\boldsymbol{\Psi} = (\mathbf{I} - \boldsymbol{\Omega})^{-1}. \tag{19}$$

Each element measures the direct and indirect expenditure of sector i on sector j. Finally, let the $N \times 1$ column vector $\boldsymbol{\lambda}$ denote the economy's (revenue-based) Domar weights, defined as

$$\boldsymbol{\lambda}' \equiv \mathbf{b}'\boldsymbol{\Psi} \equiv \mathbf{b}'(\mathbf{I} - \boldsymbol{\Omega})^{-1}. \tag{20}$$

The jth element of this vector measures the direct and indirect expenditure of the final good sector on sector j.

The next set of notation describes the direct and indirect reliance of sector-specific technology on other sectors given the technology's input requirements and the existing structure of the production network. Let the $N \times N$ matrix $\tilde{\boldsymbol{\Psi}}^{\theta} \equiv \mathbf{M}^{\theta}\boldsymbol{\Psi}$ denotes the cost-based Leontief inverse via technology θ. Intuitively, the ijth entry of this matrix measures the elasticity of the marginal cost of technology θ in sector i with respect to the price of good j given the equilibrium production network. Similarly, let the $N \times N$ matrix $\boldsymbol{\Psi}^{\theta} \equiv \big(\mathbf{I} - \mathrm{diag}(\boldsymbol{\sigma}^{\theta})\big)^{-1}\mathbf{M}^{\theta}\boldsymbol{\Psi}$ denote the revenue-based Leontief inverse via technology θ. The ijth entry of this matrix measures the direct and indirect expenditure on sector j if a firm uses technologyθ in sector i

Finally, I introduce notation to extract a specific row or column vector from the matrix objects above. For example, consider the $N \times N$ revenue-based Leontief inverse matrix $\boldsymbol{\Psi}$. I use $\boldsymbol{\Psi}^{\theta}_{k\bullet}$ to denote the $1 \times N$ row vector equal to the kth row of $\boldsymbol{\Psi}^{\theta}$. Similarly, I use $\boldsymbol{\Psi}^{\theta}_{\bullet k}$ to denote the $N \times 1$ column vector equal to the kth column of $\boldsymbol{\Psi}^{\theta}$.

3.2 Existence and Uniqueness

There are two key challenges in establishing these results. First of all, the availability of different technologies creates a discrete choice problem in each variety alongside the continuous choice problem over the quantity of labour and intermediate inputs. Second, there is feedback between prices, technology choice, emissions, and output. Despite this, I show that an equilibrium exists and is unique.

Theorem 1 (Existence and uniqueness of equilibrium)**.** *Given the exogenous set of productivity shifters, subsidy rates and technology input requirements, there exists a unique equilibrium.*

The intuition for this result can be broken down into three steps. First, I show that there is a unique set of normalised sectoral prices that satisfy equilibrium. Intuitively, this is the problem remains convex despite the discrete choice of technology. This derives from the assumptions of constant returns to scale and Cobb-Douglas production. On a technical note, since the sectoral prices depend on firm-variety prices, which in turn depend on sectoral prices, firms' optimisation behaviour creates a mapping from sectoral prices onto itself. I show that this mapping is a contraction mapping, guaranteeing existence and uniqueness of an equilibrium set of prices.

The second step of the proof maps these normalised prices to other endogenous variables for a given level of damages, D. For a given set of equilibrium normalised prices, there is a unique set of technology choices. This pins down the economy's production network from which we can derive all other elements of an equilibrium for a given level of emissions.

The final step finds the equilibrium level of emissions. Intuitively, this is another fixed point problem. Since emissions do not influence technology choices, I show that the right hand side of the equilibrium emissions equation (27) is decreasing in emissions. This guarantees a unique value of equilibrium emissions, completing the proof.

3.3 Equilibrium Relationships

As part of the proof for the existence and uniqueness of equilibrium, I derive characterisations of key endogenous variables in terms of the equilibrium values of other endogenous variables. I now discuss these corollaries in turn in order to support the discussion of my comparative statics in Section 4.

Corollary 1 (Equilibrium normalised prices)**.** *Given the equilibrium values for sectoral productivity shifters $\boldsymbol{Q}$, sectoral markdowns $(1-\boldsymbol{S})$, and cost-based Leontief matrix $\tilde{\boldsymbol{\Psi}}$, the equilibrium set of normalised prices is given by*

$$\log \overline{\boldsymbol{P}} = \tilde{\boldsymbol{\Psi}}(-\log \boldsymbol{Q} + \log(1-\boldsymbol{S})) \tag{21}$$

Proof: See Appendix

This result reveals that the normalised price of a good is a function of the average productivity shifter and markdown in its sector as well as the productivity shifters and markdowns of its direct and indirect inputs, as measured by the economy's cost-based Leontief inverse matrix. Each element of this matrix, $\tilde{\Psi}_{ij}$, measures the importance of the price of good j to the price of good i.

The next result describes the nominal value of sectoral output.

Corollary 2 (Equilibrium nominal sectoral output). *Given equilibrium aggregate output Y and the economy's revenue-based Domar weights $\boldsymbol{\lambda}$, as defined in equation (20), the equilibrium vector of nominal sector outputs is given by*

$$[P_1 Y_1 ... P_N Y_N]' = \boldsymbol{\lambda}' Y \equiv \boldsymbol{b}'(\boldsymbol{I} - \boldsymbol{\Omega})^{-1} Y \tag{22}$$

Proof: See Appendix

This result reveals that the nominal size, or revenue, of a sector depends on its revenue-based Domar weight, λ_i. Intuitively, the Domar weight captures the direct and indirect expenditure of the final good sector on a given intermediate sector. While this result looks similar to that of Acemoglu et al (2012), it differs in that Domar weights are endogenous in my model. This is because they depend on firm-level technology choices across the economy.

The following two results characterise the equilibrium wage and its relationship with the average product of labour.

Corollary 3 (Equilibrium wage). *Given the equilibrium level of damages D, cost-based Domar weights $\tilde{\boldsymbol{\lambda}}$, sectoral productivity shifters $\boldsymbol{Q}$, and sectoral markdowns $(1 - \boldsymbol{S})$, the equilibrium wage is given by*

$$\log W = \log(1 - D) + \tilde{\boldsymbol{\lambda}}'(\log \boldsymbol{Q} - \log(1 - \boldsymbol{S})) \tag{23}$$

Proof: See Appendix

The intuition for this expression is as follows. Increases in average sectoral productivity shifters or subsidies increase the marginal revenue product of labour, bidding up the wage. The size of this effect depends on a sector's cost-based Domar weight $\tilde{\lambda}_i$. Intuitively, this measures the sector's direct and indirect importance to the final good sector and, therefore, aggregate output. On the other hand, the wage is decreasing in damages as higher damages reduce labour productivity.

I now turn to the relationship between the wage and the average product of labour. Let ξ denote the wage distortion, defined as

$$\xi \equiv \frac{W}{Y/L} - 1. \tag{24}$$

The following corollary shows how this distortion depends on the set of exogenous sector-specific subsidy rates.

Corollary 4 (Equilibrium wage distortion)**.** *Given the equilibrium values for clean technology shares* $\boldsymbol{G}$ *and revenue-based Domar weights* $\boldsymbol{\lambda}$*, the equilibrium wage distortion is given by*

$$\xi = \sum_i \left(G_i \frac{\sigma_i^c}{1-\sigma_i^c} \lambda_i + (1-G_i) \frac{\sigma_i^d}{1-\sigma_i^d} \lambda_i \right) \tag{25}$$

Proof: See Appendix

Intuitively, this relationships occurs as subsidies raise the wage above the marginal product of labour. Later, I will show the aggregate implications of this distortion. Specifically, while subsidised sectors are able to afford these higher wages, other sectors reduce their demand for labour. Thus subsidies distort the allocation of labour away from their efficiency use, for a given level of damages.

Building on the results above, I now turn to real sectoral output.

Corollary 5 (Equilibrium real sectoral output)**.** *Given the equilibrium values for damages* D*, sectoral productivity shifters* $\boldsymbol{Q}$*, sectoral markdowns* $(1-\boldsymbol{S})$*, the cost-based Leontief inverse matrix* $\tilde{\boldsymbol{\Psi}}$*, and wage distortion* ξ*, the equilibrium real sectoral output in sector* i *is given by*

$$Y_i = \underbrace{\left((1-D)\prod_j Q_j^{\tilde{\Psi}_{ij}}\right)}_{\substack{\textit{Productivity of} \\ \textit{embodied labour}}} \times \underbrace{\frac{\lambda_i}{\prod_j (1-S_j)^{\tilde{\Psi}_{ij}}(1+\xi)}}_{\substack{\textit{Quantity of} \\ \textit{embodied labour}}} \times L \tag{26}$$

Proof: See Appendix

This result reveals that the real output of a given sector depends on features of the other sectors and the aggregate economy. The intuition for this result is as follows. As labour is the only factor of production, we can interpret the output of a given sector as the total embodied labour allocated to the sector and the productivity of this embodied labour

On the allocation side, a higher Domar weight $lambda_i$ or higher subsidies for itself or its direct and indirect inputs increase the allocation of labour. On the other side, an increase in the wage distortion, ξ, reduces the allocation of labour.

On the productivity side, there are two components. First, there is the reduction in labour productivity due to negative externality of emissions. The second component is a weighted average of sectoral productivity shifters according to the cost-based Leontief inverse matrix. This measures the direct and indirect importance of a given sector to another sector in terms of production. Holding all else equal, if there is an improvement in the productivity shifter for a key input sector (i.e., a high cost-based Leontief inverse entry), then there will be a large improvement in the embodied productivity and, thus, output of sector i.

Substiting the above expression for real sectoral output (26) into the emissions function (1), we obtain the next result on the equilibrium level of emissions.

Corollary 6 (Equilibrium emissions)**.** *Given the equilibrium values for emissions E, sectoral productivity shifters $\boldsymbol{Q}$, sectoral markdowns $(1 - \boldsymbol{S})$, the cost-based Leontief inverse matrix $\tilde{\boldsymbol{\Psi}}$, and wage distortion ξ, the equilibrium real sectoral output in sector i is given by*

$$E = \sum_i \left((1 - \delta(E)) \prod_j Q_j^{\tilde{\Psi}_{ij}}\right) \frac{\lambda_i}{\prod_j (1 - S_j)^{\tilde{\Psi}_{ij}} (1 + \tau)} \epsilon_i L \tag{27}$$

Proof: See Appendix

There are two key points to highlight here. First, the appearance of emissions E on the right hang side of the expression reveals the negative feedback from emissions to itself. This is because an increase in emissions leads to an increase in damages and a reduction in labour productivity. This directly reduces the output of fossil fuel sectors, which reduces aggregate emissions. Second, the output of fossil fuel sectors, and thus emissions, depends on features of other sectors. For example, holding all else equal, an increase in the sectoral productivity shifter of an upstream sector to a fossil fuel sector will lead to an increase in fossil fuel output and emissions. Further, as equation (22) above revealed, the Domar weight of a sector depends on the economy's input-output network. Therefore, technology and input choices in other sectors influences the Domar weight and, thus, size of fossil fuel sectors.

I finish this section with a characterisation of aggregate output in the following corollary.

Corollary 7 (Equilibrium aggregate output)**.** *Given the equilibrium values for damages D, sectoral productivity shifters $\boldsymbol{Q}$, sectoral markdowns $(1 - \boldsymbol{S})$, cost-based Domar weights $\tilde{\boldsymbol{\lambda}}$, and the wage distortion ξ, equilibrium aggregate output is given by*

$$\log Y = \overbrace{\underbrace{\log(1 - D)}_{\text{Damage-related efficiency}} + \underbrace{\tilde{\boldsymbol{\lambda}}' \log \boldsymbol{Q} - \tilde{\boldsymbol{\lambda}}' \log(1 - \boldsymbol{S}) - \log(1 + \xi)}_{\text{Non-damage efficiency}}}^{\text{Aggregate TFP}} + \log L \tag{28}$$

Proof: See Appendix

The corollary above shows that aggregate TFP can be decomposed into two components: damage-related efficiency, which measures the negative impact of emissions on labour productivity across the economy, and non-damage efficiency, which measures how the allocation of labour and intermediate inputs across sectors influence output holding the level of damages constant. From the discussion in this section, it is easy to see that changes in exogenous subsidy rates will influence both components of TFP and, therefore, aggregate output. In the next section, I investigate this response in more detail.

4 Comparative Statics with Subsidies

In this section, I investigate the effect of a marginal change in the sector-specific subsidy rate for the clean technology. The first set of results show how prices, technology choices, Domar weights, and the wage distortion change in response to the subsidy change. These results provide the building blocks for my main result, Theorem 7: a general characterisation of the effect of a subsidy change on aggregate output in terms of the existing production network structure, the input requirements of different technologies, the profile of marginal switchers in each sector, and other macroeconomic variables. I then apply the Theorem to study an economy with no existing subsidies.

4.1 Normalised Prices

The following result describes how normalised prices, a key determinant of technology choice, change in response to a marginal change in a sector-specific clean technology subsidy.

Theorem 2 (Change in normalised prices)**.** *The marginal change in the normalised price of sectoral good i with respect to a change in the clean subsidy in sector k is given by*

$$\frac{d \log \overline{P}_j}{d\sigma_k^c} = -\tilde{\Psi}_{jk} \frac{G_k}{1 - \sigma_k^c} \, \forall j \in \mathcal{N} \tag{29}$$

Proof: See Appendix

The result shows that the effect is entirely determined by the existing structure of the production network and the existing clean technology share in sector k. Specifically, the change in the normalised price of good j depends on its direct and indirect reliance on good k, as measured by the cost-based Leontief inverse matrix $\tilde{\boldsymbol{\Psi}}$, and the change in the price of good k due to the subsidy change, measured by $\frac{G_k}{1-\sigma_k^c}$. The intuition is as follows. The marginal increase in the clean technology subsidy in sector k immediately reduces the price of good k given that the proportion G_k of firms in the sector use the clean technology. The reduction in the price of good k propagates to downstream sectors that use it as an intermediate input, either directly or indirectly. This propagation is captured by the cost-based Leontief inverse. While the change in prices causes some varieties to switch to another production technology, this switching has no effect on prices at the margin. Consequently, the effect on normalised prices is entirely determined by existing technology choices.

4.2 Technology Choices

The next result describes how technology choices and, thus, technology shares across the economy change in response to a change in the subsidy.

Theorem 3 (Change in technology shares)**.** *The marginal impact of a subsidy to the clean* *technology in sector k on the vector of clean technology shares $\boldsymbol{G}$ is given by*

$$\frac{d\boldsymbol{G}}{d\sigma_k^c} = \overbrace{\boldsymbol{g}}^{\boldsymbol{G}'(\log \boldsymbol{\Gamma})} \circ \overbrace{\left(\boldsymbol{e}_k \frac{1}{1-\sigma_k^c} + \left(\tilde{\boldsymbol{\Psi}}^c_{\bullet k} - \tilde{\boldsymbol{\Psi}}^d_{\bullet k} \right) \frac{G_k}{1-\sigma_k^c} \right)}^{\frac{d \log \boldsymbol{\Gamma}}{d\sigma_k^c}} \tag{30}$$

where $\circ$ denotes the Hadamard product operator.
Proof: See Appendix

We see that the effect is composed of two parts: the density of firms in each sector at the margin of switching, measured by the vector $\mathbf{g}$, and the change in the competitiveness threshold in each sector due to the change in the subsidy. This latter component decomposes further into two interpretable components. First, holding normalised prices constant, the subsidy in sector k makes the clean technology more attractive. Second, the subsidy induces a change in normalised prices across the economy, as described in Theorem (2). This also changes marginal cost of producing with each technology. In particular, $\tilde{\Psi}^\theta_{ik} \equiv \sum_j \tilde{M}^\theta_{ij} \tilde{\Psi}_{jk}$ measures the total effect of the change in intermediate input prices on the marginal cost of producing with technology θ in sector i. This affects the relative cost of the two technologies, prompting some firms to change their technology choice. In more concrete terms, if the clean technology is more exposed to good k than the dirty technology (i.e., $\tilde{\Psi}^c_{ik} > \tilde{\Psi}^d_{ik}$), then its marginal cost will fall relative to the dirty technology. As a result, more firms in sector i will choose to produce with the clean technology.

4.3 Production Network

As discussed in Section 3, the Domar weights of intermediate sectors are a determinant of sector size, the level of emissions, and, ultimately, aggregate output. Building on my earlier results regarding changes in technology choices, I now characterise how Domar weights evolve in response to a change in a sector-specific subsidy. Importantly, the result highlights how changes in price and technology choice in sector k propagate along existing input-output linkages to other sectors.

Theorem 4 (Change in domar weights)**.** *The marginal change in the Domar weight of sector i, λ_i, due to a marginal change in the subsidy to the clean technology in sector k is given by*

$$\begin{aligned} \frac{d\lambda_j}{d\sigma_k^c} &= \lambda_k \frac{G_k}{1-\sigma_k^c} \Psi^c_{kj} \\ &+ \lambda_k \frac{g_k}{1-\sigma_k^c} \left(\Psi^c_{kj} - \Psi^d_{kj} \right) \\ &+ \sum_i \lambda_i g_i \left(\tilde{\Psi}^c_{ik} - \tilde{\Psi}^d_{ik} \right) \frac{G_k}{1-\sigma_k^c} \left(\Psi^c_{ij} - \Psi^d_{ij} \right) \end{aligned} \tag{31}$$

Proof: See Appendix

The intuition for this result is as follows. The first term is the additional direct and indirect expenditure of sector k on good j induced by the increase in the clean technology subsidy for the existing firms using the technology. The second and third terms capture the effects of technology switching, as described in Theorem (3). Specifically, the second term is the impact of the increase in the clean technology share in sector k. When $\Psi^c_{kj} < \Psi^d_{kj}$, the clean technology relies less on good j than the dirty technology, taking into account direct and indirect demand given the existing production network. This means the increase in the clean technology share in sector k leads to a reduction in the size of secotr j. Finally, the third term is the implication of price-induced switching on the total demand for good j. This captures the downstream propagation of prices (measured by the cost-based Leontief matrix $\tilde{\mathbf{\Psi}}$), the change in technology choice of these downstream sectors (measured by the density of firms at the margin of switching, $\mathbf{g}$), and the subsequent upstream propagation of demand for direct and indirect intermediate inputs (measured by the revenue-based Leontief inverse matrix $\mathbf{\Psi}$).

4.4 Wage Distortion

The next result considers the how the endogenous wedge between the wage and the average product of labour changes in response to a change in a sector-specific subsidy.

Theorem 5 (Change in wage distortion)**.** *The marginal impact of a subsidy to the clean technology in sector k on the wage distortion is given by*

$$\begin{aligned} \frac{d\xi}{d\sigma^c_k} = \sum_j \Bigg(& \frac{d\lambda_j}{d\sigma^c_k}\Big(G_j \frac{\sigma^c_j}{1-\sigma^c_j} + (1-G_j)\frac{\sigma^d_j}{1-\sigma^d_j}\Big) \\ & + \lambda_j \frac{dG_j}{d\sigma^c_k}\Big(\frac{\sigma^c_j}{1-\sigma^c_j} - \frac{\sigma^d_j}{1-\sigma^d_j}\Big) \\ & + \frac{\lambda_k G_k}{(1-\sigma^c_j)^2}\mathbb{1}_{j=k}\Bigg) \end{aligned} \tag{32}$$

Proof: See Appendix

Intuitively, the increase in the clean subsidy in sector k changes the demand for labour across the economy. This change in demand depends on the size of each sector, the labour expenditure in each sector, and how both of these objects change due to the subsidy change.

In general, the value and sign of the change in the wage distortion is ambiguous. In the following corollary, I consider the special case of no existing subsidies where we can derive a more precise result.

Corollary 8 (Change in wage distortion with no existing subsidies)**.** *Suppose there are no existing subsidies, $\sigma^\theta_i = 0 \,\forall\, i \in \mathcal{N}, \theta \in \{d, c\}$. Then the marginal impact of a subsidy to the* *clean technology in sector k on the wage distortion is given by*

$$\frac{d\xi}{d\sigma_k^c} = \tilde{\lambda}_k G_k \tag{33}$$

In this setting, the change in the wage distortion is always positive. The size of the effect is product of the Domar weight of sector k and the clean technology share in sector k.

4.5 Emissions

Theorem 6 (Change in emissions)**.** *The effect of a marginal change in the clean technology subsidy in sector k on total emissions $\log E$ is given by*

$$\frac{d\log E}{d\sigma_k^c} = \frac{1-\delta(E)}{1-\delta(E)+\delta'(E)E}\left\{\sum_i \frac{d\lambda_i}{d\sigma_k^c}\frac{\epsilon_i}{\overline{\epsilon}}\frac{1}{P_i} - \sum_i \frac{d\log \overline{P}_i}{d\sigma_k^c}\lambda_i\frac{\epsilon_i}{\overline{\epsilon}}\frac{1}{P_i} - \sum_i \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}\right\} \tag{34}$$

where $\frac{d\lambda_i}{d\sigma_k^c}$ is given by (31)*, $\frac{d\log \overline{P}_i}{d\sigma_k^c}$ is given by* (29)*, and $\frac{d\xi}{d\sigma_k^c}$ is given by* (32)*.*
Proof: See Appendix

4.5.1 Example: Local Input Expenditure Effect

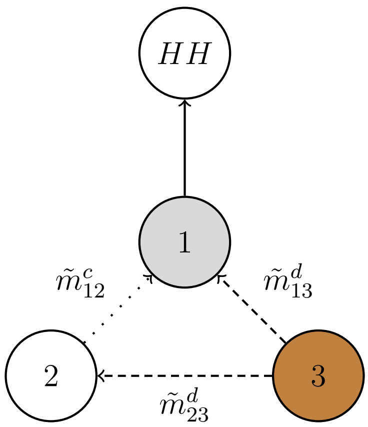

FIGURE I. STYLISED ECONOMY A

Consider the stylised three-sector economy depicted in figure I. The top node denotes the household while the remaining nodes denote intermediate sectors. Sector 3, shaded in brown, is the sole fossil fuel sector. The different arrows denote intermediate inputs for different production technologies. More specifically, the solid arrows denote inputs for the final good, the dashed arrows denote intermediate inputs for each sector's dirty technology, and the dotted arrows denote intermediate inputs for each sector's clean technology. The edge labels denote the value of the intermediate input shares. In the remainder of this section, I study changes in the clean technology subsidy in sector 1.

Definition 2 (Stylised Economy A). *The final good is produced using good* 1 *only. Thus, the consumption shares vector is given by*

$$\boldsymbol{b}' = [1,\, 0,\, 0]$$

The clean technology intermediate input shares are given by

$$\boldsymbol{M}^c = \begin{bmatrix} 0 & \tilde{m}^c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The dirty technology intermediate input shares are given by

$$\boldsymbol{M}^d = \begin{bmatrix} 0 & 0 & \tilde{m}^d_{13} \\ 0 & 0 & \tilde{m}^d_{23} \\ 0 & 0 & 0 \end{bmatrix}.$$

All subsidies are set to zero

$$\sigma_i^\theta = 0 \,\forall\, i, \theta.$$

The equilibrium clean technology shares are given by

$$\boldsymbol{G}' = [G_1,\, G_2,\, 1],$$

where $G_1, G_2 \in (0,1)$.

The following lemma applies Theorem 6 to the stylised economy under discussion. The result reveals that an intervention in sector 1 has no technology cascade effect. Instead, the change in output depends on how the network propagates the change in input expenditure in sector 1 to the rest of the economy as well as the impact of the subsidy change on the aggregate wage distortion.

Lemma 1 (Change in emissions in Stylised Economy A). *Consider the economy described by Definition 2. The effect of a marginal change in the clean technology subsidy in sector* 1 *on emissions* $\log E$ *is given by*

$$\frac{d\log E}{d\sigma_1^c} = \frac{1-\delta(E)}{(1-\delta(E)) + \delta'(E)E}\frac{1}{\tilde{\lambda}_3}\left(\tilde{\lambda}_1 G_1 \tilde{\Psi}^c_{13} + \tilde{\lambda}_1 g_1 (\tilde{\Psi}^c_{13} - \tilde{\Psi}^d_{13}) - \tilde{\lambda}_1 G_1 \tilde{\lambda}_3\right) \tag{35}$$

The proposition below characterises when a subsidy increase leads to an increase in total emissions.

Proposition 1 (Policy backfiring in Stylised Economy A). *Consider the economy described by Definition 2. A marginal increase in the clean technology subsidy in sector* 1 *has a positive* *effect on emissions* $\log E$ *if and only if*

$$\underbrace{\tilde{m}^c_{12}(1 - G_2)\tilde{m}^d_{23}}_{\tilde{\Psi}^c_{13}} > \underbrace{\tilde{m}^d_{13}}_{\tilde{\Psi}^d_{13}} \tag{36}$$

In words, if the indirect reliance of sector 1's clean technology on the fossil fuel sector is larger than the direct and indirect reliance of sector 1's dirty technology on the fossil fuel sector, then an increase in the clean technology subsidy in sector 1 will lead to an increase in equilibrium emissions and a reduction in aggretate output.

4.5.2 Example: Technology Cascade Effect

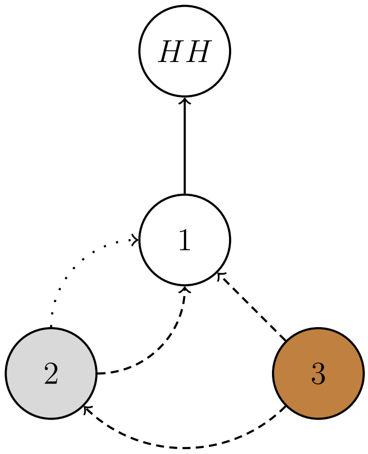

Figure II. Stylised Economy B

Consider the stylised three-sector economy depicted in figure II.

Definition 3 (Stylised Economy B)**.** *The final good is produced using good* 1 *only. Thus, the consumption shares vector is given by*

$$\boldsymbol{b}' = [1,\, 0,\, 0]$$

The clean technology intermediate input shares are given by

$$\boldsymbol{M}^c = \begin{bmatrix} 0 & \tilde{m}^c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The dirty technology intermediate input shares are given by

$$\boldsymbol{M}^d = \begin{bmatrix} 0 & \tilde{m}^d_{23} & \tilde{m}^d_{13} \\ 0 & 0 & \tilde{m}^d_{23} \\ 0 & 0 & \tilde{m}^d_{33} \end{bmatrix}.$$

All subsidies are set to zero

$$\sigma_i^\theta = 0 \,\forall\, i, \theta.$$

The equilibrium clean technology shares are given by

$$\boldsymbol{G}' = [G_1,\, 1,\, 1],$$

where $G_1 \in (0,1)$*.*

The following lemma applies Theorem 6 to the stylised economy under discussion. The result reveals that an intervention in sector 1 has no technology cascade effect. Instead, the change in output depends on how the network propagates the change in input expenditure in sector 1 to the rest of the economy as well as the impact of the subsidy change on the aggregate wage distortion.

Lemma 2 (Change in emissions in Stylised Economy B)**.** *Consider the economy described by Definition 3. The effect of a marginal change in the clean technology subsidy in sector* 1 *on emissions* $\log E$ *is given by*

$$\frac{d\log E}{d\sigma_1^c} = \frac{1-\delta(E)}{(1-\delta(E))+\delta'(E)E}\frac{1}{\tilde{\lambda}_3}\left(g_1(\tilde{\Psi}_{12}^c - \tilde{\Psi}_{12}^d)(\Psi_{13}^c - \Psi_{13}^d) - \tilde{\lambda}_2\tilde{\lambda}_3\right) \tag{37}$$

The proposition below characterises when a subsidy increase leads to an increase in total emissions.

Proposition 2 (Policy backfiring in Stylised Economy B)**.** *Consider the economy described by Definition 3. A marginal increase in the clean technology subsidy in sector* 1 *has a positive effect on emissions* $\log E$ *if and only if*

$$\tilde{m}_{12}^c < \frac{g_1 - (1-G_1)^2}{g_1 + G_1(1-G_1)}\tilde{m}_{12}^d \tag{38}$$

Intuitively, if sector 1's clean technology relies on good 2 sufficiently less than sector 1's dirty technology, then an increase in the subsidy for the clean technology in sector 2 leads to an increase in total emissions. While there is no direct emissions effect of this subsidy increase, there is a technology cascade effect, causing sector 1 to shift towards the dirty technology.

4.5.3 Example: Propagation via Two Channels

Consider the following three-sector economy. Sector 3 is the fossil fuel sector and the final good only uses good 1. Sector 1's dirty technology uses labour and good 3, while its clean technology uses labour and good 2. Sector 2's dirty technology uses labour and good 3, while its clean technology uses labour only. Finally, sector 3's dirty technology uses labour and good 3, while its clean technology uses labour only. The flow of intermediate inputs are depicted

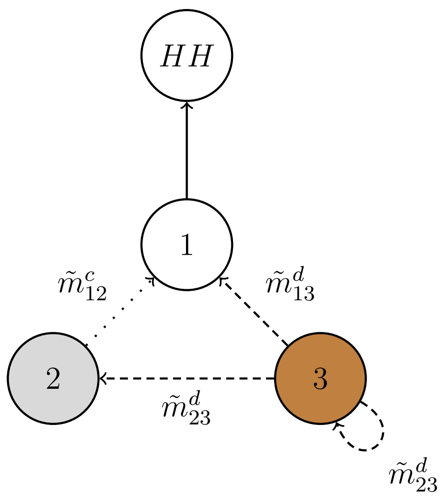

FIGURE III. STYLISED ECONOMY C

in figure III, where solid arrows indicate intermediate inputs for the final good, dashed arrows indicate intermediate inputs for the dirty technology, and dotted lines indicate intermediate inputs for the clean technology. The edge labels denote the value of the intermediate input shares. For simplicity, I assume that all subsidies are equal to zero and sector 3 only uses the dirty technology. This economy is formalised in Definition 4 below.

Definition 4 (Stylised Economy C)**.** *The consumption shares are given by*

$$\boldsymbol{b}' = [1,\, 0,\, 0]$$

The clean technology intermediate input shares are given by

$$\tilde{\boldsymbol{M}}^c = \begin{bmatrix} 0 & \tilde{m}^c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The dirty technology intermediate input shares are given by

$$\boldsymbol{M}^d = \begin{bmatrix} 0 & 0 & \tilde{m}^d_{13} \\ 0 & 0 & \tilde{m}^d_{23} \\ 0 & 0 & \tilde{m}^d_{33} \end{bmatrix}.$$

The equilibrium clean technology shares are given by

$$\boldsymbol{G}' = [G_1,\, G_2,\, 1], \quad G_1, G_2 \in (0,1)$$

All subsidies are set to zero

$$\sigma_i^\theta = 0 \,\forall\, i, \theta.$$

The following lemma applies Theorem 6 to find the emissions effect of a marginal increase in the clean technology subsidy in sector 2. Compared to the general case, only two of the three network propagation channels are active, namely: the local input switching effect, and the technology cascade effect.

Lemma 3 (Change in emissions in Stylised Economy C)**.** *Consider the economy described by Definition 4. The effect of a marginal change in the clean technology subsidy in sector* 2 *on emissions* $\log E$ *is given by*

$$\frac{d\log E}{d\sigma_2^c} = \frac{1-\delta(E)}{(1-\delta(E))+\delta'(E)E}\Bigg(\overbrace{\tilde{\lambda}_2 g_2(-\tilde{\Psi}_{23}^d)\frac{1}{\tilde{\lambda}_3}}^{\textit{Local expenditure effect}} \\ + \underbrace{\tilde{\lambda}_1 g_1(\tilde{\Psi}_{12}^c)G_2(\tilde{\Psi}_{13}^c - \tilde{\Psi}_{13}^d)\frac{1}{\tilde{\lambda}_3}}_{\textit{Technology cascade effect}} - \underbrace{\tilde{\lambda}_2 G_2}_{\textit{Wage distortion effect}}\Bigg) \tag{39}$$

Clearly, the local expenditure effect and wage distortion effect depress the level of emissions. The direction of the technology cascade effect depends on the relative size of $\tilde{\Psi}_{13}^c$ and $\tilde{\Psi}_{13}^d$ - in words, the difference in the direct and indirect use of fossil fuels by sector 1's clean and dirty technologies given the existing production network. If the dirty technology's reliance is higher ($\tilde{\Psi}_{13}^c < \tilde{\Psi}_{13}^d$), then the technology cascade effect - and the overall emissions change - is negative. On the other hand, if the clean technology's reliance is higher ($\tilde{\Psi}_{13}^c > \tilde{\Psi}_{13}^d$), then the technology cascade effect is positive. The density of firms in sector 1 at the margin of switching, g_1, determines the magnitude of this effect. If this value is large enough, it will outweigh the negative effect of the local expenditure effect and the wage distortion effect. The proposition below formalises this intuition and characterises when the subsidy increase leads to an increase in total emissions.

Proposition 3 (Policy backfiring in Stylised Economy C)**.** *Consider the economy described by Definition 4. A marginal increase in the clean technology subsidy in sector* 2 *has a positive effect on emissions* $\log E$ *if and only if the following conditions hold*

- $\tilde{\Psi}_{13}^c > \tilde{\Psi}_{13}^d$
- $g_1 > G_1\big(G_1 + \frac{\tilde{\Psi}_{13}^d}{\tilde{\Psi}_{13}^c - \tilde{\Psi}_{13}^d} + \frac{g_2}{G_2}\frac{\tilde{\Psi}_{23}^d}{\tilde{\Psi}_{13}^c - \tilde{\Psi}_{13}^d}\big)$

4.5.4 Example: Propagation via Three Channels

I now study an intervention with all three network propagation channels. Consider the following four-sector economy. There are two fossil fuel sectors: sector 2 and sector 4. Importantly, these sectors may differ in their equilibrium level of emissions per dollar of output, ϵ_i/P_i. The final good uses good 1 and good 2. Sector 1's dirty technology uses labour and good 4, while its clean technology uses labour only. Sector 2's dirty technology uses labour and good 4, while its clean technology uses labour and good 3. Sector 3's dirty technology uses labour and good

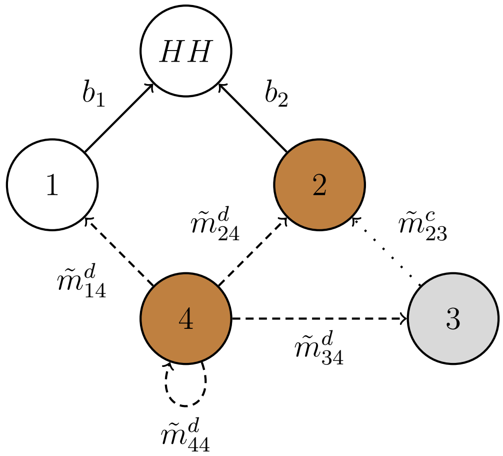

FIGURE IV. STYLISED ECONOMY C

4, while its clean technology uses labour only. Finally, sector 4's dirty technology uses labour and good 4, while its clean technology uses labour only. The flow of intermediate inputs are depicted in figure IV. For simplicity, I assume that all subsidies are equal to zero and sector 4 only uses the dirty technology. This economy is formalised in Definition 5 below.

Definition 5 (Stylised Economy D)**.** *The consumption shares are given by*

$$\boldsymbol{b}' = [b_1,\, b_2,\, 0,\, 0]$$

The clean technology intermediate input shares are given by

$$\tilde{\boldsymbol{M}}^c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{m}^c_{23} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The dirty technology intermediate input shares are given by

$$\tilde{\boldsymbol{M}}^d = \begin{bmatrix} 0 & 0 & 0 & \tilde{m}^d_{14} \\ 0 & 0 & 0 & \tilde{m}^d_{24} \\ 0 & 0 & 0 & \tilde{m}^d_{34} \\ 0 & 0 & 0 & \tilde{m}^d_{44} \end{bmatrix}.$$

The equilibrium clean technology shares are given by

$$\boldsymbol{G}' = [G_1,\, G_2,\, G_3,\, 0], \quad G_1, G_2, G_3 \in (0,1)$$

The equilibrium values for the density of firms in each sector with technology gap equal to the *competitiveness threshold are given by*

$$\boldsymbol{g}' = [g_1,\, g_2,\, g_3,\, 0], \quad g_1, g_2, g_3 \in (0, \infty)$$

All subsidies are set to zero

$$\sigma_i^\theta = 0\,\forall\, i, \theta.$$

The following lemma applies Theorem 6 to find the emissions effect of a marginal increase in the clean technology subsidy in sector 3.

Lemma 4 (Change in emissions in Stylised Economy D)**.** *Consider the economy described by Definition 5. The effect of a marginal change in the clean technology subsidy in sector* 3 *on emissions* $\log E$ *is given by*

$$\begin{aligned}\frac{d\log E}{d\sigma_2^c} =& \frac{1-\delta(E)}{(1-\delta(E))+\delta'(E)E}\Bigg(\overbrace{\tilde{\lambda}_3 g_3(-\tilde{\Psi}_{34}^d)\frac{\epsilon_4/P_4}{\bar{\epsilon}}}^{\text{Local expenditure effect}} \\ &+\underbrace{\tilde{\lambda}_2 g_2(\tilde{\Psi}_{23}^c)G_3(\tilde{\Psi}_{24}^c-\tilde{\Psi}_{24}^d)\frac{\epsilon_4/P_4}{\bar{\epsilon}}}_{\text{Technology cascade effect}}+\underbrace{\tilde{\Psi}_{23}G_3\tilde{\lambda}_2\frac{\epsilon_2/P_2}{\bar{\epsilon}}}_{\text{Productivity effect}}\underbrace{-\tilde{\lambda}_3 G_3}_{\text{Wage distortion effect}}\Bigg)\end{aligned} \tag{40}$$

Discussion is work in progress

Proposition 4 (Policy backfiring in Stylised Economy D)**.** *Consider the economy described by Definition 5. A marginal increase in the clean technology subsidy in sector* 3 *has a positive effect on emissions* $\log E$ *if and only if*

$$\frac{\epsilon_2/P_2}{\epsilon_4/P_4} > \frac{G_2 g_3\tilde{\Psi}_{34}^d + g_2 G_3(\tilde{\Psi}_{24}^d-\tilde{\Psi}_{24}^c)+G_2G_3\tilde{\lambda}_4}{G_2G_3(1-\tilde{\lambda}_2)} \tag{41}$$

Discussion is work in progress

4.5.5 Example: Intervention with Existing Subsidies

Example is work in progress

4.6 Aggregate Output

I now present my main result: a general characterisation of the impact of a subsidy change on aggregate output in terms of the existing production network structure, the input requirements of different technologies, the profile of marginal switchers in each sector, and other macroeconomic variables. The result highlights the key role played by the network in propagating the effects of the subsidy increase to other sectors, which contributes to the total effect on output.

Theorem 7 (Change in aggregate output)**.** *The marginal impact of a subsidy to the clean technology in sector k on aggregate output $\log Y$ is given by*

$$\frac{d \log Y}{d\sigma_k^c} = \frac{d}{d\sigma_k^c}\left[\log(1-D)\right] + \frac{d}{d\sigma_k^c}\left[\tilde{\boldsymbol{\lambda}}' \log \boldsymbol{Q} - \tilde{\boldsymbol{\lambda}}' \log(1-\boldsymbol{S}) - \log(1+\xi)\right] \tag{42}$$

where

$$\begin{aligned}\frac{d}{d\sigma_k^c}\left[\log(1-D)\right] = \frac{-\delta'(E)E}{(1-\delta(E))+\delta'(E)E}\Bigg(&\sum_i \frac{\lambda_k G_k}{1-\sigma_k^c}\Psi_{ki}^c \frac{\epsilon_i/P_i}{\overline{\epsilon}} \\ &+ \sum_i \frac{\lambda_k g_k}{1-\sigma_k^c}(\Psi_{ki}^c - \Psi_{ki}^d)\frac{\epsilon_i/P_i}{\overline{\epsilon}} \\ &+ \sum_i \sum_j \lambda_j g_j (\tilde{\Psi}_{jk}^c - \tilde{\Psi}_{jk}^d)\frac{G_k}{1-\sigma_k^c}(\Psi_{ji}^c - \Psi_{ji}^d)\frac{\epsilon_i/P_i}{\overline{\epsilon}} \\ &+ \sum_i \tilde{\Psi}_i k \frac{G_k}{1-\sigma_k^c}\lambda_i \frac{\epsilon_i/P_i}{\overline{\epsilon}} \\ &- \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}\Bigg)\end{aligned} \tag{43}$$

and

$$\frac{d}{d\sigma_k^c}\left[\tilde{\boldsymbol{\lambda}}' \log \boldsymbol{Q} - \tilde{\boldsymbol{\lambda}}' \log(1-\boldsymbol{S}) - \log(1+\xi)\right] = \tilde{\lambda}_k \frac{G_k}{1-\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c} \tag{44}$$

Proof: See Appendix

The intuition for the result is as follows. The change in the subsidy rate affects output via the two components of TFP: damage-related efficiency, which tracks the impact of emissions on labour productivity, and non-damaged efficiency, which considers the efficiency of the allocation of labour and intermediate inputs given the level of damages. I turn to these components in turn

The change in non-damage efficiency is composed of two components. The first term is the increase in aggregate output due to the increase in the output of sector k. Intuitively, $G_k/(1-\sigma_k^c$ is the size of the increase in sector k's output and $\tilde{\lambda}_k$ measures the importance of sector k to aggregate output. The second component of non-damage efficiency accounts for the change in output due to the change in the wage distortion term. If the distortion term increases, then there is a downward force on aggregate output.

The change in the damage-related term is made up of several components, each relating to different features of the production network. The term outside of the large brackets captures the impact of changes in fossil fuel production on damages taking into account the negative feedback effect of emissions onto itself. This term is always negative. The first term in the large brackets captures the additional indirect demand for fossil fuels by existing clean technology firms in sector k due to the subsidy change. The second term measures the change in emissions due to switching in sector k. The third term captures how the price reduction in sector k influences technology choices and, thus, demand for fossil fuels across the economy. The fourth term captures the propagation of the price reduction to any downstream fossil fuel sectors, which leads to an increase in emissions. The final term is the change in the wage distortion, which reduces output across the economy including that of fossil fuels.

In the follow corollary, I apply the above Theorem to study a simpler setting with no existing subsidies.

Corollary 9 (Change in aggregate output with no existing subsidies)**.** *Suppose there are no existing subsidies, $\sigma_i^\theta = 0 \,\forall\, i \in \mathcal{N}, \theta \in \{d, c\}$. Then the marginal impact of a subsidy to the clean technology in sector k on aggregate output $\log Y$ reduces to*

$$\frac{d \log Y}{d\sigma_k^c} = \frac{d \log(1-D)}{d\sigma_k^c} \tag{45}$$

where

$$\begin{aligned}
\frac{d}{d\sigma_k^c}\bigg[\log(1-D)\bigg] = \frac{-\delta'(E)E}{(1-\delta(E))+\delta'(E)E}\Bigg(&\sum_i \tilde{\lambda}_k G_k \tilde{\Psi}_{ki}^c \frac{\epsilon_i/P_i}{\bar{\epsilon}} \\
&+ \sum_i \tilde{\lambda}_k g_k (\tilde{\Psi}_{ki}^c - \tilde{\Psi}_{ki}^d)\frac{\epsilon_i/P_i}{\bar{\epsilon}} \\
&+ \sum_i \sum_j \lambda_j g_j (\tilde{\Psi}_{jk}^c - \tilde{\Psi}_{jk}^d) G_k (\tilde{\Psi}_{ji}^c - \tilde{\Psi}_{ji}^d)\frac{\epsilon_i/P_i}{\bar{\epsilon}} \\
&+ \sum_i \tilde{\Psi}_{ik} G_k \tilde{\lambda}_i \frac{\epsilon_i/P_i}{\bar{\epsilon}} \\
&- \tilde{\lambda}_k G_k\Bigg)
\end{aligned} \tag{46}$$

In this setting, there is no change in the non-damage component of TFP as the positive effect on aggregate output of the increase in sector k output is perfectly offset by the reduction in output across the economy due to the wage distortion. Intuitively, this occurs because, given the level of damages, resources are allocated to their most efficiency use in the absence of subsidies, which are distortionary. As a result, the change in output is entirely determined by the change in damage-related efficiency.

5 Conclusion

In this paper, I took a novel endogenous network perspective to understand how changes in sector-specific technology subsidies affect technology choices, the level of emissions, and aggregate output. To do this, I developed an analytically tractable model featuring multiple sectors, heterogeneous firms, endogenous technology choice, and climate damages.

I showed how aggregate output can be decomposed into damage-related and non-damage efficiency. I then provided a general characterisation of the impact of a subsidy change on aggregate output in terms of the structure of the production network, the input requirements of different technologies, the profile of firms at the margin of switching, and other macroeconomic variables. To illustrate the importance of the network, I discussed several examples where well-meaning policies that increase clean technology subsidies lead, perversely, to an increase in total emissions and a reduction in aggregate output.

Taken together, my results have three important policy implications. First, the existing structure of the production network determines whether, and by how much, an increase in a sector-specific clean technology subsidy reduces total emissions. Conditional on finding an intervention that reduces total emissions (and thus improves damage-related efficiency), policy makers may face a trade-off with a decrease in non-damage efficiency. Third, and finally, a policymaker interested in improving welfare should exploit the network structure to find interventions with the largest improvement in aggregate output, taking into account the changes in both damage-related and non-damage efficiency.

In ongoing work, I extend my work along two dimensions. First, I study the possibility of policy backfiring in more general settings than those discussed in Section 4. Second, I calibrate the model to data to understand the quantitative importance of the network in propagating the effects of sector-specific technology subsidies.

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A Proofs

Work in Progress