FTPL and the maturity structure of government debt in the New Keynesian Model

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Abstract

How do skyrocketing debt-to-GDP ratios and government expenditures affect inflation and how does this depend on the maturity structure of sovereign debt? In this paper, we revisit the fiscal theory of the price level (FTPL) within the New Keynesian (NK) model. We show in which cases the maturity of government debt matters for the transmission of policy shocks. The central task of this paper is to shed light on the theoretical predictions of the maturity structure on macro dynamics with an emphasis on (expected) inflation. In particular, we show how fiscal policy shocks affect interest rate and inflation dynamics. We highlight our results by quantifying the economic effects of the US COVID-19-emergency fiscal package (CARES), and shed light on the recent surge in inflation in the FTPL-NK model. In contrast, the same shocks have only small inflationary effects in the corresponding NK model with active monetary policy.

Keywords: NK models, FTPL, Government debt, Maturity structure, CARES
JEL classification numbers: E32, E12, C61

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Contents

1 Introduction

In response to the global coronavirus pandemic, governments around the world tried to cushion the economic downturn by financing large-scale fiscal support and relief packages such as the US Coronavirus Aid, Relief, and Economic Security (CARES) Act, with unprecedented volumes. For example, when including loan guarantees, the CARES Act amounts to about \$2 trillion (or 10% of US GDP) with substantial budgetary effects. The Congressional Budget Office (CBO) projects CARES to add \$1.7 trillion to deficits over the next decade.[1] In order to alleviate a deep recession, policy makers have implemented further stimulus packages (e.g., the American Rescue Plan). The funding of these unprecedentedly large fiscal programs drastically increased debt levels with yet unknown consequences (e.g., accounting for distributional effects, CARES is expected to increase the debt-to-GDP ratio by 12% in Kaplan, Moll, and Violante, 2020). This in turn led to a resurgence of policy debates and macroeconomic research about the effects of public debt and fiscal policy on macro aggregates, inflation, and inflation expectations where no consensus has been reached. One central question here is how government debt affects the transmission channels of fiscal and monetary policy. Governments face a challenging task to maintain a sustainable level and maturity structure of sovereign debt. On the one hand, fiscal policy faces a financing decision on whether to either increase the level of public debt or to raise taxes today. On the other hand, fiscal policy needs to decide on whether to issue bonds with longer maturities, or to simply roll-over maturing debt with short-term bonds. What impact can we anticipate from the recent large-scale fiscal programs, specifically, how does the maturity structure of outstanding debt affect those outcomes? This paper fills this gap in the analysis of fiscal and monetary policy.

In this paper we address the transmission of fiscal and monetary policy shocks on interest rates and inflation dynamics in a framework which combines the fiscal theory of the price level (FTPL) with the traditional New Keynesian (NK) model of inflation. Our central aims are the theoretical predictions of transitory and permanent policy shocks, which offer empirical testable implications for the role of the maturity structure of debt on the transmission of fiscal and monetary policy. Within this framework we study the effects of the recent CARES Act trough the lens of the fiscal theory. We depart from the existing literature on the effects of the maturity structure of government debt in three dimensions. First, our framework allows us to link the macro model to term-structure models in finance (Vasicek, 1977; Cox, Ingersoll, and Ross, 1985), which sheds light on a crucial facet: the distinction between temporary and permanent shocks. Our approach allows us to compute the term structure of interest rates and inflation expectations by solving a partial differential equation, which can be easily extended to nonlinear solutions,

[1] Congressional Budget Office, CARES Act, https://www.cbo.gov/publication/56334

default risk, and term premia. Second, in contrast to existing approaches[2], we compute zero-coupon bond prices for arbitrary maturities and states and then show the bounds for the effects of the maturity structure of government debt on macro dynamics and inflation decomposition. Finally, we show that the fiscal theory in the continuous-time version works through two distinct channels: (i) a direct FTPL effect through a discrete jump in the price of existing bonds and (ii) an indirect effect through changing the path of future real interest rates. While the first channel is a pure asset pricing channel, the second channel is the traditional effect present in forward-looking rational expectations models. Hence, even in the model with short-term debt, the fiscal theory has implications on the future path of the real interest rate, in particular, the term structure of interest rate, inflation expectations, and the real economy.

We calibrate a simple FTPL-NK model to match the maturity structure of outstanding US government debt and study the aggregate effects of fiscal and monetary policy instruments. We confirm that the maturity structure of existing public debt has important implications for the transmission channels of monetary and fiscal policy. Our results show how the average maturity significantly shapes the inflation response to fiscal and monetary policy shocks. First, following a transitory monetary policy shock, a longer maturity structure translates to a larger response in the real interest rate. In cases where outstanding government debt consists solely of short-term debt, the traditional negative correlation of the nominal interest rate and current inflation is reversed and term structure and inflation expectations are more sensitive to shocks. Similarly, based on the underlying maturity structure of government debt, expansionary fiscal policy leads to higher inflation and more accumulation of debt with short-term debt. Our inflation decomposition shows that with perpetuities, the inflation response to transitory shocks is dictated solely by future fiscal policy with changes in future monetary policy being soaked up by an immediate asset pricing effect. Second, we illustrate how inflation expectations and the term structure helps in identifying permanent policy shocks. Here, the maturity structure often produces some unpleasant short-term side effects. For example, a lower inflation target may even increase current inflation, inflation expectations, and nominal interest rates, but reduces long-term bond yields due to the re-evaluation of existing bonds.

Our findings confirm the hypothesis that the CARES Act with its unprecedented large-scale fiscal stimulus programs, i.e., the large cuts in primary surplus and hikes in government debt, has generated a market response with strong inflationary effects but effectively helped stimulating the real economy. However, the recent surge in inflation and medium-term inflation expectations indicate that markets do *not* expect that the newly issued debt is backed by subsequent higher future surpluses. This seems in contrast to the aftermath of the global financial crisis and raises cautionary flags as hyperinflations

[2]Among others see Leeper, Leith, and Liu (2019), Lustig, Sleet, and Yeltekin (2008), Faraglia, Marcet, Oikonomou, and Scott (2013) or Faraglia, Marcet, Oikonomou, and Scott (2019).

are widely believed to have fiscal origins (cf. Leeper and Leith, 2016). We contrast our results with a direct comparison of the same CARES Act shock in the simple NK model. In this case, the stimulus package basically reduces to a demand shock and is accompanied by a relatively small rise in the present value of future inflation rates by less than one percentage point. We show and discuss, that FTPL implies a more subtle inflationary impact because stimulus packages, on top of the direct demand effect, also have a debt and an asset pricing component.

In line with the existing literature on the fiscal theory, we confirm a prominent role of those ideas in the FTPL-NK model with a plausible maturity structure of sovereign debt (cf. Cochrane, 2001; Leeper and Leith, 2016).[3] Most theoretical studies, such as Sims (2011, 2013), Leeper and Leith (2016), and Cochrane (2018), highlight important insights, e.g., the role of long-term bonds in the simple NK model causing a 'boomerang inflation' response to monetary policy shocks. In these models, long-term bonds are used to offset an initial positive co-movement of the inflation and the interest rates.[4] Other studies focus on the low-frequency relationship between the fiscal stance and inflation in a model with long-term debt (see Kliem, Kriwoluzky, and Sarferaz, 2016) or the government spending multiplier (see Leeper, Traum, and Walker, 2017). We are not aware of a comprehensive study on the effects of fiscal and monetary policy shocks on inflation and inflation expectations, or more generally about the role of fiscal theory in the NK model with an empirically calibrated average maturity of existing sovereign debt. Unfortunately, an inflation decomposition into a direct FTPL effect and an indirect effect is tricky and less clear-cut in the discrete-time model because the price level can jump (which in the continuous-time version is determined by past inflation). Hence, a continuous-time version of the FTPL-NK model (see also Sims, 2011; Cochrane, 2018) helps isolating the effects of the maturity structure because in the model with short-term debt, as in traditional NK models with fiscal policy and sovereign debt, the direct bond pricing effect is zero and the fiscal theory works solely through the indirect effect.

We do not discuss the optimal maturity structure of debt (debt-maturity management). It is important to keep in mind that many theoretical and empirical studies recognize an important effect of the maturity structure of public debt in a broader context of optimal monetary and fiscal policies.[5] Leeper, Leith, and Liu (2019) show how high sovereign debt levels and the debt-maturity structure can increase the 'inflationary bias'. In this setup, higher debt levels and shorter maturities increase the temptation of the policy

[3]In this paper we focus on the fiscal regime and neglect potential fiscal-monetary coordination problems which may arise in a regime-switching model as in Bianchi (2012) or Bianchi and Melosi (2019).

[4]Cochrane (2023) and Liemen (2022) discuss alternative ideas and show that long-term debt is not necessary to address this counterfactual response for short-term debt in the FTPL-NK model.

[5]Other papers study the optimal debt-maturity management (cf. Buera and Nicolini, 2004; Shin, 2007; Faraglia, Marcet, and Scott, 2010; Debortoli, Nunes, and Yared, 2017; Bigio, Nuño, and Passadore, 2019). Bigio, Nuño, and Passadore (2019) show how liquidity costs can prevent an instantaneous re-balancing across maturities and identify different forces that shape the optimal debt-maturity distribution.

maker to use surprise inflation and to decrease the real value of government debt. Lustig, Sleet, and Yeltekin (2008) study the optimal policy if the fiscal authority is constrained by its ability to lend and only issues non-contingent nominal debt. In this case, optimal policy is achieved by almost the exclusive use of long-term debt. Even though the holding return on long-term debt is more volatile in contrast to short-term debt, it offers a hedge against fiscal shocks. Faraglia, Marcet, Oikonomou, and Scott (2013) analyze how inflation is affected by the maturity of sovereign debt and debt levels when fiscal and monetary policy are coordinated. They conclude that higher debt levels cause higher inflation, while a longer maturity structure increases its persistence.

Recently, Kaplan, Moll, and Violante (2020) and Bayer, Born, and Luetticke (2021) also evaluate the role of skyrocketing debt levels, following the large-scale fiscal stimulus programs within the NK models with heterogeneous agents (HANK). Focusing on the role of liquidity, Bayer, Born, and Luetticke (2021) find that the expansionary stimulus programs decreased the liquidity premium of government bonds. More closely related to our analysis is Bianchi, Faccini, and Melosi (2023), who propose a 'fiscal theory of persistent inflation', based on a framework where debt can be partially unfunded. Except for the responses to unfunded fiscal shocks, monetary policy always acts actively and fiscal policy passively. In their framework, monetary and fiscal regimes coexist. In contrast, we exclusively focus on either the standard NK or a FTPL-NK framework which clarifies the role of the maturity structure on the macro dynamics in either regime. Using empirical data, Bianchi, Faccini, and Melosi (2023) predict and match the observed rise in the inflation rate following the American Rescue Plan Act of 2021. In line with our analysis of the CARES Act, they argue that the surge of inflation rates primarily occurred due to fiscal inflation. Di Giovanni, Kalemli-Özcan, Silva, and Yildirim (2023) estimate that between December 2019 and June 2022 around one-third of US inflation is attributed to demand effects through the fiscal stimulus packages. In an empirical study for 37 OECD countries Barro and Bianchi (2023) apply the fiscal theory quantifying the economic effects of inflation: the real debt reduction through higher inflation effectively accounted, on average, for about 50 to 60 percent of government financing.

The rest of the paper is organized as follows. First, in Section 2 we formalize a simple perfect-foresight FTPL-NK model and study the effects of transitory and permanent structural zero-probability shocks. In Sections 3 and 4, we provide a thorough analysis and simulation of the CARES Act of 2020, and discuss the recent surge in inflation and differences to the aftermath of the global financial crisis. Section 5 concludes.

2 The Model

In this section, we show how the FTPL mechanism outlined in Sims (2011) and Cochrane (2018) is embedded in the continuous-time NK model (cf. Posch, 2020). For reasons of clarity, we shortly discuss the main channels of FTPL in the linear NK framework and abstract from the effects of uncertainty and nonlinearities.

2.1 Fiscal theory of monetary policy

As shown in Cochrane (2018), the presence of longer-term debt has effects on both the real economy and on how monetary policy is conducted, and more generally how government policies affect inflation. Consider the three-equation perfect-foresight NK model

$$\begin{aligned} \mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && (1) \\ \mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t && (2) \\ \mathrm{d}i_t &= \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, && (3) \end{aligned}$$

in which x_t is the output gap, y_t is output, i_t is the nominal interest rate, ρ the rate of time preference, π_t is inflation, where κ controls the degree of price stickiness with $\kappa \to \infty$ as the frictionless (flexible price) and $\kappa \to 0$ perfectly inelastic (fixed price) limits, θ controls interest rate smoothing with $\theta \to \infty$ implying the traditional feedback rule, $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1)$, and with π_t^* and i_t^* being parametric values.

Following Cochrane (2018) we implement the fiscal theory of the price level (FTPL) by closing the system with a fiscal block

$$\begin{aligned} \mathrm{d}a_t &= ((i_t - \pi_t)a_t - s_t)\mathrm{d}t && (4) \\ \mathrm{d}s_t &= f(s_t, y_t, a_t)\mathrm{d}t, && (5) \end{aligned}$$

in which a_t is the real value of sovereign debt (held by households), s_t is the primary surplus $s_t \equiv T_t - g_t$ following the fiscal rule $f(s_t, y_t, a_t)$, where T_t are lump-sum tax revenues, g_t government spending other than interest payments. It comprises the net payments to holders of bonds through interest and retirement of outstanding debt (cf. Sims, 2011). We use the notion of 'sovereign debt' and 'government bonds' interchangeably, which after all can be considered as a medium of exchange (paper money).

The central equation in the FTPL-NK model links the primary surpluses to the real value of sovereign debt. In fact, solving forward (4), the future path of primary surpluses imposes a 'constraint' for fiscal policy (government budget constraint), because

$$a_t \equiv \frac{n_t p_t^b}{p_t} = \mathbb{E}_t \int_t^\infty e^{-\int_t^u (i_v - \pi_v)\mathrm{d}v} s_u \mathrm{d}u, \tag{6}$$

where n_t denotes the number of outstanding bonds, p_t^b the bond price, and p_t the price level, which must equal its (expected) real present value.[6] In this paper, we focus on bounded

[6] Cochrane (2018) as well as Sims (2011) abstract from government consumption, g_t, in their framework,

solutions and $\lim_{T\to\infty} e^{-\int_t^T (i_v-\pi_v)\,\mathrm{d}v} a_T = 0$.[7] Rather than being a budget constraint or limiting fiscal capacity, equation (6) should be thought of as being a valuation formula as it asserts a value p_t^b to the supply of government bonds n_t and a given price level p_t.

Similar to assuming perfectly flexible prices, it is unrealistic assuming that government debt is either floating debt or perpetual debt (cf. Sims, 2011). In what follows, we refer to floating debt as short-term and to perpetuities as long-term debt. We introduce bonds with decaying coupon payments (similar to Woodford, 2001), and assume that longer-term bonds at average duration are amortized at rate δ and pay a nominal coupon $\chi + \delta$ such that at steady state the bonds sell at par and results compare to Sims (2011). No-arbitrage requires (see PDE approach Cochrane, 2005, chap. 19.4),

$$\mathrm{d}p_t^b = (i_t - ((\chi+\delta)/p_t^b - \delta))p_t^b \mathrm{d}t + \mathrm{d}\delta_{p_t^b}, \quad \mathbb{E}_t(\mathrm{d}\delta_{p_t^b}) = 0 \tag{7}$$

in which $\mathrm{d}\delta_{p_t^b}$ captures discrete changes in the bond price due to zero-probability structural shocks, with $\chi = i_{ss}$ such that $p_{ss}^b = 1$ is identical to floating debt. Note that (7) is *not* a stochastic differential equation (SDE) because the 'shocks' have zero probability. Following the literature, $\mathrm{d}\delta_{p_t^b}$ reminds us that the variable p_t^b can jump (forward-looking). In theory, we can issue floating debt which pays at $\chi = i_t$ and with $\delta \to \infty$ average duration approaches zero such that $p_t^b \equiv 1$. In contrast, for long-term bond we set $\delta = 0$ (cf. Sims, 2011). By integrating the linear approximation of equation (7), we obtain

$$p_t^b = 1 - \mathbb{E}_t \int_t^\infty e^{-(\chi+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u, \tag{8}$$

which shows that the initial response of the bond price is determined entirely by the discounted and maturity-adjusted path of the nominal interest rate. If we use the average duration of 6.8 years from the central bank's Security Open Market Account (SOMA), we calibrate $\delta = 1/6.8$ and $\chi = 0.03$ (see Del Negro and Sims, 2015).[8]

In contrast to the discrete-time model, the price level p_t cannot jump and is given by past price quotations (Calvo's insight).[9] Because the number of outstanding bonds in (6) is fixed and cannot jump either, only the bond price p_t^b reacts to changes in either future surplus s_u, or the future real interest rate $i_u - \pi_u$ for $u \geq t$ (direct FTPL effect). Because with short-term debt $p_t^b \equiv 1$, the direct FTPL requires the presence of longer-term debt. The bond price then passes on to the value of debt, inducing a jump in a_t (market value), i.e., making a_t a forward-looking variable. As we show below, the average duration $1/\delta$

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such that primary surpluses correspond to taxes, $s_t = T_t$.

[7]Hence, we focus on the standard no-bubble solution (e.g., Sims, 2011; Cochrane, 2018). There is a literature showing that a 'bubble term' can be important for the budget constraint (cf. Reis 2021).

[8]Below we use a zero-coupon bond with time-to-maturity of $1/\delta$ years interchangeably.

[9]Because no mass of firms can change prices instantaneously, the NK Phillips curve allows a jump in the inflation rate but not in the price level (cf. Cochrane, 2018, Online Appendix). Here, the price-level jump of the discrete-time model rather translates into a smooth change by affecting inflation.

of the maturity structure of outstanding government debt determines the strength of the direct FTPL effect, such that with short-term debt we eliminate jumps in p_t^b.

The path of the primary surplus on the right-hand side of equation (6) is determined by fiscal policy, so by assumption, surpluses typically do not jump if the value of sovereign debt changes (we discuss different scenarios below). Hence, changes in fiscal policy are accommodated by the real interest rate (indirect FTPL effect) such that (6) is not violated. So even without the presence of long-term debt, monetary policy must accommodate future changes in fiscal policy. Although households are indifferent with respect to the maturity of government debt because of arbitrage, the bottom line of this paper is to show that it has important implications for inflation dynamics, the term structure, inflation expectations, and the real economy. Thus, for ease of illustration, we focus on a fiscal regime (or fiscal dominance) throughout the paper, while the insights are useful for a regime-switching approach, as in Bianchi and Melosi (2019).

2.2 Simple fiscal policy rules versus policy inertia

There seems to be a consensus among economists that there is a systematic response of fiscal policy to the state of the economy. While theoretical papers often assume simple fiscal policy rules (Sims, 2011; Cochrane, 2018), most empirical studies suggest the presence of a time lag (or inertia) between the variables and the policy response (cf. Kliem, Kriwoluzky, and Sarferaz, 2016; Bianchi and Melosi, 2019). For instance, some time is typically required for changes in the tax code or the publication of a revised budget plan. In what follows, we propose a generic framework that facilitates the coherent study of different specifications for fiscal policy rules, and which enables the study of the effects of both temporary and permanent shocks. Starting with the definition of primary surplus in (5), $s_t = T_t - g_t$, which implies $\mathrm{d}s_t = \mathrm{d}T_t - \mathrm{d}g_t$, and specifying a tax rule as

$$\mathrm{d}T_t = \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t, \tag{9}$$

where ρ_τ controls the degree of inertia with $\rho_\tau \to \infty$ as the flexible limit (feedback rule), in which $T_t = T_t^* + \tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss})$. For $\rho_\tau \to 0$ we obtain the inelastic limit where $T_t \equiv T_t^*$. This fiscal policy is accompanied by a rule for government spending

$$\mathrm{d}g_t = \rho_g \left(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_t^*)\right) \mathrm{d}t, \tag{10}$$

where ρ_g controls the degree of inertia with $\rho_g \to \infty$ as the flexible limit (feedback rule), in which $g_t = g_t^* + \varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss})$. For $\rho_g \to 0$ we obtain the inelastic limit where $g_t \equiv g_t^*$. In what follows, we refer to the model parameters, or more generally, to the levels of government expenditures, taxes, and debt as 'fiscal policy', such that

$$\begin{aligned}\mathrm{d}s_t &= \rho_\tau\left(\tau_y(y_t/y_{ss}-1)+\tau_a(a_t-a_{ss})-(T_t-T_t^*)\right)\mathrm{d}t \\ &\quad -\rho_g\left(\varphi_y(y_t/y_{ss}-1)+\varphi_a(a_t-a_{ss})-(g_t-g_t^*)\right)\mathrm{d}t.\end{aligned}$$

Note that we could add others variables such as the inflation rate, π_t, which will be a function of the relevant state variables.[10] In a linearized version, such addition of variables gives rise to different parametrization of the responses to the state variables. Our results thus shed light on reasonable fiscal policy rules, which ultimately is an empirical question and beyond the scope of our analysis (cf. Kliem and Kriwoluzky, 2014).

Kliem and Kriwoluzky (2014) show that the fiscal policy rules, in which tax rates respond to the level of output, are not supported by the data. This is surprising as most papers in the theoretical FTPL literature study an output response only (cf. Sims, 2011; Cochrane, 2018).[11] Kliem, Kriwoluzky, and Sarferaz (2016) find weak empirical evidence in favor of output in fiscal policy rules, but rather evidence in favor of responses to the fiscal stance (such as the level of debt or debt-to-GDP ratios). We follow the conventional approach and focus on (locally) determinate solutions only. As shown in Leith and von Thadden (2008), this has important implications for the admissible parameter set for a particular regime, in particular the size of parameters τ_a and φ_a.

More generally, because the discussion for the appropriate fiscal policy rules applies to both tax rates and government expenditures, we conclude that no consensus has emerged yet about $f(a_t, s_t, y_t)$ in the surplus equation (5). In contrast to most central banks with a clear mandate, the fiscal policy parameters may depend on political orientation and/or institutional details. But this choice is far from being innocuous: To see the role of τ_a in determining active/passive fiscal policy, abstract from inflation dynamics, $r_t^f \equiv i_t - \pi_t$, and consider a simple feedback rule $s_t = s_{ss} - \tau_a(a_t - a_{ss})$. A linearized version is

$$\mathrm{d}a_t = (a_{ss}(r_t^f - r_t^*) + (\rho - \tau_a)(a_t - a_{ss}))\mathrm{d}t. \tag{11}$$

If $\tau_a > \rho$ in (11), the real debt dynamics would be non-explosive for bounded solutions. Following Leeper (1991), this corresponds to *passive* fiscal policy and *vice versa* for the case of $\tau_a < \rho$. As soon as fiscal policy turns passive, the fiscal policy block no longer affects other variables of the model, and the model dynamics for non-fiscal-block variables coincide with the ones of the three-equation NK model. While fiscal-block variables still respond to shocks, they remain decoupled from the underlying NK model.[12] Since our

[10]With a fiscal policy rule responding to inflation, a higher interest rate may produce lower inflation even with short-term debt (cf. Cochrane, 2023, Chap. 5.7).

[11]Note that Sims (2011) and Cochrane (2018) impose $\rho_\tau \to \infty$ (feedback rule), and the fiscal policy rule $g = s_g(y/y_{ss} - 1)$ can be replicated for $\rho_g \to \infty$ (feedback rule) and by setting $\varphi_y = s_g$.

[12]Liemen (2023) introduces distortionary taxes in a similar modelling framework. By doing so debt becomes a relevant state variable in both monetary and fiscal regimes. If the economy is relatively far

Table 1: Parametrization 1 (benchmark, similar to Kliem and Kriwoluzky 2014).

ρ	0.03	subjective rate of time preference
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady state output
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	1	output response fiscal tax rule (Sims, 2011; Cochrane, 2018)
τ_a	0	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	0	output response fiscal expenditure rule
φ_a	0	debt response fiscal expenditure rule
ρ_g	0	inertia of fiscal expenditure rule
s_g	0.1534	government consumption to output ratio (Bilbiie, Monacelli, and Perotti, 2019)
s_{ss}	0.0324	steady-state surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

focus is on the recent surge in debt levels we focus on the fiscal regime with $\tau_a < \rho$ and abstract from introducing distortionary taxes.

Our benchmark parametrization in Table 1 follows Kliem and Kriwoluzky (2014), and allows for inertia in the fiscal policy rule for tax revenues. Specifically, the tax rule in (9) has an output response $\tau_y > 0$ and an inelastic fiscal expenditure target such that $g_t \equiv g_t^*$ in (10) with $\rho_g \to 0$, and a corresponding T_t^* to match the US debt-to-GDP ratio of about 108% right before the pandemic (2020Q1).[13] We follow Bilbiie, Monacelli, and Perotti (2019) and set the steady-state government consumption-to-output ratio, s_g, to 15.34%. A higher share of government consumption-to-output of about 20%, similar to Justiniano, Primiceri, and Tambalotti (2013) and Eichenbaum, Rebelo, and Trabandt (2020), only slightly affects the model dynamics. Hence, the implied fiscal rule $f(s_t, y_t, a_t)$, in the law of motion for primary surplus (5), takes the form

$$f(s_t, y_t, a_t) \equiv y_t/y_{ss} - 1 - (s_t - s_t^*). \tag{12}$$

Market clearing and the fiscal policy rule then imply (cf. Appendix A.1.3):

$$y_t/y_{ss} - 1 = (1 - s_g)x_t. \tag{13}$$

away from its fiscal limit, dynamics are similar to the fiscal regime considered in our paper.

[13] U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, Federal Debt: Total Public Debt as Percent of Gross Domestic Product [GFDEGDQ188S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GFDEGDQ188S, January 13, 2022.

such that the equilibrium dynamics can be summarized as

$$\mathrm{d}x_t = (i_t - \rho - \pi_t)\mathrm{d}t \tag{14a}$$
$$\mathrm{d}\pi_t = (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t \tag{14b}$$
$$\mathrm{d}i_t = (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\mathrm{d}t \tag{14c}$$
$$\mathrm{d}a_t = ((i_t - \pi_t)a_t - s_t)\mathrm{d}t \tag{14d}$$
$$\mathrm{d}s_t = ((1 - s_g)x_t - (s_t - s_t^*))\mathrm{d}t \tag{14e}$$

in which x_t, π_t are forward-looking (jump) variables, and a_t satisfies (6).[14]

2.3 Solution to the linearized equilibrium dynamics

Following the FTPL literature, we solve a linearized system around the steady state for the initial values π_0 and x_0 given the state variables i_0, a_0, and s_0.[15] To this end, we use an eigenvalue-decomposition on the Jacobian matrix of the set of differential equations and study the local dynamics induced by an unexpected (zero-probability) shock on the stable manifold back to a steady state. Technically, we solve the system using the stable eigenvalues in order to find the unique (backward) solution. The jumps in forward-looking variables π_t and x_t, together with zero-probability shocks to the state variables i_t, a_t, and s_t, determine the initial values of the endogenous model variables.

In case of long-term debt, we use the bond price equation (7) and the dependence of a_t on the price in p_t^b from the valuation equation (6). Note that we need the bond price equation (7) only to pin down the initial price jump (direct FTPL effect), which translates to a shock to a_t. For example, consider a monetary policy shock $\mathrm{d}\varepsilon_i \equiv i_t - i_{t-}$ in the model with longer-term debt and store the implied initial price jump $\mathrm{d}\delta_{p_t^b} \equiv p_t^b - p_{t-}^b$. Consider then the same monetary policy shock $\mathrm{d}\varepsilon_i$ in the model with short-term debt, without bond price effects (no direct FTPL effect), and a contemporaneous shock $\mathrm{d}\varepsilon_a \equiv a_t - a_{t-} = \mathrm{d}\delta_{p_t^b}$, i.e., use the stored price jump as an additional structural shock to a_t, the short-term debt model has exactly the same solution as the model with long-term debt.

Proposition 1 (Linear solution) *The linear approximation to the system of the model's equilibrium dynamics (14) implies a set of functions for given states (i_t, a_t, s_t)*

$$x_t = \bar{x}_i(i_t - i_{ss}) + \bar{x}_a(a_t - a_{ss}) + \bar{x}_s(s_t - s_{ss}), \tag{15a}$$
$$\pi_t = \pi_{ss} + \bar{\pi}_i(i_t - i_{ss}) + \bar{\pi}_a(a_t - a_{ss}) + \bar{\pi}_s(s_t - s_{ss}), \tag{15b}$$
$$p_t^b = p_{ss}^b + \bar{p}_i^b(i_t - i_{ss}) + \bar{p}_a^b(a_t - a_{ss}) + \bar{p}_s^b(s_t - s_{ss}), \tag{15c}$$

[14] For an alternative parametrization, $f(s_t, y_t, a_t) \equiv (\tau_a - \varphi_a)(a_t - a_{ss}) - (s_t - s_t^*)$ together with a slightly changed Phillips curve (14b), our results can be found in Appendix C.1 (cf. Table D.1).

[15] Alternative approaches, which can account for non-linearities and risk, either solve the boundary value problem for a grid of state variables to approximate the policy function (cf. Posch, 2020), or use perturbation (cf. Parra-Alvarez, Polattimur, and Posch, 2021) to obtain the policy functions.

where bars denote the partial derivatives (slopes), evaluated at (i_{ss}, a_{ss}, s_{ss})*:*

$$\begin{aligned}
\bar{x}_i &= x_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_i^b v_{ss} \bar{x}_a/(1 - v_{ss}\bar{p}_a^b), \\
\bar{x}_a &= x_v(i_{ss}, v_{ss}, s_{ss}) p_{ss}^b (1 - v_{ss}\bar{p}_a^b)/(1 - v_{ss}\bar{p}_a^b + p_{ss}^b v_{ss}\bar{p}_a^b), \\
\bar{x}_s &= x_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_s^b v_{ss} \bar{x}_a/(1 - v_{ss}\bar{p}_a^b), \\
\bar{\pi}_i &= \pi_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_i^b v_{ss} \bar{\pi}_a/(1 - v_{ss}\bar{p}_a^b), \\
\bar{\pi}_a &= \pi_v(i_{ss}, v_{ss}, s_{ss}) p_{ss}^b (1 - v_{ss}\bar{p}_a^b)/(1 - v_{ss}\bar{p}_a^b + p_{ss}^b v_{ss}\bar{p}_a^b), \\
\bar{\pi}_s &= \pi_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_s^b v_{ss} \bar{\pi}_a/(1 - v_{ss}\bar{p}_a^b), \\
\bar{p}_i^b &= p_i^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b), \\
\bar{p}_a^b &= p_v^b(i_{ss}, v_{ss}, s_{ss})/(1 + v_{ss} p_n^b(i_{ss}, v_{ss}, s_{ss})/p_{ss}^b), \\
\bar{p}_s^b &= p_s^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b).
\end{aligned}$$

Here, $v_t \equiv n_t/p_t$ *defines the real number of bonds because the partial derivatives in terms of* a_t *(market value) reflect the indirect effects only, keeping fixed the price of government debt,* p_t^b*, while the total effects are visible only in terms of* v_t *(face value).*

Proof. Appendix A.4 ■

Our linearized solution (15) thus gives the policy functions in terms of v_t in Figure 1. For illustration, we also show the policy functions in terms of a_t (cf. Figure 2). Except for the bond price p_t^b, the policy functions coincide for different maturity structures and correspond in terms of a_t to the short-term debt case in terms of v_t. Figure 1 sheds light on how the maturity structure of government debt matters for the responses of macro aggregates with changes in the state variables. Probably the most striking result is the link between inflation and interest rates: For the average duration of government bonds in the data (blue solid), we obtain the traditional negative link between interest rates and current inflation rates. This shows that the fiscal regime is crucial to the traditional effect of monetary policy. A knife-edge case exists in which the direct FTPL effect offsets the indirect effect and interest rates would have no contemporaneous effect on inflation. Figures A.1 and A.2 show the corresponding policy functions in the simple NK model without FTPL. In this case, the policy function coefficients for debt and taxes are equal to zero and maturity would not matter for non-fiscal policy block variables.

2.4 Term structure of interest rates

The term structure of interest rate, defined as the yield of zero-coupon bonds as a function of their maturity, reveals important insights on expectations about the future path of macro aggregates and inflation. Given the equilibrium prices, we can price any asset. The no-arbitrage condition implies that the asset prices adjust such that the households will be indifferent in their portfolio decision. Let us consider a nominal (zero-coupon) bond

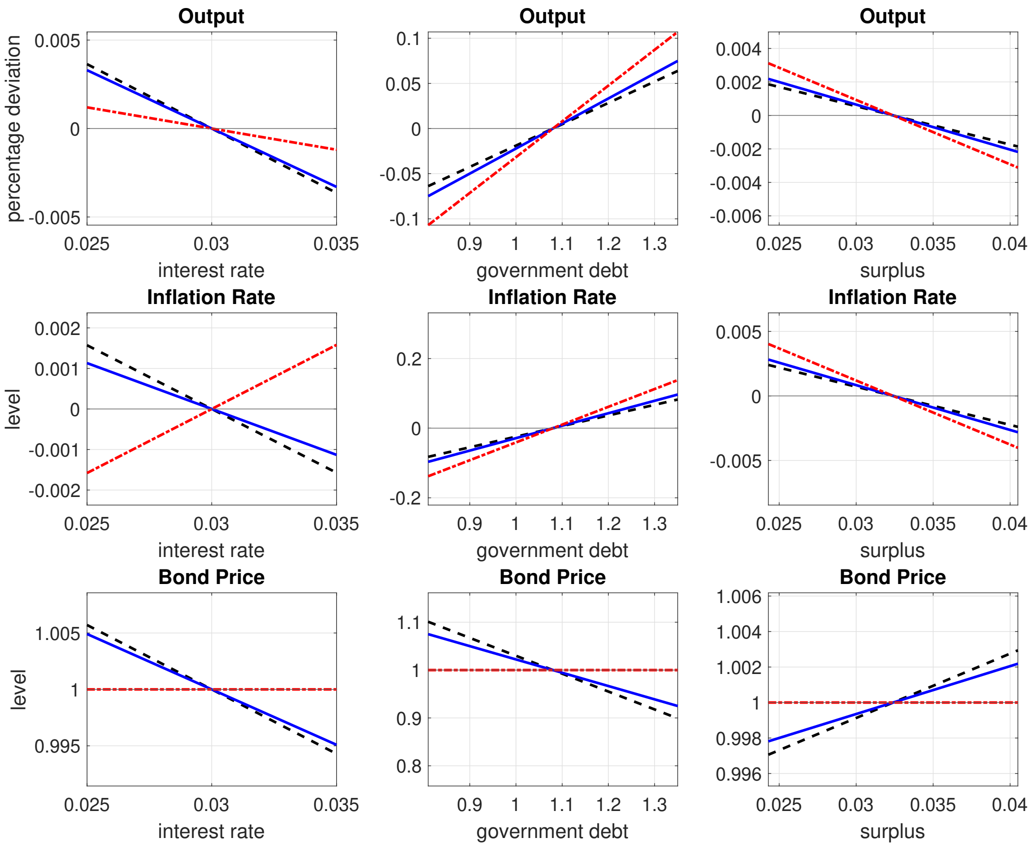

Figure 1: Policy functions for the parametrization in Table 1, showing the total response in terms of v_t (indirect and direct effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

with unity payoff at maturity N,

$$p_t^{(N)} = \mathbb{E}_t\left(e^{-\rho N}\lambda_{t+N}/\lambda_t e^{-\int_t^{t+N}\pi_u du}\right), \tag{16}$$

where λ_t is the marginal value of wealth, or the current value shadow price, consistent with equilibrium dynamics of macro aggregates. Note that the equilibrium price p_t^b can be computed along the same lines (because the maturity distribution is approximately exponential with a duration of $1/\delta$, the average-maturity bonds will share the same properties as zero-coupon bonds at maturity $1/\delta$). The equilibrium bond price can be obtained from the fundamental pricing equation for the price $p_t^{(N)}$ (Cochrane, 2005, chap. 19.4),

$$\mathbb{E}_t\left((\mathrm{d}p_t^{(N)})/p_t^{(N)}\right) - \left(1/p_t^{(N)}(\partial p_t^{(N)}/\partial N) + i_t\right)\mathrm{d}t = 0. \tag{17}$$

We derive the term structure of the interest rate in Section A.5. While in this paper, we focus on the expectation channel and abstract from other determinants such as risk premia and liquidity, an extension to include risk and term premia in the analysis is

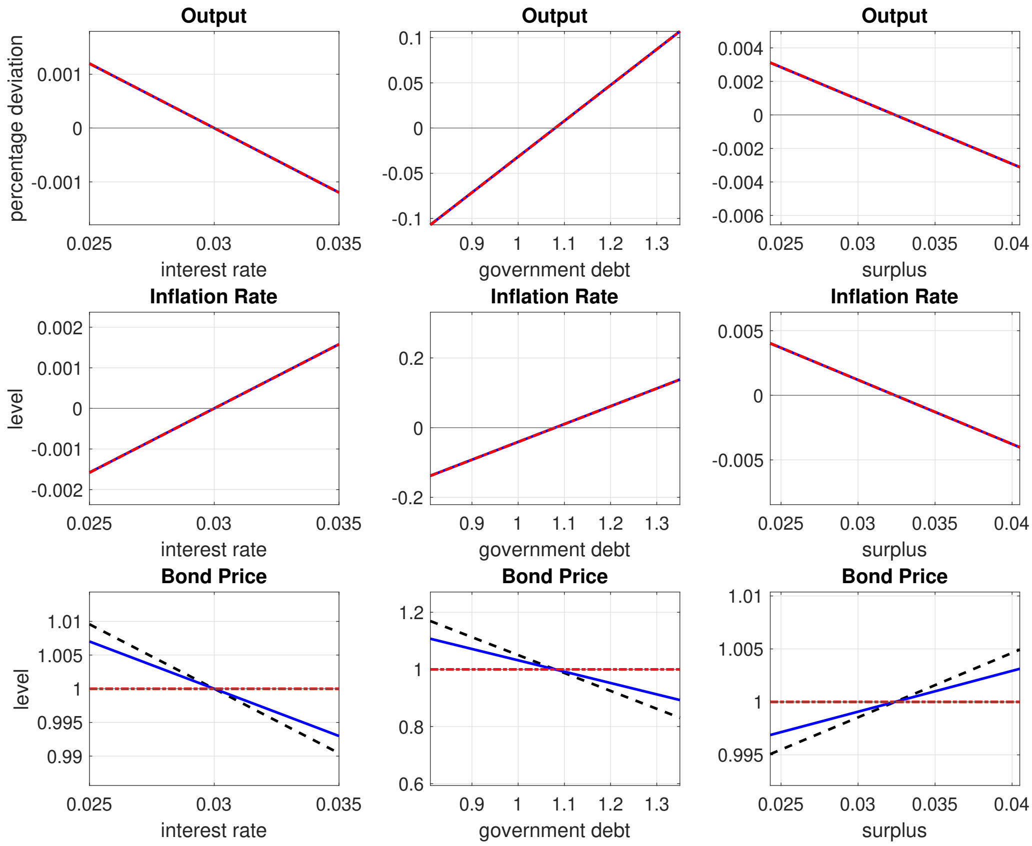

Figure 2: Policy functions for the parametrization in Table 1, showing the partial response in terms of a_t (indirect effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

straightforward (cf. Posch, 2020). In particular we want to study the effects of temporary and permanent zero-probability shocks on the term structure of interest rates.

2.5 Inflation decomposition and expected inflation

Inflation and expected inflation are key determinants of monetary policy. In what follows we decompose the total effects of structural shocks on those key variables from their theoretical impulse response functions (IRFs). By the inflation decompositions we answer the question how much structural shocks contribute to the observed responses and which channels reinforce or dampen the inflationary effects of the shocks. In a fiscal-theoretic interpretation, our decompositions answer "what changes in variables caused the observed inflation" (our decomposition follows Cochrane, 2022a, 2023).

Inflation decomposition. For our decomposition based on the IRFs, we start with the linearized debt evolution using $r \equiv i_{ss} - \pi_{ss} = \rho$ and $s_{ss} = \rho a_{ss}$

$$\mathrm{d}(a_t/a_{ss} - 1) = (i_t - \pi_t + r(a_t/a_{ss} - 1) - s_t/a_{ss})\mathrm{d}t$$

and

$$a_t/a_{ss} - 1 = \mathbb{E}_t \int_t^\infty e^{-r(u-t)} s_u/a_{ss} \mathrm{d}u - \mathbb{E}_t \int_t^\infty e^{-r(u-t)}(i_u - \pi_u)\mathrm{d}u,$$

which is the linearized present value formula corresponding to (6). The real value of debt is the present value of surpluses, discounted at the (steady-state) real interest rate.

From the linearized definition (6), the real value of sovereign debt (market value) can be decomposed into

$$a_t/a_{ss} - 1 = v_t/v_{ss} - 1 + p_t^b/p_{ss}^b - 1, \tag{18}$$

either by changes in debt issued or valuation (direct effects). Hence, we get the identity

$$\begin{aligned} \int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u &= \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^b - 1 + v_t/v_{ss} - 1 \end{aligned} \tag{19}$$

in the perfect-foresight model, which allows us, for example, to decompose the effects of zero-probability shocks on present values of future inflation into changes in the present value of future interest rates (monetary policy), the present value of changes in future surpluses (fiscal policy), and the FTPL effects (real debt decomposition). Higher inflation must correspond to higher future interest rates, lower future surpluses and the devaluation of outstanding debt, and the newly issued government bonds, such that the real value of debt equals the present value of surpluses (cf. Cochrane, 2022a).

Recall that the direct FTPL effect is strongest for perpetuities with $\delta \to 0$. Changes in future interest rates (monetary policy) will be soaked up in an initial re-evaluation of sovereign debt, and fiscal policy fully determines inflation. In contrast, in the short-term model with $\delta \to \infty$, changes in monetary policy affect future expected inflation most. For illustration, suppose that $\pi_{ss} \equiv 0$ and $\chi \equiv i_{ss} = r$, from (8) the bond price is

$$p_t^b = 1 - \int_t^\infty e^{-(r+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u.$$

Hence, the strength of the direct FTPL effect depends on both the average maturity $1/\delta$ and future monetary policy, such that (19) at $t = 0$ and for $v_0 = v_{ss}$ can be written as

$$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u = \int_0^\infty e^{-ru}\left(1 - e^{-\delta u}\right)(i_u - i_{ss})\mathrm{d}u - \int_0^\infty e^{-ru}(s_u - s_{ss})/a_{ss}\mathrm{d}u.$$

It shows that for $\delta \to 0$ monetary policy is fully captured in the bond price p_t^b, therefore irrelevant for future inflation, and fiscal policy fully determines inflation.

Inflation expectations. We can study the effects of monetary and fiscal policy shocks on the model-implied expected inflation, e.g., to confront the rational expectation forecast results with survey data. From the Phillips curve in (14b) it follows

$$\pi_t - \pi_t^* = \kappa \int_t^\infty e^{-\rho(v-t)} x_u \mathrm{d}u.$$

The inflation rate, π_t, denotes *current* expected inflation measured as deviation from its policy target rate π_t^*. Multiplying the differential equation for the inflation rate by the integrating factor and evaluating from t to $t+N$, we obtain

$$\pi_t^{(N)} \equiv \mathbb{E}_t(\pi_{t+N}) = \pi_t^* + e^{\rho N}(\pi_t - \pi_t^*) - \kappa e^{\rho N} \int_t^{t+N} e^{-\rho(u-t)} x_u \,\mathrm{d}u. \tag{20}$$

Intuitively, the model-implied inflation forecast is a forward contract to inflation, which can be more informative than using forward rates (Gürkaynak, Sack, and Wright, 2007). We compute the rational expectation forecast π_{t+N} as a function of the current state variables (i_t, a_t, and s_t) and the fixed forecasting horizon N. Hence, for the N-year ahead future expected inflation rate, we compute $\pi_t^{(N)}$ from (using Feynman-Kac)

$$\begin{aligned} \partial \pi_t^{(N)}/\partial N &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial \pi_t^{(N)}/\partial i_t)\,\mathrm{d}t \\ &\quad + (\partial \pi_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + (\partial \pi_t^{(N)}/\partial s_t)((1-s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t \end{aligned}$$

together with the known solution (15) and by imposing the boundary condition $\pi_t^{(0)} = \pi_t$. Similar to the term structure of interest rates, the solution to the PDE then implies the N-years ahead inflation expectations for a given state variable as

$$\pi_t^{(N)} = \pi^{(N)}(i_t, a_t, s_t). \tag{21}$$

We show how to derive the model-implied inflation expectations in Section A.5. Because the model time unit is years, the N-year ahead inflation forecast $\pi_t^{(N)}$ refers to the empirical NY1Y measure. As a simple approximation, we may define the weighted sum of N-year ahead inflation forecast for the successive k years $\pi_t^{(N,k)}$ as

$$\pi_t^{(N,k)} \approx (1/k) \ln\Big(\sum_{i=N}^{k} \big(1 + \pi_t^{(i)}\big)\Big). \tag{22}$$

which shed some light on the effects of model-implied inflation expectations.

2.6 Monetary and/or fiscal policy and transitional dynamics

Defining monetary policy shocks as changes in monetary policy with no exogenous changes in surplus (cf. Cochrane, 2018), we can answer the question of how maturity matters in the model for the transition of unexpected (zero-probability) shocks. Similarly, we consider

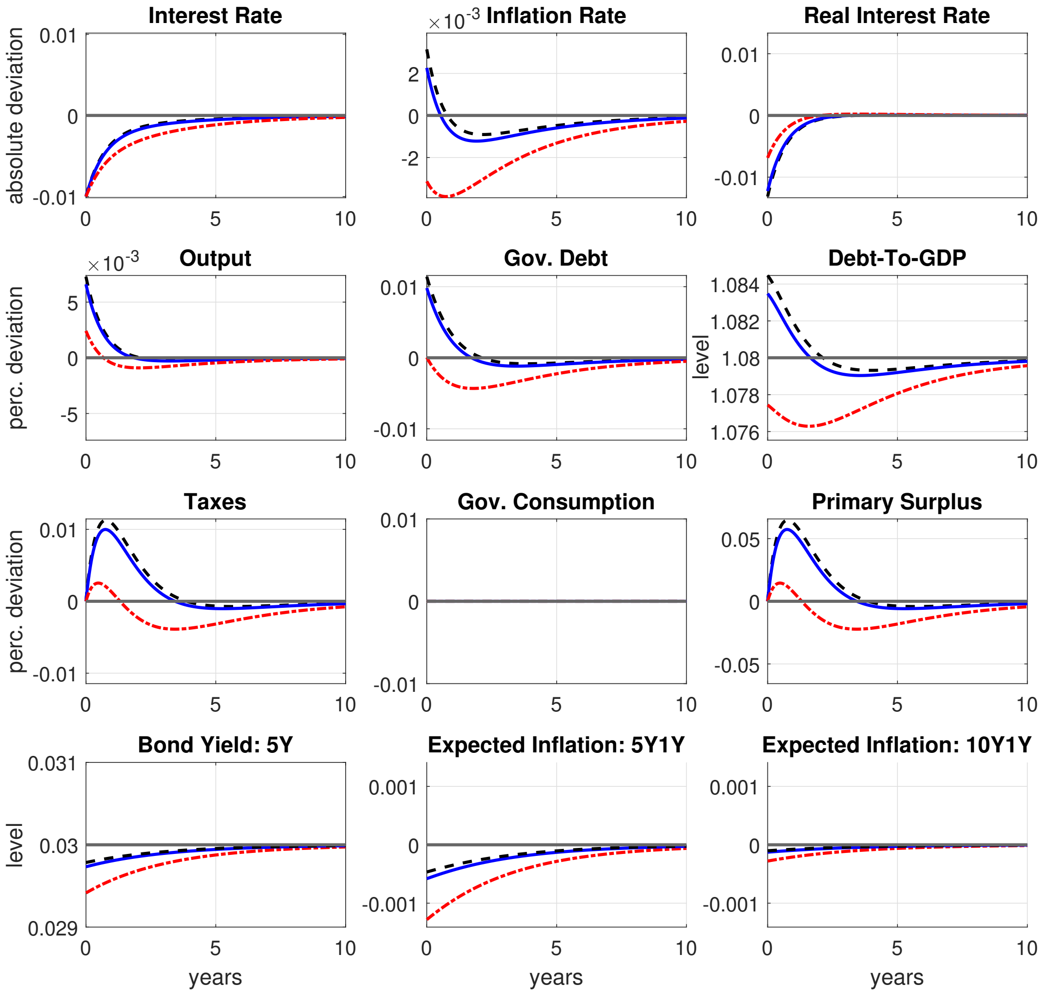

Figure 3: Transitory monetary policy shock for the parametrization in Table 1. Decrease in nominal interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

unexpected changes in fiscal policy without changing the nominal interest rate.

2.6.1 Transitory shocks

Monetary policy. Consider an accommodative transitory monetary policy shock of 100 basis points (bp), i.e., the policy rate i_t decreases by 1 percentage point. An unexpected decrease in nominal interest rates i_t initially has expansionary effects on output because the real interest rate decreases (cf. Figure 3). This effect is larger the longer the average maturity of government debt (i.e., 'stepping on a rake effect of inflation' for perpetuities). Here, the maturity structure matters because the monetary policy shock decreases the real interest rate even more for long-term bonds (black dashed) than with only short-term debt (red dotted). Because with short-term debt the direct FTPL effect is missing, the real debt does not respond immediately and we are left with the indirect FTPL effect,

Table 2: Inflation decomposition (19) for the monetary policy shock in Figure 3.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_v \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-rv} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	-0.29	-1.14	0.29	1.14
Average	-0.48	-1.25	0.21	0.98
Short-Term	-1.62	-1.91	-0.29	0

Table 3: Inflation decomposition (19) for the tax cut in Figure 4.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	1.14	0.66	-1.14	-0.66
Average	1.34	0.78	-1.05	-0.49
Short-Term	1.89	1.10	-0.79	0

which unambiguously lowers inflation on impact (cf. Cochrane, 2018).

Fiscal authorities react following the specified fiscal rule and because of the increased output this results into higher surpluses from increased tax receipts. A higher surplus then lowers inflation (cf. Figure 1), which again slowly increases the real interest rate. Note how the sign of the initial response of inflation depends on the current maturity structure, which is has been shown by the policy functions before. Future expected inflation turns negative for all maturities (as shown in Figure 3). In fact, the net present value of future expected inflation is negative, ranging from -0.29 to -1.62 percentage points depending on the maturity of government debt (cf. Table 2). Here, the negative effect on inflation can be attributed to either fiscal policy only (black dashed), where future monetary policy is soaked up by higher bond prices, or a mix of monetary and fiscal policy, which is buffered by *lower* net present value of future tax receipts (solid blue and red dotted).

The direct FTPL effect increases the value of government debt as bonds appreciate, even more than output in the case of perpetuities such that lower interest rates initially lead to a higher debt-to-GDP ratio. With short-term debt only, essentially the picture is completely reversed: government debt initially is reduced because of higher output, which leads to a substantially lower debt-to-GDP ratio – maturity matters qualitatively.

Fiscal policy. Along the same line, defining a fiscal policy shock as an unexpected

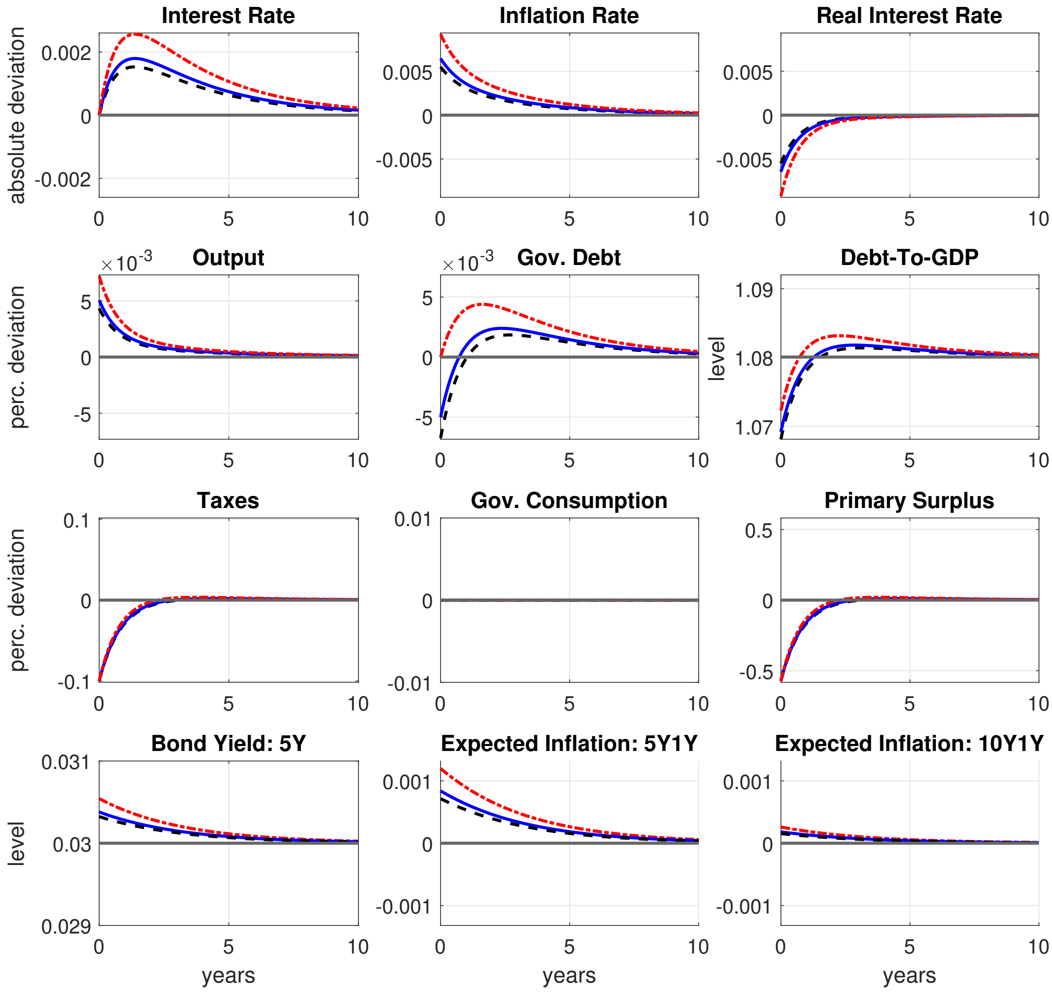

Figure 4: Transitory fiscal policy shock for the parametrization in Table 1. Decrease in taxes (surplus) by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

change in surplus (or its components), with no change in monetary policy, we can answer the question of how maturity matters in the model for the transition of zero-probability fiscal policy shocks. Consider now an expansive fiscal policy shock (cut T_t by 10 percent). An unexpected cut in taxes (decreases surplus s_t) has expansionary effects on output and thus unambiguously increases inflation and leads to higher inflation expectations, such that for a given short-term rate, the real interest rate is lower (cf. Figure 4).

Hence, expansive fiscal policy (decreased surplus) leads to more inflation and lowers the real interest rate (cf. Figure 1). This in turn causes the monetary authority, following a Taylor rule, to slightly increase nominal rates, whereas the effects on 5-year bond yields are being driven mainly by higher inflation expectations. Lower primary surpluses, after an initial devaluation of real government debt, lead to further accumulation of debt and are accompanied by higher future inflation. In fact, the net present value of future inflation is positive, ranging from 1.14 to 1.89 percentage points depending on the maturity structure of government debt (cf. Table 3). Again, the total effect on inflation can be attributed to either fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or a mix of monetary and fiscal policy (blue solid and red dotted).

After all, the maturity structure of government debt matters most for the direct FTPL effect, which dampens the effects on interest rates, inflation, and output dynamics. The unexpected fiscal policy shock devalues nominal government bonds and output increases, which initially leads even to a lower debt-to-GDP ratio. Here, the initial deficits are not repaid by subsequent surpluses or output growth but at the cost of higher inflation and more nominal debt, which is inflated away by subsequent inflation with no permanent changes in the real value of debt. In fact, this is like a 'partial default' on nominal debt. For the case of short-term debt only, higher output leads, after a decrease in the debt-to-GDP ratio, to more debt accumulation because the direct effect is missing. All deficits are being inflated away. What may seem like a deal, "the trick is to convince people that sinning once [...] is a once-and-never-again devaluation or at best a rare state-contingent default, not the beginning of a bad habit." (Cochrane, 2023, p.245).

Finally, consider a fiscal policy shock of issuing new debt (increase n_t by 3 percent). We are particularly interested in such shock because debt levels increased dramatically during the COVID-19 pandemic. Suppose that this increase in government debt leaves long-run surpluses and the average maturity unchanged (i.e., a transitory shock). Then, the newly issued debt creates unexpected inflation and higher inflation expectations because the debt is not fully paid back by subsequent surpluses and has expansionary effects through a lower real interest rate (cf. Figure 5). Depending on the average maturity, a significant portion of the newly issued debt is inflated away. In fact, the net present value of future expected inflation ranges from 2.08 to 3.49 percentage points depending on the maturity structure of government debt (cf. Table 4). It is most striking for long-term debt, where only one third of the initial debt shock is repaid by higher surpluses. Only the remainder creates unexpected future inflation, and future monetary policy is soaked up by lower bond prices (black dashed). Hence, the total effect on inflation and on inflation expectations is smallest due to direct FTPL effect. For the case of short-term debt, the direct effect does not offset monetary policy, which results in the highest net present value of future inflation, even higher than the initial debt shock (red dotted).

Again, the maturity structure of government debt matters because the direct FTPL effect devaluates long-term debt such that the initial increase in real debt (market value) is lower and the effect on inflation is largest for short-term debt. The indirect effect rises inflation and inflation expectations, which forces the monetary authority to increase nominal interest rates. Though the higher output also leads to higher tax receipts and implies a larger future primary surplus, the stimulus only partially accounts for the increased liabilities. Eventually, the unexpected increase in real debt (face value) is inflated away

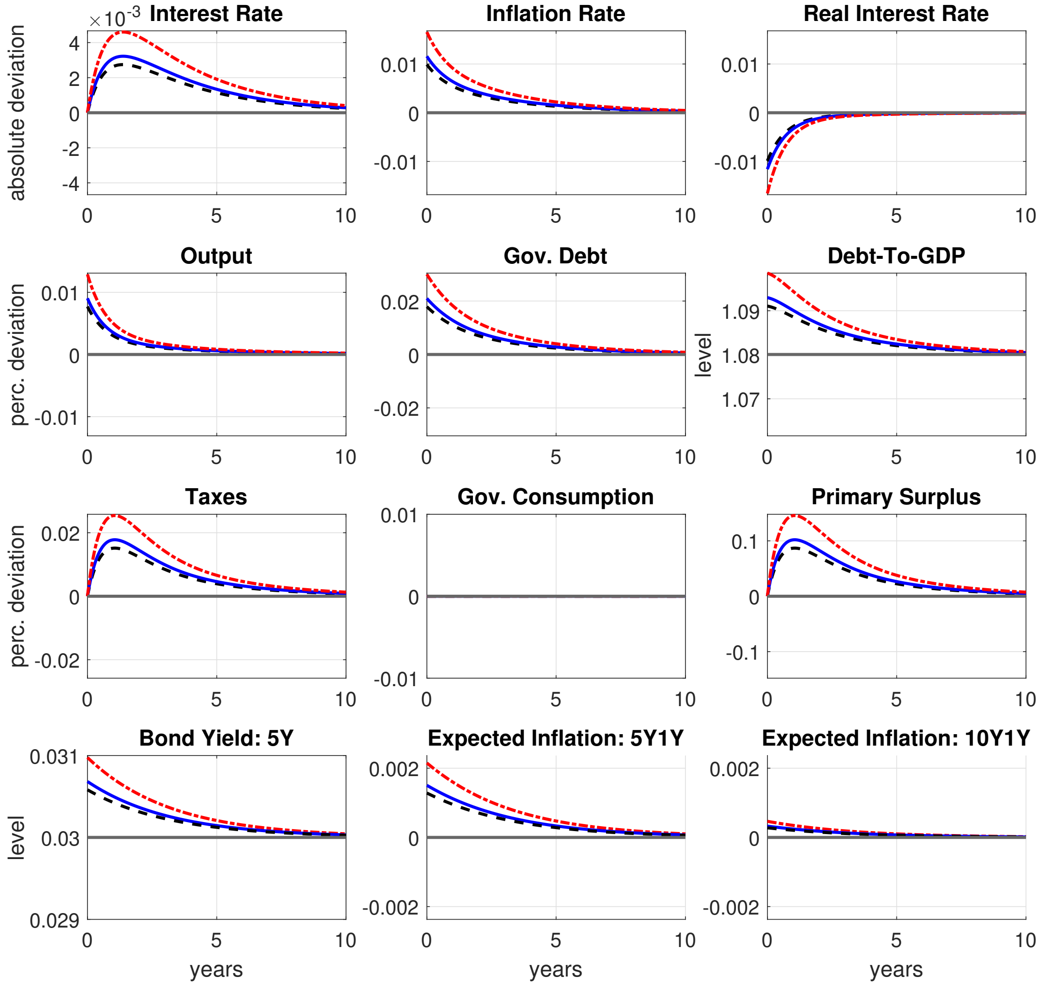

Figure 5: Transitory fiscal policy shock for the parametrization in Table 1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

by unexpected future inflation and is only partially repaid by higher surpluses. However, the number of outstanding bonds increases permanently to $n_{ss} = v_{ss}e^{\int_t^\infty \pi_u \mathrm{d}u}$.

2.6.2 Permanent shocks

Monetary policy. Consider a monetary policy shock decreasing the inflation target by 50 bp, or equivalently, the policy interest rate target (isomorphic to the inflation target), $i_{ss}^{new} = \rho + \pi_{ss}^{new}$, decreases by 0.5 percentage points. Suppose that the change is fully credible and fully observed, i.e., does not require learning and filtering. A lower inflation target or long-term interest rate then has an expansionary effect on output because it creates inflation and the real interest rate decreases (cf. Figure 6, solid blue).

In all models, independent of the maturity structure, the permanent shock clearly shows up in 10-years ahead inflation expectations and bond yields (cf. Figure 6). Because

Table 4: Inflation decomposition (19) for the tax cut in Figure 5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	2.08	1.21	0.92	-1.21	3.00
Average	2.44	1.42	1.08	-0.90	3.00
Short-Term	3.49	2.03	1.54	0	3.00

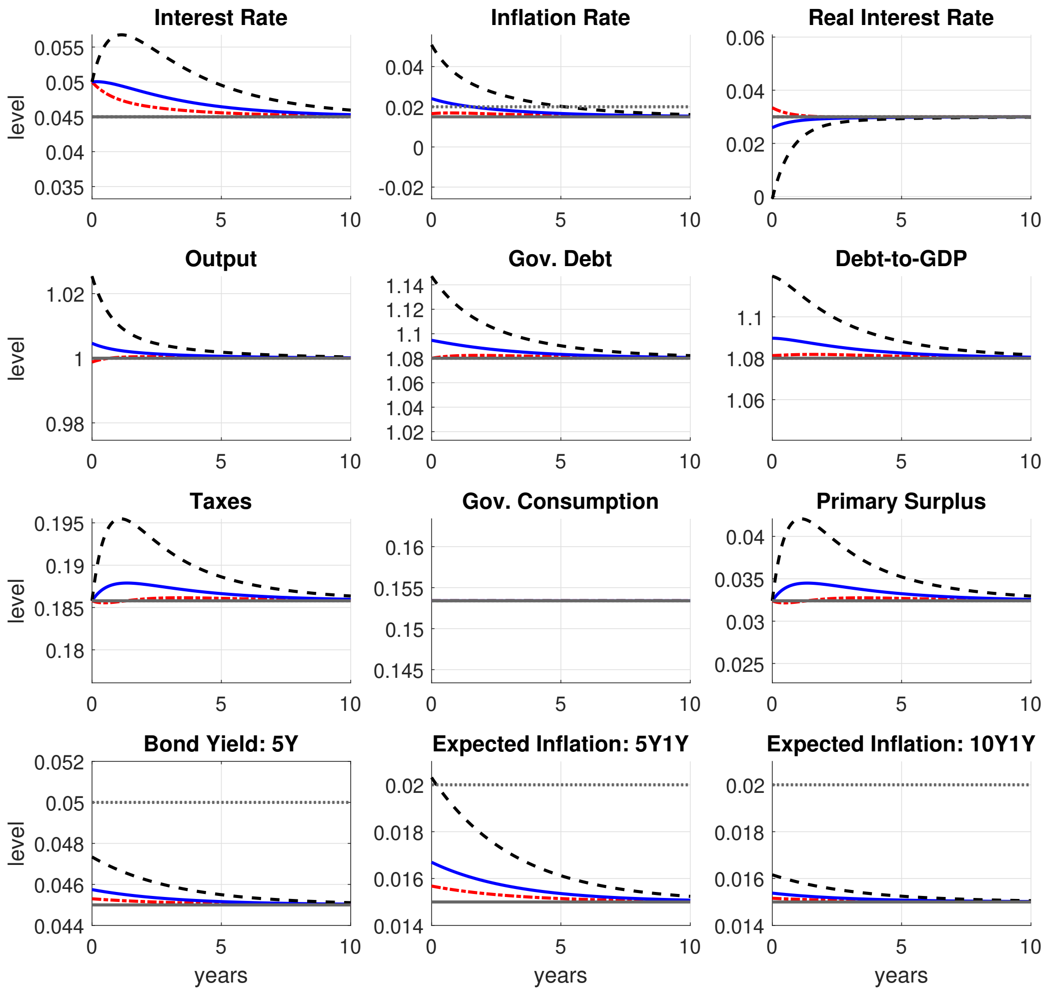

Figure 6: Permanent monetary policy shock for the parametrization in Table 1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

of transitional dynamics, the shock is not as clearly visible at the shorter end. While the permanent shock increases the 1-year bond yields up to 50 bp, it decreases 10-year bond

Table 5: Inflation decomposition (19) for the permanent target rate shock in Figure 6.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^{b,new} - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.02	5.16	3.34	−4.91	11.11
Average	2.39	1.88	0.85	−1.24	2.60
Short-Term	0.81	0.96	0.15	0	0

yields by 50 bp. However, in the model with short-term debt only, the permanent lower inflation target would be even contractionary because lower current inflation increases the real interest rate. Most importantly, the maturity structure matters because the permanent shock even *increases* current expected inflation and decreases the real interest rate (solid blue and black dashed). Because the direct FTPL effect is missing in the model with short-term debt, real debt does not respond immediately and we are left with the indirect effect. However, the direct FTPL effect substantially increases the real value of existing long-term government debt such that the lower inflation target leads to a higher debt-to-GDP ratio, higher tax receipts and thus higher primary surpluses. With short-term debt, the picture is different: initially lower tax revenues (primary surpluses) and lower output with only small changes in real debt lead to negligible effects on the debt-to-GDP ratio. Hence, the maturity effect is more pronounced the longer the average maturity of government debt (cf. Table 5). In fact, current inflation increases by more than 300 bp in the model with perpetuities with net present value of future inflation of about 8 percent.

How can we understand this dramatic response for inflation dynamics in the model with long-term debt? The simple answer is that the response of inflation is due to a price or valuation effect on existing longer-term bonds, which pay a nominal coupon $\chi + \delta$. Hence, a monetary policy shock in form of a lower inflation target $\pi_t^* \equiv \pi_{ss}^{new} = \pi_{ss} - 0.005$ translates into a higher price $p_{ss}^{b,new}$, and with no change in fiscal surplus results into a lower steady-state value of sovereign debt v_{ss}^{new}. From the decomposition (19), we get

$$\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u = \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,$$

with a new

$$p_{ss}^{b,new} = \frac{\chi + \delta}{i_{ss}^{new} + \delta}, \quad \text{and} \quad v_{ss}^{new} = a_{ss}/p_{ss}^{b,new}. \tag{23}$$

A permanent monetary policy shock leads to an implicit debt shock $v_t/v_{ss}^{new} - 1$ because

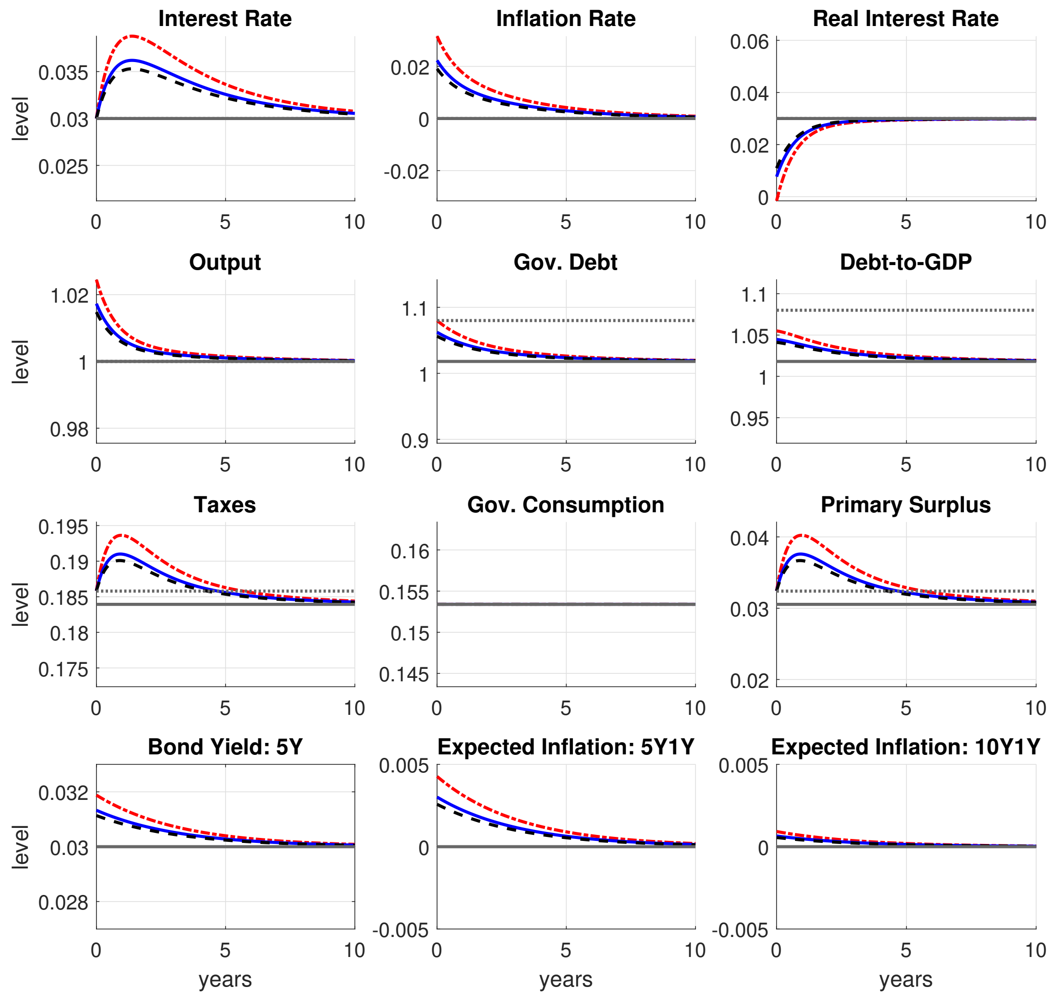

Figure 7: Permanent fiscal policy shock for the parametrization in Table 1. Decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

of existing longer-term bonds do no longer sell at *par* in steady state. Relative to the lower new steady state level of government debt v_{ss}^{new} (face value), the current debt level v_t now is above its steady-state level – because debt v_t does not jump, which thus can be interpreted as an 'implicit' expansionary fiscal policy shock (compare to Figure 5). This shock is inflationary and the shock size depends on the maturity structure (cf. Table 5). The effect is already sizable with average maturity (by 2.60 percent), and is substantial with longer maturities (up to more than 11 percent for perpetuities). Both direct effects give the change in the market value of government debt. Even the price effect is negative of about -1.24 percent (p_0^b increases, but p_{ss}^b increases even more), the implied debt shock by 2.60 percent leads to an increase of the market value by 1.36 percent.

Fiscal policy. Consider an expansive fiscal policy shock (cut T_t^* by 1 percent).[16] An

[16] A contemporaneous fiscal policy shock $T_t = 0.99T_{t-}$ with permanent effects, $T_{ss}^{new} = 0.99T_{ss}$ has a

Table 6: Inflation decomposition (19) for the permanent tax shock in Figure 7.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new}\mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	4.02	2.34	2.07	−2.34	6.08
Average	4.70	2.74	2.38	−1.74	6.08
Short-Term	6.66	3.88	3.31	0	6.08

unexpected change in future tax revenues (decreases surplus s_t^*) has expansionary effects on output today and thus increases current inflation and inflation expectations, which lowers real interest rates (cf. Figure 7). The stimulus to output quickly leads to higher tax revenues in the short run at the cost of higher inflation. In this case, the net present value of future inflation is positive, ranging from 4.02 to 6.66 percentage points depending on the maturity structure of government debt (cf. Table 6). Our fiscal policy shock leads to an instantaneous devaluation of long-term debt and dampens the effects on interest rate and inflation dynamics. Again, the total effect on inflation can be attributed either to fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or to a mix of monetary and fiscal policy (solid blue and red dotted). The indirect effect unambiguously rises inflation (decreases the real interest rate), which causes the monetary authority to adjust the nominal interest rates. Temporarily higher tax revenues (higher surplus) then lead to a further decline of government debt, and the debt-to-GDP ratio converges to its lower steady-state level.

In particular, the change in the target tax receipts, $T_t^* \equiv T_{ss}^{new} = 0.99T_{ss}$ translates into changes in the steady-state values of primary surplus, $s_{ss}^{new} = T_{ss}^{new} - g_{ss}$, and sovereign debt, $a_{ss}^{new} = s_{ss}^{new}/\rho$ or $v_{ss}^{new} = a_{ss}^{new}/p_{ss}^b$, and from the identity (19),

$$\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u = \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}^{new}\mathrm{d}u + p_t^b/p_{ss}^b - 1 + v_t/v_{ss}^{new} - 1$$

such that our permanent fiscal policy shock leads to an 'implicit' debt shock $v_t/v_{ss}^{new} - 1$, because debt v_t does not jump and is 'too high' relative to the new and lower v_{ss}^{new}. With similar arguments – because of government debt being backed by taxes – any (austerity) measure leading to *higher* tax receipts, T^*, and/or *lower* government consumption, g_t^*, such that the steady-state primary surplus, $s_t^* = T_t^* - g_t^*$, increases, eventually need to *increase* the long-run real bond supply and the real value of government debt, i.e., increase

similar decomposition and would create more unexpected inflation.

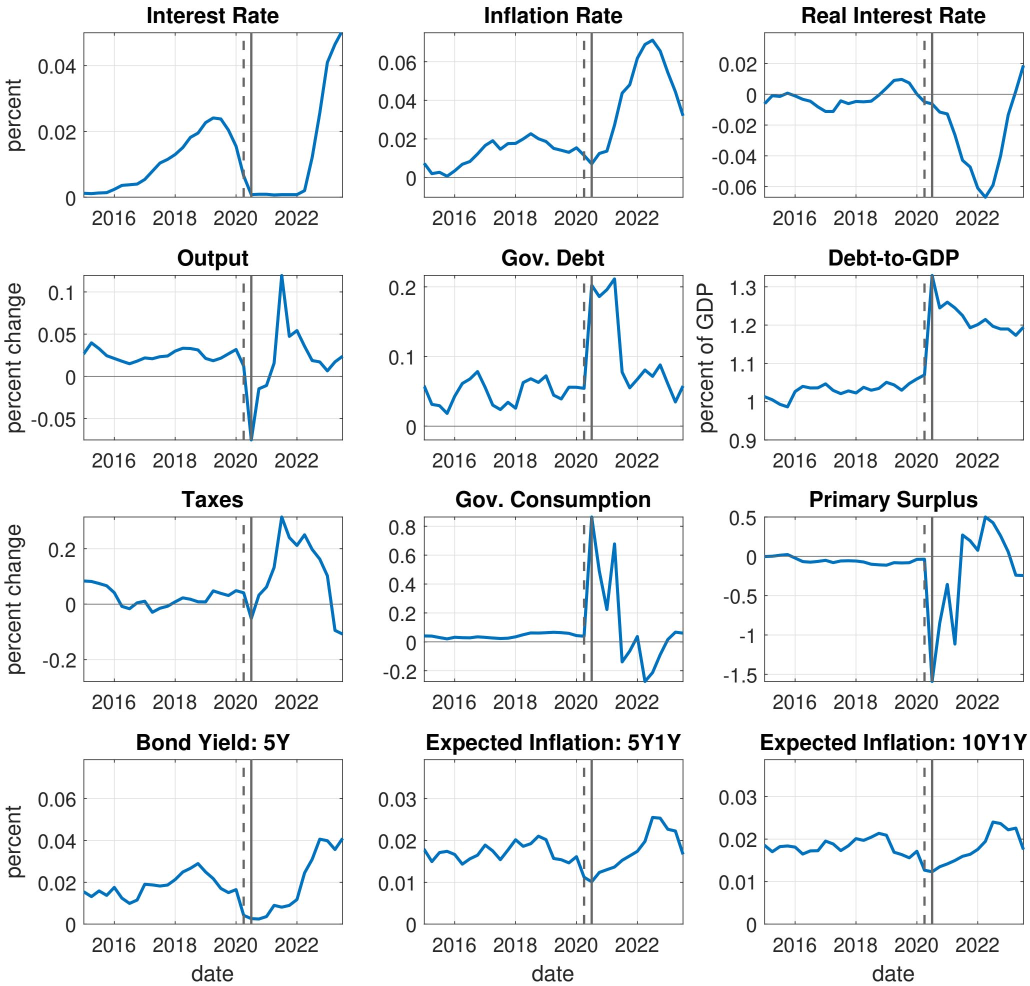

Figure 8: Time series plots of the US data from 2015Q1 through 2023Q2 from the Federal Reserve Bank of St. Louis Economic Dataset (FRED) as defined in Table A.5. Dashed line: 2020Q1, Solid line: 2020Q2 (CARES Act signed into law on March 27, 2020).

the market and face value debt-to-GDP ratio.

3 The CARES Act

The Coronavirus Aid, Relief, and Economic Security (CARES) Act is an extensive US economic stimulus package that was signed into law on March 27, 2020, in response to the COVID-19 pandemic. Its central objective was a direct and fast assistance for the real economy in order to keep it afloat and as functioning as possible. The unprecedented volume of the act is estimated to be more than $2 trillion (10% of US GDP). Because CARES includes loan guarantees, the Congressional Budget Office (CBO) projects smaller budgetary effects. Still, the CBO estimates that CARES will add $1.7 trillion to deficits between 2020 and 2030, but most effects take place until 2022.

Table 7: Upper Part: Predictions of the CARES Act by the Congressional Budget Office (CBO), the Joint Committee on Taxation, and estimated effect on debt-to-GDP ratio from Kaplan, Moll, and Violante (2020). Lower part: Translation to the theoretical model.

CARES Act: Empirical Figures

		Billions of Dollars	as % of GDP	as % of Outlays (receipts) 2019
A	Increased Mandatory Outlays	988	4.6%	22.2%
B	Increased Discretionary Outlays	326	1.5%	7.3%
C	Decreased Revenues	408	1.9%	11.8%
D	Increase of debt-to-GDP Ratio: 12% (cf. Kaplan, Moll, and Violante, 2020)			

CARES Act: Model Variables

			abs. Change	as % of GDP	as % of Steady State Value
A + **B**	$\equiv$	Shock g_t	0.061	6.1%	39.8%
C	$\equiv$	Shock T_t	-0.019	-1.9%	-10.2%
D	$\equiv$	Shock v_t/y_t by 12% (either temporary and/or permanent)			

Sources: Congressional Budget Office (2020).

3.1 Taking the model to the data

Figure 8 shows empirical time-series data for our key variables in the NK-FTPL model in the years around the CARES Act, which was signed into law on March 2, 2020.[17] In what follows, we assume that the stimulus package arrives as a (structural) zero-probability shock. Due to the emergency character of the program in response to the COVID-19 pandemic, we consider zero-probability shocks as a reasonable assumption. Table 7 shows the CBO's breakdown of the \$1.7 trillion into outlays and receipts. The size of the budgetary relevant part of the CARES Act exceeds more than 8% of US GDP. We use the Kaplan, Moll, and Violante (2020) estimate, that increased outlays (6.1% of GDP) together with decreased revenues (1.9% of GDP) are going to increase the debt-to-GDP ratio by about 12%. In the lower part of Table 7 we translate the CARES Act into zero-probability shocks in the FTPL-NK model. We attribute the increase in outlays to an unexpected rise in g_t by 6.1% of GDP, which corresponds to an increase in government consumption by about 39.8%. In the empirical data, the rise in mandatory and discretionary outlays amounts to 29.5% of total expenditures in 2019. Analogously we attribute the decrease in revenues as

[17]Data retrieved from FRED, Federal Reserve Bank of St. Louis (cf. detailed description in Table A.5).

a revenue shock by 1.9% of GDP, which translates to a decrease in tax receipts by 10.2%. Empirically, the initial decrease in revenues looks smaller, but was about the same order of magnitude (11.8% of total receipts in 2019). As a consequence of the large increase in current government expenditures and a simultaneous drop in current tax receipts in 2020Q2, the primary deficits increased by roughly 150%.[18] Both tax receipts and primary surpluses followed S-shape dynamics. In order to finance the CARES Act, the US had to take on new debt, which is reflected in the upward jump in total public debt by 20% in 2020Q2. Despite the strong increase in the deficit and the cut in the funds rate, GDP decreased substantially in the second quarter of 2020.

Let us quantify the large-scale fiscal stimulus package, henceforth CARES Act shock (cf. Table 7). We translate the empirical figures to the model variables as zero-probability shocks to government consumption, $\mathrm{d}g_t$ ($\mathbf{A} + \mathbf{B} = 6.1\%$ of GDP), to tax receipts, $\mathrm{d}T_t$ ($\mathbf{C} = -1.9\%$ of GDP), such that the primary surplus turns into a large deficit of roughly $s_0 \approx -8\%$ of GDP, or $\mathrm{d}s_t \approx -250\%$, and newly issued debt, $\mathrm{d}v_t/v_{ss} = 0.12$, which implies $\mathrm{d}(v_t/y_t) \approx 12\%$ ($\mathbf{D} = 12\%$ of GDP), together with an accommodative monetary policy shock of 150 bp.[19] Note that an increase in debt also increases output on impact, so we define $\mathbf{D}$ as newly issued debt, i.e., a shock to outstanding debt v_t (or v_t/y_{ss}) because we normalize $y_{ss} = 1$, rather than a shock directly to the debt-to-GDP ratio. We match the 12% increase for (short-term) debt-to-GDP ratio for $\mathrm{d}v_t = 0.1296$, which in fact is a lower bound given the huge increase in the observed debt-to-GDP ratio.

In order to model a realistic scenario for the US economy in 2020Q1, we employ our benchmark parametrization in Table 1, except for two modifications regarding the surplus dynamics and the level of the natural rate. First, we want to model a persistent shock to government consumption with own dynamics (and thus surplus dynamics). Hence, we set $\rho_g \equiv 1$ and assume a counter-cyclical output response of $\varphi_y \equiv -s_g$,

$$\mathrm{d}g_t = (-s_g(y_t/y_{ss} - 1) - (g_t - g_t^*))\,\mathrm{d}t, \tag{24}$$

e.g., to model policies like food stamps or unemployment insurance which imply that surplus reacts pro-cyclically (cf. Sims, 2011; Cochrane, 2023). Second, we follow Werning (2012) and consider a shock to the natural rate r_t (preference shock), i.e., the real interest rate that would prevail in the flexible-price outcome, to model that the economy is close to a liquidity trap. Hence, we introduce an autoregressive shock process d_t with $\rho_d > 0$,

[18] In order to keep the data close to the definition in the model, we abstract from interest rate costs. For an alternative definition of primary surpluses including interest rate costs see Cochrane (2023).

[19] We use the notion of zero-probability shocks and initializing the economy at particular state variables as interchangeably. Formally, we refer to zero-probability shocks, for example, $\mathrm{d}g_t \equiv (\cdot)\,\mathrm{d}t + \mathrm{d}\delta_{g_t}$ with $\mathbb{E}_t(\mathrm{d}\delta_{g_t}) = 0$, similar to (7). We refer to the Online appendix for alternative counterfactual scenarios.

which determines the persistence of an exogenous (zero-probability) shock,

$$\mathrm{d}d_t = -\rho_d(d_t - 1)\,\mathrm{d}t \tag{25}$$

such that $r_t = \rho + \rho_d(d_t - 1)$ defines the 'natural rate' of interest (cf. Posch, 2020). We initialize the size of the shock d_0 in order to generate a drop in output in 2020Q1, which implies an initial natural rate $r_0 \approx -0.1$ with persistence $\rho_d = 0.6501$. In the absence of the fiscal package, this would have implied a severe recession (as shown in Figure A.14). Moreover, keep in mind that monetary policy was not silent in response to the global coronavirus pandemic, but responded to the large drop in output and fears of deflationary pressures. In March 2020, the Federal Reserve decreased the federal funds rate in two steps from 1.58% to 0.05%. Because the timing of the rate cuts was about the same time, we model this by an accommodative monetary policy shock of 150 bp.

Given the surge in inflation rates in the aftermath of the COVID-19 pandemic, as shown in Figure 8, a significant body of literature has emerged in recent years. Many different theories try to shed light on understanding their origins and on predicting the future paths of inflation. For example, Harding, Lindé, and Trabandt (2022, 2023) argue that the inflation dynamics can be explained in terms of cost-push or supply shocks together with a quasi-kinked demand function. Closer to our analysis, Bianchi, Faccini, and Melosi (2023) show that unfunded fiscal policy shocks can predict the inflationary effects of another post-COVID stimulus package.[20] On the empirical front, Di Giovanni, Kalemli-Özcan, Silva, and Yildirim (2023) estimate that around two-thirds of US inflation can be attributed to aggregate demand shocks and the other third to supply shocks (based on an disaggregated NK model similar to Baqaee and Farhi, 2022). At least half of the total aggregate demand shocks is attributed to the fiscal stimulus packages.

A key question of the related current debate on the surge in sovereign debt is whether the unprecedented value of newly issued debt increased the long-run debt-to-GDP ratio.[21] The answer to this question is linked and contributes to the discussion on unfunded fiscal policy shocks (cf. Bianchi, Faccini, and Melosi, 2023). By looking at different scenarios, we shed light on the debate of permanent vs. temporary changes in the debt-to-GDP ratio and give more insights into the channels and predictions of the FTPL-NK model in contrast to the simple NK model (cf. Section 3.3).

[20]Bianchi, Faccini, and Melosi (2023) study the American Rescue Plan Act (ARPA), which was signed to law a year after the CARES Act. Though both large-scale fiscal packages were of similar size (ARPA was about $1.9 trillion), we are interested in the effects of the emergency character of CARES due to its unprecedented immediate upward jump in public debt in recent history (cf. Figure A.9).

[21]Note that both the debt-to-GDP ratio measured in face value v_{ss}^{new}/y_{ss} and market value a_{ss}^{new}/y_{ss} would be affected at the same order of magnitude as long as $p_{ss}^b = 1$ (see Equation 23).

3.2 The economic effects of the CARES Act shock

Let us now quantify the economic effects of the CARES Act shock in the NK-FTPL model. Figure 9 shows the effects of the CARES Act shock together with contemporaneous shocks to the natural rate $\mathrm{d}r_t = -0.1$, and a monetary policy shock $\mathrm{d}i_t = -0.015$ on our variables of interest for the next 10 years, thereby initializing the US economy to roughly match the empirical figures at 2020Q1 (cf. Figure 8).

Both shocks to the primary surplus ($\mathrm{d}g_t = 0.061$ and $\mathrm{d}T_t = -0.019$), and the shock to the debt-to-GDP ratio are expansionary ($\mathrm{d}v_t = 0.1296$). While the shock to the natural rate depresses output and thus inflation (see counterfactuals), which partly was offset by the accommodative monetary policy, the overall effects of the contemporaneous shocks were inflationary on impact about 1 bp, building up to about 4 bp, and increases the 5-year ahead inflation expectations about 2 bp, and long-term expectations about 0.5 bp, such that for a given short-term rate, the real interest rate dropped to -2.5 bp.

As a positive result, it can be noted that fiscal and monetary policy helped to avoid a deep recession in 2020Q2. In our counterfactual simulations, most of the output loss due to the natural rate shock was effectively offset by the two policies: Without the fiscal emergency package the initial response of output in the FTPL-NK model would have been more than -12.5% along with large deflationary effects. Most efficient was the CARES Act shock, effectively reducing the drop in output to about -3%. Without accommodative monetary policy, the output loss would have been slightly larger around -4%.

The dire effects on inflation are most evident by looking at the inflation decomposition (cf. Table 8). If the CARES Act shock was purely an emergency package not backed by future fiscal adjustments ('unfunded fiscal shock' in Bianchi, Faccini, and Melosi, 2023), the FTPL-NK model predicts substantial and persistent inflationary effects. Our results show that for the 12% increase in outstanding debt, together with the 3% decrease of future surplus (fiscal policy), and with the resulting 11% higher future interest rates (monetary policy), which for the prevailing average maturity of debt is partly offset -5% by the direct FTPL effect (asset pricing), the total effect on inflation is substantial and generates about 21% future inflation. Modifying our initial assumption of a constant average maturity, the economic effects of the CARES Act shock on future inflation would have been between 17% (long-term debt) and 26.5% (short-term debt) as shown in Table 8 and illustrated in Figure 9. Note that with long-term debt only, the higher future interest rates are fully anticipated by a devaluation of nominal government bonds. With short-term debt only, this asset pricing effect is not present, and therefore the inflationary effects of lower future surpluses and higher government debt are highest.

Note that the debt-to-GDP ratio in the FTPL-NK model is shown as a_t/y_t. Because the total public debt reported by the government typically refers to the face value of outstanding obligations, the corresponding figure is the dotted red line in the IRFs for the

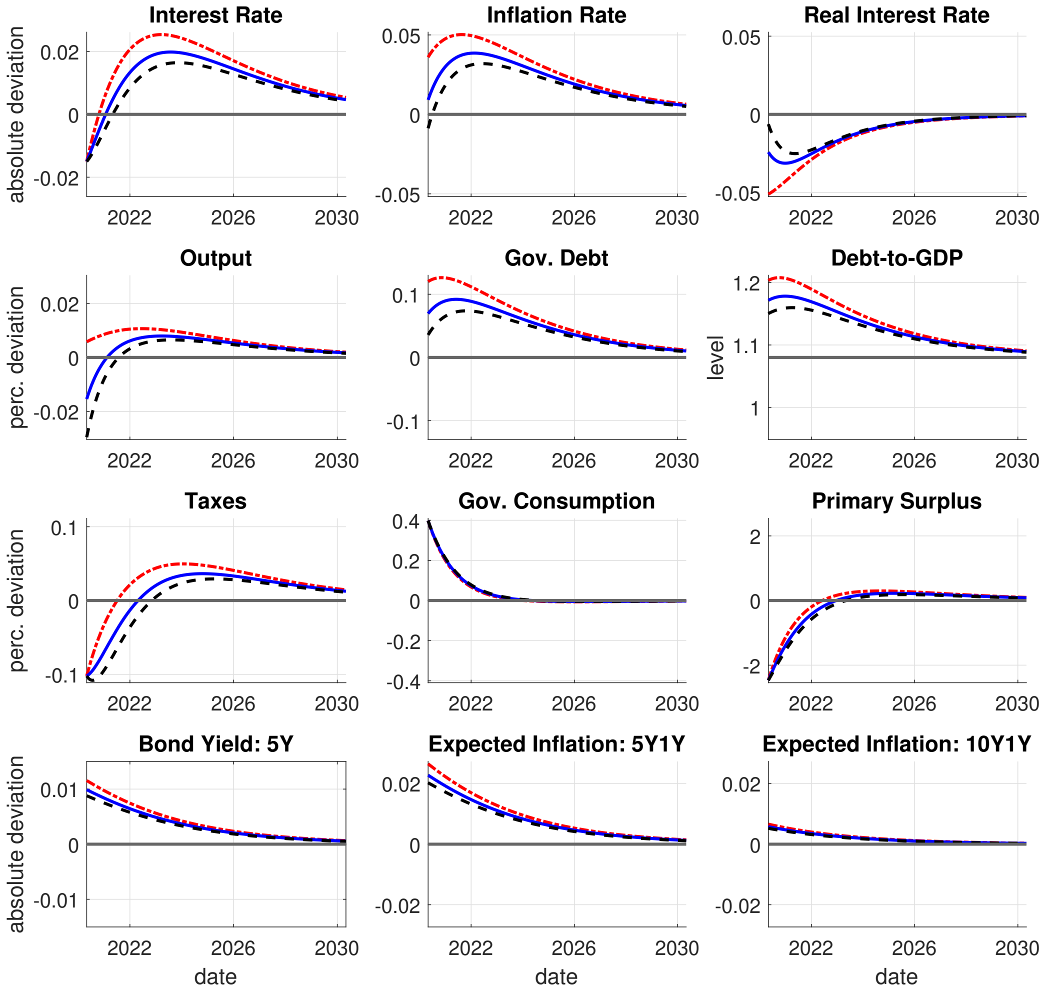

Figure 9: CARES Act shock and monetary policy shock, parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, increase in debt by 12 percent and interest rate cut by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

government debt and the debt-to-GDP ratio (short-term debt) together with the average maturity (solid blue) in the remaining plots of Figure 9.

Hence, the CARES Act shock (decreased surplus and increased debt) unambiguously led to higher bond yields, inflation, and inflation expectations, which eventually forced the monetary authority to increase nominal rates. Nevertheless, real interest rates will persistently remain below their equilibrium value, even after the dissipation of the natural rate shock. Through the lens of fiscal theory, this unprecedented large-scale fiscal program, which is not followed by sufficiently higher subsequent surpluses, was expected to spur inflation and inflation expectations. Even though primary surplus turns slightly positive, the newly issued debt will be fully deflated away by higher future inflation. The impact on future inflation would have been even more pronounced with a shorter average maturity

Table 8: Inflation decomposition (19) for the CARES Act shock in Figure 9.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	16.99	8.44	−4.99	−8.44	12.00
Average	20.82	10.67	−3.21	−5.06	12.00
Short-Term	26.55	14.01	−0.54	0	12.00

of government debt. The primary lesson from this experiment is that, in avoiding a deep recession, there is a trade-off: a surge in inflation due to the significant, unexpected build-up of government debt and the expansionary surpluses in both outlays and revenues.

3.3 A permanent shock scenario?

A key question is whether the observed large-scale fiscal operations are funded or backed by subsequent higher future surpluses. In what follows we address the case if the CARES Act shock was (partially) backed by future fiscal adjustments. What do responses to inflation and inflation expectations tell us about agents' beliefs at the core of the fiscal theory? From the fiscal theory point of view, this question translates to whether the increase in debt is followed by a subsequent higher future surplus. While the higher future surplus does not necessarily have to be permanent, possibly the cleanest analysis is to ask whether the CARES Act shock is considered permanent or transitory. In what follows, we consider a scenario in which the CARES Act shock does have a permanent component causing a higher long-run debt-to-GDP ratio. Because the debt level is ultimately determined by future surpluses, a higher debt level $a_{ss}^{new} \equiv s_{ss}^{new}/\rho$ requires higher surpluses s_{ss}^{new}. Put differently, the *real* debt level or debt-to-GDP ratio increases permanently only if agents presume that the newly issued debt is financed by either higher revenues and/or lower government consumption (i.e., backed by higher future surpluses).

Suppose the economy is at the steady-state at $t = 0$. In what follows, we define unfunded fiscal shocks based on the identity in (19) as follows: A funded fiscal policy shock to s_t demands $\int_0^\infty s_u/a_{ss}\,\mathrm{d}u = 1 - v_0/v_{ss}$. Any fiscal policy shock $\int_0^\infty s_u/a_{ss}\,\mathrm{d}u < 1 - v_0/v_{ss}$ would be partially unfunded. Similarly, as long as $v_0/v_{ss} - 1 > \int_0^\infty s_u/a_{ss}\,\mathrm{d}u$, the newly issued debt is (partially) unfunded. Based on this definition, similar to Bianchi, Faccini, and Melosi (2023), funded fiscal shocks are irrelevant for inflation, while unfunded fiscal shocks lead to an increase in inflation accommodated by the monetary authority. Note that our definition is consistent as long as the fiscal rule $f(s_t, y_t, a_t)$ is unchanged. Of course, we may think of scenarios where the surplus rule is changed without changing the long-run debt-to-GDP ratio such that a fiscal policy shock is funded.

Consider now a situation in which a fraction α of the newly issued debt is funded by subsequent higher revenues, so that $v_{ss}^{new} = v_{ss} + \alpha(v_0 - v_{ss})$. Then α is interpreted as the fraction of the newly issued debt $v_0 - v_{ss} = \mathbf{D}v_{ss}$ backed by higher future surpluses. If the observed shock to debt v_t (face value) was permanent, i.e., the newly issued debt was backed by higher future surpluses, we set $\alpha = 1$. If a fraction α of the newly issued debt $\alpha\mathbf{D}$ is backed by higher future surpluses, we may restrict $\alpha \geq 0$. The case of $\alpha = 1$ shows that from the fiscal theory point of view, an initial shock to v_t does not lead to an unexpected 'debt shock'. In fact, the effective 'debt shock' size in our inflation decomposition (19) is $(1 + \mathbf{D})/(1 + \alpha\mathbf{D}) - 1 \geq 0$. Any value of $\alpha > 1$ implies that the long-run increase in outstanding debt-to-GDP ratio would be higher than the initial shock to v_t.

For illustration, suppose that half (or all) of the newly issued debt is permanent, i.e., backed by subsequent higher future surpluses, $\alpha = 0.5$ (or $\alpha = 1$), which for $\mathbf{D} = 0.12$ implies a 'debt shock' of 5.66% (or 0%). In our simulations we find that output decreases by about 3% (or 5%) and the initial impact on inflation would be even negative in those scenarios. Nevertheless, the CARES Act shock would have caused about 15.5% (or 10.5%) future inflation. Similarly, under what conditions for α would the CARES Act shock be considered a funded fiscal policy shock? In fact, to have a funded fiscal policy shock, we would need α to be about 2.25, or put differently a long-run debt-to-GDP ratio of $v_{ss}^{new}/y_{ss} = 1.33$ (i.e., a 25% higher debt-to-GDP ratio).[22] In this situation, both the debt shock and the surplus shocks would be funded, albeit at the expense of a significant economic downturn and deflation, comparable in magnitude to the counterfactual scenario. Comparing the transitory shock to the permanent scenarios, we may conclude that only the CARES Act shock in which the newly issued debt is *not* sufficiently backed by higher future surpluses leads to a surge in future expected inflation similar to the observed response. Our results confirm Bianchi, Faccini, and Melosi (2023), who attribute the economic rebound at the end of 2020 to the CARES Act to combat the consequences of the pandemic crisis, and that the package was partially unfunded.

4 Further discussion

Similar to Sims (2011), Leeper and Leith (2016), and Cochrane (2018), our benchmark parametrization in Table 1 with policy functions in Figures 1 and 2 suggests that sovereign debt with average maturities $1/\delta > 0$ is crucial for obtaining the traditional negative relationship between (current) inflation and the interest rate in the FTPL-NK model. It

[22]Note that the results are shown in the Appendix, for $\alpha = 0.5$ in Figure A.15 and Table A.11, and for $\alpha = 1$ in Figure A.16 and Table A.12, respectively. The counterfactual analysis of no CARES Act shock is shown in Figure A.14 and Table A.10. The funded shock scenario is shown in Figure (A.17 and Table A.13.

should be clarified, however, that long-term debt is useful but neither a necessary nor a sufficient condition. Cochrane (2023) shows that a contractionary monetary policy shock can initially decrease the inflation rate even in the presence of short-term debt when we allow for a direct inflation response in the fiscal policy rule. While this specification might be controversial within empirically estimated fiscal policy rules, the consequences are intriguing and point toward the need to intensify research on fiscal policy rules.

We may replicate Cochrane (2023) for the parametrization in Table 1, but extending either fiscal policy rule by allowing for an explicit inflation response, e.g., replacing (9) by

$$\mathrm{d}T_t = \rho_\tau(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) + \tau_\pi(\pi_t - \pi_t^*) - (T_t - T_{ss}))\mathrm{d}t \qquad (26)$$

which is yet another specification of $f(s_t, y_t, a_t)$ in the dynamics of primary surplus (5) as long as for $x_i \neq 0$. Figure 10 shows the corresponding policy functions for $\tau_\pi \equiv 1$. In fact, a negative slope $\bar{\pi}_i$ is obtained not only for longer-term debt but also for short-term debt. Otherwise, an inflation response $\tau_\pi = 1$ does not qualitatively change the policy functions. Liemen (2022) shows how to obtain the negative inflation response with short-term debt in a FTPL-NK model with capital. In either way, the average maturity plays a role, and the introduction of longer-term bonds shapes model dynamics, as discussed in the previous sections. In other words, the maturity structure matters for macro dynamics.

Somewhat different to the CARES Act shock, fiscal policy does not inflate away debt but largely consists of borrowing and credibly promising future surpluses to repay debt (see Cochrane, 2023, p.12). Hence, a today's surplus decline must turn around and rise later on: a particular function $f(s_t, y_t, a_t)$ to which Cochrane refers an "S-shaped" surplus response. As discussed in Section 3.3, the degree to which debt is backed by higher future surpluses determines the degree to which the net present value of primary surpluses dampen or magnify the present value of future inflation. Clearly, in order to pay back the newly issued debt the primary surplus has to follow an S-shape. If a fiscal shock creates stimulus to output through unexpected inflation, it typically creates higher tax revenues and larger primary surpluses following the shock (cf. Figure **??**). Such 'built-in' S-shaped dynamics are not sufficient, however, because they simply reflect that fiscal policy was *not* fully funded by subsequent higher future surpluses. As long as agents 'believe' that a fiscal policy is not fully backed by higher future surpluses, a policy shock will create unexpected inflation (compare Table A.15 to Tables A.3 and A.4). Alternatively, Cochrane (2023) introduces a latent state variable to replicate a 'typical' fiscal policy. However, what what matters is only the change in the net present value which creates unexpected inflation (cf. Figure A.3 for different scenarios).[23]

A more subtle issue is the assumption of perfect foresight. Thus, the absence of risk implies that there is no term premium and/or default risk premium. In particular, our

[23]See also Cochrane (2022b) for a simple discrete-time version with partially-repaid debt.

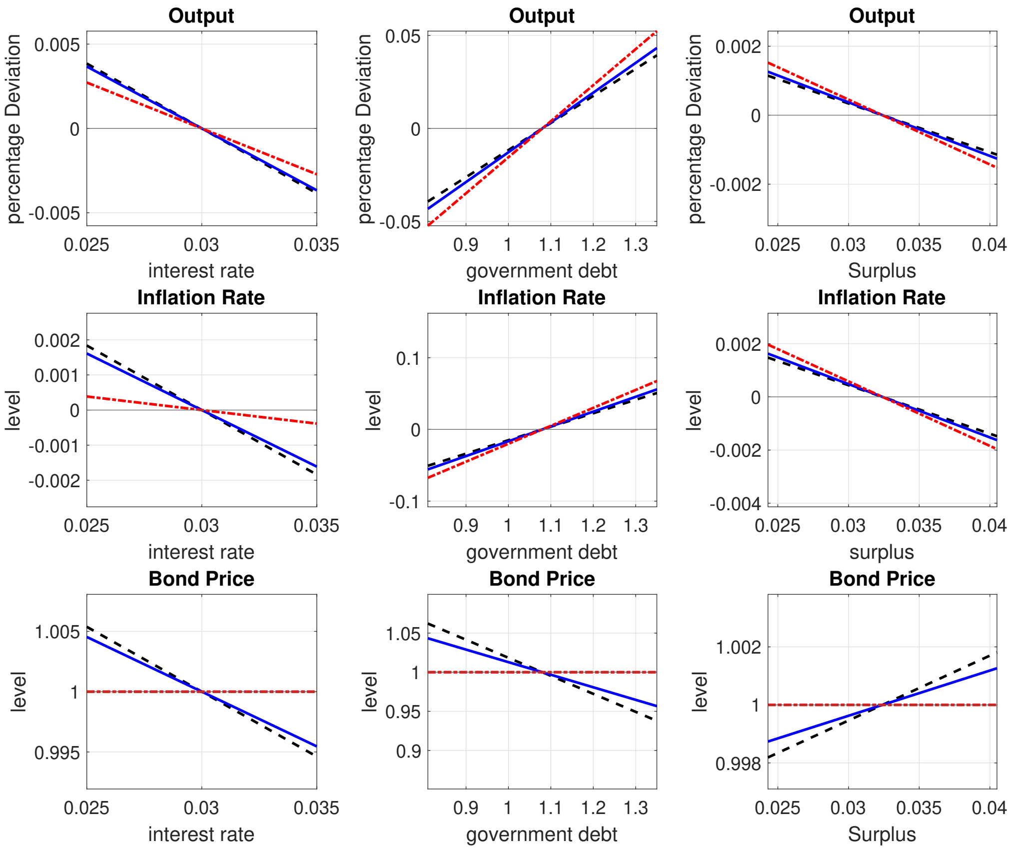

Figure 10: Policy functions for the parametrization in Table 1 together with an explicit inflation response in (26) in terms of v_t. Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

analysis neglects a potential feedback of the fiscal stance on risk premia. Though it goes beyond the scope of the present analysis, this limits the insights for the term structure and inflation expectation analysis (cf. Posch, 2020, for the effects of risk in the NK model). In crisis periods, governments can only 'devalue' via inflation rather than default explicitly. Because sovereign bonds are valued by the present value formula, changes of default risk due to fiscal shocks may have substantial effects on the price of existing bonds.

5 Conclusion

We revisit the fiscal theory and extend the simple NK model with a fiscal block in order to analyze the role of the maturity structure of sovereign debt on interest rates and inflation dynamics. Our results suggest that the average maturity of existing debt has a prominent role for the propagation of transitory and permanent policy shocks in the FTPL-NK model. We show how the effects translate to the term structure of interest rate and to model-implied inflation expectations. Our finding justifies a critical assessment of neglecting the direct FTPL effect in the traditional NK framework. Through the lens of the fiscal theory, we decompose the present value of future inflation into indirect effects (changes in future monetary policy and fiscal policy) and a direct FTPL effect, which basically is an asset pricing re-evaluation of existing bonds. In particular, we highlight that sovereign debt, with an empirically plausible average maturity for the US, largely offsets the impact of monetary policy on the present value of future inflation.

Given that high post-COVID inflation rates have their origins in a complex interplay of various economic factors, our starting point is the fact that the standard NK models likely miss to address a key feature of the CARES Act. By introducing the fiscal theory to the simple NK model, a fiscal stimulus shock does not only have a demand but also a government debt or asset pricing component and depends on the maturity structure. Thus, our analysis suggests that FTPL is a reasonable starting point to understand and explain a great proportion of the empirically observed rise in the inflation rates.

Though a profound analysis, which requires estimating the structural parameters and potentially latent state variables, is beyond the scope of the paper, the experiment mimics a low interest rates environment, a situation which seems more plausible for the US at the outset of the great pandemic. It shows that fiscal theory identifies the large-scale fiscal packages as the source of the recent surge in inflation.

Our application simulates the CARES Act of 2020, which we translate to shocks to the primary surplus of about 8 percent of GDP and to the debt (face value) by 12 percent. Without a credible future (S-shaped) policy change, the FTPL-NK model predicts a surge in inflation, which amounts to an increase of the net present value of future inflation about the same size as the increase of newly issued debt. We show how this dramatic inflation response not only depends on the average maturity of existing bonds, but also primarily on the perception of agents whether the large-scale fiscal operations are ultimately backed by a higher future surplus or not. In contrast to the aftermath of the global financial crisis of 2008, where the inflation response was not as strong or inflation even declined, the recent surge in inflation and medium-term inflation expectations indicates that the newly issued debt is not considered as being backed by subsequent higher surpluses.

We believe that this paper is a promising starting point for using the fiscal theory in more elaborate models, including regime-switching, nonlinearities, and stochastic shocks. First, our results for the term structure of interest rates and inflation expectations would be much more informative. Our setup is a natural starting point and benchmark for models with term premia (cf. Posch, 2020), convenience yield, or default risk. Second, more research is needed for the surplus dynamics, e.g., estimating parameters of the fiscal policy rule (cf. Kliem, Kriwoluzky, and Sarferaz, 2016). Third, it seems important to study the effects of maturity in the medium-size NK models including regime switches (see Bianchi and Melosi, 2019), financial frictions (cf. Brunnermeier and Sannikov, 2014), and productive capital (cf. Brunnermeier, Merkel, and Sannikov, 2021; Liemen, 2022), and to study the effects and transmission in models with heterogeneous agents (cf. Kaplan, Moll, and Violante, 2018; Bayer, Born, and Luetticke, 2021). This opens the path toward a more profound fiscal policy evaluation and to address questions of fiscal limits and sovereign defaults (fiscal sustainability).

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A Appendix

A.1 Technical details FTPL model

In this paper, we use a linear version of the micro-founded NK model (cf. Posch, 2020). The basic structure of the model is as follows. A representative household consumes, saves, and supplies labor. The final output is assembled by a final good producer, who uses as inputs a continuum of intermediate goods manufactured by monopolistic competitors. The intermediate good producers rent labor to manufacture their good and face the constraint that they can only adjust the price following Calvo's pricing rule (Calvo, 1983). Finally, there is a monetary authority that fixes the short-term nominal interest rate through open market operations following a Taylor rule and a detailed government sector with a fiscal authority that issues debt, taxes, and consumes following fiscal policy rules.

A.1.1 Households

Let the reward function of the households be given as

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left\{ \log c_t - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right\} dt, \qquad \psi > 0, \tag{A.1}$$

where ρ denotes the subjective rate of time preference, ϑ is the inverse of the Frisch labor supply elasticity, and ψ scales the disutility from working by supplying labor in terms of hours l_t (we use ψ to normalize $l_{ss} = 1$). Let n_t denote the number of shares of government bonds; assuming that each bond has a nominal value of one unit, whereas p_t^b is the equilibrium price of bonds. Suppose the household earns a disposable income of

$$\delta^c n_t + p_t w_t l_t - p_t T_t + p_t \digamma_t$$

where δ^c are coupon payments, p_t is the price level (or price of the consumption good), w_t is the real wage, T_t are lump-sum taxes, and $\digamma_t$ are the profits of the firms in the economy. Hence, the household's budget constraint reads

$$\mathrm{d}n_t = \left(\left(\delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t \right) / p_t^b - \delta n_t \right) \mathrm{d}t, \tag{A.2}$$

in which p_t^b denotes the bond price. Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ.

The first-order condition for households to maximize (A.1) subject to (A.2) is

$$\psi l_t^\vartheta c_t = mc_t, \tag{A.3}$$

which is the standard static optimality condition between labor and consumption. Hence, for the given preferences (A.1), the Euler equation for consumption reads (cf. Posch, 2020)

$$\mathrm{d}c_t = (i_t - \pi_t - \rho)c_t \, \mathrm{d}t, \tag{A.4}$$

or the linearized version

$$\mathrm{d}c_t \approx (i_t - \rho - \pi_t)c_{ss} \, \mathrm{d}t, \tag{A.5}$$

with π_t being determined in general equilibrium.

A.1.2 The final good producer

There is one final good, produced using intermediate goods with

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.6}$$

where ε is the elasticity of substitution.

Final good producers are perfectly competitive and maximize profits subject to the production function (A.6), taking as given all intermediate goods prices p_{it} and the final good price p_t. Hence, the input demand functions associated with this problem are:

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t \qquad \forall i,$$

and

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

is the (aggregate) price level.

A.1.3 Intermediate good producers

Each intermediate firm produces differentiated goods out of labor using:

$$y_{it} = l_{it}, \tag{A.8}$$

where l_{it} is the amount of the labor input rented by the firm. Therefore, the marginal cost of the intermediate good producer is the same across firms:

$$mc_t = w_t. \tag{A.9}$$

The monopolistic firms engage in price setting à la Calvo, which then gives rise to the NK Phillips curve (see, e.g., Leith and von Thadden, 2008; Posch, 2020)

$$\mathrm{d}(\pi_t - \pi_{ss}) \approx (\rho(\pi_t - \pi_{ss}) - \kappa_0(mc_t/mc_{ss} - 1)) \, \mathrm{d}t. \tag{A.10}$$

Note that from (A.3) $\psi y_t^\vartheta c_t = mc_t$ such that a linearized version is

$$mc_t/mc_{ss} - 1 \approx (c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1).$$

Moreover, for the parametrization in Table 1, we have that $g_t \equiv g_{ss}$ and thus

$$\begin{aligned} \mathrm{d}(\pi_t - \pi_{ss}) &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((y_t/y_{ss} - 1)y_{ss}/c_{ss} + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &\equiv (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t \end{aligned} \tag{A.11}$$

as in (2), where $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$ is the output gap and $\kappa \equiv \kappa_0(1 + \vartheta(1 - s_g))$ captures 'price stickiness'. Our definition of the output gap is to formulate the benchmark model as close as possible to the one used in the literature, where typically $s_g \equiv 0$.

Note that with this definition of the output gap, we obtain (1) from (A.5) as

$$\begin{aligned} \mathrm{d}(y_t - g_{ss}) &= (i_t - \rho - \pi_t)(y_{ss} - g_{ss})\,\mathrm{d}t \\ &= (i_t - \rho - \pi_t)(1 - s_g)y_{ss}\,\mathrm{d}t \end{aligned}$$

after inserting our definition $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$.

For the parametrization in Table D.1 in the online appendix, with variable government consumption,

$$\begin{aligned} mc_t/mc_{ss} - 1 &= (1 + \vartheta(1 - s_g))(y_t/y_{ss} - 1)/(1 - s_g) - (g_t/g_{ss} - 1)s_g/(1 - s_g) \\ &= (1 + \vartheta(1 - s_g))x_t - (g_t/g_{ss} - 1)s_g/(1 - s_g) \end{aligned}$$

and thus the Phillips curve in the generalized version obeys

$$\mathrm{d}(\pi_t - \pi_{ss}) = (\rho(\pi_t - \pi_{ss}) - \kappa x_t + \kappa_0 s_g/(1 - s_g)(g_t/g_{ss} - 1))\,\mathrm{d}t. \tag{A.12}$$

A.1.4 Government

We assume that the monetary authority sets the nominal interest rate i_t of short-term bonds through open market operations according to either the feedback model,

$$i_t - i_t^* = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1), \quad \phi_\pi > 0,\ \phi_y \geq 0, \tag{A.13a}$$

or the partial adjustment model (cf. Posch, 2020):

$$\mathrm{d}i_t = \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, \quad \theta > 0, \tag{A.13b}$$

which includes a response to inflation and output, and a desire to smooth interest rates.

The fiscal authority trades a nominal non-contingent bond. Let n_t be the outstanding stock of nominal government bonds, i.e., the total nominal value of outstanding debt (alternative assets are priced using arbitrage arguments but are in net zero supply). The government incurs a real primary surplus $s_t \equiv T_t - g_t$ where revenues T_t and expenditure g_t rules are given in (9) and (10). Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ. Hence, the government faces the constraint that the newly issued debt must cover amortization plus coupon payments of outstanding debt, net of the primary surplus such that the nominal value of outstanding debt follows

$$\mathrm{d}n_t = \big(((\delta + \chi)n_t - p_t s_t)\,/p_t^b - \delta n_t\big)\ \mathrm{d}t, \tag{A.14}$$

where p_t^b is the bond price.

A.1.5 Aggregation

First, market clearing demands:

$$y_t = c_t + g_t = c_t + T_t - s_t, \tag{A.15}$$

and suppose aggregate output is produced according to (e.g., in the linearized model)

$$y_t = l_t$$

in which we normalized to $y_{ss} = l_{ss} \equiv 1$ in the benchmark parametrization, and the income is generated through

$$y_t = w_t l_t + \digamma_t.$$

All outstanding sovereign debt is owned by households, so (A.2) and (A.14) yield

$$(\delta + \chi)n_t - p_t s_t = \delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t.$$

Recall that the real value of sovereign debt is defined as in (6), $a_t = n_t p_t^b / p_t$. In equilibrium,

$$i_t \mathrm{d}t = ((\chi + \delta)/p_t^b - \delta)\mathrm{d}t + (1/p_t^b)\,\mathrm{d}p_t^b$$

such that the bond price follows (7). We define the inflation rate π_t such that

$$\mathrm{d}p_t = \pi_t p_t \mathrm{d}t \tag{A.16}$$

and the (realized) rate of inflation is locally non-stochastic.

Hence, the budget constraint of the fiscal authority (6) can be written as

$$
\begin{aligned}
\mathrm{d}a_t &= (p_t^b\,\mathrm{d}n_t + n_t\,\mathrm{d}p_t^b - n_t p_t^b/p_t\,\mathrm{d}p_t)/p_t \\
&= ((\delta+\chi)n_t/p_t - s_t)\ \mathrm{d}t - \delta n_t p_t^b/p_t + n_t\,\mathrm{d}p_t^b/p_t - n_t p_t^b/p_t(1/p_t)\,\mathrm{d}p_t \\
&= ((\delta+\chi)n_t/p_t - s_t)\ \mathrm{d}t - \delta a_t\,\mathrm{d}t + a_t i_t\,\mathrm{d}t - ((\delta+\chi)n_t/p_t - \delta a_t)\,\mathrm{d}t - a_t\pi_t\,\mathrm{d}t,
\end{aligned}
$$

which is equation (4) in the fiscal block.

Similarly, the household's budget constraint (A.2) can be written as

$$
\begin{aligned}
\mathrm{d}a_t &= (p_t^b\,\mathrm{d}n_t + n_t\,\mathrm{d}p_t^b - n_t p_t^b/p_t\,\mathrm{d}p_t)/p_t \\
&= \big((\delta+\chi)a_t/p_t^b - s_t\big)\ \mathrm{d}t - \delta a_t + a_t(1/p_t^b)\,\mathrm{d}p_t^b - a_t\pi_t\,\mathrm{d}t \\
&= \big((\delta+\chi)a_t/p_t^b - s_t\big)\ \mathrm{d}t - \delta a_t + (-((\delta+\chi)/p_t^b - \delta) + i_t)a_t\mathrm{d}t - a_t\pi_t\,\mathrm{d}t \\
&= -s_t\,\mathrm{d}t + i_t a_t\mathrm{d}t - a_t\pi_t\,\mathrm{d}t \\
&= ((i_t - \pi_t)a_t + w_t l_t - c_t - T_t + \digamma_t)\,\mathrm{d}t,
\end{aligned}
$$

which again shows that the household's budget constraint coincides with the government budget constraint. Using (A.2) and (A.14), together with market clearing (A.15), the coupon payments cover payouts and amortization such that $\delta^c \equiv \delta + \chi$.

A.1.6 Steady-state values

From (1), (4), and (7), we obtain $i_{ss} = \rho + \pi_{ss}$, $a_{ss} = s_{ss}/\rho$, and $p_{ss}^b = 1$. In this model

$$
mc_{ss} = w_{ss} = \frac{\varepsilon - 1}{\varepsilon},
$$

where ε is the elasticity of substitution between intermediate goods. Moreover, condition (A.3) implies together with the market clearing condition (A.15) that

$$
\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}.
$$

Observe that $c_{ss} = y_{ss} - g_{ss} = l_{ss} - g_{ss}$, defining $s_g = g_{ss}/y_{ss}$ such that

$$
\psi l_{ss}^{1+\vartheta}(1 - s_g) = w_{ss}.
$$

Hence, we parameterize

$$
\psi \equiv w_{ss} l_{ss}^{-(1+\vartheta)}/(1 - s_g)
$$

to normalize the steady-state output $y_{ss} = l_{ss} = 1$, such that $\digamma_{ss} = 1/\varepsilon$, $c_{ss} = 1 - g_{ss}$, $T_{ss} = s_{ss} + g_{ss}$ (s_{ss} and s_g are calibrated using US targets).

A.2 Reformulation in terms of real debt in face value

Recall equation (A.14)

$$\mathrm{d}n_t = \left(\left((\delta+\chi)n_t - p_t s_t\right)/p_t^b - \delta n_t\right)\,\mathrm{d}t.$$

With the price level following

$$\mathrm{d}p_t = p_t \pi_t \mathrm{d}t.$$

Define,

$$v_t \equiv n_t/p_t \tag{A.17}$$

so that v_t is the value of debt (face value) in real terms. Differentiating,

$$\mathrm{d}v_t = \mathrm{d}\left(\frac{n_t}{p_t}\right) = \frac{\mathrm{d}n_t}{p_t} - \frac{n_t}{p_t}\frac{\mathrm{d}p_t}{p_t}.$$

or

$$\mathrm{d}v_t = \left(\left((\delta+\chi)/p_t^b - \delta - \pi_t\right)v_t - s_t/p_t^b\right)\mathrm{d}t. \tag{A.18}$$

Thus, we can rewrite our baseline model as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(A.19a)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t && \text{(A.19b)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\mathrm{d}t && \text{(A.19c)}\\
\mathrm{d}v_t &= \left(\left((\delta+\chi)/p_t^b - \delta - \pi_t\right)v_t - s_t/p_t^b\right)\mathrm{d}t && \text{(A.19d)}\\
\mathrm{d}s_t &= ((1-s_g)x_t - (s_t - s_t^*))\mathrm{d}t. && \text{(A.19e)}
\end{aligned}$$

Sims (2011) and Cochrane (2018) utilize the real value of debt, a_t, as relevant state variable in their models, which can jump due to changes in the bond price. In contrast to real market value debt a_t, real face value debt, v_t, does not jump. Thus, we can use v_t together with the bond price, p_t^b, to obtain the real debt (market value) as

$$a_t \equiv v_t p_t^b. \tag{A.20}$$

This formulation makes the reasons for jumps in a_t clearer. Furthermore, it simplifies the interpretation of shocks to debt (we can directly shock v_t). However, keep in mind that both model formulations imply the same dynamics and refer to the same model.

A.3 Linearized dynamics

In this paper use the linearized NK model, so we need to linearize the equations (A.4), (4), and (7). Let us summarize the equilibrium dynamics for our parametrization. Alternative equilibrium dynamics are summarized in the online appendix.

A.3.1 Benchmark parametrization (Table 1)

Using $\pi_t^* = \pi_{ss}$, $i_t^* = i_{ss} = \rho + \pi_{ss}$, and $s_t^* = s_{ss}$, together with the parametrization of the benchmark model (cf. Table 1), the linearized equilibrium dynamics can be written as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(A.21)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t && \text{(A.22)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_{ss}) - (i_t - i_{ss}))\mathrm{d}t && \text{(A.23)}\\
\mathrm{d}a_t &= (a_{ss}(i_t - \pi_t - \rho) + \rho(a_t - a_{ss}) - (s_t - s_{ss}))\mathrm{d}t && \text{(A.24)}\\
\mathrm{d}s_t &= ((y_t/y_{ss} - 1) - (s_t - s_{ss}))\,\mathrm{d}t && \text{(A.25)}\\
\mathrm{d}p_t^b &= \big((i_t - i_{ss}) + (\chi + \delta)(p_t^b - 1)\big)\,\mathrm{d}t, && \text{(A.26)}
\end{aligned}$$

where

$$y_t/y_{ss} - 1 = (c_t - c_{ss} + g_t - g_{ss})/y_{ss}$$

such that with $g_t = g_{ss}$ we get $\kappa \equiv (1 + \vartheta(1 - s_g))\kappa_0$, and

$$x_t = (y_t/y_{ss} - 1)/(1 - s_g) = (c_t/c_{ss} - 1)(c_{ss}/y_{ss})/(1 - s_g) = (c_t/c_{ss} - 1),$$

i.e., the consumption Euler equation can be written in terms of the output gap.

A.4 Proof of Proposition 1

Recall that in the model with long-term debt, a proper predetermined state variable (which does not jump) is v_t rather than a_t, hence, we linearize

$$a_t - a_{ss} \;=\; p_{ss}^b(v_t - v_{ss}) + v_{ss}(p_t^b - p_{ss}^b)$$

such that the real value of government debt changes due to two channels

$$\mathrm{d}a_t = p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b. \tag{A.27}$$

The partial derivatives of the policy function $x(i_t, a_t, s_t)$ show the indirect FTPL effect for a given bond price, p_t^b, such that we need to isolate the direct FTPL effect due to the re-evaluation of sovereign debt. Now, evaluating the effect of a change to i_t at some reference point, say $\bar{x}_i = x_i(i_{ss}, a_{ss}, s_{ss})$, the slope of the policy function in terms of a_t would only include the indirect effect, keeping fix the price of government debt. Note that our solution implies both $p^b_t = p^b(i_t, v_t, s_t)$ or $p^b_t = p^b(i_t, a_t, s_t)$ such that

$$\mathrm{d}p^b_t = p^b_i(i_t, v_t, s_t)\,\mathrm{d}i_t + p^b_v(i_t, v_t, s_t)\,\mathrm{d}v_t + p^b_s(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.28}$$

and $\mathrm{d}p^b_t = p^b_i(i_t, a_t, s_t)\,\mathrm{d}i_t + p^b_a(i_t, a_t, s_t)\,\mathrm{d}a_t + p^b_s(i_t, a_t, s_t)\,\mathrm{d}s_t$ and thus using (A.27)

$$\mathrm{d}p^b_t = p^b_i(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}i_t + p^b_a(i_{ss}, a_{ss}, s_{ss})(p^b_{ss}\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p^b_t) + p^b_s(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}s_t$$

or equivalently

$$\begin{aligned}\mathrm{d}p^b_t &= \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}i_t + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}v_t \\ &\quad + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}s_t \end{aligned}\tag{A.29}$$

and by matching coefficients with (A.28)

$$\begin{aligned} p^b_i(i_t, v_t, s_t) &= \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\ p^b_v(i_t, v_t, s_t) &= \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\ p^b_s(i_t, v_t, s_t) &= \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}, \end{aligned}$$

we can conclude that

$$\begin{aligned} \bar{p}^b_i &\equiv p^b_i(i_{ss}, a_{ss}, s_{ss}) = p^b_i(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}^b_a) \\ \bar{p}^b_a &\equiv p^b_a(i_{ss}, a_{ss}, s_{ss}) = \frac{p^b_v(i_{ss}, v_{ss}, s_{ss})}{1 + v_{ss}p^b_n(i_{ss}, v_{ss}, s_{ss})/p^b_{ss}} \\ \bar{p}^b_s &\equiv p^b_s(i_{ss}, a_{ss}, s_{ss}) = p^b_s(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}^b_a). \end{aligned}$$

Similarly, for the inflation rate we can utilize

$$\mathrm{d}\pi_t = \pi_i(i_t, v_t, s_t)\,\mathrm{d}i_t + \pi_n(i_t, v_t, s_t)\,\mathrm{d}n_t + \pi_s(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.30}$$

or, equivalently,

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)\,\mathrm{d}a_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t. \tag{A.31}$$

We substitute equation (A.27)

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)(p^b_{ss}\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p^b_t) + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t$$

or

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)p^b_{ss}\,\mathrm{d}v_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t + v_{ss}\pi_a(i_t, a_t, s_t)\,\mathrm{d}p^b_t.$$

Substitute by equation (A.29)

$$\begin{aligned}
\mathrm{d}\pi_t \;=\; & \left(\pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}i_t \\
& + \left(\pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}v_t \\
& + \left(\pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}s_t
\end{aligned}$$

and matching coefficients with equation (A.30)

$$\begin{aligned}
\pi_i(i_{ss}, v_{ss}, s_{ss}) &= \pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_v(i_{ss}, v_{ss}, s_{ss}) &= \pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_s(i_{ss}, v_{ss}, s_{ss}) &= \pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}.
\end{aligned}$$

Rearranging terms we arrive at

$$\begin{aligned}
\bar{\pi}_i &\equiv \pi_i(i_{ss}, a_{ss}, s_{ss}) = \pi_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{\pi}_a &\equiv \pi_a(i_{ss}, a_{ss}, s_{ss}) = \pi_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{\pi}_s &\equiv \pi_s(i_{ss}, a_{ss}, s_{ss}) = \pi_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a}.
\end{aligned}$$

We proceed analogously for the output gap, $x(i_t, v_t, s_t)$ and $x(i_t, v_t, s_t)$. Except for notation the derivations are identical to the inflation rate. Thus,

$$\begin{aligned}
\bar{x}_i &\equiv x_i(i_{ss}, a_{ss}, s_{ss}) = x_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{x}_a &\equiv x_v(i_{ss}, a_{ss}, s_{ss}) = x_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{x}_s &\equiv x_s(i_{ss}, a_{ss}, s_{ss}) = x_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a},
\end{aligned}$$

which closes the proof (inflation rates and output gap analogously).

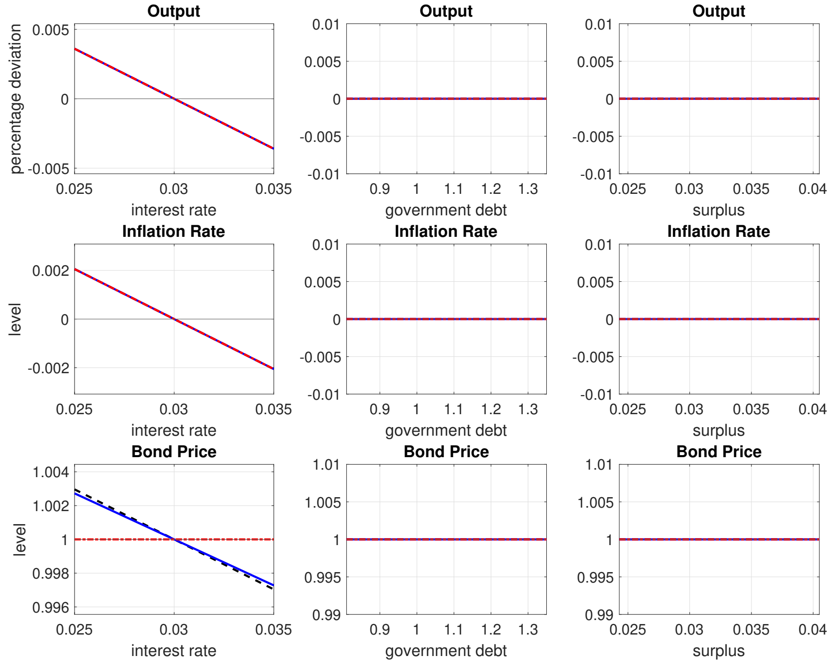

Figure A.1: Policy functions for the parametrization in Table 1 with $\phi_\pi = 1.6$ and $\tau_a = 0.04$ (Active Money/Passive Fiscal Policy), showing the total response in terms of v_t (indirect and direct effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

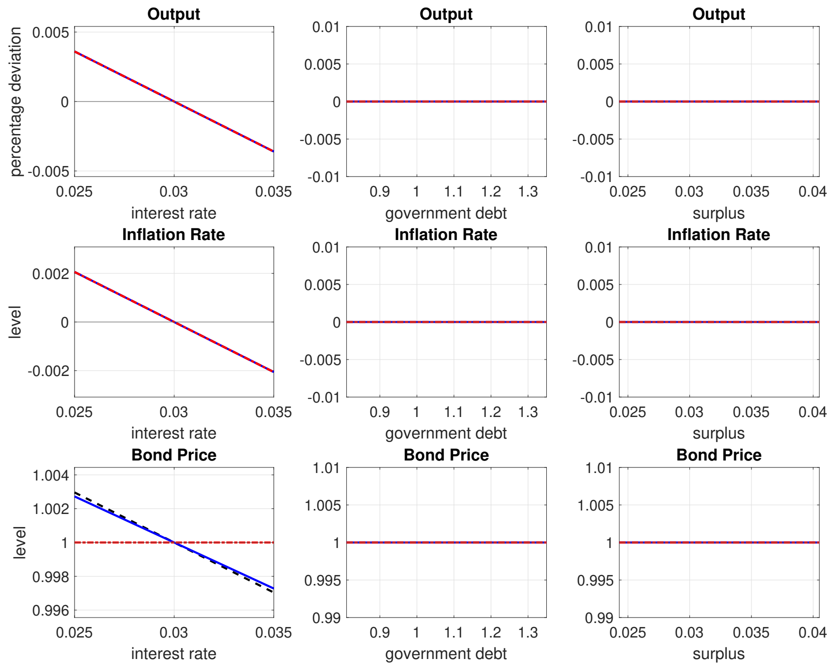

Figure A.2: Policy functions for the parametrization in Table 1 with $\phi_\pi = 1.6$ and $\tau_a = 0.04$ (Active Money/Passive Fiscal Policy), showing the partial response in terms of a_t (indirect effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

A.5 Term Structure of Interest Rates and Inflation

Observe that in equilibrium, the bond price $p_t^{(N)}$ is a function of the state variables, so $p_t^{(N)} = p_t^{(N)}(i_t, a_t, s_t)$, where from (14c), (14d), and (14e) we get

$$\begin{aligned} \mathrm{d}p_t^{(N)} &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t)\,\mathrm{d}t \\ &\quad +(\partial p_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + ((1-s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t \end{aligned}$$

together with the solution (15) and thus the PDE (henceforth *PDE approach*) reads:

$$\begin{aligned} &(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t) + ((1-s_g)x_t - (s_t - s_t^*))(\partial p_t^{(N)}/\partial s_t) \\ &+((i_t - \pi_t)a_t - s_t)(\partial p_t^{(N)}/\partial a_t) = (\partial p_t^{(N)}/\partial N) + i_t p_t^{(N)}. \end{aligned} \tag{A.32}$$

The solution to the pricing equation implies the complete term structure of interest rate for any given interest rate and maturity:

$$y_t^{(N)} \equiv y^{(N)}(i_t, a_t, s_t) = -\log p_t^{(N)}(i_t, a_t, s_t)/N. \tag{A.33}$$

Our strategy is to use collocation and we approximate the function $p_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$, in which v is an n-vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a *linear* interpolation equation,

$$\begin{aligned} &\big((1-s_g)x_t - (s_t - s_t^*))\Phi_4' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\ &\quad +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1' - i_t\Phi\big)v = 0_n, \end{aligned} \tag{A.34}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$ with boundary condition $\Phi(0, i_t, a_t, s_t)v = 1_n$. So we concatenate the two matrices and solve the linear system for the unknown coefficients.

Analogously to the above approach, we compute model-implied inflation expectations. In this case, we approximate the function $\pi_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$. The n-vector v is a vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from the *linear* interpolation equation, e.g.,

$$\begin{aligned} &\big(((1-s_g)x_t - (s_t - s_t^*))\Phi_4' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\ &\quad +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1'\big)v = 0_n, \end{aligned}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$ with the boundary condition $\Phi(0, i_t, a_t, s_t)v = 1_n \cdot \pi_t$.

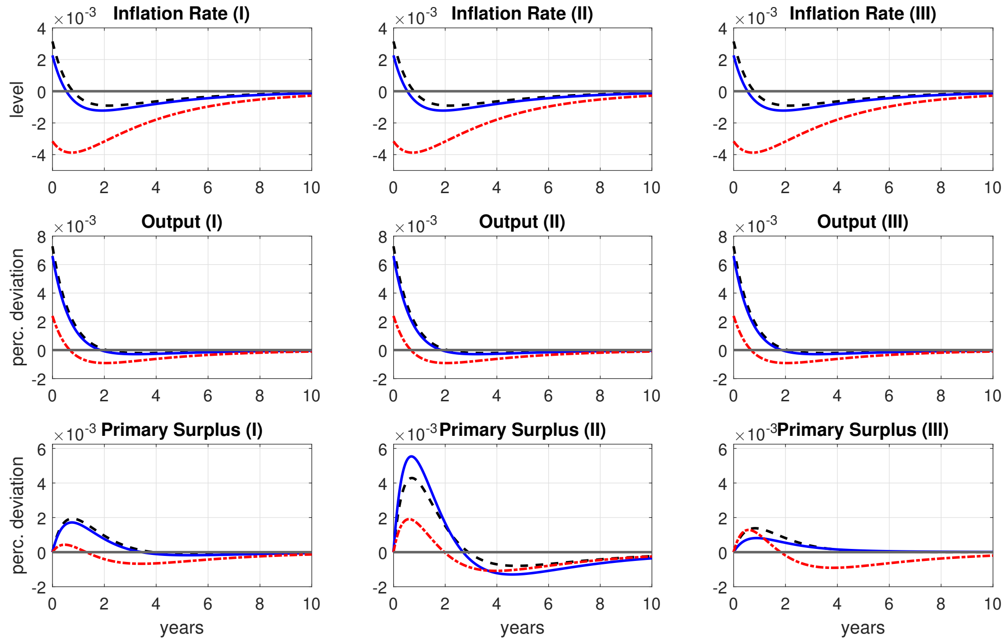

Figure A.3: Transitory monetary policy shock for the parametrization in Table 1 and different surplus dynamics. Decrease nominal interest rate by 1 percentage point. Left-hand panel: Baseline scenario, $\tau_\pi = 0$ and $\tau_y = 1$. Middle panel: $\tau_\pi = 1.02$ and $\tau_y = 3.08$. Right-hand panel: $\tau_\pi = 0.5$ and $\tau_y = -0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

A.6 Figures and Tables

Table A.1: Inflation decomposition (19) for the monetary policy shock in Figure A.3.

Surplus Rule	Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
I	Average	−0.48	−1.25	0.21	0.98
II	Average	−0.48	−1.25	0.21	0.98
III	Average	−0.48	−1.25	0.21	0.98

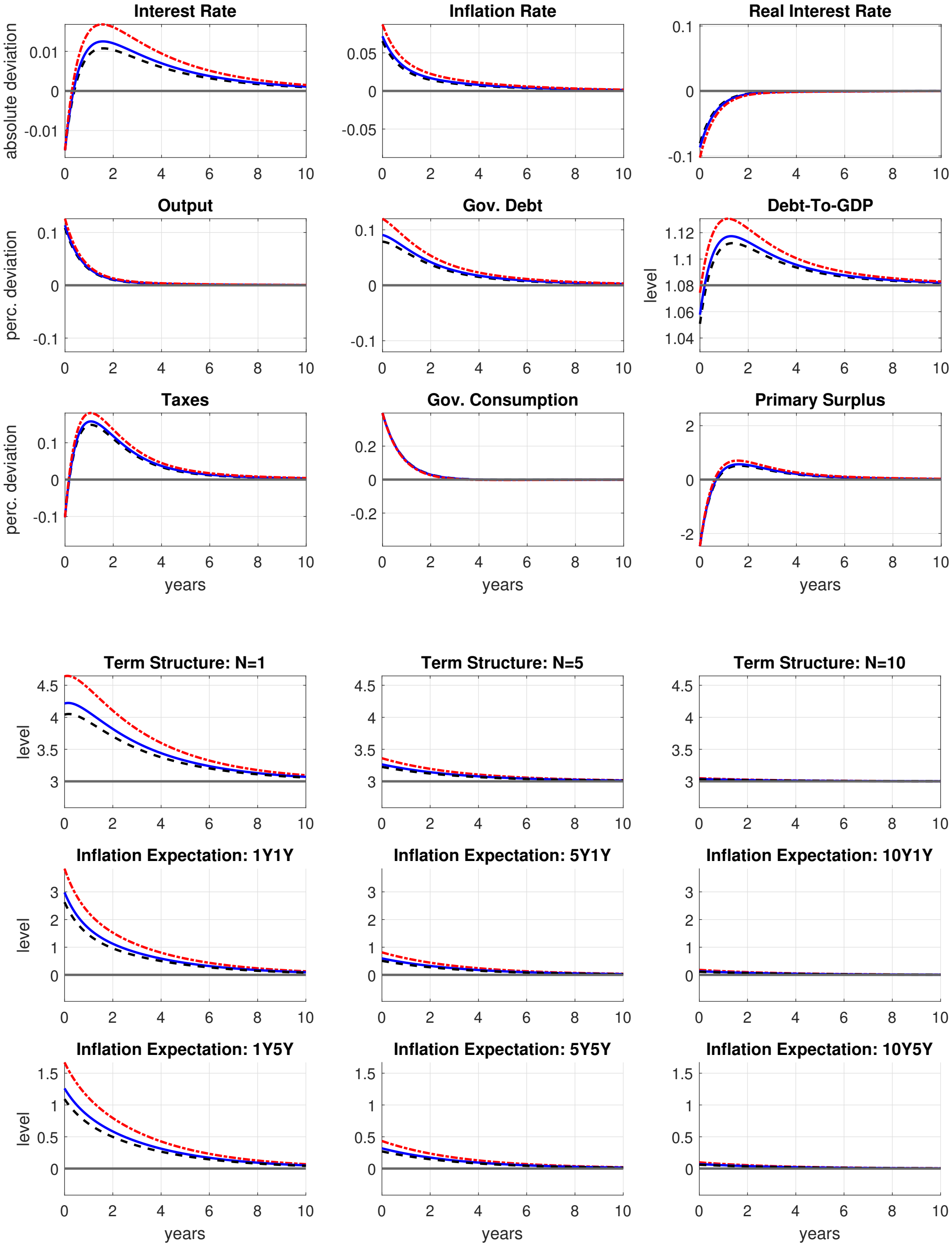

Figure A.4: CARES Act and monetary policy shock using parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, and increase in debt (face value) by 12 percent, and decrease interest rates by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

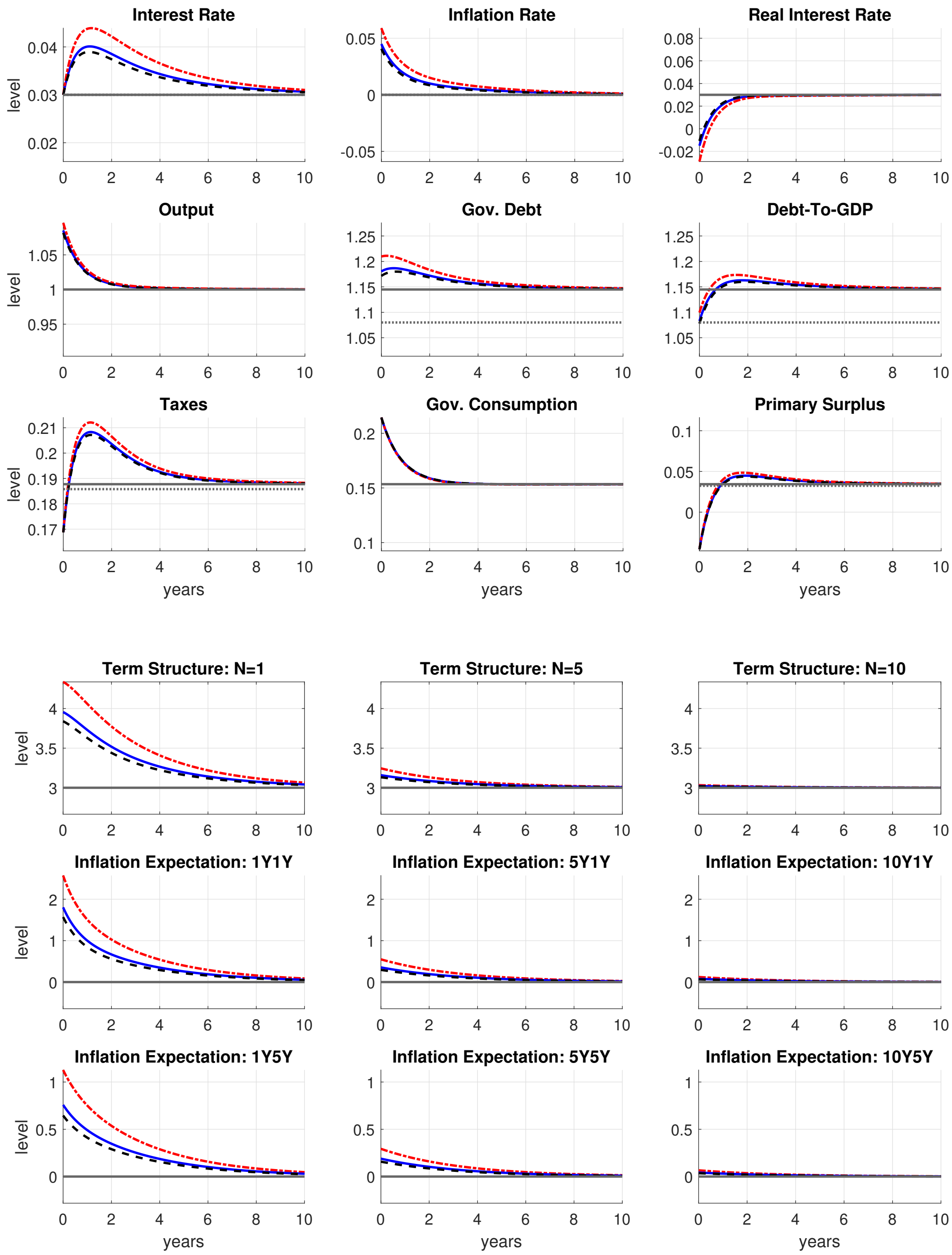

Figure A.5: CARES Act shock with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

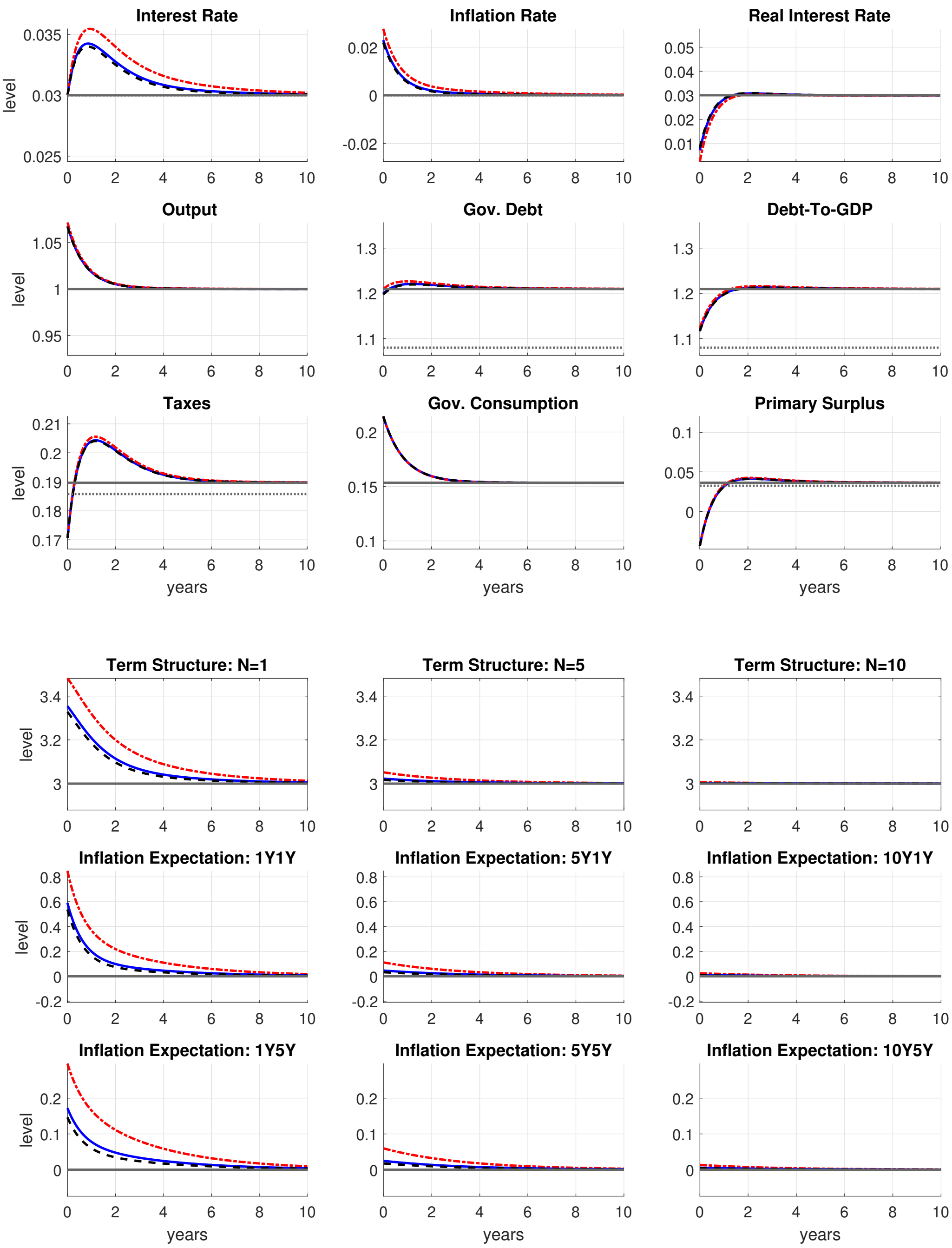

Figure A.6: CARES ACT shock with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.2: Inflation decomposition (19) for the CARES Act in Figure A.4.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	9.60	4.14	2.40	−4.14	12.00
Average	10.96	4.93	3.04	−2.93	12.00
Short-Term	14.28	6.86	4.58	0	12.00

Table A.3: Inflation decomposition (19) for the CARES Act shock in Figure A.5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	5.72	3.33	−0.06	−3.33	5.66
Average	6.63	3.86	0.34	−2.56	5.66
Short-Term	9.61	5.60	1.65	0	5.66

Table A.4: Inflation decomposition (19) for the CARES Act shock in Figure A.6.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	1.72	1.00	−1.72	−1.00	0
Average	1.93	1.12	−1.63	−0.83	0
Short-Term	2.92	1.70	−1.22	0	0

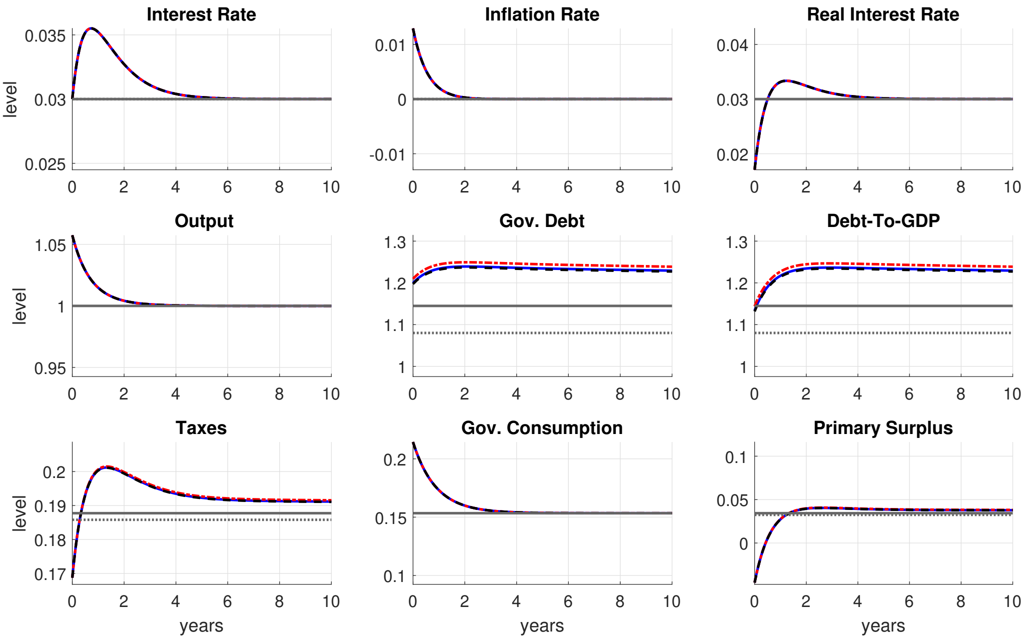

Figure A.7: CARES Act shock in the simple NK model with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$, $\varphi_y = -s_g$. To obtain the monetary regime, we further set $\tau_a = 0.04$ and $\phi_\pi = 1.6$ and. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

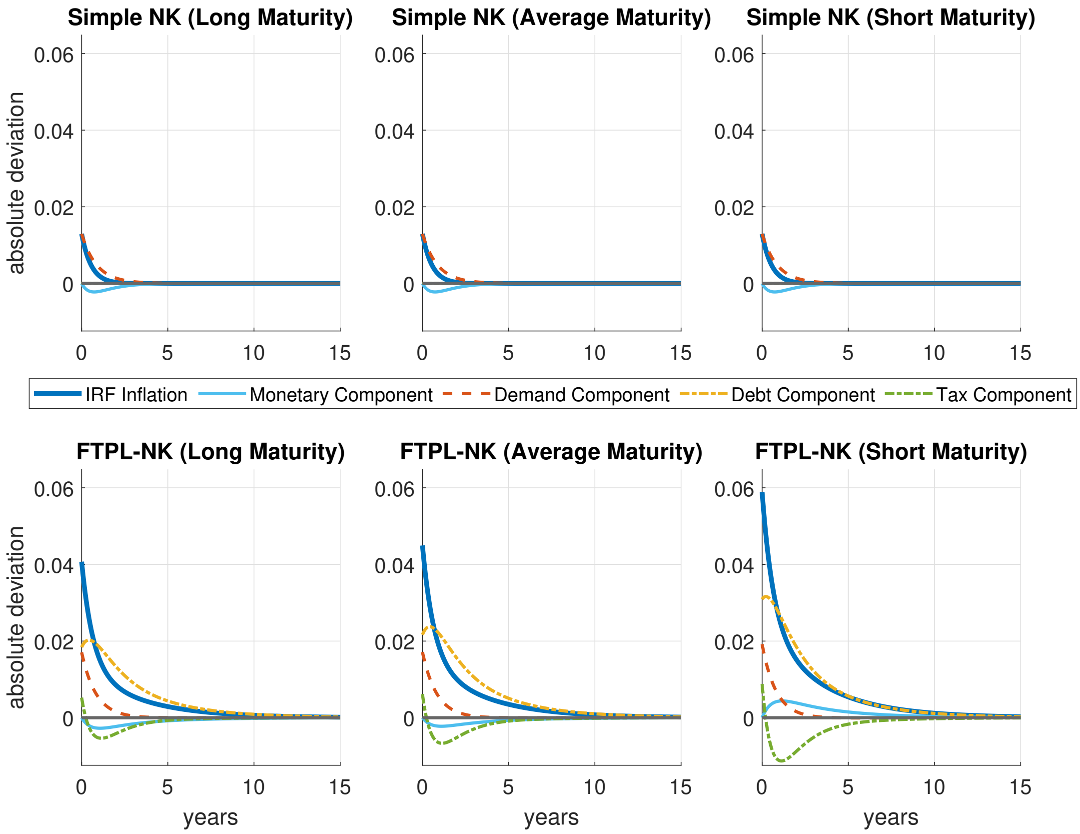

Figure A.8: Inflation rate decomposition (??) in terms of policy function coefficients for the CARES Act shock in figures A.5 and A.7. Upper panels show the decomposition and the actual inflation dynamics for the simple NK model with either long-term, average or short-term debt maturity. The lower panels show the corresponding case for the FTPL-NK model. Individual components sum up to the IRF of the inflation rate.

A.7 Empirical Data

Table A.5: Federal Reserve Bank of St. Louis Economic Dataset (FRED).
(2015Q1 through 2023Q2, retrieved on Nov 11, 2023)

Interest Rate (FEDFUNDS)	Federal Funds Effective Rate, Percent, Quarterly, Not Seasonally Adjusted, End of Period https://fred.stlouisfed.org/series/fedfunds
Inflation Rate (PCEPI)	Personal Consumption Expenditures: Chain-type Price Index, Percent Change from Year Ago, Quarterly, Seasonally Adjusted, End of Period, https://fred.stlouisfed.org/series/PCEPI
Real Interest Rate	Interest Rate - Inflation Rate
Output (GDPC1)	Real Gross Domestic Product, Percent Change from Year Ago, Quarterly, Seasonally Adjusted Annual Rate, https://fred.stlouisfed.org/series/GDPC1
Gov. Debt (GFDEBTN)	Federal Debt: Total Public Debt, Percent Change from Year Ago, Quarterly End of Period, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/GFDEBTN
Debt-to-GDP (GFDEGDQ188S)	Federal Debt: Total Public Debt as Percent of Gross Domestic Product, Percent of GDP Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/gfdegdq188S
Taxes (W006RC1Q027SBEA)	Federal government current tax receipts, Percent Change from Year Ago, Quarterly, Seasonally Adjusted Annual Rate, https://fred.stlouisfed.org/series/W006RC1Q027SBEA
Gov. Consumption (FGEXPND)	Federal Government: Current Expenditures, Percent Change from Year Ago, Quarterly, Seasonally Adjusted Annual Rate, https://fred.stlouisfed.org/series/FGEXPND
Primary Surplus	Taxes - Gov. Consumption
Bond Yield: 5Y (THREEFY5)	Fitted Yield on a 5 Year Zero Coupon Bond, Percent, Quarterly, Not Seasonally Adjusted https://fred.stlouisfed.org/series/THREEFY5
Expected Inflation: 5Y1Y (EXPINF5YR)	5-Year Expected Inflation, Percent, Quarterly, Not Seasonally Adjusted https://fred.stlouisfed.org/series/EXPINF5YR
Expected Inflation: 10Y1Y (EXPINF10YR)	10-Year Expected Inflation, Percent, Quarterly, Not Seasonally Adjusted hhttps://fred.stlouisfed.org/series/EXPINF10YR

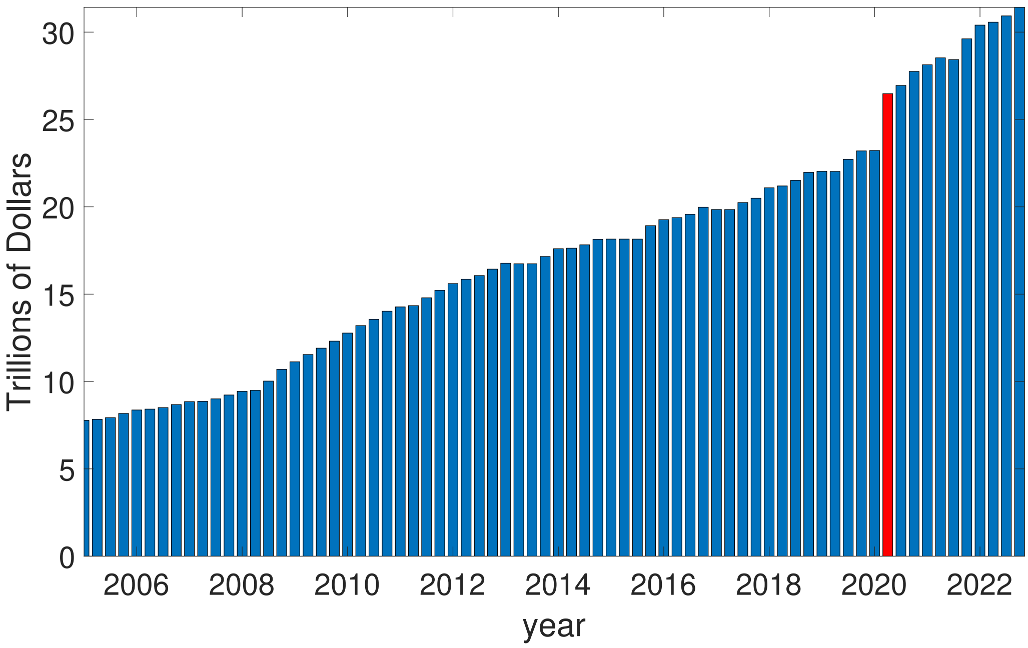

Figure A.9: Bar chart on quarterly US Federal Debt: Total Public Debt from 2001Q1 through 2022Q4 from the Federal Reserve Bank of St. Louis Economic Dataset (FRED) as defined in Table A.5. Red bar: 2020Q2 (CARES Act signed into law on March 27, 2020).

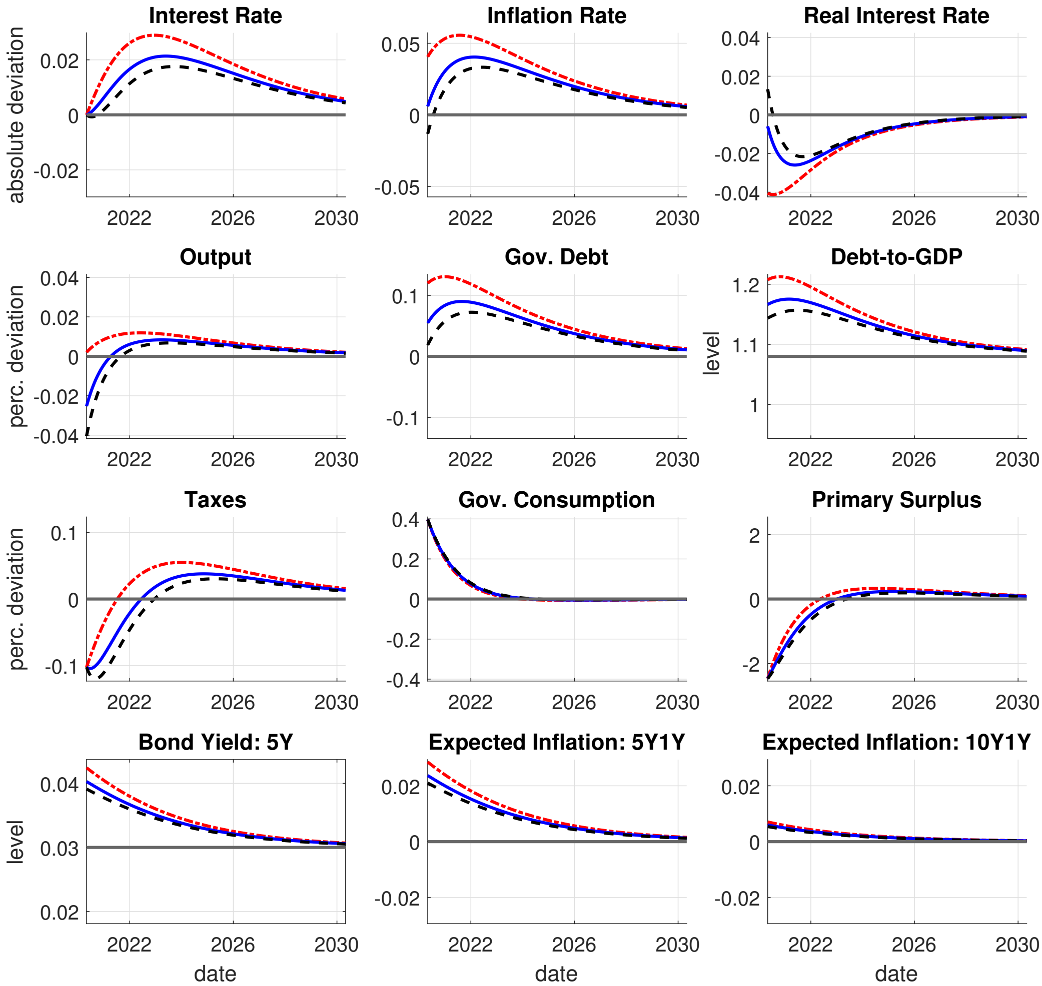

Figure A.10: Transitory CARES Act shock for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.6: Inflation decomposition (19) for the CARES Act shock in Figure A.10.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	17.44	10.16	−5.44	−10.16	12.00
Average	21.54	12.55	−3.53	−6.54	12.00
Short-Term	28.94	16.86	−0.08	0	12.00

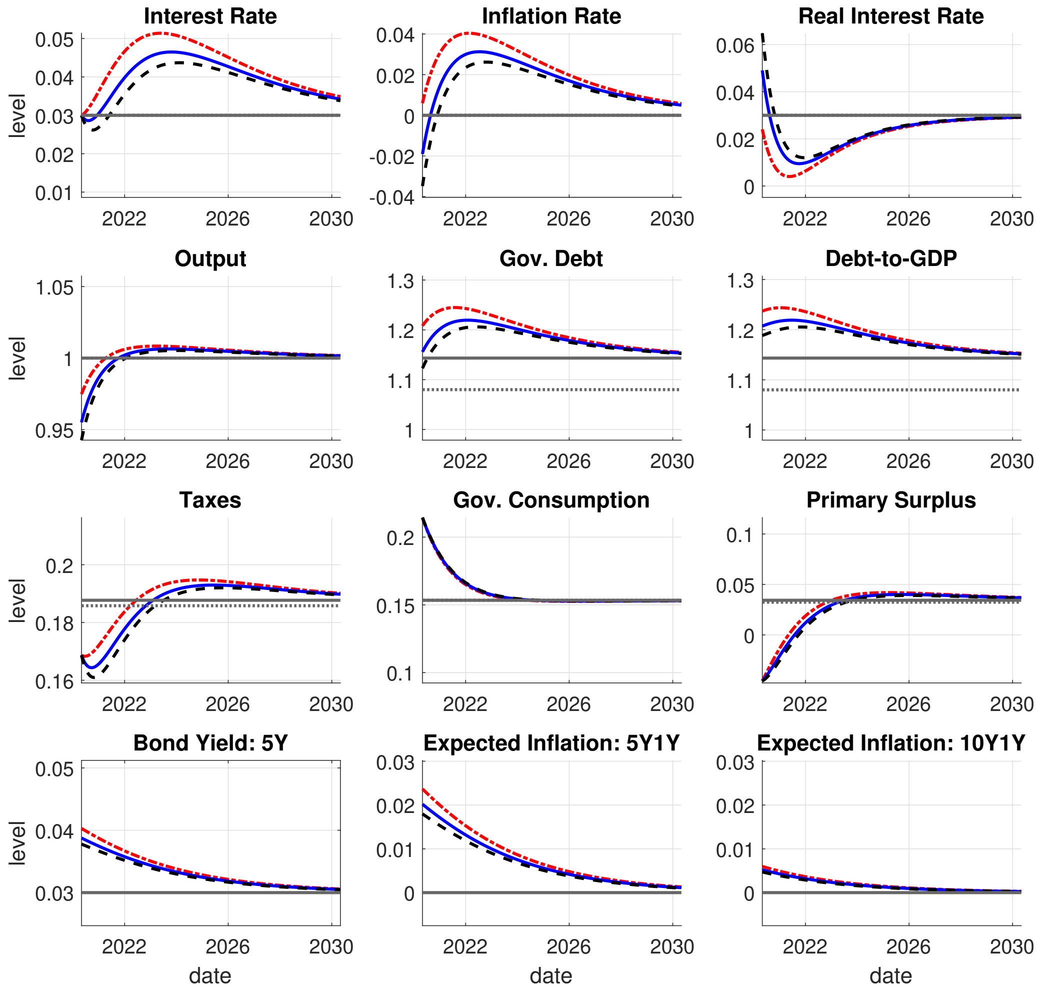

Figure A.11: CARES Act shock with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.7: Inflation decomposition (19) for the CARES Act shock in Figure A.11.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u / a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b / p_{ss}^b - 1$ direct effect	$v_0 / v_{ss}^{new} - 1$ debt shock
Long-Term	12.83	7.47	−7.16	−7.47	5.67
Average	16.21	9.44	−5.67	−4.58	5.67
Short-Term	21.55	12.55	−3.32	0	5.67

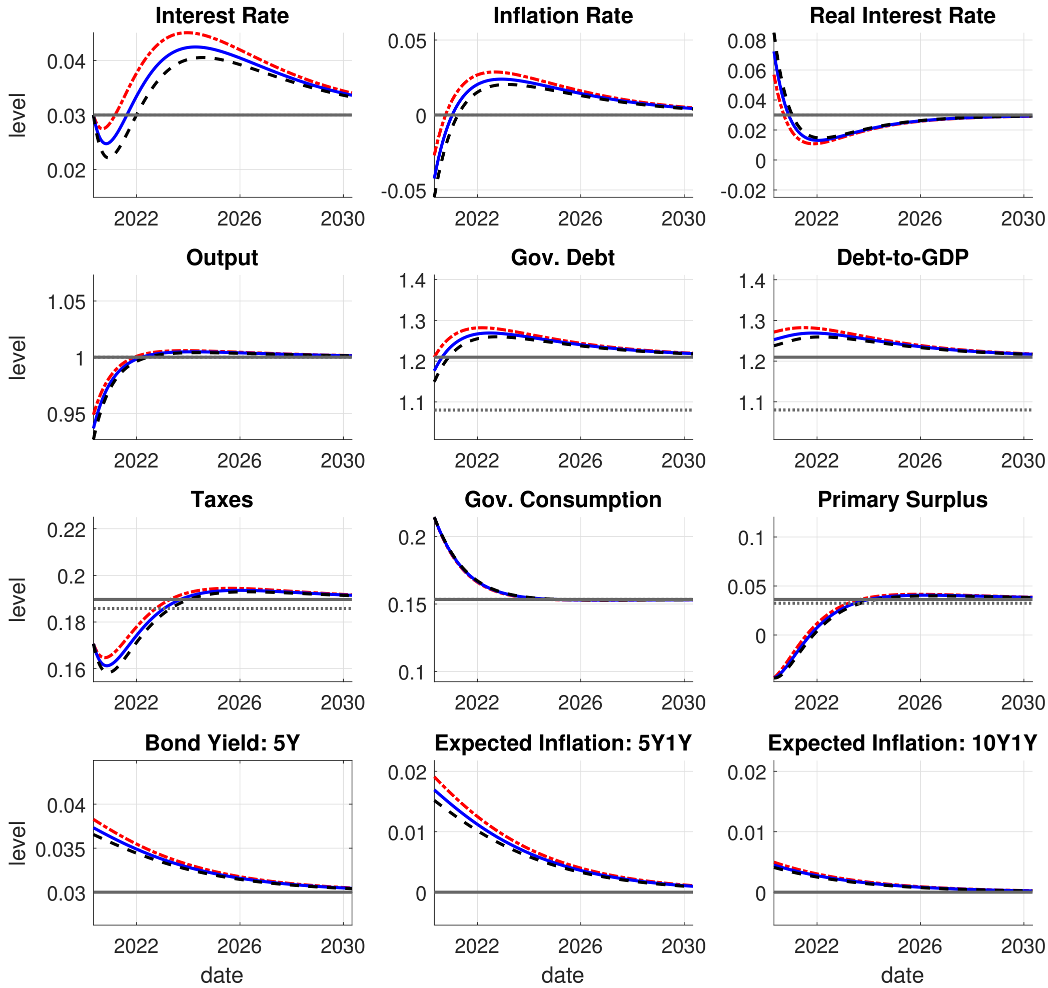

Figure A.12: CARES ACT shock with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.8: Inflation decomposition (19) for the CARES Act shock in Figure A.12.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.55	4.98	−8.55	−4.98	0
Average	11.23	6.54	−7.44	−2.75	0
Short-Term	14.52	8.46	−6.06	0	0

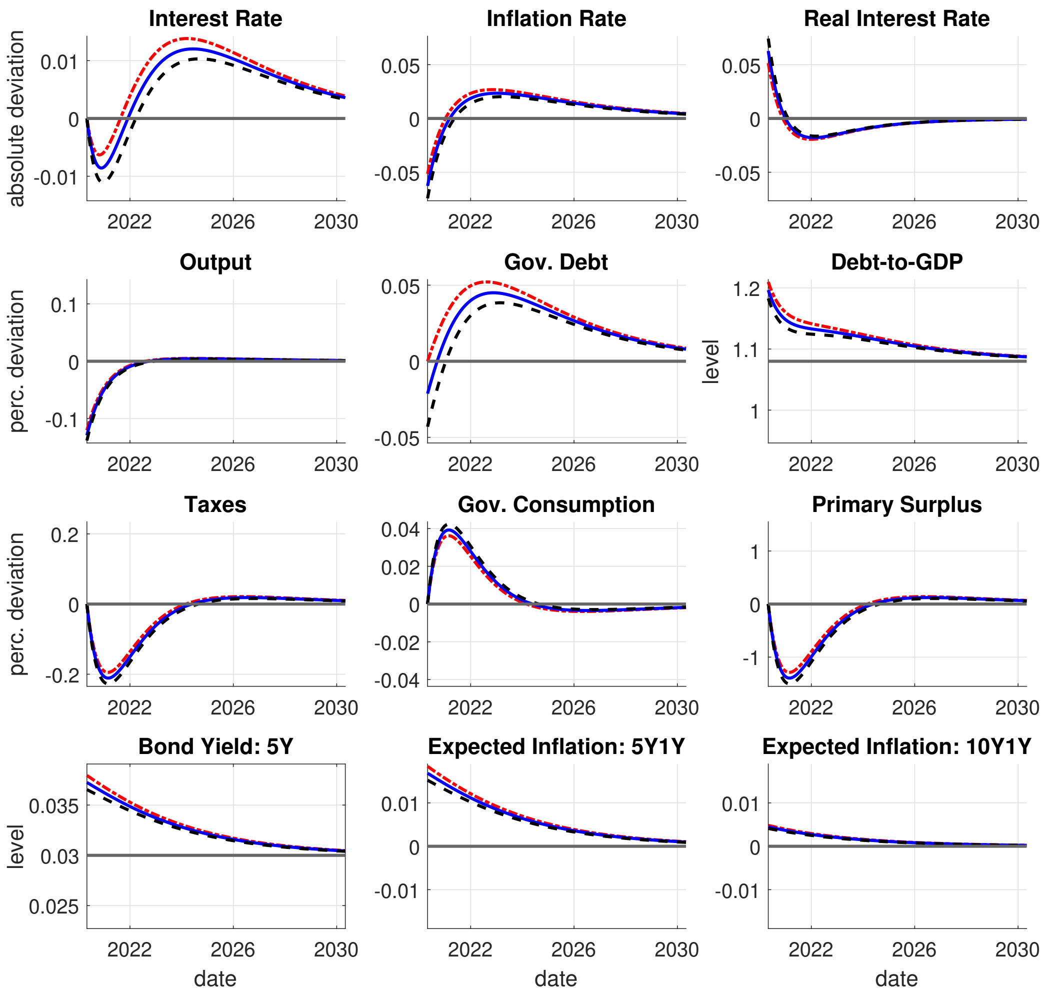

Figure A.13: Counterfactual dynamics in the absence of the CARES Act for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.9: Inflation decomposition (19) for the CARES Act shock in Figure A.13.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	7.39	4.30	−7.39	−4.30	0
Average	9.85	5.74	−6.24	−2.13	0
Short-Term	12.26	7.14	−5.12	0	0

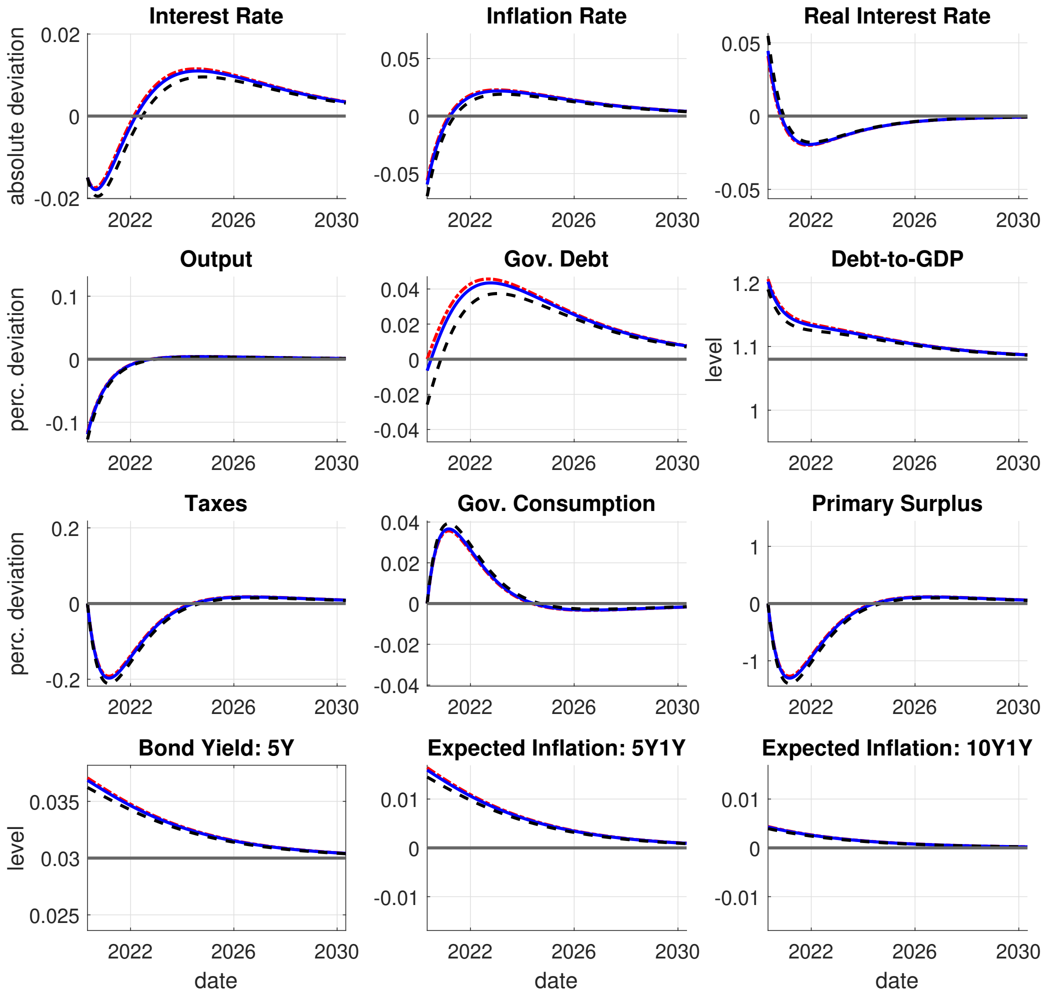

Figure A.14: Counterfactual dynamics in the absence of the CARES Act but with interest rate cut by 150 bp for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.10: Inflation decomposition (19) for the CARES Act shock in Figure A.14.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	6.94	2.59	-6.94	-2.59	0
Average	9.13	3.86	-5.92	-0.65	0
Short-Term	9.87	4.29	-5.57	0	0

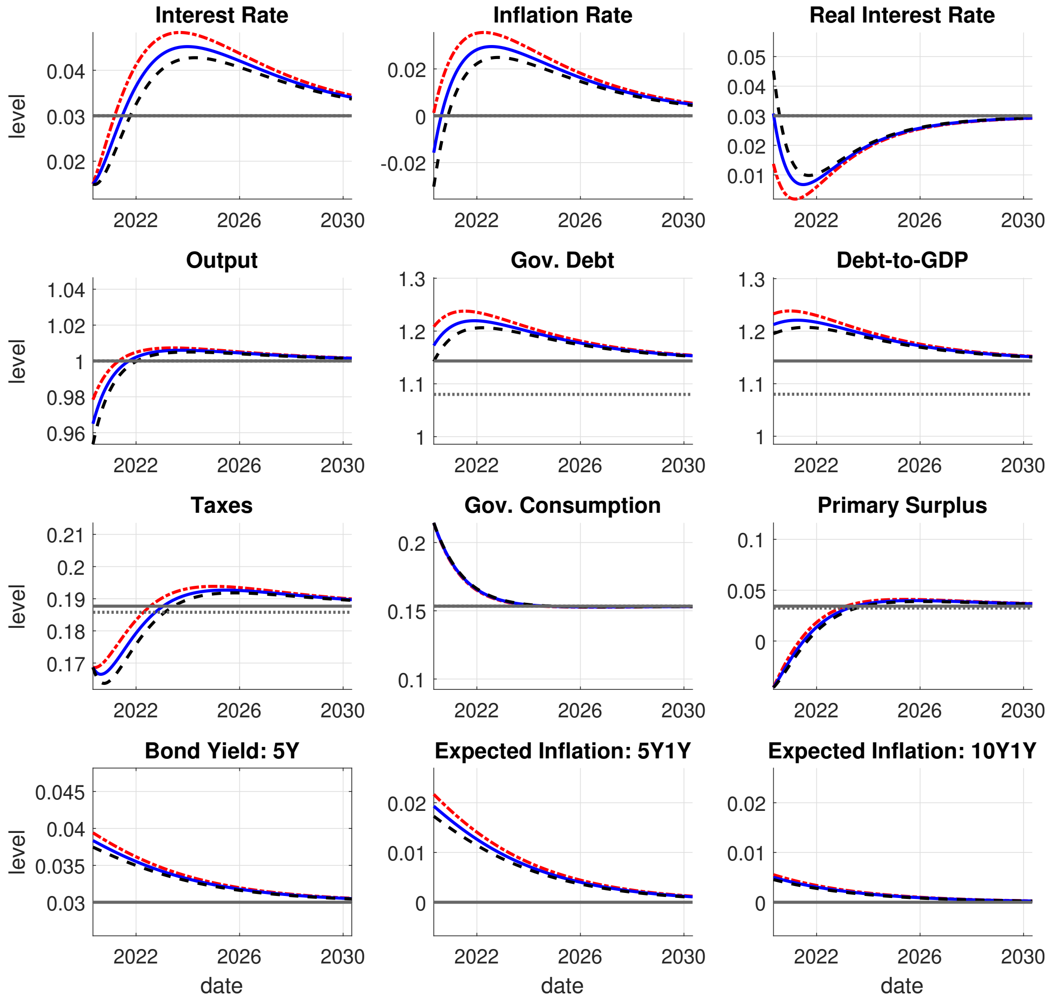

Figure A.15: CARES Act shock with monetary policy shock and with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, increase in debt (face value) by 12 percent and interest rate cut by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.11: Inflation decomposition (19) for the CARES Act shock in Figure A.15.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	12.40	5.77	−6.72	−5.77	5.67
Average	15.50	7.57	−5.36	−3.11	5.67
Short-Term	19.13	9.69	−3.77	0	5.67

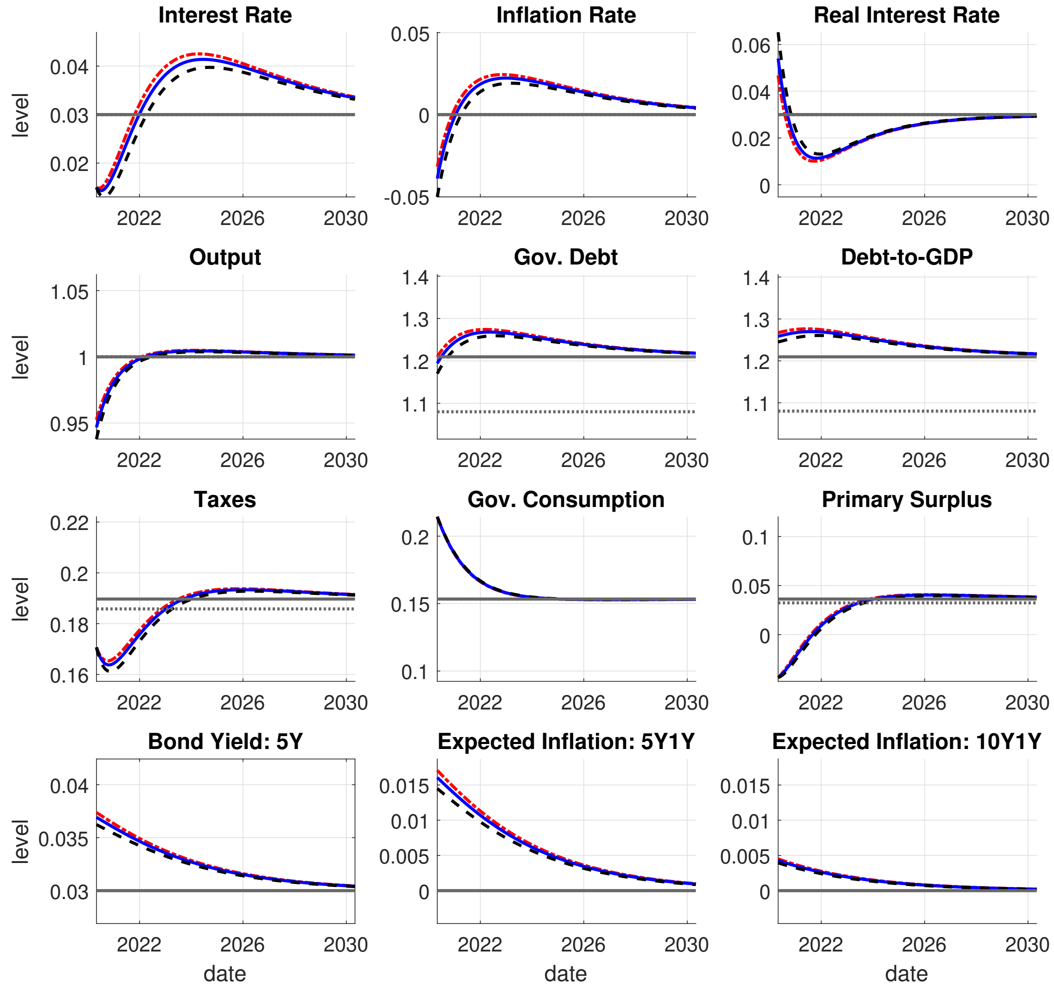

Figure A.16: CARES ACT shock with monetary policy shock and with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, increase in debt (face value) by 12 percent and interest rate cut by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.12: Inflation decomposition (19) for the CARES Act shock in Figure A.16.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.14	3.28	−8.13	−3.28	0
Average	10.53	4.68	−7.14	−1.29	0
Short-Term	12.08	5.58	−6.50	0	0

A.8 CARES Act Shock: A fully funded Scenario

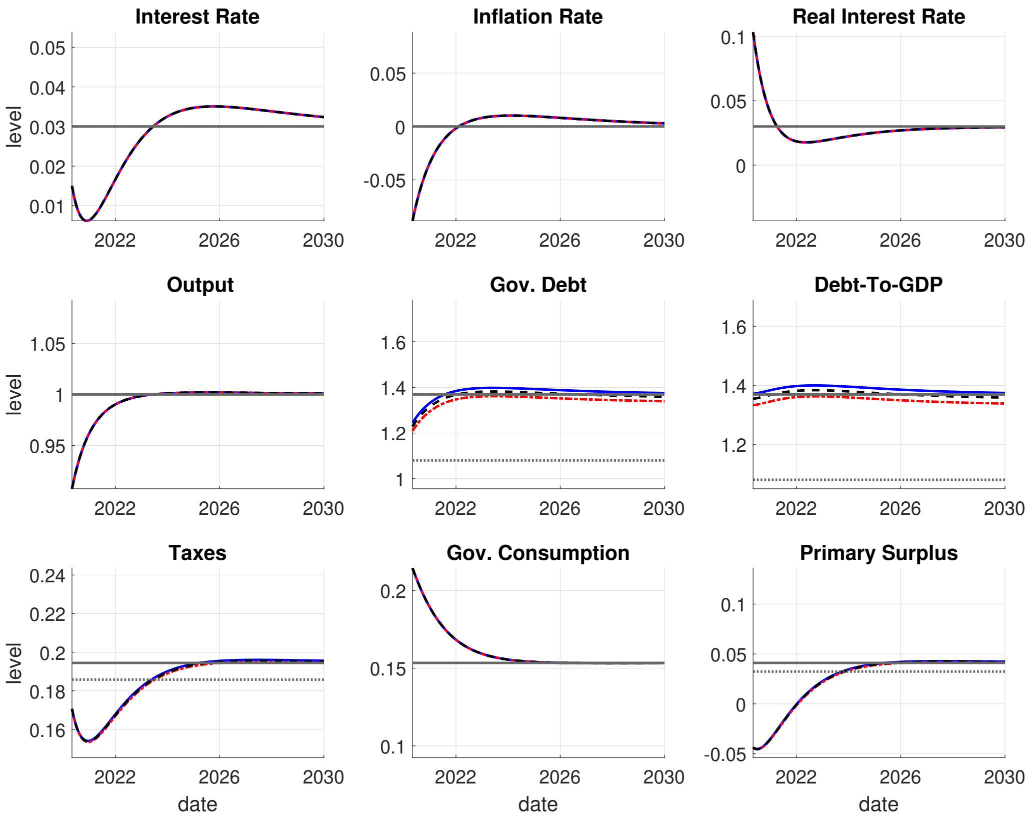

Figure A.17: Fully Funded CARES Act. Long-Term: $v_{ss}^{new} = 1.35$ for $\alpha = 2.11$. Average: $v_{ss}^{new} = 1.37$ for $\alpha = 2.23$. Short-Term: $v_{ss}^{new} = 1.33$ for $\alpha = 1.94$.

Table A.13: Inflation decomposition (19) for the CARES Act shock in Figure A.18.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	0	-1.46	-10.60	1.46	-10.60
Average	0	-1.46	-10.52	2.58	-11.64
Short-Term	0	-1.46	-10.72	0	-9.27

A.9 The CARES Act through the lens of the simple NK model

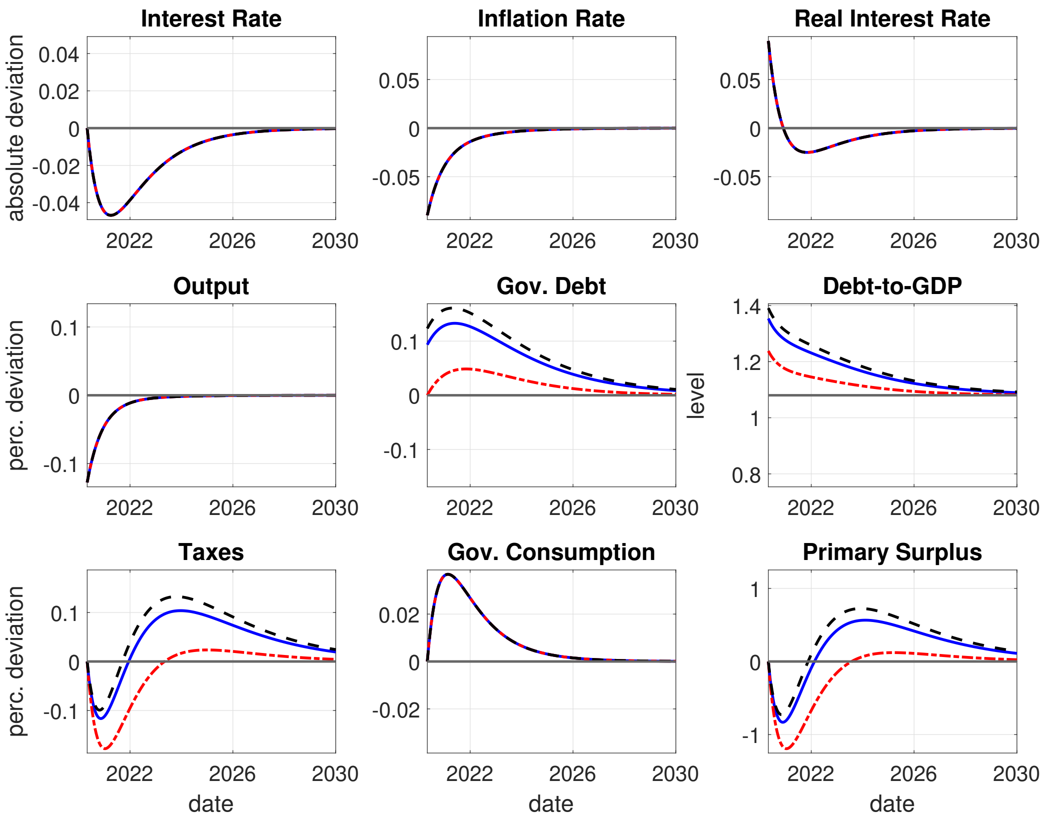

Figure A.18: Counterfactual dynamics in the absence of the CARES Act for the parametrization in Table 1 with $\rho_g = 1$, $\varphi_y = -s_g$ and active monetary policy with $\phi_\pi = 1.6$ and $\tau_a = 0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.14: Inflation decomposition (19) for the CARES Act shock in Figure A.18.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \text{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \text{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \text{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	-7.94	-12.33	7.94	12.33	0
Average	-7.94	-12.33	4.94	9.33	0
Short-Term	-7.94	-12.33	-4.39	0	0

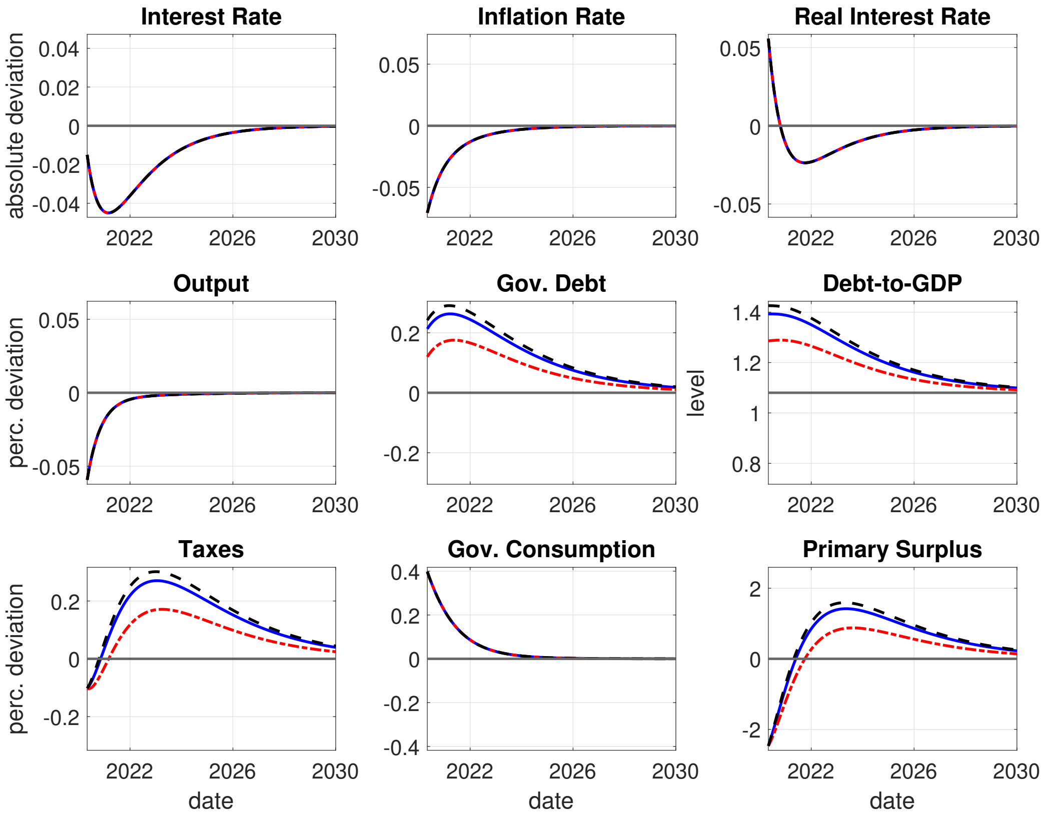

Figure A.19: Transitory CARES Act shock with monetary policy shock for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$ and active monetary policy with $\phi_\pi = 1.6$ and $\tau_a = 0.25$. Decrease in surplus by 8 percent of GDP, interest rate cut by 150 bp. and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.15: Inflation decomposition (19) for the CARES Act shock in Figure A.19.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	−6.90	−12.17	18.90	12.17	12.00
Average	−6.90	−12.17	16.02	9.29	12.00
Short-Term	−6.90	−12.17	6.73	0	12.00