Withholding Verifiable Information

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Verifiable Disclosure Games

In many economic interactions that involve communication, the messages are verifiable

- they can be vague but can never be false
- examples:
  - sellers disclosing and highlighting certain features of a product to consumers
  - political experts organizing and simplifying poll results for politicians
  - advisors condensing and distilling market research for managers

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Verifiable disclosure games

- the sender privately observes a payoff-relevant state before she sends a message
- the receiver subsequently chooses an action that affects the sender's payoff
- the sender's messages are "verifiable": every message must contain the true state

This Paper

The literature in VDG mainly focuses on the receiver's preferred (most informative) equilibria

- under some conditions, there is an eqm in which the sender fully reveals her private info

Little is known about how much can the sender benefit from verifiable communication:

- what does a sender's preferred equilibrium look like, and how well can Sender do?
- what is the full range of the sender's equilibrium payoffs?

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In a simple class of verifiable disclosure games, I find that

- there is a sender's preferred equilibrium in which on-path messages are action recommendations and have a simple structure
- a continuum of equilibria with distinct payoffs satisfying the aforementioned properties $\Longrightarrow$ caveat against focusing on fully revealing equilibria in policy debates
- under some conditions the sender can gain quite a bit from verifiable communication

The Model

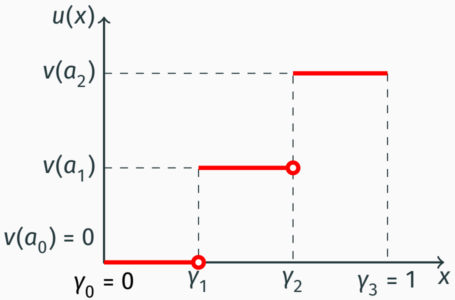
(Baseline) Model

- Two players: Sender and Receiver
- State space $[0, 1]$, generic element ω, prior F that admits a strictly positive density
- Sender privately observes the state and then sends a message to Receiver
- Observing the message, Receiver forms an expected state based on it: $x = \mathbb{E}[\omega \mid \text{message}]$

Three key assumptions:

1. Receiver has finitely many actions; today's talk: 3 actions, so $\mathcal{A} = \{a_0, a_1, a_2\}$
   - the results presented today extend to finitely many actions in a natural way
2. Receiver's optimal action only depends on the expected state
   - there exist cutoffs $0 = \gamma_0 \leq \gamma_1 \leq \gamma_2 \leq \gamma_3 = 1$ such that a_i is optimal iff $x \in [\gamma_i, \gamma_{i+1}]$
3. Sender's payoff $v : \mathcal{A} \to \mathbb{R}$ only depends on Receiver's action (state-independent preferences)
   - assume $v(a_2) > v(a_1) > v(a_0) = 0$

Sender's Payoff: Illustration



Sender's value function $u(x)$: Sender's highest attainable payoff as a function of the expected state x

The Game

- Following Grossman (1981) and Milgrom (1981), when the state is ω, Sender's message space is

$$\mathcal{M}(\omega) = \{m \subseteq [0,1] : m \text{ closed}, \omega \in m\}$$

- (Perfect Bayesian) **equilibrium** of the verifiable disclosure game:
  - Sender's and Receiver's strategies are sequentially rational
  - Receiver's belief system is updated via Bayes' Rule whenever possible
  - Observing message m, Receiver must deem any state that is not in m impossible
- **Sender's preferred equilibria** are the equilibria that attain the highest possible Sender's expected (ex ante) payoff

Equilibrium Analysis

Obedient Recommendation Equilibrium

Definition. A collection of 3 closed subsets of the state space $[0, 1]$, $\{B_0, B_1, B_2\}$, is a **partition** if $B_0 \cup B_1 \cup B_2 = [0, 1]$, and for any $i, j = 0, 1, 2$, $\mu_F\left(B_i \cap B_j\right) = 0$.

Definition. An equilibrium of the verifiable disclosure game is an **obedient recommendation equilibrium (ORE)** if there exist a partition $\{B_0, B_1, B_2\}$ such that for each $i = 0, 1, 2$,

(1) for (almost) every $\omega \in B_i$, Sender sends message B_i; and

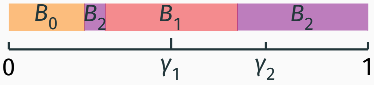
(2) upon receiving message B_i, Receiver plays action a_i.

- Every on-path message B_i in an ORE can be interpreted as a recommendation of action a_i that Receiver finds it optimal to follow

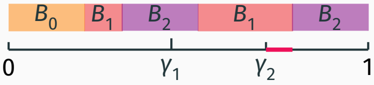
Characterizing ORE

Definition. A partition $\{B_0, B_1, B_2\}$ is

- **obedient** if $\mathbb{E}[\omega \mid \omega \in B_i] \in [\gamma_i, \gamma_{i+1}]$ for each $i = 0, 1, 2$;
- **incentive compatible** if for all i and every $\omega \in B_i$, Sender (weakly) prefers B_i to $\{\omega\}$.



A partition that is IC



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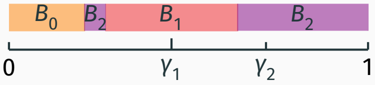

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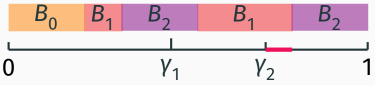

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Lemma. A partition $\{B_0, B_1, B_2\}$ is the set of on-path messages of an ORE if and only if it is both obedient and incentive compatible.

Maximally skeptical beliefs: $\mathbb{P}(\min m \mid m) = 1$ if $m \notin \{B_0, B_1, B_2\}$

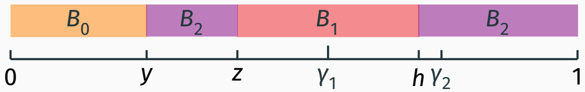
- deters all deviations except full revelation

Sender's Preferred Equilibrium

Proposition. There exists an ORE that is a Sender's preferred equilibrium in which

(i) the on-path messages $\{B_0, B_1, B_2\}$ satisfy $B_0 = [0, y]$, $B_1 = [z, h]$, and $B_2 = [y, z] \cup [h, 1]$,

(ii) Sender's expected payoff is strictly higher than in any fully revealing equilibrium.



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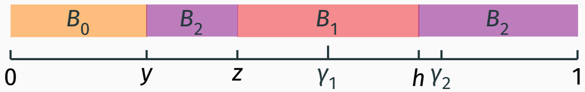

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- Show that the the "nested interval" structure is the most "deviation-proof"

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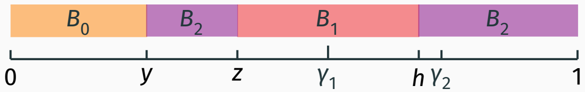

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For $n > 3$ actions, the partition has a "laminar" structure (Candogan and Strack, 2022), in which each B_i is the union of at most $\max\{1, i - 1\}$ disjoint intervals

Sender's Equilibrium Payoff Set And Other ORE

Proposition. Any payoff that is below the sender's payoff in her preferred equilibria and above her payoff in a fully revealing equilibrium can be sustained in an ORE in which the on-path messages $\{B_0, B_1, B_2\}$ satisfy $B_0 = [0, y]$, $B_1 = [z, h]$, and $B_2 = [y, z] \cup [h, 1]$. Proof idea

- Every such ORE survives the Never-a-Weak-Best-Response (NWBR) criterion proposed by Cho and Kreps (1987)
- There is a continuum of equilibria in which both Sender and Receiver play pure strategies, and the on-path messages take simple forms
- May suggest that focusing on the fully revealing equilibrium outcome in policy debates need not always be appropriate

How Well Can Sender Potentially Do?

An upper bound is given by Sender's expected payoff when she can commit on what messages to send in each of the states

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In the communication environment I study, Candogan (2019) and Arieli et al. (2023) show that the information design solution can be implemented by a partition $\{B_0, B_1, B_2\}$ such that

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Observation. A partition $\{B_0, B_1, B_2\}$ that solves the information design problem supports an ORE iff it is incentive compatible, i.e., Sender never wants to deviate to fully reveal.

- With commitment, Sender may recommend "middle action" a_1 "too often"

Sufficient Condition

Proposition. If $v(a_2) > C(\gamma_1, \gamma_2)\, v(a_1)$, the commitment payoff can be attained in an equilibrium.

- Sender's value needs to increase sufficiently fast as the expected state increases
- Guarantees that Sender would never want to recommend a_1 "too often" when she can commit
- Identifies a class of communication environments in which Sender does not benefit from commitment power

Illustration

Discussion & Summary

Discussion

Message space

- It is crucial to allow Sender to use any message that is a closed set that contains the state
- For example, the constructed Sender's preferred equilibrium need not be achieved under "truth or nothing" or closed interval message spaces

Further cheap talk opportunities

- Using a well known characterization in Lipnowski and Ravid (2020), one can show that Sender does not benefit from further cheap talk communication

More general model

- All results have natural analogs when the number of actions is more than 3
- Some results extend when either the state is multidimensional, or the receiver has a continuum of actions

Literature

Receiver's preferred / most informative equilibria in verifiable disclosure games: e.g., Grossman and Hart (1980), Grossman (1981), Milgrom (1981), Seidmann and Winter (1997), Hagenbach, Koessler, and Perez-Richet (2014), Hart, Kremer, and Perry (2017)

Whether (and to what extent) the sender can benefit from verifiable communication: Mezzetti (2020), Pram (2021), Titova (2022), Ali, Lewis, and Vasserman (2023)

(Im)possibility of attaining commitment outcome in communication environments w/o commitment: e.g., Lipnowski (2020), Best and Quigley (2023), Kuvalekar, Lipnowski, and Ramos (2022), Mathevet, Pearce, and Stacchetti (2022), Pei (2023)

- This paper is about verifiable disclosure games

Summary

I explore the extent to which the sender can benefit from verifiable communication

- find a sender's preferred equilibrium and the sender's equilibrium payoff set
  - each of these payoffs can be attained in an equilibrium where on-path messages are action recommendations and have a simple structure
  - focusing on fully revealing equilibria in policy debates need not always be appropriate
- identify a class of communication environments in which the sender can attain her commitment payoff
  - roughly, it requires that the sender's value function increases sufficiently fast

Applications: selling with quality disclosure and influencing voters

Thank you!

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Back-up Slides

Equilibrium Payoff Set: Proof Idea

Let $\{\overline{B}_0, \overline{B}_1, \overline{B}_2\}$ be the partition associated with Sender's preferred equilibrium obtained in the previous proposition

Step 1: Construct an ORE defined by partition $\{\underline{B}_0, \underline{B}_1, \underline{B}_2\}$ in which Sender's expected payoff is the same as a fully revealing equilibrium

Step 2: Every payoff strictly between Sender's payoff in her preferred equilibria and a fully revealing equilibria can be obtained in an ORE defined by a partition that is a "mixture" of $\{\overline{B}_0, \overline{B}_1, \overline{B}_2\}$ and $\{\underline{B}_0, \underline{B}_1, \underline{B}_2\}$

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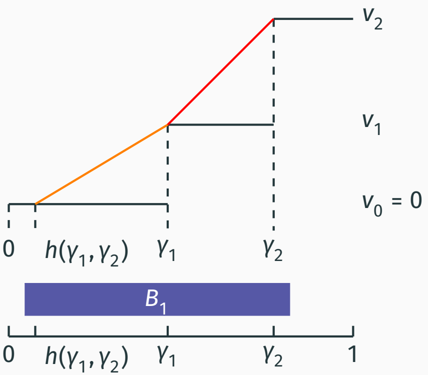
Sufficient Conditions: Illustration



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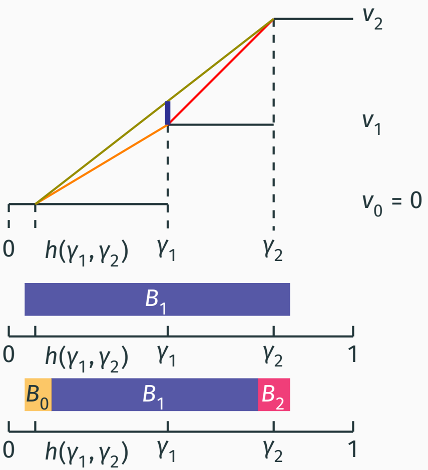

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Sufficient Conditions: A Further Result

Corollary. If f is increasing and $v_2 > 2v_1$, the commitment payoff can be attained in an equilibrium.