Exploitative Contracting in a Life Cycle Savings Model

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Motivation

1. Private pensions increasingly important:

- coverage of over 75% in 12 OECD countries;
- associated assets worth \$54 trillion in 2020 (99.9% of OECD's GDP), with 66% held in the US.

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- associated assets worth \$54 trillion in 2020 (99.9% of OECD's GDP), with 66% held in the US.

2. Individuals responsible for financial security in retirement, but savings decisions systematically influenced by behavioural factors:

- self-control problems (Ameriks et al. 2007, Ashraf et al. 2006);
- procrastination and inattention (Benartzi & Thaler 2004, 2007, Chetty et al. 2014, Choi et al. 2004, 2011, Madrian & Shea 2001).

Research Questions

This paper: A model of contracting between a present-biased saver and a profit-maximising financial provider.

• Behavioural IO perspective \implies Endogenize asset parameters r and f.

- 1 What are the qualitative properties of contracts provided by the market?
- 2 What is their quantitative impact on savings outcomes?
 - What is the impact of common regulatory policies?

Preview of Results

Question $1 \implies$ A tractable model:

▶ Naïve present-biased agents are exploited in the marketplace.

- Substitution effect dominates \implies exploitative contracts are **'inefficiently expensive'** (high-yield, high-fee).
- Income effect dominates \implies exploitative contracts are **'inefficiently cheap'** (low-yield, low-fee).

Question 2 \implies A calibrated life-cycle model with choice of a pension provider:

- ▶ The equilibrium contract is Pareto-inefficient (too 'cheap').
- ▶ This inefficiency lowers pension wealth at retirement by 8% (expected annual consumption post retirement \downarrow 3%), generating CE welfare loss of 0.23% p.a.

Literature and Contribution

Exploitative contracting with present-biased agents

- DellaVigna & Malmendier (2004), Gabaix & Laibson (2006), Grubb (2009), Heidhues & Kőszegi (2010), …, Bubb & Warren (2020), Gottlieb & Zhang (2021)
- \star Direction of the inefficient distortion to simple contracts.
- * Quantitative evaluation of exploitative contracting.

Behavioural household finance

- ..., Angeletos et al. (2001), Lusardi et al. (2017), Laibson et al. (2018)
- \star Interaction with a financial provider \rightarrow endogenous asset parameters.

Simple Model

Baseline case:

- Simple contracts P = (r, f).
- Monopolistic provider who observes the agent's characteristics.

► Timing:

- Period 0: Provider proposes a contract and the agent accepts/rejects.
- Period 1: The agent saves, given the contract parameters.
- Period 2: Retirement.

Period 1: Given contract parameters, the agent saves to maximise:

$$U_1 = u(c_1) + \boldsymbol{\beta} \,\delta u(c_2) - f$$

- $\beta \leq 1$ captures the present bias (Strotz 1955, Laibson 1997),
- ▶ $c_1 = Y s$, $c_2 = (1 + r)s$ for some Y > 0.

Period 0: The agent **evaluates the offer** according to:

$$U_0 = u(\hat{c}_1) + \delta u(\hat{c}_2) - f$$

assuming singleton beliefs $\hat{\beta} \in [\beta, 1]$ (O'Donoghue & Rabin 2001).

- For $\hat{\beta} = \beta$, an agent is called 'sophisticated'.
- For $\hat{\beta} > \beta$, an agent is called 'naïve'.

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Lemma 1. For a strictly concave and continuously differentiable $u(\cdot)$, $s(\hat{\beta}, r)$ is increasing in $\hat{\beta}$. Consequently, U_0 is increasing in $\hat{\beta}$ for any r, f.

Functional form assumptions:

1 The agent has CRRA utility:

$$u(x) = \frac{x^{1-\theta} - 1}{1-\theta}$$

with $\theta > 0$.

2 The **financial provider** maximises the profit function:

$$\pi = f - K(r, s)$$

where $K(\cdot, s)$ is increasing, continuously differentiable, and strictly convex in r. More

The optimal contract solves:

$$\max_{r,f} \pi = f - K(r,s)$$
 s.t.:

1. $s = s(\beta, r)$ 2. $V_0(s(\hat{\beta}, r), r) - f \ge \underline{u}$ The optimal contract solves:

$$\max_{r,f} \pi = f - K(r,s)$$
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depends on β

1. $s = s(\beta, r)$ 2. $V_0(s(\hat{\beta}, r), r) - f \ge \underline{u}$

depends on $\hat{\beta}$

The optimal contract solves:

1. s =

$$\begin{split} \max_{r,f} \ \pi \ = \ f - K(r,s) \quad \text{s.t.:} \\ 1. \ s = s(\beta,r) & \text{depends on } \beta \\ 2. \ V_0(s(\hat{\beta},r),r) - f \geq \underline{u} & \text{depends on } \hat{\beta} \end{split}$$

Proposition 1. In a monopolistic market with perfect observability:

- 1. A sophisticated agent obtains an efficient contract.
- 2. A naïve agent obtains an inefficient contract. The direction of the exploitative distortion is given by:

$$\frac{d\,r^*}{d\hat{\beta}} \; \begin{cases} > 0 & \text{for } \theta < 1, \\ = 0 & \text{for } \theta = 1, \\ < 0 & \text{for } \theta > 1. \end{cases}$$

3. The agent's utility and social surplus are decreasing in $\hat{\beta}$. The provider's profits are increasing in β . Policy interventions

Life-Cycle Model with Choice of a Pension Provider

Household side:

- Finite horizon. Stochastic income and survival.
- Deterministic retirement age and (t-dependent) household size.
- Two asset classes:
 - Liquid asset X allows for borrowing, positive holdings earn R^X , negative holdings cost R^{CC} .
 - Illiquid asset Z earns R^Z and imposes t-dependent penalties for withdrawal. No borrowing.
 - Budget constraints

Firm side:

- Two homogeneous providers. Spatial competition à la Hotelling (1929) with $\xi > 0$.
- Cost function:

$$K\left(R^{Z},Z_{t}\right)=\underbrace{k_{1}Z_{t}^{\gamma_{1}}}_{\text{admin cost}}+\underbrace{k_{2}(R^{Z}-R^{X})^{\gamma_{2}}}_{\text{investment cost}}$$

Calibration

Household simulated using the model of Laibson, Maxted, Repetto & Tobacman (2018).

- ▶ 'First-stage' parameters taken directly from LMRT (2018):
 - income process, survival probabilities, deterministic household size, retirement age of 64,
 - $R^X = 2.79\%$, $R^{CC} = 11.52\%$.
- \blacktriangleright To focus on pension choices, I modify how the illiquid asset Z is modelled:
 - Yields returns of $\mathbb{R}^{\mathbb{Z}}$, rather than consumption flow.
 - Liquidation costs drop sharply at the retirement age.

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1 I recalibrate the household parameters targeting the same moments as LMRT (2018).

• Benchmark estimates obtained under $\hat{\beta} = 1$.

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- **2** The firm side calibration:
 - Elasticity of admin costs (γ_1) from the literature. Remaining parameters $(k_1, k_2, \gamma_2, \xi)$ set jointly to match the cost-to-assets ratio, share of admin costs, markups, and R^{Z*} .
 - $\star\,$ In eqm, there cannot exist any mutually profitable deviations, price- or quality-based.

Table 1: Household side moments and goodness of fit

	Data	LMRT (2018)	Fixed pref. params	Recalibrated pref. params
Calibrated β	-	0.5054	0.5054	0.5030
Calibrated δ	-	0.9872	0.9872	0.9880
Calibrated θ	-	1.2551	1.2551	1.1051
Frac. borrowing, 21-30	0.815	0.598	0.610	0.608
Frac. borrowing, 31-40	0.782	0.607	0.653	0.714
Frac. borrowing, 41-50	0.749	0.586	0.792	0.838
Frac. borrowing, 51-60	0.659	0.568	0.840	0.853
Avg. debt to income, 21-30	0.199	0.232	0.232	0.241
Avg. debt to income, 31-40	0.187	0.237	0.254	0.272
Avg. debt to income, 41-50	0.261	0.216	0.297	0.319
Avg. debt to income, 51-60	0.276	0.196	0.328	0.323
Avg. wealth to income, 21-30	1.23	1.30	0.78	1.00
Avg. wealth to income, 31-40	1.86	1.83	1.06	1.47
Avg. wealth to income, 41-50	3.24	2.94	2.47	3.09
Avg. wealth to income, 51-60	5.34	5.05	5.27	6.27
Goodness-of-fit	-	250.75	276.28	221.34

Table 2: Firm side calibration

Jointly calibrated parameters	Value	Target moment	Moment value
Admin cost multiplier k_1	1.5083	Share of admin costs 0.50	
Investment cost elasticity γ_2	5.75	Eqm. interest rate R^Z	5%
Investment cost multiplier k_2	$2.649 {\times} 10^{12}$	Cost-to-assets ratio	0.005
Hotelling parameter ξ	0.4815	Markup	0.20
Set parameters	Value	Source	
Admin cost elasticity γ_1	0.5	Bateman, Mitchell 2004; Bikker et al. 2012	
CRRA parameter $ heta$	1.1051	Author's calibration (Table 1)	
Short-run discount factor eta	0.5030	Author's calibration (Table 1)	
Long-run discount factor δ	0.9880	Author's calibration (Table 1)	
Beliefs about present bias \hat{eta}	1	Laibson et al. 2018	
Risk-free interest rate R^X	2.79%	Laibson et al. 2018	

Quantification of Exploitative Contracting

The calibration targets $R^{Z*} = 5\% \implies$ Is this outcome Pareto-efficient?

- Along the provider's iso-profit, the agent's actual welfare is maximised for $R^Z = 5.25\%$. \implies The equilibrium contract is inefficiently cheap, as predicted ($\theta > 1$).
- Exploitative contracting lowers the agent's wealth at retirement by 8% and annual consumption in retirement by 3%.
 - Sophistication would \nearrow pension wealth by 36%, and eradicating the bias by 87%.
- ► The associated CE loss of consumer welfare is **0.23%** of annual consumption.



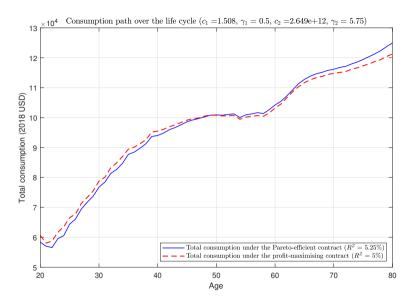


Figure 3: Contract terms and consumption paths

Policy Interventions

Table 3: Ceiling on fees

Fraction of bench- mark fee	Market R^Z	Consumer wel- fare (CE)	Provider's prof- its
1.0	5%	-	-
0.98	4.75%	-0.89%	-
0.76	4.50%	-2.03%	-
0.62	4.25%	-3.39%	-
:			
0.45	3.50%	-6.43%	-
0.40	3.50%	-6.34%	-9%
0.35	3.50%	-6.12%	-27%

Table 4: Regulating competition

Markup	Market R^Z	Consumer wel- fare (CE)	Provider's prof- its
0	5%	+0.71%	-62%
0.05	5%	+0.53%	-47%
0.10	5%	+0.36%	-31%
0.15	5%	+0.18%	-16%
0.20	5%	-	-
0.25	5%	-0.18%	+16%
0.30	5%	-0.36%	+31%

Table 5: Liquidity of pension wealth

Withdrawal penalty	Market R^Z	Efficient R^Z	Consumer wel- fare (CE)	Savings at retirement	Provider's profits
0.25	5.25%	5.25%	+2.58%	-14%	+14%
0.50	5.25%	5.25%	+1.45%	-7%	+18%
0.75	5.25%	5.25%	+0.68%	+1%	+18%
1.0	5%	5.25%	-	-	-
1.25	5%	5%	-0.34%	+6%	-13%

Conclusion

This paper: The interaction between a **present-biased saver** and a **profit-maximising financial provider**.

- 1. In a simple theoretical model:
 - Substitution effect dominates \implies exploitative contracts are 'inefficiently expensive'.
 - Income effect dominates \implies exploitative contracts are 'inefficiently cheap'.
- 2. In a calibrated life-cycle model:
 - The contract offered in market equilibrium is Pareto-inefficient ('too cheap'), lowering the agent's pension wealth by 8% and annual consumption in retirement by 3%.

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Thanks very much for your attention! sulka@dice.hhu.de

'The firm' is implicitly assumed to be a **pension fund**.

- Universal managers of pension wealth, offering tailor-made products via retail market and workplace arrangements.
- In most OECD countries, private pensions are overwhelmingly funded through pension funds:
 - 59% in the US, 54% in Canada, 100% in the UK, 97% in Australia, 61% in Japan, 100% in the Netherlands.

Empirical research on operation costs of pension funds is somewhat limited:

 \rightarrow Basu & Andrews 2014; Bateman & Mitchell 2004; Bauer et al. 2010; Bikker & de Dreu 2009; Bikker et al. 2012; Dobronogov & Murthi 2005

The literature decomposes the total cost into administrative and investment costs:

- administrative costs have substantial fixed component and increase (concavely) in the size of individual pension pot,
- determinants of investment costs are less clear, but might increase (convexly) in the rate of return and be only weakly dependent on the size of individual pension pot.

Under a ceiling on fees, firm's problem becomes:

$$\max_{r,f} \pi = f - K(r,s)$$
, s.t.:

1. $s = s(\beta, r)$ 2. $\hat{V}_0 - f \ge \underline{u}$ 3. $f \le \overline{f}$

which reduces to:

$$\max_r V_0(s(\hat{\beta},r),r) - \underline{u} - K(r,s(\beta,r))$$
, s.t.:

1. $V_0(s(\hat{\beta}, r), r) - \underline{u} \leq \overline{f}$

and Lemma 2 follows from the fact that $\frac{d\hat{V}_0}{dr} > 0$.

Lemma 2 Under an effective ceiling on fees, the optimal interest rate r^* is revised downwards. While the efficiency of a sophisticated agent's contract declines, consumer welfare is preserved. In contrast, the efficiency and consumer welfare attained by a naïve agent improve for $\theta < 1$, but decline for $\theta > 1$. In all cases, the provider's profits decrease.



Under a minimum savings requirement, firm's problem becomes:

$$\max_{r,f} \pi = f - K(r,s)$$
, s.t.:

1. $s = \max \{s(\beta, r), \underline{s}\}$ 2. $\hat{V}_0 - f \ge \underline{u}$

where $\hat{s} = \max \{ s(\hat{\beta}, r), \underline{s} \}.$

Lemma 3 The impact of a minimum savings requirement on the optimal interest rate r^* is ambiguous in general. Although for $\hat{\beta} \rightarrow 1$ the minimum savings requirement (weakly) exacerbates the original exploitative distortion, welfare loss due to naiveté might diminish.



Under perfect competition, firm's problem becomes:

$$\max_{r,f} \pi = f - K(r,s)$$
, s.t.:

1. $s = s(\beta, r)$ 2. $\hat{V}_0 - f \ge \underline{u}$ 3. $\pi = 0$

where we additionally have a zero-profit condition (ZP). ZP implies that for each r, the charged fee is set equal to the actual cost of the service $f^* = K(r, s(\beta, r))$. Thus perfect competition does away with the negative distributional effect.

If profits are strictly positive under monopoly, PC is slack, while ZP binds in equilibrium. The inefficient distortions to naifs' contracts survive, because the exploitative contracts maximise *perceived* consumer surplus.



Suppose there are two identical firms, denoted A and B, located at endpoints of a unit interval. An agent is equally likely to be located at any point along the interval. An agent located at $x \in [0, 1]$ derives the following levels of utility from the offered contracts:

$$\hat{U}_0^A = V_0(s(\hat{\beta}, r^A), r^A) - f^A - \xi x$$
$$\hat{U}_0^B = V_0(s(\hat{\beta}, r^B), r^B) - f^B - \xi (1 - x)$$

where $\xi > 0$ is parameter of 'distance aversion'.

In equilibrium, $r^{A*} = r^{B*} = r^*$ is the same as under monopoly and perfect competition.

Moreover, in a symmetric equilibrium, the two providers charge:

$$f^* = \begin{cases} \xi + K(r^*, s(\beta, r^*)) & \text{ for } \xi \in [0, f^M - K(r^*, s(\beta, r^*))] \\ f^M & \text{ for } \xi > f^M - K(r^*, s(\beta, r^*)) \end{cases}$$

where f^M denotes the profit-maximising fee charged by a monopolistic provider.

Back

With variable fees on savings, firm's problem becomes:

$$\max_{r,f,t} \pi = f + ts - K(r,s)$$
, s.t.:

1.
$$s = s(\beta, r, t)$$

2. $V_0(s(\hat{\beta}, r, t), r, t) - f \ge \underline{u}$

where

$$V_0(s(\hat{\beta}, r, t), r, t) = \frac{[Y - s(\hat{\beta}, r, t) \times (1+t)]^{1-\theta} - 1}{1-\theta} + \delta \frac{[(1+r) \times s(\hat{\beta}, r, t)]^{1-\theta} - 1}{1-\theta}$$
$$s(\beta, r, t) = \frac{1}{(1+t) + (\beta\delta)^{\frac{-1}{\theta}}(1+r)^{\frac{\theta}{\theta}}(1+t)^{\frac{1}{\theta}}} \times Y$$



With variable fees on assets, firm's problem becomes:

$$\mathrm{max}_{r,f,t} \ \ \pi = f + t(1+r)s - K(r,s) \text{, s.t.:}$$

1. $s = s(\beta, \check{r})$ 2. $V_0(s(\hat{\beta}, \check{r}), \check{r}) - f \ge \underline{u}$

where

$$V_0(s(\hat{\beta},\check{r}),\check{r}) = \frac{[Y - s(\hat{\beta},\check{r})]^{1-\theta} - 1}{1-\theta} + \delta \frac{[(1+\check{r}) \times s(\hat{\beta},\check{r})]^{1-\theta} - 1}{1-\theta}$$

and $\check{r} \equiv (1+r)(1-t) - 1$.



Suppose that share $\lambda \in [0,1]$ of agents are present-biased naifs ($\beta < \hat{\beta} = 1$), while $(1 - \lambda)$ are time-consistent ($\hat{\beta} = \beta = 1$).

Under **imperfect observability with homogeneous beliefs**, the firm's optimal pooling contract solves:

$$\max_{r,f} \mathbb{E}[\pi] = \mathbb{E}[f - K(r,s)]$$
, s.t.:

1.
$$s = s(\beta, r)$$
 with probability λ ; $s = s(1, r)$ with probability $(1 - \lambda)$
2. $V_0(s(1, r), r) - f \ge \underline{u}$



Suppose that share $\lambda \in [0,1]$ of agents are naifs ($\beta < \hat{\beta} = 1$), while $(1 - \lambda)$ are sophisticates ($\hat{\beta} = \beta < 1$).

Under **imperfect observability with heterogeneous beliefs**, the firm's optimal screening contract solves:

$$\begin{aligned} \max_{r^{N},r^{S},f^{N},f^{S}} & \mathbb{E}[\pi] = \lambda\{f^{N} - K(r^{N},s^{N})\} + (1-\lambda)\{f^{S} - K(r^{S},s^{S})\}, \text{ s.t.} \end{aligned}$$
1. $s^{N} = s(\beta, r^{N})$
2. $s^{S} = s(\beta, r^{S})$
3. $V_{0}(s(1,r^{N}),r^{N}) - f^{N} \geq \underline{u}$
4. $V_{0}(s(\beta,r^{S}),r^{S}) - f^{S} \geq \underline{u}$
5. $V_{0}(s(1,r^{N}),r^{N}) - f^{N} \geq V_{0}(s(1,r^{S}),r^{S}) - f^{S}$
6. $V_{0}(s(\beta,r^{S}),r^{S}) - f^{S} \geq V_{0}(s(\beta,r^{N}),r^{N}) - f^{N}$



Consider a simple model of **financial naiveté** about the fee f. The firm posts two fees - a 'headline fee' \underline{f} and an 'actual fee' \overline{f} . An agent is always charged \overline{f} . However, an agent believes the fee to be \underline{f} with probability $\hat{p} \in [0, 1]$ and \overline{f} with residual probability $(1 - \hat{p})$. Thus \hat{p} is the measure of agent's financial naiveté. The firm's problem is:

$$\max_{r,\underline{f},\overline{f}}\pi=ar{f}-K(r,s)$$
 s.t.:

1.
$$s = s(\beta, r)$$

2. $V_0(s(\hat{\beta}, r), r) - \hat{p}\underline{f} - (1 - \hat{p})\overline{f} \ge \underline{u}$

Consider a simple model of **financial naiveté** about the rate of return r. The firm posts two interest rates - a 'headline rate' \overline{r} and an 'actual rate' \underline{r} . An agent always earns \underline{r} . However, he believes the rate of return to be \overline{r} with probability $\hat{p} \in [0, 1]$ and \underline{r} with residual probability $(1 - \hat{p})$. Thus \hat{p} is the measure of agent's financial naiveté.

If an agent learns the true r in period 1, the firm's problem is:

$$\mathrm{max}_{\bar{r},\underline{r},f}\;\pi=f-K(\underline{r},s)\quad \text{ s.t.: }$$

1.
$$s = s(\beta, \underline{r})$$

2. $(1 - \hat{p})V_0(s(\hat{\beta}, \underline{r}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \overline{r}), \overline{r}) - f \ge \underline{u}$

If, on the other hand, an agent is uncertain about the true r in period 1, the firm's problem is:

$$\max_{\bar{r},\underline{r},f}\,\pi=f-K(\underline{r},s)\quad \text{ s.t.: }$$

1.
$$s = s(\beta, \overline{r}, \underline{r}, \hat{p})$$

2. $(1 - \hat{p})V_0(s(\hat{\beta}, \overline{r}, \underline{r}, \hat{p}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \overline{r}, \underline{r}, \hat{p}), \overline{r}) - f \ge \underline{u}$

where $s(\beta, \bar{r}, \underline{r}, \hat{p})$ maximises:

$$\mathbb{E}U_1 = \frac{[Y-s]^{1-\theta}-1}{1-\theta} + \beta \delta \{ \hat{p} \frac{[(1+\bar{r})s]^{1-\theta}-1}{1-\theta} + (1-\hat{p}) \frac{[(1+\bar{r})s]^{1-\theta}-1}{1-\theta} \}$$



Borrowing constraints are captured by:

$$Z_t \ge 0, \qquad X_t \ge -\lambda \bar{Y}_t \qquad \forall t$$

Let I_t^i denote the agent's net investment into asset *i*. Then, dynamic budget constraints are:

$$Z_{t+1} = (1 + R^Z)(Z_t + I_t^Z)$$

$$X_{t+1} = (1+R)(X_t + I_t^X)$$

where $R = R^X$ if $X_t + I_t^X \ge 0$, and $R = R^{CC}$ otherwise.

Let κ_t denote the withdrawal penalty associated with the illiquid asset. Then, the static budget constraint is:

$$C_t = Y_t - I_t^Z - I_t^X + \kappa_t \min(I_t^Z, 0)$$



Sensitivity analysis (1/3)

Target	Calibrated parame- ters	Pareto-efficient R^Z	CE welfare loss	Retirement savings
Benchmark	$k_1 = 1.5083 \gamma_1 = 0.5 k_2 = 2.650 \times 10^{12} \gamma_2 = 5.75 \xi = 0.481$	5.25%	0.23%	-8%
$\gamma_1 = 0.35$	$ \begin{array}{l} k_1 = 10.0620 \\ \gamma_1 = 0.35 \\ k_2 = 3.206 \times 10^{12} \\ \gamma_2 = 5.80 \\ \xi = 0.385 \end{array} $	5.25%	0.23%	-8%
$\gamma_1 = 0.65$		5.25%	0.23%	-8%
Cost-to-assets ratio 0.0035	$k_1 = 1.0558 \gamma_1 = 0.5 k_2 = 1.213 \times 10^{15} \gamma_2 = 7.45 \xi = 0.308$	5.25%	0.26%	-8%

Sensitivity analysis (2/3)

Cost-to-assets ratio 0.0065	$k_1 = 1.9607 \gamma_1 = 0.5 k_2 = 3.544 \times 10^{10} \gamma_2 = 4.55 \xi = 0.602$	5.25%	0.26%	-8%
Admin share 0.40	$k_1 = 1.2066 \gamma_1 = 0.5 k_2 = 2.203 \times 10^{11} \gamma_2 = 5.05 \xi = 0.385$	5.25%	0.22%	-8%
Admin share 0.60	$k_1 = 1.8099 \gamma_1 = 0.5 k_2 = 7.938 \times 10^{13} \gamma_2 = 6.70 \xi = 0.481$	5.25%	0.23%	-8%
Markup 0.15	$k_1 = 1.5083 \gamma_1 = 0.5 k_2 = 2.649 \times 10^{12} \gamma_2 = 5.75 \xi = 0.385$	5.25%	0.23%	-8%
Markup 0.25		5.25%	0.23%	-8%

Sensitivity analysis (3/3)

$R^{Z*} = 4.75\%$	$k_1 = 1.4079$	5.00%	0.31%	-11%
	$\gamma_1 = 0.5$ $k_2 = 2.554 \times 10^{12}$ $\gamma_2 = 5.60$ $\xi = 0.385$			
$R^{Z*} = 5.25\%$	$ \begin{array}{l} k_1 = 1.4079 \\ \gamma_1 = 0.5 \\ k_2 = 2.554 \times 10^{12} \\ \gamma_2 = 5.60 \\ \xi = 0.385 \\ k_1 = 1.5936 \\ \gamma_1 = 0.5 \\ k_2 = 2.306 \times 10^{12} \\ \gamma_2 = 5.85 \\ \xi = 0.481 \end{array} $	5.50%	0.26%	-6%

Robustness checks (1/3)

Check	Calibrated parameters	Pareto-efficient R^Z	CE welfare loss	Retirement savings
Benchmark	$ \begin{array}{c} k_1 = 1.5083 \\ \gamma_1 = 0.5 \\ k_2 = 2.650 \times 10^{12} \\ \gamma_2 = 5.75 \ \xi = 0.481 \end{array} $	5.25%	0.23%	-8%
$K(R^{Z}, Z_{t}) = k_{1} + k_{2}(R^{Z} - R^{X})^{\gamma_{2}}$	$ \begin{array}{l} k_1 = 793.36 \\ k_2 = 4.6947 \times 10^{12} \\ \gamma_2 = 5.90 \\ \xi = 0.2465 \end{array} $	5.25%	0.23%	-8%
$K(R^Z, Z_t) = k_1 + k_2 [(R^Z - R^X)Z(t)]^{\gamma_2}$	$k_1 = 793.36k_2 = 1.3966 \times 10^{-19}\gamma_2 = 5.45\xi = 0.7523$	5.25%	0.23%	-8%
Alt. business model	$k_1 = 1.5083 \gamma_1 = 0.5 k_2 = 4.695 \times 10^{12} \gamma_2 = 5.90 \xi = 0.3852$	5.25%	0.17%	-8%
Competition (markup of 0)	$k_1 = 1.5083 \gamma_1 = 0.5 k_2 = 2.1895 \times 10^{12} \gamma_2 = 5.70 \xi = 0.1262$	5.25%	0.26%	-8%

Robustness checks (2/3)

	1	I	1	I
Monopoly (no markup target)	$k_1 = 1.5083 \gamma_1 = 0.5 k_2 = 5.7629 \times 10^{11} \gamma_2 = 5.35 \xi = 4.4842$	6%	1.14%	-26%
Variable fee 0.005 (markup of 0.30)	$k_1 = 1.2709 \gamma_1 = 0.5 k_2 = 3.822 \times 10^{15} \gamma_2 = 7.75 \xi = 0.0414$	5.25%	0.26%	-13%
$egin{aligned} & heta &= 2.0 \ & (eta &= 0.4319, \ & \delta &= 0.9941) \end{aligned}$	$k_1 = 1.8281 \gamma_1 = 0.5 k_2 = 4.633 \times 10^8 \gamma_2 = 3.40 \xi = 0.00013$	5.50%	0.42%	-5%
$\theta = 2.0$ ($\beta = 0.4319$, $\delta = 0.9941$); monopoly	$k_1 = 1.8281 \gamma_1 = 0.5 k_2 = 1.008 \times 10^8 \gamma_2 = 3.00 \xi = 0.0012$	6.50%	2.61%	-14%
$egin{aligned} & heta = 0.5 \ & (eta = 0.7473, \ & \delta = 0.9733) \end{aligned}$	$k_1 = 1.1199\gamma_1 = 0.5k_2 = 8.426 \times 10^{15}\gamma_2 = 8.05\xi = 26.728$	5.25%	0.13%	-57%

Robustness checks (3/3)

$\theta = 0.5$ ($\beta = 0.7473$, $\delta = 0.9733$); monopoly		3.50%	1.37%	+100%
Partial naiveté ($\hat{\beta} = 0.87575$)	$k_1 = 1.5109 \gamma_1 = 0.5 k_2 = 8.114 \times 10^{14}$	5.00%	-	-
Partial naiveté ($\hat{eta}=0.7515$)	$\begin{aligned} \gamma_2 &= 7.25\\ \xi &= 0.4815\\ k_1 &= 1.5280\\ \gamma_1 &= 0.5\\ k_2 &= 5.677 \times 10^{14}\\ \gamma_2 &= 7.15\\ \xi &= 0.3852 \end{aligned}$	5.00%	-	-

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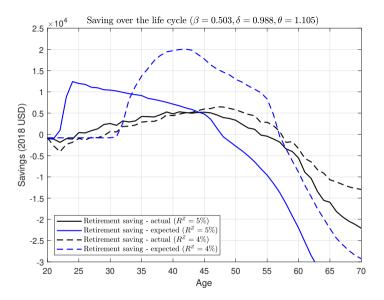


Figure 2: Retirement saving paths (expected and actual)

The comparison with a behaviour of a sophisticated present-biased agent is not as straightforward. On the one hand, sophisticates may want to compensate for their future self-control problem, but on the other hand, they take into account the fact that their future selves will use the resources in a 'suboptimal' way.

 Harris and Laibson (2001) derive an approximate solution to the intrapersonal game called Hyperbolic Euler Equation:

$$u'(C_t) \ge \mathbb{E}_t \left[(1+R^Z) \left\{ \frac{\partial C_{t+1}}{\partial Z_{t+1}} \beta \delta + (1-\frac{\partial C_{t+1}}{\partial Z_{t+1}}) \delta \right\} u'(C_{t+1}) \right]$$

which holds in the neighbourhood of $\beta = 1$.

Salanié and Treich (2006) use a three-period model to demonstrate that under the CRRA formulation, sophisticates save more than naifs for θ > 1, but the reverse is true for θ < 1. With a logarithmic utility function, savings are independent of the degree of naiveté.</p>



The firm-side calibration matches some non-targeted moments well:

- 1. The equilibrium fee is equal to 0.6% of assets, measured at the mean.
 - Fee levels observed in the pension industry are typically between 0.5% and 1.7% (Dobronogov, Murthi 2005; OECD 2017; Tapia, Yermo 2008).
- 2. The absolute administrative costs are equal to \$1511 on average.
 - Bikker and de Dreu (2009) note a substantial heterogeneity in administrative costs of the Dutch pension funds, which vary from \$53 to \$1509. These bounds are slightly narrower for a sample of larger pension funds from four countries in Bikker et al. (2012), ranging from \$30 to \$674. For Australia, Bateman and Mitchell (2004) report the range from \$105 to \$897.
- 3. The cost-to-assets ratio, which is calibrated to a value of 0.005 at the means, varies from 0.0035 to 0.0289 over the life cycle, with an average of 0.009



Simple Model

Extensions

1. Simplifying technical assumptions

- 1.1) Quasi-Linearity: Relaxing the assumption of quasi-linear utility function results in a different cutoff $\tilde{\theta} < 1$, but the results are otherwise unaffected.
- 1.2) Endogenous \underline{u} : When naifs misperceive utility from their outside option (e.g. costless bank account), the negative distributional effect is mitigated, but this does not affect the efficiency effect.

2. Competition

- Perfect competition does away with the distributional effect, but the exploitative distortion to r^* is unaffected. Firm's problem
- Hotelling-style model of imperfect competition delivers the same r^* and fees that change monotonically between the monopolistic and perfectly competitive levels. The model

- 3. Variable fees
 - Variable fee on savings shifts r^* in the direction that results in $s(\beta, r) \uparrow$, irrespective of $\hat{\beta}$. But since naif's valuation is less sensitive to the variable fee, this only amplifies the baseline distortion. Firm's problem
 - Same logic applies when the variable fee is calculated based on assets.
- Firm's problem

- 4. Alternative business model
 - When offering a defined-benefit pension, the provider can profit from the wedge between the total return on the investment r^P and the payout promised to the agent $r^C \leq r^P$:

$$\tilde{\pi} = \delta^{P}[(1+r^{P})(f+s) - (1+r^{C})s] - \tilde{K}(r^{P})$$

Then, the provider has an additional incentive to collect and re-invest high fees. This may overturn the original exploitative distortion, but only for $\theta > 1$ and steep enough cost function $K(\cdot)$.

- 5. Financial (il)literacy
 - When the provider can offer both 'headline' and 'actual' contract terms (either r or f), and an agent attaches too high a weight to the headline parameters, the distortion due to naiveté carries over in qualitative sense, in addition to the headline parameters seeming very attractive.
 - If the overoptimism is about the fees (Model), the baseline distortion is amplified. But not if the overoptimism is about returns (Model).

- 6. Menu contracts
 - Defence: Pension contracts heavily regulated (non-discrimination rules) + simple contracting space allows to link theory to a numerical life-cycle model.
 - But: Offering r(s) would only exacerbate the welfare loss from naiveté.

- 7. Imperfect observability
 - 7.1) Heterogeneous β , same $\hat{\beta}$

Firm's problem

 \implies Pooling contract can make naifs better or worse off, depending on θ and a cross-partial derivative of K(r,s).

7.2) Heterogeneous $\hat{\beta}$, same β (Firm's problem) \implies Screening contract makes naifs better off $(f_N^* \downarrow)$, but shifts sophisticate's contract (r_S^*) away from first-best.



Consider three common policy interventions.

(Timing: The firm is allowed to respond.)

1. Ceiling on fees

Firm's problem

- Under an effective ceiling, $r^{*}\downarrow$
- Sophisticate's contract no longer efficient, but welfare preserved.
- Efficiency and consumer welfare attained by a naif improve for $\theta < 1$, but decline for $\theta > 1$.
- Firm's profits decrease.

- 2. Regulating competition
 - Boils down to $\underline{u}\uparrow$, either due to lower barriers to entry/switching or more comprehensive public pensions.
 - Lower fees $f^* \downarrow$ (consumer welfare \uparrow , firm's profits \downarrow), but no impact on efficiency.
 - (Relies on quasi-linearity.)

3. Minimum savings requirement

Firm's problem

- Now $s = \max \{s(\beta, r), \underline{s}\}.$
- Whether the requirement binds, and whether the agent realises this, depends on r.
- Impact of \underline{s} on r^* is ambiguous in general.
 - \implies Deception vs. revenues from commitment.
- Although for $\hat{\beta} \rightarrow 1$ the minimum savings requirement exacerbates the original exploitative distortion, welfare loss due to naiveté might diminish.
 - \implies A naif might obtain commitment, for which she does not pay.