Optimal Long-Run Fiscal Policy with Heterogeneous Agents

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Optimal long-run level of debt?

- **Trade-off:** Liquidity benefits (e.g. self insurance) vs. distortionary taxation
- Bewley-Huggett-Aiyagari models
- Two concepts for **long-run optimality:**
  - Optimal steady state (OSS)
  - Ramsey steady state (RSS)

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- Transitional dynamics matter: discounted at β
- OSS special case with social discount factor = 1
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Lots of work on OSS!

e.g. Aiyagari McGrattan (1998)

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Little work on RSS!

Aiyagari (1995) simply assumes it…

Exceptions: Acikgöz Hagedorn Holter Wang (2022)
Chien Wen (2022), LeGrand Ragot (2023)

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- **Basic intuition:** Distortionary cost of higher debt lower during transition!

Heterogeneous-agent economy

Households

$$\max_{\{c_{it}, n_{it}, a_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it})$$

$$c_{it} + a_{it} = (1 + r_t)a_{it-1} + w_t e_{it} n_{it} \qquad a_{it} \geq 0$$

standard Markov process

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Aggregate household behavior:

	sequence space	*steady state*
Consumption c_{it}	$\mathscr{C}_t(\{r_s, w_s\})$	$\mathscr{C}^{ss}(r, w)$
Effective labor $e_{it}n_{it}$	$\mathscr{N}_t(\{r_s, w_s\})$	$\mathscr{N}^{ss}(r, w)$
Utility $u(c_{it}, n_{it})$	$\mathscr{U}_t(\{r_s, w_s\})$	$\mathscr{U}^{ss}(r, w)$

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Note: $\mathscr{C}, \mathscr{N}$ automatically consistent with aggregate household budget constraint

Production and government policy

- Representative firm: $Y_t = \mathcal{N}_t$ (similar with capital)
- Government: spends fixed $G > 0$ (can relax)
  - controls debt → implement any path $\{r_s\}$
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  - controls <u>debt</u> $\rightarrow$ implement any path $\{r_s\}$
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- **Implementability in the sequence space:**

$\{r_s\}$, $\{w_s\}$ part of an equilibrium $\iff$ $\mathscr{C}_t(\{r_s, w_s\}) + G = \mathcal{N}_t(\{r_s, w_s\})$

Optimal steady state

Optimal steady state problem

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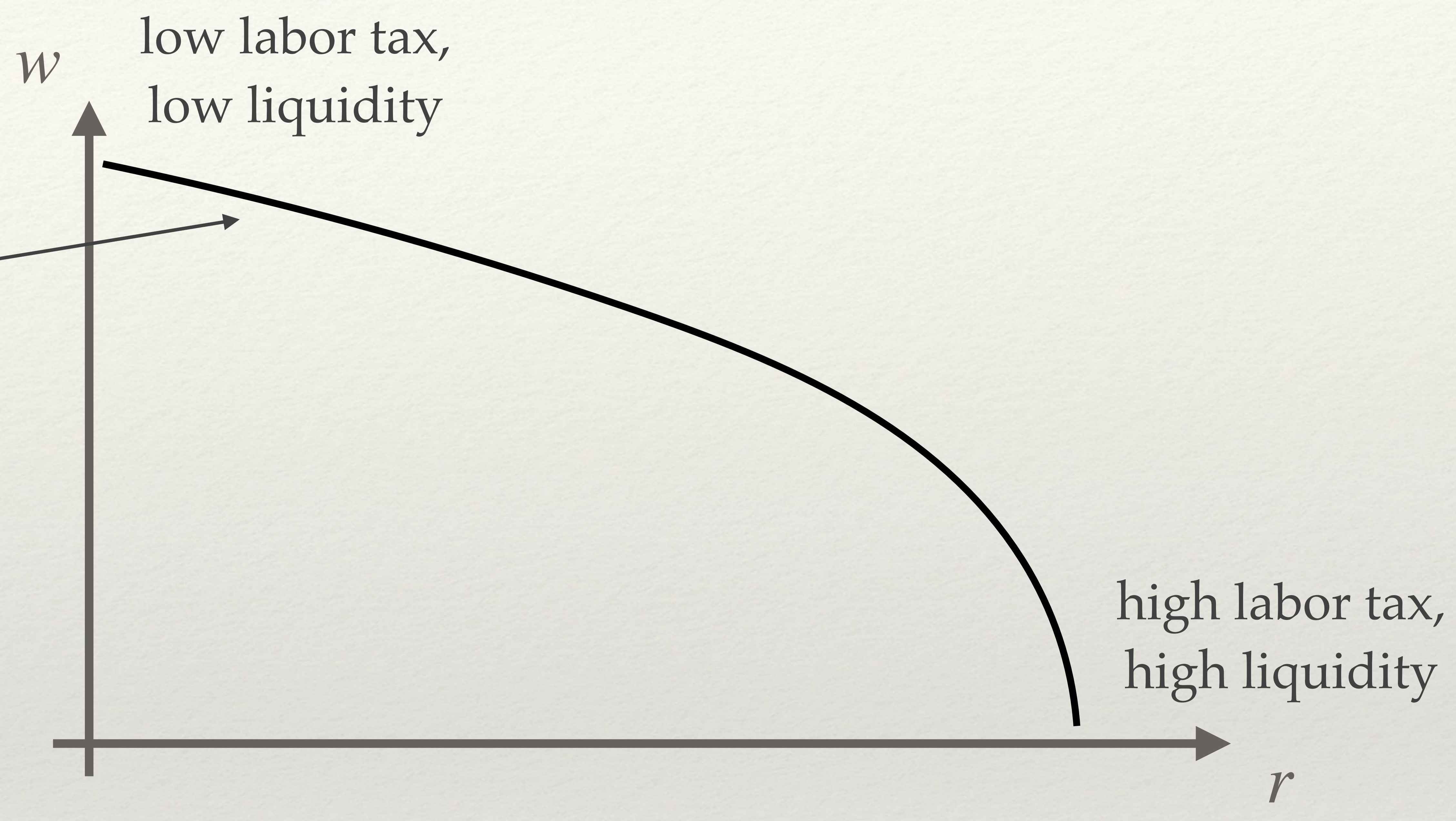

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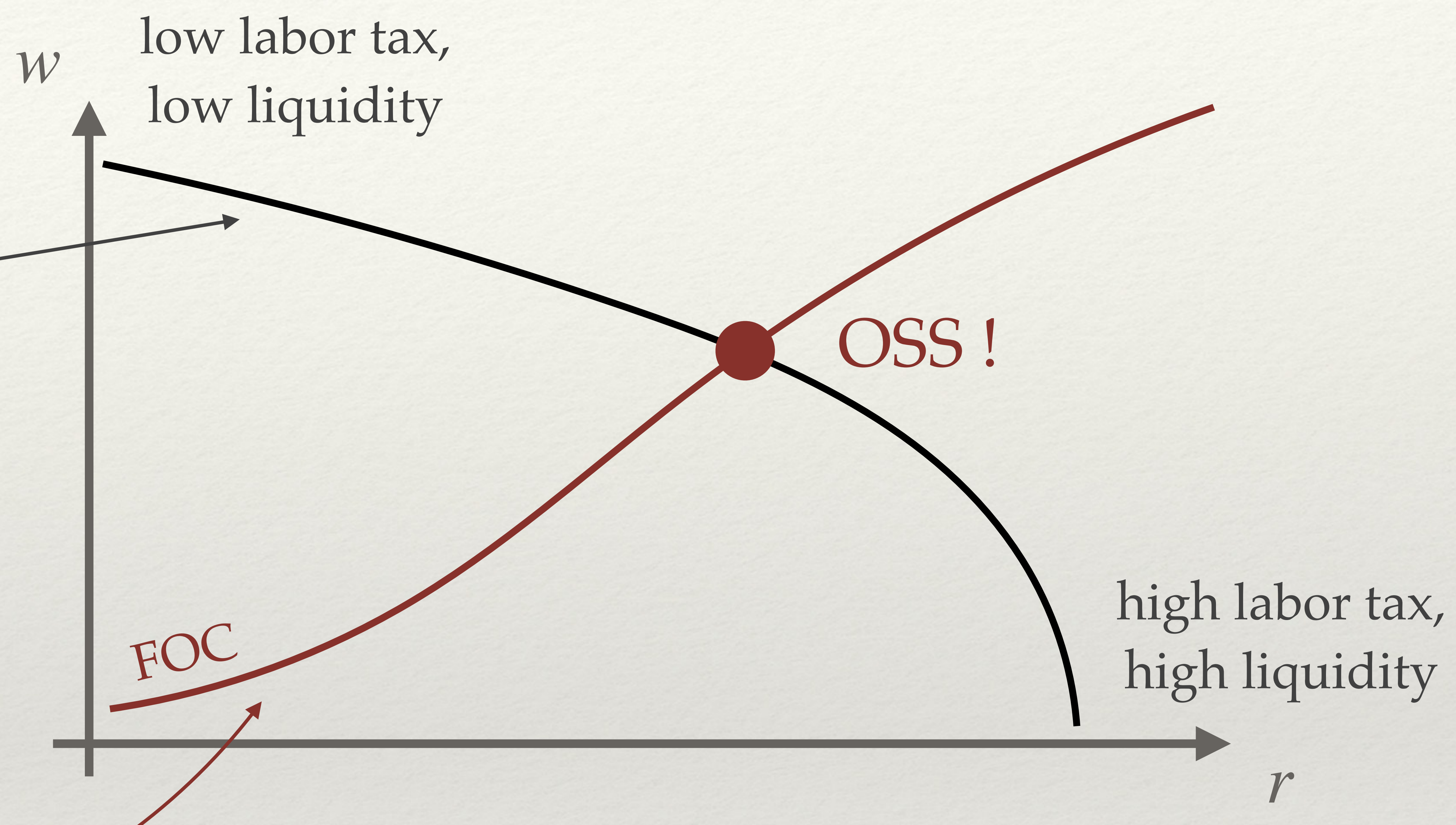

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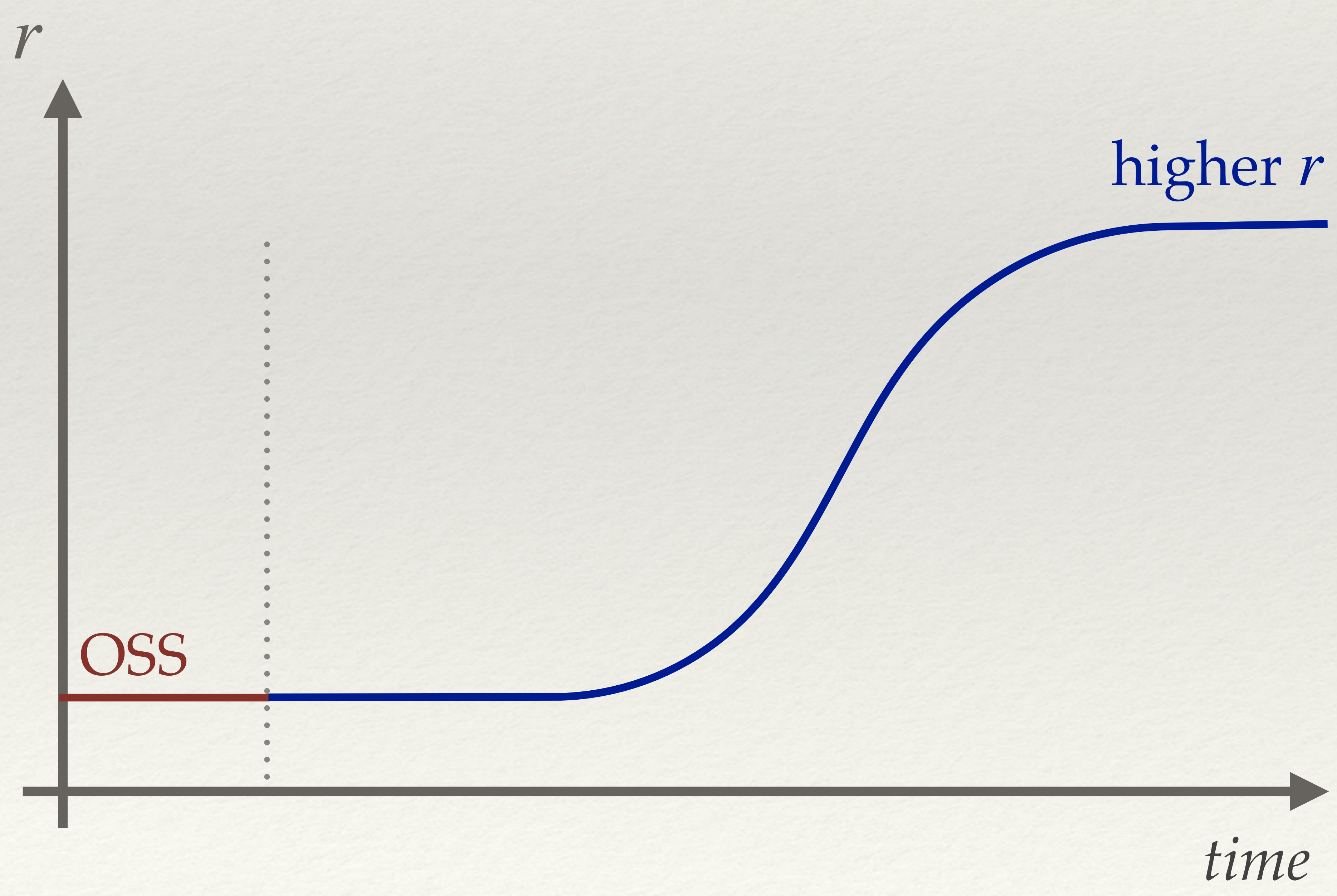
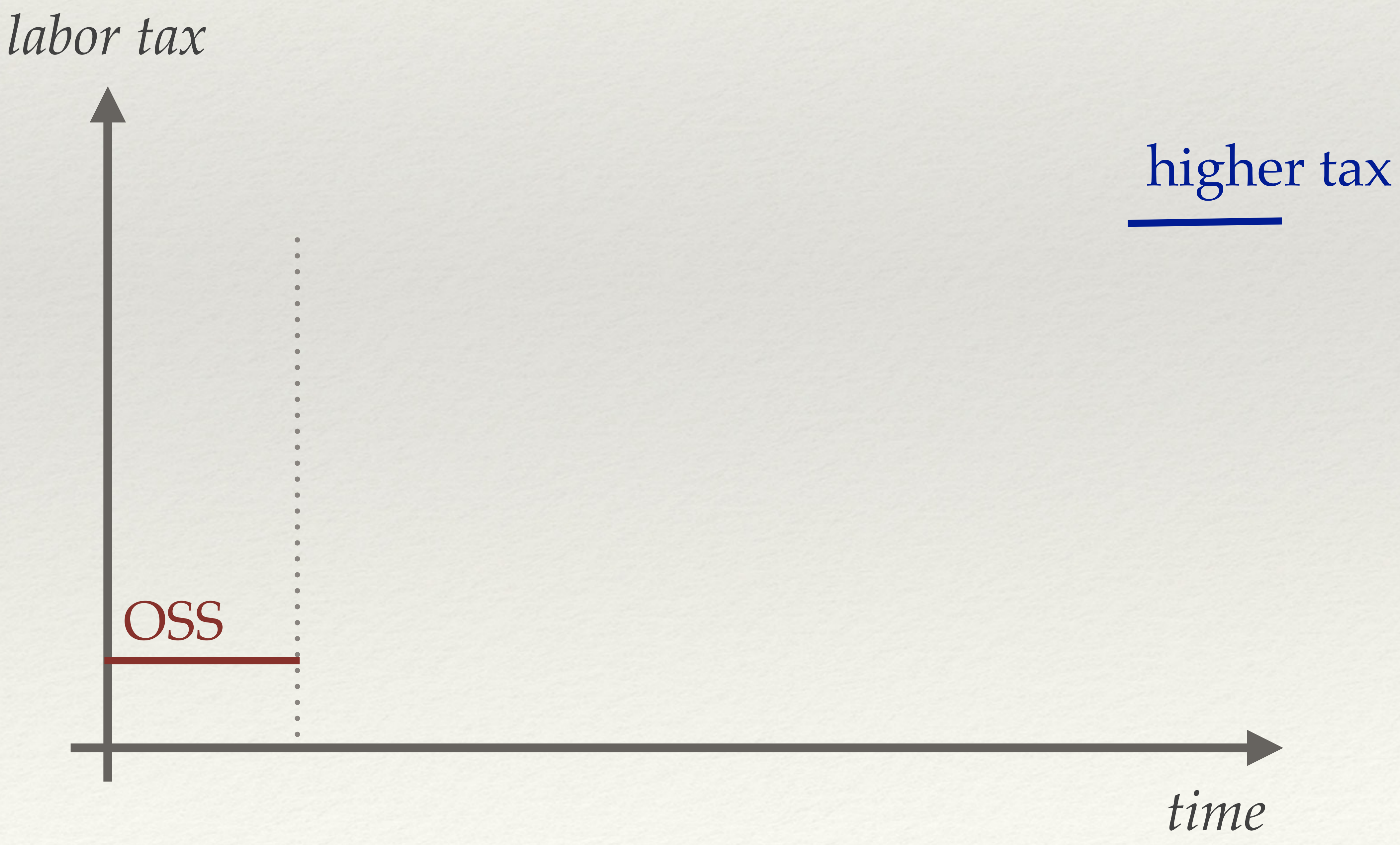
What is this missing?

- All derivatives **ignore transitions**!
- Ex: imagine starting at OSS, then contemplate increase in r





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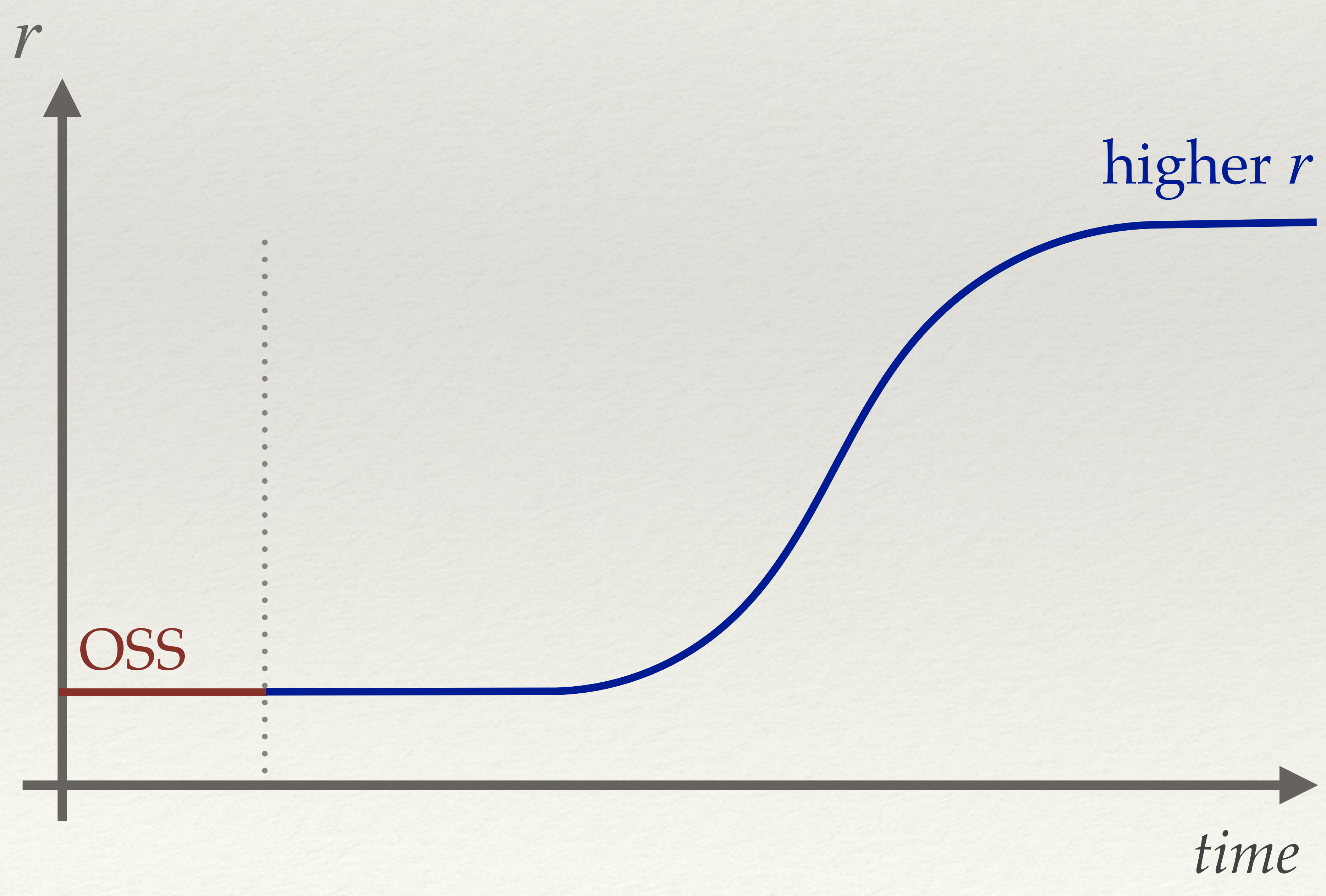

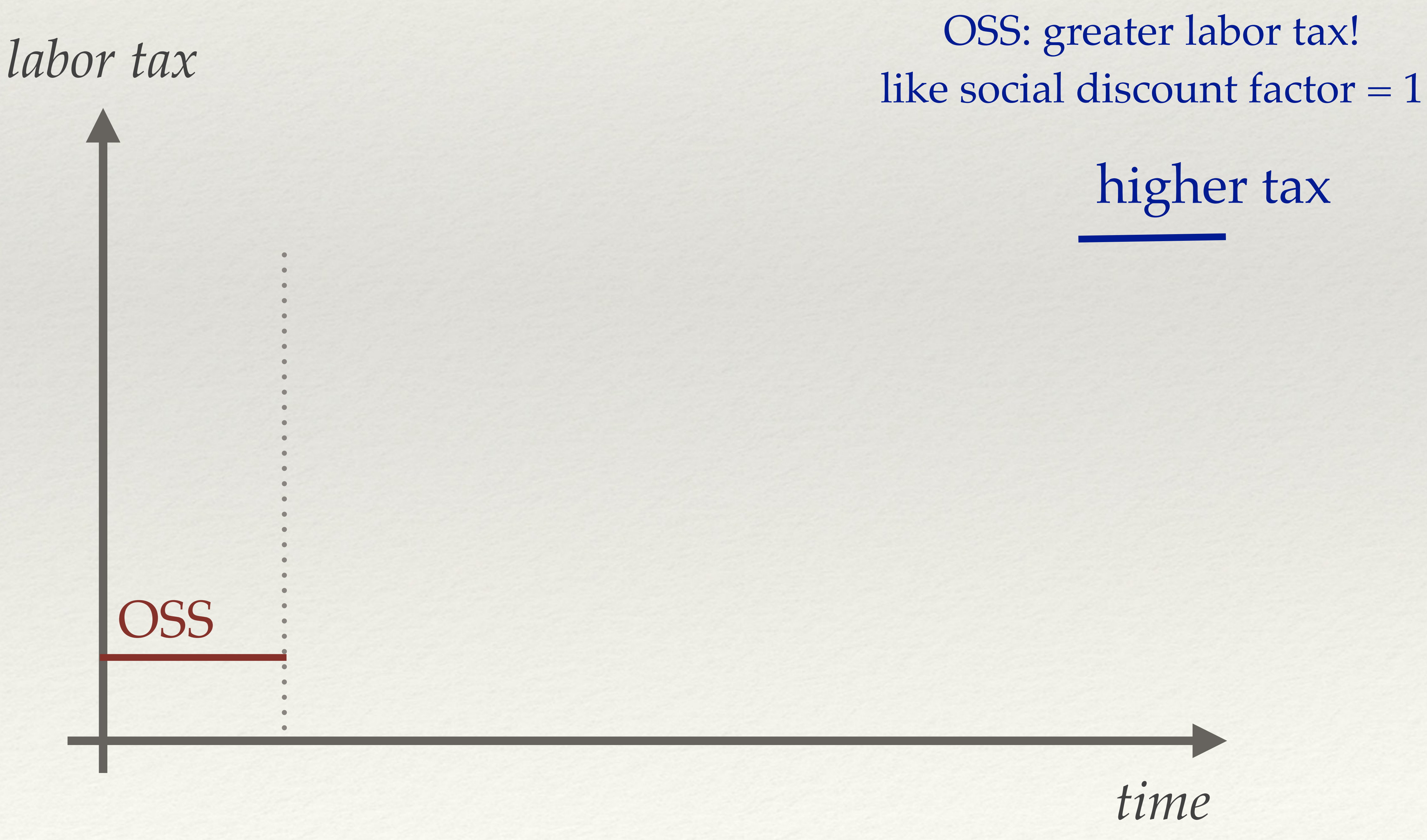

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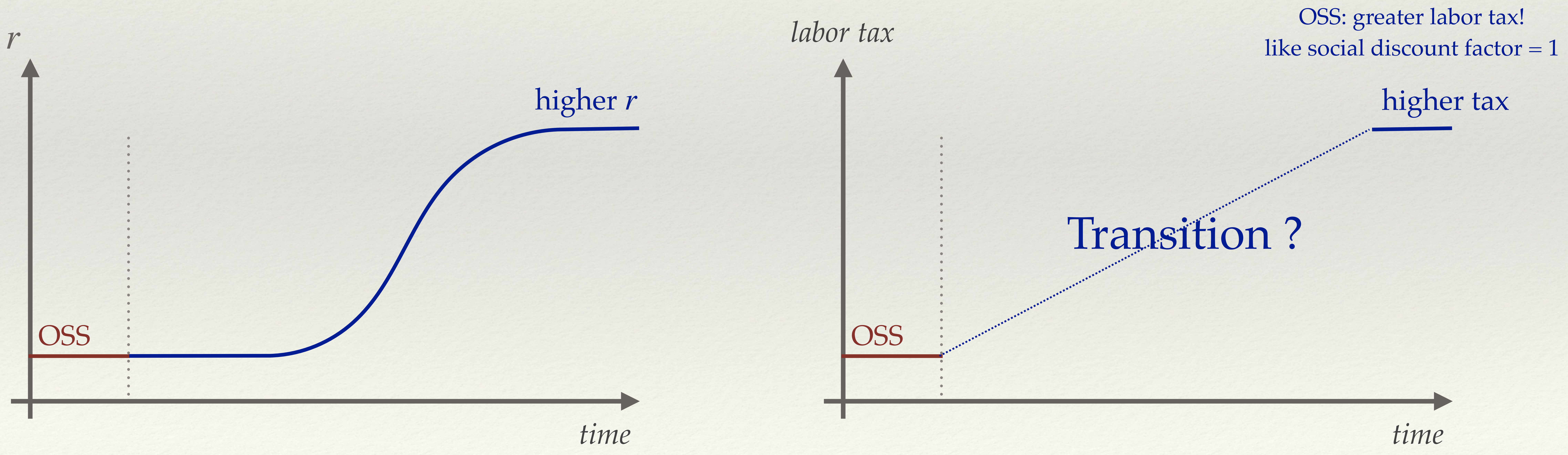

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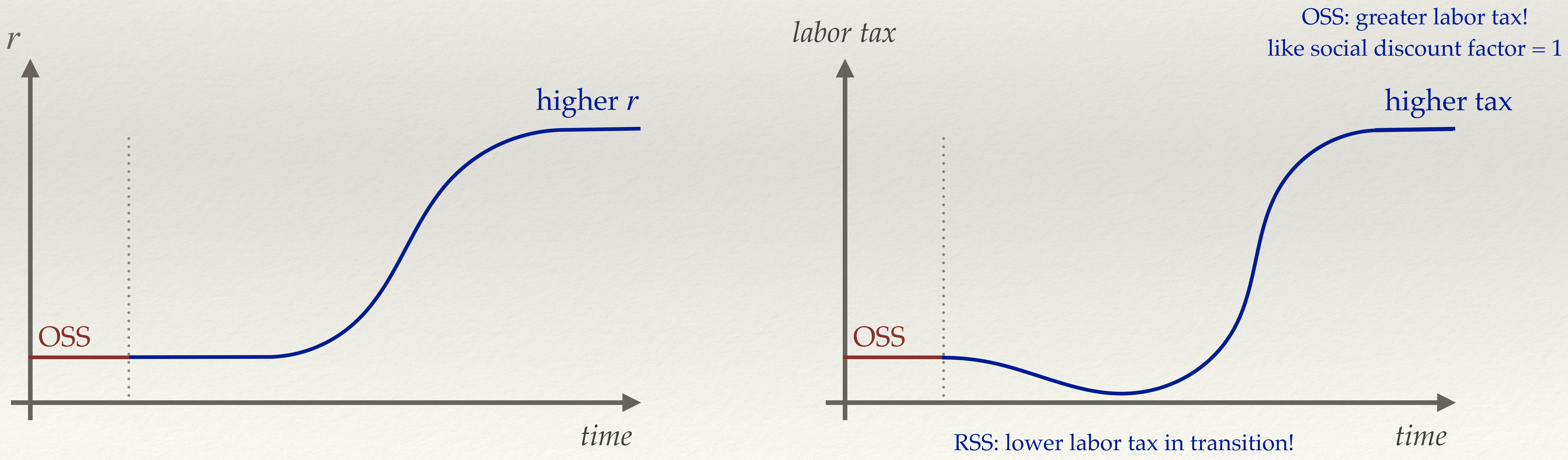

Ramsey steady state

β-derivatives

- Want to differentiate sequence-space functions in a way that
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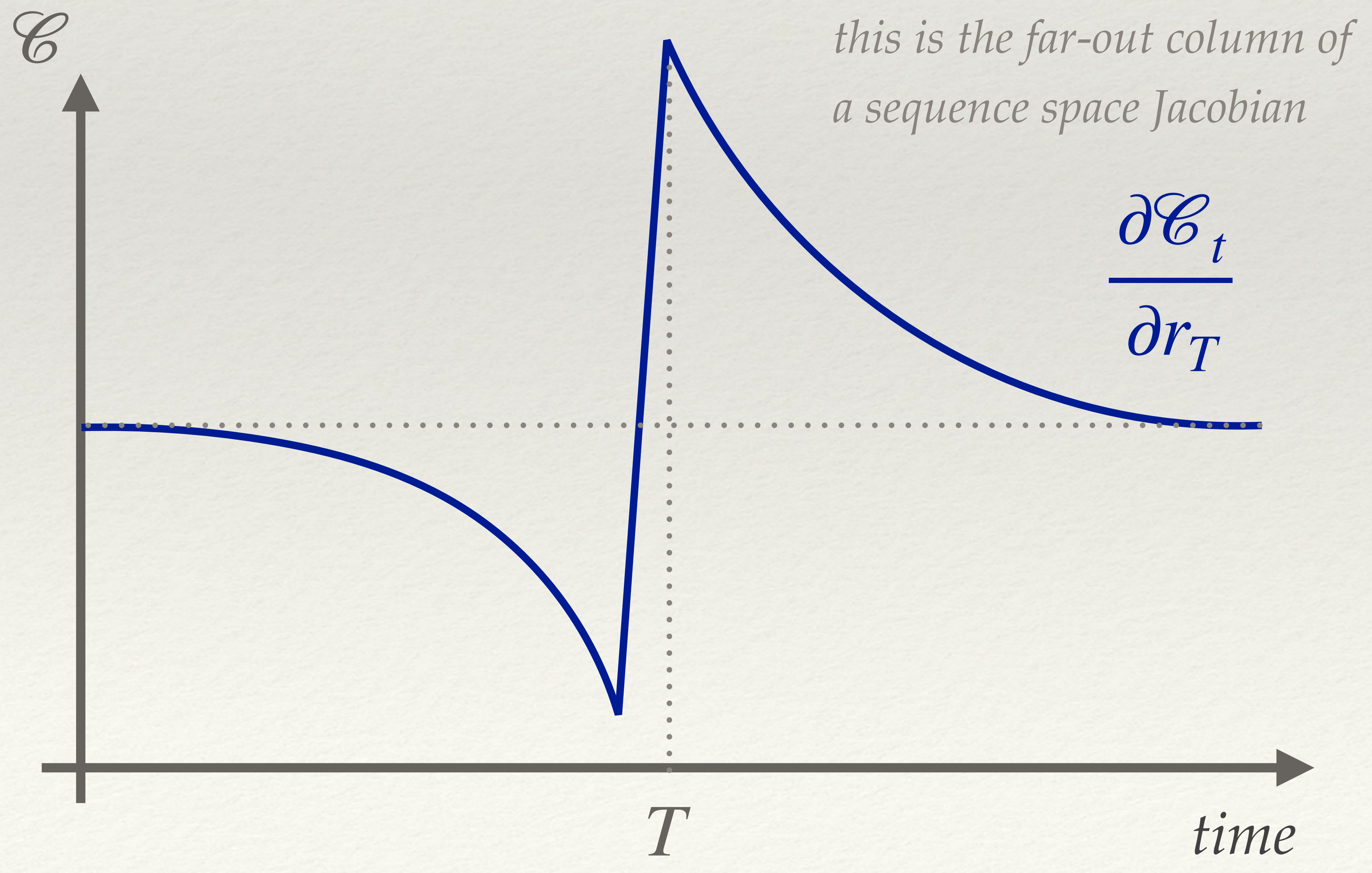

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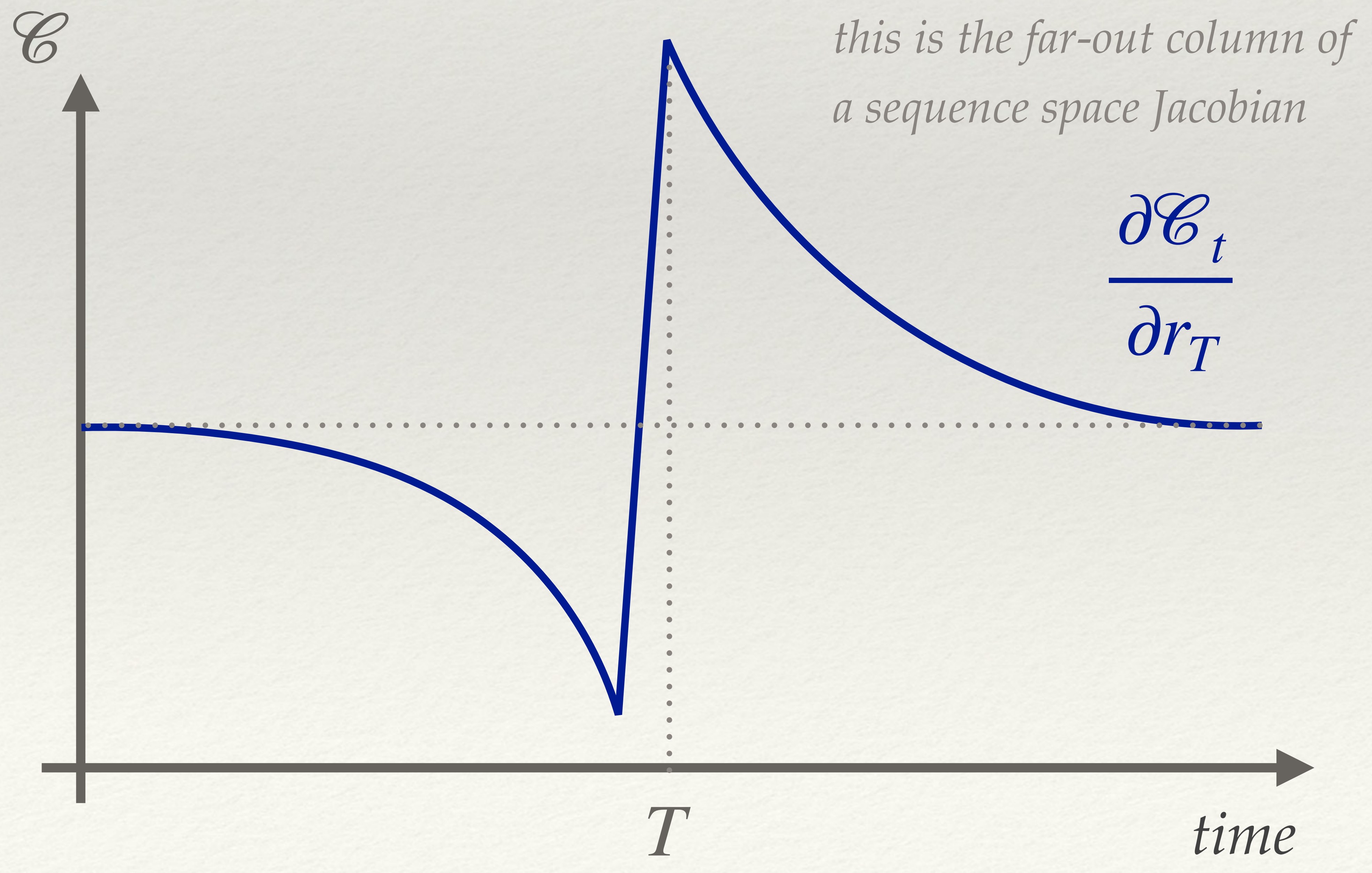

Def: *β-derivative* of $\mathscr{C}$ w.r.t. r is:

$$\mathscr{D}^{\beta}_{\mathscr{C},r} \equiv \lim_{T\to\infty} \sum_{t=0}^{\infty} \beta^{t-T} \frac{\partial \mathscr{C}_t}{\partial r_T}$$

- includes entire transition, discounted with β
- can allow for arbitrary social discount factor
- very easy to compute

Ramsey steady state

Full-commitment Ramsey problem

$$\max_{\{r_s, w_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \mathscr{U}_t(\{r_s, w_s\})$$

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<u>Result:</u> If RSS exists, it satisfies

(discounted) increase in labor tax
to cover greater r

$$\mathcal{D}^{\beta}_{\mathcal{U},r} = \frac{\mathcal{D}^{\beta}_{\mathcal{C},r} - \mathcal{D}^{\beta}_{\mathcal{N},r}}{\mathcal{D}^{\beta}_{\mathcal{C},w} - \mathcal{D}^{\beta}_{\mathcal{N},w}} \cdot \mathcal{D}^{\beta}_{\mathcal{U},w}$$

discounted liquidity
benefit from raising r

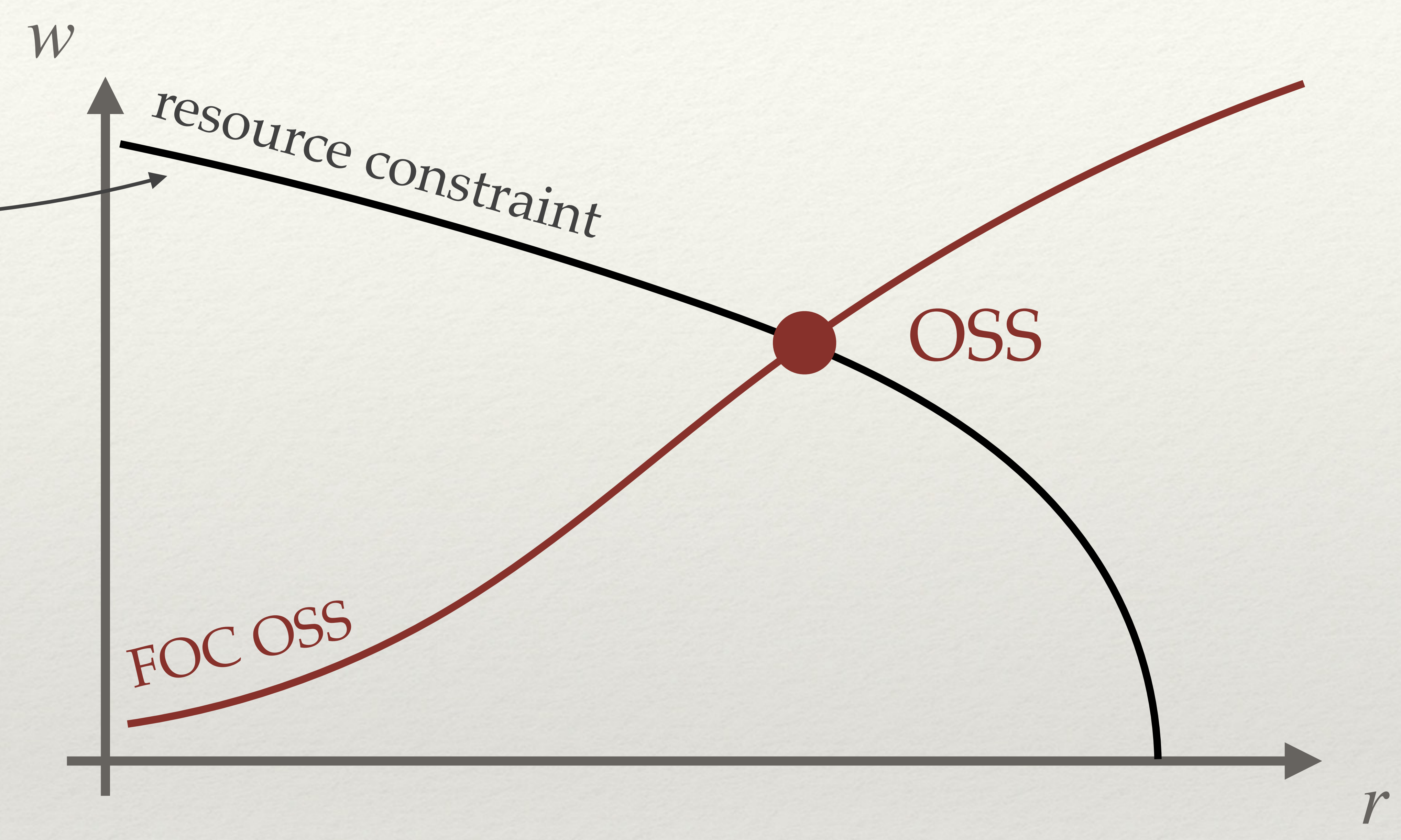
discounted utility cost
from greater labor tax

Utility functions

- What does RSS look like? Turns out to depend on utility function $u(c, n)$
- For today, focus on one of two cases:
  - $u(c, n) = \log c - v(n)$ $\leftarrow$ balanced growth compatible
  - $u(c, n) = \log(c - v(n))$ $\leftarrow$ GHH

Balanced growth: No RSS!

$$\mathscr{C}^{ss}(r,w) + G = \mathscr{N}^{ss}(r,w)$$



RSS FOC:

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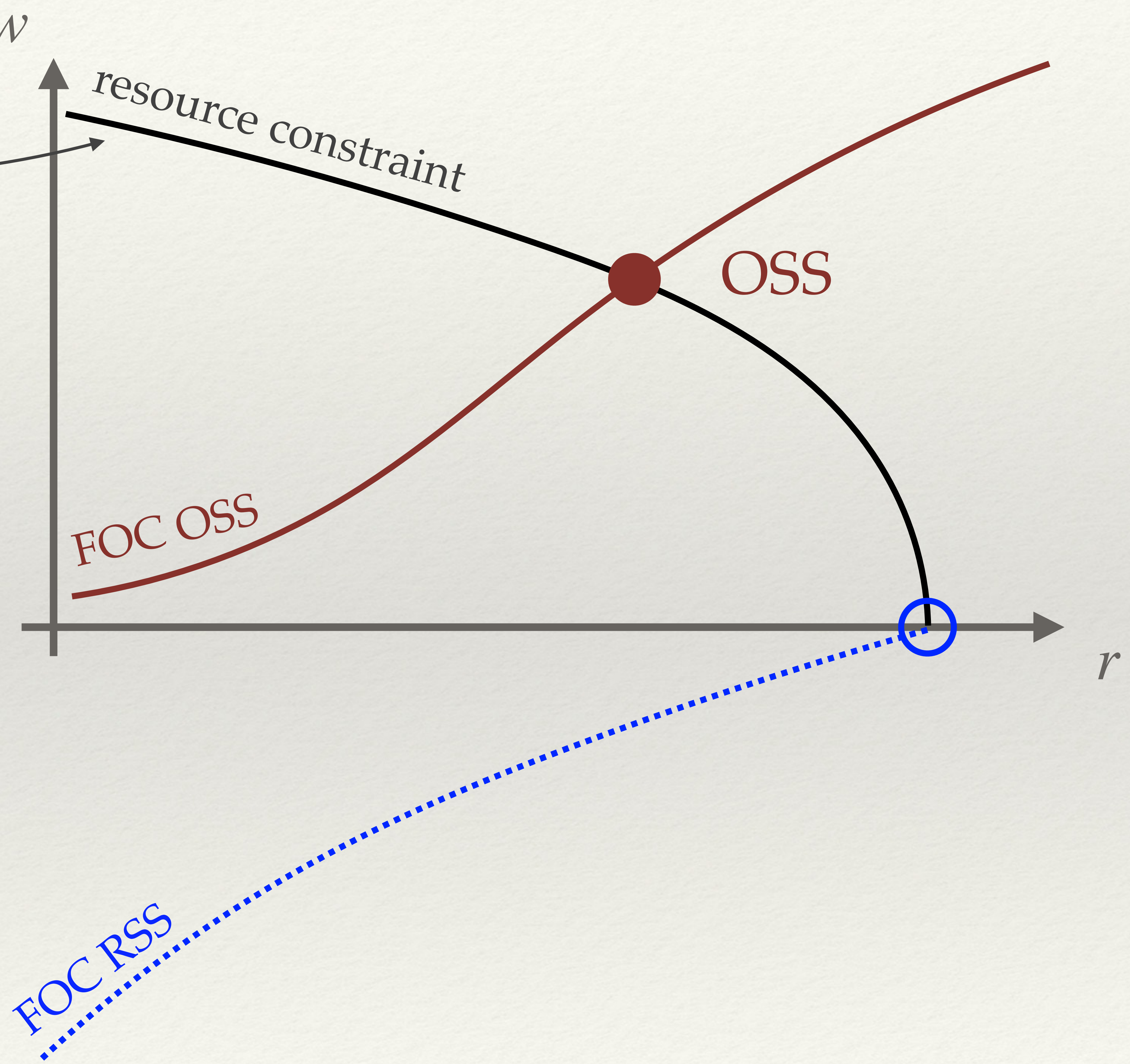

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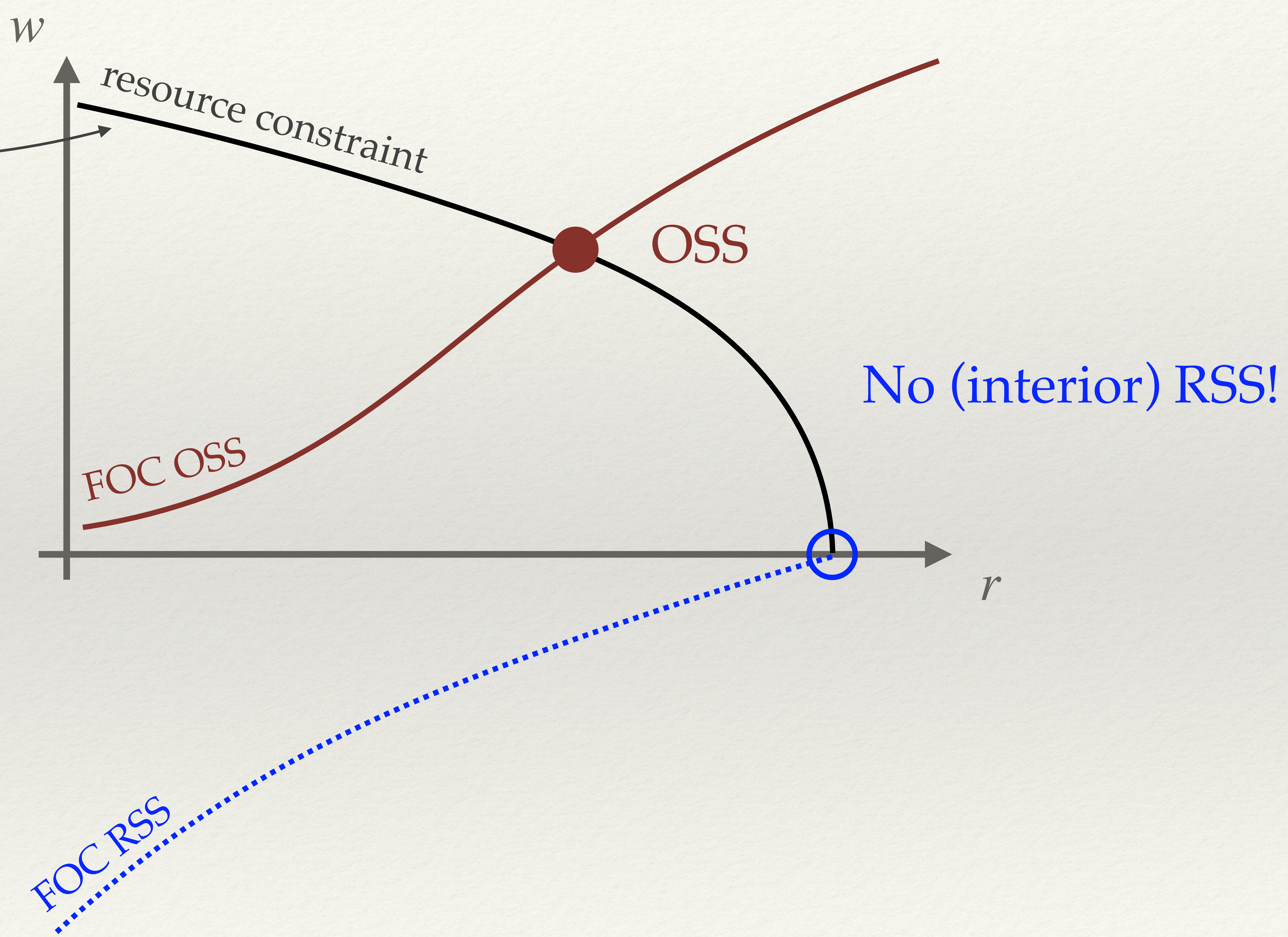

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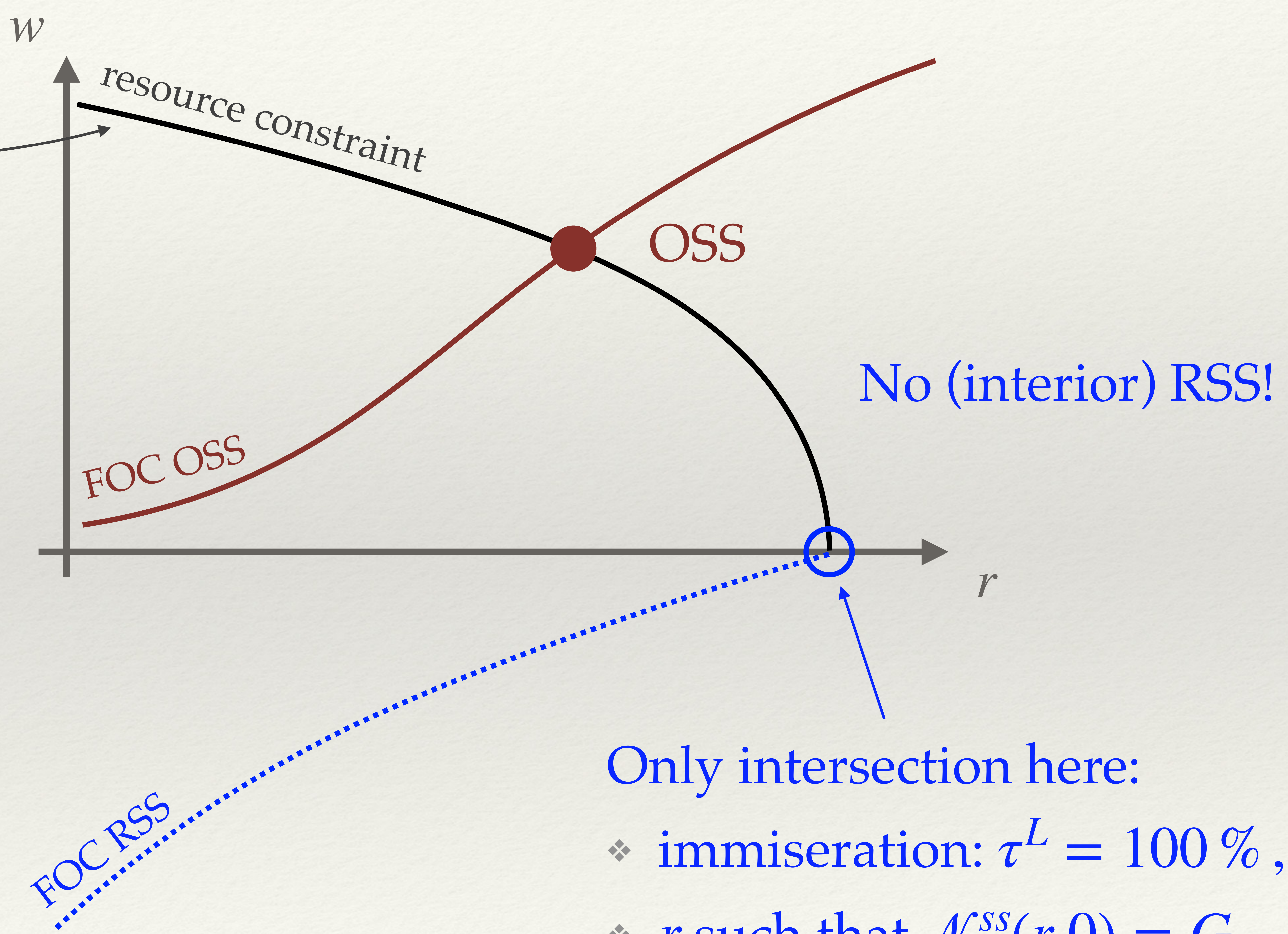

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Only intersection here:

- immiseration: $\tau^L = 100\,\%$, $C = 0$
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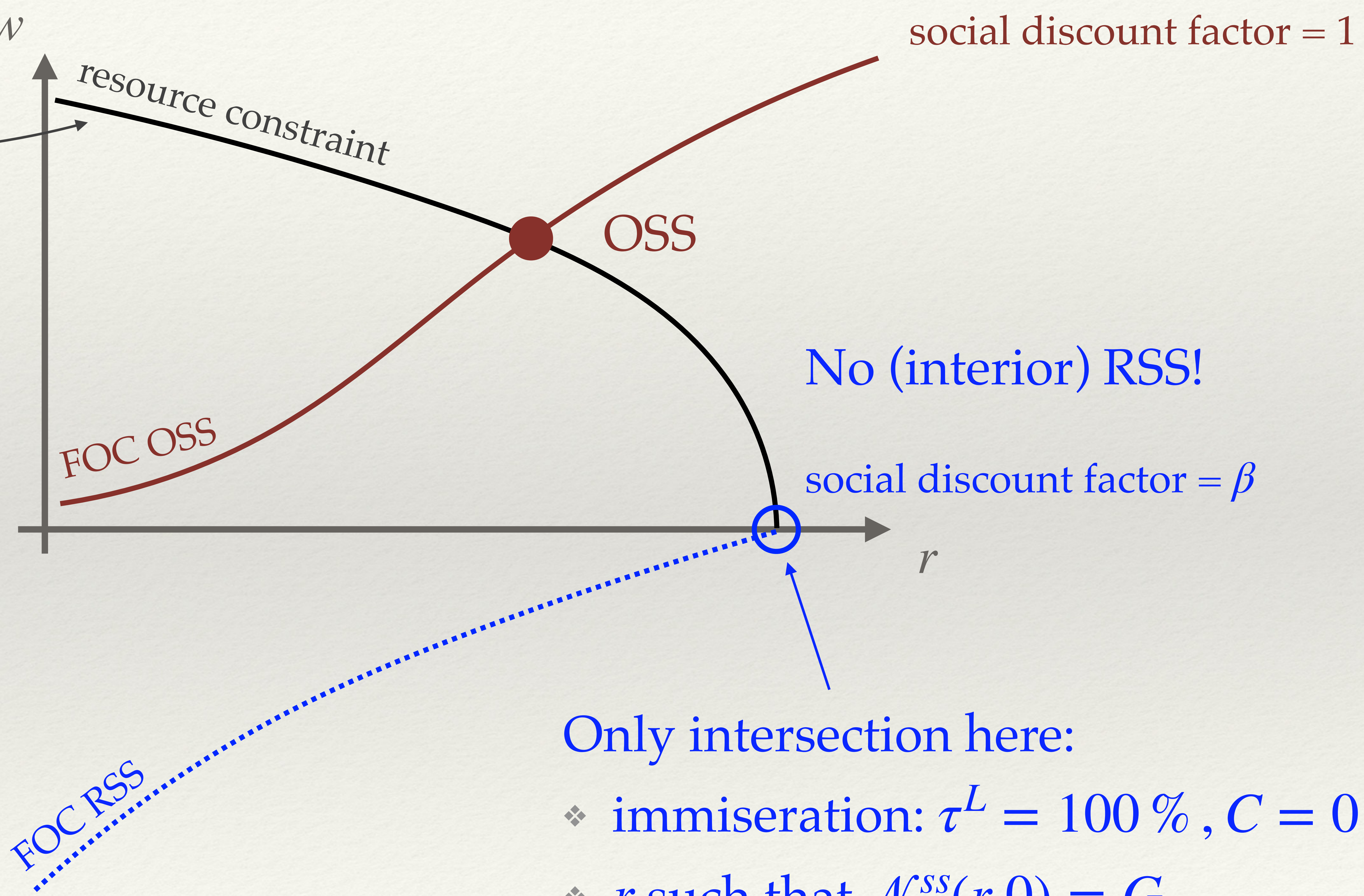

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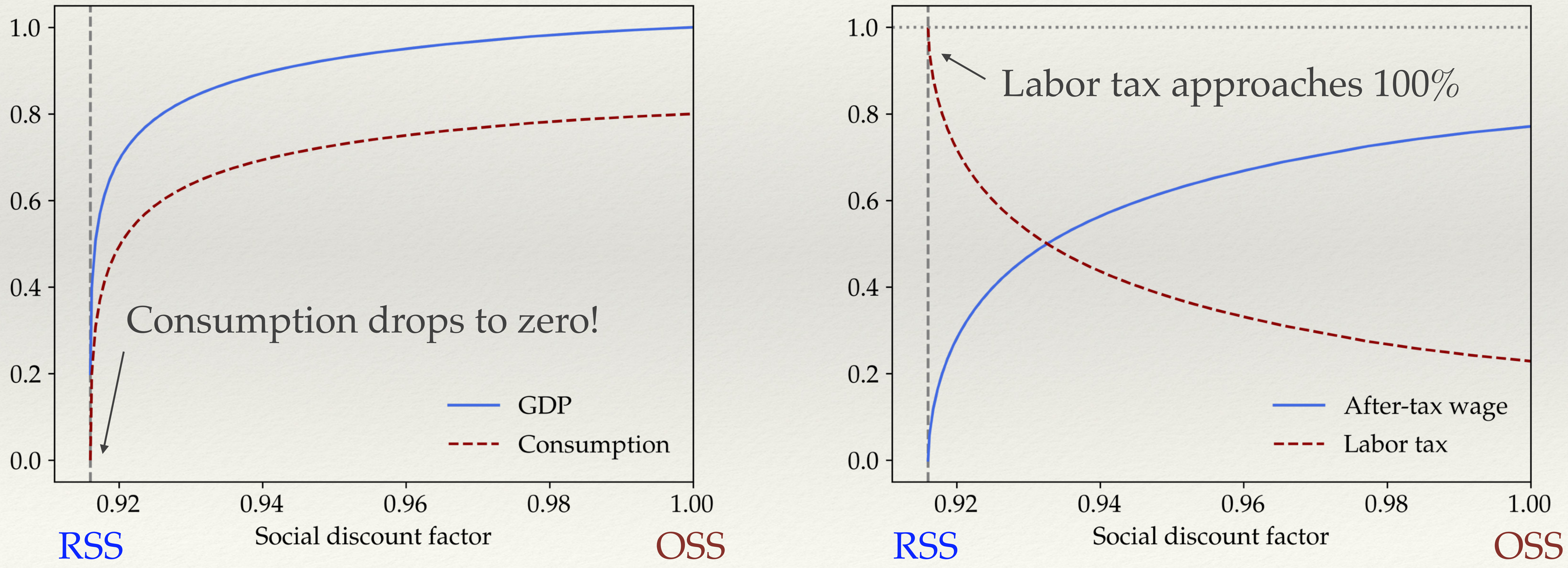
Balanced growth: No RSS!

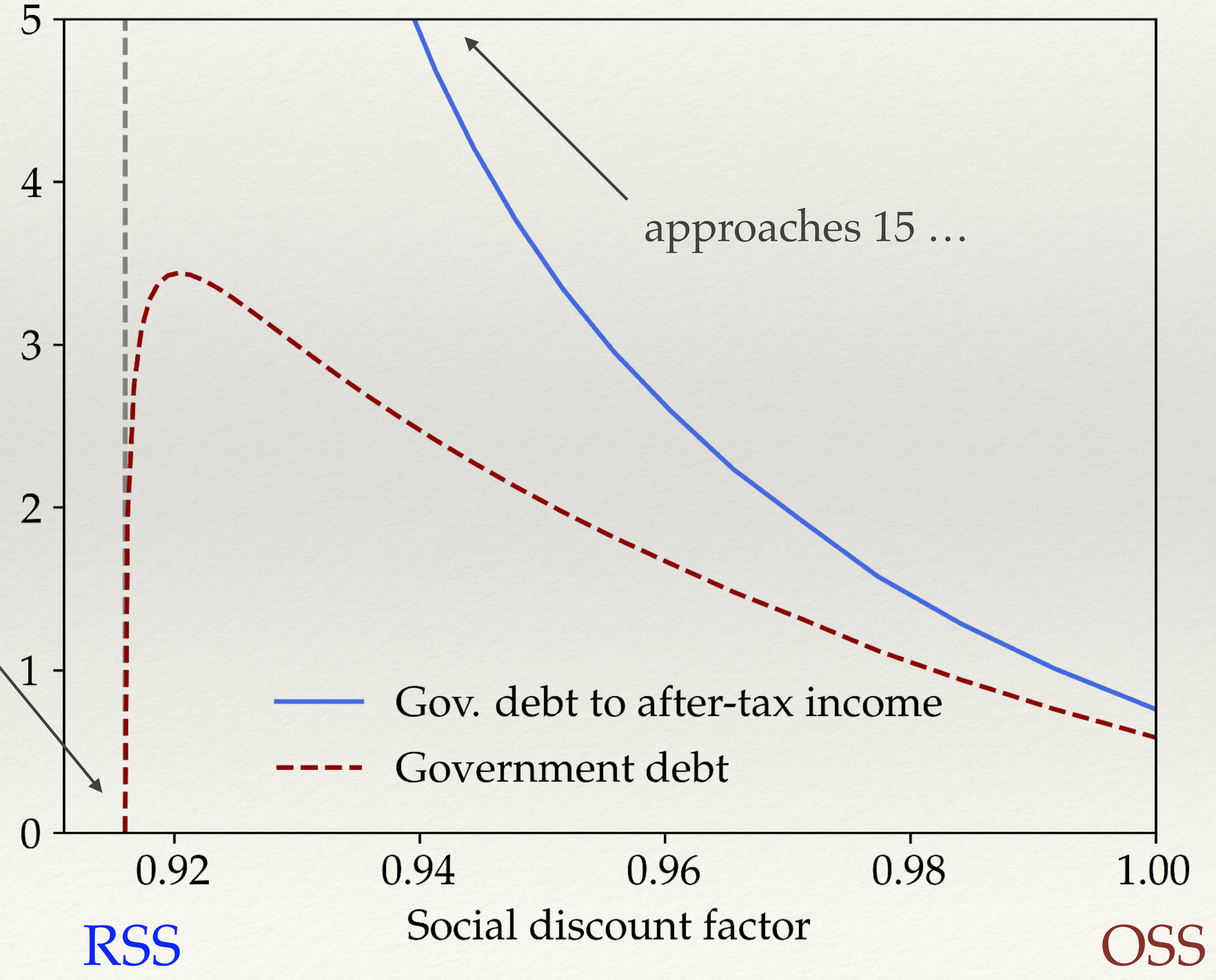
- Standard calibration, varying planner's discount factor



Balanced growth: No RSS!

- Gov. debt explodes relative to after-tax income (full insurance), however …

… in absolute terms,
gov. debt goes to zero!



Balanced growth: Why no RSS?

- Clear liquidity benefit of greater debt
- Cost of distortionary taxation depends on horizon:
  - short run **benefit**: rising debt allows to reduce distortionary taxation!
  - long run **cost**: greater debt eventually requires greater taxation
- OSS only takes cost into account. RSS both.
- Once benefit is accounted for, RSS calls for more and more liquidity

GHH: Can get RSS

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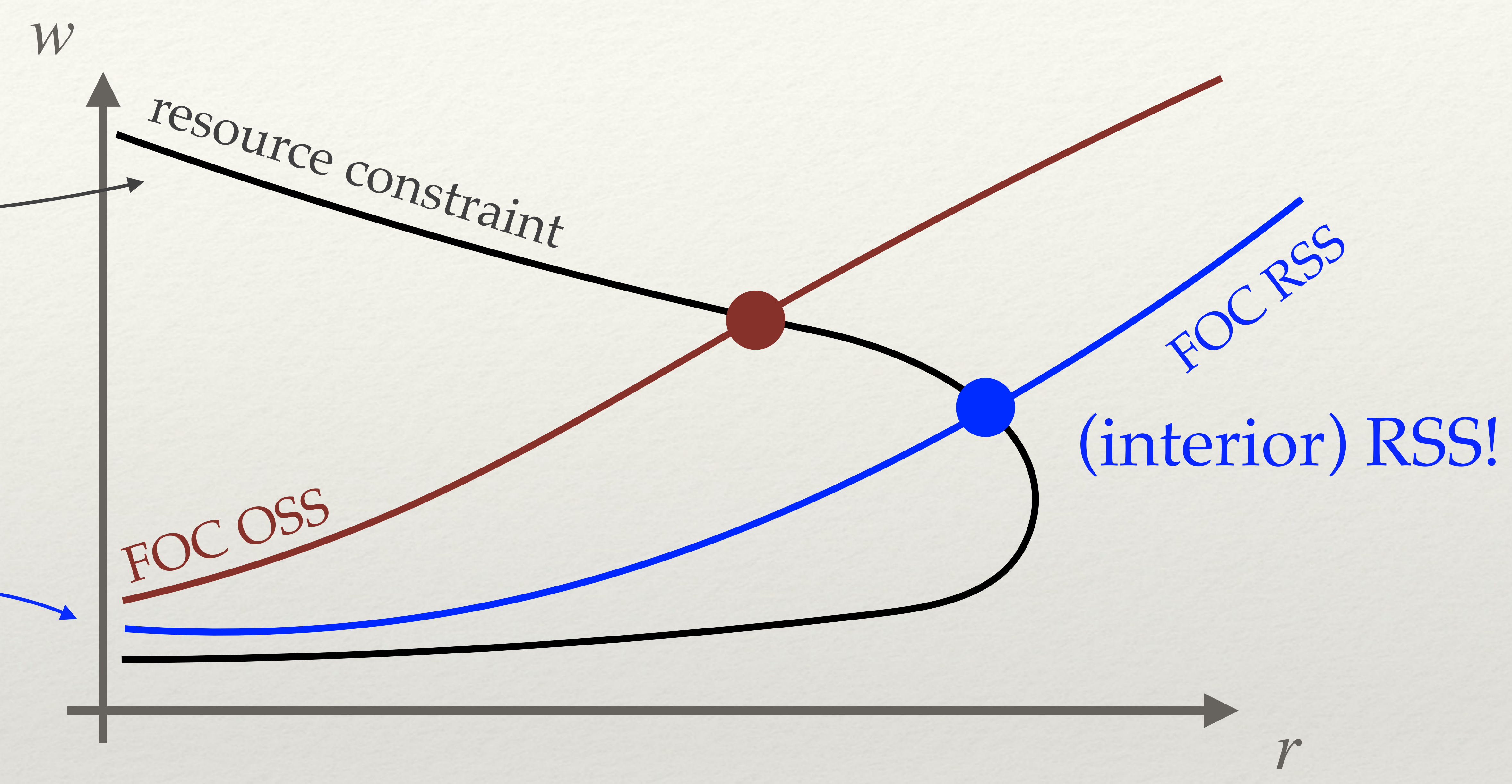
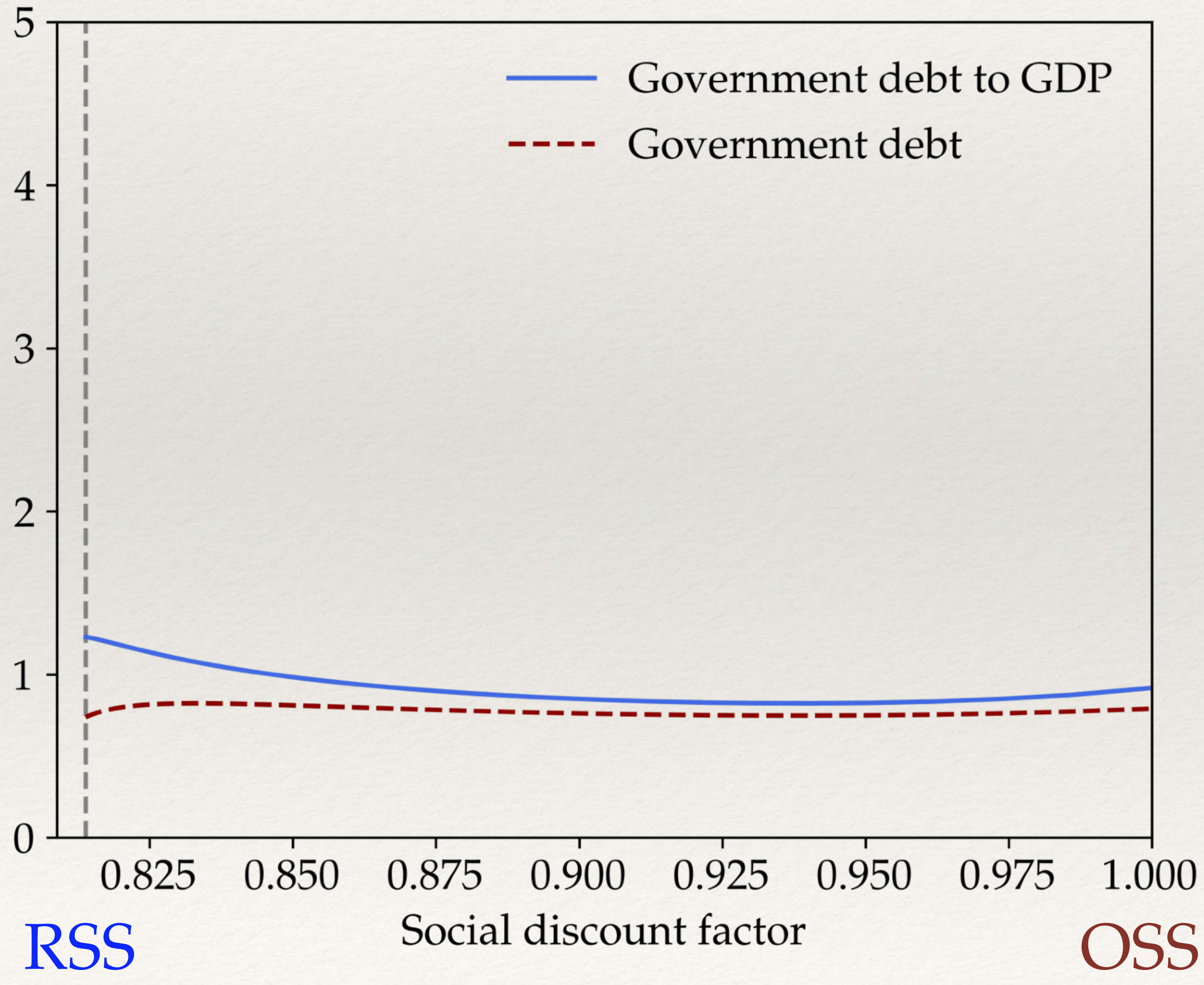
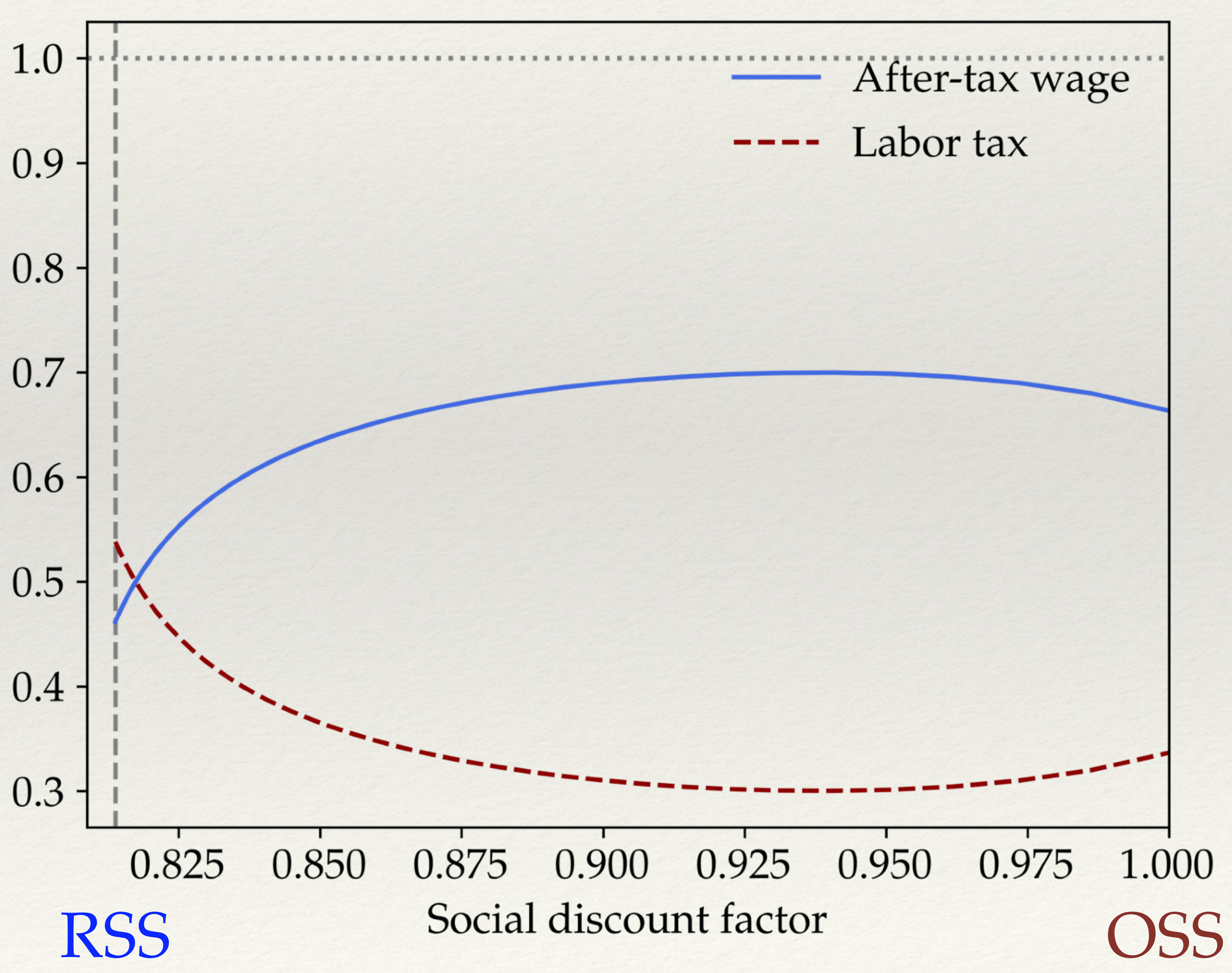


- Interior RSS because GHH is not compatible with balanced growth.
- Labor tax distortions become huge, driving labor to zero.
- Long-run cost looms large even for RSS

GHH: RSS exists!

- Standard calibration, varying planner's discount factor





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Is there something wrong with Ramsey taxation?