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Conclusior O

# Bank beliefs and firm lending: evidence from Italian loan-level data

#### by P. Farroni\* & J. Tozzo\* \*Bocconi University and Bank of Italy

EEA-ESEM Congress

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#### Motivation

# Current views about financing reflect the opinions bankers hold about the uncertainties they must face. (Hyman Minsky, Stabilizing an Unstable Economy, 1986)

From the revived Miskyan/Kindelbergerian *credit view*, financial intermediaries are central in the economic system

- Crucial understanding their expectations;
- Bankers' opinions about future uncertainty drives today's decisions about credit allocation and price;
- What if these expectations are systematically distorted?

**Challenge:** Scarcity of data on lenders' expectations and high borrower heterogeneity;

**Solution:** *New dataset* with lenders' forecasts about firms and loan-level information;

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#### Research questions and results preview

After measuring bankers' expectations we ask:

- 1. Are banks' beliefs fully rational? Do bankers' beliefs fit for a model of diagnostic expectations?
  - Data suggest banks overreaction to both micro and macro news in assessing firms' default probability;
- 2. Do banks differ in their degree of (non-)rationality?
  - We document heterogeneity in the degree of banks' distortion;
- 3. What are the real effects of (non-)rational beliefs?
  - More distorted banks reduce (increase) new loans and increase (reduce) interest rates more than its peers conditional on receiving negative (positive) news from the same borrower.

Results confirmed by counterfactual exercises conducted through a structural model of banking competition.

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#### Sketch of the mechanism



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Literat	ure					

- Lenders' beliefs: Fahlenbrach. et al. (2018), Richter and Zimmermann (2019-wp), Ma et al.(2021-wp); Contribution: granular measurement of lenders' beliefs
- Non FIRE agents: Coibion and Gorodnichenko (2012,2015), Gennaioli et al. (2012,2016); Bordalo et al. (2016, 2019, 2020); Contribution: investigate lenders' expectations
- Credit supply and sentiment: Baron and Xiong (2017), Lopez-Salido et al. (2017), Greenwood et al. (2019-wp), Krishnnamurthy and Li (2020-wp); Contribution: empirical confirmation of theoretical hypotheses with a precise measurement of credit supply-side beliefs

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Data						

Unique loan-level data from the Italian section of the European credit registry (AnaCredit):

- Very large cross section of about 700k distinct firms belonging to various Nace sectors;
- Monthly frequency;
- Detailed information on about 2 mln loan contracts each month;
- Main variable of interest: 1-year probability of default (PD);

Other datasets:

- Industrial production index (Istat);
- Italian Credit Registry, Cerved, Taxia;

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Other datasets:

- Industrial production index (Istat);
- Italian Credit Registry, Cerved, Taxia;

 $\textit{PD} \in [0,1] :=$  forecast of 1-year horizon about the borrower's default status;

- 1. PDs estimated by large and sophisticated institutions using the internal ratings-based (IRB) approach for capital requirements;
- 2. Banks must estimate a PD for each borrower in their credit portfolio;
- 3. Banks must **revise PDs periodically** and satisfy stringent requirements for screening ability;
- 4. Banks use PDs in the calculation of capital requirements (PD  $\uparrow \Rightarrow$  requirements  $\uparrow$ );
- 5. Banks must demonstrate that PDs play an essential role in their risk management, credit approval and decision making process (use test);

News distribution

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News distribution



#### PD and default rate densities: a comparison



**Takeaway**: Banks tend to overestimate defaults as much as the borrowers' riskiness increases. Higher dispersion for lower centiles.

PD boxplot

J. Tozzo

#### Bank beliefs and firm lending

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#### A model of expectations

Standard learning model adapted it to our binary setting (default/survive):

- Defaults occur if firms' cashflows/fundamentals x<sub>t</sub> fall below a given threshold a;
- **b** Banks do *not* observe directly cashflows but only noisy signal  $y_t$ ;
- ► Banks' beliefs may be rational or "diagnostic" according to a parameter  $\theta > 0$ ;

Rational 
$$\hat{x}_{t+1|t} = \rho \hat{x}_{t|t-1} + K I_t$$
  
Diagnostic  $\hat{x}_{t+1|t}^{\theta} = \rho \hat{x}_{t|t-1} + K (1+\theta) I_t$ 

$$\widehat{\mathcal{PD}}_{t+1|t}^{\theta} = \mathbb{E}^{\theta} \left[ \mathbb{I}\{x_{t+1} \leq a\} | y_t, y_{t-1}, \dots \right] = \Phi \left( \frac{a - \hat{x}^{\theta}_{t+1|t}}{\widehat{\Omega}^{1/2}} \right)$$

Details

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#### Testable implications

1. Exploiting a forecast error decomposition and linearizing the expressions for the probability of default obtained through the model, we get an expression that links **forecast errors** to **innovation** 

$$FE_{t+1|t}^{\theta} \approx \theta \underbrace{\mathcal{K}}_{|\widehat{\Omega}^{1/2}} \phi\left(\frac{a}{\widehat{\Omega}^{1/2}}\right)_{>0} I_{t} + w_{t+1}$$
(1)

 In a simple one-period loan model, borrowers promise to repay tomorrow
 a = L(1 + r) for a loan today of size L; we obtain an expression for
 interest rates depending on the borrower's probability of default:

$$r_{t} = \frac{\Phi\left(\frac{a-\hat{s}_{t+1|t}}{\widehat{\Omega}^{1/2}}\right)}{1-\Phi\left(\frac{a-\hat{s}_{t+1|t}}{\widehat{\Omega}^{1/2}}\right)}, \quad r_{t}^{\theta} \approx r_{t} - \theta \underbrace{\frac{K}{\widehat{\Omega}} \frac{\phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)}{\Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)^{2}}}_{>0} I_{t}$$
(2)

Details

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$$FE_{t+1|t}^{\theta} \approx \theta \underbrace{\mathcal{K}}_{2} \underbrace{\frac{1}{\widehat{\Omega}^{1/2}} \phi\left(\frac{a}{\widehat{\Omega}^{1/2}}\right)}_{>0} I_{t} + w_{t+1}$$
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Details

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### Predictability of Fcst Errors

Predictability of FE tested through:

$$FE_{t+12|t}^{\theta,i,b} = \beta_0 + \beta_1 News_t^{i,b(s)} + \mathbf{\Gamma}'\mathbf{X} + \epsilon_{t+12}^{i,b(s)}$$

where i and b and s denote, respectively, firms, banks and sector, X is a vector of time-, borrower-, and bank-level controls;

Where both news measures can be interpreted as positive and  $idx_t^s$  is the quarterly sectorial industrial production index.

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#### Empirical measures of news:

Micro (borrower-specific):

$$News_t^{i,b} = -(\widehat{PD}_{t+12|t}^{i,b} - \widehat{PD}_{t+9|t-3}^{i,b})$$

Macro (sector-specific):

$$News_t^s = \frac{idx_t^s - idx_{t-3}^s}{idx_{t-3}^s}$$

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#### Predictability of Fcst Errors - Micro News Results

$$\textit{FE}_{t+12|t}^{\theta,i,b} = \beta_0 + \beta_1\textit{News}_t^{i,b} + \Gamma'\textbf{X} + \epsilon_{t+12}^{i,b}$$

	F	$E_{t+12 t}^{\theta,i}$
	Panel A:	All PD News
News <sub>t</sub> (all)	0.274***	0.485***
	(0.0226)	(0.00643)
N Obs.	1036314	1034841
	Panel B: Ne	egative PD News
$News_t < 0$	0.562***	0.946***
	(0.116)	(0.0157)
N Obs.	239009	224402
	Panel C: Non-	Negative PD News
$\textit{News}_t \geq 0$	-0.113***	0.0671***
	(0.0183)	(0.0129)
N Obs.	797305	794910
Time FE	Yes	Yes
Bank FE	Yes	No
Sector FE	No	No
Province FE	Yes	No
Borrower FE	No	Yes

Takeaway: One st-dev increase in micro news makes the bankers overreact on average between 20 and 250 basis points in the determination of the PD.

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#### Predictability of Fcst Errors - Macro News Results

$$\textit{FE}_{t+12|t}^{\theta,i,b} = \beta_0 + \beta_1\textit{News}_t^s + \Gamma'\textbf{X} + \epsilon_{t+12}^{i,s}$$

	F	$E_{t+12 t}^{ heta,i}$
	Panel D: A	All Sector News
News <sub>t</sub> (all)	0.00395***	0.00449***
	(0.000938)	(0.00109)
N Obs.	`505920´	`505920´
	Panel E: Neg	ative Sector News
$News_t < 0$	0.0105*	0.0101*
	(0.00443)	(0.00433)
N Obs.	291952	291952
	Panel F: Non-N	egative Sector News
$News_t \ge 0$	0.00613***	0.00702
	(0.00140)	(0.00355)
N Obs.	`213968´	`213968´
Bank FE	No	Yes
Province FE	No	Yes

Takeaway: Banks overreact to both micro and macro news (negative and positive).

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#### Banks' heterogeneity



Takeaway: Heterogeneity in banks' diagnostic levels, by micro and macro news.

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#### Overreaction and interest rate

$$\begin{split} r_t^{i,b} &= \beta_0 + \beta_1 \textit{News}_t^{i,b} + \mathbf{\Gamma}' \mathbf{X} + \epsilon_t^{i,b} \\ r_t^{i,b} &= \beta_0 + \beta_1 D_t^b + \beta_2 \textit{News}_t^{i,b} + \gamma (D_t^b \times \textit{News}_t^{i,b}) + \mathbf{\Gamma}' \mathbf{X} + \epsilon_t^{i,b} \end{split}$$

where  $D_t^b$  is an indicator variable equal to 1 if a bank is diagnostic.

	r <sup>i,b</sup>						
		Panel A:	PD News				
Newst	-0.00694 (0.00450)	0.000338 (0.00546)	0.00556 (0.0102)	0.00471 (0.00611)			
$D_t^b$	()	0.00212*** (0.000123)	0.00166***	-0.00101*** (0.000264)			
$\mathit{News}_t  imes \mathit{D}_t^b$		-0.0279*** (0.00638)	-0.0338** (0.0166)	-0.0169* (0.00946)			
N Obs. Sector FE Province FE Time FE Borrower FE	186096 No No Yes Yes	190596 No No No No	190596 Yes Yes No No	186096 No No Yes Yes			

**Takeaway**: Negative news associated with an increase in r. Effect stronger for more diagnostic banks, which vary the level of interest rate by 3.5-7 bps when receiving a one st-dev news.

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# Overreaction and quantities

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		NC	i,b	
		Panel A: I	PD News	
Newst	0.112***	-0.0821***	-0.0702	-0.0759*
	(0.0104)	(0.0268)	(0.0508)	(0.0422)
$D_t^b$		-0.0120***	-0.00973	-0.0103*
		(0.000573)	(0.00621)	(0.00553)
$News_t \times D_t^b$		0.225***	0.210***	0.155**
L.		(0.0291)	(0.0695)	(0.0594)
N Obs.	2075790	2075790	2075790	2075747
Sector FE	No	No	Yes	No
Province FE	No	No	Yes	No
Time FE	Yes	Yes	Yes	Yes
Borrower FE	No	No	No	Yes

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Takeaway: Positive news associated with an increase in quantities. Effect stronger for more diagnostic banks.

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#### A weak Italian banking system in the years 2010s

Between 2013 and 2014 Italian default rate of Non-Financial corporations peaked to 5.5%

 $\Rightarrow$  Interesting period to study the role of banks' diagnosticity on the credit system;



#### Bank beliefs and firm lending

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#### Study the effects of diagnosticity in the 2010s

- Issue: no data on PDs;
- Solution: retrieve a "synthetic" PD from credit spread, macro conditions and borrower's information, using Credit Registry, Cerved, Taxia datasets:
  - 1.  $PD^{*1} = \alpha + \beta_1 \text{Credit Spread} + \beta_2 \text{Firm Controls} + \beta_3 \text{Macro Controls} + \varepsilon$
  - 2.  $PD^{*2} = 1 \exp(-\text{Credit Spread})$
- Re-test predictability of forecast errors;
- Study the effects on real variables and perform conterfactual exercises (to do).

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# Results (years 2010s)

$\mathit{FE}_{t+12 t}^{ heta,i}$	$FE(PD^{*1})$	$FE(PD^{*1})$	$FE(PD^{*2})$	$FE(PD^{*2})$
		Panel A:	All News	
News	0.030*	0.146***	0.270***	0.401***
	(0.016)	(0.015)	(0.016)	(0.015)
N	346104	346104	346104	346104
R2adj	3.63%	59.29%	5.29%	59.80%
		Panel B: Ne	gative News	
News	0.302***	0.261***	0.743***	0.615***
	(0.050)	(0.028)	(0.046)	(0.030)
N	Ì59483	Ì59483	Ì59483	Ì59483
R2adj	4.04%	58.36%	5.98%	59.02%
		Panel C: Non-I	Vegative News	
News	-0.040	0.082***	0.029	0.235***
	(0.028)	(0.024)	(0.028)	(0.024)
N	186621	186621	186621	186621
R2adj	3.26%	61.64%	4.76%	61.99%
Bank FE	Yes	No	Yes	No
Sector FE	Yes	No	Yes	No
Province FE	Yes	No	Yes	No
Time FE	Yes	Yes	Yes	Yes
Borrower FE	No	Yes	No	Yes

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#### Model Estimation

# We estimate a model of banking competition as in Asymmetric information and imperfect competition in lending markets, Crawford et al. - AER (2018)

#### Demand

Demand estimation is made of one equation, which relates the firm's utility from credit demand to loan price and market-bank characteristics.

$$U_{ijm}^{D} = \alpha_0^{D} + X_{jm}^{'D}\beta^{D} + \xi_{jm}^{D} + \alpha^{D}P_{ijm} + Y_{ijm}^{'D}\eta^{D} + \nu_{ijm}$$

Where  $X_{jm}$  is vector of bank-mkt characteristics;  $P_{ijm}$  is interest rate offered by bank *j* to firm *i* and market *m*;  $\xi$  are bank-market characteristics unobservables to the econometrician;  $Y_{ijm}^{'D}$  are firm-bank-market characteristics.

Supply

On the supply side banks compete a-la Bertrand Nash on prices (interest rates)  $P_{ijm}$ . Profit function is given by:

$$\Pi_{ijm} = P_{ijm}Q_{ijm}(1 - PD(\theta_j, I_i)) - MC_{ijm}Q_{ijm}$$

 $Q_{ijm}$  represents the expected demand for loan (given by probability of demand times expected amount of loan).

Assumptions GM

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#### Supply

On the supply side banks compete a-la Bertrand Nash on prices (interest rates)  $P_{ijm}$ . Profit function is given by:

$$\Pi_{ijm} = P_{ijm}Q_{ijm}(1 - PD(\theta_j, I_i)) - MC_{ijm}Q_{ijm}$$

 $Q_{ijm}$  represents the expected demand for loan (given by probability of demand times expected amount of loan).

Assumptions GMM

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#### Baseline estimation

- 1. We use benchmark regression to identify rational banks;
- We know diagnostic banks should behave differently when news is provided;
- 3. Derive an expression for the diagnostic PD that depends on the parameter  $\theta$ , which is the goal of our estimation;
- 4. Express the diagnostic PD in relation to the rational (from the model of expectations):

$$\Delta \widehat{PD} = PD_t^{\theta} - PD_t^{RE} \approx \beta_1(\theta)I_t + \varepsilon_t$$
$$\Rightarrow PD^{\theta} \approx PD^{RE} + \beta_1(\theta)I_t$$

5. Estimate the parameter of diagnosticity  $\beta(\theta)$  (negative in this formulation).

Introduction	Data	Model of expectations	Results	Back in the 10s	Structural Estimation	Conclusi
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# Results of model estimation

		Prob. borr-bank relationship
Demand param.	Tenure	1.658***
		(0.181)
	Previous rel.	1.403***
		(0.387)
	Constant	0.940
		(15.644)
	Share branches	0.988
		(1.913)
	Avg. Price	-1.442***
		(0.519)
	Borrower FE	0.899***
		(0.220)
	Age	0.888***
		(0.147)
	log Sales	0.890**
		(0.396)
	log Asset	0.890
		(1.202)
	Debt Eq.	0.899***
		(0.136)
Supply param.	Const. (news)	0.039***
		(0.000)
	News	-0.599***
		(0.018)
	Const. (Deposit int. rate)	1.003
		(0.873)
	Deposit int. rate	1.000
		(13.065)

Table: Demand and Supply Estimation - Results

#### Bank beliefs and firm lending



### Counterfactual Exercises (1)

# **Exercise 1:** what happens when doubling the estimated average $\theta$ , conditional on having a unit increase in *News*?

 Table:
 Counterfactual
 Results

	$\Delta P$	$\Delta Q$
News	$-0.419^{***}$	0.017***
	(0.162)	
Bank FE	Yes	Yes
Market FE	Yes	Yes

**Results:** drop of 42bp in interest rate, and increase of 1.7% probability of having a borrower-bank relationship.

Introduction	Data	Model of expectations	Results	Back in the 10s	Structural Estimation	Conclusion
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Counterfactual Exercises (1)

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$\Delta P$	$\Delta Q$
$-0.419^{***}$	0.017***
(0.162)	(0.003)
Yes	Yes
Yes	Yes
	Δ <i>P</i> -0.419*** (0.162) Yes Yes

Table: Counterfactual - Results

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Introduction	Data	Model of expectations	Results	Back in the 10s	Structural Estimation	Conclusion
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#### Counterfactual Exercises (2)

# **Exercise 2:** what happens if *News* increases/decreases by a standard deviation in diagnostic vs rational banks?

	$\Delta P$	$\Delta Q$
Diagn. Bnk $ \Delta News > 0$	-0.324***	0.047***
	(4.141)	(0.314)
Diagn. Bnk $ \Delta News < 0$	0.268***	$-0.051^{***}$
	(4.380)	(0.346)

**Result:** Rates diminish and bank-borrower relationships rise when positive news is given; converse is true when negative news is provided.

Introduction	Data	Model of expectations	Results	Back in the 10s	Structural Estimation	Conclusion
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Coun	terfactu	al Exercises (3)	1			

**Exercise 3**: all previously identified *diagnostic* banks see their coefficient  $\theta$  set to zero. What happens to prices and quantities when a median positive news is given to these banks, relative to the rational benchmark?

	$\Delta P$	$\Delta Q$
Median News	$1.671^{*}$	$-0.004^{*}$
Bank FE	Yes	Yes
Market FE	Yes	Yes

**Result:** Prices of banks supposed to be diagnostic is higher once they are set to rational; converse is true for quantities.

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	$\Delta P$	$\Delta Q$
Median News	$1.671^{*}$	$-0.004^{*}$
	(0.999)	(0.002)
Bank FE	Yes	Yes
Market FE	Yes	Yes

**Result:** Prices of banks supposed to be diagnostic is higher once they are set to rational; converse is true for quantities.

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- 1. We use banks' estimate of borrowers' probability of default to measure lenders' expectations;
- 2. Empirical results confirm model of diagnostic expectations fits well with our data:
  - Banks tend to overreact to news when forecasting firms' defaults, heterogeneously;
  - Banks that overreact more, on average increase (decrease) more interest rates after negative (positive) news;
  - More diagnostic banks increase (decreases) the probability of giving new loans to firms conditional on receiving good (bad) news, compared to less diagnostic peers.
- 3. Structural estimation of a competitive banking model confirms diagnosticity has impact on lending prices and quantities;
- 4. Next Steps: use 2010s data to fuel the structural model and quantify the effect of diagnosticity on the credit cycle.

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# Summary statistics

Table: Main Variables summary statistics

	Ν	Min	Max	Mean	p25	p50	p75	SD
Banks	10	1	10	-		-		
Sectors	84	0	99	-	-	-	-	-
Interest rate	1203139	0368974	.999997	.0289351	.012066	.02325	.04	.0231714
Loan size	1203139	0	7.00e+08	491562.7	25000	86250	275000	4098105
PD	1203139	0	.793926	.0220524	.003636	.008249	.0201	.0458031
News	1036279	790302	.7558669	0006854	0	0	0	.027573
News $> 0$	250150	6.74e-07	.7558669	.013541	.000838	.0036	.0119	.0328915
News $< 0$	238985	790302	-2.09e-07	0171456	014526	00433	001091	.040993

Back - PD zoom

# PD box-plot by banks



 Takeaway:
 PD interquartile ranges across banks are concentrated between

 0-0.02; presence over
 0.04 mostly due to outliers.

#### News Distribution



Back - PD zoom

#### A model of expectations

Standard learning model adapted it to our binary setting (default/survive):

- Defaults occur if firms' cashflows/fundamentals x<sub>t</sub> fall below a given threshold a;
- **b** Banks do *not* observe directly cashflows but only noisy signal  $y_t$ ;
- ▶ Banks' beliefs may be *rational* or *diagnostic* according to a parameter  $\theta > 0$ .

Back - model

#### Model of expectations - Rational Bank

In state space form the model is

$$egin{aligned} & x_t = 
ho x_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, \sigma_v^2) \ & y_t = x_t + w_t, \quad w_t \sim \mathcal{N}(0, \sigma_w^2) \end{aligned}$$

Since everything linear and Gaussian the optimal 1-period ahead cashflow forecast is

$$\hat{x}_{t+1|t} = \rho \hat{x}_{t|t-1} + KI_t$$

where  $I_t = y_t - \hat{x}_{t|t-1}$  represents the news (or innovation) and K is the Kalman gain (in steady state). Hence, a rational bank belief  $f(x, I_t)$  over future cash flows x is

$$f(x, I_t) = \phi\left(x; \hat{x}_{t+1|t}, \widehat{\Omega}\right)$$

with  $\widehat{\Omega}$  the forecast error variance.

Back - model

#### Model of expectations - Diagnostic Bank

Diagnostic bank's beliefs  $f^{\theta}(x, I_t)$  put more probability mass on events  $I_t$  which are relatively more frequent (*representative*) compared to the baseline case of no incoming news

$$f^{\theta}(x, l_t) = f(x, l_t) R(x, l_t)^{\theta} Z, \qquad R(x, l_t) := \frac{f(x, l_t)}{f(x, 0)}$$

Since everything else unchanged from rational case, exploiting normality we can characterize diagnostic beliefs as

$$f^{\theta}(x, I_t) = \phi\left(x; \hat{x}^{\theta}_{t+1|t}, \widehat{\Omega}\right)$$
$$\hat{x}^{\theta}_{t+1|t} = \rho \hat{x}_{t|t-1} + K(1+\theta) I_t$$

Given beliefs  $f(x, I_t), f^{\theta}(x, I_t)$  and the default threshold of *a*, we can define PD as

$$\widehat{PD}^{\theta}_{t+1|t} = \mathbb{E}^{\theta} \left[ \mathbb{I}\{x_{t+1} \leq a\} | y_t, y_{t-1}, \dots \right] = \Phi \left( \frac{a - \hat{x}^{\theta}_{t+1|t}}{\widehat{\Omega}^{1/2}} \right)$$

Back - model

#### Interest rates

Setting assumptions:

Contract: simple one-period loan that borrowers promise to repay tomorrow:

$$a = L(1 + r)$$
, for a loan today of size L;

Competition deprives lenders of any surplus;

Obtain: 
$$\frac{1}{1+r} = \mathbb{E}\left[\mathbb{I}\{x > a\}\right] = 1 - \widehat{PD}$$

Including the expression for PD, derived interest rate reads:

$$r_{t} = \frac{\Phi\left(\frac{a - \hat{x}_{t+1|t}}{\widehat{\Omega}^{1/2}}\right)}{1 - \Phi\left(\frac{a - \hat{x}_{t+1|t}}{\widehat{\Omega}^{1/2}}\right)}$$

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#### Testable implications

Exploiting FE decomposition and linearizing the model's equations, we link **forecast errors** to **innovation** and distorted **interest rates** to **rational** ones:

$$FE_{t+1|t}^{\theta} \approx \theta \underbrace{K \frac{1}{\widehat{\Omega}^{1/2}} \phi\left(\frac{a}{\widehat{\Omega}^{1/2}}\right)}_{>0} I_{t} + w_{t+1}$$
(3)  

$$r_{t} \approx \Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right) - \frac{1}{\widehat{\Omega}^{1/2}} \frac{\phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)}{\Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)^{2}} \hat{x}_{t+1|t} \qquad r_{t}^{\theta} \approx r_{t} - \theta \underbrace{\frac{K}{\widehat{\Omega}} \frac{\phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)}{\Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)^{2}}}_{>0} I_{t} \qquad (4)$$

$$NC_{t} \approx \Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right) - \frac{1}{\widehat{\Omega}^{1/2}} \frac{\phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)}{\Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)^{2}} \hat{x}_{t+1|t} \qquad NC_{t}^{\theta} \approx NC_{t} - \theta \underbrace{\frac{K}{\widehat{\Omega}} \frac{\phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)}{\Phi\left(\frac{-a}{\widehat{\Omega}^{1/2}}\right)^{2}}}_{>0} I_{t} \qquad (5)$$

$$EE^{\theta} := z_{t+1} = \widehat{PD}^{\theta} \quad \text{and } z_{t+1} \text{ is a binary indicator of default status}$$

where  $FE_{t+1}^{\theta} := z_{t+1} - \widehat{PD}_{t+1|t}^{\theta}$  and  $z_{t+1}$  is a binary indicator of default status. Back - implications

#### Bank beliefs and firm lending

#### Main assumptions

1 Issue: Data provides prices of signed contracts between firms and banks. But, to estimate the model we need also prices charged by banks from whom firms decided not to borrow. Solution - Step 1: estimate a price prediction model with firm fixed effects, with multi-bank borrowing:

$$P_{ijm} = \gamma_0 + \gamma_1 T_{ijm} + \gamma_2 L_{ijm} + \lambda_{jm} + \omega_i^p + \tau_{ijm}$$

where  $\omega_i^p$ ,  $\lambda_{ijm}$  are firm and bank-area-time FE;  $T_{ijm}$  is tenure of relationship between borrower *i* and the bank *j* in market *m*;  $L_{ijm}$  is loan size and  $\tau_{ijm}$  are prediction errors.

From equation above obtain predicted prices  $\tilde{P}_{ijm}$ , from borrowing firms offered from bank they have not chosen to borrow from.

**Solution** - **Step 2**: For those firms not having multiple bank relationships, we match firm FE using propensity score matching on a vector of observables.

#### Main assumptions

- 2 Assumption: We define outside option as firms not borrowing from any of the banks in the sample.
- 3 Assumption: We look only at the main line of credit within a year, for each borrower.
- 4 Assumption: We use propensity score matching (point 1) to retrieve news and PDs for borrowers that do not have a relationship with some banks.

#### Estimation: 2-step GMM - First stage

Predictive price equation can be written as

$$\begin{split} P_{ijm} &= \tilde{P}_{ijm} + \tilde{\tau}_{jm} \\ P_{ijm} &= \tilde{P}_{jm} + \tilde{\gamma}_1 T_{ijm} + \tilde{\gamma}_2 L_{ijm} + \tilde{\omega}_i^{\,p} + \tilde{\tau}_{jm} \end{split}$$

The term  $\tilde{\omega}_i^p$  is firm FE that can be used as a proxy for demand unobservable through pricing(soft information influencing pricing); relating this to the soft information influencing demand (given by parameter  $\eta^D$ , we can define all firm level covariates influencing demand:

$$Y_{ijm}^{D} = \eta_1^{D} T_{ijm} + \eta_2^{D} L_{ijm} + \eta_3^{D} Y_i + \eta_4^{D} \tilde{\omega}_i^{P}$$

Including the last two equations in the demand estimation equation yields:

$$\begin{split} U_{ijm}^{D} &= \delta_{jm}^{D} + \alpha^{D} (\tilde{P}_{jm} + \tilde{\eta}_{1} T_{ijm} + \tilde{\gamma}_{2} L_{ijm} + \tilde{\omega}_{i}^{P} + \tilde{\tau}_{jm}) + \\ &\eta_{1}^{D} T_{ijm} + \eta_{2}^{D} L_{ijm} + \eta_{3}^{D} Y_{i} + \eta_{4}^{D} \tilde{\omega}_{i}^{P} + \nu_{ijm} \\ &= (\delta_{jm}^{D} + \alpha^{D} \tilde{P}_{jm}) + (\eta_{1}^{D} + \alpha^{D} \tilde{\eta}_{1}) T_{ijm} + (\eta_{2}^{D} + \alpha^{D} \tilde{\gamma}_{2}) L_{ijm} + \\ &\eta_{3}^{D} Y_{i} + (\eta_{4}^{D} + \alpha^{D}) \tilde{\omega}_{i}^{P} + \alpha^{D} \tilde{\tau}_{jm} + \nu_{ijm} \\ &= \tilde{\delta}_{jm}^{D} + Y_{ijm}^{\prime D} \tilde{\eta}^{D} + \zeta_{ijm} \end{split}$$

#### Bank beliefs and firm lending

#### Estimation: 2-step GMM - First stage 2

Demand equation above 3 main points:

- We follow the BLP(1995) approach; errors ζ<sub>ijm</sub> = α<sup>D</sup> τ̃<sub>jm</sub> + ν<sub>ijm</sub> are composite errors of structural demand and predicted price, assumed to be distributed as EV-1;
- 2.  $\alpha^D$  does not enter independently, cannot be estimated in the first stage;

Resulting demand, i.e. probability that firm *i* chooses bank *j* in market *m* and time *t*:

$$Pr_{ijm}^{D} = \frac{\exp(\hat{\delta}_{jm}^{D}(X_{jm}^{m}, \tilde{P}_{jm}, \xi_{jm}^{D}, \alpha^{D}, \beta^{D}) + V_{ijm}^{D}(Y_{ijm}^{m}, \tilde{\eta}^{D}))}{1 + \sum_{l} \exp(\hat{\delta}_{jm}^{D}(X_{jm}^{D}, \tilde{P}_{jm}, \xi_{jm}^{D}, \alpha^{D}, \beta^{D}) + V_{ijm}^{D}(Y_{ijm}^{D}, \tilde{\eta}^{D}))}$$

where  $V^{D}_{ijm} = Y^{'D}_{ijm} \tilde{\eta}^{D}$  and  $\hat{\delta}^{D}_{jm}$  are specific constants recovered through the contraction method of the BLP(1995).

#### Estimation: 2-step GMM - Second stage

Use instrumental variable estimation to recover structural parameters in demand equation. First stage finds constants  $\hat{\delta}_{jm}^D$ , which contain bank-market-time covariates  $X_{jm}^D$  and bank-market-time specific component of predicted prices  $\tilde{P}_{jm}$ . We IV-regress constants on bank-market-time components using cost-shifters as instruments, where cost-shifters are interest rates on deposits.

$$\hat{\delta}^{D}_{jm} = lpha_{0}^{D} + lpha^{D} \tilde{P}_{jm} + X^{'D}_{jm} eta^{D} + \xi^{D}_{jm}$$