Optimal Skill Mixing Under Technological Advancements*

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Abstract

Using worker surveys and online job posting data, I document that the U.S. economy has seen a substantial increase in the mixing of skill requirements from 2005-2018, both for incumbent jobs and newly posted vacancies. American workers increasingly work in occupations that demand mixtures of analytical, computer, and interpersonal skills rather than specializing in one of them, even within granular occupations. This change occurred primarily in low- to medium-wage occupations, and the return to working in occupations or studying college majors with more mixed skills has also increased. To understand the sources of these shifts, I build a directed search model with multi-dimensional skills in which firms optimally choose occupations' skill intensities before producing with a worker, delivering endogenous specialization in skill demand. Counterfactual analysis shows that the rise in the complementarity of skills in production and in the cost of skills for occupation operation account for the majority of skill mixing shifts and corresponding wage and employment dynamics in this period.

Keywords: skill demand, technological changes, occupations, search model, worker training
JEL Codes: J21, J23, J24, J31, E24

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I Introduction

The nature of work in the United States has changed dramatically in recent decades. As technology advances, are employers shifting their demands towards specific specialized skills, or are they seeking a broader range of skills? Moreover, if there's an evident trend towards a *mix* of skill demands, how should workers adapt to this change? A vast literature documents the decline in the demand for "routine" tasks and related worker skills due to technological shifts (i.e., Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011) and the rising importance of social skills (Cortes, Jaimovich, and Siu 2021; Deming 2017). This evidence points toward changes in the mixtures of skills that appear in occupations as employers adopt new technologies. The degree of skill mixing among occupations has important and distinct implications: if occupations demand more of a particular skill, evidencing specialization in skill demand, then it benefits workers to become experts; if, however, occupations mix their use of different skills, evidencing "skill mixing," multidisciplinary schooling and training is more advantageous.

This paper studies the phenomenon of employer skill mixing, exploring its implications for workers, and seeks to understand the underlying sources of these shifts. The analysis begins with the aggregation of suitable data and the creation of measures to assess skill mixing. For this purpose, this paper primarily draws on the Occupational Information Network (O*NET), which gathers surveys from incumbent workers of their current jobs and provides the importance of different skill requirements. By considering longer time spans and focusing on continually updated occupations, I show that O*NET allows a credible analysis of longitudinal changes in skill demand. Supplementing this, the second dataset, Lightcast (formerly known as Burning Glass), provides real-time skill demand from millions of online job vacancies, enabling the measurement of the extensive margin share of jobs that require specific skills. Equipped with these datasets, I evaluate the degree of skill mixing within each occupation by calculating the cosine similarity between an occupation's skill vector and the unit vector on which skills along several domains are equally important; consequently, this "mixing index" increases as an occupation's demand for different skills gets closer.

Drawing on information about skill demand for both incumbent jobs and newly posted job vacancies, this paper presents evidence that from 2005 to 2018, occupations in the United States increasingly demand mixtures of different skills. Leveraging the O*NET dataset, I

show that even at the 7-digit occupation level, there is a sizable increase in the degree of skill mixing, particularly for analytical, computer, and interpersonal skills that are considered to be non-routine. Compared to their distributions in 2005, the skill mixing indexes of these skills in year 2018 have increased by 9 percentiles on average for the whole economy, and by 4 percentiles for occupations that are constantly updated every 6 years. The growth of skill mixing is even starker in higher-level 4-digit occupations, by 14 and 11 percentiles respectively. Such increases persist within gender, industry, and occupation groups, are unaffected by controls of workers' labor supply, and are robust to alternative measures of skills and indexes of mixing. Lightcast data further confirms the trend of skill mixing: the share of online posted vacancies demanding a mixture of the non-routine skills has increased by 5.1 percentiles in 2017 compared to 2007.

Two facts stand out for the phenomenon of skill mixing. First, a shift-share decomposition shows that skill mixing has occurred primarily within-occupation, with worker reallocation playing a minor role. For example, for the 14 percentile increase in the mixing of the analytical, computer, and interpersonal skills in the full O*NET data, within-occupation increase contributed 9.5 percentiles; in Lightcast data, 4.4 percentiles of the 4.7 percentile increase in the mixing of the four skills are attributable to within-occupation increases. Thus, the intensive margin shift in occupational skill requirements drives skill mixing, distinguishing it from other labor market changes for which worker reshuffling plays a key role, or the change is mainly across-occupation.[1] Second, the greatest increase in the mixing of the four non-routine skills appear among service and white-collar occupations, while for routine skill, mixing happened at a higher level for blue-collar occupations. Thus, low- to medium-wage occupations are the main drivers of skill mixing. While both genders experience an increase in skill mixing, male workers are more likely to see a higher rise in skill mixing in higher-wage occupations.

The phenomenon of skill mixing bears significant distributional consequences in the labor market. A notable structural shift in the U.S. labor market since the 1980s has been job polarization (Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014), a trend that continues to be evident in the data from 2005 to 2018. Skill mixing emerges as a key factor in explaining these distributional dynamics. For occupations within similar wage ranks in 2005,

[1]For example, in Autor and Dorn (2013), the polarization of the labor market is attributed to the substitution of medium-skill workers in routine jobs and their flow into service jobs; in Deming (2017) across-occupation employment shift drives the rising importance of social skills.

it is observed that those that have become more skill-mixed experience greater growth in both employment share and wages. Remarkably, the growth in employment and wages during this period is almost exclusively attributed to occupations that have seen an increase in skill mixing. Therefore, skill mixing provides a novel and multi-dimensional lens to understand these labor market transformations.

To evaluate the impact of skill mixing on workers' labor market outcomes, I estimate the wage returns to skill mixing by combining the National Longitudinal Survey of Youth 1979 and 1997 (NLSY 79 & 97), taking advantage of the rich information on participant's abilities employment and educational histories. I find a significant return to skill mixing for both occupational choices and worker skills. To assess the wage premium, I run a regression with multiple skills and their interactions, both for occupations and individual workers with two-way fixed effects in the spirit of Abowd, Kramarz, and Margolis (1999) (hereafter AKM). My preferred specifications indicate that workers in occupations that demand one standard deviation higher in the mixture of analytical and computer, or analytical and interpersonal skills, gain a 1.3 to 1.5 percent wage premium; meanwhile, workers who are more mixed of these skills earn 2.8 to 4.4 percent more. Further, leveraging the education information in NLSY, I calculate the skill content and degree of mixing for each college major. I find that workers studying a college major a standard deviation more mixed of analytical and computer or of computer and interpersonal skills earn 2.1 to 6.5 percent more.

The rich empirical findings on skill mixing necessitate a need for a comprehensive understanding of the underlying driving forces. To meet this challenge, I build a directed search model with several novel features. First, the model represents both firms and workers through multi-dimensional skills, laying the basis for an examination of skill mixing. Second, firms will need to design their occupations before meeting workers, incurring a cost upon hiring depending on their skill demand choices, as in Acemoglu (1999).[2] This endogenous occupation design is crucial in delivering the dynamic choices of skill mixing based on the skill distribution in the labor market. Third, the model incorporates non-linear production and cost technologies, departing from the common assumption of linear production functions in standard search models. This non-linearity allows the model to capture the varying degrees of

[2]The endogenous choices of the intensity of inputs were first studied in the appropriate technology literature (Atkinson and Stiglitz 1969; Basu and Weil 1998; Acemoglu and Zilibotti 2001; Jones 2005; Caselli and Coleman 2006; León-Ledesma and Satchi 2019). Several studies in the labor literature allow firms to adjust labor usage besides the quantity margin. In Lazear (2009), firms choose the weight on the skills workers supplied, and in Eeckhout and Kircher (2018), firms trade-off between more versus higher quality workers.

skill complementarity in production and the increasing marginal costs of combining skills in occupations. Despite the rich setup, the model remains tractable satisfying Block Recursivity as in Menzio and Shi (2011).

The model provides clear insights into changes in skill mixing, wages, and employment that are tied closely to the empirical observations. Central to its insights is the notion that as skills become more complementary in production or as their costs increase, firms find it more profitable to mix skills than to specialize. Consequently, jobs demanding a mixture of skills become increasingly prevalent. The model further predicts that these shifts in the production and cost environment enhance the productivity of worker-firm matches, leading to wage growth and improved job-finding probabilities for workers. Together, these model predictions offer a theoretical lens to understand the empirical trends.

I quantitatively evaluate the model to assess the relative importance of various channels that contribute to the observed skill mixing and to investigate their implications for wages and employment. Employing two periods of NLSY data, I calibrate the model parameters, which provide some key statistics on the elasticity of substitution in production and the cost structure. The results reveal that in a multi-dimensional matching framework, skills are substitutable in production, and firms face increasing marginal costs in operating occupations. Notably, sizable technology shifts have occurred: from the early 2000s to the late-2010s, there has been an increase in the complementarity of skills in production and also in firms' cost of skills for occupation operation. Meanwhile, the efficiency differential between high- and low-type workers has increased for analytical, computer, and interpersonal skills, but has declined substantially for routine skills.

Counterfactual analyses further illustrates that the technology shifts reflected in the increase in skill complementarity in production and in costs of occupation operation appear as the main drivers of skill mixing variations. For changes in wage and employment distributions, these same forces compress the high-skill occupations' wage premium, although the increased skill complementarity has positively affected high-skill occupation's employment gains. In contrast, efficiency differential changes of high-type workers negatively affects the degree of skill mixing across the periods, and contribute to the rise in relative wage and employment gains in high-skill occupations. Together, these results point to the important role technological advancements play in driving both changes in skill mixing and the wage and employment dynamics.

The rest of the paper is organized as follows. The ensuing section connects this paper to a broader set of literature and discusses the contributions. Section III presents the main empirical findings about skill mixing and many of its features. In section IV, I show the returns to mixing both at the occupation and worker levels. Section V presents a directed search model with occupation design to study the skill mixing problem and derive comparative statistics. Estimation of the model parameters and counterfactual analysis are discussed in Section VI. Section VII concludes.

II Literature Review

I study labor market dynamics emphasizing *skill mixtures* and explore new theoretical perspectives to explain them. The empirical objective aligns with the literature investigating the long-term trend of skill demand and biased technological changes (i.e., Tinbergen 1974, 1975; Katz and Murphy 1992; Autor, Katz, and Krueger 1998; Autor, Levy, and Murnane 2003; Goldin and Katz 2010; Acemoglu and Autor 2011; Autor and Dorn 2013; Deming and Kahn 2018; Deming and Noray 2020).[3] My finding that the intensive margin within-occupation changes drive skill mixing is consistent with other studies that find the major role played by within-occupation variation for aggregate job attributes (Autor and Handel 2013; Atalay et al. 2020; Freeman, Ganguli, and Handel 2020; Cortes, Jaimovich, and Siu 2021).[4] Unlike these studies, this paper studies skills in their conjunction, i.e., as mixtures, which leads to unique policy implications and broadens the understanding of the influence of technological change on the labor market.

Two papers closely related to the empirical phenomenon documented in this paper are Hershbein and Kahn (2018) and Deming (2017). The former illustrates that employers in metropolitan areas hit harder by Great Recession were more likely to post jobs demanding cognitive and computer skills, particularly in routine-cognitive occupations. My analysis

[3]The changes in relative efficiency of inputs is the focus of the skill-biased technological change (SBTC) literature, and has been shown to successful account for the major U.S. wage dynamics. See for example, Katz and Murphy (1992), Autor, Katz, and Krueger (1998), and Goldin and Katz (2010). This paper incorporates both changes in relative skill efficiency and changes in the skill complementarity, and show the latter's important role in determining skill mixing, wage shifts, and employment distribution post 2000s.

[4]Extracting task information from job ads, Atalay et al. (2020) revealed that the major change in job content during 1950-2000 occurred within-occupation, a pattern that is found to persist post-2000 by Freeman, Ganguli, and Handel (2020). Cortes, Jaimovich, and Siu (2021) discovered that from 1980 to post-2010, high-paying occupations in the United States require more social skills. Using worker-reported job tasks, Autor and Handel (2013) found that there is significant within-occupation variation in task requirements.

differs by demonstrating that skill mixing occurs for a broad set of skills, within a wide array of granular occupations, and is not specific to regions or economic downturns. Deming (2017) highlights that occupations requiring higher math and social skills based on O*NET 1997 have seen increased employment and wage growth from 1980 to 2012. In contrast, I use various versions of O*NET to capture longitudinal changes in skill demand and explore the wage and employment gains stemming from within-occupation skill mixing shifts.

Theoretically, I build a directed search model with multi-dimensional skills and endogenous occupation design, following the literature on directed search (i.e., Menzio and Shi 2010, 2011; Kaas and Kircher 2015; Schaal 2017; Baley, Figueiredo, and Ulbricht 2022; Braxton and Taska 2023). Two main contributions of this paper are: First, I allow firms to have endogenous skill demand in the spirit of Acemoglu (1999), which delivers the comparative statics regarding skill mixing. Second, I model skills in a multi-dimensional environment with non-linearity technologies. As such, the model incorporates directed search on both the worker and firm sides with high-dimensional heterogeneity on the two sides, which departs from most search models, but allows me to analyze the changes in skill mixing and the contribution of skill complementarity and cost factors.

The foundational model for worker sorting can be traced back to the seminal work of Roy (1951). Within this framework, occupations are treated as distinct categories, each requiring unique skills, and workers possess skills specific to particular occupations, preventing the exploration of skill mixing.[5] An earlier tradition, including theoretical work by Shi (2001) and empirical investigations such as Hagedorn, Law, and Manovskii (2017), adopt a single-dimensional index to represent worker heterogeneity. By design, these models preclude discussions on skill mixing. A burgeoning literature explores the multidimensional matching of workers and firms that features two-sided heterogeneity and skill transferability (i.e., Yamaguchi 2012; Lindenlaub 2017; Lise and Postel-Vinay 2020). While much of this literature focuses on the assortative nature of worker-firm matching and the evolution of worker skills[6], this study instead examines firms' endogenous skill demand trade-offs in response to technological advancements or shifts in skill supply.

A related literature, inspired by Rosen (1983), Murphy (1986), and Heckman and Sedlacek (1985), features skill indivisibility or bundling, allowing for nonlinear wage schedules and a

[5] In Roy or Ricardian type of models, workers will also specialize in a particular skill based on comparative advantages, making it harder to study skill mixing's implications for workers.

[6] A notable deviation is Ocampo (2022), which introduces the optimal combination of tasks, leading to endogenous occupational heterogeneity.

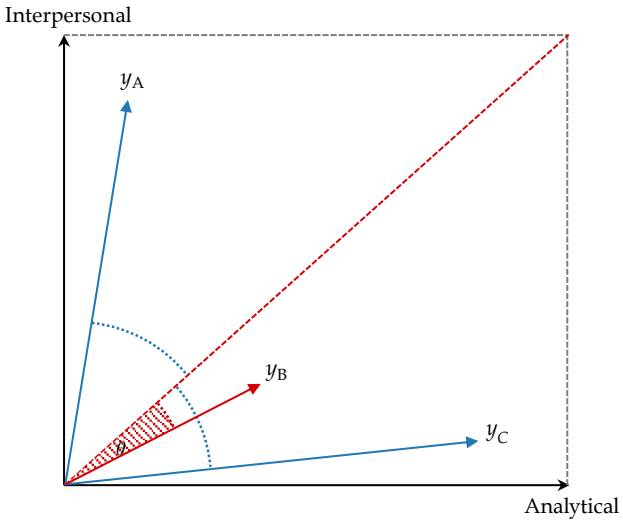
flexible degree of occupational specialization. Choné and Kramarz (2021) introduce a skill bundling framework featuring heterogeneous firms and using Swedish matched employer-employee data, they find that generalist workers earn more over time. In a separate study, Edmond and Mongey (2021) show that when skill are priced differently across occupations, firms tend to adopt technology that reflect these skill prices, leading to opposing within-occupation changes in inequality. A critical aspect of these models is the need to take a stance on the aggregation of worker skills within firms, as discussed by Eeckhout and Kircher (2018). Different from this approach, I apply a matching model to address the indivisibility of skills and endogenous skill demand at the worker level, inherently delivering nonlinear wages and skill mixing.

Quantitatively, I provide model-based identifications of the elasticity of substitution parameters among a number of different skills and the relevant cost parameters under a tractable general equilibrium model of the labor market with endogenous skill intensities. Meanwhile, These results contribute to the recent work on task-based models that has typically assumed exogenously the elasticity of substitution among different types of skills (i.e., Autor, Levy, and Murnane 2003; Autor and Dorn 2013), and also relates to studies on the elasticity of substitution among different types of workers (Johnson 1997; Heckman, Lochner, and Taber 1998; Krusell et al. 2000).

III Evidence of Skill Mixing

In this section, I examine the shifts in the extent of skill mixing in the economy. I start by showing that an angle-based index can effectively measure the magnitude of skill mixing within a multi-dimensional skill space. Using both O*NET and Lightcast data at varying levels of granularity, I explore the growth in skill mixing, decomposing it into across- and within-occupation changes. I further explore the primary sources of this variation and the differences across occupation and gender groups. Lastly, I underscore the significance of the mixing of different skills by illustrating its consequential effects on employment and wage distributions.

Figure 1: Illustrating Skill mixing



Notes: This figure contrasts three occupations—A, B, and C—in the skill space of analytical and interpersonal skills. Each occupation is characterized by its skill vector (y_A, y_B, y_C), and also by the angle between the skill vector and the 45-degree line. The angle is used to measure skill mixing as it illustrates the trade-off between skills.

III.A Measures and Data

The Degree of Skill Mixing: An intuitive approach to assess the degree of mixing of an occupation's skill demands is by examining the angular discrepancy between the occupation's skill vector and the unit vector, on which different skill requirements are equivalent. As depicted in Figure 1, occupations A and C exhibit greater specialization as their skill vectors deviate from the diagonal. In contrast, occupation B is considered to have a higher level of skill mixing with a smaller angle (θ).[7] I formalize the idea behind figure 1 by measuring skill mixing using cosine similarity.[8] Specifically:

Definition 1 (Degree of Skill Mixing of an occupation)**.** *The skill mixing index for an occupation j in a K-dimensional space characterized by the skill intensity vector $y_j = \{\alpha_{j1}, ..., \alpha_{jK}\} \in Y \subseteq \mathbb{R}^{K+}$ is the cosine similarity between its skill vector and the norm $\hat{v}$ in the skill space.*

[7] A nice feature of angle difference is that it doesn't depend on the length of the skill vector, only on the degree of skill mixing.

[8] Cosine similarity together with other measures, such as Euclidean distance and Manhattan distance, have been used to calculate the similarity between vectors (i.e., Xia, Zhang, and Li 2015). An angle-based measure is by no means the only measure of skill mixing, though it has the clearest graphical illustration of the trade-off among skills. Online Appendix A.5 discusses two alternative skill mixing indexes: inverse Herfindahl–Hirschman Index (HHI) and normalized absolute distance.

$$Mix(y_j) = \frac{y_j \hat{v}}{||y_j|| \cdot ||\hat{v}||}, where\ \hat{v} = [1, 1, ..., 1]' \subseteq \mathbb{R}^{K+} \tag{1}$$

Data Construction: To evaluate the extent of skill mixing within occupations over time, I calculate the above index using the Occupational Information Network (O*NET). Specifically, I rely on four versions of O*NET published between 2008 and 2022, and I focus exclusively on data sourced from worker surveys. Below I discuss the details of data processing.

Developed by the North Carolina Department of Commerce and administered by the U.S. Department of Labor, O*NET is a successor to the Dictionary of Occupational Titles (DOT). It has become a primary resource for analyzing occupational skill requirements and work environments (i.e., see Acemoglu and Autor 2011; Yamaguchi 2012; Deming 2017). O*NET offers an comprehensive picture of occupations, covering approximately 270 descriptors categorized into nine modules.[9] While the earlier versions of O*NET include legacy ratings from DOT analysts, a shift occurred in 2003 when O*NET began sourcing responses from random samples of workers (job incumbents). To ensure consistent measurement, I choose descriptors from questionnaires updated based on these worker surveys.[10]

A key challenge when using O*NET comes from employing the longitudinal variation in occupation descriptors. Specifically, while each version of O*NET contains roughly 970 7-digit occupations, an average of 110 occupations undergo updates annually.[11] Such a pattern of updates could introduce selection bias when constructing measures of skill demand based on the descriptors. Contrasting prior research, which often explores worker reallocation across occupations using a single O*NET version, this paper's emphasis on the dynamics of skill demand requires the examination of these longitudinal changes.

To examine these longitudinal shifts in skill demand via O*NET data, I employ two approaches, following works such as Ross (2017); Freeman, Ganguli, and Handel (2020). First, I focus on broader year intervals. For the time frame spanning 2005 to 2018, I analyze the differences in skill requirements between the start and end of this period, during which most occupations are updated at least twice. To capture more granular time patterns, I use 4-year intervals, ensuring updates to cover over half of the occupations within these intervals. Given

[9]For a comprehensive overview of O*NET, refer to online Appendix A.1, and for a discussion on the descriptors employed, see online AppendixA.2.

[10]Specifically, I use descriptors from the Work Context, Work Activities, Knowledge, and Skills questionnaires.

[11]The decision of occupation updating is based on analysts' evaluations of factors such as the size of employment, the demand for labor, and alterations in the type of work involved. See Tippins and Hilton (2010) for more details.

that each ONET version retains data from prior years, I make a distinction between the release year and the represented year when integrating ONET with other datasets. Online Appendix A.1 shows the specific O*NET versions used, their release dates, and the corresponding years.[12] Second, 274 7-digit occupations consistently receive updates between 2005, 2011, and 2018. While these occupations don't represent the entire economy, their trends—under continual updates—supplements the broader occupation analysis.

Furthermore, I use data from online job postings from Lightcast (formerly "Burning Glass Technologies") for the years 2007 and from 2010 to 2017. Lightcast is a labor market analytics firm that collects and analyzes millions of online job postings and provides detailed profiles of each job posting, including education requirements and thousands of codified skills extracted and standardized from the posting text. The key advantage of Lightcast data is that it provides comprehensive and up-to-date information on labor market conditions, and many recent studies have used this dataset to analyze trends in job skill demand (see, i.e., Deming and Kahn (2018); Hershbein and Kahn (2018); Braxton and Taska (2023)). It is essential to recognize that Lightcast's skill measurements differ from O*NET's in their information content. While O*NET gauges the level and importance of a skill (intensive margin), Lightcast identifies whether a skill is required for a job (extensive margin). Additionally, Lightcast might overlook jobs advertised through other means, possibly over-representing jobs or sectors that tend to advertise online, inherently favoring growing firms (Davis, Faberman, and Haltiwanger 2013). I employ Lightcast as a additional source to complement the picture of skill mixing changes over time.

Skill Measures: Leveraging the O*NET occupation descriptors, I first derive skill measures in line with Acemoglu and Autor (2011).[13] Specifically, I construct two non-routine skills, analytical and interpersonal; to streamline the dimensions of skills for clear analysis, I consolidate routine skills, both cognitive and manual into one. Meanwhile, non-routine analytical and interpersonal skills have been kept distinct.[14] To capture the rise of computer technology post-2000, I construct a computer skill measure based on two components related to operating a computer. Appendix Table A2 shows the detailed composing descriptors for

[12]Specifically, O*NET versions 13.0, 18.0, 22.0, and 25.0 of were released in 2008, 2013, 2017, and 2022, respectively. These versions are interpreted as representing the years 2005, 2009, 2013, and 2018.

[13]Their approach is widely applied and facilitates comparisons with other studies.

[14]Since I only use descriptors updated by job incumbents in this study, I didn't use non-routine manual skill since part of the composing descriptors comes from surveys of job analysts exclusively.

each of the skill measures.

While these four skill measures serve as the core of this study's analysis, I've also introduced two novel skills that are relatively non-routine—leadership and design—to provide a more comprehensive perspective on evolving skill demands. To enhance the reliability of these skill measures, especially for granular longitudinal time patterns, I apply principal component analysis (PCA) on the chosen descriptors following Guvenen et al. (2020) and Yamaguchi (2012). The final skill measures are linearly rescaled to lie in [0,1].[15] As a check of validity, online Appendix Table A3 shows that my constructed skills correlate highly with other similar skill measures used in the literature. In Online Appendix A.2, I build "broader" skill measures that each include more relevant descriptors, which are also highly correlated with the benchmark ones. Along with the discussion of my empirical results, I demonstrate their robustness to using alternative measures of skills and indexes of skill mixing.

Regarding the Lightcast data, I directly use the measures from Braxton and Taska (2023), which in turn are based on the methodology of Hershbein and Kahn (2018). Specifically, for the years 2007 and 2010-2017 of Lightcast data that this study uses, a vacancy is defined to use the analytical skill if any of the codified job skills contain keywords such as "research", "analy", and "decision". Similarly, a vacancy is defined to require interpersonal skill if the codified job skills contain keywords such as "communication" or "teamwork".[16] Each occupation's skill measure is then determined by the proportion of vacancies demanding that specific skill, capturing the extensive margin of firm skill demand. To classify occupations within the Lightcast data, I used a 4-digit consistent census occupation code, as developed by Autor and Dorn (2013) to ensure matching with other datasets.

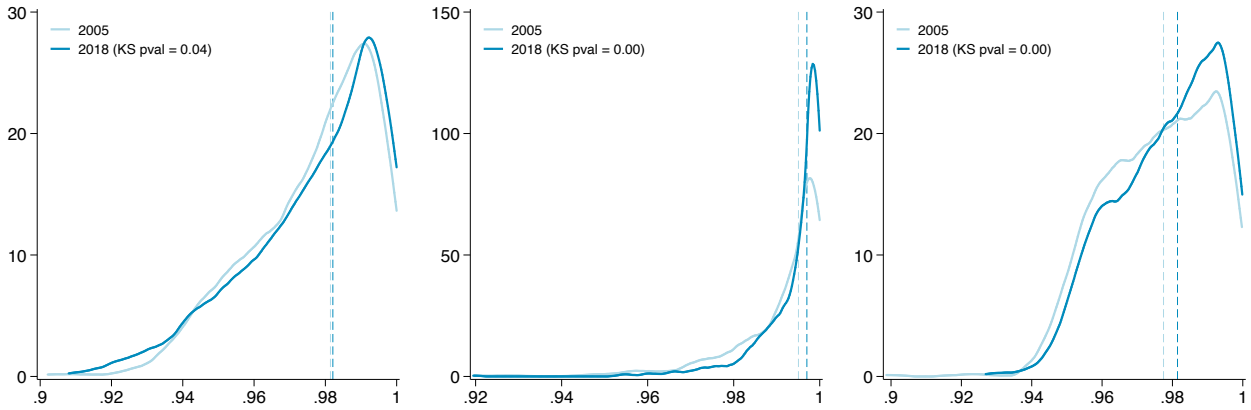
III.B Aggregate Trends

From 2005 to 2018, occupations in the U.S. economy has seen a significant increase in skill mixing, with the rise particularly pronounced for non-routine skills. I start by showing

[15] Based on Definition 1, it is crucial that skill vectors are in the positive real space for an angle-based measure to be appropriate. In that regard, normalization by standard deviation won't work unless with additional re-normalization, and linear transformation to a positive interval appears most desirable as it also retains the cardinal information that is likely to be useful for an easily interpretable skill comparison (e.g., Autor and Handel (2013), Deming (2017), and Lise and Postel-Vinay (2020)). Alternative measures of skills and skill mixing are discussed in online Appendixes A.5 and A.6.

[16] More specifically, the keywords used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The keywords used to capture interpersonal skills are "communication", "teamwork", "collaboration", "negotiation", and "presentation".

Figure 2: Density for Skill Mixing Indexes (Cosine Distances), 2005 vs. 2018



(1) RNR Skills (2) Non-routine Skills (3) Other Non-routine

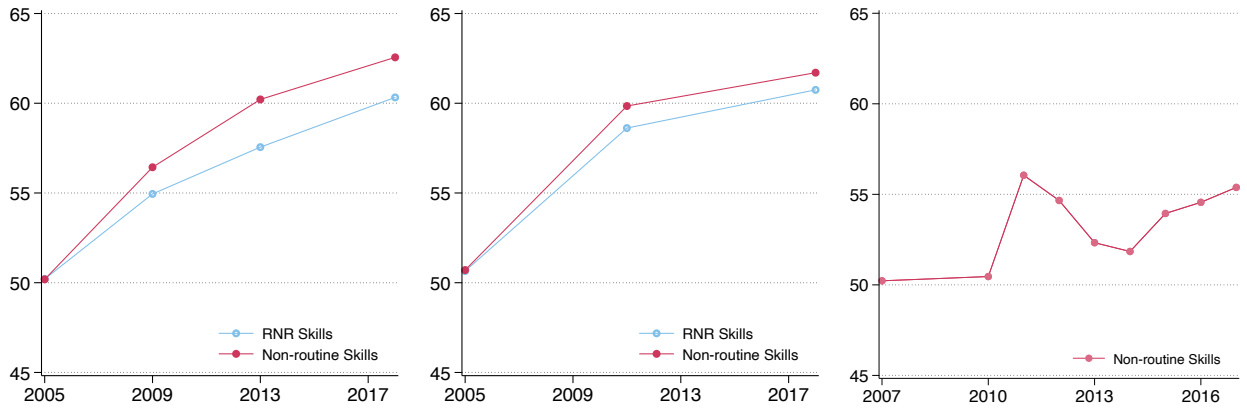
*Notes: These figures plot the kernel density of different skill mixing indexes in 2005 (light blue line) and 2018 (dark blue line). The x-axis displays the value of skill mixing indexes with a maximum of 1 by construction. "RNR" indicates routine and non-routine skills that are defined by Acemoglu and Autor (2011). Non-routine skills include non-routine analytical and interpersonal skills, as well as computer skill, as detailed in online Appendix table A1. "Other non-routine" include leadership and design skills to the aforementioned non-routine skills. These plots are created using O*NET at 7-digit occupations without employment weighting.*

that this trend happens at even granular 7-digit occupations from O*NET data. Figure 2 depicts the density and median values of three skill mixing indexes: one combining routine and non-routine (RNR) skills, a second focusing solely on non-routine skills, and a third examining other non-routine skills. As these indexes remain unweighted by employment, the shifting distributions solely represent changes in occupations' skill mixing, unaffected by worker reallocation.

Panel (1) first reveals a modest rightward shift in the density of skill mixing index for RNR skills during this period. The Kolmogorov-Smirnov (KS) test confirms the distributional difference as statistically significant at the 5 percent level. Nonetheless, the density shift is even more pronounced for analytical, computer, and interpersonal skills (Panel 2) and if one includes other non-routine leadership and design skills (Panel 3). By 2018, the density of the skill mixing indexes of these skills peaks at a much higher value than 2005. The KS test shows these differences in distributions are significant at the 1 percent level, indicating a substantial growth in occupations demanding a high level of skill mixing of non-routine skills.

This trend of escalating skill mixing becomes even sharper when accounting for the evolving composition of labor force across occupations. In online A2, I merge O*NET data with employment weights from the Occupational Employment and Wage Statistics (OEWS) at

Figure 3: Trend of Skill Mixing in the US Economy, 2005-2018



(1) Full O*NET (2) Constant Updates (3) Lightcast

*Notes: These figures plot the employment-weighted skill mixing indexes in the U.S. economy from 2005-2018. The y-axis is the percentile of skill indexes in year 2005. By construction, each index has a mean of 50 percentiles in 2005; succeeding points are employment-weighted means mapped to its percentile in 2005. Panel (1) and (2) combine O*NET and ACS data with consistent 4-digit occupation codes from Autor and Price (2013) and developed by Deming (2017). The matching of different O*NET releases and ACS years are detailed in online Appendix Table 1. Panel (1) show the trend for the universe of occupations while Panel (2) only include 274 7-digit occupations that are constantly updated between 2005, 2011, and 2018. Panel (3) combines Lightcast job posting data and ACS with same occupation coding. Employment weights from ACS are the total hours of work aggregated to sex-education-industry-occupation cells.*

the 6-digit level. The rightward shift of all the skill mixing indexes becomes more pronounced when weighted by employment shares over the years.[17] This result implies that occupations with larger employment have exhibited a more significant rise in skill mixing; therefore, a greater portion of the labor market now demands a higher degree of skill mixing.

In addition to index-based evaluation of skill mixing, one can also non-parametrically examine occupation skill requirements in two-dimensional spaces. Online Appendix A.3 discusses and presents non-parametric plots for six skill pairs from both 2005 and 2018, confirming the observed increase in skill mixing, particularly for non-routine skills.

Time Pattern: To more carefully examine the time profile of the changing skill mixing and understand the sources of variation, I combine the longitudinal variation in skill mixing from O*NET as well as ACS to conduct further analysis. I show the trend pattern at 4-year intervals so that more than half of the occupations (about 60% of employment) are updated

[17]The OEWS uses 6-digit SOC codes, while O*NET uses 7-digit occupation codes that are based on 6-digit SOC. I match OEWS with O*NET at a 6-digit SOC level and distribute the employment weight evenly for 7-digit O*NET occupations within a 6-digit occupation.

between observations. By construction, each index has a mean of 50 percentile in 2005; succeeding points are employment-weighted means of each index mapped to its percentile in 2005. I weight each skill requirement from O*NET by the total hours of work in each sex-education-industry-occupation cell in ACS to implicitly control for changes in task inputs due to variations in gender, education, industry, and occupation mixes in the U.S. economy (see Autor, Levy, and Murnane (2003) and Deming (2017) for other examples).

Figure 3 demonstrates that the degree of mixing of non-routine skills has risen substantially and steadily between 2015 and 2018. By 2018, the degree of mixing among non-routine skills for an average occupation in the economy is 12.6 percentile higher than its 2005 level. The degree of skill mixing of RNR has also increased steadily to a slightly lesser extent, averaging 10.3 percentiles higher. As mentioned, a potential concern of using O*NET data to obtain the longitudinal variation of skill demand is that the trend could be affected by the inconsistency in occupation updating. In Panel (2) of Figure 3, I compute these trends focusing solely on 274 7-digit occupations that are constantly updated between 2005, 2011, and 2018, thus reflecting a consistent updating of skill demand among these occupations. The same qualitative pattern holds, that is, there has been a sharp increase in the degree of skill mixing, particularly of non-routine skills between 2005 and 2018. Nonetheless, for the constantly updated occupations, the shift is mostly pronounced before 2011.[18]

In Panel (3) of Figure 3, I complement the picture of changing degree of skill mixing using the Lightcast data through a similar paring with O*NET data starting in 2007, the first year when the company starts to collect job postings. Overall, firms are also more likely to post job requirements that contain more mixed-skill demands. By 2017, the degree of skill mixing in job postings averaged 5.1 percentiles higher compared to 2007. The time pattern of skill mixing among online job postings appears to be more volatile, first peaking in 2011, then sliding down until 2014, before dramatically rising afterward until 2017. Despite the greater variance, the same qualitative pattern holds that occupations have a higher demand for the mixing of non-routine skills.[19]

[18]In online Appendix Figure A3, I show employment percentages and hourly wages across various job categories in the full and the sample for constantly updated occupations. The hourly wage rates across the categories seem fairly consistent between the full and selected samples with minor discrepancies, only that the selected sample has less presence of professionals and sales occupations.

[19]The higher degree of volatility is partly driven by the nature of the measure and the data. The measures of skills from job postings are whether firms require a particular skill in the text of job ads, naturally noisier than the question on level and importance from O*NET. Moreover, firm job posting is more influenced by firm entry and exit patterns.

Table 1: Time Trend of Skill Mixing Indexes

	Skill Groups	6-digit Occupations			4-digit Occupations		
		total	within	across	total	within	across
Full O*NET	RNR Skills	6.78	4.93	1.85	12.23	9.26	2.97
	Non-routine Skills	9.21	5.62	3.59	14.07	9.53	4.54
Constant Updates	RNR Skills	5.59	6.73	-1.14	9.70	10.57	-0.87
	Non-routine Skills	4.05	5.33	-1.29	10.58	9.50	1.09
Lightcast	Non-routine Skills				4.66	4.37	0.28

Notes: This table shows the shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index h over two periods t and τ, its change $\Delta T_{h\tau} = T_\tau - T_t$ which can be decomposed to $\Delta T_h = \sum_j \left(\Delta E_{j\tau}\alpha_{jh}\right) + \sum_j \left(E_j \Delta\alpha_{jh\tau}\right) = \Delta T_h^a + \Delta T_h^w$ where $E_{j\tau}$ is employment weight in occupation j in year τ, and $\alpha_{jh\tau}$ is the level of mixing index h in occupation j in year τ, $E_j = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_{jh} = \frac{1}{2}(\alpha_{jht} + \alpha_{jh\tau})$. ΔT_h^a and ΔT_h^w then represent across-occupation and within-occupation change.

One may be concerned about that the overall patterns shown so far are driven by the choice of skill measures or the choice of skill mixing index. To address this, in online Appendix A.5 and A.6, I demonstrate the robustness of these trends across varying skill measures, alternative skill mixing indexes, and skill mixing indexes of distinct skill pairs rather than multi-dimensional ones. For example, using standardized (or broader) measures of skills, the increase in the degree of mixing of non-routine skills is 6 (or 13) percentiles from 2005 to 2018; using inverse Herfindahl-Hirschman Index, the increases in the mixing indexes of any given skill pairs are above 10 percentiles during the same period. Across these checks, the qualitative picture remains consistent: there has been a notable rise in the degree of skill mixing, particularly for non-routine skills.

Decomposing the Sources: To gain a deeper understanding of the variations underlying changes in skill mixing, I undertake two exercises. First, I decompose the longitudinal changes in skill mixing in the U.S. economy, differentiating between intensive margin skill mixing index changes within occupations and extensive margin employment shifts across occupations. This analysis reveals that within-occupation skill mixing shifts play a more influential role in driving skill mixing than across-occupation employment shifts. Second, I perform a regression analysis that include extensive controls, including various skill supply measures, as well as gender, industry, and occupation fixed effects. I find that the pronounced

Table 2: Time Trend of Skill Mixing Indexes

	RNR Skills	Non-routine Skills
Full O*NET	0.70***	0.71***
	[0.10]	[0.09]
Constant Updates	0.75***	0.65***
	[0.11]	[0.11]
Lightcast		0.33**
		[0.15]
Sex × Industry × Occ. FE	X	X
Exp. and edu. controls	X	X

*Notes: This table reports the results of regressing values of RNR skills and Non-routine skills on a time trend variable (year values) for the full O*NET, Constant Updates, and Lightcast datasets combined with ACS. See online Appendix A.1 and A.5 for the data construction. The regressions include controls for sex—industry-occupation fixed effects, as well as 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects, polynomials of years of work experience up to power 4, and the interaction of experience polynomials and education fixed effects and gender. Robust standard errors are reported in brackets. *** p<0.01, ** p<0.05, and * p<0.1.*

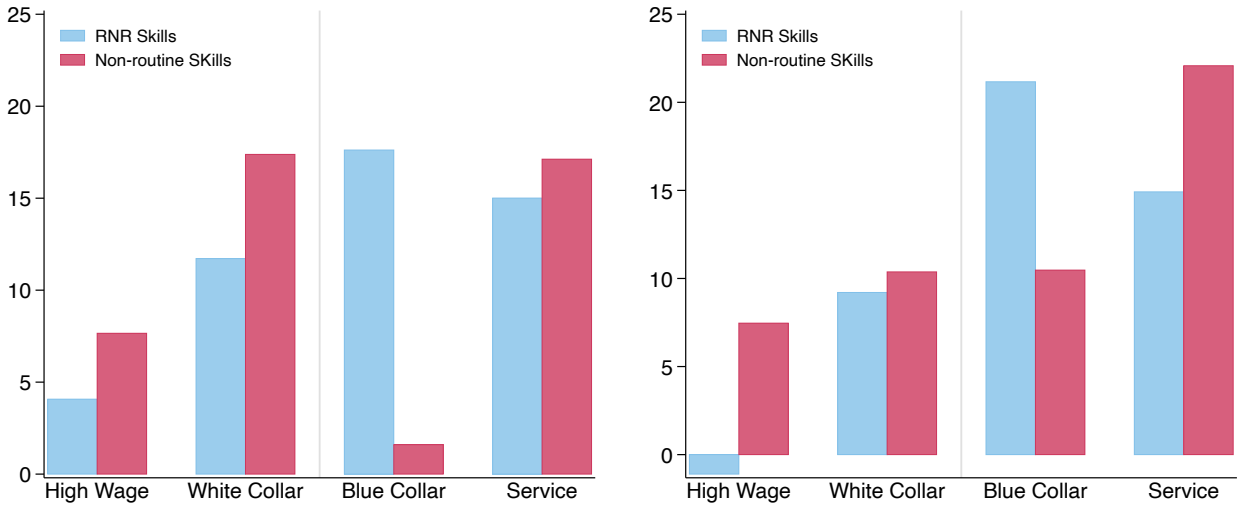
trend of increasing skill mixing persists.

Table 1 shows the decomposition of the changes in the employment-weighted skill mixing indexes into within-occupation index shifts and across-occupation employment changes, at both 6-digit SOC and 4-digit census occupation levels. I conduct the analysis both for the full ONET dataset and the subset of persistently updated occupations, alongside the Lightcast dataset. Irrespective of the dataset or skill groupings, within-occupation variation predominantly drives the rise in skill mixing. For example, for the 9.2 percentile increase in the mixing of non-routine skills in the full O*NET data, within-occupation increase contributes 5.6 centiles while only 1.1 percentiles stem from worker reallocation; for the 4.7 percentile increase in the mixing of non-routine skills in Lightcast data, within-occupation increase accounts for 4.4 percentiles.[20] Interestingly, for the constantly updated occupations at 6 digits, worker reallocation actually contributes negatively to the increase in skill mixing. This pattern implies that for these granular occupations under regular updates, the contribution of within-occupation variation to the skill mixing increase is even stronger. At 4-digit occupations, worker reallocation does contribute positively to the increase in the mixing of non-routine skills, but the influence is still marginal compared to within-occupation variation.

An alternative explanation of the employer-side shifts in accounting for skill mixing could

[20]Online Appendix A4 shows the decomposition results using skill mixing indexes for different skill pairs and a similar result holds.

Figure 4: Skill Mixing Index Change by Occupation Groups and Gender, 2005-2018



(1) Male (2) Female

*Notes: These figures plots the changes in skill mixing indexes across different occupation groups for male and female workforce. The units of the index changes are percentiles of their distributions in 2000. Workers are categorized into four occupation groups – High Skill, White Collar, Blue Collar, and Service following Acemoglu and Autor (2011). O*NET and ACS data are combined for these figures with consistent occupation codes Autor and Price (2013) and developed by Deming (2017).*

be that even within occupations, the supply of labor might have changed, due to, i.e., rising human capital, or labor force participation of female workers. To further shed light on the sources, Table 2 shows a regression of skill mixing indexes on linear time trend (years) across combinations of O*NET and Lightcast with ACS data. I further control the interaction of gender, industry, and occupation effects, and flexible polynomials and interactions of years of education and experiences. The table shows a universal increase in the degree of skill mixing in the magnitude of 0.65 to 0.75 percentile per annum using O*NET data, and 0.33 per annum using Lightcast data. Moreover, this increase persists within gender, industry, and occupation groups and is unaffected by controls of worker's labor supply. This finding suggests that the primary drivers for skill mixing are not from the worker side, but rather highly related to demand-side dynamics.

III.C Skill Mixing Changes by Occupation

Beneath these general trends of skill mixing are diverse patterns among occupations and gender groups. Figure 4 illustrates the changes of skill mixing indexes from 2005 to 2018 across four primary occupation categories, grouped by wage levels and encompassing all non-

agricultural employment in the U.S.[21] This is done for both male and female workers, with changes presented in percentiles relative to their respective 2005 counterfactual distributions, similar to the approach in Figure 3.

Considering the occupational trends, service and white-collar occupations predominantly contribute to the increase in the mixing of non-routine skills. In contrast, blue-collar occupations demonstrate the most pronounced rise in mixing RNR skills, with service occupations following closely. This pattern highlights that the bulk of skill mixing occurs in lower-wage professions, especially within the service sector. In terms of gender differences, male workers in white-collar and high-wage roles exhibit a greater rise in skill mixing across both skill categories. Conversely, female workers experience more pronounced growth in skill mixing within lower-wage blue-collar and service roles.[22] This suggests the trend that male workers gravitate towards higher-wage roles with increased skill mixing, whereas females are more commonly found in lower-wage roles that become more skill mixed.[23]

In online Appendix A.4, I show the decomposition of skill mixing by industries, and a similar pattern holds. The private service sector followed by retail trade and construction lead others in the growth of skill mixing, while public, social and professional services sectors demonstrate only modest increases, particularly for RNR skills.[24][25] I also show the decomposition of occupations' changing mixing of distinct skill pairs, which confirms that non-routine skills drive skill mixing in all occupations, while routine skills are only more mixed with other skills in blue-collar and service occupations.

Distributional Implications: One of the key structural changes in the U.S. labor market post-1980 is the pronounced job polarization or hollowing out of middle-skill employment and wage growth, due potentially to the routine-biased technological change and offshoring (Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014). To see how much skill mixing can relate to these distributional dynamics, Figure 5 depicts the smoothed observed changes in both the share of total hours worked and log wage in 2005-2018 for occupations

[21] The grouping is based on Acemoglu and Autor (2011).

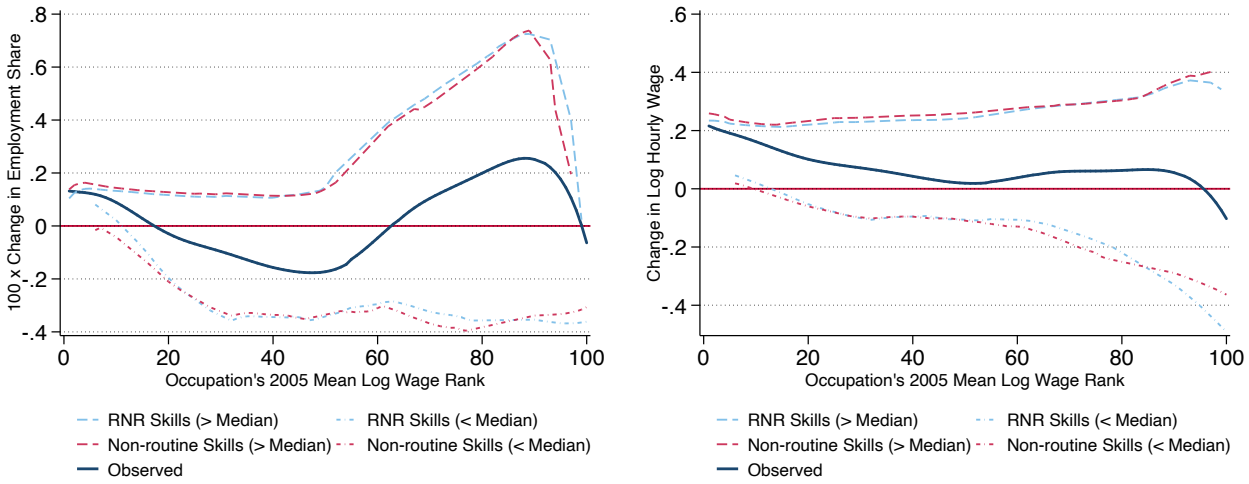
[22] To clarify this distinction, a separating line is presented in the figure.

[23] While a comprehensive discussion on the gender gap is beyond this paper's scope, it's plausible that males might have greater access to mixed-skill roles at the higher end of the wage distribution due to persistent gender segregation. For a deeper analysis into gender segregation trends, see Blau, Brummund, and Liu (2013).

[24] The sectors that have the least growth in skill mixing are public and education and social services.

[25] This result is consistent with Hershbein and Kahn (2018) that industries with locally consumed goods are more likely to change skill demand.

Figure 5: Smoothed Employment and Wage Changes by Skill Percentile, 2005-2018



Notes: These figures plot the smoothed observed as well as counterfactual changes of employment share (Panel A) and hourly wage (Panel B) for occupations between 2005-2018. On the x-axis, occupations are ranked into 100 percentiles by the average log wages of workers in those occupations in 2000. The changes in the share of hours worked and percent wage growth are then calculated for each percentile, which fit into smoothed lines using cubit polynomial fit. Counterfactual lines are the smoothed employment/wage changes only for occupations with above-median increases in the skill mixing indexes.

ranked by their hourly wage percentiles in 2005. I reconstruct these smoothed employment/wage changes for two groups of occupations: those with above-median increases in skill mixing indexes and those below the median.

Figure 5 first confirms the inverted bell shape (polarization) of observed employment and to a lesser extent, the change of wages. Furthermore, it illustrates key differences for occupations that have become more skill mixed. For occupations within similar wage ranks in 2005, those that become more mixed in skill use have a higher increase in employment share and wage growth. In fact, almost the entirety of employment and wage growth is accounted for by occupations that have become more skill mixed during this period. Therefore, relating to polarization, the differential growth in employment and wage among occupations at the top and bottom end of 2005's wage distributions entirely stems from skill-mixing occupations during this period. Besides being an important phenomenon for labor market dynamics, skill mixing also provides a unified and multi-dimensional perspective of the polarization changes.

IV Returns to Skill Mixing

In order to better understand the influence of skill mixing on workers' labor outcomes, this section will examine the wage returns associated with skill mixing in relation to occupational choices and inherent worker skills. Additionally, the return on investment for a college major with a more mixed skill set will be discussed.

IV.A Data and Measurement

To assess wage returns associated with skill mixing, I use the National Longitudinal Survey of Youth (NLSY) datasets from both 1979 and 1997 cohorts, which offer comprehensive records of the participant's employment and educational histories. I combine these two cohorts to increase the sample size, limiting to the period from 2005 to 2019 to align with the timing of my skill mixing measurements from O*NET as discussed in the previous section.[26] The NLSY data is connected with O*NET via the census occupation data in NLSY and the crosswalk formulated by Autor and Dorn (2013). My principal focus is the real log hourly wage, adjusted to 2013 dollars. As Altonji, Bharadwaj, and Lange (2012), I trim values of the real hourly wage below 3 or above 200. The results of wage returns are robust to considering alternative sample constructions, such as excluding respondents over the age of 55 or using the unprocessed real hourly wage. The key advantage of NLSY is that it is a worker-level panel, and also contains information on worker's pre-market abilities. This allows for the control of worker characteristics in assessing occupational wage returns to skill mixing and also facilitates the evaluation of return to a worker-level degree of skill mixing. The selected measures of worker abilities are chosen to align well with the skill measures in O*NET: the Armed Forces Qualifying Test (AFQT) scores represent analytical skill; the social skills measure developed by Deming (2017) is employed to represent interpersonal skill.[27]; routine skill is measured by the workers' Armed Services Vocational Aptitude Battery (ASVAB)

[26]The NLSY 1979 and NLSY 1997 are nationally representative surveys of youth, capturing data from individuals aged 14 to 22 in 1979 and 12 to 16 in 1997 respectively. During my sample period, the median age is 37, and 91 percent of the sample aged below 50.

[27]I use the AFQT scores constructed by Altonji, Bharadwaj, and Lange (2012) that are consistent across NLSY waves and account for age-at-test, test format, and other peculiarities. For interpersonal skills, I use the social skill measure developed by Deming (2017) assessing extraversion, which is constructed based on sociability in childhood and adulthood in NLSY79, and two questions from the Big 5 inventory in NLSY97 respectively

mechanical orientation scores.[28] As NLSY offers scant information on workers' computer skills, I adopt the worker's occupation or college major's computer skill value in the year 2005 as a proxy for the worker's initial endowment of computer skill. Online Appendix Table 10 lists the corresponding measures.

Moreover, the college education information in NLSY allows me to evaluate the return to studying college majors with different degrees of skill mixing.[29] I calculate for workers who studied a particular major, the employment weighted average of skill intensities and skill mixing indexes of their occupations contained in O*NET, to be used as measures of skills and skill mixing for that major.[30] In Online Appendix Table A8, I list the top majors both in terms of the levels and changes in the degrees of skill mixing for different skill pairs. Agriculture and Natural Resources stand out as it is the highest in mixing all different skills. Two other majors: Architecture and Environmental Design and Mathematics are among the top majors in mixing analytical, computer, and interpersonal skills. Whereas Engineering and Law surpass other majors in becoming more mixed of routine skills and other skills.

IV.B Wage Returns

To test for the returns to skill mixing, I regress the log wage of worker i in occupation j on the levels of different skills k required by that occupation, as well as their degrees of mixing:

$$ln(wage_{ijt}) = \sum_k \sum_{h \neq k} \left[\beta_k \text{skill}_{k,jt} + \beta_{kh} Mix(\{\text{skill}_{k,jt}, \text{skill}_{h,jt}\}\right] + \sum_k \sum_{h \neq k} \left[\omega_k \text{skill}_{k,ijt} + \omega_{kh} Mix(\{\text{skill}_{k,ijt}, \text{skill}_{h,ijt}\})\right] + \gamma X_{ijt} + \delta_j + \delta_t + \delta_i + \epsilon_{ijt} \quad (2)$$

$$\text{where skill}_k, \text{skill}_h \in \{\text{ananlytical, computer, mechanical, interpersonal}\}$$

Conditional on skill levels, the coefficients on skill mixing indexes β_{kh} identify the returns to working in occupations more mixed of those skill combinations. To further examine the worker level returns to skill mixing, I add to the right-hand side the levels of the skills that

[28] ASVAB test scores are only available for the NLSY79 survey. For NLSY97, I impute their ASVAB scores using a regression model with indicators for gender and ethnicity, and fixed effects that include age, year, census division, metropolitan area, and urbanity.

[29] There are some inconsistencies in NLSY's field of study coding: NLSY79 uses its own major codes that contain 25 two-digit categories, while NLSY97 uses another set codes for years leading to 2010 and transfers to National Center for Education Statistics (NCES)'s 2010 College Course Map (CM10) for years after 2010. For consistency, I map the two different types of major codes in NLSY97 to the 25 two-digit major categories in NLSY79. Online Appendix Table A9 shows the crosswalk of different types of major field of study codes.

[30] I take the first field within a year as representing a worker's major field in the case of multiple fields.

workers have, and their degrees of mixing. The coefficients on worker-level mixing indexes ω_{kh} then identify the wage premium to the mixing of worker skills conditional on occupation skill requirements. Throughout all the specifications, I include ethnicity by gender, age, metropolitan status, individual year, years of education, census region, and urbanity fixed effects. I also include occupation fixed effect to control time-invariant differences across occupations and focus on how the changes in skill requirements within the occupation are affecting wage returns, consistent with the empirical finding that this margin is the main driver of skill mixing. Standard errors are clustered at the individual worker level to account for within-group correlation and heteroskedasticity among repeated observations at the individual level. To focus the discussion on the wage returns to skill mixing, I only present the results on the skill mixing index β_{kh} and ω_{kh}, and discuss the returns to individual skills and how they interact with skill mixing in online Appendix A.7.

Table 3: Return to Skill Mixing of Occupation and Worker Skills

Dependent: ln(hourly wage)	(1)	(2)	(3)	(4)
Occupation Skills				
Mix (analytical + computer)	0.007	0.011**	0.013***	0.011
	[0.005]	[0.005]	[0.005]	[0.008]
Mix (analytical + interpersonal)	0.016***	0.016***	0.015***	0.027***
	[0.005]	[0.005]	[0.004]	[0.007]
Mix (computer + routine)	-0.022**	-0.029***	-0.021***	-0.026**
	[0.009]	[0.009]	[0.008]	[0.012]
Mix (computer + interpersonal)	-0.008	-0.012**	-0.014***	-0.011
	[0.006]	[0.006]	[0.005]	[0.009]
Mix (routine + analytical)	-0.050***	-0.056***	-0.050***	-0.057***
	[0.008]	[0.009]	[0.008]	[0.012]
Mix (routine + interpersonal)	0.023***	0.029***	0.019**	0.023*
	[0.008]	[0.009]	[0.008]	[0.012]
Worker Skills				
Mix (afqt + computer)		0.044*		0.021*
		[0.023]		[0.013]
Mix (afqt + social)		0.028*		-0.081***
		[0.015]		[0.021]
Mix (computer + asvab mech)		0.013		-0.081***
		[0.025]		[0.027]
Mix (computer + social)		0.008		0.065***
		[0.013]		[0.020]
Mix (asvab mech + afqt)		0.001		0.115***
		[0.009]		[0.042]
Mix (asvab mech + social)		-0.040***		-0.064
		[0.011]		[0.044]
Ethnicity × Gender, Age, Region, Edu FE	X	X	X	X
Occupation FE	X	X	X	X
Worker FE			X	X
Observations	87,655	78,719	87,655	50,580
R-squared	0.426	0.439	0.758	0.762

*Notes: This table reports the result of estimating equation (2) using pooled NLSY79&97 data for employed workers from 2005-2019. Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. The worker-level skill measures are constructed as in Table 10 and skill mixing indexes are then calculated accordingly. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.*

Occupation and Worker Level Returns: Table 3 shows the wage returns to skill mixing at both the occupation and individual levels and indicates a positive premium of mixing

non-routine skills. Column (1) shows that workers in occupations that become one standard deviation more mixed of analytical and interpersonal skills earn a wage gain of 1.6 percent per year, significant at the 1 percent level. Workers in occupations that become more mixed of analytical and computer skills also reap wage increases though not significant. Other the other hand, workers in occupations more mixed of routine and other skills saw a wage decline, except for routine and interpersonal, the mixing of which gives a 2.3 percent wage gain.

One potential concern is that workers sort into occupations in which their skills are rewarded higher, making it difficult to estimate the returns to the mixing of occupational skills across workers. In column (2), I further include worker abilities and their degree of mixing. Such a specification serves two purposes. On the one hand, the inclusion of additional worker characteristics improves the precision of the identified wage premiums at the occupation level; on the other, it sheds light on the return to skill mixing at the worker level conditional variations in the workers' occupations. The results indicate that on the occupational level, the wage premium for analytical and interpersonal, as well as routine and interpersonal skills persist, while for analytical and computer has become stronger, at 1.1 percent per year, and significant at 5 percent level. Turning to the worker side, workers that are a standard deviation more mixed of analytical and computer, as well as analytical and interpersonal also earn a wage premium of 3-4 percent, though is not very precisely estimated.

In column (3), I further restrict the analysis to within worker variation by adding worker fixed effects; along with the occupation fixed effects, this specification essentially resembles an AKM model.[31] The magnitudes of the returns to skill mixing shown in column (3) are similar to those in column (2). Workers who are in an occupation that are one standard deviation more mixed of analytical and computer as well as analytical and interpersonal skills see their wage increase by 1.3 and 1.5 percent respectively, and those who are in occupations that are one standard deviation more mixed of mechanical and interpersonal skills earn 1.9 percent more. Contrarily, workers in occupations that are more mixed with the rest of the skill combinations are associated with a wage reduction of 1.4 to 5 percent.

In online Appendix Table A6, I show that the occupational level returns to skill mixing

[31] Using within worker variation to study wage growth has been discussed and applied in i.e., Neal (1999); Gibbons et al. (2005); Lazear (2009) and Deming (2017). Choné and Kramarz (2021) found that under a worker assignment model with bundled skills, the implied wage equation also has an AKM form.

conditional on worker fixed effects is robust to alternative measures of skills and mixing. Across all these robustness checks, a similar qualitative pattern holds. Specifically, there is about a 1 to 2.5 percent wage premium for working in occupations more mixed of analytical and computer as well as analytical and interpersonal skills and a 3 percent of a wage premium for routine and interpersonal skills. The mixing of other skill combinations is associated with negative wage returns at the occupation level. Further, in online Appendix Table A6 I show that there is also a positive employment premium for workers with a more mixed skill set: workers with a more mixed level of all the skill pairs except for routine and interpersonal are also more likely exit unemployment.[32]

Returns to College Major's Skill Mixing: With the calibrated skill accumulation of workers, I test the return to obtaining a more mixing skill set in column (5), conditional on occupational skill requirements and worker fixed effects. Workers don't seem to earn a wage premium as they become more mixing of most skills in switching occupations, except for mechanical and interpersonal skills, the mixing of which brings a 10.8 percent return.[33] At the same time, the positive and significant returns to switching to an occupation more mixing of analytical and computer as well as analytical and interpersonal skills persist conditional time-varying worker characteristics, validating their robustness.

To focus on the role that schooling and college education play in workers' human capital formation and return to skill mixing, in column (6) I represent a worker's human capital by the skill content of a worker's accumulated education experience.[34] Such a designation necessarily restricts the analysis to those who have entered college and brings up selection concerns; however, controlling worker fixed effects and fixed and time-varying occupation attributes, the evidence will imply whether it is rewarding to transition to a more mixing major conditional on one's job choices. The result in column (6) shows a positive return of around 5 percent to switching to a college major a standard deviation of more mixing of

[32]Throughout my analysis, I classify a worker as employed if the worker earns a wage greater than zero and has held one or more jobs since the last NLSY interview, consistent with Altonji, Bharadwaj, and Lange (2012) and Deming (2017). Further, workers without a paying job for 24 months are considered to be out of the labor force.

[33]This is consistent with the evidence in Autor and Dorn (2013) that low-skill service occupations attract workers from mid-skill routine occupations and are associated with a wage gain; the result here indicates that such transition is likely to be also accompanied by the mixing of mechanical and interpersonal skills on the workers' side.

[34]I apply rolling averages of skill and mixing measures of a worker's entire education history to represent that worker's education skill content, therefore, there is worker-level variation.

analytical and computer or of mechanical and interpersonal skills. The wage premium to skill mixing appears to be larger for college major choices relative to occupation choices.

V A Directed Search Model with Occupation Design

The rich empirical findings on skill mixing pose challenges in understanding their driving forces. In what follows, I attempt to provide an overarching framework to investigate the mechanisms. For this purpose, I build a directed search model with several unique features: First, both firms and workers are represented by multi-dimensional skills; Second, firms must make decisions about occupation design before meeting workers, a process that involves a cost payable upon meeting as in Acemoglu (1999);[35] Third, the model incorporates non-linear production and operation cost technologies. Despite the rich setup, the model remains tractable satisfying Block Recursivity as in Menzio and Shi (2011). Under these specifications, the model offers clear insights regarding changes in skill mixing, wages, and employment that are linked to the empirical findings.

V.A Environment

Workers: Time is discrete. At each period t, there is a unit measure of heterogeneous workers that lives forever. Each worker is characterized by a vector of multi-dimensional skills $\mathbf{x} = \{x^1, ..., x^k, \ldots, x^K\} \in S \subset \mathbb{R}^K$, where K is the dimension of a closed skill space S. Workers are risk-neutral, have linear utilities equal to their consumption, and discount the future with a factor β.

Firms: On the other side of the market, there is an endogenous measure of risk-neutral firms each running one vacancy. Potential entrant firms pay a cost c to post their vacancies across different occupations $j = \{1, ..., J\}$, with $J \geq 2$. Each occupation is characterized in the same multi-dimensional skill space as workers' skills, $\mathbf{y} = \{y^1, ..., y^k, \ldots, y^K\} \in S \subset \mathbb{R}^K$, which has the interpretation of a vector skill requirement or skill importance for each of the worker skills. Firms share workers' discount factor β. In the exposition below, I subsume time subscript in describing worker and firm characteristics.

[35] As such, the model incorporates directed search on both the worker and firm sides with high-dimensional heterogeneity on the two sides.

The production function of each worker-firm pair takes a CES form of the skill inputs of workers and skill requirements of occupation j that the firm runs the job:

$$f(\mathbf{x}, \mathbf{y}) = \left[\sum_{k=1}^{K} (x^k y^k)^{\sigma} \right]^{\frac{1}{\sigma}}, \tag{3}$$

where the elasticity of substitution between the two skills is $\frac{1}{1-\sigma}$.[36] This production technology represents an extension of the production technology used in the multi-dimensional skill matching literature (i.e., Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022), where there is within-match complementarity between worker and firm attributes, but I also allow complementarity across skills regulated by σ. When multi-dimensional skill distributions of workers and firms are available, such a production technology gives a clearer portrayal of the interaction between skill demand and supply.

A unique feature of this model is that I allow firms to actively *design* the job before meeting the worker (Acemoglu (1999)), delivering a flexible degree of skill mixing. Specifically, firms with both filled and unfilled vacancies design their occupations by optimally choosing the occupational skill requirements $y(j)$ in each period.[37] Such an intensity choice of each skill in an occupation alters the efficiency of that skill will and essentially leads to different production technologies for that occupation, capturing the overall quality of the occupation and the optimal degree of skill mixing. In designing the occupation, both worker skill profiles and skill complementary play an important role, as firms would want to exploit what skills that the workers supply given the technology.[38] Nonetheless, such a job design incurs a cost $C(\mathbf{y})$ that is payable upon meeting with a worker. This cost increases in the skill level that the firm chooses ($C'(\mathbf{y}) > 0$), and represents the necessary expenses to operate an occupation. This cost can be understood as an operation cost as in Hopenhayn (1992) that is depends on the skill-level of the occupation.[39]

[36] Since labor is the only input in the model, it can be understood as "equipped" labor, and occupations' skill requirement or importance $\mathbf{y}$ takes a factor augmenting form.

[37] This feature is consistent with the empirical finding that both incumbent jobs and vacancies have changing degrees of skill mixing.

[38] For example, in designing an occupation (i.e., sales persons) for lower-skill workers who might have a greater supply of interpersonal than analytical skill, firms may want interpersonal skill to be more intensively used to take advantage the labor supply; on the contrary, if online marketing has increased the complementarity between analytical and interpersonal skills, firms may adjust accordingly to let the analytical skill to be more intensive. In equilibrium, this endogenous intensity will depend on other forces in the model.

[39] For example, to design and operate an occupation that employs high-skill workers, a firms will need to incur higher expenses in terms of better offices and equipment rentals.

Labor Market: There is a continuum of submarkets that are indexed by worker and occupation skill profiles $(\mathbf{x}, \mathbf{y})$, as well as the share of worker-firm output ω that firms promise to workers.[40] Workers with skill profile $\mathbf{x}$ direct their search towards different occupations and promised utilities, meeting one vacancy at a time. Matching between workers and firms is frictional and is regulated by a standard constant to scale matching function. Under this directed search environment, each submarket has a separate tightness (vacancy-unemployment ratio), denoted by $\theta(x, y, \omega)$. In each submarket, workers find job with probability $p(\theta(x, y, \omega))$, and firms fill the vacancy with probability $q(\theta(x, y, \omega)) = p(\theta(x, y, \omega))/\theta(x, y, \omega)$.[41]

The timing of the model evolves as follows. At the beginning of each period, a fraction δ of worker-firm pairs separate exogenously. Before the labor market opens and unlike standard search models, firms will first need to design the occupations at this stage before they post vacancies across submarkets. The labor market then opens, and both unemployed and employed search for unfilled vacancies and form matches with firms under the constant return to scale matching technology. The labor market then closes, firms produce the output, pay the occupation operation cost, as well as the wage which is a share of output to workers. Unemployed workers receive a transfer with a value of b. Lastly, workers are able to be learning-by-doing, and their skills evolve according to the Markov process depending on their employed occupations.

Aggregate and Individual State: The aggregate state of the economy is the distribution of workers across employment status, skill profiles, occupational skill requirements, and promised utilities, denoted as $\psi \in \Psi$. I subsume aggregate state in the exposition of model equilibrium in the next section and show that in fact, the model equilibrium is independent of the aggregate state.

Nonetheless, in the model, I allow workers to learn on the job, and their subsequent skill profiles are contingent upon their current employment status, as in Lise and Postel-Vinay (2020). Specifically, considering each skill j in the worker's skill profile $\mathbf{x}$ as an element of the

[40] This arrangement can be considered as an employment contract simply specifies the output share ω promised to the worker contingent on the state for the current period, as well as the continuation value of the match in the subsequent period (see next section). The contract is assumed to be fully committed by both the workers and firms.

[41] Functions p and q also satisfy usual regularity conditions: twice continuously differentiable; $p'(\theta) > 0, p''(\theta) < 0, p(0) = 0$; $q'(\theta) < 0, q''(\theta) > 0, q(0) = 1$.

finite set S, the evolution of this skill follows a Markov process $\pi(x_j'|x_j, y_j)$, conditional on the worker's current skill level and employed occupation. If a worker is matched with an occupation that requires a skill level exceeding his or her own ($x_j < y_j$), the worker's skill j will adjust upward in the next period: $x_j' > x_j$, and the inverse applies for a worker whose skill is lower than the requirements of their current occupation. The probability, and thereby the speed of skill adjustment, is contingent upon the specific j. For unemployed workers, they are treated such that their present occupation demands a zero level for all skills. The calibration of the skill adjustment probability is discussed in Section VI.

V.B Model Equilibrium

I shall now characterize the optimal strategies for workers' job search and firms' job creation and continuation. The value functions for workers are described at the point of the production stage when the labor market comes to a close, while for firms I also consider the job design stage before the labor market opens.

Worker's Problem: Let $U(\mathbf{x})$ denote the value of being unemployed and searching for a worker $\mathbf{x}$ at time t. Similarly, let $W(\mathbf{x}, \mathbf{y}, \omega)$ be the total discounted returns from holding a job of skill requirements $\mathbf{y}$ and output share ω at time t. These values can be written as:

$$\begin{aligned} U(\mathbf{x}) &= b + \beta E\Big\{\max_{y',\omega'} \; p(\theta_{t+1}(\mathbf{x}', y', \omega'))W_{t+1}(\mathbf{x}', y', \omega') \\ &\qquad + \big[(1 - p(\theta_{t+1}(\mathbf{x}', y', \omega'))\big]U_{t+1}(\mathbf{x}')\Big\} \\ W(\mathbf{x}, \mathbf{y}, \omega) &= \omega f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y}) + \delta U_{t+1}(\mathbf{x}') + \beta(1-\delta)E\Big\{\max_{\mathbf{y}',\tilde{\omega}'} \; p(\theta_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}')) \\ &\qquad + W_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}')\big[(1 - p(\theta_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}'))\big]W(\mathbf{x}', \mathbf{y}, \omega)\Big\} \end{aligned} \tag{4}$$

Unemployed workers gain a utility b through the current period's transfer. In the subsequent period, their skills may transition to $\mathbf{x}'$, which are likely to depreciate due to their unemployed status. Meanwhile, within the submarket that aligns with their skill profiles, workers engage in the search for vacancies that span a variety of occupations $\mathbf{y}$ and output shares ω, looking for the highest continuation value. In choosing $\mathbf{y}$ and ω, workers face the tradeoff between the value of employed and the success probability of a match $p(\theta_{t+1}(\mathbf{x}', y', \omega')$, both of which hinge on the occupation and output share that the

workers target. Should the match prove successful, the workers enjoy the continued value that employment offers; otherwise, their status of unemployment persists.

Workers currently employed in a firm characterized by $(\mathbf{y}\omega)$ receive a wage equivalent to the share *omega* of the output from their match. When the subsequent period arrives, they face a probability *delta* of an exogenous separation, in which case they become unemployed with a value $U_{t+1}(\mathbf{x}')$ and engage in job search immediately. Employed workers perform on-the-job searches in their current match for new occupations and output shares $(\mathbf{y}', \tilde{\omega}')$, on the premise that there is a positive probability $p(\theta_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}'))$ that the continuation value from the new match offers exceeds that of the original firm. In the absence of such possibilities or if the transition is not successful, the worker remains with the initial firm.

Firm's Problem: Consider a firm running occupation $\mathbf{y}$, offering output share ω, and employing worker $\mathbf{x}$. Let $J(\mathbf{x}, \mathbf{y}, \omega)$ denote the total discounted profits to this firm:

$$\begin{aligned} J(\mathbf{x}, \mathbf{y}, \omega) = \max_{\mathbf{y}} & (1-\omega) f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y}) \\ & + \beta(1-\delta) E\Big\{ (1 - p(\theta_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}')) J(\mathbf{x}', \mathbf{y}, \omega) \Big\} \end{aligned} \tag{5}$$

In the current period, firms receive a portion $(1-\omega)$ of the worker-firm output, after paying the workers their wages. In addition, firms also need to cover the occupation operation cost $C(\mathbf{y})$, which depends on the skill levels required by the occupation in which the firm is engaged. The labor market operates under free entry for firms, hence, maintaining a vacancy bears no value. In the case of exogenous separation, or with a probability $p(\theta_{t+1}(\mathbf{x}', \mathbf{y}', \tilde{\omega}'))$ that the worker finds another job at an optimal occupation $\mathbf{y}'$ and output share $\tilde{\omega}'$ through on-the-job search, the firm accrues no profits. In the case where the match persists, the firm continues to acquire discounted profits from the match.

$$c = \beta E\Big\{ q(\theta(\mathbf{x}, \mathbf{y}, \omega)) J(\mathbf{x}, \mathbf{y}, \omega) \Big\} \tag{6}$$

The free-entry condition further highlights firms' choice of optimal degree of skill mixing and the tradeoff that agents face in the model. Prior to the opening of the labor market in each period, incumbent and entrant firms re-design the occupation, taking into consideration the overall production technology and worker skills within their respective submarkets.[42]

[42]Considering that incumbent firms and new entrants utilize identical production technologies and confront

Given that the value of a vacancy is zero, firms will opt for an optimal skill mixing that equates the firm's anticipated discounted profits to the cost of vacancy posting as in equation (6). This condition implicitly pins down market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$. If an occupation for a specific worker type becomes more profitable, the number of vacancies posted will increase, leading to a rise in market tightness but at the same time a reduction in the job-filling rate.[43]

The free entry condition also reflects the tradeoff faced by workers. Since workers receive the remaining output claimed by the firms, in markets with higher job-finding probabilities (i.e., tighter markets), the value of employment is likely to be lower. Workers' job finding probability also feedback to firms' discounted profits through worker on-the-job search and the chance that the firm attracts other employed workers.

Block-recursive Equilibrium: Despite the multi-dimensional skill setup, the model still achieves great analytical tractability by relieving the dependence on the entire distribution of agents across aggregate states in characterizing agents' value functions and the market tightness. Such a convenient feature was coined as "block-recursive" in Menzio and Shi (2010) and Menzio and Shi (2011) for a broad range of directed search models.[44] This is a result of two features of the model. First, as search is directed and workers choose optimally the occupation and output share, their life utility does not depend on their outside options, and workers do not need to forecast the wage depending on the entire distribution of employment. Second, there are separate markets for workers of different profiles, and workers search for jobs within their own submarket, in which firms carry different occupations. This additional degree of seperability implies that the market tightness of a submarket is independent of the worker distribution in other markets, relieving the burden of workers and firms to forecast other markets in making their decisions.[45] In online Appendix B.2, I formally define a Block-recursive equilibrium for the economy and show its existence and uniqueness.

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the same worker skills within each submarket, their choices align.

[43] As in other directed search models, only a portion of submarkets may open in equilibrium, depending on firm's value and corresponding market tightness in different markets

[44] Block recursivity allows not only analytical tractability but also enables standard numerical techniques to solve the model. The framework considered in this paper involves more heterogeneity and requires an additional degree of directness, as discussed.

[45] Such additional directness implies that, i.e., computer scientists only confront other computer scientists in job search, while sales clerks only compete with other sales clerks. In reality, the degree of separability will depend on specific occupations and the overall economic condition. As reported by Osberg (1993), search directedness is procyclical and is higher when the market is tight. In bringing the model to the data, I use economic recovery periods and more coarse occupations to be consistent with the model.

Skill Mixing, Wages and Employment: The model yields several predictions regarding changes in skill mixing, wages, and employment that align closely with the empirical findings detailed in Sections III and IV. These predictions emphasize the role of skill complementarity within a production framework that features indivisible skills. The formal propositions and proofs of these outcomes can be found in online Appendix B.1, and a concise discussion is provided here.

Under the production technology described in equation (3) and the occupational design cost outlined in (7), increased complementarity in production or a higher degree of increasing marginal costs leads firms to find it more profitable to employ a mixture of different skills rather than specializing in one, leading to increased skill mixing. Additionally, the supply of skills by workers influences these outcomes: as workers supply a more diverse set of skills, it becomes more efficient to design jobs that require this mix of skills. In terms of wages and employment, if skills become more complementary in production or less costly to combine, the output of the worker-firm match rises, leading to wage increases. Through the free entry condition specified in equation (6), this increased joint worker-firm value results in a tighter labor market and an elevated job-finding probability for workers. I quantitatively calibrate the model and test these predictions in the next section.

VI Quantitative Analysis

In this section, I calibrate the model to evaluate the quantitative importance of different channels contributing to the phenomenon of skill mixing, and explore implications for wages and employment. I first describe data construction and measurement, followed by a discussion of calibration strategy, and estimated parameters. I then perform counterfactual analyses and show that technological shifts embodied in the changes in complementarity of skills in production, the operation cost of occupation operation, as well as the efficisency differential of high-type workers have played a major role in driving the variation in skill mixing. I then show that these forces also account for a significant part of wage and employment shifts.

VI.A Measurement and Calibration

Data and Measurement: I apply the same combination of NLSY 79 & 97 along with O*NET data as in Section IV to calibrate the model. The datasets provide counterparts to the

model variables such as worker skills ($\mathbf{x}$), occupational skill requirements ($\mathbf{y}$), employment distribution, and wage levels. The model is calibrated to two periods of data from early 2000s to late 2010s separately, which coincides with a substantial shift in skill mixing and abstracts from the financial recession. Specifically, the steady-state of the model is fitted to the data from 2005–2006 and 2016–2019 to ensure comparability of sample sizes across these two periods, and I restrict to those workers with information on their skills.[46] Finally, for both worker and job skill profiles, I consider the same set of skills (analytical, computer, interpersonal, routine) as in Section IV, only combining analytical and computer skills into one to have a skill dimension of three to be feasible for quantitative analysis.

Considering the potential influence of skill supply variation on skill mixing, I calibrate two key aspects of it. First, worker skill supply variation *across* the two data periods is introduced to align with the workers' choices of occupation and college major (if attended), as well as their employment status, following the approach of Lise and Postel-Vinay (2020). Specifically, a worker accumulates γ_j times the gap between the worker's endowment and an occupation's requirement of skill j in each year, with the value of γ_j depending on learning or depreciation (upward or downward accumulation).[47] Second, the Markov process $\pi(x'_j|x_j, y_j)$ of worker skill adjustment, detailed in Section V, is calibrated such that if an occupation demands a higher (or lower) skill level than the worker possesses, the worker skill will adjust upwards (or downward) in the next model period. Given that a model period equates to a quarter, the yearly skill adjustment rates are divided by 4 ($\frac{\gamma_j}{4}$). This value is further scaled by the gap between the worker's skill set and the occupation's demands to compute the adjustment probability.[48] Online Appendix B.4 provides further details of the skill supply calibration.

[46] NLSY 1997 was conducted annually during 2005-2006, but only biannually in 2016-2019, so does NLSY 1979 for the later period. The same sizes for the two selected periods are 30,654 and 43,340 respectively.

[47] The speed of skill adjustment γ_j is calibrated using the estimates from Lise and Postel-Vinay (2020) as in online Appendix Table 10. Workers' skills can be lost when not employed but can't be lower than their initial endowments. For skill changes while in school, I specify that workers spend on average 3 years learning the skills of their majors.

[48] In particular, the Markov probability of upward adjustment is determined by: $\frac{x_j^{up}-x_j}{y_j-x_j}\mathbf{1}(x_j^{up} < y_j) \times \frac{\gamma_j^{up}}{4}$, and of downward adjustment is given by: $\frac{x_j^{down}-x_j}{y_j-x_j}\mathbf{1}(y_j < x_j^{down}) \times \frac{\gamma_j^{down}}{4}$. Here, x_j represents the current grid value of worker skill j, while x_j^{up} or x_j^{down} denotes the value of worker skill j up or down a grid, respectively. The indicator variables $\mathbf{1}(y_j < x_j^{down})$ or $\mathbf{1}(x_j^{up} < y_j)$ evaluates whether the skill j grid value of the worker's current employed occupation is greater or smaller than the value of the worker's skill j grid. This means that a worker will only adjust up or down a grid if the occupation's skill is larger or smaller than the corresponding up or down grid value for the worker's skill.

Functional Forms: The model is parameterized as follows. The multi-dimensional skill production function is defined as in equation (3), which accommodates cross-skill complementarity controlled by σ and enables a sensible interaction between skill demand and supply, in line with the multi-dimensional matching literature (i.e.,Lise and Postel-Vinay 2020; Lindenlaub 2017; Ocampo 2022). I allow a flexible form of occupation operation cost $C(\mathbf{y})$ as in equation (7), where ρ regulates the degree of increasing or decreasing marginal cost of elevated skill requirements, and τ governs the scale of the cost of operating an occupation.[49]

$$C(\mathbf{y}) = \tau[\sum_{k=1}^{K}(y^k)^\rho] \tag{7}$$

The matching function assumes a standard Cobb-Douglas form, $M(s,v) = \mu s^\eta v^{1-\eta}$, indicating that η is the elasticity of matches concerning total search effort, and μ is the matching efficiency. This function form leads to the job finding rate being $p(\theta) = \mu\theta^{1-\eta}$ and the vacancy filling rate being $q(\theta) = \mu\theta^{-\eta}$.

Calibration Strategy: The calibration of parameters falls into two categories. For parameters that regulate the search environment, I follow closely the conventions of the search and matching literature. However, for those parameters linked to the technology for skills in production, as well as the efficiency differential of high-type workers, I estimate them internally through Simulated Methods of Moments (SMM).

The model period is one quarter. Given that all agents are risk-neutral, the discount rate β is assigned a value of 0.99, corresponding to an annual interest rate of approximately 4 percent. The job separation rate δ is set at 10 percent per quarter, as in Shimer (2005). For employed workers, their share of output ω is set at 0.6, mirroring the labor share of GDP in 2005. For unemployed workers, the unemployment benefit b is set at 0.25 as in Braxton and Taska (2023), which equates to about 51 percent of worker consumption in the calibrated model. The elasticity of the matching function η is set at 0.5 as is standard, and the matching efficiency μ is set to 0.65, as in Mercan and Schoefer (2020). Table 5 panel A summarizes these externally calibrated parameters.

Before describing the protocol of internal estimation, I outline how to map occupations and workers in the model to the data and the choice of grid points. I classify occupations

[49]Besides technical convenience, the functional form (7) also implies that for a given cost, firms need to tradeoff the choice of altering different skill intensities.

Table 4: Moments and Model Match

	First Period		Second Period	
	Data	Model	Data	Model
Worker moments				
Relative wage of high type				
Analytical/computer	1.30	1.29	0.95	1.02
Interpersonal	1.00	1.00	1.25	1.28
Routine	1.52	1.53	1.54	1.40
Unemployment rate	0.05	0.06	0.04	0.04
Occupation moments				
Relative wage of high skill	1.30	1.30	1.56	1.41
Employ. share (low skill)	0.43	0.42	0.37	0.32
Employ. share (high skill)	0.57	0.58	0.63	0.68
$100 \times$ Skill mixing (low skill)	97.54	96.83	98.96	99.10
$100 \times$ Skill mixing (high skill)	95.74	96.84	94.12	95.11

Notes: This table reports the average values of the targeted moments both in the data and through model simulation. The data used for the moment calculation and for SMM estimation are two periods of pooled NLSY79&97 for employed workers: period 1 from 2005–2006 and period 2 from 2016–2019. Two types of moments are included. The worker moments include the relative wage of high type workers as well as the unemployment rate. The occupation moments include the relative wage of high skill occupations, the employment share and the skill mixing index of RNR skills in low and high skill occupations.

into high- and low-skill, as in Section IV, with the former group including high-wage and white-collar jobs, and the latter blue-collar and service jobs. The grid point for an occupation's requirement of a skill y_j is set such that moving up one grid corresponds to 50 percent of the median observed value of y_j for that occupation.[50] On the worker side, for each of the worker skill x_j, a worker belongs to the high type if the skill is above the median value among all workers.[51] For those workers classified as low-type in any skill, their skill grid is set to be the average observed value within that category. I then internally estimate the skill level of high-type worker relative to low-type for each skill ($\alpha_k, k = \{\text{analytical/computer } (a), \text{interpersonal } (p), \text{routine } (r)\}$).[52] Since labor is the only input in production, these relative efficiencies represent the "efficiency differential" of high-type workers that leads to higher productivity (i.e., Katz and Murphy 1992; Autor, Katz, and

[50] As the model calibration uses data of two periods with a consistent grid, I determine grid points by averaging the occupation's median values across both periods.

[51] With three chosen skills, there are 8 worker types in the model.

[52] As I merge analytical and computer skills into one for calibration using their average values, I denote this combined skill as "analytical/computer".

Krueger 1998; Goldin and Katz 2010).[53]

The SMM procedure starts with solving the steady state policy for the agents given the parameters of the model. A panel of workers, each with a lifespan of 80 quarters, is then simulated. These workers start as unemployed and draw their skill supply from the calibrated empirical distribution. As each model period concludes, the workers' skills evolve following the calibrated Markov process, $\pi(\mathbf{x}'|\mathbf{x},\mathbf{y})$. Each iteration thus results in a distribution of employment statuses and corresponding labor market outcomes. The parameters are then estimated minimizing the distance between simulated and empirical moments by searching over the parameter space, offering local identification. Online Appendix B.5 provides further details on the numerical implementation.

The estimation targets 9 moments as shown in Table 4 for both periods of data that include: i) the relative wage of the high-type worker for each skill; ii) the unemployment rate; iii) the relative wage of high-skill occupation; iv) the share of employment across occupations; and iv) the skill mixing indexes of occupations. All moments are directly computed from the two periods of data from NLSY, except for unemployment, for which I use the statistics from Bureau of Economic Analysis (BEA) to avoid the age composition effects present in NLSY.[54] The model parameters are jointly identified from the moments, for which a concise summary of the key information for identification is given below with more detailed discussion in online Appendix B.3. I first identify complementarity parameter of skills in production σ targeting the correlation of within occupation relative wages and worker skills. The cost parameters ρ is then estimated leveraging the firm's optimization conditions in skill mixing. Conditional on parameters estimated at the production side, the model's employment distribution further aids in estimating τ. Lastly, unemployment rate and relative wage structures disciplines the vacancy posting cost and the efficiency differential of high-type workers.

Estimation Results: Table 5 panel B presents the internally estimated parameters, which indicates considerable technological shifts between the two periods. For the initial period, the estimated σ is 0.5, suggesting that skills are substitutable in production. Firms encounter

[53] As such, the quantitative analysis explores both the role of changes in relative input efficiency that is the focus of SBTC literature, and changes in skill complementarity.

[54] The cohort's age profile in NLSY data influences the reported employment rate. For example, by the late 2010s, a substantial segment of the NLSY 79 cohort are above age 50, making them more likely to be out of the labor force. Additionally, the unemployment rate from NLSY, derived from numbers of job held since the last survey, averages 9 percent, notably higher than BEA data. However, this decision primarily affects vacancy posting cost parameter.

Table 5: Parameter Estimates

Parameter	Description	Value	
	A. Externally Calibrated		
β	Discount Rate	0.99	
δ	Job separation rate	0.1	
ω	Worker share of surplus	0.6	
b	Unemployment benefit	0.25	
η	Elasticity of the matching function	0.5	
μ	Matching efficiency	0.65	
	B. Internally Estimated	Period 1	Period 2
σ	Elasticity parameter of skills in production	0.5	0.3
τ	Scaler of occupation operation cost	1.4	1.9
ϕ	Rate of increasing marginal cost	1.2	1.7
α_a	Efficiency differential of analytical/computer skill	1.2	1.6
α_p	Efficiency differential of interpersonal skill	1.0	1.5
α_r	Efficiency differential of routine skill	1.2	1.1
c	Vacancy posting cost as a share of output	0.1	0.4

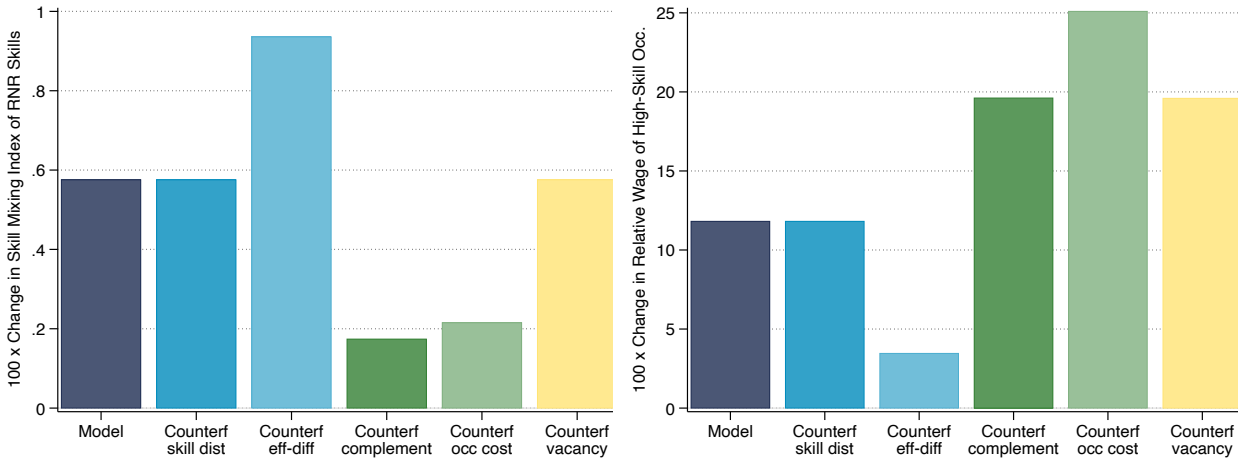
Notes: This table shows the exogenously calibrated as well as internally estimated parameters. The data used for the internal estimation are two periods of pooled NLSY79&97 for employed workers: period 1 from 2005–2006 and period 2 from 2016–2019.

rising marginal costs as they increase skill requirements ($\rho \geq 1$). When it comes to efficiency differentials, high-type workers in analytical/computer, interpersonal, or routine skills are 1.5 to 2.3 times as productive as their low-type counterparts. Vacancy posting cost is 10 percent of the quarterly output of a worker-firm match, close to the estimates of Silva and Toledo (2009).[55]

In the second period, there is a significant rise in skill complementarity in production, reflected in the reduction of σ. During this period, the occupation operation cost for firms also rises, reflected in the increase of both the scale and the convexity of the cost function. As discussed in Section VI, this increased complementarity as well as cost of skills intensifies firms' incentives to mix skills. Moreover, efficiency differential increases for analytical/computer and interpersonal skills, whereas it has considerably declined for routine skills. This pattern suggests that wage premiums for high-type workers of routine skills become much lower compared to other skills; it also implies that the incentive to mix routine with other skills in

[55] Silva and Toledo (2009) estimate that in early 2000s, the vacancy posting cost is 14 percent the average quarterly compensation of workers. Given that my calibration sets the worker output share at 60 percent, this suggests that the vacancy posting cost equates to 16 percent of the quarterly worker compensation.

Figure 6: Model Counterfactual



Notes: These figures plot the model generated changes in skill mixing in low-skill occupations (panel 1) and changes in relative wage of high-skill occupation (panel 2). Different model channels are shut down individually by eliminating the relative calibrated values to highlight the contribution of each channel. The full model has all the model features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differential, and vacancy posting cost across the two periods are shown in Table 5. Worker skill supply distribution variation across the periods are calibrated according to Table 10.

high-skill occupation will likely decline. Lastly, the cost of posting vacancies has increased post 2010s, consistent with the finding of Schaal (2017).[56]

VI.B The Drivers of Skill Mixing and Implications for Wages and Employment

In this section, I employ the model to perform a series of counterfactual experiments to assess the relative significance of each model channel in explaining the shifts in the degree of skill mixing. Additionally, I evaluate the influence of these channels on the changes in earnings and employment distribution. Specifically, I focus on the roles of changes in skill supply, efficiency differentials, skill complementarity in production, occupation operation cost, and vacancy posting cost in generating moment variations that align with the data. To gauge the importance of each of these channels, I individually suspend the shifts in parameter values associated with each of these channels and observe the impact on different outcomes.

Counterfactual Skill Mixing: I begin by assessing how different channels contribute to

[56] Schaal (2017) shows that when incorporating data from these later years, the vacancy posting cost is twice that of the estimate of Silva and Toledo (2009).

the growth in skill mixing within low-skill occupations, which has been noticeably observed in the data.[57] The first panel of Figure 6 illustrates that the full model predicts a rise in skill mixing within low-skill occupations over the two periods. In the absence of changes in the supply of worker skills across these periods, the predicted increase in skill mixing remains unchanged. On the other hand, if we ignore changes in the efficiency differential of high-type workers, the predicted increase in skill mixing increases by nearly 50 percent. This result implies that the growing efficiency differential of analytical-computer and interpersonal skills, along with the large decline in the efficiency differential for routine skills, makes their combination in low-skill occupation less favorable, consistent with our prediction.[58]

The subsequent counterfactual results indicate that the rise in the complementarity of skills in production and in the occupational operation costs are the main determinants of skill mixing. First, if we suppress the increase in skill complementarity while preserving the variation in parameters, the growth in skill mixing becomes a third of the level under the full model. Similarly, if we eliminate the changes in occupational operation cost, the growth in skill mixing reduces by nearly two thirds. These experiments underscore that the joint evolution of skill complementarity and occupational operation cost is the central force driving the increase in skill mixing over the two periods.[59] Lastly, the costs associated with posting job vacancies appear to have little impact on skill mixing.

Wage and Employment Effects: I proceed to investigate how the same model channels influencing skill mixing also impact wage and employment distribution. The second panel of Figure 6 illustrates the unweighted relative wage changes between high-skill and low-skill occupations from the early 2000s to the late 2010s, with the model predicting an approximate 12 percent increase. As observed with skill mixing, variations in worker skill supply and vacancy posting cost appear to have minimal effects. In contrast, the efficiency differential between worker types emerges as a key driver; without it, the wage premium for high-skill occupations would have declined to only 3.5 percent. These results are intuitive: as efficiency

[57] I focus on changes within occupation for my counterfactual analysis, primarily because this appears to be the main area where skill-mixing changes occur, as shown in Section IV. Additionally, in the quantitative part of this study, the number of occupations is far fewer than in the actual data, leading to more fluctuations in employment shares. This could distort the view of skill mixing if worker re-allocation is taken into account.

[58] The result is similar for high-skill occupation. In online appendix B.6 I show that if we disregard changes in the efficiency differential, skill mixing rises above the level predicted by the full model in high-skill occupation, suggesting that these changes actually reduce skill mixing.

[59] Online Appendix B.6 investigates the implications of τ and ϕ individually for skill mixing changes and results show that they are equally important.

differentials increase, the absolute skill difference between high- and low-type workers grows, leading to higher wage gains in high-skill occupations that more likely to employ high-type workers.

Regarding wage distribution, the rise in the complementarity of skills in production and in occupation operation costs predominantly compress the wage gap between high- and low-skill occupations. Ignoring skill complementarity while maintaining variation in other parameters would have shifted the high-skill occupation's wage premium from a 12 percent increase to a 20 percent increase. Similarly, if we only vary occupational operation costs, the increase would become 25 percent. These two factors collectively work to diminish wage disparities across occupations. In online Appendix B.6, I discuss the effects of these channels on the employment gains in high-skill occupation. The impact of changes in efficiency differentials and occupation operation costs on employment mirrors their effects on wages. However, for high-skill occupation's employment gains, increased skill complementarity has been a positive driver, while growing vacancy posting cost has reduced them.

In summary, the growth in complementarity of skills in production and in cost of skills for occupation operation emerge as the main drivers of skill mixing changes, whereas efficiency differential changes of high-type workers negatively affects the degree of skill mixing across the periods. For changes in wage and employment distributions, the growth in skill complementarity and occupation costs compress the high-skill occupations' wage premium, although the increased skill complementarity has positively affected high-skill occupation's employment gains. Furthermore, the changes in efficiency differentials contribute to the rise in wage and employment gains in high-skill occupations.

VII Conclusion

Skills are inevitably embedded in workers and understanding the demand for skill *mixtures* is important in studying the dynamics of the labor market. I present a rich set of empirical findings on the phenomenon of "skill mixing", and show that between 2005-2018, there has been a sizable growth in the degree of mixing, particularly for analytical, computer, and interpersonal skills. To understand the heterogeneous within-occupation variation in skill mixing of different occupations, I provide an integrated explanation incorporating insights from the directed search and endogenous technological change literature. Bringing the model to the data, I estimate parameters that are important in understanding the interaction among

skills and show that technological change is the main driver of skill mixing.

The phenomenon of skill mixing brings forth very different policy implications for worker training and college consulting. Using NLSY 79 and 97 combined with O*NET data, I show that workers in occupations that become more mixed of analytical, computer, and interpersonal skills earn a positive wage premium. Further, I calculated the degree of mixing for each college major and show that students who have studied a college major more mixed of analytical and interpersonal skills earn as much as 5 percent more. In sum, this paper's results suggest that in a world with an increasing trend of skill mixing with positive wage premiums due to technological advancements, educators and policymakers should consider providing more "mixed" skills to workers rather than focusing solely on nurturing expertise and specialization.

References

Abowd, J. M., Kramarz, F., and Margolis, D. N. (1999). High wage workers and high wage firms. *Econometrica*, 67(2):251–333.

Acemoglu, D. (1999). Changes in unemployment and wage inequality: An alternative theory and some evidence. *American economic review*, 89(5):1259–1278.

Acemoglu, D. and Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier.

Acemoglu, D. and Zilibotti, F. (2001). Productivity differences. *The Quarterly Journal of Economics*, 116(2):563–606.

Altonji, J. G., Bharadwaj, P., and Lange, F. (2012). Changes in the characteristics of american youth: Implications for adult outcomes. *Journal of Labor Economics*, 30(4):783–828.

Atalay, E., Phongthiengtham, P., Sotelo, S., and Tannenbaum, D. (2020). The evolution of work in the united states. *American Economic Journal: Applied Economics*, 12(2):1–34.

Atkinson, A. B. and Stiglitz, J. E. (1969). A new view of technological change. *The Economic Journal*, 79(315):573–578.

Autor, D. H. and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American economic review*, 103(5):1553–97.

Autor, D. H. and Handel, M. J. (2013). Putting tasks to the test: Human capital, job tasks, and wages. *Journal of labor Economics*, 31(S1):S59–S96.

Autor, D. H., Katz, L. F., and Krueger, A. B. (1998). Computing inequality: have computers changed the labor market? *The Quarterly journal of economics*, 113(4):1169–1213.

Autor, D. H., Levy, F., and Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly journal of economics*, 118(4):1279–1333.

Autor, D. H. and Price, B. (2013). The changing task composition of the us labor market: An update of autor, levy, and murnane (2003). *Working Paper*.

Baley, I., Figueiredo, A., and Ulbricht, R. (2022). Mismatch cycles. *Journal of Political Economy*, 130(11):2943–2984.

Basu, S. and Weil, D. N. (1998). Appropriate technology and growth. *The Quarterly Journal of Economics*, 113(4):1025–1054.

Blau, F. D., Brummund, P., and Liu, A. Y.-H. (2013). Trends in occupational segregation by gender 1970–2009: Adjusting for the impact of changes in the occupational coding system. *Demography*, 50(2):471–492.

Braxton, J. C. and Taska, B. (2021). Technological change and the consequences of job loss.

Braxton, J. C. and Taska, B. (2023). Technological change and the consequences of job loss. *American Economic Review*, 113(2):279–316.

Caselli, F. and Coleman, Wilbur John, I. (2006). The world technology frontier. *American Economic Review*, 96(3):499–522.

Castex, G. and Kogan Dechter, E. (2014). The changing roles of education and ability in wage determination. *Journal of Labor Economics*, 32(4):685–710.

Choné, P. and Kramarz, F. (2021). Matching workers' skills and firms' technologies: From bundling to unbundling. *Working Paper*.

Cortes, G. M., Jaimovich, N., and Siu, H. E. (2021). The growing importance of social tasks in high-paying occupations: implications for sorting. *Journal of Human Resources*, pages 0121–11455R1.

Costello, A. B. and Osborne, J. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical assessment, research, and evaluation*, 10(1):7.

Davis, S. J., Faberman, R. J., and Haltiwanger, J. C. (2013). The establishment-level behavior of vacancies and hiring. *The Quarterly Journal of Economics*, 128(2):581–622.

Deming, D. and Kahn, L. B. (2018). Skill requirements across firms and labor markets: Evidence from job postings for professionals. *Journal of Labor Economics*, 36(S1):S337–S369.

Deming, D. J. (2017). The growing importance of social skills in the labor market. *The Quarterly Journal of Economics*, 132(4):1593–1640.

Deming, D. J. and Noray, K. (2020). Earnings dynamics, changing job skills, and stem careers. *The Quarterly Journal of Economics*, 135(4):1965–2005.

Edmond, C. and Mongey, S. (2021). Unbundling labor. *Working Paper*.

Eeckhout, J. and Kircher, P. (2018). Assortative matching with large firms. *Econometrica*, 86(1):85–132.

Freeman, R. B., Ganguli, I., and Handel, M. J. (2020). Within-occupation changes dominate changes in what workers do: A shift-share decomposition, 2005–2015. In *AEA Papers and Proceedings*, volume 110, pages 394–99.

Gibbons, R., Katz, L. F., Lemieux, T., and Parent, D. (2005). Comparative advantage, learning, and sectoral wage determination. *Journal of labor economics*, 23(4):681–724.

Goldin, C. and Katz, L. F. (2010). *The race between education and technology*. harvard university press.

Goos, M., Manning, A., and Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American economic review*, 104(8):2509–2526.

Guvenen, F., Kuruscu, B., Tanaka, S., and Wiczer, D. (2020). Multidimensional skill mismatch. *American Economic Journal: Macroeconomics*, 12(1):210–44.

Hagedorn, M., Law, T. H., and Manovskii, I. (2017). Identifying equilibrium models of labor market sorting. *Econometrica*, 85(1):29–65.

Heckman, J. J., Lochner, L., and Taber, C. (1998). Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. *Review of economic dynamics*, 1(1):1–58.

Heckman, J. J. and Sedlacek, G. (1985). Heterogeneity, aggregation, and market wage functions: an empirical model of self-selection in the labor market. *Journal of political Economy*, 93(6):1077–1125.

Hershbein, B. and Kahn, L. B. (2018). Do recessions accelerate routine-biased technological change? evidence from vacancy postings. *American Economic Review*, 108(7):1737–1772.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150.

Jin, X. (2017). The returns to specialization: Evidence from education-occupation match in the us from 1993 to 2017. *Working Paper*.

Johnson, G. E. (1997). Changes in earnings inequality: the role of demand shifts. *Journal of economic perspectives*, 11(2):41–54.

Jones, C. I. (2005). The shape of production functions and the direction of technical change. *The Quarterly Journal of Economics*, 120(2):517–549.

Kaas, L. and Kircher, P. (2015). Efficient firm dynamics in a frictional labor market. *American Economic Review*, 105(10):3030–3060.

Katz, L. F. and Murphy, K. M. (1992). Changes in relative wages, 1963–1987: supply and demand factors. *The quarterly journal of economics*, 107(1):35–78.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., and Violante, G. L. (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1053.

Lazear, E. P. (2009). Firm-specific human capital: A skill-weights approach. *Journal of political economy*, 117(5):914–940.

León-Ledesma, M. A. and Satchi, M. (2019). Appropriate technology and balanced growth. *The Review of Economic Studies*, 86(2):807–835.

Lindenlaub, I. (2017). Sorting multidimensional types: Theory and application. *The Review of Economic Studies*, 84(2):718–789.

Lise, J. and Postel-Vinay, F. (2020). Multidimensional skills, sorting, and human capital accumulation. *American Economic Review*, 110(8):2328–76.

Menzio, G. and Shi, S. (2010). Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, 145(4):1453–1494.

Menzio, G. and Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510.

Mercan, Y. and Schoefer, B. (2020). Jobs and matches: Quits, replacement hiring, and vacancy chains and vacancy chains. *American Economic Review: Insights*, 2(1):101–124.

Murphy, K. (1986). Specilization and human capital. *PhD dissertation, University of Chicago - Department of Economics*.

Neal, D. (1999). The complexity of job mobility among young men. *Journal of labor Economics*, 17(2):237–261.

Ocampo, S. (2022). A task-based theory of occupations with multidimensional heterogeneity. *Working Paper*.

Osberg, L. (1993). Fishing in different pools: job-search strategies and job-finding success in canada in the early 1980s. *Journal of labor economics*, 11(2):348–386.

Ransom, M. R. and Phipps, A. (2017). The changing occupational distribution by college major. In *Skill Mismatch in Labor Markets*. Emerald Publishing Limited.

Rosen, S. (1983). Specialization and human capital. *Journal of Labor Economics*, 1(1):43–49.

Ross, M. B. (2017). Routine-biased technical change: Panel evidence of task orientation and wage effects. *Labour Economics*, 48:198–214.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford economic papers*, 3(2):135–146.

Schaal, E. (2017). Uncertainty and unemployment. *Econometrica*, 85(6):1675–1721.

Shi, S. (2001). Pricing and signaling with frictions. *Journal of Economic Theory*, 98:232–260.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, 95(1):25–49.

Silva, J. I. and Toledo, M. (2009). Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics*, 13(S1):76–96.

Thompson, B. and Daniel, L. G. (1996). Factor analytic evidence for the construct validity of scores: A historical overview and some guidelines.

Tinbergen, J. (1974). Substitution of graduate by other labour. *Kyklos: international review for social sciences*.

Tinbergen, J. (1975). *Income differences: recent research*.

Tippins, N. T. and Hilton, M. L. (2010). *A database for a changing economy: Review of the Occupational Information Network (O* NET)*. National Academies Press.

Xia, P., Zhang, L., and Li, F. (2015). Learning similarity with cosine similarity ensemble. *Information Sciences*, 307:39–52.

Yamaguchi, S. (2012). Tasks and heterogeneous human capital. *Journal of Labor Economics*, 30(1):1–53.

Appendix for Online Publication

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A ADDITIONAL EMPIRICAL RESULTS

A.1 Data Construction

In this section, I give more details on data construction for the two primary datasets on job skill demand employed in Section III and IV, namely O*NET (Occupation Information Network) and Lightcast (previously known as "Burning Glass"). Specifically, I discuss strategies for leveraging the longitudinal information in these datasets with higher precision. I also present an overview of their inherent characteristics, advantages and disadvantages, and how they are cross-walked with other datasets used in the analysis.

O*NET: Administered by the U.S. Department of Labor, O*NET is a replacement to the Dictionary of Occupational Titles (DOT). It is more comprehensive and more frequently updated and has been used widely to analyze occupation skill requirements and work settings (i.e., Acemoglu and Autor 2011; Yamaguchi 2012; Autor and Price 2013).

Nonetheless, to use the longitudinal variation from O*NET, the key challenge concerns partial updating – each new version of O*NET only update an average of 110 of targeted occupations among the 970 7-digit occupations. Online Appendix Table A1 lists different versions of O*NET, the release year, and the year composition for 3 of the modules. Specifically, for each release of O*NET, I assign a "Considered Year" such that at least 55% to 60% of occupations are updated after that year.

Moreover, I use 4-year intervals. The last column of online Appendix Table A1 show the percent of occupations that are updated from the last considered year of data included in the analysis. On average, more than 50 percent of the occupations are updated across the succeeding years included in the analysis.

O*NET contains around 270 descriptors about occupations that are grouped into 9 modules: abilities, knowledge, skills, work context, work activities, experience/education requirement, job interest, work values, and work styles. For my main analysis, I only use descriptors from 3 modules: work context, work activities, and knowledge that are more interpretable as the skill requirements, and are consistently evaluated by incumbent workers for each new release. These descriptors come as importance, level, extent, and relevance. To interpret the skill measures as gauging the intensity, I use the importance information, similar to i.e., Acemoglu and Autor (2011) and Guvenen et al. (2020), but the level and importance

pieces of information are highly correlated and don't affect the qualitative patterns of skill mixing shown in the paper.

In Section III, I show the longitudinal changes in skill mixing by combining O*NET and ACS datasets. O*NET uses SOC 2000 occupation classification for releases between 2000 and 2010 and SOC 2010 for years after 2010. To link O*NET and ACS, I first bridge SOC codes to the census' OCC 2000 and OCC 2010 codes respectively using crosswalks provided by the Analyst Resource Center and the Bureau of Labor Statistics. Then different years of OCC codes are homogenized using a balanced and consistent panel of occupation codes developed by Autor and Dorn (2013) and updated by Deming (2017). The same code is also used for combining all years of ACS and O*NET data.

Lightcast: Lightcast (formerly "Burning Glass Technologies") is an analytics software company that has developed a comprehensive and detailed dataset derived from online job postings, capturing real-time labor market information, reflecting the current demand for skills and occupations. One of the key advantages of Lightcast data is its extensive coverage and high-frequency updates. By examining over 40000 online job boards and company websites, it provides near universe of online posted vacancies; moreover, it provides a level of detail that is rarely matched by other sources of labor market data, such as job titles, employer information, and specific skill requirements. This allows for a very granular analysis of job skill requirements and labor market dynamics.across different industries and regions.

The information that Lightcast collected is then parseed and deduplicated into a systematic list of thousands of codified skills. Similar to Hershbein and Kahn (2018) and Braxton and Taska (2023), the dataset that this study uses defines different skills if the codified skills from Lightcast contains relevant key words. Specifically, the key words used to capture analytical skill are: "research", "analy", "decision", "solving", "math", "statistic", and "thinking". The key words used to capture interpersonal skill are "communication", "teamwork", "collaboration", "negotiation", and "presentation". For each occupation, the share of posted vacancies that require a particular skill is then the measure of skill for that occupation, capturing the extensive margin of firm skill demand.

However, like any data source, Lightcast data also has its limitations. For instance, it only covers online job postings, which may not represent the entire labor market, especially for low-skilled jobs or jobs in small firms that do not typically advertise online. It may also have

a bias towards certain types of jobs or industries that use online job advertisements more frequently, and online vacancies by nature overrepresent growing firms (Davis, Faberman, and Haltiwanger 2013). One note of Lightcast data is that the measure of skill as introduced above focuses on the extensive margin – whether a job uses a skill or not – this is very different than the level and importance information that O*NET contains.

Table A1: O*NET Versions and Corresponding Years

	Released Year	Division	Work Context	Work Activities	Knowledge	Considered Year	Percent Updated
O*NET 13.0	2008	Post 2005	73.79%	73.79%	73.79%	2005	–
		Before 2005	26.21%	26.21%	26.21%		
O*NET 18.0	2013	Post 2009	57.15%	57.21%	57.21%	2009	59.8
		Before 2009	42.85%	42.79%	42.79%		
O*NET 22.0	2017	Post 2013	57.84%	57.67%	57.67%	2013	45.8
		Before 2013	42.16%	42.33%	42.33%		
O*NET 25.0	2022	Post 2018	54.52%	54.52%	54.52%	2018	64.2
		Before 2018	45.48%	45.48%	45.48%		

*Notes: The table summarizes different versions of the O*NET (Occupational Information Network) database, along with their released year, year division for the 5 modules (work context, work activities, knowledge, skills, abilities), and the considered year for each version. The "Post" and "Before" rows indicate whether the data in each version was collected post or before a particular year. The "Considered Year" column represents the year considered to be corresponding to each release of O*NET based on the year division of data.*

A.2 Details of Skill Measures

In this section I briefly discuss the choice of skill measures used in the main analysis. Specifically, I show the composition of descriptors of each skill used in the main analysis. I also discuss the composite skill measures' validity and correlation with other measures used in the literature.

Table A2: O*NET Skill Measures and Composing Descriptors

Non-routine Analytical	**Routine**
• Analyzing data/information	• Importance of repeating the same tasks
• Thinking creatively	• Importance of being exact or accurate
• Interpreting information for others	• Structured v. Unstructured work (reverse)
Non-routine Interpersonal	• Pace determined by speed of equipment
• Establishing and maintaining personal relationships	• Controlling machines and processes
• Guiding, directing and motivating subordinates	• Spend time making repetitive motions
• Coaching/developing others	**Leadership**
Computer	• Making Decisions and Solving Problems
• Interacting With Computers	• Developing Objectives and Strategies
• Programming	• Organizing, Planning, and Prioritizing Work
• Computers and Electronics	• Coordinating the Work and Activities of Others
Design	• Developing and Building Teams
• Design	• Guiding, Directing, and Motivating Subordinates
• Drafting, Laying Out, and Specifying Technical Devices, Parts, and Equipment	• Provide Consultation and Advice to Others

Table A2 lists the O*NET descriptors for each of the constructed composite skill measures. The analytical measure corresponds to "non-routine cognitive analytic" and the interpersonal measure corresponds to "non-routine interpersonal" from Acemoglu and Autor (2011). I collapse Acemoglu and Autor (2011)'s "routine cognitive" (the first three items under Routine) and "routine manual" (the last three items under Routine) into a big routine skill, as occupations using these skills have been shown to have had similar labor market dynamics (Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011). I didn't include the "non-routine manual" from Acemoglu and Autor (2011), since it involves descriptors from the "Abilities" module of O*NET that is evaluated solely by job analysts, and for consistency purposes I focus on occupation descriptors that are evaluated by incumbents workers.

Further, I include two additional composite skills that are be non-routine. First, I include a "leadership" composite skill that is comprised of descriptors of problem-solving, strategic thinking, teamwork, and communication. They all demand an ability to guide and manage teams, strategize and plan, solve problems, coordinate activities, and communicate effectively within a team or organizational context. Second, I include a "design" composite skill measure revolves around technical proficiency and creativity. The composing descriptors necessitate a strong understanding of design principles, the ability to draft and layout specifications for technical devices.

Table A3: Correlations Among Skill Measures

Skill Measures	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) analytical	1.00								
(2) routine	-0.45	1.00							
(3) interpersonal	0.44	-0.49	1.00						
(4) computer	0.92	-0.27	0.25	1.00					
(5) math skill	0.50	-0.11	0.12	0.46	1.00				
(6) social skill	0.34	-0.54	0.61	0.24	0.09	1.00			
(7) analytical (broader)	0.84	-0.59	0.55	0.68	0.63	0.57	1.00		
(8) mechanical (broader)	-0.43	0.58	-0.24	-0.38	-0.11	-0.38	-0.49	1.00	
(9) interpersonal (broader)	0.10	-0.35	0.73	0.02	-0.09	0.70	0.28	-0.22	1.00

*Notes: This table reports the correlation among different skill measures constructed using O*NET data from 2000-2020. The first four skills measures in rows (1) to (4) are the ones used in the main text and are constructed using the O*NET descriptors shown in Table A1. The next two measures in rows (5) to (6), math skill and social skill are constructed based on Deming (2017). Math skill is the average of 1) mathematical reasoning ability, 2) mathematics knowledge, and 3) mathematics skill. Social skill consists of the average of four variables, 1) social perceptiveness, 2) coordination, 3) persuasion, and 4) negotiation. Rows (7) to (9) contain the broader analytical, mechanical, and interpersonal skills constructed using factor analysis as discussed in online Appendix A.5 with their specific component variables.*

Table A3 shows the correlation among the chosen skills used in the main analysis, as well as math skill and social skill are constructed based on Deming (2017), and broader skill measures skills constructed using factor analysis as discussed in online Appendix A.5. It reveals the analytical skill (row 1), exhibits a strong positive correlation with computer skills (0.92) and a moderate correlation with math skills (0.50). This pattern suggests that positions requiring analytical skills frequently necessitate computer and mathematical proficiency. Interpersonal skills (row 3) indicate a moderate-to-strong positive correlation with social skills (0.61) and broader interpersonal skills (0.73). This correlation suggests that occupations demanding interpersonal skill also emphasize social abilities. These results validate the

interpretation of the analytical and interpersonal skills with strong positive correlation with math and social skills used in other studies.

On the other, a strong negative correlation exists between routine and interpersonal skills (-0.49) and between routine and interpersonal skills (-0.45), indicating that these skill sets rarely overlap in job requirements. The broader skill categories (rows 7 to 9) align well with their narrower counterparts, reinforcing the validity of these categorizations. In sum there exist specific, identifiable skills in the labor market, some of which are more align with each other, but they tend not to overlap, reflecting distinct competencies.

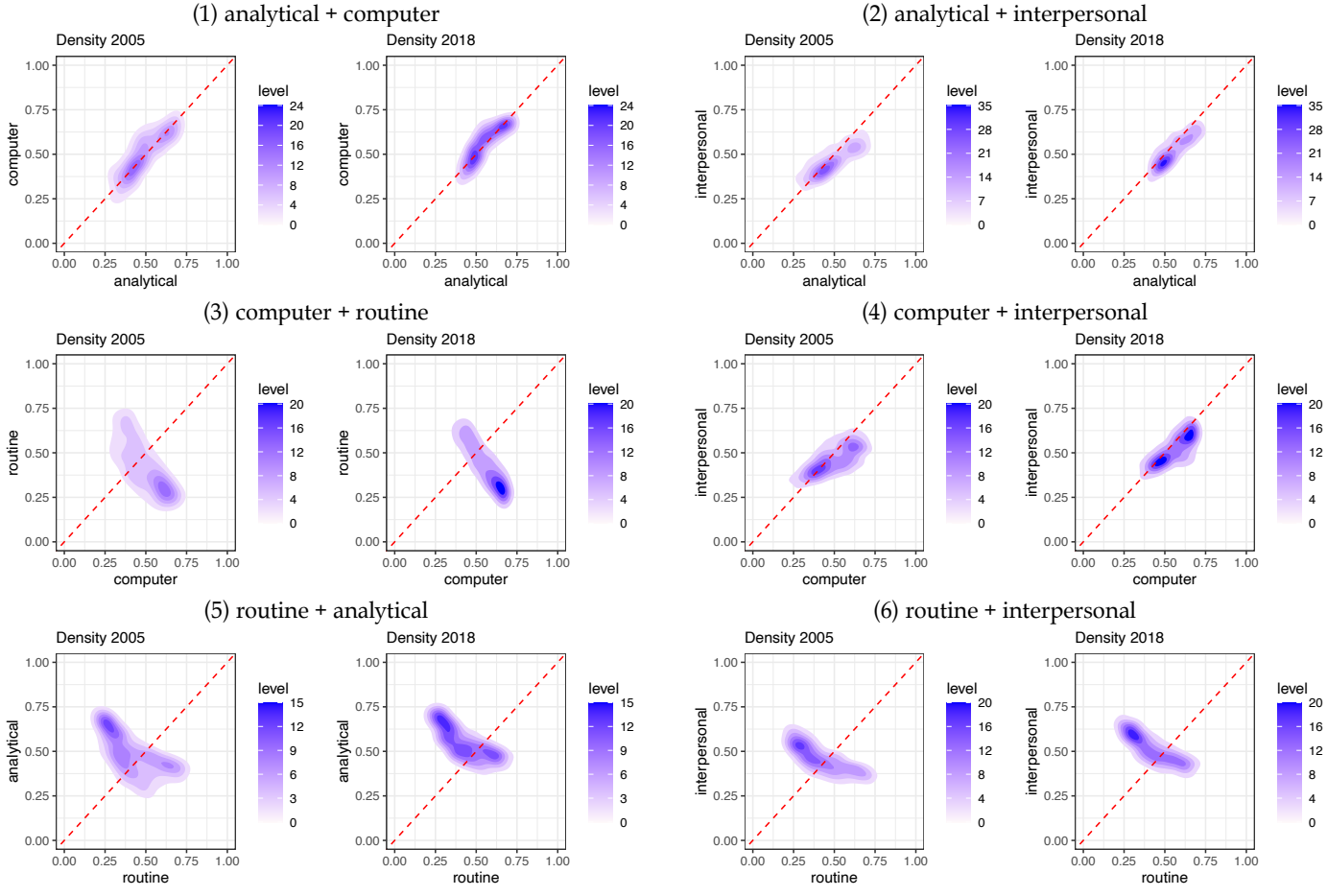
A.3 Alternative Non-parametric Examination of Trend

An intuitive alternative check of the changes in the degree of skill mixing across periods is to non-parametrically plot the density of skill intensities in different skill dimensions. Figure A1 depicts the density of skill requirements of six skill pairs out of the four constructed skills in 2005 and 2018 respectively using O*NET data combined with ACS. As in previous studies of job attributes, I aggregate the ACS to sex-education-industry cells that implicitly control for changes in task inputs due to variations in skill and industry mixes in the U.S. economy. Employment weights are obtained as the total hours of work aggregated to each cell. The ACS then supplies the O*NET data with employment across worker types to present an overarching picture of skill intensities in the economy.

From the figure, there is a clear shift towards mixed skill requirements in panel (1) pertaining to analytical and computer skills where these skills are positively correlated. Two salient changes happened in this period: first, the entire distribution of skill intensities moves near the 45-degree line; second, there is a significant increase in density around the 45-degree line. Both of these changes will lead to an increased degree of skill mixing, according to how it is defined based on the position of skill vectors relative to the 45-degree line. Such a change is also salient for other non-routine skill combinations: in the analytical and interpersonal skills space (panel 2), as well as in the computer and interpersonal skills space (panel 4).

On the other hand, one can scarcely observe changes towards mixing in the routine skill spaces, as shown in panel (3),(5) and (6). From these three plots, there is an increase in density towards the non-routine direction, losing density in routine skill, and the resulting change in relationship with the diagonal doesn't indicate a strengthening of mixing.

Figure A1: Non-parametric Depiction of Skill Intensities, 2005 vs. 2018

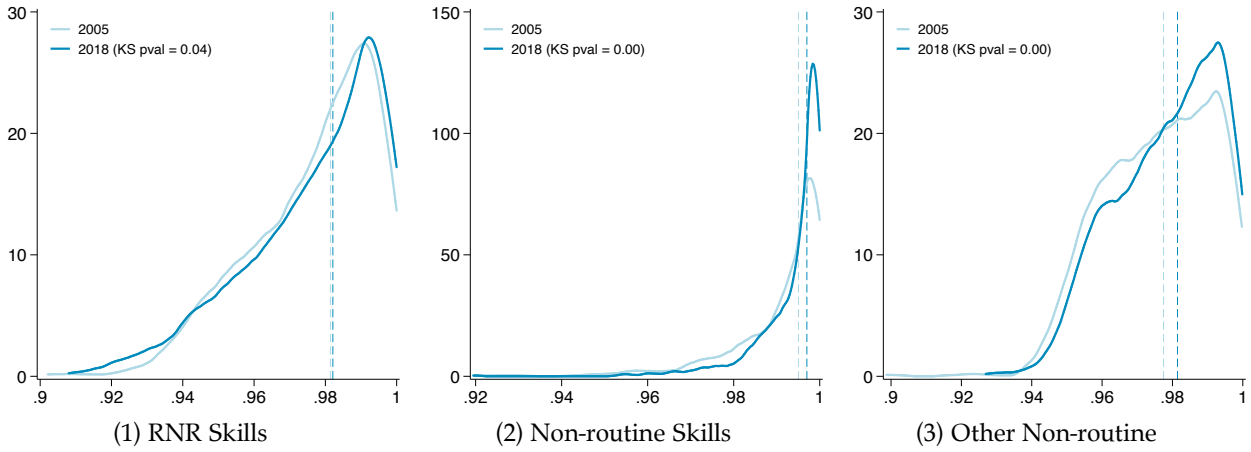


*Notes: These density plots show the intensity of occupation skill requirements across the U.S. economy in 2005 (column 1) and 2018 (column 2) in six two-dimensional skill spaces, as illustrated in the six panels. Darker colors indicate higher density and the 45-degree line is also plotted. O*NET and ACS data are combined for the construction of these plots. The two datasets are merged using consistent occupation codes constructed by Autor and Price (2013) and further developed by Deming (2017). Skill measures are constructed using the O*NET descriptors shown in Table A1. All measures are normalized to [0,1].*

A.4 Robustness of Trend Results to Different Weights and Groupings

In this section, I discuss the robustness of the trend results to different weighting, granularities, and groupings. In particular, I show the density results using weighted skill mixing indexes instead of unweighted in the main analysis; the trend of skill mixing using indexes for different skill pairs, instead of high-dimensional indexes; the heterogeneity of skill mixing increases across occupations using indexes for different skill pairs; and the differential changes in skill mixing across industries.

Figure A2: Density for Skill Mixing Indexes (Weighted Cosine Distances), 2005 vs. 2018



(1) RNR Skills (2) Non-routine Skills (3) Other Non-routine

*Notes: These figures plot the PDF of different mixing indexes in 2005 (light blue line) and 2018 (dark blue line). The x-axis displays the value of mixing indexes with a maximum of 1 by construction. These plots are created using O*NET and ACS data merged with occupation codes constructed by Autor and Price (2013) and developed by Deming (2017).*

On concern of analysis of skill mixing shown in 2 is that as it shows the changes in the density of skill mixing indexes without weighting, it might not accurately represent the overall picture of mixing in the whole economy. In A2, I weight the skill mixing indexes using employment weight at 6-digit SOC level occupation from OEWS. The results show a similar message that there is sizable increase in skill mixing particularly for non-routine skills. The only difference is that with employment weighting, the increase in the skill mixing of RNR skill is more discernable. This implies relative higher weight of occupations intensive in RNR skills that also increase in skill mixing in these skills.

Next, I discuss the changes in skill mixing using indexes of different skill pairs instead of high-dimensional indexes. Figure A5 panel (1) shows the results. The figure shows similar results as the main analysis: there is a stronger increase in skill mixing among non-routine

skills. For the skill combinations involving routine skills, the change in skill mixing is negligible.

In Table A4, I show the decomposition results of the changes in the skill mixing indexes for different skill pairs across different datasets. A similar pattern as the main analysis in 1, that is within-occupatoin variation surpassed across-occupation variation in accounting for the increase for skill mixing. This is particularly true using constantly updated occupations at 6-digit occupatoin level for non-routine skill pairs, and also quite apparent in the Lightcast data. The only slight difference is that for full O*NET data at 6-digit level, across-occupatoin variation do contribute to a comparable amount to the change in skill mixing for skill pairs with routine skill.

Additionally, in Figure A3, I show employment percentages and hourly wages across various job categories in the full and the sample for constantly updated occupation. This information gives the occupational structure and return for these two samples. It can be seen that while professionals make up a smaller percentage in the selected sample, they exhibit a higher average wage, suggesting a focus on higher-earning professionals in the selected sample. Conversely, the sales category shows a drastic reduction in the selected sample, indicating its limited representation. The hourly wage rates across the categories seem fairly consistent between the full and selected samples, with minor discrepancies.

Table A4: Decomposition of Mixing Indexes' Changes by Skill Pairs

	Skill Groups	6-digit Occupations			4-digit Occupations		
		total	within	across	total	within	across
Full O*NET	analytical + computer	10.52	6.40	4.12	10.49	6.60	3.89
	analytical + interpersonal	5.36	2.90	2.46	8.17	4.08	4.09
	computer + routine	4.38	2.41	1.97	5.16	2.94	2.22
	computer + interpersonal	7.23	3.60	3.63	11.81	7.51	4.30
	routine + analytical	4.00	2.29	1.71	4.23	3.16	1.07
	routine + interpersonal	1.93	0.12	1.81	2.35	1.08	1.26
Constant Updates	analytical + computer	5.59	6.03	-0.44	6.42	5.89	0.53
	analytical + interpersonal	3.53	4.58	-1.05	4.00	3.00	1.00
	computer + routine	2.88	3.69	-0.81	0.52	1.93	-1.42
	computer + interpersonal	0.78	1.86	-1.09	6.86	5.93	0.93
	routine + analytical	2.04	2.13	-0.09	1.48	3.60	-2.12
	routine + interpersonal	0.81	0.82	-0.01	-0.33	1.47	-1.80
Lightcast	analytical + computer				12.64	11.74	0.90
	analytical + interpersonal				2.51	2.20	0.31
	computer + interpersonal				-4.18	-3.79	-0.39

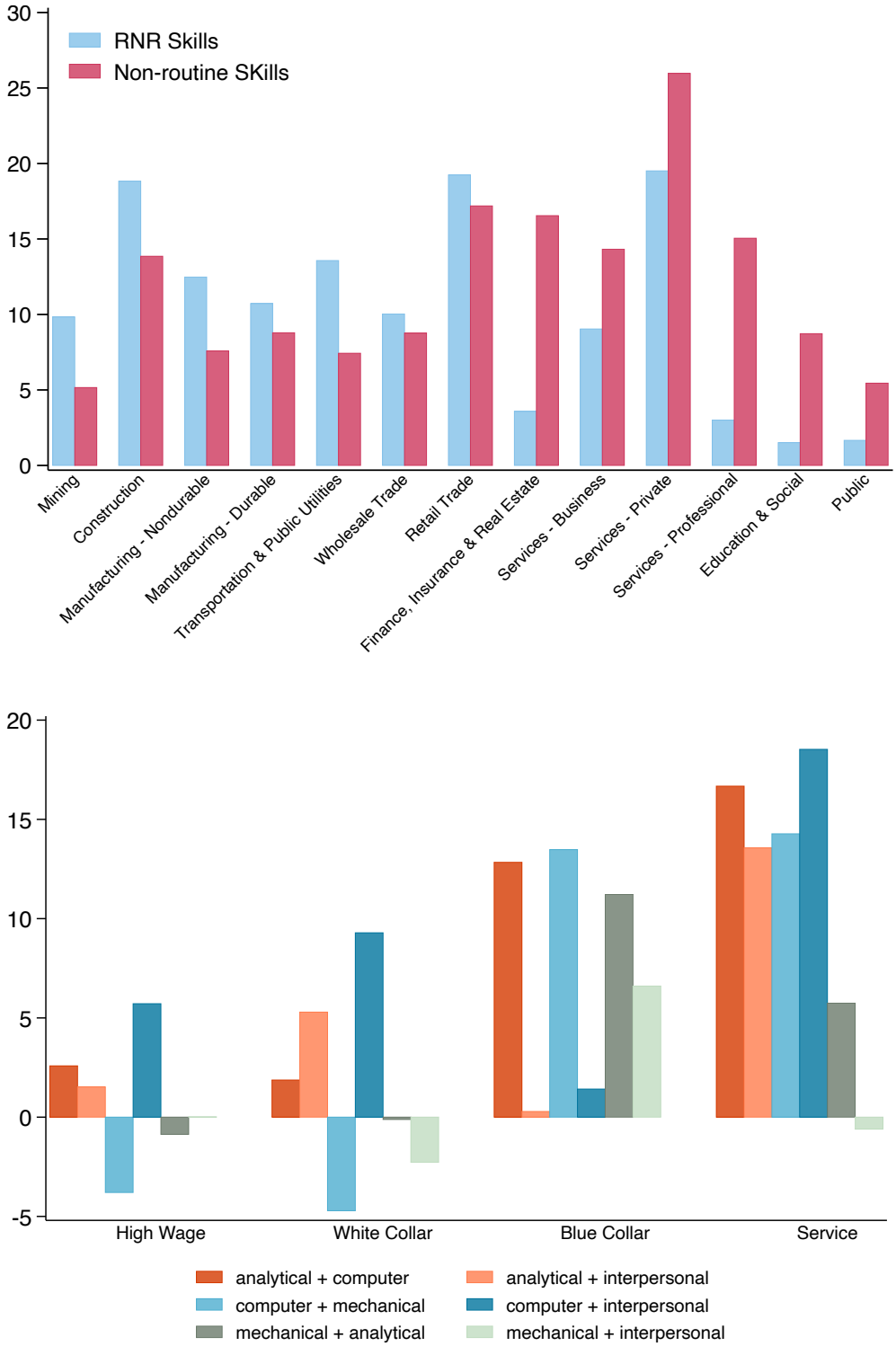
Notes: This table shows the shift-share decomposition of changes in the average level of different mixing indexes between 2005-2018 in percentile units. Specifically, for a change in the percentile of a mixing index h over two periods t and τ, its change $\Delta T_{h\tau} = T_\tau - T_t$ which can be decomposed to $\Delta T_h = \sum_j \left(\Delta E_{j\tau}\alpha_{jh}\right) + \sum_j \left(E_j \Delta\alpha_{jh\tau}\right) = \Delta T_h^a + \Delta T_h^w$ where $E_{j\tau}$ is employment weight in occupation j in year τ, and $\alpha_{jh\tau}$ is the level of mixing index h in occupation j in year τ, $E_j = \frac{1}{2}(E_{jt} + E_{j\tau})$ and $\alpha_{jh} = \frac{1}{2}(\alpha_{jht} + \alpha_{jh\tau})$. ΔT_h^a and ΔT_h^w then represent across-occupation and within-occupation change.

Figure A3: Mixing Index Change by Industry and Occupation Groups, 2005-2018

Now, I turn to discuss the robustness of the occupation heterogeneity in skill mixing changes. Figure A3 provides a detailed view of the changes in the skill mixing of different skill pairs across various occupations. Overall, the increase in the degree of mixing of non-routine skill pairs is higher than the increase in the mixing of skill pairs that include routine skills. Service and blue-collar occupations experience the highest increases in skill mixing of different skills, surpassing white-collar and high-wage occupations. When it comes to routine skills, blue-collar jobs lead other occupations in terms of increase in mixing.

Figure A3 also provides a detailed view of the changes in the skill mixing of RNR and non-routine skills across various industries. The main patterns indicate that the private service sector, followed by retail trade and construction, leads others in the growth of skill mixing, while public, education, social, and professional services experience the least increases in skill mixing. There is also noticeable heterogeneity across industries in terms of the skills that are mixed. For instance, in finance, real estate, and professional services, there is much higher mixing in non-routine skills relative to RNR skills; conversely, in industries like mining, transportation, public utilities, and construction, RNR skills are mixed in a higher degree.

Figure A4: Mixing Index Change by Industry and Occupation Groups, 2005-2018



A.5 Robustness of Trend Results to Measures of Skills

In this section, I discuss the robustness of the trend results to using alternative measures of skills. Specifically, I present alternative trend results using different ways of processing skill descriptors from O*NET, such not using PCA, and standardizing rather than rescaling. I also show the robustness using broader skill measures than those applied in the main analysis.

Alternative Construction of Skills: Since O*NET contains a large number of descriptors, many of which capture the same dimensions of skill requirements, it becomes standard practice to first abstract useful information from the descriptors to construct lower-dimensional measures of skills. The first approach, as in Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011) and Deming (2017), takes the average of a subset of variables and assumes that such average represents a particular broader skill intensity and not others. The other approach, as in Lise and Postel-Vinay (2020), applies PCA to the entire set of variables, which assumes that each of the variables contains information about underlying components that are orthogonally distributed. Both approaches impose different assumptions, with the first one giving more easily interpretable skill groups while the second being more data-driven. A third approach, as in Yamaguchi (2012), first picks descriptors that are ex-ante most easily interpretable with respect to each skill dimension, and then conducts PCA on those descriptors to abstract the most relevant variation. The main body of the paper adopts the third approach; here I show robustness checks using alternative skill measures.

Online Appendix Figure A5 presents the trend results using skill measures constructed by taking an average of the descriptors without imposing PCA (panel 2) and using skill measures normalized by standard deviation rather than linearly scaled to $[0,1]$ (panel 3). Normalizing by standard deviation necessarily creates negative values for the skills; since the mixing index is defined based on positive real values, having these negative values invalidates the mixing index in measuring skill mixing. One solution is to add a positive number to the skill measures. As any number chosen is essentially arbitrary, here I added the negative of the smallest value such that the re-scaled measure lies exactly above 0. For both of these robustness exercises, the main message is similar to the main text: there is a significant increase in mixing for non-routine skills, and less so for RNR skills.

Skill Measures: Another concern is that by using skill measures from Acemoglu and Autor (2011), each of which is constructed from a few descriptors, the resulting skill measures could

be relatively “narrow” and don’t provide a comprehensive depiction of the skill spaces. To alleviate this concern, I construct skill measures using a broader set of descriptors, similar to that of Lise and Postel-Vinay (2020). I first select descriptors from abilities, knowledge, skills, and work activities files that are more relevant for job skill demand, leaving me with around 163 descriptors. I then combine each year’s O*NET data with ACS and conduct PCA on the merged data from the year 2005-2018.

The result from this approach supports the choice of analytical, routine, and interpersonal skills in the main text The first three factors out of PCA explain around 60% of the variation across all the descriptors for years. The first factor has a strong positive association with reason and math skills, such as "Deductive Reasoning", "Inductive Reasoning" and "Mathematics", while the second factor relates more to motor coordination and mechanical work, such as "Multi-limb Coordination", "Mechanical" and "Equipment Maintenance". The third factor is clearly more associated with interacting with other people, such as "Selling or Influencing Others" and "Resolving Conflicts and Negotiating with Others". I interpret the second factor as "mechanical" rather than routine for the broader skill measures.

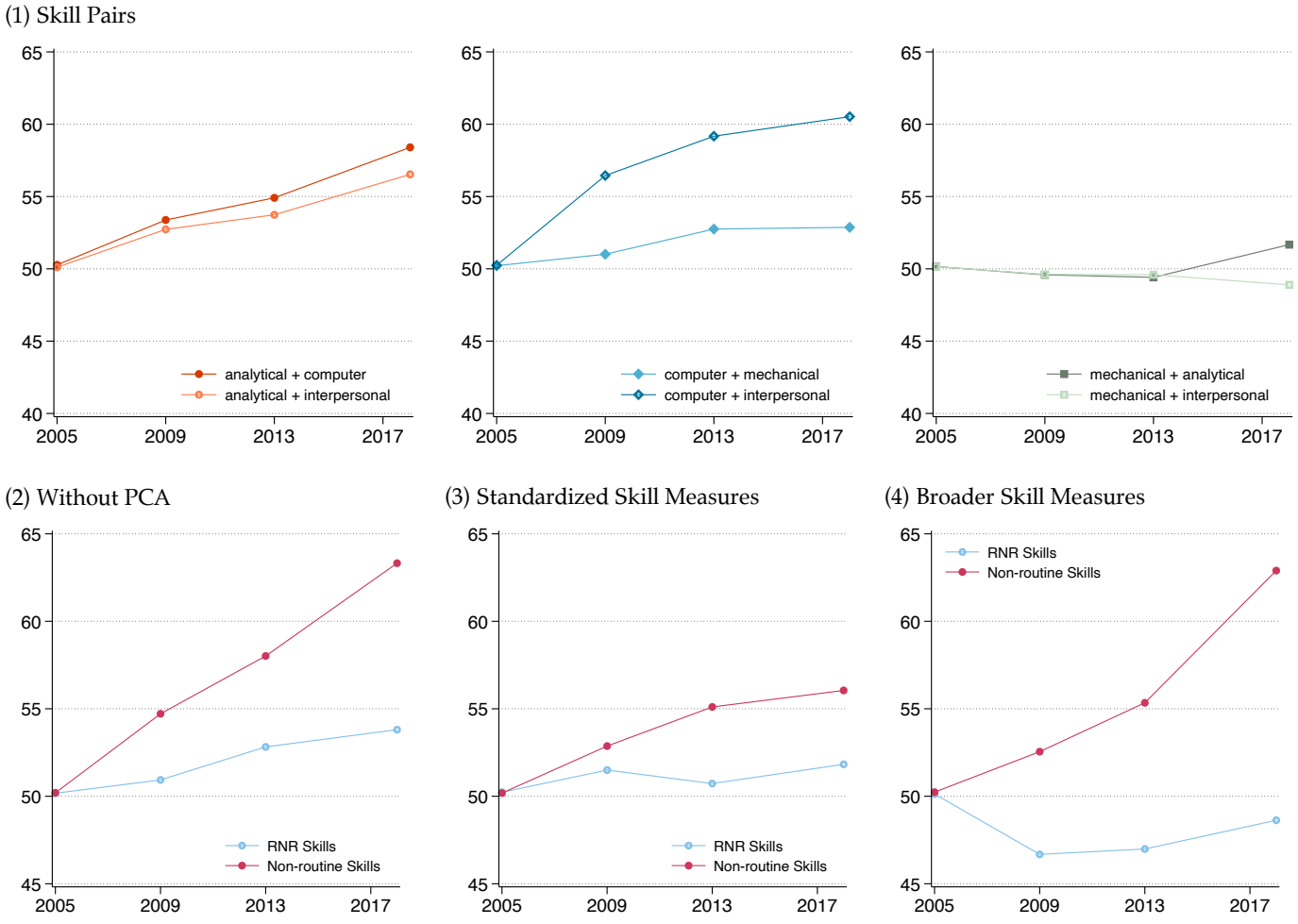
After conducting PCA, one could directly extract the factors imposing the assumptions that these factors are orthogonal to each other. While this is obviously quite convenient, it nevertheless creates the challenge of interpretability, since each of the factors has been constructed such that it is positively or negatively correlated with all of the 163 descriptors, and the assumption of orthogonality appears strong if the underlying skills are complementary in production across occupations. To take a fine balance between comprehensibility and interpretability, I adopt an approach similar to the measurement validation literature (Costello and Osborne 2005; Thompson and Daniel 1996), where I first conduct PCA/factor analysis to reveal the underlying dimensionality and structure of the measure (as has been done in the previous step). Guided by the factor loadings, I then hand-pick the skill descriptors into three broad groups “analytical”, “mechanical” and “interpersonal” without imposing the orthogonality assumption.

Online Appendix Table A5 illustrates the selected descriptors for each of the composite skill measures. These descriptors are broadly in line with Acemoglu and Autor (2011) but have several distinctions. First, the descriptors coming from factor analysis lean more toward reasoning, comprehension, and expression. Second, the mechanical skill used in the main text is the average of two ASVAB test scores that are constructed by the weighted

average of 26 O*NET descriptors. The ASVAB "Mechanical Comprehension" tests contestants' "understanding of the principles of mechanical devices, structural support, and properties of materials" and the ASVAB Electronics Information tests contestants' "understanding of electrical current, circuits, devices, and systems", both stressing one's knowledge basis. On the other hand, the descriptors chosen by conducting PCA relate more to physical control, coordination, and machine operation aspects rather than mental perception. Third, the descriptor choices for interpersonal skill from factor analysis also emphasize interactions with others as in Acemoglu and Autor (2011) but are more comprehensive.

Online Appendix Figure A5 panel (4) illustrates the trend results using these broader skill measures. The message on the growth of skill mixing remains the same as the main text, that is there is strong growth of skill mixing for non-routine skills. Nontheless, for RNR skills, the degree of skill mixing has decreased using the broader measures.

Figure A5: Trend of Skill Mixing with Alternative Skill Measures



*Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. Panel (1) shows the changes in skill mixing indexes of 6 distinct skill pairs of the 4 skills. In panel (2), skill mixing indexes are calculated using skill measures without using PCA, and in panel (3), skill measures are normalized to have mean 0 and standard deviation 1. Panel (4) shows the changes in mixing indexes using broader skill measures as described in online Appendix A.5.*

Table A5: Components of Broader Skill Measures

Analytical	**Mechanical**	**Interpersonal**
• Deductive Reasoning	• Multilimb Coordination	• Assisting and Caring for Others
• Inductive Reasoning	• Speed of Limb Movement	• Selling or Influencing Others
• Mathematical Reasoning	• Mechanical	• Resolving Conflicts and Negotiating
• Number Facility	• Performing General Physical Activities	• Coaching and Developing Others
• Mathematics	• Handling and Moving Objects	• Staffing Organizational Units
• Economics and Accounting	• Controlling Machines and Processes	• Service Orientation
• Reading Comprehension	• Operate Vehicles, Mechanized Devices or Equipmnt	• Administration and Management
• Writing	• Repairing and Maintaining Mechanical Equipment	• Customer and Personal Service
• Speaking	• Repairing and Maintaining Electronic Equipment	
• Oral Comprehension	• Installation	
• Written Comprehension	• Equipment Maintenance	
• Oral Expression	• Repairing	
• Written Expression	• Production and Processing	

*Notes: This table lists the O*NET descriptor components for each of the constructed (broader) composite skill groups as discussed in online Appendix A.5.*

A.6 Robustness of Trend Results to Measures of Skill Mixing

I introduce two additional measures and show the robustness of the trend results using these alternative mixing measures.

A first commonly used measure for concentration or specialization based on the share of a total quantity is the Herfindahl–Hirschman Index (HHI).[60] Equation (8) shows how to use inverse HHI to measure skill mixing for an occupation represented by $(\alpha_{ja}, \alpha_{js})$. Observe that this index is maximized when $\alpha_{ja} = \alpha_{js}$, exactly corresponding to the case when the skill vector lies on the unit vector and becomes most mixed. If one skill's intensity is greater than the other, the occupation becomes less mixed and this index becomes smaller. Similar to an angle-based mixing index, this measure is insensitive to the length of a skill vector, since each skill is normalized by the total quantity of skills in that occupation.

$$\left[\left(\frac{\alpha_{ja}}{\alpha_{ja}+\alpha_{js}}\right)^2+\left(\frac{\alpha_{js}}{\alpha_{ja}+\alpha_{js}}\right)^2\right]^{-1} \tag{8}$$

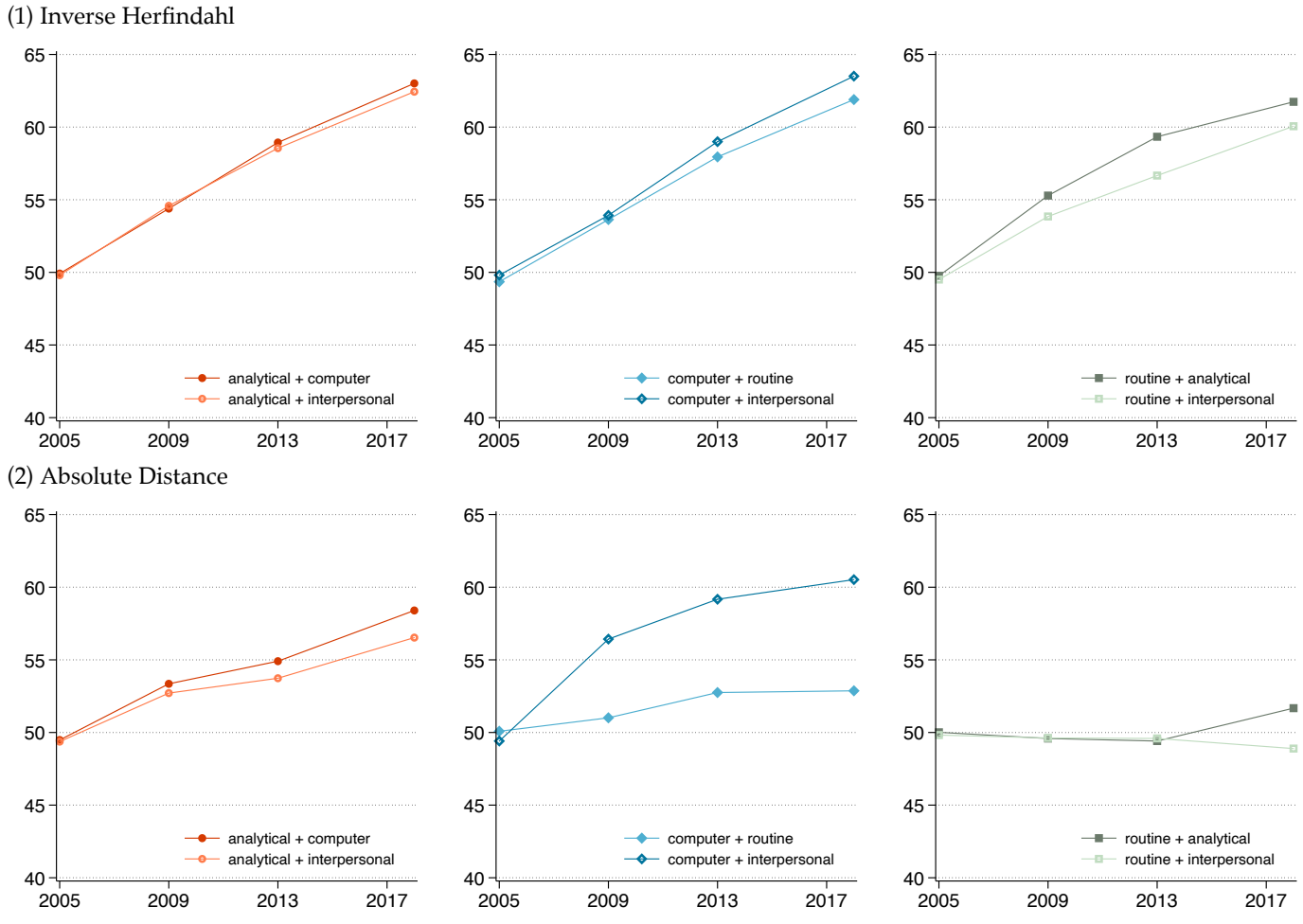
$$-\frac{|\alpha_{ja}-\alpha_{js}|}{\alpha_{ja}+\alpha_{js}} \tag{9}$$

Under a similar vein, the degree of skill mixing could also be measured by normalizing the absolute distance between skill intensities for a skill vector: as this distance decreases, the overall skill portfolio becomes more balanced; normalization then eliminates the effect of the length of the skill vector. Equation (9) gives a particular specification of such a measure. As can be seen from this construction, as the absolute distance between skill intensities decreases and the degree of mixing increases, this measure also increases, though from the direction of $(-\infty, 0)$.

In Online Appendix Figure A6, I show the robustness of the trend results using these alternative measures in panels (1) and (2). Both measures deliver the same message as the cosine mixing index in the main text, that is, there is a sizable increase in skill mixing, particularly for non-routine skills. The only difference is that for the HHI skill index, there is also comparable increase in skill mixing for RNR skills.

[60] For applications in the labor literature, Ransom and Phipps (2017) and Jin (2017) use the inverse of HHI as the "variety index" to examine how diverse the jobs held for students who graduated from a certain major. Similar logic can be applied to the measurement of skill mixing: in the context of 2-dimensional skill space, the more "varied" skills an occupation uses essentially means that skills are more mixed.

Figure A6: Trend of Skill Mixing with Alternative Indexes



*Notes: These three panels plot the employment-weighted mixing indexes of different skills in the U.S. economy from 2005-2018 using O*NET and ACS data. In panels (1) and (2), mixing indexes are calculated using the Inverse Herfindahl index and Absolute Distance as discussed in online Appendix A.6.*

A.7 Additional Results on Wage and Employment Returns

In this section, I provide more detailed results on wage returns, and results relating employment to occupation skill mixing. I also provide robustness results to the analysis of the returns to skill mixing in the main paper.

Detailed Results on Wage and Employment Returns: I first check the returns to individual skills and how they interact with the returns to skill mixing. Table A6 columns (1) shows that in a cross-sectional regression analytical and computer skills both have significant positive returns. Workers employed in occupations requiring a higher degree of these two skills earn more. Nonetheless, worker in occupations that requiring a higher level of interpersonal skills have a wage reduction.

Column (2) of Table A6 shows that by restricting to within-occupation variation and including skill mixing measures and an important pattern appears: the coefficients for most individual skills become slightly more negative (except for routine skill),[61] while the skill mixing indexes of analytical paired with interpersonal skills, as well as of routine and interpersonal skills show significant positive returns. Such a pattern persists in column (3) and (4) including worker skills and fixed effects, only that the skill mixing of analytical and computer skills are more precisely estimated to have a positive return. This indicates that the mixing of skills earns separate and additional rewards beyond those predicted by individual skills.

Turning to employment, there is also a positive employment premium for workers with a more mixed skill set. Column (6) of Table A6 shows that workers with a more mixed level of computer and interpersonal routine skills, or computer and interpersonal skills, or routine and interpersonal skills, are more likely to move from unemployment to employment. Workers with a more mixed level of analytical and computer, or analytical and interpersonal skills, are also more likely to exit unemployment, but the results are not precisely estimated. On the other hand, workers with a more mixed level of routine and interpersonal skills are less likely to find employment.

[61] The insignificant or even negative return to analytical skill over time also finds support from the literature. Lise and Postel-Vinay (2020) shows a strong negative 14.4 percent return on cognitive skill using NLSY data with 3-digit occupation fixed effects. Deming (2017) found that the return to cognitive skills has declined across the NLSY79 and NLSY97 cohorts, similar to Castex and Kogan Dechter (2014).

Robustness Checks: Table A7 shows the robustness checks to the results in Table 3. Specifically, Columns (1) and (2) utilize the Absolute Distance and Inverse Herfindahl measures to formulate mixing indexes (refer to online Appendix A.6 for details), while Columns (3) and (4) employ standardized and broader measures of skills (refer to online Appendix A.5 for details).

The findings presented in Table A7 clearly indicate a consistent trend: workers experience a positive return when they are employed in occupations that are more mixed of analytical with computer skills, analytical with interpersonal skills, and routine with interpersonal skills. Specifically, a notable increase in wage is observed with workers are in occupations more mixed of analytical and computer skills, especially when applying standardized and broad skill measures; similarly, occupations becoming more mixed of analytical and interpersonal skills, when assessed using the Absolute Distance and Inverse Herfindahl measures, also show a significant positive return. The mixing of routine and interpersonal skills exhibits a positive return as well across the different measures.

On the other hand, the mixing of computer and routine skills, computer and interpersonal skills, and routine and analytical skills all exhibit significant negative wage returns at the occupational level. These negative coefficients may indicate that the combination of these particular skills is less beneficial or leads to inefficiency

Table A6: Return to Skill Mixing Full Table with Individual Skills

Dependent: ln(hourly wage)	(1)	(2)	(3)	(4)
Occupation Skills				
Analytical	-0.019**	-0.019**	-0.012	-0.033***
	[0.009]	[0.009]	[0.008]	[0.011]
Computer	-0.002	-0.008	-0.003	-0.017
	[0.010]	[0.011]	[0.009]	[0.013]
Interpersonal	-0.019**	-0.022**	-0.021***	-0.027**
	[0.009]	[0.009]	[0.008]	[0.011]
Routine	0.027***	0.035***	0.025***	0.047***
	[0.010]	[0.011]	[0.009]	[0.015]
Mix (analytical + computer)	0.007	0.011**	0.013***	0.012
	[0.005]	[0.005]	[0.005]	[0.008]
Mix (analytical + interpersonal)	0.016***	0.016***	0.015***	0.028***
	[0.005]	[0.005]	[0.004]	[0.007]
Mix (computer + routine)	-0.022**	-0.029***	-0.021***	-0.026**
	[0.009]	[0.009]	[0.008]	[0.012]
Mix (computer + interpersonal)	-0.008	-0.012**	-0.014***	-0.012
	[0.006]	[0.006]	[0.005]	[0.009]
Mix (routine + analytical)	-0.050***	-0.056***	-0.050***	-0.058***
	[0.008]	[0.009]	[0.008]	[0.012]
Mix (routine + interpersonal)	0.023***	0.029***	0.019**	0.023*
	[0.008]	[0.009]	[0.008]	[0.012]
Worker Skills				
Afqt (analytical)		0.065***		-0.038
		[0.012]		[0.023]
Computer		0.045***		0.017
		[0.006]		[0.023]
Social (interpersonal)		0.015***		-0.003
		[0.005]		[0.029]
ASVAB (routine)		-0.008		-0.012
		[0.016]		[0.022]
Mix (afqt + computer)		0.044*		0.017
		[0.023]		[0.013]
Mix (afqt + social)		0.028*		-0.075***
		[0.015]		[0.020]
Mix (computer + asvab mech)		0.013		-0.070***
		[0.025]		[0.026]
Mix (computer + social)		0.008		0.061***
		[0.013]		[0.019]
Mix (asvab mech + afqt)		0.001		0.096**
		[0.009]		[0.039]
Mix (asvab mech + social)		-0.040***		-0.045
		[0.011]		[0.042]
Ethnicity*Gender, Age/Year, Region, Edu FE	X	X	X	X
Occupation FE	X	X	X	X
Worker FE			X	X
Observations	87,655	78,719	87,655	50,580
R-squared	0.426	0.439	0.758	0.761

Notes: See Table 3 notes.

Table A7: Robustness Checks of Return to Skill Mixing

Dependent: ln(hourly wage)	(1)	(2)	(3)	(4)
Occupation Skills				
Analytical	-0.014*	-0.008	-0.009	-0.013
	[0.008]	[0.033]	[0.008]	[0.008]
Computer	-0.002	0.069**	0.002	-0.038***
	[0.009]	[0.027]	[0.009]	[0.010]
Interpersonal	-0.019**	-0.118***	-0.018**	-0.014*
	[0.008]	[0.030]	[0.008]	[0.008]
Routine	0.026***	0.091***	0.005	0.010
	[0.009]	[0.017]	[0.008]	[0.008]
Mix (analytical + computer)	0.007	-0.040	0.008*	0.020***
	[0.005]	[0.036]	[0.005]	[0.007]
Mix (analytical + interpersonal)	0.010**	0.156***	0.006	0.025***
	[0.004]	[0.042]	[0.004]	[0.005]
Mix (computer + routine)	-0.028***	-0.045***	-0.021**	-0.087***
	[0.007]	[0.015]	[0.008]	[0.013]
Mix (computer + interpersonal)	-0.011**	-0.019	-0.013***	-0.021***
	[0.005]	[0.033]	[0.005]	[0.008]
Mix (routine + analytical)	-0.033***	-0.080***	-0.041***	-0.041**
	[0.007]	[0.015]	[0.008]	[0.018]
Mix (routine + interpersonal)	0.010	0.033**	0.033***	0.026**
	[0.007]	[0.016]	[0.006]	[0.012]
Ethnicity × Gender, Age, Region, Edu FE	X	X	X	X
Occupation FE	X	X	X	X
Worker FE	X	X	X	X
Observations	87,655	87,655	87,655	87,655
R-squared	0.757	0.757	0.757	0.758

*Notes: This table reports the robustness checks to the results in Table 3. Columns (1) and (2) use Absolute Distance and Inverse Herfindahl measures to construct mixing indexes (see online Appendix A.6 for details) and Columns (3) and (4) use standardized and broad measures of skills (see online Appendix A.5 for details). Log hourly wages are the outcome variables and person-year is the unit of observation. The occupational skill and skill mixing measures come directly from O*NET and are merged to NLSY79&97 based on census occupation codes. All measures of skill and skill mixing are normalized to have mean 0 and standard deviation 1. Ethnicity-by-gender, age, year, census region, urbanicity, and a 5-category (no high-school, high-school graduate, some college, college graduate, post-college) education fixed effects are included for all regressions, with additional fixed effects as indicated in the table. Standard errors are clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10.*

Table A8: Top College Majors in Skill Mixing

Mixing Index – Level	Mixing Index – Change
analytical + computer	
Physical Sciences	Agriculture and Natural Resources
Social Sciences	Mathematics
Area Studies	Letters
analytical + interpersonal	
Education	Mathematics
Military Sciences	Architecture and Environmental Design
Theology	Agriculture and Natural Resources
computer + routine	
Military Sciences	Agriculture and Natural Resources
Health Professions	Law
Mechanics	Engineering
computer + interpersonal	
Physical Sciences	Agriculture and Natural Resources
Education	Letters
Psychology	Architecture and Environmental Design
routine + analytical	
Mechanics	Agriculture and Natural Resources
Military Sciences	Area Studies
Transportation	Engineering
routine + interpersonal	
Mechanics	Law
Military Sciences	Theology
Transportation	Mathematics

*Notes: This table lists the top 3 college majors for each mixing index both in terms of levels and in terms of changes from 2005 to 2019. To calculate the degree of skill mixing for college majors, I first map the occupation level degree of skill mixing contained in the O*NET data to NLSY, and then calculate for each college major's students, the employment weighted average of skill intensities and mixing indexes of their occupations. I use both NLSY79&97 to get the employment weight on occupations.*

Table A9: Return to Skill Mixing Full Table with Individual Skills

NLSY97 Code (before 2010)	Major Field of Study	NLSY79 Code
0	None, no major yet (didn't/don't) have to declare yet;	.
1	Agriculture/Natural resources	1
2	Anthropology	22
3	Archaeology	22
4	Architecture/Environmental design	2
5	Area studies	3
6	Biological sciences	4
7	Business management	5
8	Communications	6
9	Computer/Information science	7
10	Criminology	22
11	Economics	22
12	Education	8
13	Engineering	9
14	English	15
15	Ethnic studies	3
16	Fine and applied arts	10
17	Foreign languages	11
18	History	22
19	Home economics	13
20	Interdisciplinary studies	49
21	Mathematics	17
22	Nursing	12
23	Other health professions	12
24	Philosophy	15
25	Physical sciences	19
26	Political science and government	21
27	Pre-dental	4
28	Pre-law	14
29	Pre-med	4
30	Pre-vet	4
31	Psychology	20
32	Sociology	22
33	Theology/religious studies	23
36	Nutrition/Dietetics	4
37	Hotel/Hospitality management	5
38	Other - Recoded to Liberal Arts and Sciences	49
39	Other - Recoded to Automobile/Automotive Mechanics Technology/Technician	24
40	Other - Recoded to Human Services, General	21
41	Other - Recoded to Social Work	21
42	Other - Recoded to Electrical/Electronics Maintenance and Repair Technology	24
43	Other - Recoded to Geography	22
44	Other - Recoded to International Relations & Affairs	21
45	Other - Recoded to transportation & materials moving	25
46	Other - Recoded to security and protective services	21
47	Other - Recoded to legal support services	14
48	Other - Recoded to other sciences/applied sciences	49
99	UNCODABLE	99

NLSY97 Code (CM10)	Major Field of Study	NLSY79 Code
1	Agriculture, agriculture operations, & related sciences	1
3	Natural resources and conservation	1
4	Architecture and related services	2
5	Area, ethnic, cultural, gender, and group studies	3
9	Communications, journalism, and related programs	6
10	Communications technologies/technicians & support services	6
11	Computer & information sciences & support services	7
12	Personal and culinary services	49
13	Education	8
14	Engineering	9
15	Engineering technologies & engineering-related fields	9
16	Foreign languages, literatures, and linguistics	11
19	Family and consumer sciences/human sciences	13
22	Legal professions and studies	14
23	English language and literature/letters	15
24	Liberal arts and sciences, general studies & humanities	49
25	Library science	16
26	Biological and biomedical sciences	4
27	Mathematics and statistics	17
28	Military science, leadership, and operational art	18
29	Military technologies and applied sciences	18
30	Multi/interdisciplinary studies	49
31	Parks, recreation, leisure, and fitness studies	21
32	Basic skills development/remedial education	8
33	Citizenship activities	21
34	Health-related knowledge and skills	12
35	Interpersonal and social skills	6
36	Leisure and recreational activities	49
37	Personal awareness and self-improvement	8
38	Philosophy and religious studies	15
39	Theology and religious vocations	23
40	Physical sciences	19
41	Science technologies/technicians	24
42	Psychology	20
43	Homeland security, law enforcement, firefighting, and related protective services	18
44	Public administration and social service professions	21
45	Social sciences	22
46	Construction trades	24
47	Mechanic and repair technologies/technicians	24
48	Precision production	24
49	Transportation and materials moving	25
50	Visual and performing arts	10
51	Health professions and related programs	12
52	Business, management, marketing, & related support services	5
53	High school/secondary programs and certificates	8
54	History	22
60	Residency programs	12
999	Uncodable	99

NLSY79 Code	Major Field of Study
0	None, General Studies
1	Agriculture and Natural Resources
2	Architecture and Environmental Design
3	Area Studies
4	Biological Sciences
5	Business and Management
6	Communications
7	Computer and Information Sciences
8	Education
9	Engineering
10	Fine and Applied Arts
11	Foreign Languages
12	Health Professions
13	Home Economics
14	Law
15	Letters
16	Library Science
17	Mathematics
18	Military Sciences
19	Physical Sciences
20	Psychology
21	Public Affairs and Services
22	Social Sciences
23	Theology
24	Mechanics
25	Transportation
49	Interdisciplinary Studies
99	Other

B THOERY AND QUANTITATIVE

B.1 Propositions and Proofs

Lemma 1. *An occupation* $\mathbf{y} = \{y^1, ..., y^k, \ldots, y^K\} \in S \subset \mathbb{R}^K$ *within a closed skill space S of dimension K is more mixed in skills based on Definition 1 if for any pair of skills* (j, k)*, the ratio of* $\frac{y_j}{y_k}$ *becomes closer to 1.*

Proof of Lemma 1: For the occupation $\mathbf{y}$ and we want to establish how the degree of skill mixing changes if the skill dimensions for j and k are to vary. The lemma can be simply proved by considering the skill mixing index for for this occupation. Let $y_k = ry_j$ and denote y_j by y, the mixing index for $\mathbf{y}$ is:

$$\frac{y + ry + A}{\sqrt{K}\sqrt{y^2 + r^2y^2 + B}},$$

where A and B are two constants that don't depend on y_k and y_j. The above equation is maximized at $r = 1$. Therefore, for any y_j, the occupation is more skill-mixed if the ratio r is close to 1. This completes the proof. *Q.E.D.*

Proposition 1 (Changes in Skill Mixing)**.** *Consider an occupation* $\mathbf{y}_t = \{y_t^1, ..., y_t^k, \ldots, y_t^K\} \in S \subset \mathbb{R}^K$ *within a closed skill space S of dimension K. Assume that firms operate the occupation with a production technology as described by equation (3) and under an occupational rental cost defined by equation (7). Under these conditions, occupation* $\mathbf{y}_t$ *will show an increased degree of skill mixing given the following conditions:*

(i) The skills within the vector $\mathbf{y}_t$ *demonstrate an rise in complementarity in production (a decrease*

in σ), provided that σ does not undergo a change in sign.

(ii) The skills within the vector $\mathbf{y}_t$ exhibit an higher increasing marginal cost (an increase in ϕ), under the condition that $\phi > \sigma$.

Additionally, occupation $\mathbf{y}_t$ will exhibit a increased degree of skill mixing in the (y_t^k, y_t^h) dimension if the ratio between (x_t^k, x_t^h) approaches unity.

Proof of Proposition 1: Lemma 1 posits that an occupation $\mathbf{y}_t$ exhibits greater skill mixing if the ratio across all skill dimensions approximates 1. Therefore, establishing the influence of the ratio on the degree of skill mixing suffices. The initial step concentrates on any two skills within the vector (y_t^k, y_t^h).

The firm value function indicates that the firm re-optimizes the choice of $\mathbf{y}_t$ in each period. Consequently, within a given submarket at a particular time instance $(\mathbf{x}_t, \mathbf{y}_t)$, the firms' choices of $\mathbf{y}_t$ remain uninfluenced by the continuation value, rendering it a static problem. Time subscript is subsumed in the subsequent proof.

By deriving the first-order condition of firms' optimization problems in the submarket $(\mathbf{x}, \mathbf{y})$ and taking ratios, one obtains the following condition: $\frac{y_h}{y_k} = (\frac{x_h}{x_k})^{\frac{\sigma}{\rho-\sigma}}$. Therefore, the ratio of firms' optimal skill requirement choices for any two skills (y_h, y_k) is influenced by three variables: the elasticity parameter of substitution in production σ, the degree of increasing marginal occupational rental cost ρ, and the ratio of worker skills in the submarket (x_h, x_k).

From the equation, it is evident that as σ decreases, indicating an increase in skill complementarity in production, $\frac{y_h}{y_k}$ will converge to 1 for any two skills (y_h, y_k), under the assumption that σ does not change sign. Similarly, as ρ increases, $\frac{y_h}{y_k}$ will approximate 1 for any two skills (y_h, y_k), given that $\rho - \sigma$ does not change sign.

The influence of worker skill bias on the degree of skill mixing of $\mathbf{y}$ presents a more complex scenario, as a change in the ratio $\frac{x_h}{x_k}$ does not directly imply a change in the ratio of other skill pairs. Consequently, to gauge its impact on the degree of skill mixing, the focus must remain on the (y_h, y_k) dimension. For this specific dimension, if $\frac{x_h}{x_k}$ converges to 1, then $\frac{y_h}{y_k}$ also approaches 1. *Q.E.D.*

Proposition 2 (Changes in Wage and Job Finding)**.** *Consider an occupation $\mathbf{y}_t \in S \subset \mathbb{R}^K$ within a closed skill space S of dimension K. Assume that firms operate the occupation with a production technology as described by equation 3 and under an occupational rental cost defined by equation 7. Also, these firms offer a output share ω to workers and have value functions described by equation 5. Further, let worker value described by equation 4. Under these conditions, workers in occupation*

$\mathbf{y}$ *will earn a higher wage and have a higher job finding probability under condition (i) and (ii) of Proposition 1*

Proof of Proposition 2:

Wages: To establish the change in wages, one need to show that the output of the worker-firm match increases as the elasticity parameter σ decreases and approaches 0 from 1, or if σ decreases in the negative range, consistent with skills becoming more complementary in production. At a particular output share rate ω, such value changes of σ will lead to higher wages.

Now, let's obtain the first derivative of ρ for the production function 3. WLOG, let's consider the case of two skills, and express y_1x_1 and y_2x_2 as m and n. The output of a worker-firm match can be expressed as $q = (m^\rho + n^\rho)^{1/\rho}$. We can take log of the production function $\ln(q) = \frac{1}{\rho}\ln(m^\rho + n^\rho)$ and then take logarithmic differentiation that gives the following:

$$\frac{1}{q}\frac{\partial q}{\partial \rho} = -\frac{1}{\rho^2}\ln(m^\rho + n^\rho) + \frac{1}{\rho}\frac{1}{m^\rho + n^\rho}(m^\rho \ln(m) + n^\rho \ln(n))$$

Solving for $\frac{\partial q}{\partial \rho}$ gives:

$$\frac{\partial q}{\partial \rho} = q[-\frac{1}{\rho^2}\ln(m^\rho + n^\rho) + \frac{1}{\rho}\frac{1}{m^\rho + n^\rho}(m^\rho \ln(m) + n^\rho \ln(n))]$$

$$\frac{\partial q}{\partial \rho} = q[-\frac{1}{\rho}\ln(q) + \frac{1}{\rho}q^{-\rho}(m^\rho \ln(m) + n^\rho \ln(n))]$$

In the case of the calibration of the model, since m, n, and y are all in the range of $[0, 1]$, one can show that the above derivative is negative when $0 < \rho < 1$ or when $\rho < 0$.

With respect to (ii) of Proposition 1, it is easy to see that since for the analysis of this paper, both $(\mathbf{x}, \mathbf{y})$ are in the range $[0, 1]$, therefore the occupation rental cost is decreasing in ϕ, so wage should increase as occupation rental cost increases.

Employment: For job finding probability, it suffices to to show that $p(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ is increasing in σ and ϕ. This becomes simpler, since the above proof establishes that worker-firm output is increasing in both σ and ϕ, so does firm's value $J(\mathbf{x}, \mathbf{y}, \omega)$. By the free entry condition in equation (6), at a fixed vacancy posting cost, an increase in $J(\mathbf{x}, \mathbf{y}, \omega)$ implies a decrease in $q(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ and therefore implies an increase in $p(\theta_t(\mathbf{x}, \mathbf{y}, \omega))$ under constant return to scale matching technology. *Q.E.D.*

B.2 Equilibrium Definition and Block Recursivity

In this section I define the a block-recursive equilibrium (BRE) for the economy following Menzio and Shi (2011). I further show that the equilibrium of the economy is unique and is block recursive.

Definition 2 (Block-recursive Equilibrium)**.** *Let $\psi \in \Psi$ be the aggregate state of the economy, which is a distribution of agents across employment status $e = U, W$, skill profiles $\mathbf{x}$, occupational skill requirements $\mathbf{y}$, and output shares ω.*

A block-recursive equilibrium for this economy consists of value functions for both unemployed and employed workers $U(\mathbf{x}) : S \to \mathbb{R}$, $W(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0,1] \to \mathbb{R}$, and their respective policy functions $y'_U(\mathbf{x}) : S \to S \times [0,1]$, $y'_W(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0,1] \to S \times S \times [0,1]$; firms' policy function $J(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0,1] \to \mathbb{R}$ and corresponding policy function $y'_J(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0,1] \to S \times S \times [0,1]$; labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega) : S \times S \times [0,1] \to \mathbb{R}_+$; and aggregate state $\psi \in \Psi$ such that:

1. *The worker's value functions $U(\mathbf{x})$ and $W(\mathbf{x}, \mathbf{y}$ satisfy (4) for all states $\psi \in \Psi$ and $y'_U(\mathbf{x})$, $y'_W(\mathbf{x}, \mathbf{y}, \omega)$ are the associated policy functions respectively*

2. *Firms' value function $J(\mathbf{x}, \mathbf{y}, \omega)$ satisfy (5) for all states $\psi \in \Psi$ and $y'_J(\mathbf{x}, \mathbf{y}, \omega)$ is the associated policy function*

3. *The labor market tightness $\theta(\mathbf{x}, \mathbf{y}, \omega)$ in each submarket $(\mathbf{x}, \mathbf{y}, \omega)$ for all states $\psi \in \Psi$ is consistent with free-entry condition in equation (6)*

From the above definition of block-recursive equilibrium that agents' value functions and policy functions, as well as the market tightness are independent of the aggregate state, only requiring that they are consistent with the aggregate state distribution of agents. Such an equilibrium is easier to characterize analytically and solve numerically. Note a key difference between the above definite of BRE and the one defined in Menzio and Shi (2011). In the economy studied in this paper, because I use the model to study the steady state equilibrium, the value functions, policy functions, and market tightness are entirely independent of the aggregate state. Whereas Menzio and Shi (2011) studies out-of-steady-state dynamics, so the value functions, policy functions, and market tightness are still depend on the aggregate productivity shocks, but are independent of the distribution of agents across employment status and match-specific shocks.

Now, I show that a block-recursive equilibrium exists and is unique.

Proposition 3 (Existence and Uniqueness of BRE)**.** *Under the model specification of linear utility and invertible and constant returns to scale matching function, also assume that the support for worker and occupation skill profiles S has bounded, then: i) all equilibria are block recurve as defined in definition 2; ii) there exists a unique block-recursive equilibrium.*

Proof of Proposition 3:

The proof first establishes the uniqueness of value functions (U, W, J), as well as policy functions and market tightness $(y'_U, y'_W, y'_J, \theta)$; then, the proof establishes their independence from aggregate state.

Uniqueness: I first show that the value functions for workers and firms as defined in equation (4) and (5) are contractions. Let $\Theta = S \times S \times [0, 1]$, which is bounded based on the assumption that S is bounded. Let $B(\Theta)$ the space of bounded functions $V : \Theta \to \mathbb{R}$ and the operator associated with worker or firm value functions denoted by $T : B(\Theta) \to B(\Theta)$. It is straightforward to verify that T satisfy monotonicity and discounting properties:

1. (monotonicity) For $V, V' \in B(\Theta)$, $V \leq V'$ implies $T(V) \leq T(V')$
2. (discounting) For $V \in B(\Theta)$ and $\epsilon > 0$, $T(V + \epsilon) =$

The above conditions establish that the operator T associated with either firm or worker values functions is a contraction bunder Blackwell's sufficient conditions. Therefore, the optimal values workers and firms obtain through dynamic optimization problems are unique.

Next, I show that the policy functions and market tightness are also unique. Since the optimal values firms and worker obtain for their dynamic optimization problems (4) and (5) are unique, the associated policy functions (y'_U, y'_W, y'_J) are also unique due to concavity of the production function defined in equation (3) and workers have linear utility over consumption. To show the uniqueness of market tightness, first note that since it is assumed that the matching function is invertible, one may directly obtain market tightness through the market clearing condition (6) with $\theta > 0$. The uniqueness of θ then follows from the uniqueness of firms' value function.

Independence of Aggregate State: In the model economy, workers with different skill profiles $\mathbf{x}$ search in their own market, and firms with different skill requirements $\mathbf{y}$ post jobs

in these separated markets, therefore, one can establish that the value functions of firms, workers and the market tightness are all independent of the aggregate state ψ. I establish this argument more rigorous through a backward induction argument as in Braxton and Taska (2021). For this purpose, I introduce back time subscript in the notation.

At the terminal period $t = T$, for an employed worker, the continuation value is zero for $T + 1$ onward, so the worker's dynamic programming problem does not depend on the aggregate distribution across states, and is equal to the worker's share of output $W_T(\mathbf{x}, \mathbf{y}, \omega) = \omega f(\mathbf{x}, \mathbf{y})$.

Similarly, the firm's value function also remains independent of the aggregate distribution $J_T(\mathbf{x}, \mathbf{y}, \omega) = (1 - \omega) f(\mathbf{x}, \mathbf{y})$. As a result, through the free entry condition in equation (6), the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$ is also independent of the aggregate distribution.

Firms at $T - 1$ make occupation design choices $\mathbf{y}$ to solve the firm dynamic programming problem in equation (5); workers at $T - 1$ make labor market search choices over occupations $\mathbf{y}$ to solve the worker dynamic programming problem in equation (4); As long as $\mathbf{y}$ is within a bounded interval bounded interval, the extreme value theorem assures at least one solution to this problem. This process is repeated stepping back from $t = T - 1, ..., 1$, which completes the proof. *Q.E.D.*

B.3 Identification of Parameters

First, I focus on identifying the elasticity parameters in production and occupation operation cost, denoted by σ and ρ. As highlighted by Caselli and Coleman (2006), the challenge arises when allowing for the endogenous choice of the efficiency of inputs under constraints, as the elasticity parameters cannot be separately identified. To overcome this challenge, I begin by estimating σ using the *relative wage within occupation* instead of relying on absolute wage levels.

Specifically, based on the model, the wage that workers receive per period is given by the share ω of the output of the worker-firm match, reduced by the occupation design cost, formulated as $w(\mathbf{x}, \mathbf{y}) = \omega f(\mathbf{x}, \mathbf{y}) - C(\mathbf{y})$. Consequently, within each occupation, the difference in wage relative to a base worker type $\Delta w(\mathbf{x}, \mathbf{y})$ can be articulated as follows:

$$\Delta w(\mathbf{x}, \mathbf{y}) = \omega \left[\sum_{k=1}^{K} (x^k y^k)^\sigma \right]^{\frac{1}{\sigma}} - A, \tag{10}$$

where A is occupation-specific and does not depend on the cost parameter τ or ρ. This formulation enables the identification of σ independent of the cost parameters. To carry out the estimation equation (10), I first adjust the wage for occupation fixed effects in order to account for A and ω, and for other factors like ethnicity by gender, age, metropolitan status, individual year, years of education, census region, and urbanity fixed effects. These adjustments help control for other variables that might influence the worker-firm output and enhance the precision of the estimation. Next, I compute the within-occupation difference of the adjusted wage relative to the lowest skill type worker.[62] Last, I employ the maximum likelihood estimation (MLE) method to calculate σ.

I now turn to the identification of the cost parameters ρ and τ. To begin with, note that the first-order condition of firms' optimization problems in the submarket $(\mathbf{x}, \mathbf{y})$ can be simplified in ratios to: $\frac{y_h}{y_k} = \left(\frac{x_h}{x_k}\right)^{\frac{\sigma}{\rho-\sigma}}$, a relationship that exclusively depends on the parameters σ and ρ. With σ already estimated through MLE, the Simulated Method of Moments (SMM) then targets the skill ratio y_j/y_k, which aligns with the moment of the degree of hybridization of occupations. Further, for employed workers, the distribution of employment across various occupations is governed by wages $w(\mathbf{x}, \mathbf{y})$. Given the parameters

[62] Refer to Section VI for an in-depth discussion on how worker skill types are calibrated.

described above, this functional relationship allows the estimation of τ.

I'll now discuss how the vacancy posting cost c and the relative skill level of the high-skill worker α_k are identified. Given the calibrated unemployment benefit b, the parameters of the matching function, and the parameters of the production and cost function, the identification process unfolds as follows. First, the free entry condition (6) and the value function of unemployed workers (4) reveal that the probability of exiting unemployment is solely determined by the vacancy posting cost. By targeting the level of unemployment, c is thus identified. Second, the identification of α_k is achieved by examining the relative wage of high- to low-skill workers, expressed as $\frac{w(\mathbf{ff}\mathbf{x}^{\mathbf{l}},\mathbf{y})}{\mathbf{w}(\mathbf{x}^{\mathbf{l}},\mathbf{y})}$, where $\mathbf{x}^{\mathbf{l}}$ is the calibrated skill level of the low-skill worker. Given the previously calibrated parameters, this relative wage depends only on the relative skill level of the high-skill worker, leading to the identification of α_k. This completes the identification of all the parameters in the model.

B.4 Calibration of Skill Supply

I carry out the calibration of two key aspects of skill supply variation: the Markov probability of worker skill adjustment over a single model period and the variation in worker skill supply spanning two data periods that the model aims to align with two steady states. I'll first delve into the details of the skill variation between data periods and then explore the skill evolution within a model period as guided by the Markov process, following the approach of Lise and Postel-Vinay (2020).

Across-period Skill Supply Variation: Considering the potential influence of skill supply variation on skill mixing, I calibrate the model to reflect workers' choices in occupation, college major (if attended), and employment status, in line with the approach of Lise and Postel-Vinay (2020). This calibration introduces variation in worker skill supply across two periods. Worker skills are adjusted based on the requirements of an occupation or a college major; they increase if the requirements exceed the original skills and decrease if the requirements are lower or if the worker is unemployed. The speed of this adjustment is asymmetric and skill-specific.

Specifically, following the estimates from Lise and Postel-Vinay (2020), as presented in online Appendix Table 10, a worker's skills accumulate at a rate of γ_j times the gap between the worker's skill j and the occupation's requirement for that skill each year. The value of γ_j depends on whether it relates to learning or depreciation (upward or downward accumulation). Additionally, workers can lose skills when not employed, with unemployment treated as requiring a zero level for all skills. However, I specify such that a worker's skill level cannot fall below their initial endowments. For changes in skills while in school, I specify that workers spend an average of three years learning the skills of their majors.

I incorporate two modifications into this framework. First, since Lise and Postel-Vinay (2020)'s estimates are based on weekly data, I adjust them by multiplying by the number of working weeks, set at 47. Second, I align Lise and Postel-Vinay (2020)'s estimates of cognitive, interpersonal, and manual skills with my analysis's categories of analytical, interpersonal, and routine skills.[63] Since Lise and Postel-Vinay (2020)'s estimates do not include computer skills, I use their cognitive skill estimates as a proxy.

[63]Their exclusion restriction imposes that (i) the ASVAB mathematics knowledge score only reflects cognitive skills; (2) the ASVAB automotive and shop information score only reflects manual skills; (3) the Rosenberg self-esteem score only reflects interpersonal skills.

In calculating the skill adjustment, I first standardize both worker skills and occupation skill requirements. Then, for example, if a worker is employed in an occupation that requires a standard deviation higher in analytical skill compared to the worker's analytical skill, the worker will accumulate 0.36 standard deviations of analytical skill in a year due to learning on the job. Conversely, if a worker's interpersonal skill is higher than required, it will decrease by only 3×10^4 standard deviations, almost remaining unchanged, as interpersonal skills are estimated to be very hard to lose.

Markov Skill Supply Adjustment: I now discuss the Markov process of skill adjustment. Specifically, considering each skill j in the worker's skill profile $\mathbf{x}$ as an element of the finite set S, the evolution of this skill follows a Markov process $\pi(x_j'|x_j, y_j)$, conditional on the worker's current skill level and employed occupation. If a worker is matched with an occupation that requires a skill level exceeding his or her own ($x_j < y_j$), the worker's skill j will adjust upward in the next period: $x_j' > x_j$, and the inverse applies for a worker whose skill is lower than the requirements of their current occupation.

The calibration of the Markov adjustment probability is conducted in a similar manner to that of the across-period skill supply variation. Since a model period corresponds to a quarter, the annual adjustment rates for different skills are divided by 4, resulting in $\frac{\gamma}{4}$. This value represents the rate at which worker skills approach to occupation skill requirements, and it is regarded as the probability that a worker's skill j will adjust to the corresponding value.

The key challenge in this calibration process arises when quantifying the model: both worker skill and occupation skill requirements are discretized as grid values. To accommodate this discretization, the probability that a worker moves up or down a grid for skill j based on the occupation is scaled as below.

The Markov probability of upward adjustment is determined by:

$$\frac{x_j^{up} - x_j}{y_j - x_j} \mathbf{1}(x_j^{up} < y_j) \times \frac{\gamma_j^{up}}{4}$$

Similarly, the Markov probability of downward adjustment is given by:

$$\frac{x_j^{down} - x_j}{y_j - x_j} \mathbf{1}(y_j < x_j^{down}) \times \frac{\gamma_j^{down}}{4}$$

Here, x_j represents the current grid value of worker skill j, while x_j^{up} or x_j^{down} denotes the value of worker skill j up or down a grid, respectively. The indicator variables $\mathbf{1}(y_j < x_j^{down})$ or $\mathbf{1}(x_j^{up} < y_j)$ evaluates whether the skill j grid value of the worker's current employed occupation is greater or smaller than the value of the worker's skill j grid. This means that a worker will only adjust up or down a grid if the occupation's skill is larger or smaller than the corresponding up or down grid value for the worker's skill. This process specifies the interplay between skill adjustment and occupation requirements, and allows for a precise calibration within the model's framework.

Table 10: Annual Skill Learning and Depreciation Rate

O*NET Measure	NLSY Measure	γ_{school}^{learn}	γ_j^{up}	γ_j^{down}
analytical	AFQT score	0.33	0.36	0.10
interpersonal	Deming (2017) social skill	0.33	0.05	0.00003
routine	ASVAB	0.33	1	0.36
computer	OCC/Major's 2005 Value	0.33	0.36	0.10

*Notes: This table illustrates for each O*NET skill measure, its corresponding skill measure using NLSY79&97 data, and the learning and depreciation rate for these different skills. The AFQT is the same as the one used by Altonji, Bharadwaj, and Lange (2012) followed by Deming (2017), which controls for age-at-test, test format, and other idiosyncrasies. Deming (2017)'s social skill measure consists of sociability in childhood and sociability in adulthood in NLSY79, and two questions from the Big 5 inventory gauging the extraversion in NLSY97. The average of workers' ASVAB mechanical orientation and electronics test scores are used for mechanical skill. Since ASVAB scores are not available for the NLSY97 survey, they are imputed based on predictive regression using the NLSY79 survey. Workers' occupations' or college majors' O*NET computer skill scores in the year 2000 are used as their endowed computer skill. The skill accumulation/depreciation rate is directly from Lise and Postel-Vinay (2020)'s estimates based on monthly data converted to annual values. Skill learning/depreciating while attending college is specified to be 33% per year.*

B.5 Algorithm and Solution Method

The quantitative method used for estimation is SMM. Given the parameters in the model that are internally estimated $\Theta = \{\sigma, \rho, \phi, c, \alpha_k\}$, each iteration of SMM first solves the steady state firm and worker policy function, after which a panel of worker is simulated to obtain the equilibrium distribution of labor market outcomes.

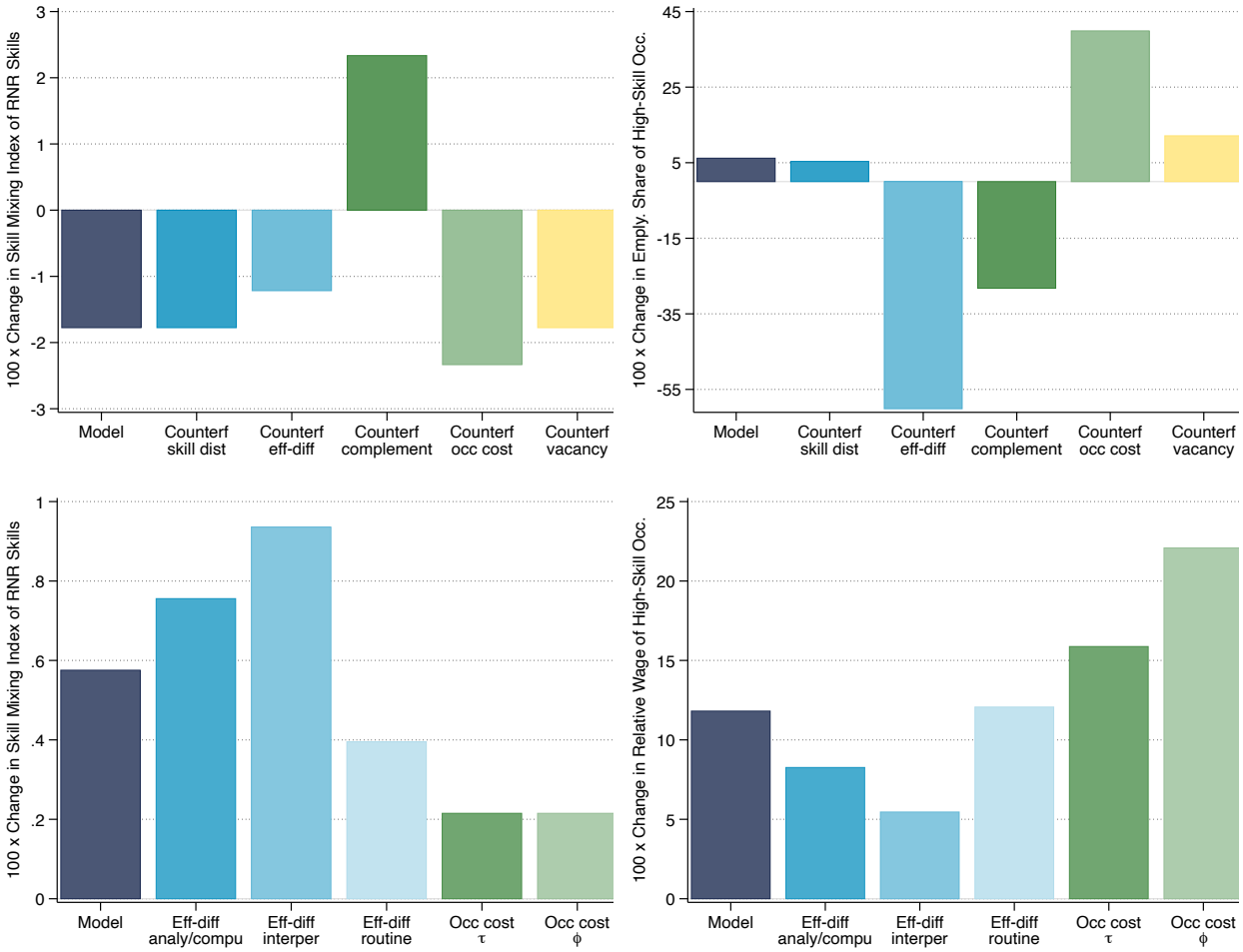
Specifically, to find the steady state policy of agents, I use value function iteration:

1. Fix the number of periods T
2. Starting from the terminal period T, solve the firm problem as in equation (4)
3. Use the free entry condition in equation (6) to obtain the market tightness $\theta_T(\mathbf{x}, \mathbf{y}, \omega)$
4. With the market tightness, solve the worker dynamic programming problem in equation (4)
5. Repeated stepping back from $t = T - 1, ..., 1$
6. Check if the difference in worker value $U_{t+1} - U_t$, $W_{t+1} - W_t$ and the firm value $J_{t+1} - J_t$ is less than a predetermined tolerance level. If yes stop, if not increase T and go back to first step

Next, I simulate 10,000 workers for $T(T > 200)$ periods, burning the first 40 periods to obtain distribution of labor market outcomes across different occupations and worker types. Finally, the SMM procedure minimizes the Euclidean distance between the model-implied moments and the same moments obtained from data.

B.6 Additional Counterfactual Results

Figure 7: Model Counterfactual



Notes: These figures plot the model generated changes in skill mixing in high-skill occupations (panel 1) and changes in employment share of high-skill occupation (panel 2). Different model channels are shut down individually by eliminating the relative calibrated values to highlight the contribution of each channel. The full model has all the model features. The values of skill complementarity in production, cost of skills in occupation operation, efficiency differential, and vacancy posting cost across the two periods are shown in Table 5. Worker skill supply distribution variation across the periods are calibrated according to Table 10. Panel (3) shows the model generated changes in skill mixing in low-skill occupation and the relative wage of high-skill occupations by shutting down the skill efficiency differential for analytical/computer, interpersonal, and routine skills individually; also by shutting down τ and ϕ individually.