Non-market allocation mechanisms

Optimal design with investment incentives

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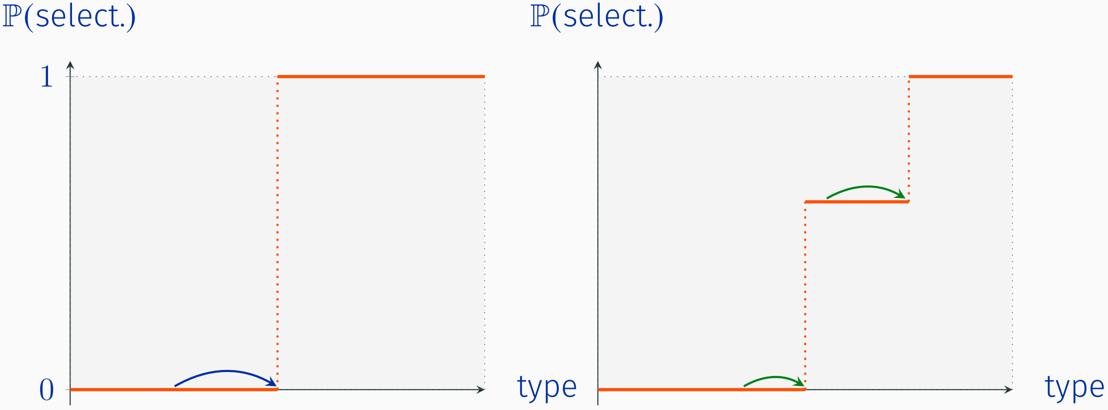
This paper

How should allocation/selection mechanisms be designed, when accounting for agents' investment incentives?

Randomization

Intuitively, randomization might help by spreading investment incentives (but complicated tradeoff).



(a) Pass-fail

(b) Some randomization.

Contribution

Main theorem

If the investment cost is quadratic and the principal is selecting in the upper tail, a pass-fail rule is optimal for the principal.

- A firmer foundation for the use of such rules.
- A possible explanation for the prevalence of pass-fail rules.
- We also characterize the optimal pass-fail and provide some comparative statics.

Extensions

Three extensions:

(i) Capacity constrained principal.

(ii) Utilitarian welfare maximization (weight on the agent).

(iii) Implementation through information design.

Results

- Pass-fail rules remain optimal in (i) and (ii).
  (i) Capacity constraint lowers the optimal cutoff.
  (ii) Accounting for agents' costs increases the optimal cutoff.
- Relaxing the principal's commitment power as in (iii) does not reduce her optimal payoff.

Model

Setup

Agent

- Natural type: $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} < 0 < \bar{\theta}$.
- Distribution: cdf $F : \Theta \to [0, 1]$.
- Final type: $t \in T = \mathbb{R}$.
- Cost: $\gamma c(t, \theta)$ where $\gamma > 0$.

Principal

- Can observe t
- Cannot observe θ but knows F.
- Allocation/Selection: $a \in \{0, 1\}$.

Payoffs

Agent's payoff

Allocation net of investment cost:

$$v(a,t) = a - \gamma c(t,\theta).$$

Principal's payoff

Final type conditional on allocation:

$$\pi(a,t) = at$$

Timing and incentive-compatibility

1. The principal commits to a selection rule $\sigma : T \to [0, 1]$ which is publicly revealed.
2. Agents observe θ and choose an investment rule $\tau : \Theta \to T$.
3. Agents selected with probability $\sigma(\tau(\theta))$

Definition (Incentive-compatibility)

- An investment rule τ is incentive-compatible under selection rule σ if, for all $\theta \in \Theta$:

$$\tau(\theta) \in \arg\max_{t \in T} \sigma(t) - \gamma c(t, \theta).$$

- An investment rule τ is implementable if there exists a selection rule σ under which τ is incentive-compatible.

Assumptions: Cost

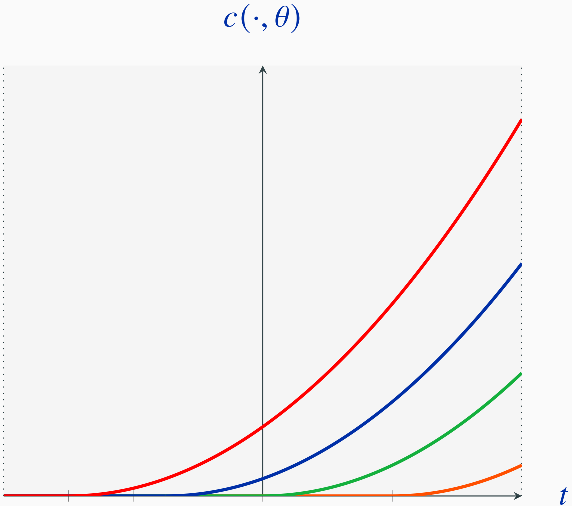
Assumption (Quadratic cost)

The cost function $c : T \times \Theta \to \mathbb{R}_+$ *is given by:*

$$c(t, \theta) = \frac{(t-\theta)^2}{2} \mathbb{1}_{t \geq \theta}.$$

Define θ_0 *by:*

$$\gamma c(0, \theta_0) = 1 \iff \theta_0 = -\sqrt{2/\gamma}$$



Assumptions: Distribution

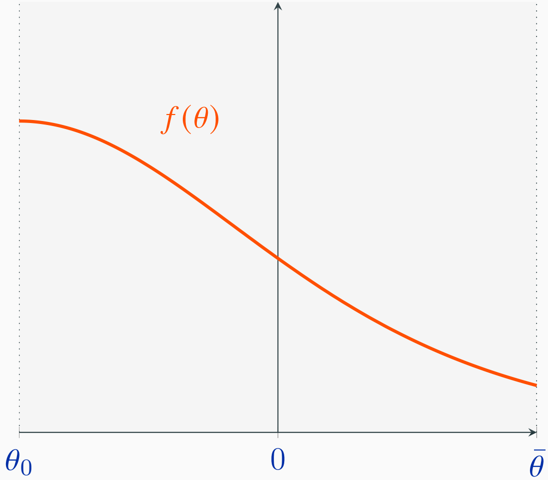
Assumption (Selection in the upper tail)

The cdf F admits a density function f which is:

(i) *strictly positive on Θ;*

(ii) *differentiable;*

(iii) *decreasing: $f'(\theta) \leq 0$ for $\theta \geq \theta_0$.*



Principal's problem

Principal's ex-ante expected payoff is:

$$\int_{\underline{\theta}}^{\bar{\theta}} \tau(\theta)\,\sigma\big(\tau(\theta)\big)\,f(\theta)\,\mathrm{d}\theta$$

Principal's program

$$\begin{aligned} \underset{\sigma,\tau}{\text{maximize}} \quad & \int_{\underline{\theta}}^{\bar{\theta}} \tau(\theta)\,\sigma\big(\tau(\theta)\big)\,f(\theta)\,\mathrm{d}\theta \\ \text{subject to} \quad & \tau(\theta) \in \arg\max_{t\in T} \sigma(t) - \gamma c(t,\theta),\ \forall\theta\in\Theta \end{aligned}$$

Main result

Main result

Definition

A selection rule σ is a $t^{\dagger}$-pass-fail rule if there exists a selection cutoff $t^{\dagger} \in T$ such that, for almost every $t \in T$:

$$\sigma(t) = \mathbb{1}_{t \geq t^{\dagger}}.$$

Theorem

For any $\gamma > 0$, there exists a strictly positive selection cutoff t^{*}_{γ} such that the t^{*}_{γ}-pass-fail rule is optimal.

Overview of the proof

Monotone selection rules

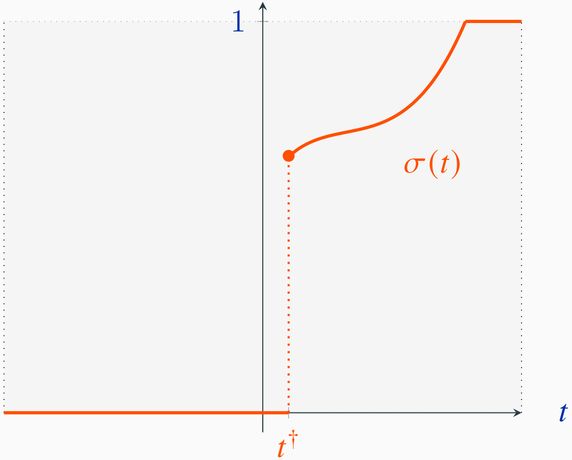
Lemma

We can restrict attention w.l.o.g. to selection rules such that:

(i) $\sigma(t) = 0$ *for all* $t < 0$, *and;*

(ii) σ *is non-decreasing;*

Let $t^\dagger = \inf\{t \in T \mid \sigma(t) > 0\}$.



Agent's pseudo utility

- Indirect utility and pseudo utility:

$$U(\theta) := \max_{t \in T} \sigma(t) - \gamma c(t, \theta)$$
$$= \gamma \left(\underbrace{\max_{t \in T} \left\{ t\theta + \frac{\sigma(t)}{\gamma} - \frac{t^2}{2} \right\}}_{:= u(\theta)} - \frac{\theta^2}{2} \right)$$

- Then $u(\theta)$ is convex and the Envelope theorem implies

$$u'(\theta) = \tau(\theta) \quad \text{a.e.}$$

- Furthermore, we have:

$$\sigma\big(\tau(\theta)\big) = \gamma \left(u(\theta) + \frac{u'(\theta)^2}{2} - \theta u'(\theta) \right) \quad \text{a.e.}$$

Variational program

Principal's program

$$\underset{u \in \mathcal{U}}{\text{maximize}}\ V(u) := \int_{\theta_0}^{\bar{\theta}} u'(\theta)\left(u(\theta) + \frac{u'(\theta)^2}{2} - \theta u'(\theta)\right) f(\theta)\,\mathrm{d}\theta$$

where, $u \in \mathcal{U}$ (feasible pseudo utility functions) iff.:

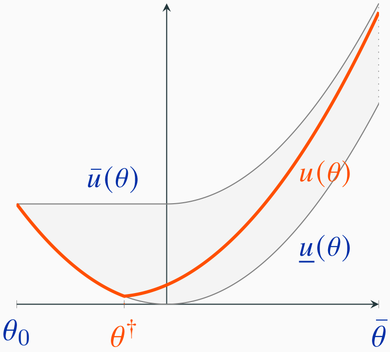
(i) u is a convex function.

(ii) $\underline{u}(\theta) := \theta^2/2 \leq u(\theta) \leq 1/\gamma + (\theta^2/2)\mathbb{1}_{\theta \geq 0} =: \bar{u}(\theta)$

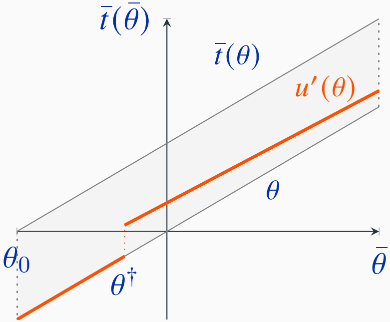
(iii) $\theta \leq u'(\theta) \leq \theta + \sqrt{2/\gamma} =: \bar{t}(\theta)$

(iv) $u(\theta) + u'(\theta)^2/2 - \theta u'(\theta) \leq 1/\gamma$

Feasible pseudo utilities: illustration



(a) An admissible pseudo-utility u.



(b) Its induced investment rule u'.

Solutions as extreme points

Let $\mathcal{U}(\theta^\dagger) := \left\{u \in \mathcal{U} \,\middle|\, \theta^\dagger = \sup\{\theta \in \Theta \mid u(\theta) = \underline{u}(\theta)\}\right\}$ for any $\theta^\dagger \in [\theta_0, \bar{\theta}]$.

Principal's program

$$\underset{\theta^\dagger \in [\theta_0, \bar{\theta}],\, u \in \mathcal{U}(\theta^\dagger)}{\text{maximize}} \quad V_{\theta^\dagger}(u) = \int_{\theta^\dagger}^{\bar{\theta}} u'(\theta)\left(u(\theta) + \frac{u'(\theta)^2}{2} - \theta u'(\theta)\right) f(\theta)\, \mathrm{d}\theta$$

Lemma

- *For any $\theta^\dagger \in [\theta_0, \bar{\theta}]$, the set $\mathcal{U}(\theta^\dagger)$ is convex and compact.*
- *For any $\theta^\dagger \in [\theta_0, \bar{\theta}]$, the functional $V_{\theta^\dagger} : \mathcal{U}(\theta^\dagger) \longrightarrow \mathbb{R}$ is upper-semicontinuous, and, if f is decreasing, it is also convex.*
- *Therefore $V_{\theta^\dagger}(u)$ has a maximizer that is an extreme point of $\mathcal{U}(\theta^\dagger)$.*

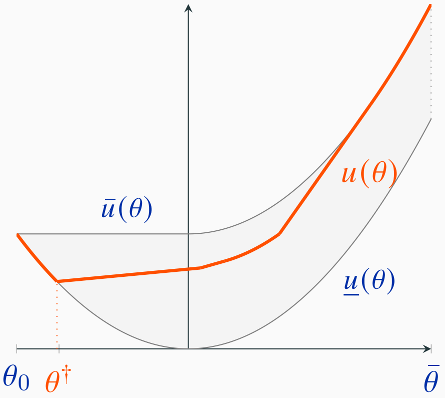
Extreme points: necessary conditions

Proposition

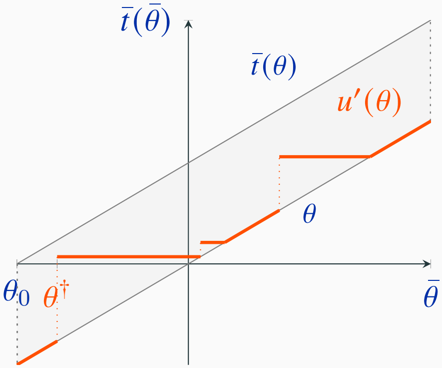
If u is an extreme point of $\mathcal{U}(\theta^\dagger)$, it must be a sequence of affine and quadratic pieces. Moreover, u cannot be quadratic below zero.

- Affine: $u(\theta) = a\theta + b$ for some a, b.
- Quadratic: $u(\theta) = \frac{\theta^2}{2} + c$ for some $c \in [0, \gamma/2]$.
- Let $\mathcal{C}(\theta^\dagger)$ be the set of such functions = our set of candidates.

Extreme points: example

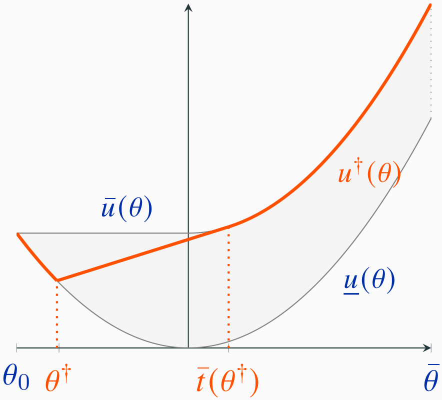


(a) An extreme point u.

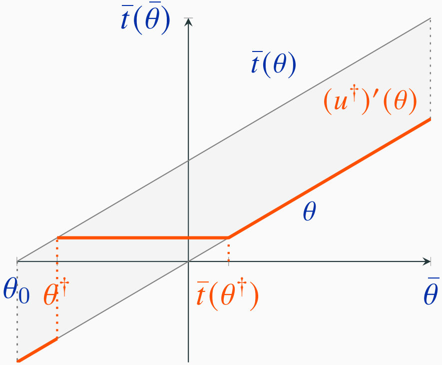


(b) Its induced investment rule u'.

Our candidate: pass-fail



(a) Pseudo-utility under pass-fail.



(b) Investment under pass-fail.

Pass-fail optimality

- Let $\mathrm{D}V_{\theta^\dagger}(u)(h)$ be the Gâteaux derivative of $V_{\theta^\dagger}$ at u in direction h.
- By convexity of $V_{\theta^\dagger}$, for any $u \in \mathcal{C}(\theta^\dagger)$:

$$V_{\theta^\dagger}(u^\dagger) - V_{\theta^\dagger}(u) \geq \mathrm{D}V_{\theta^\dagger}(u)(u^\dagger - u).$$

- We prove that, for any $u \in \mathcal{C}(\theta^\dagger)$:

$$\mathrm{D}V_{\theta^\dagger}(u)(u^\dagger - u) = \int_{\theta^\dagger}^{\bar{\theta}} \Big(\underbrace{\alpha(\theta)}_{\geq 0} f(\theta) + \underbrace{\beta(\theta)}_{\leq 0} f'(\theta) \Big) \underbrace{\big(u^\dagger(\theta) - u(\theta)\big)}_{\geq 0} \, \mathrm{d}\theta \geq 0.$$

□