Reverse Selection in Asymmetric Trade with Heterogeneous Firms

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July 13, 2023

Abstract

In a simple model of monopolistic competition with heterogeneous firms, we show that trade among asymmetric countries can revive less productive firms to specialize in export, a pattern we call "reverse selection". While reverse selection contradicts the common belief that trade only favors more productive firms, we show that the two patterns complement each other: Whenever reverse selection takes place in some countries, trade also crowds out less productive firms in some of their trading partners. The two contrasting patterns are driven by different purchasing power in different countries, which can arise endogenously from country asymmetry in fundamentals such as population size or overall productivity level. Our paper echoes existing empirical findings, offers a rich set of implications for understanding many salient phenomena in trade, and reveals new factors that can shape the measured impacts of trade on firm performances.

Keywords: firm heterogeneity, asymmetric trade, reverse selection, crowd-in
JEL codes: F10, F12, F14

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1 Introduction

The literature on trade with heterogeneous firms argues that trade reallocates resources from low productivity firms to high productivity firms by crowding out less productive firms and allowing more productive firms to expand from home to foreign markets. This argument, known as the selection hypothesis, was formalized by Melitz (2003) in a model of monopolistic competition, and was subsequently reaffirmed by, for instance, Helpman, Melitz, and Yeaple (2004) and Melitz and Ottaviano (2008), all in a setting where incomes are equal among trading partners. The reality, however, is that countries differ greatly in income. With income disparity, does trade necessarily crowd out less productive firms and give foreign markets only to the more productive ones?

To answer this question, we develop a simple model with endogenous income disparity, and find that a "reverse selection" phenomenon may arise in poorer countries where less productive firms are crowded in by trade and, once crowded in, become specialized in export. By "crowd in", we mean that firms unviable under autarky begin to produce under trade, and by "reverse selection", we mean that exporters can be less productive than non-exporters, the opposite of what is predicted by the selection hypothesis. In richer countries, by contrast, trade always crowd out less productive firms, and exporters always sell both at home and abroad, just as they do in all countries when these countries are symmetric and equal in income.

The reason behind reverse selection is intuitive. A richer country has a higher demand and hence consumes more varieties than a poorer country does. In consequence, less productive firms in poorer countries may be able to serve a richer foreign market but not the domestic market, thus becoming exporters that specialize in trade. Compared to autarky, two forces are in play here. The first force is the classical one: trade tends to raise the opportunity cost of resources. This is the force that causes less productive firms to be crowded out when countries are symmetric. The second force comes from income disparity and hence disparity in demand: trading with a richer country means that a poor country faces a higher demand from abroad. Combining these two forces together, we are able to show that, as long as there is income disparity, some producers in a poor country will specialize in exports, and these exporters will be less productive than those that sell both at home and abroad. When the disparity becomes sufficiently large, the country's marginal firm under autarky will be able to thrive in foreign markets and in foreign markets only, giving rise to crowd-in and hence reverse selection.

Our analysis would have been incomplete and hence unsatisfactory without the recognition that income disparity itself is a product of trade. Trade determines countries' income and hence shapes income inequality around the world. Therefore, a further question must be addressed: Will trade result in an income disparity across countries that is so large that it induces reverse selection?

The literature offers two approaches to determine countries' income in trade. One is to exogenously pin down wage rates across countries by assuming the existence of an equilibrium in which all countries produce a tradable numeraire good using constant returns to scale technology, while at the same time assuming a specific form of free entry so that producers in every economy have zero profit in equilibrium. The other is to employ extra nuts and bolts, such as specific forms of consumer preferences and specific distribution functions of firms' productivity, to calculate explicit equilibrium solutions. In this paper, we develop a different approach by introducing a concept called (per capita) purchasing power. We show that, in equilibrium, purchasing power corresponds (albeit not identical) to a country's income, and hence its disparity reflects cross-country income inequality. This approach enables us to truly endogenize each country's income and generate interesting testable implications, while at the same time allows us to carry out the analysis through reasoning without relying on those extra nuts and bolts, so that we can crystalize all the driving forces behind our results.

We endogenize purchasing power and its disparity by considering two kinds of exogenous asymmetry among countries: population size and overall productivity (to capture the level of economic development). We show that, other things being equal, smaller countries and more productive countries will enjoy a larger per capita purchasing power than their respective counterparts. In addition, we show that a country with a larger purchasing power can be either more or less competitive on the global market depending on the source behind its richness. A more advanced economy, after taking into account of its superior overall productivity, will be more competitive, giving the country an edge over its less advanced trading partners in serving their respective home market. In contrast, a smaller country will be less competitive, allowing its larger counterpart to better penetrate its market than vice versa.

From there we derive two central insights of our paper. (1) Holding overall productivity constant, reverse selection happens in a country if it is sufficiently larger than its trading partners. (2) Holding population size constant, reverse selection happens in a country if it lags sufficiently behind in overall productivity *and* its labor can flow freely between the tradable and non-tradable sectors. In other words, the disparity in the level of economic development alone cannot produce reverse selection without resource reallocation between the tradable and non-tradable sectors.

We further trace out the evolution of trade impacts in a backward country depending on economic development and globalization around the world, captured in our model by the rise in overall productivity in the country itself and in advanced countries, as well as changes in the number of these two types of countries involved in trade. In particular, there exists a critical level of overall productivity which we refer to as the trade frontier. Below the frontier, there is no

trade; beyond it, trade starts to flow, which creates a tradable sector in a backward country that initially specializes in exporting to more advanced countries. There will be zero bilateral trade between such a country and countries with similar or lower overall productivity at this stage of its development (Helpman, Melitz, and Rubinstein (2008), Baldwin and Forslid (2010)). With a further rise in its overall productivity, the sector will serve both at home and abroad, with less productive firms initially crowed in (as compared to autarky) to completely specialize in export, but eventually crowded out (as compared to autarky) after the country's overall productivity rises to a level close to that of more advanced countries.

We also show that the expansion of global trade raises the trade frontier; and as some backward countries grow and join the global trade, it will become more difficult for the remaining backward countries to join the global trade. Our paper thus suggests a divergent force in trade and that it pays for a backward country to participate in global trade earlier. As backward countries close the gap in overall productivity with advanced nations, the former's presence at the home market will expand whereas the latter's presence at its own domestic market will decline, reflecting part of the recent dynamics between China and the U.S.

Our analysis reveals that *equilibrium* purchasing power disparity depends on the structure of overall productivity across countries, on how countries can reallocate resources between their tradable and non-tradable sectors, and on how countries are able to spend their income across their trading partners. Accordingly, our paper brings to light a host of factors that can potentially affect the measured impacts of trade on firm performances in a particular country. They include the level of economic development of the country itself as well as that of its trading partners; how the country and its trading partners move up or down the global productivity ladder; should trade alter a country's overall productivity, whether trade induces developed and developing countries to converge or diverge; how fluid the internal factor market is in the country under study, and if not, whether there is domestic market liberalization in company with the trade liberalization; and whether the trading partners of the country under study have their trade exposures altered subsequent to the trade liberalization of the country under study. In doing so, our paper offers a possible reconciliation between the stylized empirical patterns and the possibility of reverse selection, which is a logical extension of the existing theory that has offered a powerful explanation for the empirical patterns.

An empirical phenomenon often observed in developing countries is that many exporters, known as export-oriented firms, are specialized in exports. Recent research has documented that exporters in China, one of the largest exporters in the world, tend to be less productive and that a key reason is that a large portion of these exporters are export-oriented (Lu (2012), Lu, Lu, and Tao

(2010), Dai, Maitra, and Yu (2016), Manova and Yu (2016), and Chen and Sun (2019)). While this observation is at odds with the selection hypothesis,[1] it is rather consistent with what our model would predict, even though the model is not intended as a study about China per se. Our paper implies that the China findings can be traced to what China is about: her large population and low productivity relative to many of its trading partners at a time pertinent to those findings; and to what China has done: opening up to the world and reforming domestically as well, which among other things allowed rural labor to migrate and become urban workers. Our analysis implies that it is the combination of all these factors that had propelled the emergence and expansion of China's export-processing sector. From this understanding a corollary can be readily drawn: The recent decline of the export-processing sector in China can be explained by the country's rapid convergence to developed countries along with her slow-growing labour force.

At the core of the reverse selection phenomenon is the possibility for trade to crowd in, rather than crowding out, less productive firms. Such a possibility was also highlighted by Zhelobodko et al. (2012) and Mrázová and Neary (2017), who demonstrate how crowd-in may arise for non-constant substitution preferences when countries are symmetric. We approach the problem via country asymmetry rather than preferences and, as a result, are able to offer a completely different set of testable and arguably more interesting implications. Later we will elaborate more about how our approach differs from these two papers.

Other researchers have also attempted to extend the original Melitz (2003) model to accommodate country asymmetry. Helpman, Melitz, and Yeaple (2004), Chaney (2008), Helpman, Melitz, Rubinstein (2008), Melitz and Ottaviano (2008), Demidova and Rodriguez-Clare (2009, 2013), Arkolakis (2010), Eaton, Kortum, and Kramarz (2011), Bertoletti and Epifani (2014), Simonovska (2015), and Bertoletti, Etro, and Simonovska (2018) did so to offer a richer characterization of patterns of trade; Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008), Baldwin and Forslid (2010), Arkolakis, Costinot, and Rodriguez-Clare (2012), Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019) did so to size up the gains from trade under monopolistic competition in more complex settings. Our paper differs from these research efforts by focusing on how trade among asymmetric countries contributes to disparity in income and hence disparity in purchasing power that ultimately leads to the reverse selection phenomenon. By linking the purchasing power disparity to differences in the overall productivity across countries, our paper also differs from

[1] These researchers (except Lu 2012) point out, quite rightly, that even when countries are symmetric, the selection hypothesis replies crucially on the assumption that it is less costly to trade at home than abroad, and that should this assumption be reversed, less productive firms will be able to export but not sell domestically. Lu (2012), on the other hand, attributed the China findings to the country's relative abundance in one factor of production as compared to its trading partners. Offered in a symmetric trade setting, however, this last reasoning begs the question of why the same argument does not apply symmetrically to China's trading partners.

these research efforts by providing a dynamic portrait of how trade impacts on firm performances may evolve with economic development around the world.

We would like to point out that country asymmetry does not necessarily translate into income inequality across countries. Many models in the literature assume a specific form of free entry so that trade among countries of different population sizes becomes isomorphic to trade among a different number of countries with identical population size. This allows population asymmetry to be completely absorbed by something called copies or masses of firms, thus leaving firm and consumer behaviors intact regardless of country asymmetry. To focus on empirically measurable implications, our paper gets rid of such assumption of free entry and hence the concept of copies or masses, which allows asymmetry in population size to have a bite in the trade equilibrium.

The removal of the free entry assumption allows firms to have positive profits in our model, as in Chaney (2008). Different from Chaney (2008) where profits are distributed globally, we assume that the profits are distributed domestically as in Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011). Later we will highlight the role of such a profit distribution in shaping a country's purchasing power and hence the trade impacts on firm performances.

The rest of the paper is organized as follows. We lay out our model in Section 2 and reproduce in a symmetric trade setting the classical insight that trade crowds out less productive firms and allows more productive firms to sell both at home and abroad. In Section 3, we first analyze, assuming an exogenous disparity in purchasing power, what kind of firms will specialize in export. We then endogenize such disparity by introducing country asymmetry in population size and in overall productivity (to capture the level of economic development) respectively. Finally, we compare trade and autarky to analyze whether less productive firms are crowded in or out, and draw further implications from such comparative statics. Section 4 concludes by discussing a wealth of interesting implications arising from our simple model. All the proofs are collected in a separate file referred to as the Online Appendix.

2 A Baseline Model of Symmetric Trade

We begin by introducing a very simple model to reproduce the crowd-out insight in a symmetric trade setting. Consider a representative economy. In its tradable sector, variety $j \in [0, \infty)$ is produced by a monopolistic firm at marginal cost $c(q, j)$ that is weakly increasing in output q and strictly increasing in j with $\lim_{j\to\infty} c(0, j) = \infty$. The cost is in the form of labor, the only factor of production. There is no fixed cost of production, trade cost, setup cost, or any cost to draw the

productivity index j.[2]

The economy has a unit measure of homogeneous consumers, who collectively own all the firms and each consumer supplies a fixed amount of labor, l. An individual consumer's preference is represented by

$$U(q(i)_{i\in(0,\infty)}) \equiv \int_0^\infty u(q(i))di,$$

corresponding to the utility derived from the tradable sector, with $q(i)$ being the (per-capita) consumption quantity of variety i, and $u(.)$ the utility from a given variety. We assume that $u' > 0$ and $u'' < 0$.

Since varieties enter a consumer's utility symmetrically, consumption quantity $q(i)$ varies across varieties only due to their price differentials. Provided that each variety's price increases monotonically in cost (which will be shown to be indeed the case), varieties sorted in an ascending order in production cost corresponds to varieties sorted in a descending order in consumption quantity. In autarky, therefore, the ith variety in a consumer's consumption basket corresponds to the ith least costly variety in the economy. In trade, a consumer sources his consumption from the global market and such correspondence must be amended, as will be shown in a moment.

With the assumption of costless trade, we characterize the impact of trade using a discrete comparison between autarky and free trade, rather than an incremental reduction in trade cost. In doing so, our analysis can be readily applied to an economy that unilaterally opens itself to trade.[3] We assume that the sets of varieties produced by different countries do not overlap. Following the literature, we assume monopolistic competition with uniform pricing within each country.

Let $q(j)$ denote the (per-capita) amount of the jth variety *produced in a country and sold in its domestic market*. In a symmetric trade equilibrium involving m such representative economies,[4] variety j's total production and global sales is $mq(j)$ given the unit measure of population in each country. Accordingly, in such an equilibrium, firm j (the producer of variety j) chooses $p(j)$, the price of variety j, to maximize

$$\pi(j) \equiv p(j)mq(j) - \int_0^{mq(j)} c(q, j)dq.$$

[2]For expositional simplicity, we present our analysis without assuming any cost in trade in the main text of our paper. We incorporate trade cost in the Online Appendix of this paper, which demonstrates that despite the presence of trade cost, the "reverse selection" phenomenon can emerge whenever countries are sufficiently different.

[3]Demidova and Rodriguez-Clare (2009, 2013) analyze unilateral trade liberalization in the form of a marginal reduction in trade cost.

[4]For ease of exposition, we will present our analysis of a symmetric equilibrium here, and relegate to the Appendix a sketch of proof for the non-existence of any asymmetric equilibrium. In such a symmetric equilibrium of trade, the autarky outcome can be obtained by setting $m = 1$. In the rest of the paper when a symmetric trade outcome is explicitly compared with an autarky outcome, we use superscript m to indicate trade and superscript c to indicate autarky. When discussing a symmetric trade equilibrium for any general $m \geq 1$, of which autarky is a special case ($m = 1$), we do not attach superscripts.

Since a consumer sources his consumption from the global market, the ith variety *consumed* becomes the $j = \frac{i}{m}$th variety *produced* in each country:

$$x(i) = q(\frac{i}{m}).$$

Rewrite

$$U = \int_0^\infty u(x(i))di = \int_0^\infty u(q(\frac{i}{m}))di = m\int_0^\infty u(q(j))dj,$$

where the last equality obtains after re-indexing $i = mj$. Consumer optimization is therefore

$$\max_{q(j)_{j\in[0,\infty)}} m\int_0^\infty u(q(j))dj) \text{ s.t. } \int_0^\infty mp(j)q(j)dj \leq l + \pi,$$

where l is each consumer's endowment of labor, and π is the profit of all firms in a country shared equally among all consumers within the country. Since countries are symmetric here, wage rate is normalized to one for all countries.

We do not impose the free entry assumption, which would have implied zero profit after netting the entry cost for all firms. Instead, firms are able to earn profits in our model and we assume that domestic firms are owned domestically so that firm profits are distributed to domestic consumers in an equal fashion.

Let λ be the usual Lagrangian multiplier, which in our model measures the utility bought by an additional unit of income, or the shadow price of income. Consumer maximization then becomes

$$\max_{q(j)_{j\in[0,\infty)},\lambda} m\int_0^\infty u(q(j))dj + \lambda\left[l + \pi - \int_0^\infty mp(j)q(j)dj\right].$$

For any variety j such that $q(j) \geq 0$, consumer optimization yields:

$$u'(q(j)) = \lambda p(j).$$

The demand for variety j is therefore

$$p(j) = \frac{1}{\lambda}u'(q(j)). \tag{1}$$

Given such a demand, firm j's optimization equates the marginal cost with the (per-capita) marginal revenue:

$$\frac{1}{\lambda}r(q(j)) = c(mq(j), j),$$

where

$$r(q) \equiv u'(q) + qu''(q).$$

Note that the marginal revenue, $\frac{1}{\lambda}r(q(j))$, consists of two complementary parts: $r(q)$, which measures marginal revenue in utility term, and $\frac{1}{\lambda}$, which measures how much income is needed for an increase in utility, with λ being the aforementioned shadow price of income. Thanks to our additively separable preferences, the first component depends only on a firm's output. The second term, on the other hand, is independent of any individual firm's production. As we will see, such a decomposition will play a crucial role in our analysis.[5] For now, it suffices to note that, other things being equal, when income increases, the shadow price of income λ will decrease given that marginal utility is diminishing, and hence $\frac{1}{\lambda}$ will increase too.

We impose two properties with regards to the preferences. The first says that u has finite marginal utility.

Property (1*) $u' < \infty$.

This property sets our preferences apart from, for example, the constant elasticity of substitution (CES) preferences, and enables us to do away with fixed cost to greatly simplify our analysis.[6] In the case of CES preferences, marginal utility is unbounded: $u'(0) = \infty$, making it impossible to analyze firms' shutdown decision without invoking fixed costs of production.

The second property says that the variety-specific part of the marginal revenue is decreasing in consumption quantity whenever it is positive:[7]

Property (2*) $2u''(q) + qu'''(q) < 0, \forall q$ such that $u'(q) + qu''(q) > 0$.

Denote by κ the production threshold, i.e., the index of the last variety produced in each country. The consumption threshold (i.e., the index of the last variety consumed) is $m\kappa$. By definition, $q(\kappa) = x(m\kappa) \equiv 0$. At κ, we thus have

$$\frac{1}{\lambda}r(0) = c(0, \kappa).$$

A symmetric trade equilibrium is then characterized by three unknowns, λ, κ, and $q(j)$ for all

[5] Zhelobodko et al. (2012), Mrázová and Neary (2017), and Dhingra and Morrow (2019) also focus on a similar decomposition for their analyses.

[6] Melitz and Ottaviano (2008) and Arkolakis, Costinot, Donaldson, and Rodraguez-Clare (2019) adopted preferences with the same property.

[7] Properties (1*) and (2*) are not demanding restrictions on preferences. They are met, for example, by CARA and quadratic preferences.

$j \in [0, \kappa]$, which can be solved from the following three equilibrium conditions:

$$\frac{1}{\lambda} r(q(j)) = c(mq(j), j), \quad \text{for } j \in [0, \kappa] \tag{2}$$

$$\frac{1}{\lambda} u'(0) = c(0, \kappa), \tag{3}$$

$$\int_0^{\kappa} \int_0^{mq(j)} c(q, j) dq dj = l. \tag{4}$$

Equation (2) equates the marginal revenue and the marginal cost. Equation (3) is the zero profit condition for the threshold firm. It applies equation (2) with zero output to that firm. The fact that the marginal revenue can be decomposed into $\frac{1}{\lambda}$ and a variety-dependent component implies that trade shrinks or expands the threshold through its impact on $\frac{1}{\lambda}$. Equation (4) is the labor market clearing condition. It also corresponds to the following binding budget constraint, at both the individual and the national levels:

$$l = \int_0^{\kappa} mp(j)q(j)dj - \pi = \int_0^{\kappa} mp(j)q(j)dj - \int_0^{\kappa} \left[mp(j)q(j) - \int_0^{mq(j)} c(q, j)dq \right] dj.$$

Trade is trivially balanced since all countries are symmetric.

The equilibrium can be easily solved in three steps. For any given shadow price of labor, the intensive margin (i.e., $q(j)$) is determined by equation (2), and the extensive margin (i.e., κ) by (3). These two will generate a total labor demand as a function of labor's shadow price, the equilibrium value of which will then be solved from the labor market clearing condition (4).

With Properties (1*) and (2*) imposed, we can reproduce a key insight of Melitz (2003) and prove it using a simple logic:

Proposition 1 *When countries are symmetric, there exists a unique equilibrium in autarky and trade respectively. Compared to autarky, in the trade equilibrium,*

a) less productive firms cease operation ($\kappa^m < \kappa^c$);

b) more firms are crowded out when more countries participate in trade (κ^m decreases in m).

Proof. By the concavity of $u(.)$, both $q(j)$ and κ are strictly decreasing in λ. Hence, the left-hand side of (4) is a strictly decreasing function of λ. It is evident that the labor demand approaches infinity when $\lambda \to 0$, and zero when $\lambda \to \infty$, hence the existence and uniqueness of the trade equilibrium.

Fixing λ and hence κ, $mq(j)$ increases in m (otherwise, q must decrease, and by Property (2*), the LHS of (2) must increase while the RHS of (2) decreases). Therefore, the demand for labor

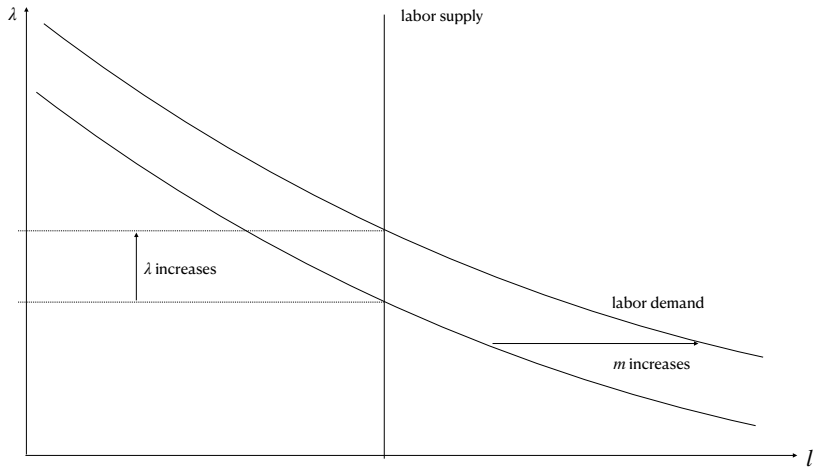

Figure 1: Trade raises the shadow price of income (λ)

must increase in m. In order for the labor market to clear, λ must increase, meaning that κ must decrease in m. *Q.E.D.*

We have reproduced the key insight obtained in Melitz (2003) that trade (among symmetric countries) crowds out less productive firms. The intuition boils down to a simple supply-demand relationship as shown in Figure 1, where the labor demand reflects the left hand side of equation (4), and the labor supply corresponds to the right hand side. As shown in the figure, the labor demand is decreasing in λ. Intuitively, a higher shadow price of income reduces the willingness to pay (as shown in equation (1)), which in turn reduces both the per-capita sales quantity q and the cutoff κ for any given m (as shown in equation (2)), thus the demand for labor. Meanwhile, fixing λ, an increase in m evidently increases the labor demand, i.e., the left hand side of equation (4). Therefore, λ must increase as a result of trade. This reduces the marginal revenue, making it unprofitable for the marginal firms to continue to operate (see equation (3)). Hence the crowd-out.

In reproducing the classical crowding-out result, we have stripped away elements such as trade cost, fixed cost of production, and free-entry, which all prove to be non-essential to this insight in the symmetric setting. We do so in order to easily extend our analysis to the case of asymmetric trade and to make clear connections between these two scenarios.[8]

[8]Different from many existing papers which are concerned with whether a firm that sells at home also exports, our analysis focuses on whether exporting firms also sell at home and whether these are less productive firms crowded in by trade. For such a research aim, the fixed cost of production is not essential to our model. However, in order to connect our study with the existing literature, we will entertain an example with such costs in the Appendix of the paper to demonstrate the robustness of our insights.

3 Trade among Asymmetric Countries

We now turn to the case of asymmetric trade. First, some explanation of the notations. In a trade setting, subscript indicates the origin country and superscript indicates the destination country. For example, q_y^x represents (per capita) sales quantity from origin y in destination x. A country-wide variable is denoted by a superscript without any subscript, for example λ^x is the shadow price of income in country x. In an autarky setting, we will use superscript c to represent autarky and a subscript to index the corresponding country. For example, λ_x^c is country x's shadow price of income under autarky, which is different from λ^x in trade. Second, we assume that each firm can practice price discrimination across different countries and hence price its variety differently. Together with Property (2*), this assumption implies that every firm faces a marginal revenue that is decreasing in its sale in each market.

With asymmetric trade, wage rates are no longer equalized across countries. Let w^x be country x's wage rate measured in some common international denominator, and N^x be the country's population size. Consumers in destination country x chooses $q_y^x(j)$ for product j from origin country y to maximize

$$\sum_{y=1}^{m} \int_0^{\infty} u(q_y^x(j))dj,$$

s.t.

$$w^x l + \int_0^{\infty} \frac{\pi^x(j)}{N^x} dj = \sum_{y=1}^{m} \int_0^{\infty} p_y^x(j) q_y^x(j) dj.$$

Once again, let λ^x be the Lagrangian multiplier for country x. Consumer maximization becomes

$$\max_{\lambda^x, q_y^x(j)_{j\in[0,\infty);h=1,2,\ldots m}} \sum_{y=1}^{m} \int_0^{\infty} u(q_y^x(j))dj + \lambda^x (w^x l + \int_0^{\infty} \frac{\pi^x(j)}{N^x} dj - \sum_{h=1}^{m} \int_0^{\infty} p_h^x(j) q_h^x(j) dj)$$

The optimization yields: $u'(q_y^x(j)) = \lambda^x p_y^x(j)$, and the resulting demand from destination country x for variety j of origin country y is:

$$p_y^x(j) = \frac{1}{\lambda^x} u'(q_y^x(j)). \tag{5}$$

Given such a demand, firm j of origin country y will choose its optimal price in destination x such that

$$\frac{1}{\lambda^x} r(q_y^x(j)) = w^y c_y(\sum_{h=1}^{m} N^h q_y^h(j), j)$$

whenever $q_y^x(j) > 0$, with $c_y(q, j)$ being the marginal cost in origin country y.

An asymmetric trade equilibrium is characterized by $\left(\lambda^x, \{\kappa_y^x\}_{y=1,2,\ldots m}, \{q_y^x(j) : j \in \kappa_y^x\}_{y=1,2,\ldots m}\right)_{x=1,2,\ldots m}$,

which in turn are solved from the following equilibrium conditions for $x = 1, 2, ...m$:

$$\frac{1}{\lambda^x} r(q_y^x(j)) = w^y c_y(\sum_{h=1}^{m} N^h q_y^h(j), j), \quad \forall j \in [0, \kappa_y^x], \forall y \in \{1, 2, ...m\} \tag{6}$$

$$\frac{1}{\lambda^x} u'(0) = w^y c_y(\sum_{h=1}^{m} N^h q_y^h(\kappa_y^x), \kappa_y^x), \quad \forall y \in \{1, 2, ...m\} \tag{7}$$

$$N^x l = \int_0^{\infty} \int_0^{\sum_{h=1}^{m} N^h q_x^h(j)} c_x(q, j) dq dj, \tag{8}$$

$$\sum_{h=1}^{m} \frac{N^h}{\lambda^h} \int_0^{\infty} \int_0^{q_x^h(j)} r(q) dq dj = \sum_{h=1}^{m} \frac{N^x}{\lambda^x} \int_0^{\infty} \int_0^{q_h^x(j)} r(q) dq dj. \tag{9}$$

Equations (6) through (8) correspond to equations (2) through (4) in the symmetric case. Note that in equation (8), $q_x^h(j) = 0$ when $j > \kappa_x^h$. Equation (9) says that the total expense a country incurs must equal the total revenue it earns. Since the revenue earned from the domestic market always equals the expense made domestically, equation (9) is essentially the trade balance condition.

3.1 Firms Specialized in Export are Less Productive

To characterize the equilibrium under asymmetric trade, we first introduce the key notion of our paper: *the (per capita) purchasing power*. In any destination country x, the demand and hence the marginal revenue for any product can be decomposed into two components as discussed earlier. The first component is related to $u(.)$. Since preferences are the same across all destination countries, in terms of (per-capita) marginal revenue function, different destination countries can differ only in the second component: λ^x, the shadow price of income.

In Figure 2, the downward sloping solid curves represent the (per-capita) demand functions per equation (5), the downward sloping dotted curves represent the corresponding (per-capita) marginal revenue functions following the left-hand-side of equation (6), and the horizontal lines indicate marginal cost of production which is assumed to be constant in output in this figure. The figure depicts how firms in country y sell in the home market y and a foreign destination x, where the foreign demand is assumed to be higher than the home demand: $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$. As a result, the export cutoff (κ_y^x) is larger than the domestic cutoff (κ_y^y). Firm κ_y^y makes no profit in the domestic market, but makes a positive profit (the size of the shaded area) when exporting to the foreign destination x.

In sum, the inverse of the shadow price of income, $\frac{1}{\lambda}$, serves the role of shifting the (per-capita) demand (i.e., willingness to pay) up and down, reflecting how much income a consumer in a country is willing to spend for a marginal increase in utility. We shall from hereon refer to $\frac{1}{\lambda}$, the

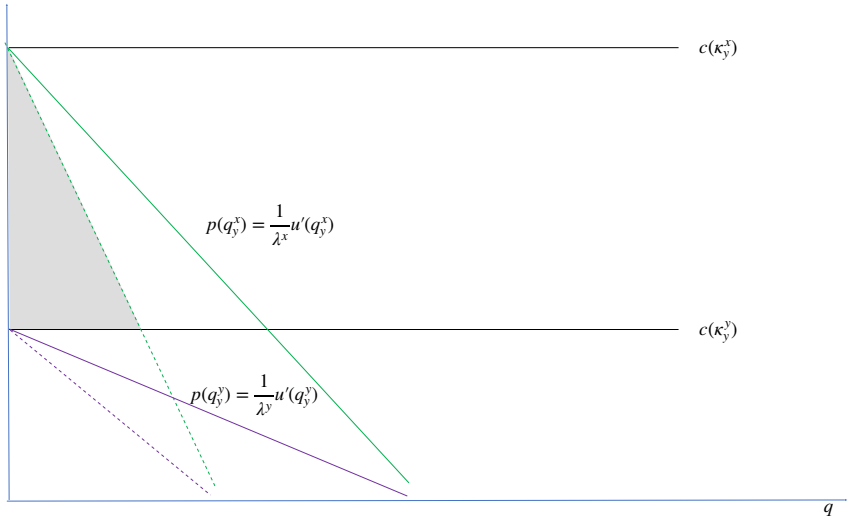

Figure 2: $\frac{1}{\lambda}$ shifts the demand and marginal revenue curves

inverse of the shadow price of income of a country, as the country's *(per capita) purchasing power*. If a country has a larger purchasing power than another, the former is richer than the latter in the sense that a consumer of the former consumes more products from each origin than a consumer of the latter. This purchasing power will be determined endogenously in equilibrium, as will be shown later, but here it suffices to draw the following observations.

Consider an origin country y. Equation (7) indicates that the country faces m thresholds: κ_y^x for $x = 1, 2, ...m$, in a trade equilibrium. As long as there exists a foreign destination country x such that $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$, then we have per equation (7)

$$w^y c_y(\sum_{h=1}^{m} N^h q_y^h(\kappa_y^x), \kappa_y^x) > w^y c_y(\sum_{h=1}^{m} N^h q_y^h(\kappa_y^x), \kappa_y^y).$$

This implies that

$$\kappa_y^x > \kappa_y^y,$$

meaning all firms with productivity $j \in (\kappa_y^y, \kappa_y^x]$ will export to the foreign market x but not sell in the domestic market. Apparently these firms are less productive than those firms that do sell in the domestic market, as $j > \kappa_y^y$. In other words,

Proposition 2 *Suppose that there exists a foreign destination country which has a higher purchasing power*

than the origin country in a trade equilibrium. Then there will be firms in the origin country that specialize in exports and these firms will be less productive than firms that sell both at home and abroad.

The proof of Proposition 2 is straightforward and is thus omitted. The proposition stands in contrast to the existing literature, according to which exporters are more productive than others, and less productive firms will either be crowded out or sell at home only. Proposition 2 says instead that, when a country trades with its richer trade partners, there will be firms with the export share being 100 percent and these firms will not be the more productive ones.[9]

Why are less productive firms crowded out in operation when countries are symmetric, but may become specialized exporters when countries are asymmetric? To understand the reason, let's first examine what happens to the productivity threshold on the domestic market in this asymmetric trade setting.

Proposition 3 *In every country, trade crowds out less productive firms on the domestic market:* $\kappa_x^x < \kappa_x^c$ *for all* x.

A comparison of equation (6) with equation (2) helps explain the intuition behind Proposition 3 . The labor demand in country x is determined by $w^x\lambda^x$ in the case of trade and λ_x^c under autarky. When countries are symmetric, w^x can be normalized to one, and hence $w^x\lambda^x$ can be reduced to λ^x. Note that an increase in $w^x\lambda^x$ reduces the willingness to pay measured in labor (as shown in equation (6)). Therefore, as trade increases the demand for labor, $w^x\lambda^x$, reflecting the marginal utility brought by an additional labor income or the shadow price of income, must increase: $w^x\lambda^x > \lambda_x^c$, regardless of whether trade is symmetric or not. As a result, the marginal revenue on the domestic market must fall, per equation (7), causing less productive firms to be crowded out *from the domestic market*.

In symmetric trade, *in every market* the marginal revenue for any firm is the same before trade and drops equally after trade. Since marginal revenue are identical across all markets for any given firm, less productive firms being crowded out from the domestic market must imply that they are crowed out from all markets, hence ceasing their operation.

By introducing trade cost, the existing literature adds a bit complexity to the above picture: less productive firms will be crowded out from foreign markets before they are crowded out from the domestic market, thus resulting in the zero export share for these firms. This brings the question

[9]Some may argue that even though these export-oriented firms may have more costly operations, the additional costs are often dispensed to deliver a higher quality that meets the standard and regulations in the foreign markets. We can easily incorporate the quality differential in our model, and Proposition 2 will remain valid. When a firm's marginal cost and marginal utility both depend on the firm's variety, a firm's productivity ranking will be sorted by the ratio between marginal cost and marginal utility rather than marginal cost alone. Then a higher foreign demand will still result in less productive firms specializing in export.

of whether Proposition 2 comes from our assumption of zero cost to trade in foreign markets. The answer is negative. Notice that for any firm to sell in a particular market, the amount of per-capita gross profit (before netting out the cost of trade) the firm can reap, as illustrated by the shaded area in Figure 2, is increasing in the purchasing power of that market. Therefore, given the cost of trade in foreign markets, should the purchasing power of some foreign destination be sufficiently higher than that of the domestic market, less productive firms can overcome the cost of trade (whether variable or fixed) and specialize in export, while more productive firms will serve both at home and abroad.

An immediate implication of Proposition 3 is that, should trade ever crowd in firms (as we will analyze next), those new firms must be less productive and specialize in export.

Corollary 1 *If crowd-in takes place as a result of trade, then the crowded-in firms must specialize in export and must be less productive than existing firms under autarky.*

The proof of this corollary is omitted. To see the logic behind this corollary, note that crowd-in takes place in origin country y only when there exists some destination country x such that $\kappa_y^x > \kappa_y^c$. Since $\kappa_y^y < \kappa_y^c$ per Proposition 3, we can conclude that

$$\kappa_y^x > \kappa_y^y,$$

meaning that all firms that are crowded in, i.e., $j \in (\kappa_y^c, \kappa_y^x]$, must be less productive than firms operating under autarky, and these firms must specialize in export: $(\kappa_y^c, \kappa_y^x] \subset (\kappa_y^y, \kappa_y^x]$, which is the set of all firms specializing in export. In other words, trade never brings back firms to serve the domestic market (Proposition 3); hence if trade ever brings some firms back to operation, these firms must emerge to serve foreign markets and must be less productive than those who are able to survive on the domestic market.

Another implication of Proposition 3 is that trade must crowd out less productive firms in the richest country, i.e., the country with the largest purchasing power:

Corollary 2 *Trade crowds out less productive firms in countries with the largest (per capita) purchasing power.*

The reason is straightforward: for a country x with the largest (per capita) purchasing power, the domestic market must offers the highest marginal revenue for its own firms; hence $\kappa_x^x = \max\{\kappa_x^y\}_{y=1,2,...m}$. It then follows from Proposition 3 that crowd-out must take place in the richest countries.

Before we move on to endogenize the purchasing power differences, let's draw a few more implications from the equilibrium conditions (6) and (7). First, for any origin country, there is a monotonicity relationship between the destination country's purchasing power and the productivity threshold, κ, that the origin country can achieve in that destination:

Corollary 3 *For any origin country, a destination country with a higher purchasing power has a lower productivity cut-off. That is, for any h, $\kappa_h^x > \kappa_h^y$ for any $j \leq \kappa_h^y$ if and only if $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$.*

Corollary 3 in turn implies that, if a firm sells in country x, then it must also sell in all countries that have a higher purchasing power than x. Conversely, if a firm does not sell in country x, then it does not sell in any countries that have a lower purchasing power than x.

Conditions (6) and (7) collectively reveal another monotonicity relationship between the productivity threshold, κ (i.e. the extensive margin) and the output level for a given variety, q (i.e., the intensive margin) attained in any destination by a given origin country:

Corollary 4 *Extensive margins and intensive margins are positively correlated. That is, for any origin country h, if $\kappa_h^x > \kappa_h^y$, then $q_h^x(j) > \kappa_h^y(j)$ for all $j \leq \kappa_h^y$.*

Assume constant marginal cost in output and cross-country differences in technology in the form of $c_y(j) = \beta_y c(j)$ with $\beta_y > 0$ for all y, j. We can then derive form (6) and (7) the following observations:

Corollary 5 *Suppose that the marginal cost is constant in output and that $c_y(j) = \beta_y c(j)$ with $\beta_y > 0$ for all y, j. Then*

1) for any two countries, x, and y, the following relationship holds:

$$c(\kappa_y^x)c(\kappa_x^y) = c(\kappa_x^x)c(\kappa_y^y); \tag{10}$$

2) for any origin-destination pair, a larger extensive margin implies a larger intensive margin. That is, if $\kappa_h^x > \kappa_k^y$, then $q_h^x(j) > \kappa_k^y(j)$ for all $j \leq \kappa_k^y$ for any x, y, h, k.

Our equilibrium characterization thus far merely describes how, in a trade equilibrium, the pattern of (per capita) purchasing power and the pattern of how firms are selected into trade are *correlated*. The existing literature often interprets the pattern of (per capita) purchasing power as an exogenous determinant of the pattern of how firms are selected into trade. In fact, without trade, it does not even make sense to compare one country with another in purchasing power. In the next three subsections, we will show how purchasing power disparity may arise endogenously from fundamentals such as population size and overall productivity.

3.2 Crowd-in due to Differences in Population Size

We begin with differences in population size. To focus on population, we assume that all countries face the same distribution of production cost, i.e., for any two countries x and y, $c_x(q, j) = c_y(q, j)$ for any given q and j. Accordingly, we will drop the country index for the marginal cost and write it as $c(q, j)$ for all countries.

Our first observation is that, other things being equal, larger countries will be poorer (having a smaller per-capita purchasing power) but more competitive (having a lower wage rate) than smaller countries.

Proposition 4 *In a trade equilibrium, a larger country has a lower wage rate and a smaller (per-capita) purchasing power:* $w^x < w^y$ *and* $\frac{1}{\lambda^x} < \frac{1}{\lambda^y}$ *if* $N^x > N^y$.

Compared to a smaller country, a larger country supplies more labor to meet the same global demand. Therefore, the larger country must be more competitive (i.e., $wc(q, j)$ must be lower) in order to clear its labor market. This in turn requires its labor to be cheaper (hence a lower wage rate) given the same $c(q, j)$ across all countries.

Because a larger country is more competitive in the trade equilibrium, its total earnings must be greater than that of a smaller country (per equation (6)). Trade balance then requires that the larger country must also spend more than its smaller counterpart in total (i.e., aggregating per-capita spending over its all population). However, doubling a nation's labor force cannot double its earnings. This is because marginal cost is increasing (weakly increasing in output and strictly increasing in variety) while marginal revenue is decreasing, and all countries face the same set of marginal revenue functions and have the same technology. In accordance, the per capita spending in the larger country must be smaller than that of a smaller country, which can be attained only when the larger country has a smaller (per-capita) purchasing power, given that both types of countries face the same set of marginal cost functions (per equation (6)).

The trade balance and labor market clearing conditions both play key roles in endogenously determining the wage rates and purchasing power.Our model assumes that domestic firms are owned by domestic people on an equal basis, so that the earnings by domestic firms determine the earning of domestic people, which equals their spending when trade is balanced. This allows us to build a clear and intuitive linkage among wage rate, per capita purchasing power, and country size. A country's supply of labor, or country size, determines wage rate, which in turn shapes the marginal cost of the country, which further determines the earning of that country and, by trade balance, the spending of the country, and hence per capita purchasing power in equilibrium.

Since λ, the shadow price of income, is decreasing in income, we can conclude from Proposition

4 that a smaller country has a larger income, that is, wage income and per capita profits combined. The wage income is apparently higher in a smaller country according to Proposition 4. Proposition 4 also implies that, with their ability to access cheaper labor at home, firms in a larger origin country must earn more profits than firms in a smaller origin country. That is, $\sum_{h=1}^{m} \pi_x^h(j) \geq \sum_{h=1}^{m} \pi_y^h(j)$ for any given j if $N^x > N^y$. Despite that, thanks to its smaller size in population, on a per capita basis, profit income in a smaller country can be higher than that in a larger country.

Assuming constant marginal cost in output, Proposition 4 implies that smaller countries, being both richer (a higher per-capita purchasing power) and less competitive than their larger counterparts, will be a larger export destination for larger countries than the other way around: $\kappa_y^x < \kappa_x^y$. Combining this observation with Proposition 4 and Corollaries 3 and 4, we arrive at the following corollary which characterizes the order of all the productivity thresholds and output levels of all corresponding varieties between two countries of different sizes:

Corollary 6 *Suppose that the marginal cost is constant in output. Then for any two countries x and y, $\kappa_y^x < \min\{\kappa_y^y, \kappa_x^x\} < \max\{\kappa_y^y, \kappa_x^x\} < \kappa_x^y$ and $q_y^x(j) < \min\{q_y^y(j), q_x^x(j)\} < \max\{q_y^y(j), q_x^x(j)\} < q_x^y(j)$ for all $j \in [0, \kappa_y^x]$ if $N^x > N^y$.*

Tracing country size as one source behind purchasing power difference across countries, Proposition 4 also lays out a foundation for us to explore whether the two countervailing forces of trade, as highlighted by Propositions 2 and 3, will in balance generate the crowd-in of less productive firms when the size disparity among countries becomes sufficiently large. Proposition 3 suggests that trade will reduce marginal revenue on the domestic market (as compared to autarky), causing less productive firms to quit from that market: i.e., $\kappa_x^x < \kappa_x^c$. On the other hand, Proposition 2 suggests that, when there exists some foreign country y with a larger purchasing power in trade, i.e., $\frac{1}{\lambda^y} > \frac{1}{\lambda^x}$, trade also allows less productive firms to operate and export to the richer foreign market: $\lambda_x^y > \lambda_x^x$. What remains to be determined is whether the country size disparity can propel κ_x^y beyond κ_x^c to cause less productive firms to emerge as a result of trade—the crowd-in. The answer is yes.

Proposition 5 *Suppose that marginal cost is constant in output. Suppose further that there are m_L large country with population size $N > 1$, and m_S small countries with population size normalized to one. Fixing (m_L, m_S), there exists $\hat{N} < \infty$ such that trade crowds in less productive firms in large countries if $N > \hat{N}$.*

Proposition 5 considers two groups of countries: large and small. It states that fixing the number of countries within each group, if the size of a large country is sufficiently larger than that of a small country, less productive firms will be crowded in by trade in large countries.

To understand the intuition, note that according to equations (6) and (7), per capita spending of any destination country x at any origin country y is linked to the productivity threshold of the origin country serving the destination country: κ_y^x. Suppose crowd-in never takes place at all, $\kappa_y^x \geq \kappa_y^c$ for all x. Since κ_y^c, the productivity threshold under autarky, is independent of characteristics of the country's trading partners, it implies that, should trade crowd out less productive firms in an origin country, per capita spending of its trading partner on this origin country must be capped from above, independent of these trading partners' populations. In addition, by assuming constant marginal cost, we can show that $\kappa_x^c = \kappa_y^c$; the productivity thresholds under autarky are independent of country size. Therefore, should crowd-in never take place, per capita spending of a small country on a large country's products must be capped from above, independent of the relative size between the two.

However, because a variety from a small country serves (if at all) every consumer in a large country, per capita earning by a small country from a large country must correspond to the relative size of the latter. Should the per capita spending of a small country on a large country be capped from above, the per capita earning of the small country from the large country must grow to violate trade balance when the large country becomes sufficiently large. To put it simply, when the population size of large countries is sufficiently large, the need for small countries to serve every consumer in large countries will raise small countries' per capita purchasing power so much that less productive firms in large countries must be crowded in to balance the trade.

Proposition 5 is further illustrated by the following numerical example. There are two countries in this example, L and S, with population sizes $N^L \geq N^S$ and per capita labor endowment equal to $l = 1$ in both countries. The two countries share the same preferences $u(q) = q - \frac{q^2}{4}$ and the same production technology $c(j) = 1 + j$. We solve for the trade equilibrium and different productivity thresholds in each country. Figure 3 depicts these thresholds in correspondence to the relative population size, $\frac{N^L}{N^S}$.

As shown in the figure, the productivity thresholds in the two countries are identical under autarky regardless of their size difference: $\kappa_L^c = \kappa_S^c$. This is due to the fact that the production technology features constant marginal cost in output. When the two countries open for trade, trade crowds out less productive firms in both countries if their population sizes are the same ($\frac{N^L}{N^S} = 1$). When the large country's population size grows relative to the small country, i.e., when $\frac{N^L}{N^S}$ increases, the productivity thresholds in the small country, κ_S^L and κ_S^S, decrease, suggesting that crowd-out takes place in the small country. This pattern corresponds to Proposition 4 and Corollary 2: the small country is the richest between the two, and hence trade must crowd out less productive firms in the small country. The decrease in both κ_S^L and κ_S^S suggest that the small country becomes

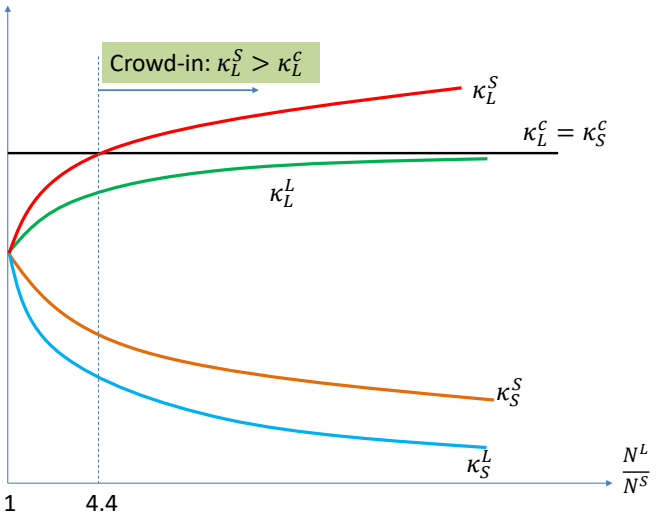

Figure 3: Crowd-in: asymmetry in population size

increasingly specialized in its most productive varieties. This phenomenon contrasts interestingly with that in the large country, which becomes increasingly diversified in its production portfolio, as both κ_L^L and κ_L^S increase with its own size relative to its smaller trading partner. Despite the diversification, crowd-out always takes place in the domestic market of the large country, as predicted by Proposition 3. Meanwhile, consistent with Proposition 2, the example shows that as long as the two countries differ in size, $\kappa_L^S > \kappa_L^L$, indicating that less productive firms become specialized in export. Lastly and most importantly, the example confirms what Proposition 5 predicts; that is, when the large country becomes sufficiently large relative to the small country (more than 4.4 in this example), crowd-in will take place in the large country: $\kappa_L^S > \kappa_L^c$.

3.3 No Crowd-in with Differences in Productivity Alone

Besides difference in population size, countries can also differ in their level of technological development, the quality of their institutions, infrastructure, and so on, that can impact the productivity of individual firms across board within each country. These differences can lead to, in the context of our model, firms' marginal costs of production in one country to be consistently lower (or higher) than in another. To capture this reality, we assume that among all countries in the world, there exist at least two countries, x and y, such that the marginal cost of any jth firm in the two countries can be rewritten as

$$c_x(q, j) = \beta_x c(q, j), \quad c_y(q, j) = \beta_y c(q, j),$$

where β_i reflects the overall productivity of country $i \in \{x, y\}$. We say that country x dominates country y in overall productivity if $\beta_x < \beta_y$, that the overall productivity is in the rise when β becomes smaller, and we assume that β is bounded below by $\beta^* > 0$.[10]

To focus on the impact of productivity difference, we assume that all countries have the same population size. As we shall see later, this assumption is not innocuous.

Our first observation says that, other things being equal, a more productive country will be richer (having a larger per-capita purchasing power) and more competitive (after taking into account of the wage differential):

Proposition 6 *Suppose that, among all countries in the world, there exists two countries x and y such that country x dominates country y in productivity: $\beta_x < \beta_y$. In a trade equilibrium, the more productive country x is more competitive and enjoys a larger (per-capita) purchasing power: $w^x\beta_x < w^y\beta_y$ and $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$.*

Like Proposition 4, the intuition of Proposition 6 can be reasoned combining the labor market clearing condition and the trade balance condition, and once again the assumption that domestic firms are owned by domestic people on an equal basis plays an important role in our reasoning. First, a more productive country must be richer (larger $\frac{1}{\lambda}$). Should a more productive country be (weakly) poorer, it will not spend as much as a country that is less productive. Per trade balance, this means that the more productive country must earn not as much revenue as its less productive counterpart. Since both countries face the same demand in the world, this in turn implies that the world must buy a smaller quantity from the more productive country. Given that countries have the same population size, the labor market cannot be cleared in both countries. Second, given that a more productive country is richer, it must spend more. Per trade balance, it must earn more as well. Hence, it must be more competitive in the sense that $w^x\beta_x < w^y\beta_y$.

Although it does not specify whether a more productive country will have a higher wage rate, Proposition 6 states that a more productive country will indeed be more competitive, in terms of $w\beta$ combined. Recall that when countries differ in size but not in productivity, a richer country (one with a higher per capita purchasing power due to its smaller size) is less competitive (see Proposition 4). In contrast, Proposition 6 shows that when countries differ in productivity but not in size, a richer country due to its superior productivity is more, rather than less, competitive.

Being more competitive, firms in a more productive country make more profits: $\sum_{h=1}^{m} \pi_x^h(j) \geq \sum_{h=1}^{m} \pi_y^h(j)$ for any given j if $\beta_x < \beta_y$. Even though Proposition 6 does not pin down the wage rate

[10] We only assume a subset of countries are rankable by productivity dominance because, unlike population size, not all countries can be ranked in terms of productivity dominance in reality: some countries may be more productive in some varieties but less so in some other varieties, for example. However, for the purpose of our analysis, it suffices that there exists some countries among which one dominates another in overall productivity.

and hence wage income, given that λ remains the shadow price of income, as every individual faces a fixed budget in this subsection, a higher per capita purchasing power implies a higher income. Therefore, rather unsurprisingly, a more productive country is also richer.

Assuming constant marginal cost in output, Proposition 6 further implies that a more productive country, being richer, is a larger export destination as compared to a less productive country for any origin country in the world, just as in the case of a smaller country. However, being richer and more competitive at the same time no longer guarantees that a more productive country, as compared to its less productive counterpart, is a bigger export destination between the two, as Corollary 6 would predict. Instead, being richer and more competitive at the same time implies that a more productive country must enjoy an edge in serving the home market. Further combining Proposition 6 with Corollaries 4 and 5, we are then able to characterize the order of all the productivity thresholds and output levels of all corresponding varieties between two countries with one dominating another in productivity:

Corollary 7 *Suppose that, among all countries in the world, there exists two countries x and y such that country x dominates country y in productivity: $\beta_x < \beta_y$. Suppose in addition that the marginal cost is constant in output. Then $\kappa_x^x > \max\{\kappa_x^y, \kappa_y^x\} > \min\{\kappa_x^y, \kappa_y^x\} > \kappa_y^y$ and $q_x^x(j) > \max\{q_x^y(j), q_y^x(j)\} > \min\{q_x^y(j), q_y^x(j)\} > q_y^y(j)$ for all $j \in [0, \kappa_y^y]$.*

It is interesting to compare Corollary 7 with Corollary 6. When the purchasing power disparity is driven by differences in population size, richer countries and poorer countries diverge more in their penetration into the markets of each other than in their penetration of their own home markets. Among their home markets and the export market of their opponents, poorer (larger) countries demonstrate the strongest capacity in accessing the market of richer countries while richer (smaller) countries the weakest capacity in entering the market of its counterparts ($\kappa_x^y > \max\{\kappa_x^x, \kappa_y^y, \kappa_y^x\}$ and $\kappa_y^x < \min\{\kappa_x^x, \kappa_y^y, \kappa_y^x\}$ when $N^x > N^y$). In contrast, when the purchasing power disparity is driven by differences in the overall productivity, richer countries and poorer countries diverge more in their penetration into their own home markets than in their penetration of the markets of each other. Among their home market and the export market of their opponents, richer (advanced) countries demonstrate the strongest capacity in serving their home markets while poorer (backward) countries the weakest ability in filling the market of their own ($\kappa_x^x > \max\{\kappa_x^y, \kappa_y^x, \kappa_y^y\}$ and $\kappa_y^y < \min\{\kappa_x^x, \kappa_x^y, \kappa_y^x\}$ when $\beta^x < \beta^y$).

The next question is, can productivity gap among countries become so large that the two forces as highlighted by Propositions 2 and 3 in balance will cause less productive firms to be crowded in? The answer is no.

Proposition 7 *Suppose that all countries have the same size in population. Then regardless of whether there exist some countries that can be ranked in productivity dominance, trade never crowds in less productive firms.*

Why does crowd-in not take place when countries differ only in productivity? The answer is simple. In terms of demand, all destination countries differ only in the demand shifter, i.e., the per-capita purchasing power. Should there be one destination country y that crowds in less productive firms in an origin country x, the marginal revenue of every x-firm earned from market y in trade must be greater than that from market x itself under autarky. Given that all countries have the same size of population, this must create more demand for labor in trade than in autarky. However, given that the labor force in the tradable sector remains the same both before and after trade, the labor market clearing condition cannot hold both before and after trade.

From the reasoning above, we can see why the assumption that all countries have the same population size is not innocuous, as is highlighted in Proposition 7. Because population size is the same across countries, when per-capita consumption of an origin country's products in a foreign destination goes beyond the origin country's autarky consumption level, so goes the total consumption and, hence, the global demand for labor in that origin country.

Meanwhile, the above reasoning also highlights the role of fixed labor supply in the tradable sector. Because the labor supply in the tradable sector is fixed in the origin country, the labor market cannot be cleared both before and after trade, thus Proposition 7. If the tradable sector can somehow draw in more labor resources from somewhere in the economy, productivity difference may be able to generate the reverse selection phenomenon even when countries have the same population. To explore this possibility, we extend our model by incorporating a non-tradable sector and allow labor to be endogenously allocated between the tradable and non-tradable sectors. The analysis is presented next.

3.4 Crowd-in due to Productivity Differences and Labor Reallocation between Tradables and Non-Tradables

This extension introduces a non-tradable good that must be foregone when labor is devoted to the tradable sector. We rewrite an individual's preference as

$$U(q(j)_{j\in(0,\infty)}) - l \equiv f(\int_0^\infty u(q(j))dj) - l,$$

where the first term is the utility derived from the tradable sector, and the second term represents the loss of utility in the consumption of the non-tradable good when labor l is devoted to the

tradable sector. One may think of the non-tradable good as leisure and $-l$ as the loss of leisure time, but the interpretation can be broader, as will be explained shortly.

The function $f(.)$ is assumed to be strictly monotone, continuously differentiable, and strictly concave. We also assume $f'(.) < \infty$ to entertain the possibility of an inactive tradable sector, for a purpose that should become clear soon. To focus on the cross-country difference in overall productivity, we assume that countries do not differ in population size.

Consumers in destination country x choose l^x and $q_y^x(j)$ for product j from origin country y to maximize

$$f(\sum_{y=1}^{m} \int_0^{\infty} u(q_y^x(j))dj) - l^x,$$

subject to the following budget constraint:

$$\sum_{y=1}^{m} \int_0^{\infty} p_y^x(j) q_y^x(j) dj = w^x l^x + \int_0^{\infty} \frac{\pi^x(j)}{N^x} dj. \tag{11}$$

A few remarks are in order for the model setting, in particular the budget constraint (11). First, in the budget constraint, income consists of profits from domestic firms and wage earnings from the tradable sector, and the income is spent entirely on tradable goods. This is certainly sensible when the non-tradable good is interpreted as leisure, but the interpretation can be broader. For a general non-tradable good, a consumer's budget constraint should include her wage income from the non-tradable sector as well as her spending on the non-tradable good. If we assume a constant returns to scale technology for the sector, the product's price must equal to the equilibrium wage rate. Then the consumer's earnings from and spending on the sector will cancel out, and (11) remains valid.

Second, individuals are homogeneous within a country in our model. As the non-tradable good is never traded across border, in any equilibrium all individuals within a country must consume the same amount of the non-tradable good and must devote the same amount of labor into the non-tradable sector. This gives our non-tradable sector a self-sufficiency feature: a unit of labor moved away from the sector means a corresponding loss of consumption of the non-tradable good for every individual in any equilibrium. Therefore, our preferences can be viewed as a short-handed representation that also fits for a broad interpretation of non-tradable good.

Third, to simplify the analysis, we have introduced a quasi-linear preference such that the marginal opportunity cost of labor supplied to the tradable sector is constant. Despite such quasi-linearity, however, there will be income effect for the tradable sector and hence purchasing power will matter. The income effect arises not because all income, tradable and non-tradable combined,

is spent on tradable goods. Rather it is because of the self-sufficiency feature, which makes the spending on tradable goods to be constrained by earnings from the tradable sector.

We can then rewrite the consumer optimization problem as

$$\max_{q_y^x(j)_{j\in[0,\infty),y=1,2,...m}} f(\sum_{y=1}^{m}\int_0^{\infty} u(q_y^x(j))dj) - \sum_{h=1}^{m}\int_0^{\infty}\frac{p_y^x(j)}{w^x}q_y^x(j)dj + \int_0^{\infty}\frac{\pi^x(j)}{w^x N^x}dj$$

to generate the following demand function:

$$f'(\sum_{y=1}^{m}\int_0^{\infty} u(q_y^x(i))di)u'(q_y^x(j)) = \frac{p_y^x(j)}{w^x}. \tag{12}$$

Despite the apparent difference between the endogenous labor setting here and the fixed labor setting in the main model, the analysis turns out to be analogous. In particular, define λ such that

$$\frac{1}{\lambda^x} \equiv w^x f'(\sum_{y=1}^{m}\int_0^{\infty} u(q_y^x(j))dj). \tag{13}$$

We can then reduce equation (12) back to equation (5):

$$p_y^x(j) = \frac{1}{\lambda^x}u'(q_y^x(j)).$$

It is immediately clear that $\frac{1}{\lambda^x}$, which will be determined endogenously in equilibrium, is once again the shifting factor that moves up or down the willingness to pay. As before, it corresponds to how much an additional payment is worthy for an extra utility. Therefore, $\frac{1}{\lambda^x}$ remains the (per capita) purchasing power of a country.

Given such a measure, the equilibrium conditions for this extension are (6), (7), (9), except for the labor market clearing condition, (8), which will now be replaced by the identity equation (13). We implicitly assume that in equilibrium, some labor is always devoted to the non-tradable sector so that the labor market clearing constraint is never binding for the tradable sector.

When countries are symmetric, the identity equation (13) can be simplified as

$$\frac{1}{\lambda} \equiv f'(\int_0^{\kappa} mu(q(j))dj).$$

Given $f(.)$ being concave, it is easy to verify that the right hand side of the identity is decreasing in $\frac{1}{\lambda}$, whereas the left hand side is increasing in $\frac{1}{\lambda}$. As the right hand side is decreasing in m, we can then conclude that, as shown in Melitz and Ottaviano (2008), trade must reduce the purchasing

power, thus crowding out less productive firms from the tradable sector, had the sector being non-empty under autarky.

Melitz and Ottaviano (2008) attribute this crowd-out in symmetric trade to what they refer to as "tougher competition" on the product market. Our analysis above reveals that the tougher product competition in Melitz and Ottaviano (2008) and the increased factor market competition in Melitz (2003) essentially work through the same channel of purchasing power, $\frac{1}{\lambda}$.

Now suppose that countries are asymmetric. As long as the tradable sector is not empty during trade, Propositions 2, 3, and Corollaries 1 through 5 continue to hold. It is also easy to show that Proposition 6 continues to hold here: if country x dominates country y in productivity, then the former has a higher per-capita spending power and is more competitive than the later:[11]

$$w^x\beta_x < w^y\beta_y \text{ and } \frac{1}{\lambda^x} > \frac{1}{\lambda^y} \text{ if } \beta_x < \beta_y. \tag{14}$$

According to (14), firms in a more productive country will continue to make more profits than their counterparts in a less productive country as in Section 3.3. Therefore, profit income is higher in a more productive country. Furthermore, having a higher per capita purchasing power implies that a more productive country x must have a larger $\sum_{h=1}^{m}\int_0^\infty u(q_h^x(j))dj$. Therefore according to the definition of the per capita purchasing power $\frac{1}{\lambda}$ in this extension, i.e., (13), we can conclude that the wage rate in a more productive country must be higher:[12]

Corollary 8 *Suppose that, among all countries in the world, there exists two countries x and y such that country x dominates country y in productivity: $\beta_x < \beta_y$. Then $w^x > w^y$.*

Furthermore, given the optimal choice of labor devoted to the tradable sector, we can show that a higher per capita purchasing power $\frac{1}{\lambda}$ for a more productive country must correspond to

[11]The proof is slightly different from that of Proposition 6, and is presented in the Appendix. Chaney (2008) also shows that a more productive country will have a higher wage rate and a higher income. Different from our model, Chaney (2008) assumes that countries differ in their productivity in an *outside opportunity*, i.e., a homogeneous tradable good which is produced by all countries in equilibrium. This allows the wage rates to be pinned down exogenously. Chaney (2008) then pins down different incomes in different countries by assuming further that each country shares the *global* profits according to its wage rate. Chaney's (2008) approach has the merit of being much simpler and more direct than ours. However, it also bears the implication that a country with a higher marginal cost in producing heterogeneous tradable goods shares more profit from heterogeneous tradable goods production, and that the heterogeneous tradable goods sector in a more productive country will be smaller as higher productivity is associated with the *outside opportunity*. In contrast, in our model, both wage and income are endogenously determined, with the implication that a country with a higher marginal cost in producing the tradable goods earns less profit from the tradable goods production, and the tradable goods sector in a more productive country will indeed be larger.

[12]Despite higher wage rate and higher per capita firm profits, to figure out whether, in equilibrium, an individual in a more productive country has a higher income (from the tradable sector), we need information about the amount of labor allocated to the tradable sector. We can no longer conclude directly from a larger $\frac{1}{\lambda}$ that the income is higher at an individual level, for the λ defined in the current extension does not correspond to a Lagragian multiplier for a fixed budget.

a larger budget and hence a larger income for its consumers in equilibrium.[13] Finally, given that Proposition 6 continues to hold, it should also be evident that Corollary 7 must remain true too.

With a foundation for the purchasing power disparity thus established, we can once again examine whether the two forces, as highlighted by Propositions 2 and 3, will in balance cause less productive firms to be crowded in when productivity gap among countries becomes sufficiently large. To expedite the analysis, we restrict our attention to a world made of $m_x > 0$ number of country x and $m_y > 0$ number of country y, with x dominating y in productivity: $\beta_x < \beta_y$. In addition, we assume for simplicity that the marginal cost of production is constant in output.

We begin with the critical value for β_y above which the tradable sector of country y will be inactive under autarky (i.e., $\kappa_y^c = 0$):

$$\beta(0) \equiv \frac{f'(0)r(0)}{c(0)}.$$

Apparently $\kappa_y^c = 0$ if and only if $\beta_y \geq \beta(0)$. It should also be evident that, per Proposition 1, if trade is open among countries y only, there will be no trade among these countries of the same type at $\beta_y = \beta(0)$: $\kappa_y^y = 0$, and that, if trade is open among both types of countries, there will be no trade among countries y either according to Proposition 3: $\kappa_y^y = 0$.

Define $\beta_1(\beta_x)$ as the threshold of β_y for a given $\beta_x \in [\beta^*, \beta(0))$ such that, fixing β_x, trade takes place between country x and country y with zero volume:

$$\beta_1(\beta_x) = \min\{\beta_y : \kappa_y^x(\beta_x, \beta_y) = 0\}.$$

We refer to $\beta_1(\beta_x)$ as the trade frontier for the less productive country y as its trade sector will operate if and only if $\beta_y \leq \beta_1(\beta_x)$.

It is straightforward to show that $\beta_1(\beta_x) > \beta(0)$.[14] That is, when a country is at the verge of having an active tradable sector under autarky, trading opportunities with more productive countries whose tradable sectors are live even under autarky will pull its tradable sector into action, but serve only the export market of those more productive countries. In fact, trade with more productive countries will diminish its purchasing power with respect to products of its peers and its own (i.e., as trade diminishes $\frac{\frac{1}{\lambda^y}}{w^y} = f'(.)$ in country y), making the home market as well as

[13] To see this, note that, in equilibrium, all firms from every origin country anticipate $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$ when $\beta_x < \beta_y$ per (14). Therefore, each of them sets a higher price for its product in the more productive country x than in the less productive country y. If the equilibrium income in a more productive country, after individuals choose their labor allocation optimally, is equal to or lower than that in a less productive country, there must exist at least an origin country h such that $q_h^x(j) \leq q_h^y(j)$ for all $j \leq \kappa_h^x$ and $\kappa_h^x < \kappa_h^y$ (per Corollary 4), which contradicts Corollary 5, given that $\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$. Therefore, we can conclude, without any surprise, that a more productive country must have a higher income (from the tradable sector) than a less productive country.

[14] Suppose not, we will have $f'(m_x \int_0^\infty u(q(i))di)u'(0) \leq \beta_x c(0)$, implying that $\kappa_x^x = 0$ at β_x, contradicting the fact that $\beta_x < \beta(0)$.

the markets of its peers now strictly unprofitable.

Finally, define $\beta_2(\beta_x)$ as the threshold of β_y for a given $\beta_x \in [\beta^*, \beta(0))$, beyond which the crowd-in takes place in country y:

$$\beta_2(\beta_x) = \max\{\beta_y : \kappa_y^x(\beta_x, \beta_y)) = \kappa_y^c(\beta_y)\}.$$

Proposition 8 *Consider a world made of $m_x > 0$ number of country x and $m_y > 0$ number of country y, with country x dominating country y in productivity: $\beta_x < \beta_y$, and the marginal cost of production is constant in output. Then fixing (m_x, m_y), there exist $\beta_1(\beta_x) > \beta(0)$ and $\beta_2(\beta_x) \in (\beta_x, \beta(0))$ for any $\beta_x \in [\beta_0, \beta(0))$, such that*

a) $\kappa_y^x > \kappa_y^c > \kappa_y^y \geq 0$ *if* $\beta_y \in (\beta_2(\beta_x), \beta(0))$,

b) $\kappa_y^x > \kappa_y^c = \kappa_y^y = 0$ *if* $\beta_y \in [\beta(0), \beta_1(\beta_x))$, *and*

c) $\kappa_y^x = \kappa_y^c = \kappa_y^y = 0$ *if* $\beta_y > \beta_1(\beta_x)$.

Proposition 8 describes how the tradable sector of the less productive country y responds to the change in overall productivity in either itself or its more productive counterpart, country x. The proof of Proposition 8 is straightforward and is omitted. We use Figure 4 to illustrate this proposition. In Figure 4, the tradable sector of country y is inactive under autarky when $\beta_y \geq \beta(0)$, and likewise for country x when $\beta_x \geq \beta(0)$. The 45^o dotted line AE represents the symmetric case where $\beta_x = \beta_y$. Along this symmetric case, trade crowds out less productive firms from the tradable sector if the tradable sector is active under autarky (i.e., if $\beta_x = \beta_y < \beta(0)$), and so for (β_x, β_y) near this 45^o dotted line. Along the bold vertical line AC, $\beta_y = \beta(0)$. The tradable sector in country y is inactive under autarky when β_y falls to the right of this vertical line (i.e., if $\beta_y > \beta(0)$), and becomes active when β_y falls to the left (i.e., if $\beta_y < \beta(0)$). Two shaded areas straddle around this vertical line. The left one is bounded by AB representing $\beta_2(\beta_x)$, and the right by AD representing $\beta_1(\beta_x)$.

Reverse selection takes place in these two shaded segments. In the segment immediately to the right of $\beta(0)$, i.e., in the shaded ACD area, trade brings the otherwise dead tradable sector of country y into action, with all firms in the sector selling only to country x: $\kappa_y^x > \kappa_y^y = 0$. There is no bilateral trade among country x and its peers. In the segment immediately to the left of $\beta(0)$, i.e., in the shaded ABC area, the tradable sector is active under autarky: $\kappa_y^c > 0$, but trade crowds in less productive firms into the sector: $\kappa_y^x > \kappa_y^c$. Although there is bilateral trade among country y and its peers, all these crowded-in firms in country y sell to country x only.

Proposition 8 gives a concrete example of zero bilateral trade in correspondence to Helpman, Melitz, and Rubinstein (2008) and Baldwin and Forslid (2010). In the shaded ACD segment immediately to the right of $\beta(0)$, all firms in the tradable sector of country y sell to more productive

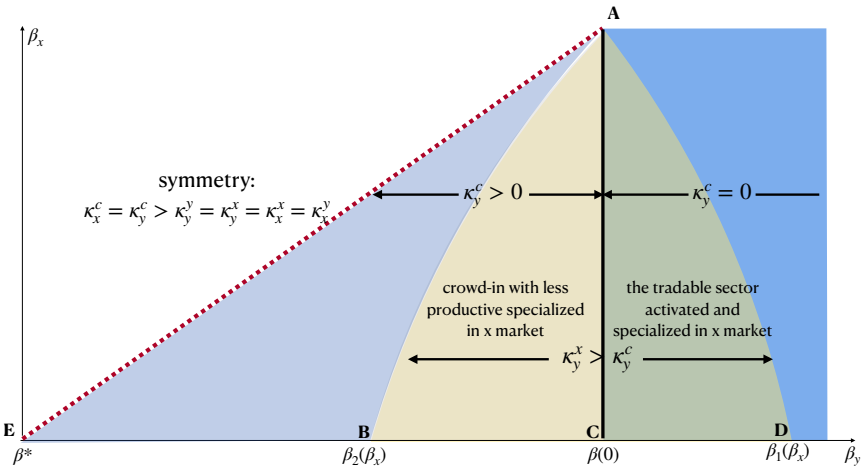

Figure 4: The pattern of trade impact in less productive country y

countries. Accordingly, there is no bilateral trade between country y and its symmetric peers. It is in fact straightforward to generalize the observation here: whenever a country's tradable sector becomes completely specialized in export, there will be no bilateral trade between this country and its symmetric peers. Given that firms in its tradable sector do not serve the domestic market, they will not serve other markets with the same purchasing power. In that case, the country and its symmetric peers will trade only with a group of richer countries.

Proposition 8 can be further illustrated using the following numerical example. In the example, there are two countries x and y with population size one. Preferences in both countries are characterized by $u(q) = q - \frac{q^2}{4}$ and $f(\int_0^\infty u(q(j))dj) = \ln(\int_0^\infty u(q(j))dj + 1)$, with the marginal cost being $c(j) = \frac{1}{10} + \frac{j}{10}$. We fix β_x and calculate how various thresholds evolve in response to changes in β_y. The results are presented in Figure 5.

As shown in the figure, under autarky, the size of the tradable sector in country x is independent of β_y, whereas that in country y (i.e., k_y^c) decreases in β_y. Trade always crowd out less productive firms from the domestic market in both countries: $\kappa_x^x < \kappa_x^c$ and $\kappa_y^y < \kappa_y^c$, just as Proposition 3 predicts. Trade also always crowds out less productive firms in the more productive country x (and hence the richest country in this two-country world): $\kappa_x^c > \kappa_x^x > \kappa_x^y$, as Corollary 2 predicts.

Figure 5 also shows that the tradable sector of country y disappears under autarky ($\kappa_y^c = 0$) when β_y is very large. However, when β_y is at the verge of making the tradable sector inactive in country y under autarky, trade helps revive the sector: $\kappa_y^x > 0$, as $\beta_1(\beta_x) > \beta(0)$. In other words,

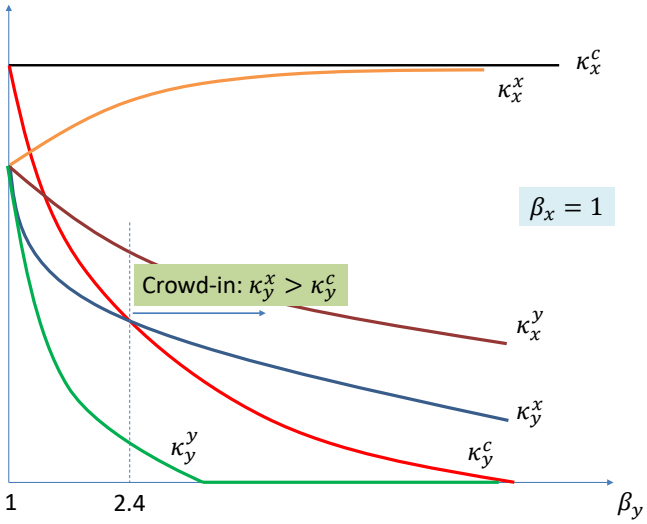

Figure 5: The pattern of trade impact in the more productive country x and the less productive country y

the entire tradable sector is crowded in as a result of trade. As country y becomes even more productive, the tradable sector becomes active even under autarky, $\kappa_y^c > 0$, and yet trade will induce the tradable sector to be entirely devoted to export with $\kappa_y^y = 0$; and with $\kappa_y^x > \kappa_y^c$, marginal firms operating under trade would obviously have remained inactive under autarky. Raising the country's purchasing power and making its products more competitive, further progress in productivity in country y eventually leads to an increase in extensive margin both at home and abroad (κ_y^y and κ_y^x increase as β_y decreases), along with the corresponding intensive margins. However, as the extensive margin in export, κ_y^x, remains larger than κ_y^c, the extensive margin under autarky, trade in comparison to autarky continues to crowd in less productive firms in country y. That is, not until β_y falls below the critical value of 2.4 (conditional on $\beta_x = 1$), where κ_y^c begins to exceed κ_y^x: trade crowds out less productive firms in country y as it does in country x. Accompanying this entire process of development in country y is the increased engagement in trade by country x, from almost self-sufficient to an equal partner of y when $\beta_y = \beta_x = 1$.

We close this section with three interesting observations for a world that consists of two types of countries only: more productive and less productive, with labor resources flowing freely between the tradable and the non-tradable sector in all countries.

Corollary 9 *Consider a world made of $m_x > 0$ number of country x and $m_y > 0$ number of country y, with country x dominating country y in productivity: $\beta_x < \beta_y$. Then $\kappa_x^y < \kappa_y^x$.*

Corollary 9 says that the more productive countries will be the larger export destination than the less productive countries are. Note that the more productive countries are more competitive

than the less productive countries, but also have a higher demand. Corollary 9 suggests that the demand factor dominates the competitiveness factor. Corollary 9 can be easily derived from the trade balance condition (9), which in this case of two types of countries with equal size and constant marginal cost in output is reduced to:

$$\frac{1}{\lambda^y}\int_0^{\kappa_x^y}\int_0^{q_x^y(j)} r(q)dqdj = \frac{1}{\lambda^x}\int_0^{\kappa_y^x}\int_0^{q_y^x(j)} r(q)dqdj.$$

As the more productive countries have a higher purchasing power ($\frac{1}{\lambda^x} > \frac{1}{\lambda^y}$), trade balance (along with Corollary 4) requires that $\kappa_x^y < \kappa_y^x$, mirroring what our numerical example above has illustrated.

Together with Corollary 7, Corollary 9 completes the characterization of how different productivity thresholds (i.e., extensive margins) and the output level for each varieties in different markets (i.e., intensive margins) are ranked against each other in a world with countries of two levels of overall productivity, $\beta_x < \beta_y$:

$$\begin{aligned} &\kappa_x^x > \kappa_y^x > \kappa_x^y > \kappa_y^y \qquad \text{and} \\ &q_x^x(j) > q_y^x(j) > q_x^y(j) > q_y^y(j) \quad \text{for all } j \in [0, \kappa_y^y]. \end{aligned}$$

Substituting (13) into condition (6), the following conditions must hold in equilibrium at $\beta_1(\beta_x)$:

$$\begin{aligned} w^x f'(m_x \int_0^{\kappa_x^x} u(q_x^x(i))di)u'(0) &= w^y \beta_1(\beta_x)c(0), \\ w^y f'(0)u'(0) &= w^x \beta_x c(0). \end{aligned}$$

Putting these two conditions together, we have:

$$\frac{f'(0)(u'(0))^2}{(c(0))^2} = \frac{\beta_x \beta_1(\beta_x)}{f'(m_x \int_0^{\kappa_x^x} u(q_x^x(i))di)}, \tag{15}$$

which implies that the trade frontier $\beta_1(\beta_x)$ is decreasing in m_x:

Corollary 10 *Consider a world made of $m_x > 0$ number of country x and $m_y > 0$ number of country y, with country x dominating country y in productivity: $\beta_x < \beta_y$, and the marginal cost of production is constant in output. Fixing β_x, $\beta_1(\beta_x)$ is constant in m_y and is decreasing in m_x.*

Corollary 10 has two complementary interpretations. First, when a larger number of rich countries engage in trade (m_x increases while holding m_y constant), it will become more difficult

for poor countries to be pulled into trade: they will have to become more productive ($\beta_1(\beta_x)$ decreases) to seize the trade opportunities.

Second, since the trade frontier $\beta_1(\beta_x)$ is independent of m_y, as more poor countries develop and transform into rich countries (m_x increases as a result of reduction in m_y), it will become harder for the remaining poor countries to make use of the trade opportunities. Should trade be a venue for poor countries to transform themselves and converge with rich countries, some poor countries taking up the path will make it increasingly more difficult for the rest of the poor countries to follow the same path. In this regard, opening to trade *early* pays off.

It is not immediately evident whether rich countries becoming richer can make it easier to engage poor countries in trade. This is because, without further restrictions on preferences, a strict monotonicity cannot be assured for the trade frontier $\beta_1(\beta_x)$. However, there does exist a substituting relationship in β_x and β_y that enables the less productive country y to expand in its home market beyond a given level:

Corollary 11 *Consider a world made of $m_x > 0$ number of country x and $m_y > 0$ number of country y, with country x dominating country y in productivity: $\beta_x < \beta_y$, and the marginal cost of production is constant in output. There exists an increasing function $\beta_y(\beta_x)$ such that, given $\hat{\kappa}_y^y \in [0, \kappa_x^c(\beta_x))$, the following equation holds in equilibrium when $\beta_y = \beta_y(\beta_x)$:*

$$\frac{1}{\lambda^y} r(0) = w^y \beta_y c(\hat{\kappa}_y^y);$$

Likewise, there exists an increasing function $\beta_y^*(\beta_x)$ such that, given $\hat{\kappa}_x^x \in [0, \kappa_x^c(\beta_x))$, the following equation holds in equilibrium when $\beta_y = \beta_y^*(\beta_x)$:

$$\frac{1}{\lambda^x} r(0) = w^x \beta_x c(\hat{\kappa}_x^x).$$

Corollary 11 says that countries of a particular group (in terms of overall productivity) will have a bigger presence at their home markets, either as a result of improved overall productivity in themselves or due to a deteriorating overall productivity in the other group of countries. Corollary 11 is a reflection of what has been taking place between, for example, China and the U.S., which has caused a recent rift in their trade relationship. As China closes in on U.S. in terms of overall productivity, the Chinese firms will have a bigger presence at home whereas the American firms will begin to lose grounds in their domestic market.

3.5 The Driving Force of Crowd-in

At the core of reverse selection are conditions under which less productive firms will be crowded into business as a result of their country moving from autarky to trade. To better understand crowd-in, it is useful to compare Propositions 5, 7, and 8.

Both Propositions 7 and 8 are about productivity differences with identical population size, yet crowd-in takes place in Proposition 8 but not in Proposition 7. This is because labor resources flow between a country's tradable and non-tradable sectors only in Proposition 8. If labor supply is fixed, the labor market clearing condition would preclude the possibility for a destination country's purchasing power in trade to go beyond that of an origin country under autarky. If, by contrast, labor supply is plastic, trade can induce labor in a less productive country to flow to its tradable sector, bringing the sector back into life in some cases and crowding in less productive firms in some other cases.

In Proposition 5, crowd-in takes place (as in Proposition 8) even though labor supply is fixed (as in Proposition 7). This is because Proposition 5 allows countries to differ in population size so that the labor market has an additional channel to be cleared: demand differential relates to per-capita demand for labor, which is multiplied by the population size to generate the total demand for labor.

The analysis reveals the importance of the fluidity of the domestic factor market in determining trade impacts. When a developing country opens up to trade, some less productive firms will begin focusing on export. However, whether there will emerge new firms specializing in export depends on how freely resources move in the economy, which in turn depends on government regulatory policies as well as the time it takes for resources to reallocate. Hence, Proposition 7 may be capturing firms' responses to trade in the short-term, whereas Proposition 8 a long-term response. For developing countries, therefore, domestic factor market liberalization can magnify the impact of trade liberalization.

A careful reader may point out that Propositions 7 pertains to a setting where labor supply is fixed in all countries, whereas Proposition 8 refers to the case where labor supply is endogenous in all countries. To apply the policy reform interpretation, it is necessary to extend Propositions 7 into a setting where, except for the country where the policy reform is concerned, resources are able to flow freely between the tradable and non-tradable sectors in the rest of the world. It turns out that, thanks to the parallel between the fixed and endogenous labor supply settings for the per capita purchasing power, $\frac{1}{\lambda}$, Propositions 7 can indeed be easily extended as follows:

Corollary 12 *Suppose that labor supply to the tradable sector in country y is fixed, while labor is endoge-*

nously allocated between the tradable and non-tradable sectors in the rest of the world and that the size of population of country y is no larger than that of the rest of the world. Suppose in addition that country y and the rest of the world can be ranked in productivity dominance. Then trade always crowds out less productive firms in country y.

To further understand crowd-in, it is useful to compare our model with Zhelobodko et al. (2012) and Mrázová and Neary (2017), who also show that trade can crowd in less productive firms, yet their countries are symmetric, whereas we have shown that crowd-in takes place only for an asymmetric setting. Trade generates two countervailing effects: it expands the number of markets where a firm can operate, but lowers the marginal revenue from any given market. The net effect on crowd-in depends on country symmetry and fixed cost of production.

Suppose that countries are symmetric. If there is no fixed cost of production as in our model, "marginal" firms (i.e., our less productive firms) are characterized by "marginal" (i.e., zero) output in every identical market. Opportunities to expand globally under trade mean little to such firms; consequently the marginal revenue reduction effect always dominates the market expansion effect, meaning that less productive firms must be crowded out. In Zhelobodko et al. (2012), Mrázová and Neary (2017), and Melitz (2003), by comparison, firms operate with a fixed cost of production. Therefore, output is never zero even for marginal firms under autarky. Since these firms can enjoy a substantial market expansion effect, there is no guarantee that this effect is dominated. Indeed, Zhelobodko et al. (2012) and Mrázová and Neary (2017) show that the market expansion effect can dominate (and therefore crowd-in may take place) for certain non-CES preferences, whereas Melitz (2003) demonstrate that it will not be possible if preferences are CES.

When countries are asymmetric, "marginal" firms at the domestic market will be able to sell a substantial positive amount of output in markets with a larger per capita purchasing power. As a result, the marginal revenue reduction effect may no longer dominate the market expansion effect for less productive firms. Hence the possibility of crowd-in.

Such a possibility, however, is ruled out by Melitz and Ottaviano (2008), who also have zero fixed cost and yet still find that country asymmetry will not lead to crowd-in. In their model, a transition from autarky to free trade would be equivalent to an increase in market size. In other words, it makes no difference whether a country triples its size, or trade with a country twice of its size, or trade with two countries with the same size. This comes from a specific free-entry assumption adopted by many papers in the literature including Melitz and Ottaviano (2008) and Melitz (2003). The assumption implies that firms of every productivity type is multiplied by an equal proportion, which adjusts endogenously to trade. Country asymmetry does not change the symmetry in firm decisions and consumer decisions. Trade can be easily balanced without any

need for firms and consumers to alter their decisions in response to changes in country size, as the size changes can be simply offset by the changes in the number of copies or masses.[15] In such a setting, country asymmetry translates into an asymmetry in firm copies but not in individual firm behavior. Per capita purchasing power remain the same across countries of different sizes, and the impact of free trade remains the same as that when these countries are symmetric: less productive firms will be crowded out.

We do not adopt such an assumption. Without the extra dimension of copies to make adjustments, when countries change in their size in our model, trade balance can be achieved only through firms and consumers adjusting their behaviors, and so does the per capita purchasing power. In doing so, we are able to derive predictions about firm responses' to trade that have clear empirical correspondence in the literature, instead of copies or masses that do not.

While the possibility of crowd-in may arise with neither fixed cost in production nor (fixed) cost in trade, some readers may be curious whether reverse selection will continue to emerge when such costs exist. We have argued earlier that, provided that these costs are modest, a sufficiently large purchasing power disparity will continue to result in reverse selection. Due to space limit, we relegate to Appendix to elaborate on the robustness of our argument in the presence of trade cost and fixed cost of production.

4 Discussions and Concluding Remarks

Our analysis offers a rich set of implications. In addition to those already mentioned, let's highlight some more before wrapping up the paper.

First, the impact of trade on firm performances is not uniform as the selection hypothesis would suggest; instead it is country-specific. In a trade network with large disparity, a country with a smaller purchasing power may witness reverse selection. This implies a potential sample selection problem for studies aiming to measure trade impacts. In particular, if a study focuses on countries with high purchasing power (say because these countries are more advanced in their institutions and hence have better organized firm and trade data), one may be led to conclude that trade always crowds out less productive firms. Likewise, if a study looks at firms' responses

[15] Specifically, fixing $N_x + N_y$ constant as $2N$, with $x \neq y \in \{1, 2, ...m\}$ as two countries with distinctive sizes, so that the total size of the two remains constant $2N$ as when they are symmetric in size. Let α be the equilibrium number of copies in each country when the countries are symmetric in size. The adjustment in copies in response to the reshuffling in size between the two countries, while keeping the total sizes of the two countries fixed at $2N$ and all decisions by firms and consumers the same as when countries are symmetric in size, is simply solved by $\alpha_x + \alpha_y = 2\alpha$ and $\alpha_x = \frac{N_x}{N_y}\alpha_y$. The details can be obtained from the authors upon request. Note that, when $N_x \to 0$, country y approaches autarky with $\alpha_y \to 2\alpha$ and $\alpha_x \to 0$. Thus, the equivalence between two economies of total size $2N$ in trade and an economy of size $2N$ in autarky.

to trade with a subset of trade partners, conclusions may be very different depending on which trade partners it focuses on. Firms serving destinations with similar or lower purchasing power than that at home will always exhibit crowd-out, whereas firms serving destinations with higher purchasing power may display crowd-in.

Second, a country's purchasing power is shaped by its overall productivity, which is determined by the country's technological advancement, human capital accumulation, the quality of its institutions, its infrastructure, and so forth, and can therefore change over time. This in turn implies that the impact of trade on a country's firm performances is time specific. For example, the impact of China retreating from global trade today on Chinese firm performances will be drastically different from the impact of China joining the world economy three decades ago. As Figure 4 may suggest, if China today has its overall productivity β_y advanced to a level below the threshold $\beta_1(\beta_x)$, then retreating from global trade is likely to crowd in less productive firms, as $\kappa_y^c > \kappa_y^x$.[16] In contrast, if China had an overall productivity β_y above the threshold three decades ago, then it is her opening to the world that allowed less productive firms to be crowded in as well, as $\kappa_y^c < \kappa_y^x$.

Third, the trade impact on a country's firm performances depends not only on the country's specifics, such as its opening to trade, macro environment, and the underlying economic development; it also depends on the world that the country opens to trade with. Imagine that the overall productivity of country y remains stagnant, while its trade partner x undergoes a change in β_x. This may crowd in less productive firms in country y initially and then crowd them out later, or vice versa. Either way, it creates the possibility for the true impact of trade on country y's firm performance to be wrongly measured, should factors shaping its trading partners' overall productivity be ignored.

Fourth, our model assumes that a country's overall productivity is independent from trade. In reality, trade can shape various determinants behind a country's overall productivity as well as that of its trade partners. Opening to trade can serve as a political commitment to battle against embedded interests that impede domestic reforms, as exemplified in China's effort to join WTO (Brandt, Van Biesebroeck, Wang, and Zhang 2017). Of course, trade is not necessarily a blessing all the time; it could cause dislocation of resources, spelling troubles for a nation, as illustrated by the trade disputes between U.S. and Japan in the past and China nowadays. The true impact of trade on a country's firm performances may therefore be decomposed into three effects: the direct effect on the country's firm performances, the indirect effect via how trade affects the country's overall

[16] In the context of our analysis in Section 3.4, less productive firms can be either crowded in or out when a country withdraws itself from global trade, depending on its overall productivity falls below or above the threshold. In contrast, when countries are symmetric, retreating from the global market always crowds in less productive firms.

productivity, and the indirect effect via how trade affects overall productivity of the country's trading partners. It is exactly because trade may not always crowd out less productive firms that the two indirect effects of trade are relevant in particular. Likewise, trade policy can also alter a country's overall productivity by exerting an impact on the country's macro-economic environment. Trade sanction against Iran, for example, can be a double hit according to our analysis: not only does it mean that the window to trade gets closed, but it can also result in a deterioration in Iran's aggregate economic conditions, i.e., an increase in β_y in Figure 4. If the pre-sanction β_y was already above the threshold, this could then aggravate the crowding out of less productive firms as the door to trade is slammed shut. If the pre-sanction β_y was marginally below the threshold, a deterioration in Iran's macro economic conditions could induce less productive firms to be crowded out in the midst of the sanction ($\kappa_y^c < \kappa_y^x$), whereas without such deterioration, the sanction would have triggered less productive firms to be crowded into Iran's domestic market ($\kappa^c > \kappa_y^x$).

Fifth, recognizing purchasing power disparity is not only important for our understanding of trade impacts on a particular country, it also highlights the relevance of the country's trading partners. To see this, we need to perform comparative statics on m, the number of countries of a particular characteristics (size of population or overall productivity) that participate in trade.[17]

Take Section 3.4 for instance. We can show that there exists $\{m_x, m_y\}$ such that κ_x^y and κ_y^x decrease when m_y or m_x increases.[18] This means that when measuring trade impacts on a country's firm performances over a period of time, subsequent trade liberalization by other countries can affect the measurement. In particular, suppose that a country y opens to trade in year t and the true (and hence immediate) impact is crowd-in. However, if the empirical measure covers all the way to $t+s$, and during these extra s years, some other countries also open to trade, then these developments may push down the productivity threshold of exporters in the country under study, causing the trade impact to appear to be crowd-out.

This potential non-monotonicity in trade impact caused by subsequent trade liberalization by *other countries* is not present when countries are assumed to be symmetric, however. Recall that when countries are assumed to be symmetric, having more countries participating in trade will further crowd out less productive firms in countries that are already in the game (Proposition 1). In other words, trade crowds out less productive firms in a country regardless of whether it is the country itself or it is the country's trade partners that expand their trade exposure.

[17] As we have highlighted earlier, with the assumption of costless trade, the comparative statics of how trade impacts the firm performance in a country remains the same whether the country unilaterally opens to trade with its trading partners or the country and its trading partners simultaneously move from autarky to trade.

[18] To see the existence of such $\{m_x, m_y\}$, it suffices to note that κ_x^y and κ_y^x decrease in both m_y and m_x if $\beta_y = \beta_x$ (per Proposition 1), and that when either m_y or m_x becomes sufficiently large, both κ_x^y and κ_y^x approach to zero.

Sixth, that trade crowds out less productive firms, thus allowing resources to be reallocated to more productive firms, is the underlying force behind the new gains from trade with firm heterogeneity. Therefore, it is natural to ask whether reverse selection, i.e., crowd-in of less productive firms to specialize in export, means a loss in welfare. The answer is no.

To see this, consider the case presented in Proposition 8. When trade with a more productive country x crowds in less productive firms in a less productive country y, say $\kappa_y^x = \kappa_y^c + \epsilon$, where $\epsilon > 0$ is a small positive value. Then the amount of labor devoted to producing for export in country y is about the same as country y would do for itself under autarky.[19] As we show in Corollary 9, $\kappa_x^y > \kappa_y^x$; in other words, by engaging the same amount of labor for export as the country would do for itself under autarky, each variety in country y will enjoy more consumption. Of course, when this is the case, country y also allocates labor in the tradable sector for itself: $\kappa_y^y > 0$ (see Proposition 8 and Figure 4). However, by revealed preferences, each consumer must be better off by spending extra on these domestically produced varieties, and firms producing these domestic varieties must make profits as well. Hence, when $\kappa_y^x = \kappa_y^c + \epsilon$, country y, which witnesses crowd-in under trade, must be strictly better off than under autarky.

On the other hand, it would be wrong to conclude that the new gains from trade with heterogeneous firms has nothing to do with the selection hypothesis. According to our analysis, the selection hypothesis never fails everywhere in a network of trade partners. If reverse selection takes place somewhere in the network, the selection hypothesis must hold somewhere else in the network. In this sense, the new gains from trade with heterogeneous firms hinges on the fact that trade crowds out less productive firms somewhere in a trade network.

Seventh, in analyzing the endogenous determination of the purchasing power, our paper has relied on the assumption that profits generated by domestic firms accrue to domestic people. This assumption, albeit a seemingly natural one, plays an important role in shaping a country's (per capita) purchasing power and its wage rate in our model. The assumption can be relaxed, for instance, to allow a redistribution of firm ownership cross-border in exchange for, say, foreign technology investment. Due to space limit, we will leave it to a future paper to entertain how such a rearrangement of firm ownership may shape the trade impact (such as shifting the trade frontier $\beta_1(\beta_x)$ and resulting in twin deficits for advanced nations).

[19] This is because $\kappa_y^x = \kappa_y^c$ implies that $\frac{1}{w^y \lambda^x} = \frac{1}{\lambda_y^c}$.

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