Price & Choose

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An example: Selecting arbitrators

Arbitration is a private dispute resolution method that does not involve courts.

The (two) involved parties choose the arbitrator who will resolve the dispute.

Structured selection procedure to help the parties exercise their right of choice, such as the American Association of Arbitrators (using vetoes, points, etc.).

Perfect information : two involved parties know each other well

An example: Selecting arbitrators

Two recent papers try to improve the procedures used by practitioners:

- de Clippel et al. (2014) proposes a “shortlisting” mechanism. Shortlisting works in only two stages, and the paper verifies its validity in the lab.

- Barberà and Coelho (2022) considers procedures with more steps, but achieving less inequality among players.

What about transfers?

- Moore and Repullo (1988) suggest using dynamic deterministic mechanisms with transfers (see Aghion et al. (2013), Fehr et al.(2022), Chen et al. (2023))

Our Contribution: Price & Choose

P&C is a two-stage mechanism fully implementing the efficient allocations.

Short mechanism where backward induction often works as a good predictor in experiments.

Extensions : equal split of the surplus, adversarial behavior, number of players, preferences over money.

Literature

- Transfers and externalities: Varian (1994) and Duggan and Roberts (2002)
- Wicksell Unanimity: Wicksell (1896), Louis and Xefteris (2022)
- Multibidding mechanism with lotteries and complex tie-breaking rule : Wettstein and Perez-Castrillo (2002)
- Many players: Quadratic Voting (Eguia and Xefteris 2022)
- Cake cutting literature: Brown and Velez (2016),..
- Auction for determining the roles: Moulin (1984),…

Plan of the talk

1. Price & Choose

2. Equal split of the surplus

3. Robustness to maxmin behavior

Not in this talk

4. Extensions to many players and non quasi-linear preferences

Plan of the talk

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Setting

- Options $A = \{a_1, a_2, \dots, a_k\}$
- Players $N = \{1,2\}$
- Payoff $u_i(a) + t_i$ with a the outcome and t_i the transfer.
- Average utility of player *i*: $Avg_i = \frac{\sum_{j=1}^{j=k} u_i(a_j)}{k}$
- Allocation (a, t_1, t_2)
- x is efficient if $u_1(x) + u_2(x)$ is maximal
- Allocation is Pareto optimal if no other allocation is preferred by both players and strictly preferred by some player

Setting

- A mechanism θ specifies an action set A_i for each player and chooses an allocation for each action profile

- The mechanism θ implements the efficient options in subgame-perfect equilibrium if:
  - the outcome of any subgame-perfect Nash equilibrium σ is efficient;
  - and, conversely, for any efficient outcome a, there is a subgame-perfect Nash equilibrium σ that selects a

Price & Choose mechanism

Price & Choose:

- The first-mover sets up a price vector with $\sum_{a \in A} p_a = 0$
- The second-mover chooses the outcome x
- The second-mover pays the corresponding price to the first-mover.

Proposition 1: **P&C implements the set of efficient options**

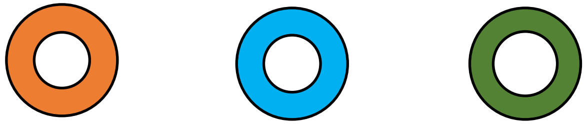
Remark 1: First-mover is a market maker

Remark 2: Sum of the prices does not alter the implementation

Price & Choose: How does it work?

- In equilibrium, P1 makes P2 indifferent between all options to « extract » all surplus: for each $x, u_2(x) - p_x = Avg_2$.
- P2 selects the option that maximizes P1's payoff, despite being indifferent
  - If P2 is indifferent and does not select the efficient one, P1 can still induce P2 to choose the most efficient one by sacrificing some payoff
- In equilibrium, payoffs are $(u_1(x) + u_2(x) - Avg_2, Avg_2)$ with x being efficient

Options
$\{a, b, c\}$



Utilities

	a	b	c	Avg_i
u_1	**12**	**9**	**0**	**7**
u_2	0	10	5	**5**

Efficiency

b unique efficient option : 19 sum of utilities

Options

$\{a, b, c\}$

Utilities

	a	b	c	Avg_i
u_1	**12**	**9**	**0**	**7**
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Efficiency

b unique efficient option : 19 sum of utilities

Prices

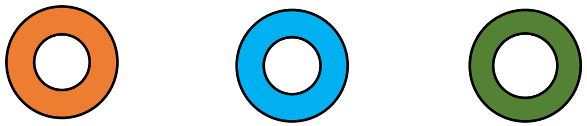
p_a € p_b € p_c €

Payoffs

u_1+p_x	**$12+p_a$**	**$9+p_b$**	**$0+p_c$**
u_2-p_x	$0-p_a$	$10-p_b$	$5-p_c$

Equilibrium : P1 makes P2 indifferent and P2 selects b

Options
$\{a, b, c\}$



Utilities

	a	b	c	Avg_i
u_1	**12**	**9**	**0**	**7**
u_2	0	10	5	**5**

Efficiency

b unique efficient option : 19 sum of utilities

Prices

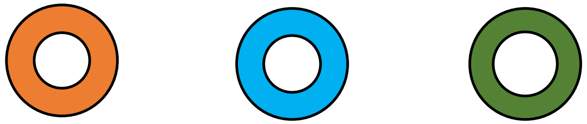
-5 € | 5 € | 0 €

Payoffs

u_1+p_x	**7**	**14**	**0**
u_2-p_x	5	5	5

If P2 selects an inefficient option, then P1 has a profitable deviation

Options
$\{a, b, c\}$



Utilities

	a	b	c	Avg_i
u_1	**12**	**9**	**0**	**7**
u_2	0	10	5	**5**

Efficiency

b unique efficient option : 19 sum of utilities

Prices

-5 €	5-0.01 €	0+0.01 €

Payoffs

u_1+p_x	**7**	**14-0.01**	**0+0.01**
u_2-p_x	5	5+0.01	5-0.01

Plan of the talk

1. Price & Choose
2. Equal split of the surplus
3. Robustness to maxmin behavior

Equal split of the surplus

- Surplus: S=$\boldsymbol{u_1(x) + u_2(x) - Avg_1 - Avg_2}$ when x is efficient
- In equilibrium, $\boldsymbol{(u_1(x) + u_2(x) - Avg_2, Avg_2)}$ if 1 moves first
  $\boldsymbol{(Avg_1, u_1(x) + u_2(x) - Avg_2)}$ if 2 moves first

Equal split of the surplus

- Surplus: S=$\boldsymbol{u_1(x) + u_2(x)}$ – $\boldsymbol{Avg_1}$- $\boldsymbol{Avg_2}$ when x is efficient

- In equilibrium, $\boldsymbol{(Avg_1 + S, Avg_2)}$ if 1 moves first
  $\boldsymbol{(Avg_1, Avg_2 + S)}$ if 2 moves first

Remark: P&C gives a first-mover advantage since S is non-negative.

Equal split of the surplus

- Bid, Price & Choose:
  1. Simultaneous bids where the winner pays the loser the highest bid
  2. Winner sets-up a price vector
  3. Loser chooses an option and pays the resp. price to the winner

- Optimal bid : $\boldsymbol{S/2}$ => *Expected profit of being first-mover*
- In equilibrium, $\left(\boldsymbol{Avg_1 + \frac{S}{2}, Avg_2 + \frac{S}{2}}\right)$

Proposition 2: **BP&C implements the set of efficient options with equal split of surplus**

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Robustness of the mechanism: Adversarial Behavior 1/3

- In the P&C mechanism, cooperative behavior in equilibrium:

  When indifferent between all options, P2 chooses the best option for P1

- In dispute resolution contexts, players may typically want to minimize the payoff of the opponent.

- We consider such an entension: when Player 2 is indifferent, he chooses the worst option for P1.

Robustness of the mechanism: Adversarial Behavior 2/3

- Players ε-maximize:

Option a is an ε-maximizer for P2 iff $u_2(a) - t_a + \varepsilon \geq u_2(a') - t_{a'}$,for any $a' \neq a$. $\beta_2^{\varepsilon}(p)$ is the set of ε-maximizers for P2 at price vector p

Adversarial nature: $\sigma_2(p) \epsilon \arg\min u_1(a) + t_a$ with $a \epsilon \beta_2^{\varepsilon}(p)$

- P1 chooses an option from $\beta_1^{\varepsilon}(p)$, the ε-maximizers for P1.

- ε-robust subgame perfect equilibria

Robustness of the mechanism: Adversarial Behavior 3/3

- a_1 unique efficient option.
- Proposition 3: **For any $\boldsymbol{\varepsilon} > \mathbf{0}$ small enough, the unique $\boldsymbol{\varepsilon}$-robust subgame perfect Nash equilibrium outcome of P&C is the efficient outcome.**

- Logic: $q^* = (p_1^* - \varepsilon, p_2^* + \frac{\varepsilon}{k-1}, \dots, p_k^* + \frac{\varepsilon}{k-1})$

where p^* is the price vector that induces a payoff Avg_2 for each x

a_1 unique option in $\beta_2^{\varepsilon}(q^*)$ and Payoff of $u_1(a_1) + p_1^* - \varepsilon$ for P1.

Conclusion

- Simple two-stage mechanism that implements efficiency
  - Key idea: **Prices help agents' coordination towars efficiency**
  - Bidding as an initial stage to ensure **equal split of the surplus**

- Robust to several specifications (adversarial behavior, number of players, preferences over money)

- Experimentally:
- efficiency? to test it -> work in progress (Funaki, Koriyama, Núñez and Rostagno 2023)
- Adversarial behavior? -> Brown and Velez (2016)

Thanks !