Auctions with a multi-member bidder

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- Economic characteristics:
  1. Public good;
  2. Aggregation problem in a strategic bidding setting.

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- Group contests - the group/team wins together or loses together. E.g., Kobayashi and Konishi 2021.

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- Team mechanism=$(A, p_1, \cdots, p_n)$

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- **Order**. If $b_i \geq b_j$ implies that $p_i(b_1, \cdots, b_n, x) \geq p_j(b_1, \cdots, b_n, x)$, for every i, j, b and x.

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- **Unboundedness**. For every r there exists a b^* such that if $b_i \geq b^*$ for some i then $A(b) \geq r$.

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  - Compare B to $B + \delta$: if the latter wins and the former loses, then the price is at least $x > n$, hence i will pay $> \frac{x}{n} > 1$. [Max-report-payment]

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  - Compare B to $B + \delta$: if the latter wins and the former loses, then the price is at least $x > n$, hence i will pay $> \frac{x}{n} > 1$. [Max-report-payment]
  - In the game with truncated report-sets $[0, B]$ there is an equilibrium—it is also an equilibrium in the original game. □

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- Theorem

Suppose that $M \geq 2n$. Then the linear-proportional model has a unique equilibrium. The equilibrium is symmetric: $\beta_1 = \cdots = \beta_n = \beta^{SPA}$, where the bid function β is given by:

$$\beta^{SPA}(\theta) = max\{\theta - a, 0\},$$

where a is the unique solution to:

$$a = \frac{n-1}{n+1} \cdot (\int_a^1 tf(t)dt + aF(a)).$$

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- Proposition

  In the linear-proportional model, the equilibrium-expected-utility of a team member with type θ is:

$$\pi^*(\theta) = \frac{1}{2M} \cdot [2\theta - max\{\theta - a, 0\}] \cdot [2a + max\{\theta - a, 0\}].$$

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- a_n=the cutoff a corresponding to a bidding team of size n.

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  The cutoff a_n satisfies the following:

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  2. *$lim_{n \to \infty} a_n = 1$.*
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- Proposition

  Consider two copies of the model—one in which the type distribution is F and one in which it is H, where F first-order stochastically dominates H. Let a^z be the cutoff corresponding to $z \in \{F, H\}$. Then $a^F \geq a^H$.

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$$lim_{n\to\infty}\frac{n\times\pi^*(\theta)}{\Pi^*(\theta)}=\frac{4}{\mathbb{E}(\theta)}.$$

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  Consider the linear-proportional mechanisms with $n = 2$, and where the regular bidder's type is uniform over $[0, 1]$. Then:

  1. *Under the second-price format, the game has a symmetric equilibrium.*
  2. *Under the all-pay format, the game has no symmetric equilibrium that is equivalent to a symmetric equilibrium of the second-price game.*
  3. *Under the all-pay format, the game has equilibria with complete free riding.*
  4. *Under the second-price format, the game has no equilibrium with complete free riding.*

Future research

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- Competition between multiple teams.