Information in (and not in) interest rates surveys

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Introduction
Motivation

- Term structure models decompose long term interest rates into
  - the information about investors' expectations of the economy
  - the risks investors perceive

but identification issues can misestimate this decomposition
(Ang and Piazzesi 2003, Hamilton and Wu 2012)

→ Survey expectations on short-term rates used

  - to aid the identification of the physical parameters
    (Kim and Orphanides 2012, d'Amico, Kim and Wei 2018)
  - to proxy for expectations (Crump, Eusepi and Moench 2018)
  - to proxy for state variables (Chun 2011)

... rely on the assumption that the survey probability measure is the same as the statistical probability measure (equal dynamics assumption)

Introduction

Motivation: is this the best use of surveys for term structure models?

- Weak empirical evidence on the rational expectation hypothesis for short-term interest rates (Cieslak 2018, Farmer, Nakamura and Steinsson 2021, Piazzesi, Salomao and Schneider 2015)
- Even under rational expectations, there can be a discrepancy between survey expectations and forecasts implied by observed long-term interest rates
  - Observed interest rates may not contain all the information necessary to identify the drivers of interest rate expectations
  - The literature has focused on unspanned macroeconomic variables (Joslin, Priebsch and Singleton 2014, Coroneo, Giannone and Modugno 2016)

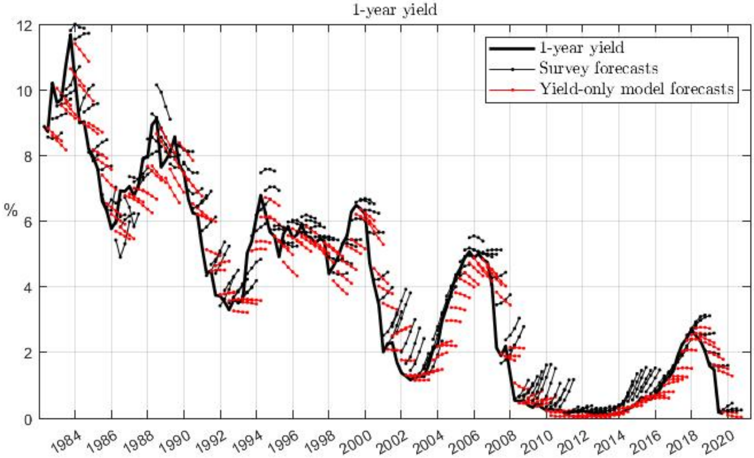
$\rightarrow$ What about surveys?

Introduction

This paper

- **This paper**: what information do interest rates surveys convey about interest rates?
- Develop a joint term structure model for observed zero-coupon yields and survey expectations that
  - Allows for three separate probability measures: risk-neutral, objective and subjective
  - Includes an additional state variable that is unspanned by observed interest rates but that drives surveys
  - Explicitly enforces a zero-lower bound on observed interest rates and interest rate surveys
- Results:
  - Rejection of the equal dynamics assumption
  - Presence of a priced unspanned factor from surveys

Stylised fact 1



Fact 1 The real-world probability distribution that drives observed interest rates is different from the subjective probability distribution that drives interest rate survey expectations. Data

Stylised fact 2

Yields: fit with first k YPC

k	1	2	3	4	5
Variance explained	97.30	99.88	99.98	100.00	100.00
Average RMSE	45.45	9.33	3.49	1.12	0.24

Surveys: fit with first k SPC

k	1	2	3	4	5
Variance explained	97.15	99.81	99.90	99.95	99.98
Average RMSE	44.95	11.68	8.40	5.90	3.94

Surveys: fit with 3 YPC and k $SPC_{\perp}$

k	0	1	2	3	4
Variance explained	97.97	99.63	99.85	99.93	99.96
Average RMSE	39.35	16.48	10.15	7.26	5.08

Fact 2 Interest rates surveys, apart from yield curve factors, are spanned by an additional, survey-specific factor.

Stylised fact 3

	$VAR(1)$			
	$YPC_{1,t-1}$	$YPC_{2,t-1}$	$YPC_{3,t-1}$	$SPC_{\perp,t-1}$
$YPC_{1,t}$	0.9835^{***}	0.0062	-0.0216^{**}	0.1204^{***}
$YPC_{2,t}$	-0.0026	0.9238^{***}	0.1250^{***}	-0.0396
$YPC_{3,t}$	0.0699^{*}	-0.0218	0.6552^{***}	-0.1515^{**}
$SPC_{\perp,t}$	-0.0086	0.0864	0.0078	0.2319^{***}

	Excess returns			
	$YPC_{1,t-1}$	$YPC_{2,t-1}$	$YPC_{3,t-1}$	$SPC_{\perp,t-1}$
$xret_{1,t}$	-1.9777^{***}	0.4302^{***}	-0.0594^{**}	-0.2456^{***}
$xret_{2,t}$	-1.9106^{***}	0.5118^{***}	-0.1041	-0.4261^{***}
$xret_{5,t}$	-1.8587^{***}	0.7612^{***}	-0.2573	-0.7565^{***}
$xret_{7,t}$	-1.8501^{***}	0.9646^{***}	-0.3107	-0.8931^{***}
$xret_{10,t}$	-1.8332^{***}	1.1786^{***}	-0.3558	-1.0314^{***}

Fact 3 The survey-specific factor (s-factor) drives the real-world dynamics of (Granger-causes) the yield curve factors.

Model: bond prices

- Shadow rate model with a $K-$dimensional state vector $\mathbf{x}_t$ More

$$r_t = \max\{\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t, \underline{r}\} \tag{1}$$

- Risk-neutral dynamics

$$\mathbf{x}_t = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_{t-1} + \mathbf{u}_t^{\mathbb{Q}},\ \mathbf{u}_t^{\mathbb{Q}} \overset{\mathbb{Q}}{\sim} i.i.d.\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{x}}) \tag{2}$$

- In the absence of arbitrage opportunities

$$p_{n,t} = \log E_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j}\right)\right] \tag{3}$$

- Physical dynamics

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_x^{\mathbb{P}} \\ \boldsymbol{\mu}_s^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Phi}_{xx}^{\mathbb{P}} & \boldsymbol{\Phi}_{xs}^{\mathbb{P}} \\ \boldsymbol{\Phi}_{sx}^{\mathbb{P}} & \boldsymbol{\Phi}_{ss}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_{x,t}^{\mathbb{P}} \\ \mathbf{u}_{s,t}^{\mathbb{P}} \end{bmatrix} \tag{4}$$

where $[\mathbf{u}_{x,t}^{\mathbb{P}}, \mathbf{u}_{s,t}^{\mathbb{P}}]' \overset{\mathbb{P}}{\sim} i.i.d.\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$ with the $K \times K$ top left matrix block of $\boldsymbol{\Sigma}$ equal to $\boldsymbol{\Sigma}_{\mathbf{x}}$ defined in Eq.(2).

Model: surveys

- The $h-$period ahead survey expectation at time t of a yield on a zero-coupon bond with an $n-$period tenor is

$$y^{s}_{n,t,h} \approx y(E^{\mathbb{S}}_{t}[\mathbf{x}_{t+h}], n; \mathbf{\Psi}) \tag{5}$$

- Dynamics under $\mathbb{S}$

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}^{\mathbb{S}}_{x} \\ \boldsymbol{\mu}^{\mathbb{S}}_{s} \end{bmatrix} + \begin{bmatrix} \mathbf{\Phi}^{\mathbb{S}}_{xx} & \mathbf{\Phi}^{\mathbb{S}}_{xs} \\ \mathbf{\Phi}^{\mathbb{S}}_{sx} & \mathbf{\Phi}^{\mathbb{S}}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}^{\mathbb{S}}_{x,t} \\ \mathbf{u}^{\mathbb{S}}_{s,t} \end{bmatrix}, \tag{6}$$

where where $[\mathbf{u}^{\mathbb{S}}_{x,t}, \mathbf{u}^{\mathbb{S}}_{s,t}]' \overset{\mathbb{S}}{\sim} i.i.d.\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$.

Model

Model specifications

Is s_t a risk factor under $\mathbb{P}$? $\mathbf{\Phi}^{\mathbb{P}}_{xs} \neq \mathbf{0}$	Equal dynamics ($\mathbb{S} \sim \mathbb{P}$) $\boldsymbol{\mu}^{\mathbb{P}}_x = \boldsymbol{\mu}^{\mathbb{S}}_x$ and $\mathbf{\Phi}^{\mathbb{P}}_{xx} = \mathbf{\Phi}^{\mathbb{S}}_{xx}$	No equal dynamics ($\mathbb{S} \nsim \mathbb{P}$) $\boldsymbol{\mu}^{\mathbb{P}}_x \neq \boldsymbol{\mu}^{\mathbb{S}}_x$ and $\mathbf{\Phi}^{\mathbb{P}}_{xx} \neq \mathbf{\Phi}^{\mathbb{S}}_{xx}$
No	**Case 1** surveys informative for econometric identification of $\mathbb{P}-$parameters	**Case 3** surveys not informative
Yes	**Case 2** surveys informative for both the physical risk factors and $\mathbb{P}-$parameters	**Case 4** surveys informative for physical risk factors

Data

- Data: sample 1983:Q1 to 2020:Q3.

- End-of-quarter interest rates, for maturities 3 and 6 months (FRED), and 1, 2, 5, 7 and 10 years (FRB)

- Consensus Blue Chips Financial Forecasts at 1 to 5 quarters ahead for 3-month, and 1, 5 and 10-year yields BCFF Estimation

Model fit: RMSE

Yields

Model	3m	6m	1y	2y	5y	7y	10y	Av
Case 0 (yield-only model, no s_t)	4.16	1.87	4.33	2.21	4.81	2.44	4.35	3.45
Case 1 ($\mathbb{S} \sim \mathbb{P}$, no s_t)	4.15	1.87	4.34	2.26	4.81	2.47	4.36	3.47
Case 2 ($\mathbb{S} \sim \mathbb{P}$, s_t is $\mathbb{P}-$risk factor)	4.11	1.86	4.24	2.32	4.84	2.57	4.44	3.48
Case 3 ($\mathbb{S} \nsim \mathbb{P}$, s_t is not $\mathbb{P}-$risk factor)	4.13	1.87	4.26	2.30	4.84	2.55	4.44	3.48
Case 4 ($\mathbb{S} \nsim \mathbb{P}$, s_t is $\mathbb{P}-$risk factor)	4.10	1.86	4.24	2.33	4.84	2.57	4.45	3.48

Surveys

Model	h	3m	1y	5y	10y	Av
Case 0 (yield-only model, no s_t)	1q	37.97	44.58	41.38	41.85	41.45
	2q	40.67	53.87	48.57	44.46	46.89
	3q	47.63	64.97	59.42	51.78	55.95
	4q	60.20	79.78	72.96	62.91	68.96
Case 1 ($\mathbb{S} \sim \mathbb{P}$, no s_t)	1q	37.79	41.90	39.87	42.21	40.44
	2q	38.06	45.10	39.71	40.79	40.92
	3q	37.42	44.49	38.84	39.67	40.10
	4q	37.91	42.45	38.49	40.16	39.75
Case 2 ($\mathbb{S} \sim \mathbb{P}$, s_t is $\mathbb{P}-$risk factor)	1q	18.69	20.21	12.63	17.85	17.34
	2q	14.64	18.23	10.89	15.71	14.87
	3q	12.84	17.26	12.83	16.18	14.78
	4q	15.67	16.43	15.72	18.49	16.58
Case 3 ($\mathbb{S} \nsim \mathbb{P}$, s_t is not $\mathbb{P}-$risk factor)	1q	18.47	20.18	12.60	17.40	17.17
	2q	14.49	18.10	11.08	15.35	14.75
	3q	12.64	17.08	13.19	15.96	14.72
	4q	15.36	16.09	16.03	18.47	16.49
Case 4 ($\mathbb{S} \nsim \mathbb{P}$, s_t is $\mathbb{P}-$risk factor)	1q	18.45	20.12	12.59	17.56	17.18
Survey fit	2q	14.43	18.14	11.05	15.56	14.80
	3q	12.70	17.22	13.07	16.12	14.78
	4q	15.55	16.33	15.86	18.52	16.56

Likelihood ratio test

Model	Restrictions (# of restr.)	Total L	$L^{\mathbb{Q}}$	$L^{\mathbb{S}}$	$L^{\mathbb{P}}$	$p-$value
Case 4	$\mathbb{S} \nsim \mathbb{P}, s_t$ is $\mathbb{P}-$risk factor None (0)	17,120.52	3,729.22	10,866.16	2,525.14	-
Case 2	$\mathbb{S} \sim \mathbb{P}, s_t$ is $\mathbb{P}-$risk factor $\boldsymbol{\mu}_x^{\mathbb{P}} = \boldsymbol{\mu}_x^{\mathbb{S}}$, $\mathbf{\Phi}^{\mathbb{P}} = \mathbf{\Phi}^{\mathbb{S}}$ (20)	17,080.31	3,729.54	10,855.72	2,495.04	0.0000
Case 3	$\mathbb{S} \nsim \mathbb{P}, s_t$ is not $\mathbb{P}-$risk factor $\mathbf{\Phi}_{xs}^{\mathbb{P}} = \mathbf{0}$ (3)	17,082.57	3,728.84	10,865.14	2,488.59	0.0000

Table: Likelihood values and likelihood ratio $p-$values for different cases.

Out-of-sample interest rates forecasts under $\mathbb{P}$

Case 4 vs	Horizon\Yield	3m	1y	5y	10y
Case 1	1q	0.791*	0.774*	0.842*	0.898*
($\mathbb{S} \sim \mathbb{P}$, no s_t)	2q	0.785**	0.790**	0.888	0.914
	3q	0.795**	0.781**	0.887	0.922
	4q	0.768**	0.744**	0.819*	0.872
Case 2	1q	1.031	0.932	1.014	0.974
($\mathbb{S} \sim \mathbb{P}$, s_t is $\mathbb{P}-$ risk factor)	2q	0.979	0.870**	0.933	0.976
	3q	0.915**	0.806**	0.869**	0.938
	4q	0.860**	0.758**	0.819**	0.910*
Case 3	1q	0.776*	0.780*	0.803*	0.862*
($\mathbb{S} \nsim \mathbb{P}$, s_t is not $\mathbb{P}-$risk factor)	2q	0.783**	0.822*	0.897	0.903
	3q	0.808*	0.842	0.950	0.955
	4q	0.802*	0.826*	0.909	0.931
Random walk	1q	0.666*	0.722*	0.868*	0.924
	2q	0.671**	0.778*	0.975	0.989
	3q	0.710**	0.811**	1.051	1.061
	4q	0.719**	0.808**	1.021	1.047

Table: Out-of-sample *RMSFE*s for the model-implied objective expectations of Case 4 relative to the ones of the other cases and the random walk for the period from 2004:Q4 to 2019:Q3. The one-side significance of the Diebold and Mariano (1995) test of equal predictive ability is established by fixed$-b$ asymptotics as in Coroneo and Iacone (2020). Risk Premium

What drives the $s-$factor?

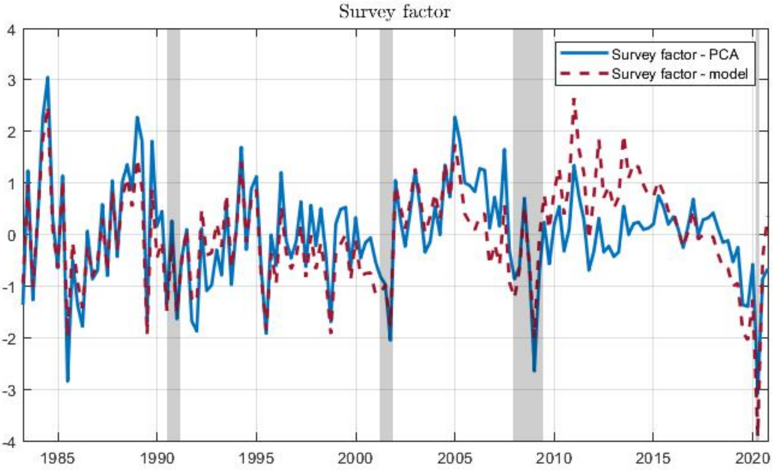


Figure: $PC-$based $s-$factor (continuous blue line) and the $s-$factor extracted from the term structure model specified as Case 4 (red dashed line). Shaded areas denote the NBER recessions.

What drives the (model-implied) $s-$factor?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
IP	13.084**						8.830**
	(5.246)						(4.451)
CPI		15.884					11.919
		(15.731)					(16.059)
EPU			-0.002*				0.002
			(0.001)				(0.001)
EPUMP				-0.006***			-0.004**
				(0.002)			(0.002)
VIX					-0.034**		-0.008
					(0.014)		(0.015)
EMnews						-0.823**	-0.577*
						(0.330)	(0.300)
Constant	-0.062	-0.103	0.280*	0.571***	0.703**	0.320*	0.519
	(0.137)	(0.189)	(0.165)	(0.198)	(0.313)	(0.175)	(0.389)
Observations	151	151	140	140	120	140	120
$\overline{R}^2$	0.034	0.002	0.034	0.135	0.067	0.084	0.166

Table: The dependent variable is the model-implied s-factor. Newey-West standard errors with 4 lags in parentheses. $*$, $**$ and $***$ denote significance at the 10%, 5% and 1%, respectively. PCA

Conclusion

1. Subjective and objective dynamics are very different
   - Augmenting the standard model with surveys under the assumption of equal dynamics distorts objective expectations and thus the implied risk premium
2. Surveys contain information about a risk factor unspanned by interest rates
   - The survey-specific factor contributes to the physical dynamics
   - Accounting for the survey-specific factor is important to obtain a reliable measurement of the expectation component (and of the risk premium)

THANK YOU!

Model

Shadow rate term structure model

- Shadow rate

$$ssr_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t \tag{7}$$

- Short-term interest rate

$$r_t = \max\{ssr_t, \underline{r}\} \tag{8}$$

- Wu and Xia (2016) show that given (7)–(8), a good approximation to the time t one period forward rate for a loan starting at $t+n$ is

$$f_{n,t} \approx \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + \mathbf{b}_n' \mathbf{x}_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right), \tag{9}$$

where the function $g(z) = z\mathtt{N}(z) + \mathtt{n}(z)$ with $\mathtt{N}(z)$ and $\mathtt{n}(z)$ the CDF and the PDF of z, respectively, and $\sigma_n^{\mathbb{Q}}$, a_n and $\mathbf{b}_n$ are known coefficients, given the risk-neutral parameters. Back

Estimation

- Estimation by maximum likelihood using the factor extraction method of Golinski and Spencer (2022)
  - Generalization of the estimation technique by factor rotation with observable factors introduced by Joslin, Singleton and Zhu (2011) to non-linear models
- Denote $\mathbf{\Theta} \equiv \{\boldsymbol{\mu}^{\mathbb{Q}}, \mathbf{\Phi}^{\mathbb{Q}}, \mathbf{\Sigma_y}, \boldsymbol{\mu}^{\mathbb{S}}, \mathbf{\Phi}^{\mathbb{S}}, \mathbf{\Sigma_s}, \boldsymbol{\mu}^{\mathbb{P}}, \mathbf{\Phi}^{\mathbb{P}}, \mathbf{\Sigma}\}$
- Conditional likelihood function

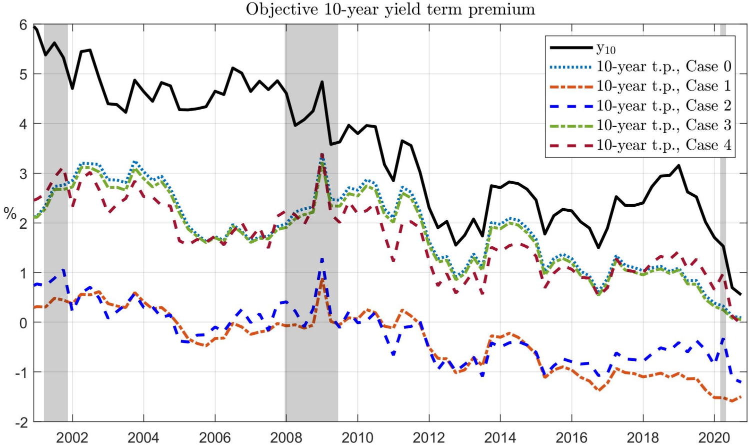
$$\log \mathcal{L}(\mathbf{\Theta}) = \sum_{t=2}^{T} \log \ell(\mathbf{Y}_t, \mathbf{Y}_t^s | \mathbf{Y}_{t-1}, \mathbf{Y}_{t-1}^s; \mathbf{\Theta})$$

- Under the JSZ parametrization, the time$-t$ conditional likelihood can be decomposed as

$$\begin{aligned} \ell(\mathbf{Y}_t, \mathbf{Y}_t^s | \mathbf{Y}_{t-1}, \mathbf{Y}_{t-1}^s; \mathbf{\Theta}) &= \ell^{\mathbb{Q}}(\mathbf{Y}_t | \mathbf{q}_{y,t}; \boldsymbol{\mu}^{\mathbb{Q}}, \mathbf{\Phi}^{\mathbb{Q}}, \mathbf{\Sigma_y}, \mathbf{\Sigma}) \\ &\quad \times \ell^{\mathbb{S}}(\mathbf{Y}_{s,t} | \mathbf{q}_{y,t}, \mathbf{q}_{s,t}; \boldsymbol{\mu}^{\mathbb{Q}}, \mathbf{\Phi}^{\mathbb{Q}}, \boldsymbol{\mu}^{\mathbb{S}}, \mathbf{\Phi}^{\mathbb{S}}, \mathbf{\Sigma_s}, \mathbf{\Sigma}) \\ &\quad \times \ell^{\mathbb{P}}(\mathbf{q}_{y,t}, \mathbf{q}_{s,t} | \mathbf{q}_{y,t-1}, \mathbf{q}_{s,t-1}; \boldsymbol{\mu}^{\mathbb{P}}, \mathbf{\Phi}^{\mathbb{P}}, \mathbf{\Sigma}). \end{aligned}$$

- Conditional of the $\mathbb{Q}$ and the $\mathbb{S}-$parameters, $\boldsymbol{\mu}^{\mathbb{P}}$ and $\mathbf{\Phi}^{\mathbb{P}}$ can be estimated by *OLS*. Back

Risk premium under $\mathbb{P}$



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Model fit: surveys

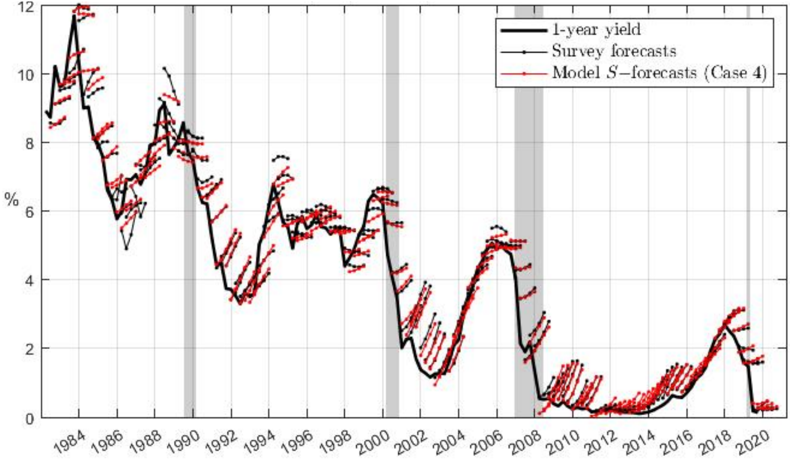
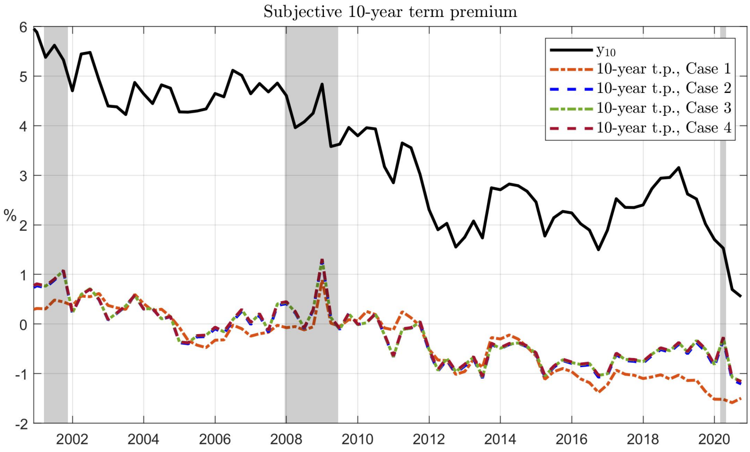


Figure: Fit of expectations extracted from the model with different subjective and objective probability measures and with priced survey specific factor (Case 4) to survey expectations for 1−year yield. Back

Risk premium under $\mathbb{S}$



Back

What drives the ($PC-$based) $s-$factor?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
IP	17.063***						4.664
	(4.447)						(4.032)
CPI		41.314***					23.659*
		(12.765)					(13.451)
EPU			-0.004***				0.000
			(0.001)				(0.001)
EPUMP				-0.007***			-0.005***
				(0.002)			(0.002)
VIX					-0.047***		-0.018*
					(0.011)		(0.010)
Emnews						-0.624**	-0.118
						(0.257)	(0.213)
Constant	-0.081	-0.268*	0.469***	0.646***	0.880***	0.219	0.657**
	(0.107)	(0.141)	(0.154)	(0.156)	(0.253)	(0.139)	(0.296)
Observations	151	151	140	140	120	140	120
$\overline{R}^2$	0.062	0.051	0.140	0.214	0.179	0.052	0.286

Table: The dependent variable is the model-implied s-factor. Newey-West standard errors with 4 lags in parentheses. $*$, $**$ and $***$ denote significance at the 10%, 5% and 1%, respectively. Back