The Changing Polarization of Party Ideologies: The Role of Sorting

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Abstract

U.S. congressional roll-call voting records show that as polarization of the two parties along the economic dimension changes, polarization along the social/cultural dimension tends to change in the opposite direction. A model of party competition within a two-dimensional ideology space is developed in which party platforms are determined by voters who compose the party. It is shown that if distribution of voter preferences is radially symmetric, polarization of party ideologies along the two dimensions are inversely related, as observed. The model gives a remarkably good quantitative account of the historically observed movements in polarization along the two dimensions.

Keywords: polarization, primaries, partisan sorting, political economy
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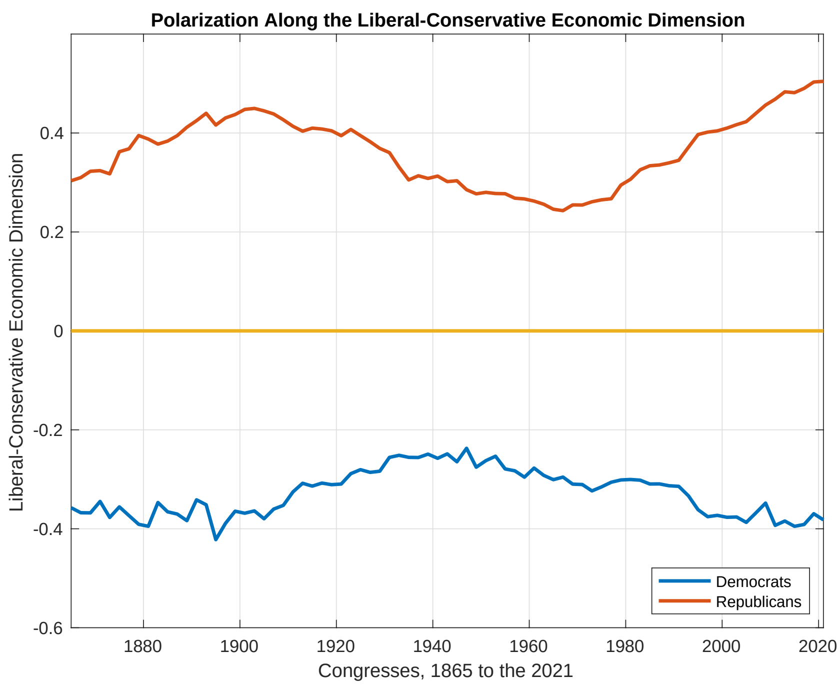
1 Introduction

Much of the analysis of the Poole and Rosenthal (1997) roll-call voting data has focused on the polarization of political parties along the liberal-conservative economic dimension. Figure 1 shows

Figure 1:
Polarization Along the Liberal-Conservative Dimension

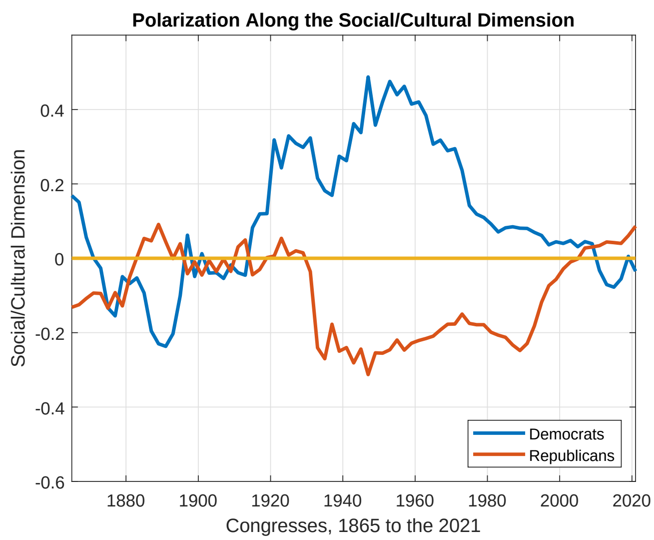


Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022)

the evolution of polarization along this dimension since 1865. It plots the mean ideological position of Democratic and Republican representatives in Congress, from the 39th Congress to the (current) 117th Congress. Throughout this period, the Republican party has occupied the conservative end of the spectrum and the Democratic party the liberal end. That said, the ideological distance between the two parties has waxed and waned over time: The distance shrank during the first eight decades of the 20th century but has been rising since 1980. Currently, the distance is the widest it has been since 1865.

In contrast, the ideological distance along the second dimension has received less attention. We refer to the second dimension as the dimension of social/cultural issues, with positive numerical values indicating social conservatism and negative numerical values indicating social liberalism (Figure 2). At the end of the Civil War, the Democratic party was the socially conservative party, while the Republican party was socially liberal. However, this changed during the final decades of the 19th century as the Democratic party became socially liberal and the Republican party socially conservative. In the first couple of decades of the 20th century, there was hardly any difference between the two parties along this dimension. Between 1920 and 1960, the ideological difference widened, with Democrats becoming increasingly socially conservative. The trend reversed starting in the mid-1960s and the ideological distance between the two parties is currently low. That said, the Republican party is now the socially conservative party, while the Democratic party is the socially liberal one.

Figure 2:
Polarization Along the Social/Cultural Dimension

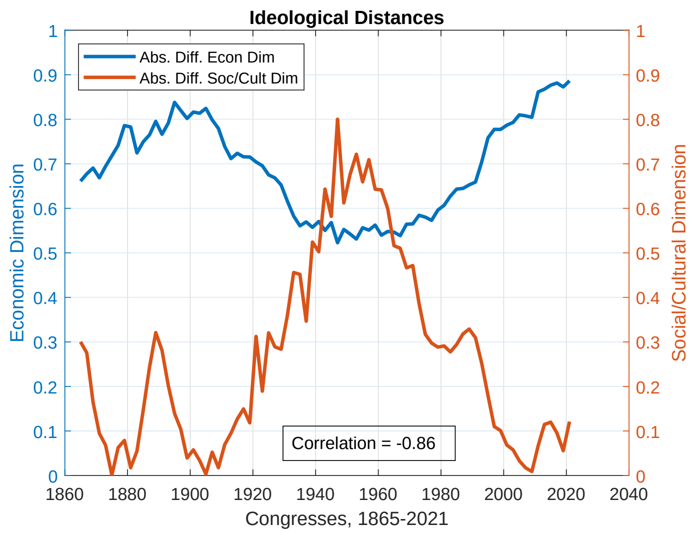


Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022)

For the purposes of this paper, there are two key takeaways of these plots. The first is the manner in which these ideological distances move relative to each other: When ideological distance increases along one dimension, it tends to *decrease* along the other dimension. This pattern of inverse association is easily seen in time plots of the absolute difference in the mean ideological positions of the two parties for each dimension, as shown in Figure 3: Periods when ideological distance along the social/cultural dimension rose tend to be periods when the ideological distance along the economic dimension fell.[1] Indeed, the correlation between the two series is -0.86.

[1]The optimal classification (OC) technique (Poole (2005)) that locates a congress-person on the two-dimensional ideology space implies only that the person's ideological position must lie in the square $[-1,1] \times [-1,1]$. The inverse association shown in Figure 3 is not implied by the mechanics of OC but is, instead, a feature of the roll-call voting patterns.

Figure 3:
Ideological Distance Between Parties Along
Economic and Social Dimensions



Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation

The second takeaway is that the ideological orientation of a party — liberal or conservative — along a dimension is not necessarily fixed. While the orientation of each party on economic issues has not changed since the Civil War, orientation on social issues has changed. The Democratic party currently espouses a liberal social ideology but for the better part of the post-Civil War period its representatives have, on average, voted conservatively on social issues. Similarly, the reverse holds for the Republican party.

The main contribution of this paper is to show that the negative association between the two distance measures shown in Figure 3 might not be a historical accident. In fact, it is implied by a spatial model of voting in which party ideologies are formed in primary elections that precede the national election.

In our model, the ideological stance of a party is determined by the composition of voters in a party's primary election. Voters are heterogeneous regarding their preferences (equivalently, ideologies) and sort into the two parties depending on what they believe each party stands for. The primary elections then turn the preferences of primary attendees into party platform and ideology for the national election. Importantly, the desire of each party to win the national election results in party platforms that trade off the preferences of party adherents against the mean preferences of the polity at large. It is this balancing act that accounts for Figure 3: When polarization along the social dimension increases, polarization along the economic dimension shrinks so as to keep the overall extremeness of a party's ideology — measured as the Euclidean distance of a party's ideology from the mean ideology of the polity — constant.

In our model, the changes in mean ideology of the two parties shown in Figures 1 and 2 is attributed entirely to re-sorting, i.e., to changes in the composition of voters making up the two parties. There is no change in the distribution of voter preferences or in any other fundamental. What, then, triggers a change in the composition of voters making up a party? It is caused by a self-fulfilling change in voters' (common) beliefs about party ideologies. Specifically, if the distribution of voter preferences is radially symmetric (a common assumption in spatial voting models which we also make), our model implies the existence of a circle in the two-dimensional ideology space such that any two diametrically opposite points on that circle constitute *equilibrium* beliefs. Meaning, if voters associate the two parties with those (diametrically) opposing ideologies, the resulting partisan sorting and competition for national power will generate party ideologies that reproduce those beliefs.

Although the stability of voter preferences over such a long span of time is surely not exactly true, the sorting and re-sorting of a stable distribution of voters into the two parties is consistent with the malleability of party ideologies evident in the figures. More importantly, we show that our model of belief-based partisan sorting gives a surprisingly good *quantitative* account of the evolution of ideological distances seen in Figure 3.

The paper is organized as follows. In Section 2, we present a brief discussion of the extant literature in economics and politics that relates to our paper. Section 3 lays out the model environment. Section 4 solves for equilibria that display polarization along one dimension (motivated by Figure 1) but may or may not display polarization along the other dimension (motivated by Figure 2). This section explains why there is an inverse association between the two ideological distances shown in Figure 3. Section 5 takes this insight further and shows that the variation in ideological distances shown in Figure 3 can be well-accounted for, in a quantitative sense, simply by the *direction* of party ideologies implicit in Figures 1 and 2. Section 6 concludes. The Appendices contain proofs of some propositions and other supplementary materials.

2 Connections to the Literature

Our paper contributes to the interrelated literatures on party competition and polarization. In Downsian models of party competition (Downs (1957)), both parties declare the same platform (convergence of policies/platforms). As pointed out by many, this prediction is at odds with the facts as parties typically offer different platforms and it is quite inconsistent with Figures 1 and 2 which show that the two US parties have consistently offered distinct ideologies.

Given this dissonance, an extensive theoretical literature has developed to explain policy divergence.[2] One strand of this literature, beginning with Wittman (1973), sought to locate policy divergence in exogenous differences in party preferences. More recently, Roemer (2001) proposed an extension of Wittman's model in which differences in party preferences are derived from the different preferences of voters who compose these parties. Our modeling approach is in this spirit, but our theory of how preferences of party members is aggregated to produce the party platform is different. We assume this preference aggregation occurs via primary elections while in Roemer (2001) it comes about through bargaining between party factions.

The rise in party polarization since the early 1980s was documented by Poole and Rosenthal (1997). Some papers that explain policy divergence also suggest reasons why polarization has increased in recent decades. When the focus is on a single policy dimension, some set of external factors must change to account for increasing polarization. Along these lines, the literature has pointed to several possible causes.

McCarty, Poole, and Rosenthal (2006) focus on the fact that the two parties are currently much more stratified by income than in the past: Higher income individuals identify with the Republican party (and lower income individuals with the Democratic party) more so now than 50 years ago. They view this development as the likely cause of increasing polarization of the two parties.[3]

A second force that has attracted attention are campaign contributions. Herrera, Levine, and Martinelli (2008) present a model in which increasing uncertainty about election outcomes increases the size of campaign contributions as well as polarization of party platforms. Drautzburg, Livshits, and Wright (2021) show how increases in legal contribution limits to political campaigns

[2]See De Donder and Gallego (2017) for a recent survey of the literature on the positioning of political parties in unidimensional and multidimensional policy spaces.

[3]See Barber and McCarty (2013) for a recent survey of the more empirical research on the causes and consequences of polarization.

can lead to greater polarization in candidate positions, provided campaign contributions generate private benefits for contestants. Konishi and Pan (2020) study a model in which policy choices have two dimensions: social and economic. In their model, third parties lobby *both* contestants to ensure the economic dimension is not politically salient. This leaves contestants to differentiate themselves by espousing different social policies, and divergence along this dimension can increase if lobbying on the economic dimension becomes more intense.

A third force that has attracted attention is the role of imperfect understanding of the true state of affairs. Dixit and Weibull (2007) present a model in which policy failures lead some individuals to want to reverse course and others to advocate an even stronger form of the failed policy. Along similar lines, although not linked directly to political polarization, Azzimonti and Fernandes (2021) present a model in which the advent of social media as a news source causes polarization of beliefs.

In contrast to these explanations, our paper connects changes in polarization along the economic dimension to changes in polarization along the social/cultural dimension (and vice versa). Similar to us, Krasa and Polborn (2014) have examined how polarization along the economic dimension is affected by polarization along the social/cultural dimension. They present a model in which contestants have opposing views on cultural issues but choose positions along the economic dimension. Under certain conditions, increased divergence between the candidates along the social dimension leads to increased polarization along the economic dimension. Our approach differs from theirs in that a party does not inherit the views of the candidate representing it, rather, candidates commit to the platform/ideology that primary attendees collectively want them to espouse. And, consistent with the inverse association between the ideological distances along the social and economic dimensions documented earlier, our model predicts that economic polarization rises when social polarization *falls*.

Political polarization has also attracted the attention of macroeconomists. This literature assumes two types of agents, each represented by a political party, where the probability of a party becoming the ruling party is either given or determined in a general election.[4] This framework has been used to study the impact of polarized preferences on fiscal policy, in Alesina and Tabellini (1990); on aggregate investment, in Azzimonti (2011); on sovereign borrowing and default, in

[4]For a closely related literature that models fiscal policy as the outcome of legislative bargaining game, see Battaglini and Coate (2008)

Cuadra and Sapriza (2008); on allocative inefficiencies, in Acemoglu, Golosov, and Tsyvinski (2011); on policy extremism and reelection probabilities, in Chatterjee and Eyigungor (2020). All these studies are explicitly dynamic and most are quantitative. Our paper shares the quantitative focus of this literature but abstracts from explicit dynamics in order to delve deeper into the formation of party preferences.

3 The Model

Our model consists of a continuum of voters and two political parties. The parties contest a general election to determine which party's policies are adopted. Leading up to the general election, the parties determine their respective policy platforms in separate primary elections.

3.1 Voters

There is a set of *voters* who care about two issues. In the space of issues, a voter's *type* is defined by $(x,y) \in \mathbb{R}^2$. If policies chosen by the government is (w,z), the utility of a type (x,y) voter is:

$$U(x,y;w,z) = -[(w-x)^2 + (z-y)^2]. \tag{1}$$

Thus (x,y) denotes the voter's ideal policies. The density of the distribution of voter types is $q(x,y) \geq 0$. For the moment we assume only that this distribution is symmetric around $(0,0)$, i.e.,

$$q(x,y) = q(-x,-y) \quad \forall\, (x,y) \in \mathbb{R}^2. \tag{2}$$

Later on, $q(x,y)$ will be specialized to disk centered at $(0,0)$.

3.2 Political Parties and the National Election

There are two political parties referred to as the D party and the R party. We denote their respective platforms by (w_k, z_k), $k \in \{D,R\}$. These are the policies each party promises to implement if elected to govern. In this (sub)section we take these policies are given and discuss how they affect the outcome of the national elections.

Let $\mathcal{P}$ denote the 4-tuple (w_D, z_D, w_R, z_R). The party that gets to govern is determined in a national election via majority vote. A type (x, y) voter votes for party D if

$$U(x,y;w_D,z_D) + A \geq U(x,y;w_R,w_D). \tag{3}$$

Otherwise, she votes for party R. Here $A \in \mathbb{R}$ is a net preference for the D party *per se* that is realized at the time of the election. It is a random variable symmetrically distributed around 0 with a CDF $F(A)$ continuous in A.

Given (3), D party will win the national election if A exceeds some $\mathcal{P}$-dependent threshold $\bar{A}(\mathcal{P})$.[5] We assert that $A(\mathcal{P})$ is the value of A for which the mean voter, i.e., the voter of type $(0,0)$, is indifferent between the two parties. To see why, suppose, without loss, that $w_D \neq w_R$. For each y, let $x(y, A, \mathcal{P})$ be such that the voter of type $(x(y, A, \mathcal{P}), y)$ is indifferent between the two parties. From (3) we get

$$x(y,A,\mathcal{P}) = \frac{\left[(z_D - y)^2 - (z_R - y)^2\right] - A + \left[w_D^2 - w_R^2\right]}{2\left[w_D - w_R\right]}. \tag{4}$$

The value of A for which the voter of type $(0,0)$ is indifferent between the two parties solves $x(0, A, \mathcal{P}) = 0$. This yields:

$$\bar{A}(\mathcal{P}) = [w_D^2 + z_D^2] - [w_R^2 + z_R^2]. \tag{5}$$

Next, observe that (4) and (5) together imply

$$x(y, \bar{A}(\mathcal{P}), \mathcal{P}) = y\left[\frac{z_R - z_D}{w_D - w_R}\right]. \tag{6}$$

Equation (6) implies that if $\tilde{x} > x(\tilde{y}, \bar{A}(\mathcal{P}), \mathcal{P})$, then $-\tilde{x} < x(-\tilde{y}, \bar{A}(\mathcal{P}), \mathcal{P})$. Therefore, given $A = \bar{A}(\mathcal{P})$, every pair of voters with symmetrically opposite preferences will vote for different parties. From the symmetry of $q(x, y)$ it follows that the measure of voters voting for the two parties must be

[5] If $w_D > w_R$, we may verify that $U(x,y;w_D,z_D)+A-U(x,y;w_R,z_R)$ is increasing in x. In this case a voter of type (x,y) votes for the D party if $x \geq x(y,A,\mathcal{P})$ and votes for the R party if $x < x(y,A,\mathcal{P})$. On the other hand, if $w_D < w_R$ then $U(x,y;w_D,z_D)+A-U(x,y;w_R,z_R)$ is decreasing in x and a voter of type (x,y) votes for the D party if $x \leq x(y,A,\mathcal{P})$ and votes for the R party if $x > x(y,A,\mathcal{P})$. Regardless, an increase in A expands the set of voters who vote for D party.

exactly one-half. Since the l.h.s. of (3) is increasing in A, the D party will achieve a majority if $A > \bar{A}(\mathcal{P}) = [w_D^2 + z_D^2] - [w_R^2 + z_R^2]$.[6]

3.3 Primaries and the Formation of Party Platforms

We turn now to the formation of party platforms. We imagine that voters have common expectations about the policy platform each party will ultimately declare. Based on these expectations, each voter participates in the primary of his or her preferred party.

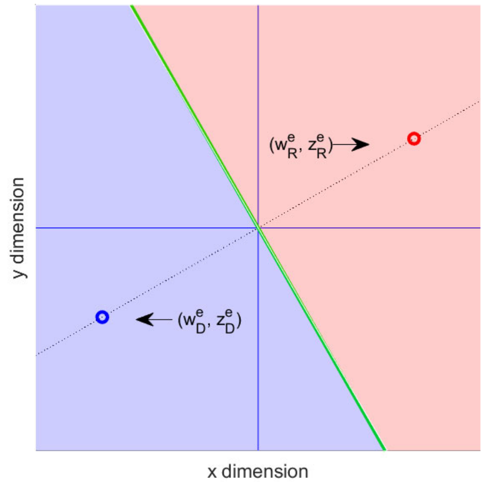
The (common) beliefs of voters determine which primary election they participate in. Let $\mathcal{P}^e = (w_D^e, z_D^e, w_R^e, z_R^e)$ denote the voters' (common) expectations of party platforms. Define set of voters who weakly prefer the platform (w_k^e, z_k^e) as $H_k(\mathcal{P}^e) = \{(x,y) \in \mathbb{R}^2 : U(x,y; w_k^e, z_k^e) \geq U(x,y; w_{\sim k}^e, z_{\sim k}^e)\}$, where $\sim k = R(D)$ if $k = D(R)$. As shown in Figure x, these sets are separated by the straight line passing through the midpoint of the line joining (w_D^e, z_D^e) and (w_R^e, z_R^e) and perpendicular to it:

$$\bar{x} = \frac{{w_D^e}^2 + {z_D^e}^2 - [{w_R^e}^2 + {z_R^e}^2]}{2(w_D^e - w_R^e)} - \left[\frac{z_D^e - z_R^e}{w_D^e - w_R^e}\right] y. \tag{7}$$

Utility is the negative of the square of the Euclidean distance from (w_k^e, z_k^e), so all voters on this line are indifferent between the two (expected) platforms. Voters on either side have a strict preference for one of the parties and, so, attend the primary of that party. We assume that voters on the line attend both primaries.

In each primary, two candidates vie to represent their party in the national election. Candidates are office motivated, i.e, care only about winning the primary election. By the Downsian logic, the candidates offer the same platform. If policies do not fully pin down votes for the two candidates because of random variation in how much voters like a candidate independent of his or her proposed policy, then, as shown in Lindbeck and Weibull (1987), the equilibrium platform coincides with the solution of the utilitarian social welfare maximization problem.

[6] If $w_D = w_R$ but $z_D \neq z_R$, an analogous argument establishes (5). If $w_D = w_R$ and $z_D = z_R$, then all voters (not just the one with mean preferences) are indifferent between the two parties and the D party will win if $A > 0$. Thus $\bar{A}(\mathcal{P}) = 0$ and (5) remains true.



With this equivalence in mind, we assume that the D-party platform is determined by the following programming problem:

$$\max_{(w,z)} \left\{ \begin{array}{c} \pi(w,z,w_R,z_R)\,\mathbb{E}_{(x,y)}\left(\left[-(w-x)^2-(z-y)^2\right]\mathbb{1}_{\{(x,y)\in \mathbb{H}_D(\mathcal{P}^e)\}}\right) \\ +\left[1-\pi(w,z,w_R,z_R)\right]\mathbb{E}_{(x,y)}\left(\left[-(w_R^e-x)^2-(z_R^e-y)^2\right]\mathbb{1}_{\{(x,y)\in \mathbb{H}_D(\mathcal{P}^e)\}}\right) \end{array} \right\}. \tag{8}$$

Here $\pi(w,z,w_R,z_R)$ is the probability of the D party winning the national election given these policies. Thus, we assume that a party's social welfare takes account of the likelihood of the party winning the national election on basis of any given platform.[7] Note that the social welfare of the D party depends only on the voters who weakly prefer the D party given beliefs $\mathcal{P}^e$, i.e., those who belong to $H_D(\mathcal{P}^e)$.

Using the fact that the probability of a D party win is $(1-F([w^2+z^2]-[w_r^2+z_R^2])$, the choice problem can be expressed as

$$\max_{(w,z)} \left\{ \begin{array}{c} -L(w_R,z_R)+ \\ \left[1-F\left(w^2+z^2-[{w_R}^2+{z_R}^2]\right)\right]\times \\ m(H_D(\mathcal{P}^e))\mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)}\left\{-\left[(w-x)^2+(z-y)^2\right]+\left[(w_R-x)^2+(z_R-y)^2\right]\right\} \end{array} \right\}, \tag{9}$$

where $m(H_D(\mathcal{P}^e))$ is the measure of the set people who attend the D-party primary, $\mathbb{E}$ is expectation taken with respect to the distribution of (x,y) conditional on $(x,y)\in H_D(\mathcal{P}^e)$, and $L(w_R,z_R)$ is

[7] At this choice stage, the parties contemplate platforms that are different from what voters expect. In equilibrium the chosen platforms will coincide with expected ones

shorthand for $\mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)}[(w_R-x)^2+(z_R-y)^2]$. Ignoring the constant term L, the objective function is thus the product of the probability of D party winning the national election and the total gain of the people attending the primary from enjoying D party's policies rather than those of the R party.

Analogously, but recognizing that R party's probability of winning the election is $F([w_D^2+z_D^2]-[w^2+z^2])$, R party's choice problem reduces to

$$\max_{(w,z)}\left\{\begin{array}{c} -L(w_D,z_D)+ \\ F\left(w_D^2+z_D^2-[w^2+z^2]\right)\times \\ m(H_R(\mathcal{P}^e))\mathbb{E}_{(x,y)|H_R(\mathcal{P}^e)}\left\{-\left[(w-x)^2+(z-y)^2\right]+\left[(w_D-x)^2+(z_D-y)^2\right]\right\} \end{array}\right\}. \tag{10}$$

If the k party cared *only* about winning the national election, its optimal policy would be to set $(w_k,z_k)=(0,0)$ because this choice maximizes the probability of a k-party win. On the other hand, if it cared *only* about maximizing the total gain conditional on a win, its optimal policy would be set to $(w_k,z_k)=(\mathbb{E}(x|H_k(\mathcal{P}^e)),\mathbb{E}(y|H_k(\mathcal{P}^e))$ as $\mathbb{E}_{(x,y)|H^k(\mathcal{P}^e)}\left[(w-x)^2+(z-y)^2\right]$ is minimized if w_k and z_k are set to the relevant conditional means of x and y, respectively. Generally speaking, these two goals are mutually incompatible and the party's best response strikes a balance between them.[8]

For more intuition, first-order conditions are helpful. If at the optimum $1-F(\cdot)>0$ and $F'(\cdot)$ exists, k party's optimal policy will satisfy the following marginal conditions:

$$(1+\phi_k)w=\mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}x \quad \text{and} \quad (1+\phi_k)z=\mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}y, \tag{11}$$

where

$$\begin{aligned}\phi_D=&\left[\frac{F'\left(w^2+z^2-w_R^2-z_R^2\right)}{1-F\left(w^2+z^2-w_R^2-z_R^2\right)}\right]\times \\ &\qquad\mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)}\{-(w-x)^2-(z-y)^2+[(w_R-x)^2+(z_R-y)^2]\},\end{aligned} \tag{12}$$

[8]In Roemer (2001), these two extremes figure as motivations ascribed to "opportunists" (who care only about winning) and "militants" (who care only about the gain, conditional on winning). What we are referring to as the social planner is called "reformists".

and

$$\phi_R = \left[\frac{F'\left(w_D^2 + z_D^2 - w^2 - z^2\right)}{1 - F\left(w_D^2 + z_D^2 - w^2 - z^2\right)} \right] \times$$
$$\mathbb{E}_{(x,y)|H_R(\mathcal{P}^e)}\{-(w-x)^2 - (z-y)^2 + [(w_D - x)^2 + (z_D - y)^2]\}. \tag{13}$$

The terms ϕ_D and ϕ_R can be negative only if the expectation terms in (12) and (13), respectively, are negative. But these terms are the net gain from winning the election for party D and R, respectively. At an optimum, these net gains can never be negative because a party always has the option of choosing the other party's policies and ensuring a zero net gain. Therefore, at an optimum, $(1 + \phi_k) \geq 1$. It follows that the optimal (w, z) for each party is on the line segment connecting $(0,0)$ to $(\mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}x, \mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}y)$. This clearly shows that parties balance the two polar objectives mentioned above, namely, maximizing the gain conditional on a win and maximizing the probability of a win. Furthermore, the closeness of (w_k, z_k) to the origin depends in an intuitive way on the distribution of A at the optimum. The more concentrated the distribution, i.e., the larger F', the larger is ϕ_k and closer is k party's optimal choice to the origin: The party is willing to put up with a platform closer to the origin if moving towards its mean preferences lowers the probability of a win substantially.

3.4 Equilibrium

An equilibrium is a 4-tuple $(w_D^*, z_D^*, w_R^*, z_R^*)$ such that if $\mathcal{P}^e = (w_D^*, z_D^*, w_R^*, z_R^*)$ then (w_D^*, z_D^*) solves (9) and (w_R^*, z_R^*) solves (10).

4 Polarized Equilibria and the Inverse Association

We turn now to explaining the inverse association. We focus on equilibria that are symmetric and polarized along at least one dimension, i.e., equilibria in which $(w_D^*, z_D^*) = -(w_R^*, z_R^*)$ and $(w_k^*, z_k^*) \neq (0,0)$.[9] We assume that A is uniformly distributed on the interval $[-\alpha, \alpha]$, $\alpha > 0$, and that voter ideal points are uniformly distributed on a disk with radius $\theta > 0$ centered at $(0,0)$.

The determination of symmetric equilibria is done in three steps. In the first step, the mean preferences of voters attending each primary is solved, given common (symmetric) beliefs about

[9] A symmetric equilibrium in which both parties offer $(0,0)$ is possible, but we ignore this equilibrium as it's not relevant for addressing the facts.

party platforms. In the second step, optimal policies of the two parties are solved, given mean preferences. In the final step, the first two steps are combined to isolate the set of beliefs that generate optimal policy platforms that, in turn, confirm those beliefs.

Step 1. *Beliefs to Conditional Means*

We consider all $\mathcal{P}^e$ such that $(w_D^e, z_D^e) = -(w_R^e, z_R^e) \neq (0,0)$, i.e., beliefs that are symmetrically polarized along at least one dimension. For concreteness, we will assume that $w_D^e = -w_R^e \neq 0$. The solution for the $w_k^e = 0$ (but $z_k^e \neq 0$) case is given by the limit of the solution as $w_k^e \to 0$.

Given $w_D^e \neq 0$ and symmetry of beliefs, it follows from (7) that the line separating D-party primary attendees from R-party primary attendees is given by

$$\bar{x}(y;\mathcal{P}^e) = -\left[\frac{z_D^e - z_R^e}{w_D^e - w_R^e}\right] y = -\left[\frac{z_D^e}{w_D^e}\right] y.$$

This is a straight line that goes through the origin and divides the circular support of the voters ideal point distribution into two semi-circles.

Figure 4:
Partisan Sorting Based on Beliefs

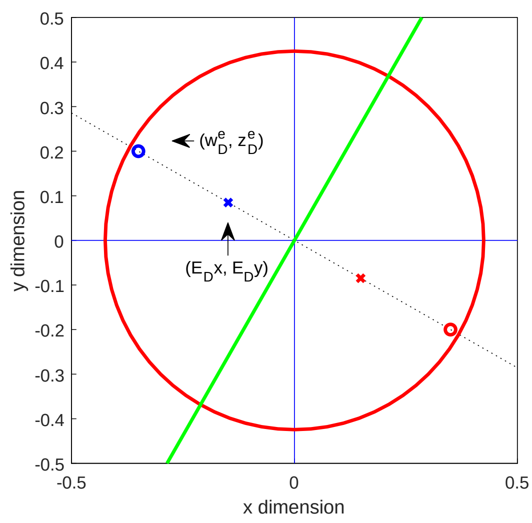


Figure 4 shows an example where points marked by circles represent the common symmetric beliefs about the platforms of the D and R parties, respectively. In this figure, $w_D^e < 0$ and $z_D^e > 0$ and, so, the line separating voters is the positively sloped straight line going through the origin and perpendicular to the dotted line joining the circle points. All voters with ideal points that lie in the semi-circle to the left of the straight line attend the D-party primary and all voters with ideal points in the semi-circle to the right of the straight line attend the R-party primary.

Since the distribution of the voter ideal points is uniform over each semi-circle, the points $(\mathbb{E}_D x, \mathbb{E}_D y)$ and $(\mathbb{E}_R x, \mathbb{E}_R y)$ coincide with the center of gravity, or centroid, of the D-party and R-party semi-circles, respectively. This allows for an easy computation of conditional means $(\mathbb{E}_k x, \mathbb{E}_k y)$ using the fact that the centroid of a semi-circle of radius θ lies on the radius that is perpendicular to the base of the semi-circle and a distance $4\theta/3\pi$ from the center. In Figure 4, the conditional means of the D- and R-party semi-circles are denoted by points marked by 'x'.

To determine the coordinates of 'x', note that since the distance of 'x' from the origin is $\left(\frac{4\theta}{3\pi}\right)$, we must have that $(\mathbb{E}_k x)^2 + (\mathbb{E}_k y)^2 = \left(\frac{4\theta}{3\pi}\right)^2$. And, since the conditional mean is located on the radius perpendicular to the base, we must have $\mathbb{E}_k y/\mathbb{E}_k x = w_k^e/z_k^e$. Denote w_k^e/z_k^e by β. Then,

$$(\mathbb{E}_k x)^2 = \left[\frac{4\theta}{3\pi}\right]^2 \frac{1}{1+\beta^2} \text{ and } (\mathbb{E}_k y)^2 = \beta^2 (\mathbb{E}_k x)^2. \tag{14}$$

These equations determine the magnitudes of $\mathbb{E}_k x$ and $\mathbb{E}_k y$ as a function of β^2 but not their signs. For their signs, note that $(\mathbb{E}_k x, \mathbb{E}_k y)$ is on the line connecting $(0,0)$ to (w_k^e, z_k^e) and, so, the signs of $\mathbb{E}_k x$ and $\mathbb{E}_k y$ must be the signs of w_k^e and z_k^e, respectively. Combining,

$$\begin{bmatrix} \mathbb{E}_k x \\ \mathbb{E}_k y \end{bmatrix} = \begin{bmatrix} \operatorname{sgn}(w_k^e)\frac{4\theta}{3\pi}\frac{1}{\sqrt{1+\beta^2}} \\ \operatorname{sgn}(z_k^e)\frac{4\theta}{3\pi}\sqrt{\frac{\beta^2}{1+\beta^2}} \end{bmatrix}. \tag{15}$$

Note that as $\beta^2 \to \infty$

$$\begin{bmatrix} \mathbb{E}_k x \\ \mathbb{E}_k y \end{bmatrix} \to \begin{bmatrix} 0 \\ \operatorname{sgn}(z_k^e)\frac{4\theta}{3\pi} \end{bmatrix}, \tag{16}$$

which is exactly the outcome when $w_k^e = 0$ and $z_k^e \neq 0$.

Step 2. *Conditional Means to Party Platforms*

In a symmetric equilibrium, $(w_k^*, z_k^*) = (-w_{\sim k}^*, -z_{\sim k}^*)$. Making this substitution in the k party's first-order condition (11), using the fact that $F'(0) = 1/[2\alpha]$ and $F(0) = 1/2$, and substituting $h^* \mathbb{E}_k x$

and $h^*\mathbb{E}_k y$ for w_k^* and z_k^* respectively, lead to the following pair of first-order conditions:

$$[\alpha + 4h^*(\mathbb{E}_k x)^2 + 4h^*(\mathbb{E}_k y)^2]h^*\mathbb{E}_k x = \alpha\mathbb{E}_k x \tag{17}$$

$$[\alpha + 4h^*(\mathbb{E}_k x)^2 + 4h^*(\mathbb{E}_k y)^2]h^*\mathbb{E}_k y = \alpha\mathbb{E}_k y. \tag{18}$$

From (14), the sum of the squared conditional means is $[4\theta/3\pi]^2$ (for both parties). Furthermore, at least one of the conditional means is non-zero. Eliminating the non-zero conditional mean from the appropriate first-order condition leads to a quadratic in h^*:

$$4\left[\frac{4\theta}{3\pi}\right]^2 h^{*2} + \alpha h^* - \alpha = 0 \tag{19}$$

The quadratic has positive and negative roots. Since h^* is just $(1+\phi)^{-1}$ and $\phi \geq 0$ at an optimum, the positive root is the relevant one. Thus, the solution is:

$$h^* = \frac{-\alpha + \sqrt{\alpha^2 + 16\alpha[4\theta/3\pi]^2}}{8[4\theta/3\pi]^2} < 1 \tag{20}$$

and the optimal party platforms are given by

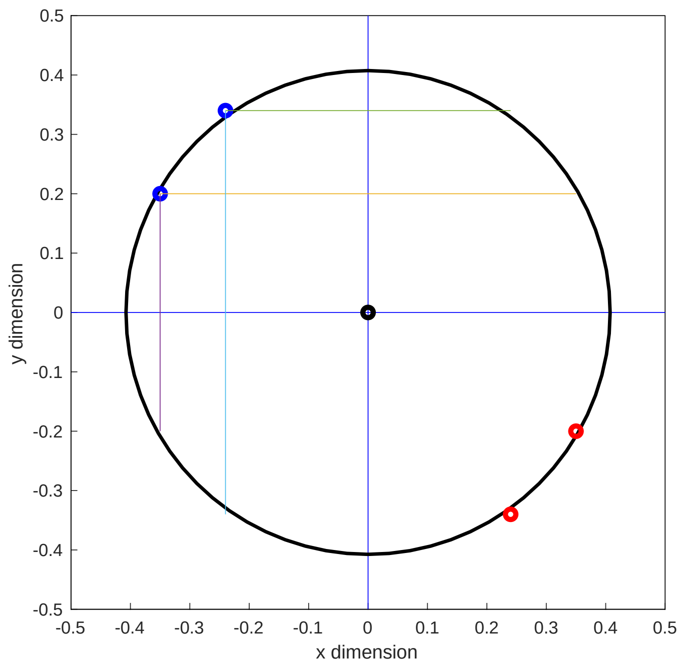
$$\begin{bmatrix} w_k^* \\ z_k^* \end{bmatrix} = \begin{bmatrix} \operatorname{sgn}(w_k^e)\frac{4\theta}{3\pi} \cdot h^* \cdot \frac{1}{\sqrt{1+\beta^2}} \\ \operatorname{sgn}(z_k^e)\frac{4\theta}{3\pi} \cdot h^* \cdot \sqrt{\frac{\beta^2}{1+\beta^2}} \end{bmatrix} \quad k \in \{D, R\}. \tag{21}$$

Step 3. *Equilibrium*:

Equation (21) says that optimal platforms depend only the direction of the belief vector, not its position. Therefore, to construct an equilibrium we may choose any direction for the belief vector, determine the optimal policy platforms, and then set the *position* of the belief vector to match the optimal platforms. Thus, there is a continuum of equilibria indexed by beliefs. The equilibrium set is a circle of radius $[4\theta/3\pi]h^*$ centered at $(0,0)$.

We come now to the key equilibrium implication of the model: Across equilibria there is an inverse association between the ideological distances of the two parties along x and y dimensions. The reason can be seen easily in Figure 5, which plots the circular equilibrium set. In this figure, any two diametrically opposed points is a symmetric polarized equilibrium. Moving along the circle we trace out different equilibria. Comparing across equilibria, it is evident that the ideological distance along the x dimension can increase only at the expense of the ideological distance along the y dimension and vice versa.

Figure 5:
Partisan Sorting and Inverse Association



More formally,

Proposition (Inverse Association)**.** *Let $\mathcal{P}^e$ and $\mathcal{P}'^e$ be any two symmetric equilibrium belief vectors, with w_D^e and $w_D'^e$ not equal to zero. Then, $\delta_x > \delta_x'$ if and only if $\delta_y < \delta_y'$.*

Proof. Let $\beta = z_D^e / w_D^e$ and $\beta' = z_D'^e / w_D'^e$. By Proposition 5 and 6, we have that

$$|w_D^e| = h^* \left[\frac{4\theta}{3\pi}\right] \frac{1}{\sqrt{1+\beta^2}} \text{ and } |w_D'^e| = h'^* \left[\frac{4\theta}{3\pi}\right] \frac{1}{\sqrt{1+\beta'^2}}, \tag{22}$$

$$|z_D^e| = h^* \left[\frac{4\theta}{3\pi}\right] \sqrt{\frac{\beta^2}{1+\beta^2}} \text{ and } |z_D'^e| = h'^* \left[\frac{4\theta}{3\pi}\right] \sqrt{\frac{\beta'^2}{1+\beta'^2}}. \tag{23}$$

By equations (20) and (14), we have that

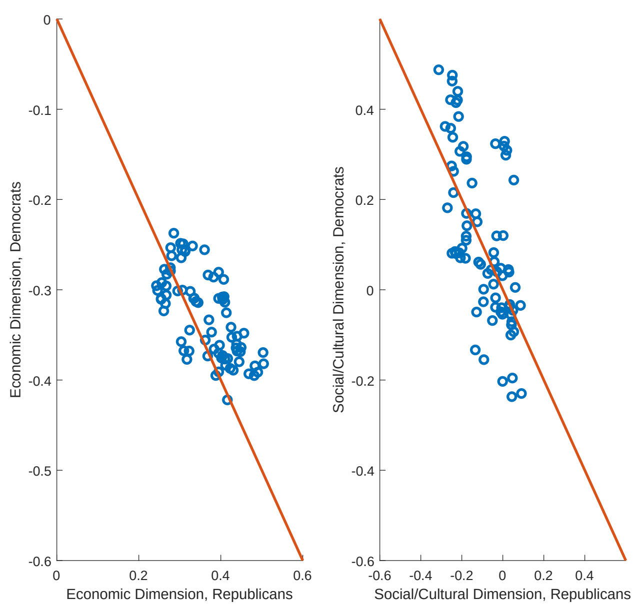
$$h^* = \frac{-\alpha + \sqrt{16\alpha\left[\frac{4\theta}{3\pi}\right]^2 + \alpha^2}}{8\left[\frac{4\theta}{3\pi}\right]^2} = h'^*. \tag{24}$$

Therefore, if $\delta_x > \delta'_x$ then $\beta < \beta'$ and so $\delta'_y < \delta_y$. Conversely, if $\delta'_y < \delta_y$ then $\beta < \beta'$ and, so, $\delta_x > \delta'_x$. □

5 Secular Changes in Polarization of Political Parties in the US

Up to this point, we have presented a model in which polarization along one dimension interacts with polarization along the other dimension. The interaction arises through the value of β^2. To recall, as β^2 increases, polarization along the x dimension decreases and that along the y dimension increases. In this section, we ask if this implication of the model has quantitative bite. That is, if the model is supplied with an empirical analog of β^2 for each Congress, can it reproduce the ideological distances for each Congress we see in Figures 1 and 2?

Figure 6:
Approximate Symmetry of Ideological Positions



Source: https://voteview.com/ and authors' calculations

We are motivated to do this because historical experience seems reasonably consonant with a key maintained assumption of the theory, namely, that policy outcomes are symmetric. Figure 6 gives scatter plots of the ideological positions of the two parties for the two dimensions. In each panel, the solid downward sloping line has a slope of -1 and passes through the origin $(0,0)$. If the ideological positions of the two parties for a dimension were exactly symmetrically opposed, all points would lie on the solid line. While not exactly symmetric, the actual points cluster around the (negative) 45 degree line. Furthermore, the fact that ideological positions of the two parties have varied considerably while remaining roughly symmetric suggests that the assumption of a radially symmetric distribution of voter ideal points is not too far off the mark.

For a given Congress, we take the empirical analog of β^2, denoted $\hat{\beta}^2$, to be $([D2-R2]/[D1-R1])^2$, where $D1$ and $D2$ are the mean ideological position of Democrats for the liberal-conservative and social/cultural dimensions, respectively. In the theory, these correspond to w_D and z_D, respectively. Similarly, $R1$ and $R2$ are the mean ideological positions for the liberal-conservative and social/cultural dimensions for the Republicans and w_R and z_R are the theory counterparts. The theory counterpart of $\hat{\beta}^2$ is β^2 since $w_R = -w_R$ and $z_D = -z_R$ and $z_D/w_D = \beta$.

In order to connect $\hat{\beta}^2$ to the theoretically predicted ideological distances, we need values of θ and α. For this, note that in any equilibrium the (common) Euclidean distance of a party's platform from the origin is a constant given by

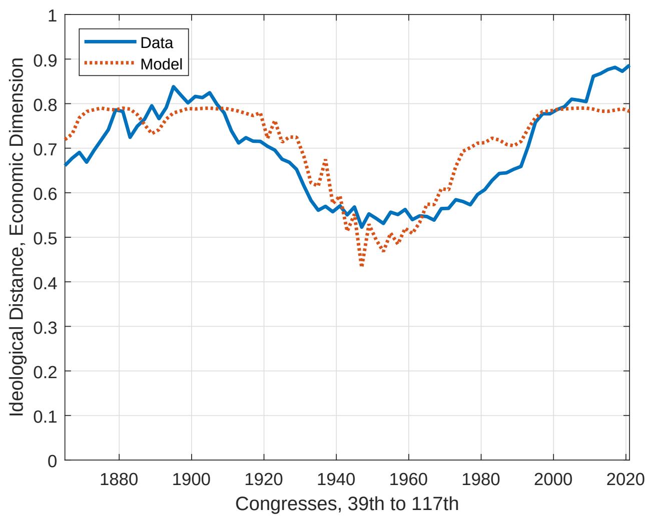
$$\begin{aligned}\sqrt{(w_k^*)^2+(z_k^*)^2} =& \sqrt{h^{*2}[(\mathbb{E}_k x)^2+(\mathbb{E}_k y)^2]}\\ =& h^*\left[\frac{4\theta}{3\pi}\right]\\ =& \frac{-\alpha+\sqrt{16\alpha\left[\frac{4\theta}{3\pi}\right]^2+\alpha^2}}{8\left[\frac{4\theta}{3\pi}\right]}.\end{aligned} \tag{25}$$

In actuality, the Euclidean distance (from the origin) of a party's ideology has not been constant over time and neither have the Euclidean distances of the two parties been exactly equal to each other at all times. However, it is the case that the distances of the D-party platform averaged over time is quite close to the distances of the R-party platform averaged over time, these being 0.39 and 0.40, respectively. This near-equality of the time averages is another reflection of the rough symmetry of ideological positions of the two parties. We set $\theta = 1$ and pick α so that the expression in (25) is equal to $(0.39+0.40)/2$.[10] With these settings of θ and α, we use time series

[10]If we had picked a different value for θ but altered the value of α so that the expression in (25) evaluated to 0.395, the theoretically predicted equilibrium outcomes would remain unchanged. In this sense, the choice of θ is a normalization. We note, however, that the method employed by the DW-Nominate program to infer the ideological positions

on $\hat{\beta}^2$ to get the theoretically predicted time series for δ_x and δ_y using the expressions in (22) and (23).

Figure 7:
Ideological Distance, Economic Dimension
Data and Model Predictions



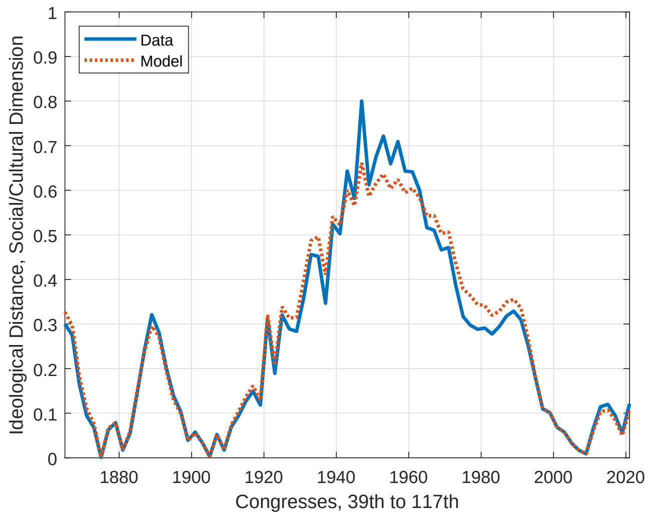
Source: https://voteview.com/ and authors' calculations

Figures 7 and 8 plots δ_x along with $|D1-R1|$ and δ_y along with $|D2-R2|$, respectively. In these figures, the solid line is history and the dotted line is the prediction of the calibrated model. In both figures, the predicted ideological distance tracks the actual ideological distances, with the correspondence being remarkably close for δ_y and $|D2-R2|$. These plots reveal that knowledge of the *ratio* $[(D2-R2)/(D1-R1)]^2$, which is $\hat{\beta}^2$, allows prediction of the *levels* of $|D2-R2|$ and $|D1-R1|$. Thus, during the '40s and '50s, when $\hat{\beta}^2$ was rising, the model predicts that $|D1-R1|$ must fall as observed, and in the post-1965 era when $\hat{\beta}^2$ fell, the model predicts that $|D1-R1|$ should rise, as observed.

The premise on which this effect is based is diversity with respect to the social/cultural ideology among both economic liberals and economic conservatives. When a polity is polarized along the social dimension, it is possible — i.e., it can be an equilibrium outcome — that modest economic conservatives (liberals) who are socially very conservative (liberal) can align themselves with the economically *liberal* (*conservative*) party because the economically liberal (conservative)

of members of Congress assumes that the position is always contained in $[-1,1]\times[-1,1]$. Thus, by construction, the mean ideological position of a party along any dimension cannot exceed 1 in absolute value. For our model to be consistent with this restriction, θ cannot exceed $3\pi/4 = 2.37$; if it did, there will exist values of α and β^2 for which the equilibrium ideological position of parties along at least one dimension will exceed 1.

Figure 8:
Ideological Distance, Social/Cultural Dimension
Data and Model Predictions



Source: https://voteview.com/ and authors' calculations

party happens also to be the more socially conservative (liberal) party. When they do, the alignment leads to *less* sorting, i.e., *more* homogeneity, along the economic dimension across the two parties and, therefore, smaller ideological distance between the two parties along the economic dimension.

As an example of this type of sorting, Figure 9 displays the ideological positions of members of the House of Representatives for 88th (1963-1965) Congress. Note the presence of economic conservatives in the Democratic party and economic liberals in the Republican party. As a contrast to this type of sorting, Figure 10 displays the ideological map of the 117th (2021-2023) Congress. We no longer see any economic conservatives in the Democratic party or economic liberals in the Republican party. The result of this sorting pattern is that the two parties are more ideologically distant along the economic dimension.

These findings bring a novel perspective on the decline and subsequent rise in polarization along the economic dimension in the post-WWII era. In the years leading up to the Civil Rights Act of 1964, the ideological distance between the Democratic and Republican parties widened along the social/cultural dimension. According to our theory, this widening was a key factor in accounting for the *narrowing* of ideological distance along the economic dimension during the same period. Similarly, the narrowing of ideological distance along the social/cultural dimension

Figure 9:
Member Ideologies, 88th Congress (1963-1965)

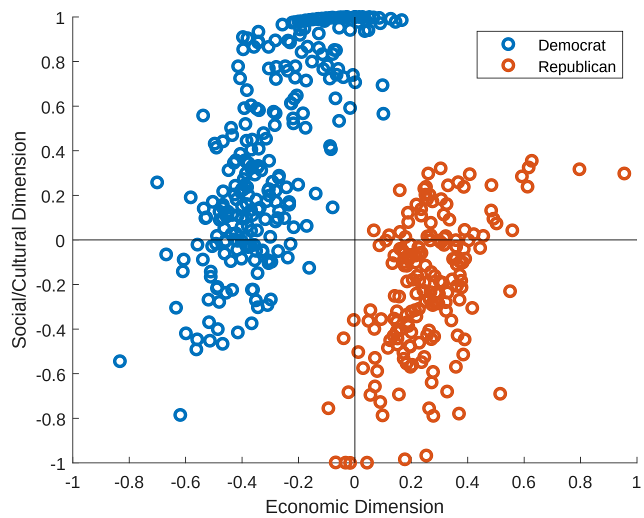


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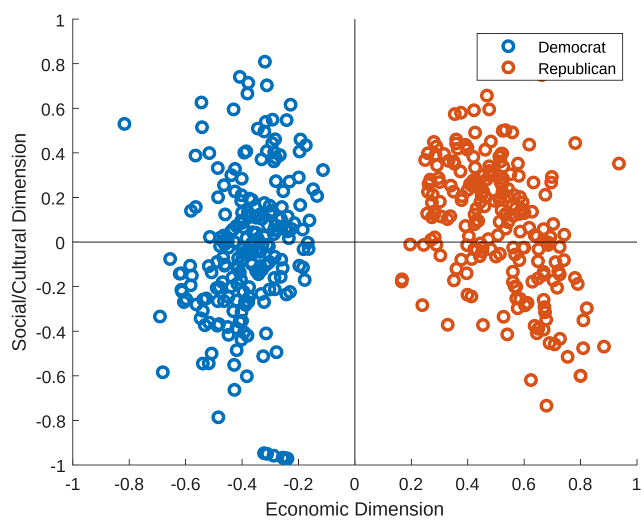
Figure 10:
Member Ideologies, 117th Congress (2021-2023)



Source: https://voteview.com/

is an important ingredient in the *widening* of ideological distance along the economic dimension since 1965.

Finally, we note that the increase in polarization that has occurred since 2000 along the first dimension is not explained by changes in the ideological distance along the second dimension. Since the turn of the 21st century, changes in the ideological distance along the second dimension have been small and these changes do not imply significant changes to polarization along the first dimension, which is predicted to be essentially flat. Thus our results are not in conflict with studies (cited earlier) that have focused on reasons (inequality, campaign contributions, biased beliefs) to account for increasing polarization along the first dimension in recent decades.

6 Summary

Viewed over a long stretch of time, roll call voting records for Democratic and Republican party representatives in Congress show an inverse association between the mean ideological distance along the social/cultural dimension and the mean ideological distance along the economic dimension.

We presented a model of party competition with a two-dimensional ideology space to explain this fact. In our model, voters have (common) beliefs about the ideological stance of the two parties and join a party based on these beliefs. Each party chooses its national platform by balancing the preferences of the voters who compose the party and the preferences of the polity at large since a party can come to power only if it wins the general election.

If the distribution of voter ideologies is radially symmetric, there is a continuum of polarized equilibria indexed by voter beliefs. Moving from an equilibrium in which the two parties are more polarized along the social/cultural dimension to one in which they are less polarized along this dimension implies an *opposite* movement for polarization along the economic dimension, i.e., from a less economically polarized equilibrium to a more economically polarized equilibrium. This occurs as a result of the differential sorting into the two parties induced by the different beliefs about what each party stands for.

Remarkably, the sorting and re-sorting of a stable distribution of voters into the two parties gives a very good *quantitative* account of the variation in ideological distances along the two dimensions over a long stretch of history. This finding brings a new perspective on the potential causes of political polarization that highlights the role of changes in beliefs as opposed to changes in preferences, law or technology.

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