Revealing Features from Optimal Choice

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Introduction

	SAMSUNG GALAXY S6 (AND EDGE)	HTC ONE M9	iPHONE 6	LG G3	MOTO X
STARTING PRICE (ON 2-YEAR CONTRACT)	N/A	N/A	$199	AT&T: $150, Verizon: Free, Sprint: $50	$99
SCREEN SIZE/ RESOLUTION	5.1 in, 2560 x 1440	5.1 in, 1920 x 1080	4.7 in, 1334 x 750	5.5 in, 2560 x 1440	5.2 in, 1920 x 1080
THICKNESS	S6: 6.8 mm, Edge: 7 mm	9.6 mm	6.9 mm	8.89 mm	3.81-9.9 mm
WEIGHT	S6: 138 g, Edge: 132 g	157 g	129 g	151.9 g	144 g
STORAGE	32/64/128 GB	32 GB	16/64/128 GB	32 GB	16/32/64 GB
OPERATING SYSTEM	Android 5.0 Lollipop with TouchWiz	Android 5.0 Lollipop with HTC Sense 7	iOS 8.1	Android 4.2.2 KitKat	Android 5.0 Lollipop
BATTERY LIFE	N/A (2550 mAh capacity for S6, 2600 mAh for S6 Edge)	N/A (2840 mAh capacity)	Up to 14 hours talk on 3G, up to 50 hours music, up to 11 hours video	Up to 21 hours talk, up to 28 days standby	Mixed usage up to 24 hours

Introduction

- Alternatives can be described in terms of their features
  - Features are measurable characteristics
  - **Examples:** consumer products, political parties, investment options etc.
- DM aggregates relevant features into a preference
- Analyst observes choices, but does not observe relevant features
- **What can we learn about which features are relevant from DM's choices?**

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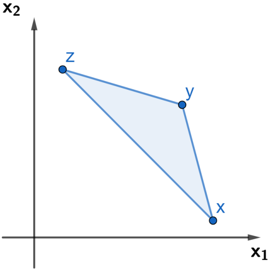
Introduction

- We develop a theory of relevant feature identification
- Useful for understanding consumption, investment, political decisions etc
    - **Example:** company launching a new product

Literature

- Alternatives = bundles of characteristics demanded by consumers (Lancaster, 1966). Objectively known and same for all
- We focus on identification
- Remain agnostic why DM neglects some features
  - Bounded rationality (N too large)
  - Preference

Example



- Features x_1 and x_2
- Preferences are strictly monotone (incr or decr) with respect to relevant features (not observed)
- **Choice of** y reveals both features as relevant
- **Choice of** x reveals nothing

Preview of results

- Characterise pairs (X, x^*) (= feasible set, observed choice) that reveal set of relevant features
    - partial identification
    - full identification
- Minimal data: single observation, multiple observations
- Minimal assumptions on behaviour: "Pareto optimality" w.r.t relevant features

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Framework

- $F = \{1, \dots, N\}$: set of possible features
- Nonempty $I \subseteq F$ is a type (not observed)
  - Type I only cares for features in I
- $X \subseteq \mathbb{R}^N$: set (nonempty closed bounded convex) of feasible alternatives
- Analyst observes X and choice $x^* \in X$, wants to identify DM's type I

Behavioural assumption

- Evaluation function $e : F \to \{-1, 0, 1\}$ captures how features matter
  - $e(i) = 1$: i matters positively
  - $e(i) = -1$: i matters negatively
  - $e(i) = 0$: i does not matter

Definition

$x \in X$ is e-admissible if

$$(\forall i : e(i)y_i \geq e(i)x_i) \ \& \ (\exists i : e(i)y_i > e(i)x_i) \ \Rightarrow \ y \notin X$$

Possible / identified type

Definition

A type $I \subseteq F$ is possible at $x^* \in X$ if there exists an evaluation function $e : F \to \{-1, 0, 1\}$ such that

(i) $\text{supp}(e) = I$

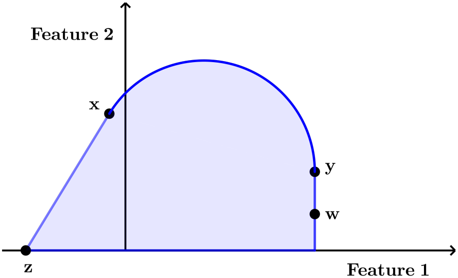
(ii) x^* is e-admissible

Definition

At x^*, the type is:

- fully identified = exactly one possible type at x^*
- partially identified = some type is possible at x^* but not all
- not identified = all types are possible at x^*

Example



- The type is
  - fully identified at x: type $\{1,2\}$
  - partially identified at y: types $\{1\}$, $\{1,2\}$
  - not identified at z: types $\{1\}$, $\{2\}$, $\{1,2\}$

Structure of possibility

- Types I, J are possible $\implies$ $I \cup J$ is possible?
  - Yes

Theorem

For any nonempty collection $\mathcal{I} \subseteq 2^F$ of types,

(i) *there exist $X \subseteq \mathbb{R}^N$ and $x^* \in X$ such that the set of possible types at x^* is $\mathcal{I}$ iff*

(ii) *$\mathcal{I}$ is closed under union*

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Partial identification

Partial identification is important in applications

- **Example:** alternatives = stocks, features = past returns, partial identification = DM looks only at the past two years of history

Theorem (partial identification)

The following statements are equivalent, for $x^* \in \partial X$: (∂X = boundary of X)

(i) *The type is partially identified at x^**

(ii) *There exists an $i \in F$ such that the elementary type $\{i\}$ is not possible at x^**

(iii) *Cone of feasible directions at x^* is not contained in any orthant*

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Elementary type condition

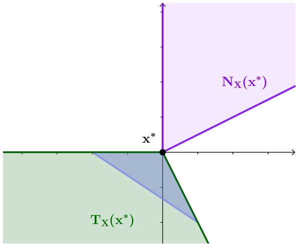
(ii) *There exists an* $i \in F$ *such that the elementary type* $\{i\}$ *is not possible at* x^*

- Reduces from $2^N - 1$ to N the number of types to check

Orthant condition

(iii) *Cone of feasible directions at x^* is not contained in any orthant*

- Cone of feasible directions at x^* is $\{\alpha\,(y - x^*)\,|y \in X,\, \alpha \geq 0\}$
- In R^2, orthant is a quadrant



- Enough richness in set of feasible tradeoffs btw the various dimensions at x^*
- Shape matters: "fatter" shape around x^* is better than "sharper" shape
- Orientation matters too

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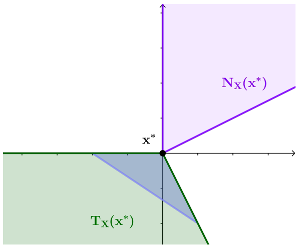

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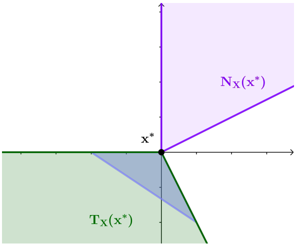

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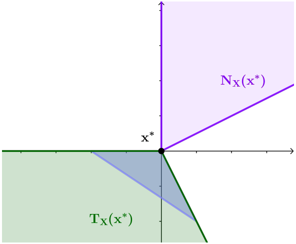

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Full identification

- **Recall:** full identification = exists exactly one possible type

Theorem

The type is fully identified at $x^* \in \partial X$ iff either

(i) $\dim X = N$ *and the cone of feasible directions at* x^* *contains*

$$\left\{x \in \mathbb{R}^N \mid (x_{i_1}, \ldots, x_{i_k}) \in K\right\},$$

where $\{i_1, \ldots, i_k\}$ *is the identified type and* K *is a closed convex cone in* $\mathbb{R}^k$*,* $k \leq N$*, such that int* $K \cup \{0\}$ *(int = interior) contains an orthant of* $\mathbb{R}^k$

(ii) $\dim X = N - 1$ *and* $x^* \in ri\, X$*. (ri = relative interior)*

Orthant condition

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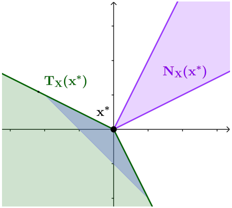
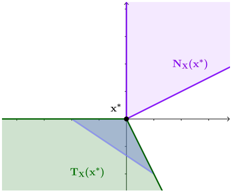


Yes: only $\{1, 2\}$ possible

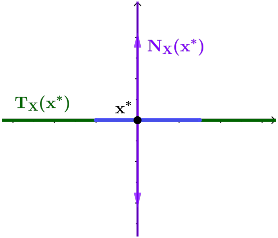


No: both $\{2\}$ and $\{1, 2\}$ possible

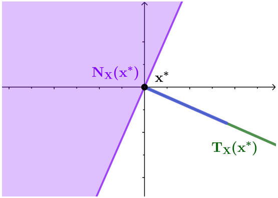
- Cone of feasible directions at x^* contains an orthant in its interior
- More subtle in higher dimensions

Interior condition

(ii) $\dim X = N - 1$ and $x^* \in ri\, X$.



Yes: only {2} possible ($x^* \in ri\, X$)



No: all types possible ($x^* \notin ri\, X$)

Corollary (dimensionality constraint)

Full identification is only possible if $\dim X \geq N - 1$

Interior condition

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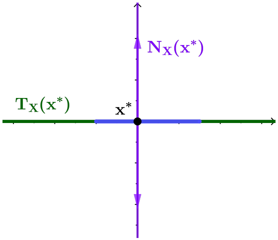

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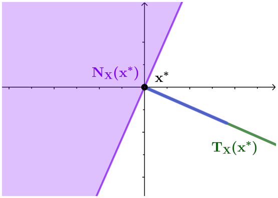

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Linear case

- Special case: X is a polytope
- DM maximises a linear objective function (wlog)
- Algebraic conditions for identification

Proposition (partial identification)

The type is partially identified at x^* $\iff$ *there is a column in* $\left(\bar{B}^T(x^*)\right)^{-1}$ *that contains both a positive and a negative entry*

- Similar conditions for X finite

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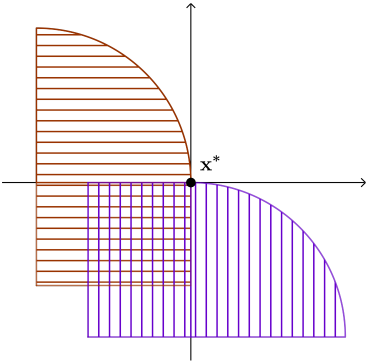
Multiple observations

- Additional observations may improve identification



- Type partially identified in both sets, but fully identified with 2 obs

Applications

- Understanding the motivations behind economic activities
- More accurate selection of observed characteristics
- Recommendation algorithms
- Design of experiments, political polls and market surveys (control over the feasible set)

Future work

- More on multiple observations
- Population of agents
- Stochastic choice
- Non-convex sets