Credit Rating under Ambiguity

Christian Hilpert[1] **Stefan Hirth**[2] Jan Pape[3] Alexander Szimayer[3]

[1] Lingnan College, Sun Yat-sen University
[2] Aarhus University and Danish Finance Institute
[3] Universität Hamburg

ESEM 2023, Barcelona

31 August 2023

Motivation

- Rating agencies need to aggregate opinions of several analysts into one rating.

  "Moody's ratings are initially determined or subsequently changed through committee. [...] At minimum, the committee includes a managing director or other designated individual and the lead analyst. The committee may be expanded to include as many perspectives and disciplines as are needed to address all analytical issues relevant to the issuer and the security being rated." (Moody's, 2009)

- Corporate decisions in groups with heterogeneous beliefs
  $\rightarrow$ ambiguous decision problems (Garlappi, Giammarino and Lazrak, 2017)

- How can we better understand the dynamic effect of the lack of agreement among analysts (ambiguity) on credit rating and default strategies?

This Paper: What do we do?

- Dynamic rating game between firm and rating agency (RA)
- RA aims to precisely rate firm to minimize reputation costs.
- Group of several credit analysts with different assessments of firm intangibles.
- Feedback effect between rating and financing conditions.
- Dynamic learning by RA and its analysts.

What do we get? Ambiguity affects rating strategy

- RA minimizes maximum reputation costs over analyst beliefs.
- Rating differs from the most conservative member's assessment. Rather, RA dynamically aggregates multiple prior beliefs in weighted average of the ratings of two members of credit analyst group.
- The more extreme and the lower the information content of the prior's belief, the more likely it is to be selected into the average.
- Possibly, only one prior with the lowest information content enters rating.
- As information unfolds and RA learns from firm's survival of apparent distress, it dynamically adjusts decision weights over time.

What do we get? Ambiguity feeds back into firm decisions

Impact of feedback effects hinges on relative direction of analyst beliefs.

- For commonly pessimistic beliefs, RA selects least pessimistic belief.

  $\rightarrow$ Ambiguity delays firm's strategic default.

- For balanced disagreement, RA balances beliefs.
- For commonly optimistic beliefs, RA selects least optimistic belief.

  $\rightarrow$ Ambiguity accelerates firm's strategic default.

Where do we stand? Feedback effects and credit risk

- Debt valuation in structural models: Leland (1994), Goldstein, Ju and Leland (2001), Duffie and Lando (2001)
- Learning from strategic actions: Grenadier, Malenko and Malenko (2016)
- Feedback effects: Manso, Strulovici and Tchistyi (2010), Manso (2013), Hilpert, Hirth and Szimayer (2022)
- Ambiguity in corporate finance:Garlappi, Giammarino and Lazrak (2017), Malenko and Tsoy (2020), Izhakian, Yermack and Zender (2022)
- Ambiguity in credit risk: Boyarchenko (2012), Augustin and Izhakian (2020)

Model: Players, state process and information structure

- Two players: firm and RA
- Firm's true cash flow X, geometric Brownian motion
- RA observes cash flow imperfectly

$$D_t = \tilde{\theta} X_t \, , t \geq 0$$

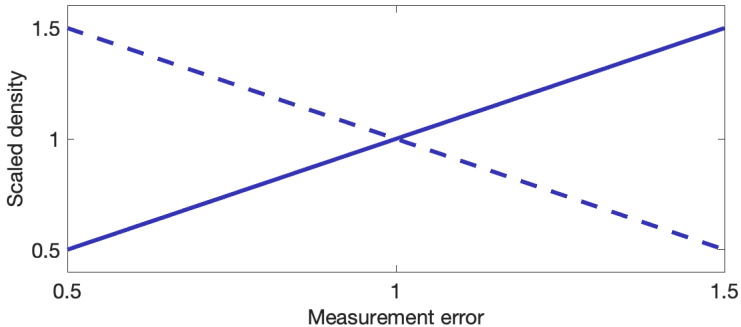
- Distortion $\tilde{\theta}$ is persistent.

Learning and belief updating

- RA has n priors π_i with density ϕ^{π_i} about the law of $\tilde{\theta}$ on the identical support $\Theta = [\underline{\theta}, \overline{\theta}]$, for each group member $i = 1, ..., n$.
- RA learns from survival of apparent distress: If firm does not default despite low cash flows, RA constrains how much it can have overestimated cash flow

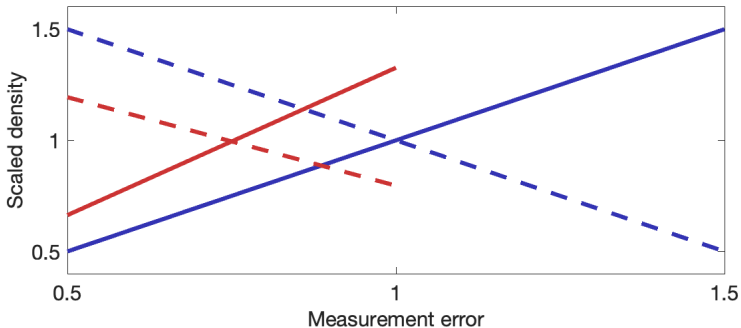
  $\Rightarrow$ eliminate high distortions θ.
- Information generating process:
  Running minimum of observed cash flow $E_t = \inf_{0 \leq s \leq t} D_s$.

Learning under multiple priors ($n = 2$)



- optimist (dashed), pessimist (solid)

Learning under multiple priors ($n = 2$)



- RA updates belief π_i with prior density ϕ^{π_i} on measurement error

Rating strategy

- Rating = estimated distance to default
  $\leftrightarrow$ predicted default cash flow level $\hat{D}_t^\star$.

$$R_t = \frac{D_t}{\hat{D}_t^\star}$$

- Minimize reputation costs of RA under ambiguity aversion

$$U_{RA}^{\pi}(D^\star, \hat{D}^\star) = -\mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(\max_{i=1,\dots,n}\int_\Theta \left[\hat{D}_t^\star - D^\star(\theta)\right]^2 \phi^{\pi_{i,t}}(\theta)d\theta\right)dt\right]$$

where $D^\star(\theta)$ = firm's default level.

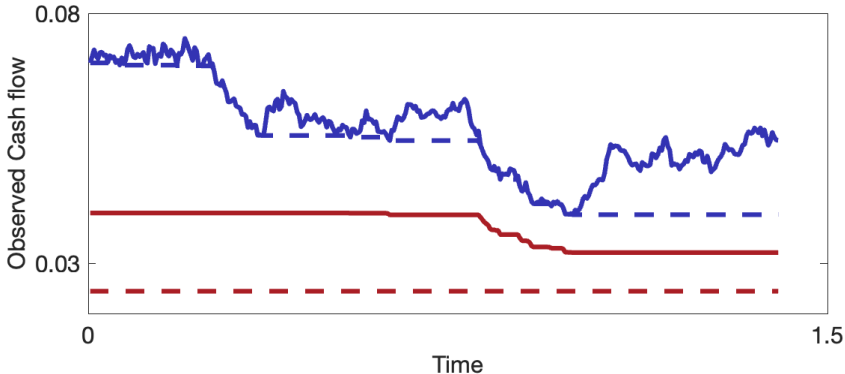
Firm strategy

- Strategy: measurement error-dependent default level $D^\star(\theta)$.
- Aim: maximize equity value

$$U_F^{(\theta)}(D^\star, \hat{D}^\star) = \mathbb{E}\left[\int_0^{\tau(\theta)} e^{-rt}\Big(D_t/\theta - C(D_t/\hat{D}_t^\star)\Big)dt\right]$$

- Feedback loop between RA with rating strategy $\hat{D}^\star$ and firm with default strategy $D^\star$ driven by interest payment C.

The model at work ...



- observed cash flow D (blue solid) and its running minimum (blue dashed)
- estimated default level $\hat{D}^\star$ (red solid) and true default level $D^\star(\theta)$ (red dashed)

Theory results

- Best response of firm: cutoff rule
- Best response of RA: learning, analyst-weighting
- Equilibrium I: existence
- Equilibrium II: uniqueness and ODE

Rating agency's best response

- RA's strategy

$$\hat{D}^\star(e; D^\star) = \arg\min_{D>0} \left\{ \max_{i=1,\ldots,n} \underbrace{\left(D - \hat{D}_i^\star(e; D^\star)\right)^2}_{\text{bias}^2} + \underbrace{\nu_i^2(e; D^\star)}_{\text{variance}} \right\},$$

minimizes maximum mean squared error ($\text{bias}^2 + \text{variance}$) over analysts.

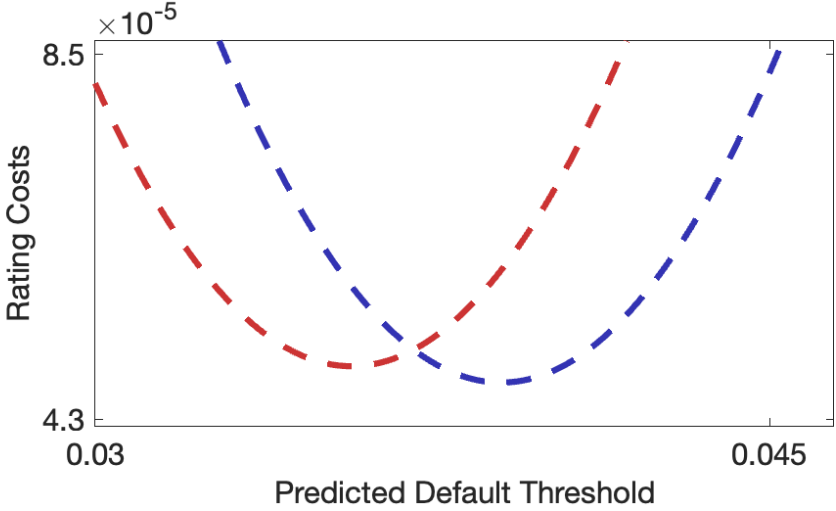
Rating agency's best response

- Think of i as most pessimistic and j as most optimistic prior.
- Best response can be restated in terms of weighted average of two best responses $\hat{D}_i^\star$ and $\hat{D}_j^\star$ as follows

$$\begin{aligned} \hat{D}^\star &= w\,\hat{D}_i^\star + (1-w)\,\hat{D}_j^\star\,, \\ w &= \frac{1}{2} + \mathbf{1}_{i\neq j}\,\frac{\nu_i - \nu_j}{2\left[\hat{D}_i^\star - \hat{D}_j^\star\right]^2}\,. \end{aligned}$$

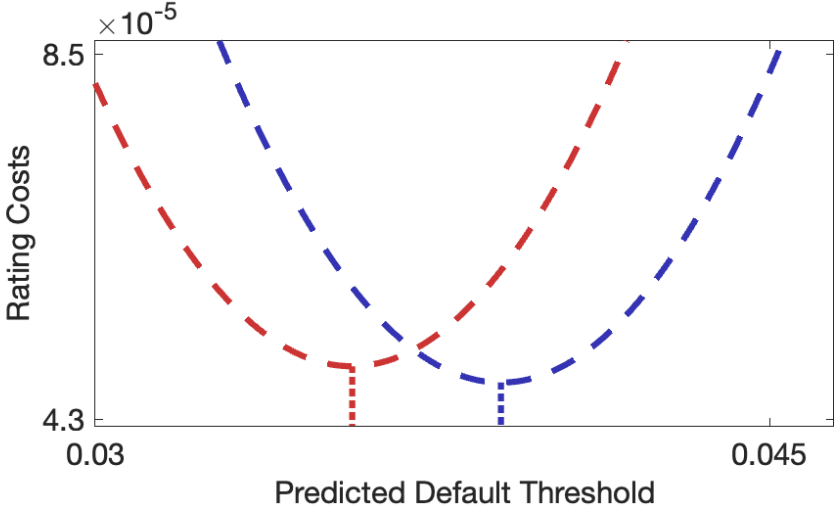
- Weight w for prior i increases (decreases) relative to prior j, if prior i...
  - has higher (lower) residual variance,
  - that is, it is less (more) informative.

Rating agency balances disagreeing analysts



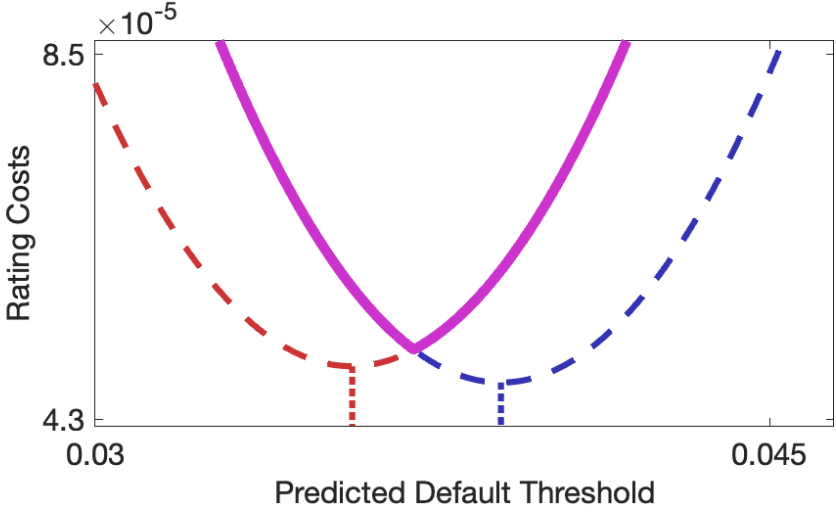
- Balanced disagreement: pessimist (blue), optimist (red)

Rating agency balances disagreeing analysts



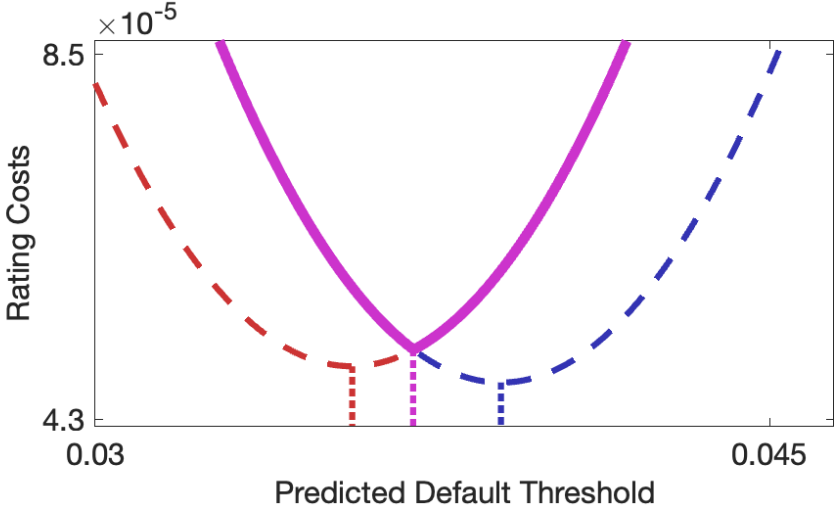
- Balanced disagreement: pessimist (blue), optimist (red)

Rating agency balances disagreeing analysts



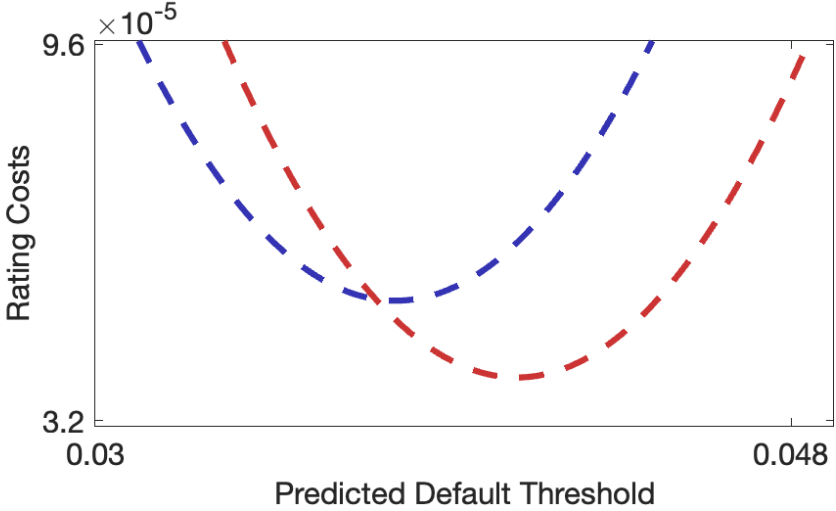
- Balanced disagreement: pessimist (blue), optimist (red)

Rating agency balances disagreeing analysts



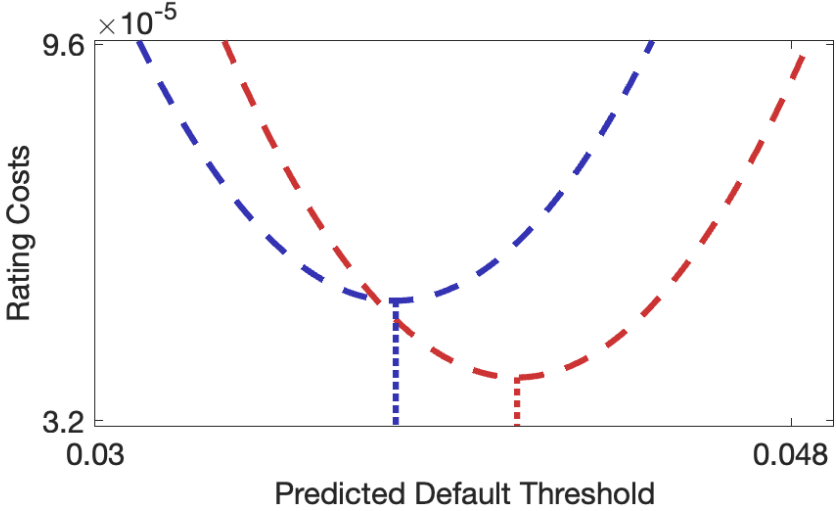
- Balanced disagreement: pessimist (blue), optimist (red)

Rating agency follows less extreme analysts



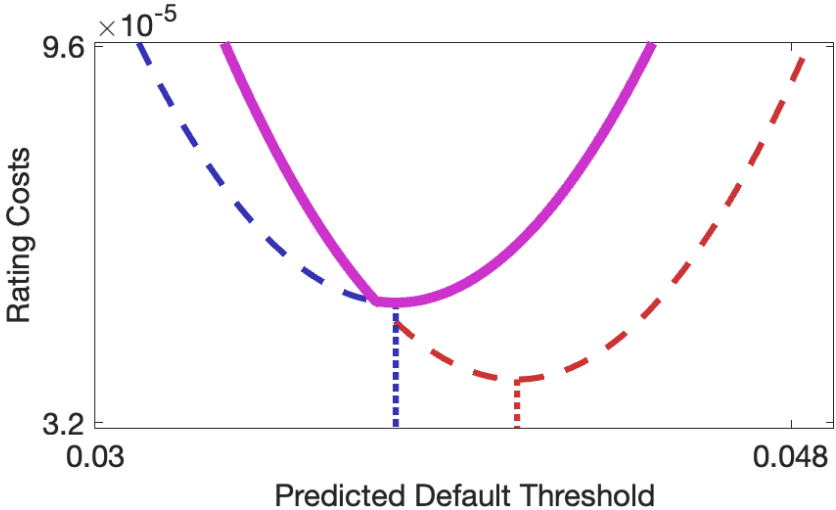
- Commonly pessimistic beliefs: pessimist (blue), super pessimist (red)

Rating agency follows less extreme analysts



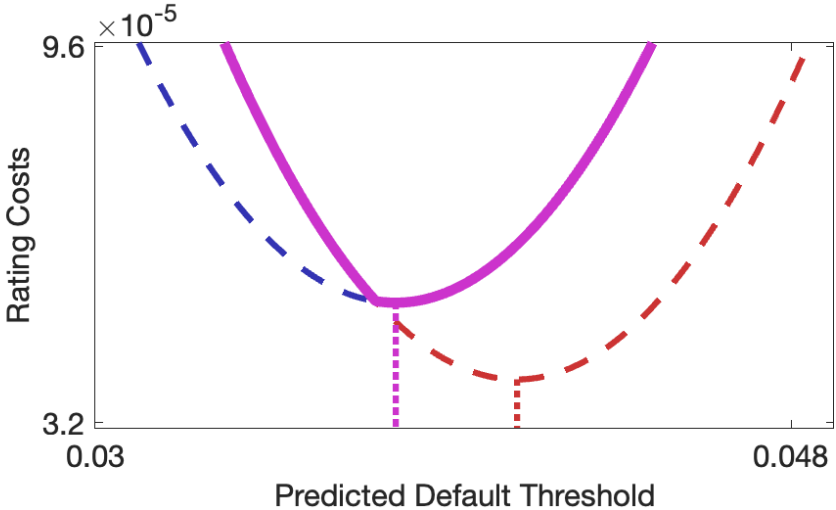
- Commonly pessimistic beliefs: pessimist (blue), super pessimist (red)

Rating agency follows less extreme analysts



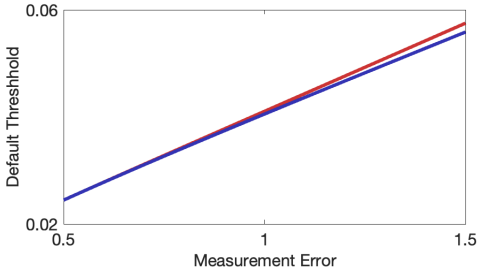
- Commonly pessimistic beliefs: pessimist (blue), super pessimist (red)

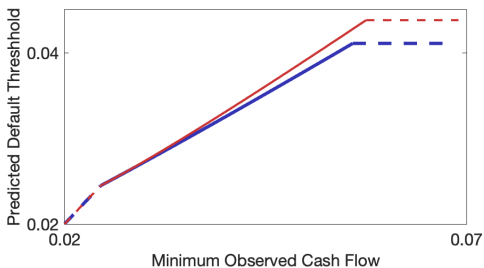
Rating agency follows less extreme analysts



- Commonly pessimistic beliefs: pessimist (blue), super pessimist (red)

Commonly pessimistic beliefs: Ambiguity delays default





- Taking average prior as benchmark clarifies the effect of ambiguity.
- Ambiguity implies higher ratings (lower $\hat{D}^{\star}$) and delays firm default.

Let's wrap up

- Ambiguity has (ex ante) surprising effects on credit ratings in equilibrium.
- Ambiguity-averse rating agency balances beliefs of credit analyst team instead of taking extreme belief as worst case.
- Rating agency gives more weight to less informative analysts.
- If common direction: Follow the least extreme analyst.
- Relative direction of analyst opinions...
  - affects impact of feedback effects.
  - feeds back into corporate decisions.

  $\Rightarrow$ ambiguity can delay as well as accelerate default.