The Extensive Margin of Bayesian Persuasion

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ESEM

Motivation

Information does not come for free (Simon '96).

Information provider (*Sender*) faces a heterogeneous audience.

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Information provider (*Sender*) faces a heterogeneous audience.

I study the persuasion of an **inattentive** Receiver who is **privately informed** about her cost and benefit of information.

Example

A seller designs a signal S of the product's quality θ to persuade a buyer to buy.

Increasing the correlation between S and θ has two effects:

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Results:

1. Characterization of the extensive margin.
2. Signals are equivalent to persuasion mechanisms.
3. Optimal signal in applications.

Model

Receiver's payoff from **action** $a \in \{0,1\}$ and **state** $\in [0,1]$ is

$$U_R(a,\theta,e;c,\lambda) = \underbrace{a(\theta - c)}_{\text{material payoff } u_R} - \underbrace{\lambda k(e)}_{\text{effort cost}},$$

in which:

- $c \in [0,1]$ is the threshold for action.
- $\lambda \in [0,1]$ is the attention cost.
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- $\theta \sim$ abs. cont. CDF F_0, with mean x_0.

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Sender's payoff is $U_S(a) = a$.

Receiver is privately informed about her **type**: $(c,\lambda) \in T$, drawn from CDF H and independent of θ.

Timing

1. Sender publicly commits to a signal $\sigma\colon \Theta \to \Delta M$ ($M = [0,1]$, meas. σ).
2. Receiver chooses an **effort** e, knowing her **type**.
   - 3.1 Nature draws the state θ from F_0;
   - 3.2 Nature draws a message m from $\sigma(\theta)$.
4. **With probability** e, Receiver observes the message m; and then chooses action a.

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Optimal Action given belief μ, of type-(c, λ) Receiver, is:

$$a^\star = \mathbf{1}\{\mathbb{E}_\mu \theta \geq c\}.$$

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Marginal Benefit of Effort given belief distribution p, of type-(c, λ) Receiver, is:

$$A(c) = \mathbb{E}_p\big[\mathbb{E}_\mu u_R(a^\star, c, \theta)\big] - u_R(\mathbf{1}\{x_0 \geq c\}, c, x_0).$$

Literature

Persuasion of privately informed Receiver, with costless access to signal. (Rayo-Segal '10; Kolotilin *et al.* '17; Kolotilin '18; Guo-Shmaya '19; ...)

Persuasion of inattentive Receiver, without private information. (Bizzotto *et al.* '20; Wei '21; Matyskova-Montes '23.)

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Attention management: $U_S = u_R$. (Lipnowski *et al.* '20, '22.)

Incomplete-Information beauty contests (Myatt-Wallace '14; Chahrour '14; Galperti-Trevino '20.)

Signals as Information Policies

We identify a signal with its induced **information policy**.

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1. I_F is convex $\leftarrow$ F is nondecreasing.
2. $I_{\overline{F}} \leq I_F \leq I_{F_0}$, in which:
   - $\overline{F}$ is posterior mean's CDF induced by an **uninformative signal**.
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Every signal induces an information policy of the CDF of the posterior mean.

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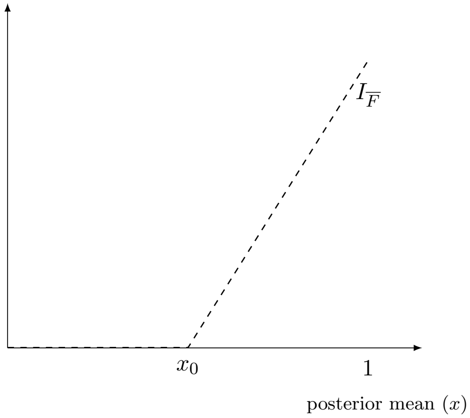
Fact 1 (Gentzkow-Kamenica '16; Kolotilin '18).
If $I\colon \mathbb{R}_+ \to \mathbb{R}_+$ satisfies 1. and 2., then: I is the information policy of the CDF of the posterior mean for some signal.

Informativeness



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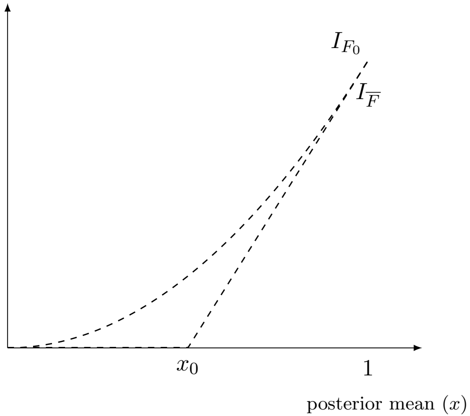

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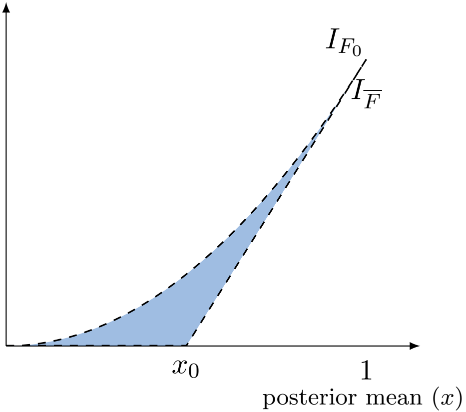

Blackwell's ranking of information policies
$I \geq J$ iff I is more Blackwell informative than J.

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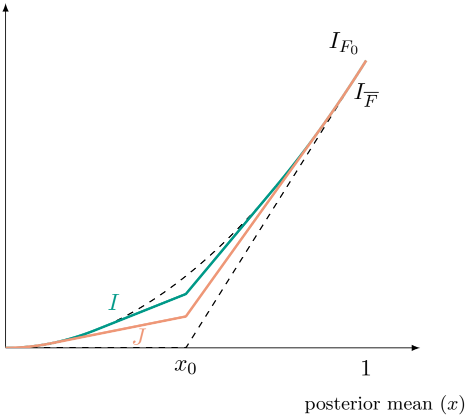

Extensive Margin

"Net informativeness" is denoted by

$$\Delta I = I - I_{\overline{F}}.$$

Lemma 1 (Marginal Benefit of Effort) The marginal benefit of effort given information policy I satisfies:

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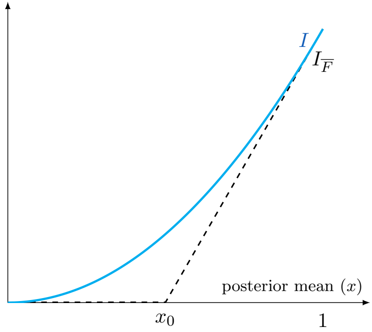

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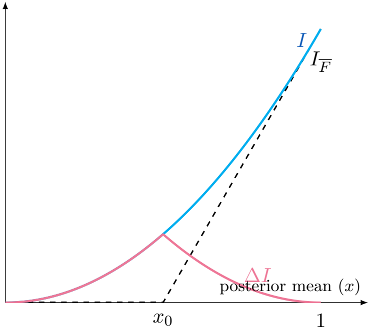

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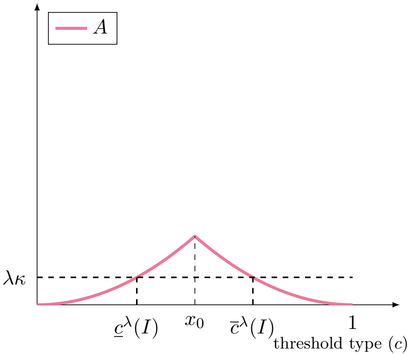

Linear Cost ($k(e) = \kappa e$)
There exist *cutoff types*, given attention-cost λ: $\underline{c}^{\lambda}(I)$, $\overline{c}^{\lambda}(I)$.

Supermodularity

Receiver's *value of information policy I* is her interim payoff, given her type:

$$V_R(\Delta I(c), \lambda) := \max_{e \in [0,1]} e \Delta I(c) - \lambda k(e).$$

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A *persuasion mechanism* is a menu of information policies: $I_\bullet = (I_r)_{r \in T}$.

A persuasion mechanism $I_\bullet$ is *incentive compatible* (IC) if:

$$V_R(\Delta I_{(c,\lambda)}(c), \lambda) \geq V_R(\Delta I_{\widetilde{(c,\lambda)}}(c), \lambda)$$

for all types $(c, \lambda) \in T$ and reports $\widetilde{(c, \lambda)} \in T$.

Equivalence

An IC persuasion mechanism $I_\bullet$ and an information policy J induce the *same effort distribution* if:

$$\arg\max_{e\in[0,1]} e\Delta J(c) - \lambda k(e) = \arg\max_{e\in[0,1]} e\Delta I_{(c,\lambda)}(c) - \lambda k(e),$$

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$$\underbrace{1 - J'(c^-)}_{\text{Prob. of } \{a^\star = 1\}} = 1 - I'_{(c,\lambda)}(c^-),$$

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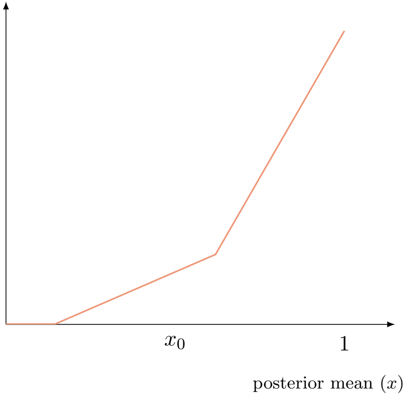
Theorem 1

For every IC persuasion mechanism $I_\bullet$ there exists an information policy J that induces the same effort and action distributions.

Equivalence

Key step: The upper envelope of the information policies in $I_\bullet$ is an information policy.



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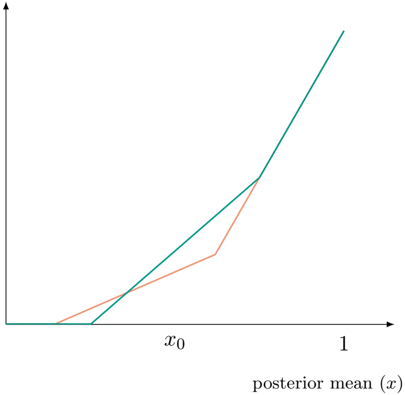

If $\lambda = 0$ is known to Sender: Kolotilin *et al.* '17.

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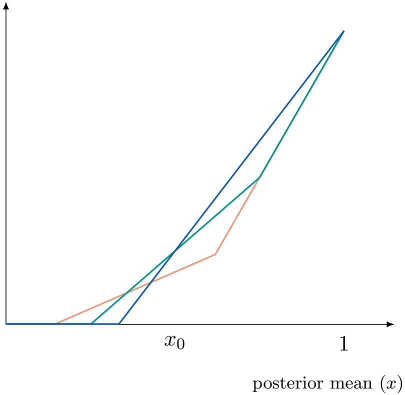

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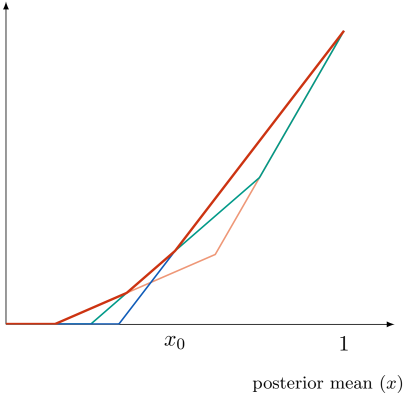

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Sender's Maximization

The Sender's expected payoff from information policy I, if Receiver's cost k is linear, is:

$$V_S(I) = \int_T \underbrace{\mathbf{1}\{\underline{c}^\lambda(I) \leq c \leq \overline{c}^\lambda(I)\}}_{\substack{\text{Extensive margin} \\ e>0}} \underbrace{\left[1 - I'(c^-) - \mathbf{1}\{x_0 \geq c\}\right]}_{\text{Intensive margin}} \mathrm{d}H(c,\lambda).$$

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I is an optimal information policy if it solves the Sender's problem:

$$\sup_{I:\, \mathbb{R}_+ \to \mathbb{R}_+} V_S(I)$$

subject to:

1. I is convex.
2. $I_{\overline{F}} \leq I \leq I_{F_0}$.

Optimal Signal

Assumption (SPness)

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2. Threshold c admits an abs. cont. PDF f that is single-peaked, with CDF F.

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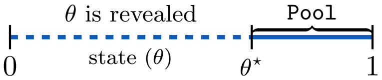

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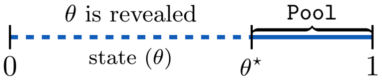

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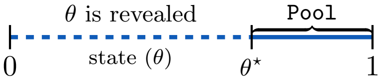

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Key step

Sender's value functional:

$$V_S(I) = \int_{\lambda=0}^{1} \int_{c=0}^{1} V_R(\Delta I(c), \lambda) h_c'(c) \,\mathrm{d}c h_\lambda(\lambda) \,\mathrm{d}\lambda.$$

Application

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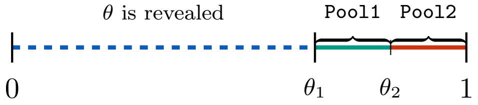
Lemma 2 (Media Censorship)
Under 1., 2. and 3., and SPness, there exists an optimal information policy that is induced by a bi-upper censorship signal.



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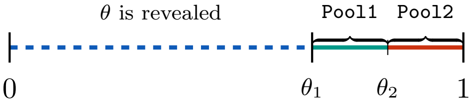

- Kolotilin *et al.* '22: no media market.
- Gehlbach-Sonin '14: Sender knows c, and does not know λ.

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