Financial Constraints and Firm Size: Micro-Evidence and Aggregate Implications

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Motivation

Size and financial constraints

According to standard theory, small firms are constrained, large firms are unconstrained.

Gertler and Gilchrist, 1994

Research questions:

- Is this an oversimplification?
- Does it matter for aggregate outcomes?

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What we do

Empirical

- Use credit data on the universe of Portuguese firms
- Provide a novel stylized fact
- Validate model mechanism

Theoretical

- Model with financial frictions and richer productivity structure
- Importance of large constrained firms for shocks propagation
- Use the model to assess aggregate outcomes

What we find

I. Empirical fact: Constrained firms across the entire size distribution

II. Quantitative model

1. Distribution of constrained firms in line with the data
2. Top 10% of large constrained firms explain 2/3 of aggregate effects of financial shocks
3. Ignoring the joint size-constrained distribution can lead to underestimating effects of financial shocks by up to 3 times

III. Mechanism validation: Constrained firms have higher elasticity to different shocks

Structure of the talk

1. Introduction
2. Literature
3. Empirics and stylized fact
4. Structural model
5. Results
6. Mechanism validation

Literature

Literature

1. **Firm dynamics:** Gertler & Gilchrist (1994) , Kashyap et al. (1994), Cooley & Quadrini (2001), Khan & Thomas (2013), Kudlyak & Sanchez (2017), Ottonello & Winberry (2018), Cloyne et al. (2023), Mehrotra & Sergeyev (2020), Crouzet & Mehrotra (2020), Pugsley et al. (2021)

   → **Contribution**: Importance of joint size and financial constraints distribution for aggregate dynamics

2. **Measuring financial constraints:** Kaplan & Zingales (1997), Lamont & Polk (2001), Whited & Wu (2006), Farre-Mensa & Ljungqvist (2016), Bodnaruk et al. (2015), Buehlmaier & Whited (2018), Hadlock & Pierce (2010), Hennessy & Whited (2007)

   → **Contribution**: Measurement based on detailed, firm-specific credit information

Empirics and stylized fact

Data

- Use *Informação Empresarial Simplificada (IES)* data on the universe of Portuguese firms between 2006 and 2017
- Matched with Bank of Portugal's credit register that records individual bank relationships and respective credit situations
- This data set is very **granular**:
  - Any (potential) loan amounting to 50 Euros or more is recorded in the credit register
  - Several types of credit, including potential, overdue, maturing and secured credit

▸ Details on cleaning ▸ Definition of credit variables ▸ Descriptive stats

Proxies for intensive margin financial constraints

Our main measure for financial constraints is:

$$\text{Constrained I}_{i,t} = \mathbb{1}(\text{Potential Credit}_{i,t} = 0 \ \& \ \Delta\text{Effective credit}_{i,t} \leq 0)$$

Intuition:

- Quantity based measure of financial constraints
- Firms exhausted all credit lines from banks
- No increase in debt across years

Some robustness measures:

- Const. $\text{II}_{i,t} = \mathbb{1}(\text{Potential Credit}_{i,t} = 0 \ \& \ \text{Overdue Credit}_{i,t} > 0)$
- Const. $\text{III}_{i,t} = \mathbb{1}(\text{Potential Credit}_{i,t} = 0 \ \& \ \Delta\text{Overdue Credit}_{i,t} > 0)$
- Const. $\text{IV}_{i,t} = \mathbb{1}(\text{Secured Credit / Assets}_{i,t} > p(90th))$ (see e.g. Rampini and Viswanathan, 2021)
- Const. $\text{V}_{i,t} = \mathbb{1}(\text{Credit} < 1 \text{ Year Maturity / Assets}_{i,t} > p(90th))$

▸ Relation to firm observables: Linear Probability Model

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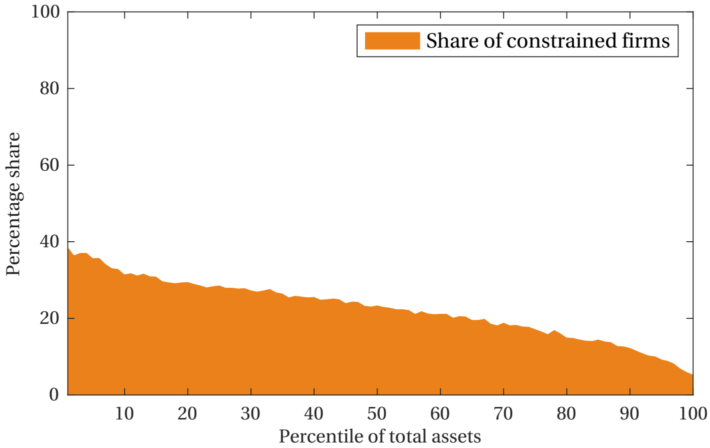
Relation to firm observables: Linear Probability Model

Main Empirical Fact: Constrained firms exist across the entire size distribution



Main Empirical Fact

Constrained firms exist across the entire size distribution

▸ All measures ▸ Const. Due ▸ Const. Due 0 ▸ Const. Secured ▸ Const. Maturing

Structural model

Structural model - overview

- Heterogeneous firms model with financial frictions
- Representative household, chooses labour and consumption ▸ details
- Firms produce according to the following production function:

$$y_{i,t} = \varphi_{i,t} k_{i,t}^{\alpha} l_{i,t}^{\nu} \quad \text{where} \quad \alpha + \nu < 1$$

- Idiosyncratic productivity schedule:

$$\ln \varphi_{i,t} = \underbrace{\theta_i}_{\text{Permanent}} + \underbrace{w_{i,t}}_{\text{Transitory}}$$

where

$$\theta_i \sim \mathcal{N}\left(\mu_\theta, \sigma_\theta^2\right)$$
$$w_{i,t} = \rho_w w_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right),$$

▸ Sketch ▸ Permanent productivity

Structural model - overview

- Firms maximize profits subject to a collateral constraint

$$\underbrace{b_{i,t+1}}_{\text{Borrowing choice}} \leq \xi \underbrace{x_{i,t}}_{\text{Cash on hand}}$$

- The **optimal, unconstrained choice** of capital $k^*(\varphi_i)$ solves the following equation

$$\beta \mathbb{E}_t \left[\frac{d\pi(\varphi_{i,t+1}, k_{i,t+1})}{dk_{i,t+1}} | \varphi_{i,t} \right] + \beta(1-\delta) = 1$$

We consider all firms that cannot implement $k^*(\varphi_{i,t})$ as constrained.

- Standard entry assumptions ▸ details

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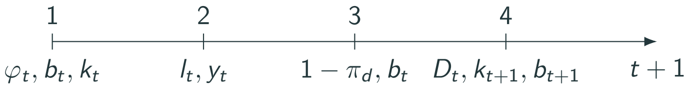
Incumbents - Within-period timing

The within-period timing of an incumbent firm i can be illustrated as follows (dropping respective subscript below):



1. Observes its idiosyncratic productivity φ_t, current stock of debt b_t and capital k_t x
2. Chooses labour input l_t and production y_t
3. If hit by the stochastic, exogenous death shock it repays outstanding debt b_t and exit
4. Conditional on survival, firm chooses its investment k_{t+1} and borrowing b_{t+1} subject to the borrowing constraint $b_{t+1} \leq \xi x_t$

▸ Details on the labour choice ▸ Details on the firm's problem

Calibration

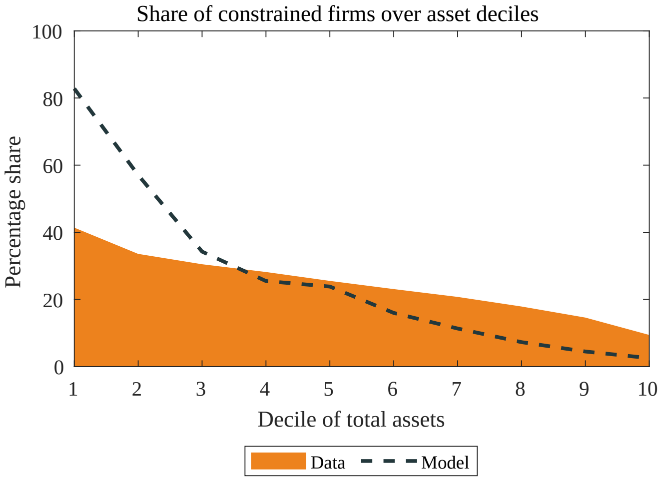
- **Objective:** Discipline joint distribution of size and financial constraints
- 6 free parameters ξ, σ_θ, ρ_w, σ_w, σ_{ke} and μ_{ke}
- 7 moments

Table 1: Calibrated model fit

Moment	Data	Model
Size of 90th percentile / median	9.440	9.218
Average leverage	0.626	0.330
Std. dev. of value added	1.559	1.644
1-year autocorrelation of value-added	0.924	0.928
5-year autocorrelation of value-added	0.818	0.762
Std. dev. of value-added growth	0.382	0.384
% of constrained firms	0.244	0.250

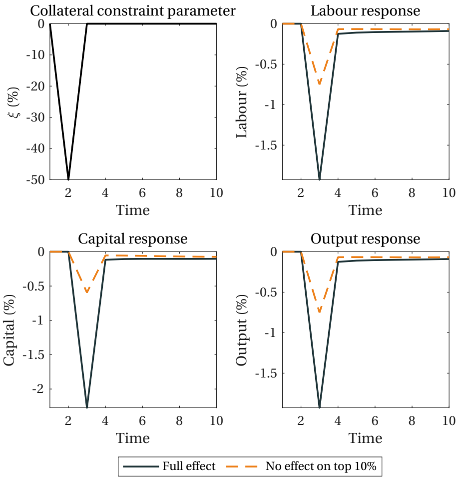
Notes. All constrained firms moments are calculated using constrained measure I.

Steady state distributions - constrained and unconstrained



Results

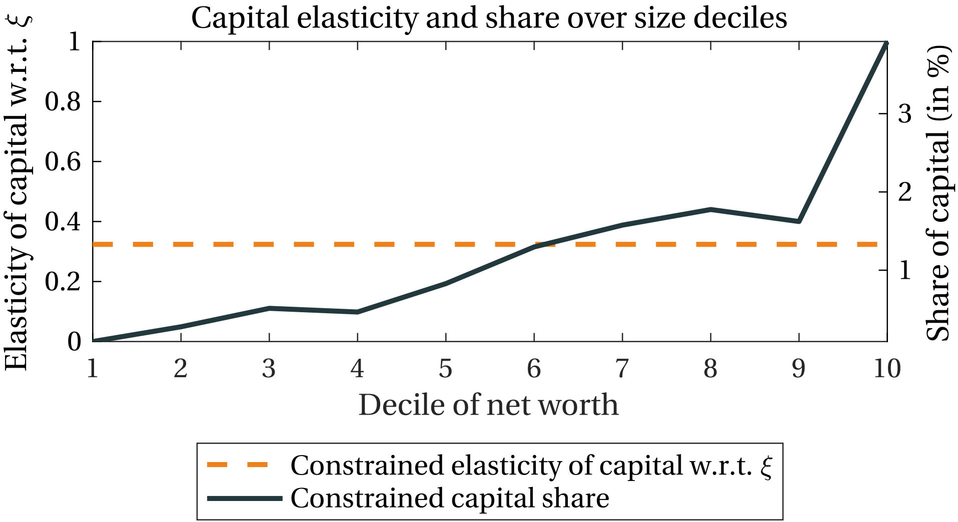
Aggregate response financial shock



Result

Effects of financial shocks are mainly driven by large constrained firms

Mechanisms at play



Mechanism

Large firms have a higher share of total capital while having the same elasticity

Additional results

- Importance of matching the joint constrained-size firm distribution
  1. Shut down permanent productivity component Alternative calibration 1 Alternative calibration distribution
  2. Directly target the distribution Alternative calibration 2
- TFP shock produces qualitatively similar results but quantitatively smaller Alternative calibration: Share constrained

Mechanism validation

Constrained firms respond more to shocks

- Run the following regressions, in the spirit of Crouzet & Mehrotra (2020):

$$g_{i,t} = \kappa u_{i,t} + \sum_{j \in \mathcal{J}} (\alpha_j + \beta_j u_{i,t}) \mathbf{1}_{i \in \mathcal{S}_t^{(j)}} + (\zeta + \eta u_{i,t}) \text{Const.} n_{i,t}$$
$$+ \gamma_I + \delta_t + \lambda_{It} + \alpha_i + \epsilon_{i,t},$$

- $g_{i,t}$ is the year-on-year log change in turnover or employees
- the set $\mathcal{S}_t^{(j)}$ is the jth size group, e.g. all firms above the 90th but below the 99th percentile
- $\text{Const}_{i,t}$ refers to the indicator measuring financial constraints
- u_{it} takes the form of three different shocks
    1. Year-on-year GDP growth;
    2. TFP estimated as in Ackerberg et al. (2015)
    3. Bank shocks as in Amiti and Weinstein (2018)

Cyclicality of Turnover

	Un-conditional	Constrained measure				
		I	II	III	IV	V
% Δ GDP	2.316	0.311	1.495	0.882	0.085	-0.145
	(0.056)	(0.054)	(0.175)	(0.217)	(0.103)	(0.102)
TFP shock	0.086	0.016	0.076	0.075	0.068	0.065
	(0.001)	(0.004)	(0.009)	(0.010)	(0.007)	(0.008)
Fin. shock	0.054	0.014	0.155	0.128	0.179	0.073
	(0.005)	(0.013)	(0.049)	(0.057)	(0.040)	(0.035)

Constrained firms exhibit higher sensitivity to GDP, TFP and financial shocks, conditional on size

▸ GDP ▸ TFP ▸ Financial Shock

Robustness

Robustness

- Employment ▸ GDP ▸ TFP ▸ Financial Shock
- Include time fixed effects ▸ GDP
- Exclude firms with zero potential credit in all periods ▸ GDP ▸ TFP ▸ Financial Shock
- Control for supply side effects ▸ GDP ▸ TFP ▸ Financial Shock

Conclusion

Three main contributions

1. Financially constrained firms exist across the entire size distribution
2. Large constrained firms account for 2/3 of the aggregate response to financial shocks
3. Importance of matching firm joint size-constrained distribution

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Thank you for your attention!

Appendix

Gertler and Gilchrist, 1994

Gertler and Gilchrist, 1994:

> *"The informational frictions that add to the costs of external finance apply mainly to younger firms, firms with a high degree of idiosyncratic risk, and firms that are not well collateralized. These are, on average, smaller firms."*

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Tables

Details on firms in Portugal

Variable	Mean	Median	Std. Dev.	Size group median			
				<90th	90th-99th	99-99.5th	>99.5th
Total Assets	3.15	0.28	85.10	0.25	5.06	42.71	135.70
Turnover	1.86	0.23	33.59	0.21	3.25	19.93	27.94
Potential credit	0.19	0.03	4.56	0.03	0.14	0.95	2.95
Effective credit	0.53	0.04	5.96	0.04	1.15	6.93	126.73
Leverage	0.28	0.20	0.38	0.20	0.24	0.17	0.08
Liquidity ratio	0.14	0.06	0.19	0.06	0.02	0.01	0.01
Age	15.01	12.00	12.26	12.00	21.00	23.00	21.50
Employees	14.47	4.00	130.58	4.00	25.00	95.00	98.00
# Banks	2.45	2.00	1.89	2.00	4.00	4.00	5.00

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Debt and financial shocks

	(1)	(2)	(3)
Bank shock: Effective credit	0.161***	0.0496***	-0.00680
	(0.0118)	(0.0147)	(0.0133)
Const.		-0.284***	
		(0.00296)	
Const. × Bank shock: Effective credit		0.117***	
		(0.0217)	
Const. Adj. Eff.			-0.766***
			(0.00333)
Const. Adj. Eff. × Bank shock: Effective credit			0.0461*
			(0.0194)
Industry × Year FE	Yes	Yes	Yes
Clustering	Firm	Firm	Firm
N	1071731	1071731	1071731

Test 1

Constrained firms increase debt by more in response to credit supply shocks

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Details on constrained firms

Variable	Constrained					Unconstrained				
	all	<90th	90th-99th	99-99.5th	>99.5th	all	<90th	90th-99th	99-99.5th	>99.5th
TFP	3.15	0.28	85.10	0.25	5.06	42.71	135.70			
	1.86	0.23	33.59	0.21	3.25	19.93	27.94			
# bank relations	0.19	0.03	4.56	0.03	0.14	0.95	2.95			
	0.53	0.04	5.96	0.04	1.15	6.93	126.73			

Back

Cyclicality of turnover

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.000	0.001	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
>99.5 × GDP Growth	-0.005	-0.004	-0.004	-0.004	-0.005	-0.005
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.213				
		(0.054)				
Const. Overdue × GDP Growth			1.426			
			(0.166)			
Const. Overdue Inc. × GDP Growth				0.988		
				(0.205)		
Const. Maturing × GDP Growth					0.666	
					(0.098)	
Const. Secured × GDP Growth						0.310
						(0.096)
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1326447	1326447	1326447	1326447	1088781	1088781

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Non-targeted moments

Table 2: Untargeted moments

Moment	Data	Model
Share of const. firms in bottom 20%	0.33	0.65
Size of const. firms 90th percentile / median	7.35	9.72
Size of unconst. firms 90th percentile / median	9.67	9.05
Asset share of const. firms	0.07	0.10
Share of const. firms in top 10% vs. bottom 20%	0.36	0.05
Percentage of const. firms in top 1%	0.09	0.01

Notes. All constrained firms moments are calculated using constrained measure I.

back

Turnover and TFP

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.273	0.268	0.265	0.267	0.266	0.266
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)
[90, 99] × TFP shock	0.096	0.098	0.100	0.099	0.089	0.089
	(0.011)	(0.011)	(0.011)	(0.011)	(0.013)	(0.013)
[99, 99.5] × TFP shock	0.014	0.017	0.019	0.018	0.059	0.057
	(0.029)	(0.029)	(0.029)	(0.029)	(0.036)	(0.036)
¿99.5 × TFP shock	0.002	0.007	0.010	0.008	0.011	0.010
	(0.041)	(0.041)	(0.041)	(0.041)	(0.049)	(0.050)
Const. Adj. Eff. × TFP shock		0.016				
		(0.004)				
Const. Overdue × TFP shock			0.076			
			(0.009)			
Const. Overdue Inc. × TFP shock				0.075		
				(0.010)		
Const. Maturing × TFP shock					0.068	
					(0.007)	
Const. Secured × TFP shock						0.065
						(0.008)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1011102	1011102	1011102	1011102	816841	816841

back

Turnover and financial shocks

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	0.015	-0.001	0.002	0.007	0.022	0.025
	(0.007)	(0.008)	(0.007)	(0.007)	(0.009)	(0.009)
[90, 99] × Bank shock	0.078	0.089	0.085	0.084	0.069	0.071
	(0.028)	(0.029)	(0.028)	(0.028)	(0.033)	(0.034)
[99, 99.5] × Bank shock	-0.050	-0.037	-0.036	-0.040	0.151	0.152
	(0.170)	(0.170)	(0.169)	(0.169)	(0.198)	(0.198)
¿99.5 × Bank shock	-0.147	-0.133	-0.133	-0.139	-0.028	-0.035
	(0.127)	(0.127)	(0.127)	(0.127)	(0.107)	(0.107)
Const. Adj. Eff. × Bank shock		0.0135				
		(0.013)				
Const. Overdue × Bank shock			0.155			
			(0.049)			
Const. Overdue Inc. × Bank shock				0.128		
				(0.057)		
Const. Maturing × Bank shock					0.179	
					(0.041)	
Const. Secured × Bank shock						0.073
						(0.035)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1196505	1196505	1196505	1196505	980796	980796

back

Cyclicality of employment

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
[99, 99.5] × GDP Growth	-0.000	0.000	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
¿99.5 × GDP Growth	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Const. Adj. Eff. × GDP Growth		0.074				
		(0.033)				
Const. Overdue × GDP Growth			0.764			
			(0.087)			
Const. Overdue Inc. × GDP Growth				0.455		
				(0.108)		
Const. Maturing × GDP Growth					0.110	
					(0.052)	
Const. Secured × GDP Growth						-0.011
						(0.053)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1360304	1360304	1360304	1360304	1116621	1116621

back

Employment and TFP

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	-0.053	-0.055	-0.056	-0.055	-0.060	-0.059
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
[90, 99] × TFP shock	0.011	0.012	0.013	0.012	0.010	0.010
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
[99, 99.5] × TFP shock	0.035	0.036	0.036	0.036	0.031	0.031
	(0.009)	(0.009)	(0.009)	(0.009)	(0.010)	(0.010)
¿99.5 × TFP shock	0.013	0.015	0.015	0.014	0.023	0.023
	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)
Const. Adj. Eff. × TFP shock		0.005				
		(0.001)				
Const. Overdue × TFP shock			0.017			
			(0.004)			
Const. Overdue Inc. × TFP shock				0.010		
				(0.004)		
Const. Maturing × TFP shock					0.008	
					(0.003)	
Const. Secured × TFP shock						0.003
						(0.003)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1014676	1014676	1014676	1014676	819792	819792

back

Employment and financial shocks

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shockt	0.001	-0.004	-0.006	-0.003	0.011	0.014
	(0.005)	(0.005)	(0.004)	(0.004)	(0.006)	(0.006)
[90, 99] × Bank shock	0.036	0.039	0.039	0.039	0.023	0.023
	(0.014)	(0.014)	(0.013)	(0.013)	(0.016)	(0.016)
[99, 99.5] × Bank shock	-0.007	-0.004	-0.002	-0.003	0.034	0.035
	(0.068)	(0.068)	(0.068)	(0.068)	(0.078)	(0.078)
¿99.5 × Bank shock	0.030	0.035	0.031	0.034	0.058	0.055
	(0.065)	(0.065)	(0.065)	(0.065)	(0.099)	(0.099)
Const. Adj. Eff. × Bank shock		-0.011				
		(0.009)				
Const. Overdue × Bank shock			0.092			
			(0.025)			
Const. Overdue Inc. × Bank shock				0.087		
				(0.031)		
Const. Maturing × Bank shock					0.046	
					(0.020)	
Const. Secured × Bank shock						-0.006
						(0.018)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1230781	1230781	1230781	1230781	1011230	1011230

back

Cyclicality of turnover

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.000	0.001	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
¿99.5 × GDP Growth	-0.005	-0.004	-0.004	-0.004	-0.005	-0.005
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.213				
		(0.054)				
Const. Overdue × GDP Growth			1.426			
			(0.166)			
Const. Overdue Inc. × GDP Growth				0.988		
				(0.205)		
Const. Maturing × GDP Growth					0.666	
					(0.098)	
Const. Secured × GDP Growth						0.310
						(0.096)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1326447	1326447	1326447	1326447	1088781	1088781

back

Cyclicality of turnover

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.001	0.001	0.001	0.001	0.001	0.002
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
¿99.5 × GDP Growth	-0.004	-0.004	-0.004	-0.004	-0.004	-0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.119				
		(0.069)				
Const. Overdue × GDP Growth			1.760			
			(0.212)			
Const. Overdue Inc. × GDP Growth				1.251		
				(0.265)		
Const. Maturing × GDP Growth					0.706	
					(0.102)	
Const. Secured × GDP Growth						0.363
						(0.103)
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1161130	1161130	1161130	1161130	955844	955844

back

Turnover and TFP

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.108	0.101	0.100	0.102	0.098	0.099
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
[90, 99] $\times$ TFP shock	0.056	0.059	0.059	0.058	0.039	0.040
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
[99, 99.5] $\times$ TFP shock	-0.020	-0.015	-0.017	-0.017	-0.009	-0.011
	(0.015)	(0.015)	(0.015)	(0.015)	(0.017)	(0.018)
¿99.5 $\times$ TFP shock	-0.024	-0.018	-0.017	-0.019	-0.027	-0.032
	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.016)
Const. Adj. Eff. $\times$ TFP shock		0.034				
		(0.003)				
Const. Overdue $\times$ TFP shock			0.137			
			(0.009)			
Const. Overdue Inc. $\times$ TFP shock				0.151		
				(0.011)		
Const. Maturing $\times$ TFP shock					0.078	
					(0.006)	
Const. Secured $\times$ TFP shock						0.069
						(0.005)
Industry $\times$ Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	907128	907128	907128	907128	738944	738944

back

Turnover and financial shocks

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	-0.047	-0.070	-0.063	-0.058	-0.030	-0.038
	(0.006)	(0.007)	(0.006)	(0.006)	(0.008)	(0.008)
[90, 99] × Bank shock	0.076	0.093	0.085	0.084	0.102	0.108
	(0.025)	(0.025)	(0.025)	(0.025)	(0.031)	(0.031)
[99, 99.5] × Bank shock	-0.101	-0.082	-0.090	-0.090	-0.013	-0.006
	(0.163)	(0.163)	(0.163)	(0.163)	(0.167)	(0.167)
¿99.5 × Bank shock	-0.073	-0.052	-0.059	-0.064	-0.012	-0.010
	(0.117)	(0.117)	(0.117)	(0.117)	(0.105)	(0.106)
Const. Adj. Eff. × Bank shock		0.005				
		(0.015)				
Const. Overdue × Bank shock			0.132			
			(0.058)			
Const. Overdue Inc. × Bank shock				0.106		
				(0.071)		
Const. Maturing × Bank shock					0.212	
					(0.039)	
Const. Secured × Bank shock						0.105
						(0.034)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1071731	1071731	1071731	1071731	883661	883661

back

Cyclicality of turnover

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × Δ GDP	-0.003	-0.003	-0.002	-0.002	-0.004	-0.004
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
[99, 99.5] × Δ GDP	-0.002	-0.002	-0.001	-0.001	-0.007	-0.007
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
¿99.5 × Δ GDP	0.002	0.002	0.004	0.005	-0.008	-0.008
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Const. Adj. Eff. × Δ GDP		0.205				
		(0.0641)				
Const. Overdue × Δ GDP			1.955			
			(0.483)			
Const. Overdue Inc. × Δ GDP				1.706		
				(0.594)		
Const. Maturing × Δ GDP					1.036	
					(0.281)	
Const. Secured × Δ GDP						0.502
						(0.288)
Industry × Δ GDP FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	1187112	1187112	1187112	1187112	976408	976408

back

Turnover and TFP

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.112	0.104	0.103	0.105	0.101	0.103
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
[90, 99] × TFP shock	0.056	0.060	0.059	0.059	0.039	0.040
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
[99, 99.5] × TFP shock	-0.023	-0.017	-0.019	-0.019	-0.011	-0.013
	(0.015)	(0.015)	(0.015)	(0.015)	(0.018)	(0.018)
¿99.5 × TFP shock	-0.027	-0.020	-0.020	-0.020	-0.029	-0.034
	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.017)
Const. Adj. Eff. × TFP shock		0.033				
		(0.003)				
Const. Overdue × TFP shock			0.125			
			(0.008)			
Const. Overdue Inc. × TFP shock				0.136		
				(0.010)		
Const. Maturing × TFP shock					0.078	
					(0.005)	
Const. Secured × TFP shock						0.070
						(0.005)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	892702	892702	892702	892702	725166	725166

back

Turnover and financial shocks

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	-0.025	-0.050	-0.041	-0.039	-0.020	-0.025
	(0.007)	(0.008)	(0.007)	(0.007)	(0.009)	(0.009)
[90, 99] × Bank shock	0.067	0.082	0.072	0.074	0.122	0.127
	(0.029)	(0.029)	(0.029)	(0.029)	(0.038)	(0.038)
[99, 99.5] × Bank shock	-0.106	-0.086	-0.096	-0.093	-0.010	-0.004
	(0.168)	(0.168)	(0.168)	(0.168)	(0.171)	(0.171)
¿99.5 × Bank shock	-0.125	-0.102	-0.108	-0.111	-0.043	-0.051
	(0.184)	(0.184)	(0.184)	(0.184)	(0.249)	(0.248)
Const. Adj. Eff. × Bank shock		0.061				
		(0.016)				
Const. Overdue × Bank shock			0.146			
			(0.049)			
Const. Overdue Inc. × Bank shock				0.169		
				(0.061)		
Const. Maturing × Bank shock					0.216	
					(0.038)	
Const. Secured × Bank shock						0.091
						(0.033)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	1153335	1153335	1153335	1153335	947734	947734

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Linear probability model

	Constrained binary			
	(1)	(2)	(3)	(4)
Age	-0.034			
	(0.000)			
Total assets		-0.066		
		(0.000)		
Leverage			-0.008	
			(0.000)	
Liquidity ratio				0.007
				(0.000)
Constant	0.246	0.245	0.244	0.244
Observations	1,365,913	1,365,913	1,365,913	1,365,913
R-squared	0.006	0.024	0.015	0.000

Every cell reports the coefficient $\hat{\beta}$ of the following (pooled) linear probability model:

$$Const_{i,t} = \alpha + \beta x_{i,t} + \epsilon_{i,t}$$

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Parameters statistical model

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Total	0.425	0.799	0.904	0.369	0.748	0.708	0.305	0.185
Unconstrained	0.431	0.770	0.884	0.399	0.769	0.744	0.311	0.158
Constrained	0.493	0.874	0.911	0.255	0.655	0.641	0.265	0.176

$$\underbrace{\log \tilde{n}_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}$$

where

$$\begin{aligned}
u_{i,a} = &\quad \rho_u u_{i,a-1} + \theta_i, && u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), && \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), && |\rho_u| \leq 1 \\
v_{i,a} = &\quad \rho_v v_{i,a-1}, && v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), && && |\rho_v| \leq 1 \\
w_{i,a} = &\quad \rho_w w_{i,a-1} + \varepsilon_{i,a}, && w_{i,-1} = 0, && \varepsilon_{i,a} \sim iid\left(0, \sigma^2_\varepsilon\right), && |\rho_w| \leq 1 \\
z_{i,a} \sim &\quad iid\left(0, \sigma^2_z\right)
\end{aligned}$$

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Calibration

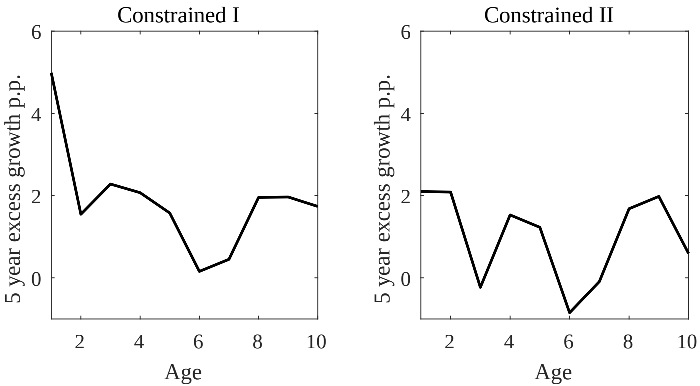
Table 3: Parameter values benchmark calibration

Parameter	Description	Value	Source
β	Discount factor	0.96	K&T (2013)
α	Returns on capital	0.30	K&T (2013)
η	Returns on labor	0.60	K&T (2013)
δ	Depreciation rate	0.065	K&T (2013)
ψ	Labour preference	2.15	K&T (2013)
π_d	Exogenous probability of exit	0.02	Data
μ_θ	Average: permanent productivity	0	Normalized
μ_w	Average: transitory shock	0	Normalized
Model			
ξ	Collateral constraint	0.50	Calibrated
σ_θ	Std. dev.: permanent productivity	0.20	Calibrated
ρ_w	Persistence of transitory shock	0.43	Calibrated
σ_w	Std. dev: transitory shock	0.11	Calibrated
μ_{ke}	Relative size of entrants	0.01	Calibrated
σ_{ke}	Standard deviation of entrants	1.35	Calibrated

Notes. K&T (2013) is short for Khan & Thomas (2013).

Figures

Constrained firms growth



Test 2

Constrained firms grow faster

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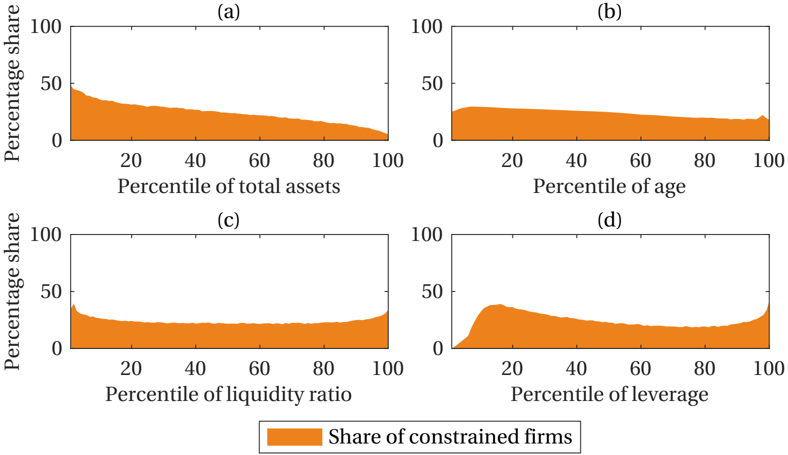
Fact: Robustness



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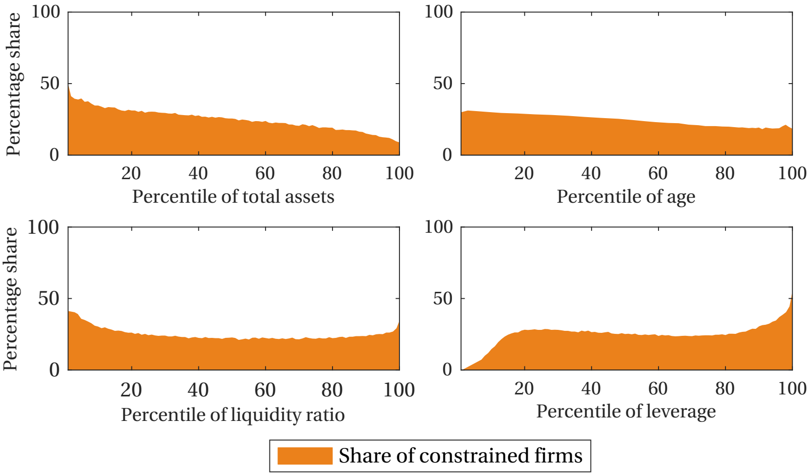
Fact: Robustness



$$\text{Constrained}_{t,i} = \mathbb{1}(\text{Potential Credit}_{t,i} = 0 \ \& \ \Delta\text{Effective credit}_{t,i} < 0)$$

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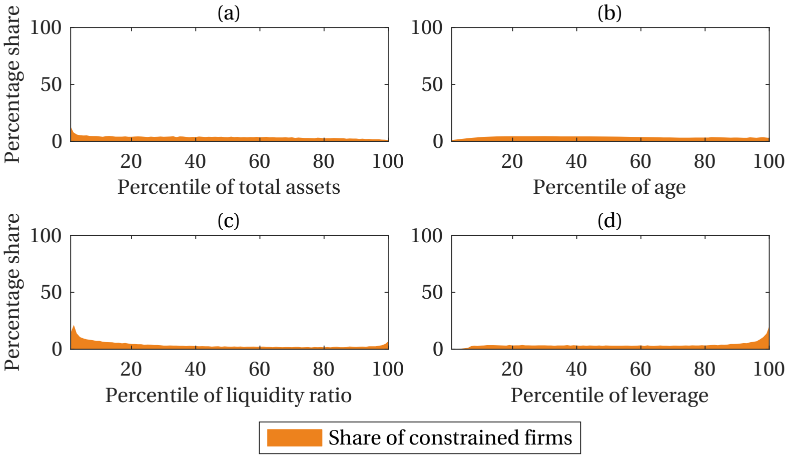
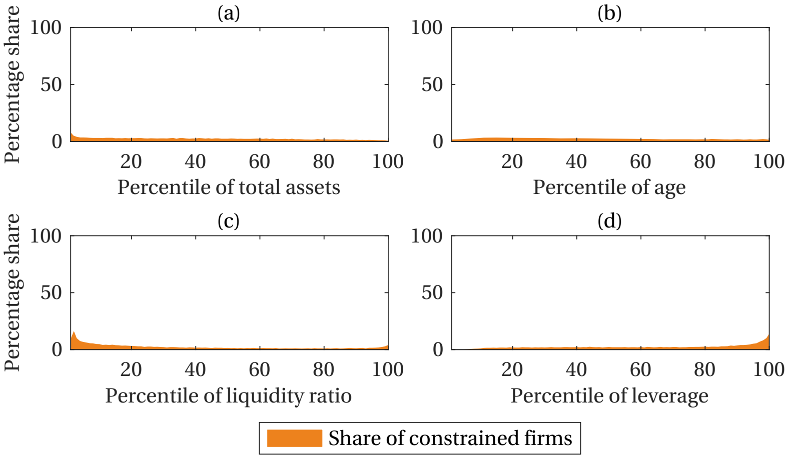
Fact: Robustness



$$\text{Constrained} = \mathbb{1}(\text{Potential Credit}_t = 0 \ \& \ \text{Overdue Credit}_t > 0)$$

back

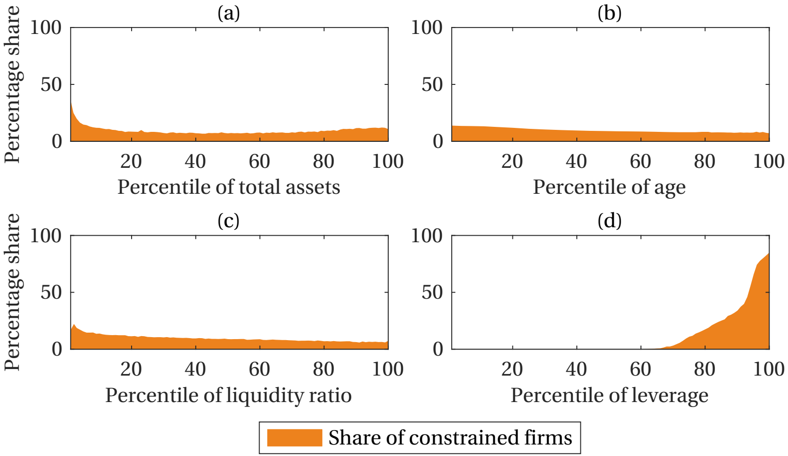
Fact: Robustness



$$\text{Constrained} = \mathbb{1}(\text{Potential Credit}_t = 0 \ \& \ \Delta\text{Overdue Credit}_t > 0)$$

back

Fact: Robustness



$$\text{Constrained} = \mathbb{1}(\text{Secured Credit } / \text{ Assets}_{i,t} > p(90th))$$

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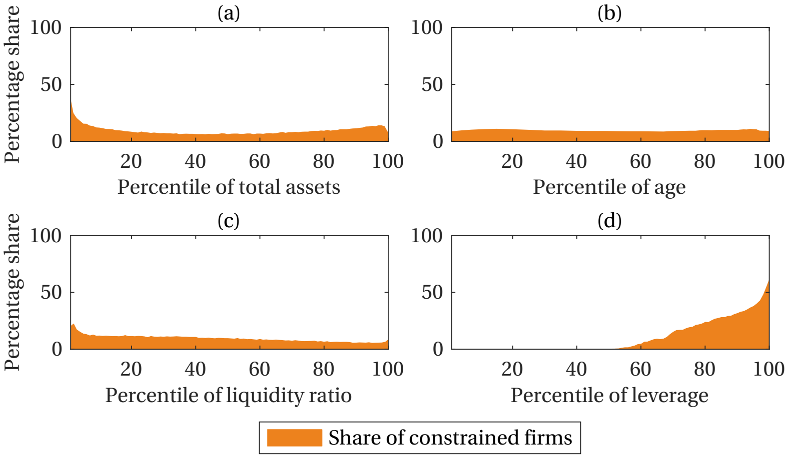
Fact: Robustness



$$\text{Const. Maturing}_{i,t} = \mathbb{1}(\text{Credit ¡ 1 Year Maturity / Assets}_{i,t} > p(90th))$$

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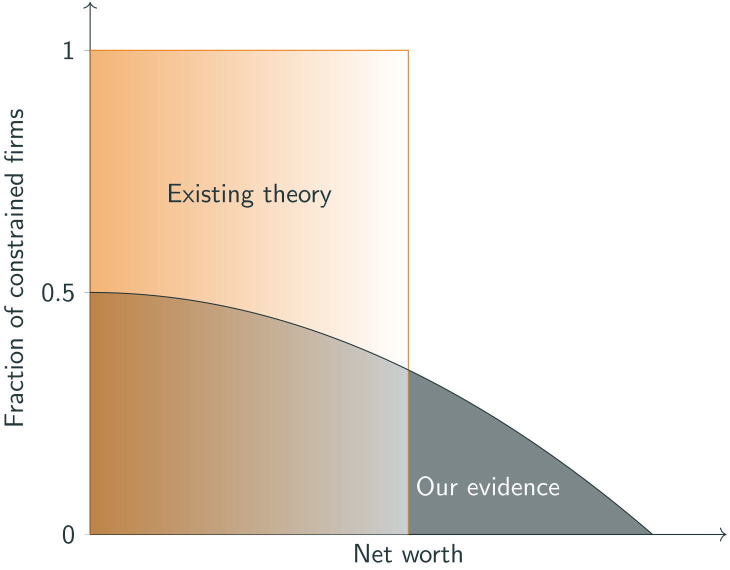
Evidence v Theory



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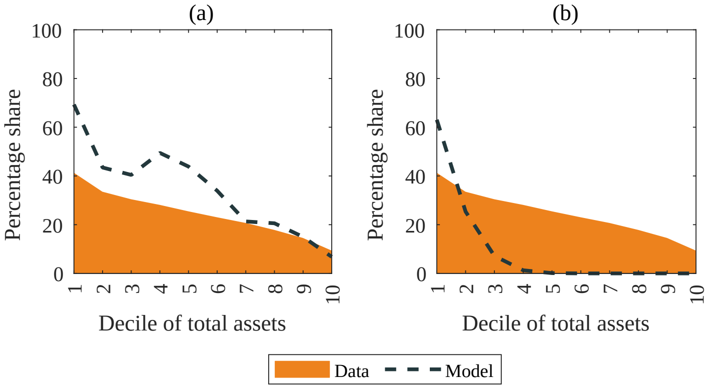
Joint size-constrained distribution - Alternative calibration



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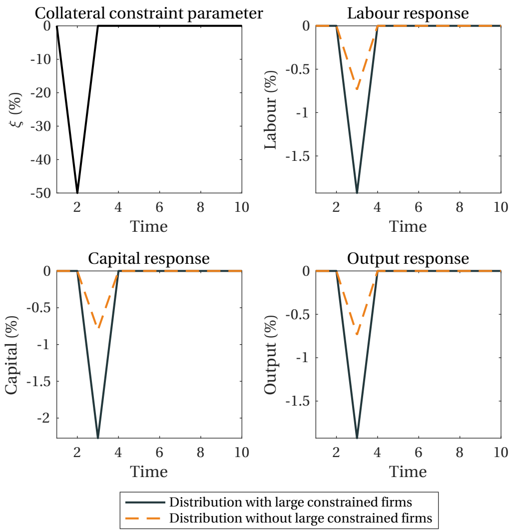
Aggregate response financial shock - Alternative Calibration 1



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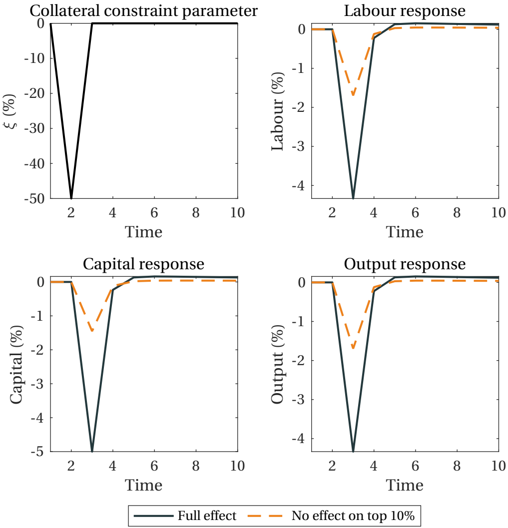
Aggregate response financial shock - Alternative Calibration 2



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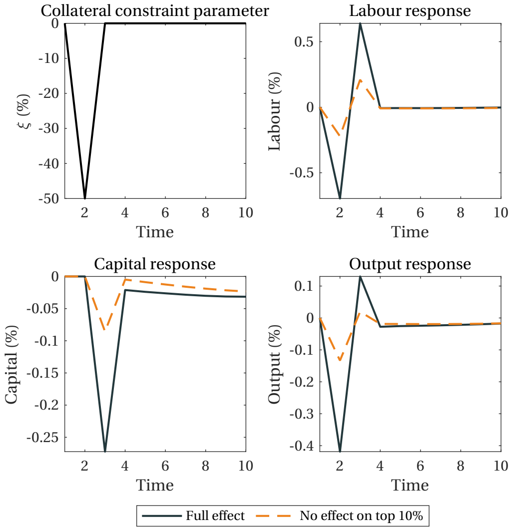
Aggregate response financial shock - GE



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Statistical model

Ex-ante heterogeneity - summary

- Look at the employment growth process within and across firms
- **Prior**: Standard deviation decreasing with age as firms reach uniform potential
  - **Result**: Standard deviation increasing with age ▸ Std
- **Prior**: Age autocorrelation tending to zero as firms grow older
  - **Result**: Autocorrelation converging to values larger than 0 ▸ Autocorr
- **Prior**: Ex-ante variance contribution small and tending to zero
  - **Result**: Ex-ante variance still explains 50% of employment variance after 10 years ▸ Ex-ante
- **Prior**: Ex-ante variance should affect more constrained than unconstrained
  - **Result**: Ex-ante contribution larger for unconstrained firms ▸ Ex-ante const

▸ Employment ▸ Statistical model ▸ Back

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- **Prior**: Standard deviation decreasing with age as firms reach uniform potential
  - **Result**: Standard deviation increasing with age ▸ Std
- **Prior**: Age autocorrelation tending to zero as firms grow older
  - **Result**: Autocorrelation converging to values larger than 0 ▸ Autocorr
- **Prior**: Ex-ante variance contribution small and tending to zero
  - **Result**: Ex-ante variance still explains 50% of employment variance after 10 years ▸ Ex-ante
- **Prior**: Ex-ante variance should affect more constrained than unconstrained
  - **Result**: Ex-ante contribution larger for unconstrained firms ▸ Ex-ante const

Ex-ante heterogeneity - methodology

- Information on firm growth needed for assessment of firm heterogeneity
- **Here:** Look at the employment growth process within and across firms
- In order to control for sector and birth year effects we estimate

$$\log n_{i,a,j,t} = \mu_j + \lambda_{t-a} + \epsilon_{i,a,j,t}$$

- Use $\log \tilde{n}_{i,a,j,t} \equiv \widehat{\epsilon_{i,a,j,t}}$ as our measure of employment in the analysis below

▸ Constrained vs unconstrained ▸ Balanced Panel ▸ Back

Standard deviation of employment

Prior

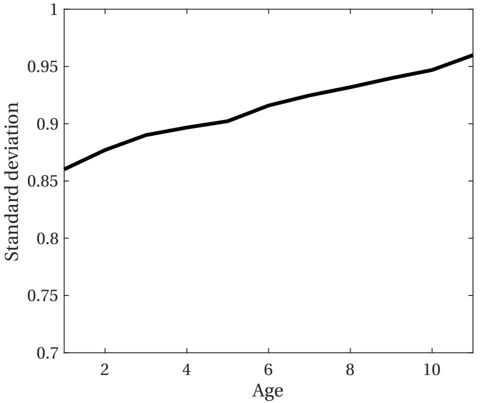
Standard deviation decreasing with age as firms reach uniform potential

Constrained vs unconstrained | Balanced Panel | Back

Standard deviation of employment

Prior

Standard deviation decreasing with age as firms reach uniform potential



▸ Constrained vs unconstrained ▸ Balanced Panel ▸ Back

Autocorrelation of employment

Prior

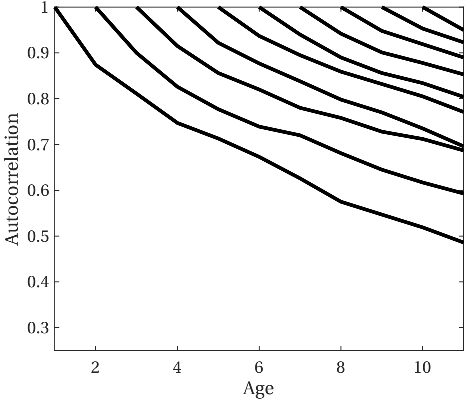
Age autocorrelation tending to zero as firms grow older

▸ Constrained vs unconstrained ▸ Balanced Panel ▸ Back

Autocorrelation of employment

Prior

Age autocorrelation tending to zero as firms grow older



Lines plot $Corr(\log \tilde{n}_{i,a}, \log \tilde{n}_{i,h})$ where $h \leq a$

▸ Constrained vs unconstrained ▸ Balanced Panel ▸ Back

Statistical model

More evidence from a flexible statistical model (Pugsley et al, 2021)

$$\underbrace{\log \tilde{n}_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}$$

where

- $u_{i,a}$ is a permanent part that converges to a certain level θ_i
- $v_{i,a}$ is a transitory part that converges to zero
- $w_{i,a}$ captures persistent ex-post shocks
- $z_{i,a}$ captures iid ex-post shocks

We calibrate the model to match the autocovariance profile of employment

▸ Details ▸ Figure: model fit ▸ Parameters ▸ Back

Derivations and Descriptions

Cleaning the dataset

- Drop all firms with less than €10K Credit throughout (Buera and Karmakar)
- Drop all firms that are out of business
- Drop all firms that are not privately or publicly held
- Drop all firms that report less than 5 years
- Drop all firm observations that have zero or less employees
- Drop all firm observations with liquidity and leverage ratio which is larger than 10 (Ottonello & Winberry (2018))

◂ Back

Credit measures

- **Effective Credit** is credit used in a regular situation, without payment delays in the effective contract
- **Potential Credit** represents irrevocable commitments of the participating entities.
- **Global credit** is the sum of effective and potential credit
- **Overdue credit** All outstanding credit exposures recorded as non-performing (including overdue, written off, renegotiated credit, overdue credit in litigation, and written off credit in litigation) are aggregated to calculate overdue credits
- **Short-Term Credit** - both with residual and original maturity less than one year
- **Long-Term Credit** - both with residual and original maturity more than a year
- **Secured Credit** is credit that is secured by real collateral, financial collateral or personal liability (state, bank or individual)

Back

Statistical model

More evidence from estimating the Pugsley et al. (2021) model in Portugal

$$\underbrace{\log \tilde{n}_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}$$

where

$$\begin{aligned}
u_{i,a} &= \rho_u u_{i,a-1} + \theta_i, & u_{i,-1} &\sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i &\sim iid\left(\mu_\theta, \sigma^2_\theta\right), & |\rho_u| &\leq 1 \\
v_{i,a} &= \rho_v v_{i,a-1}, & v_{i,-1} &\sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & & |\rho_v| &\leq 1 \\
w_{i,a} &= \rho_w w_{i,a-1} + \varepsilon_{i,a}, & w_{i,-1} &= 0, & \varepsilon_{i,a} &\sim iid\left(0, \sigma^2_\varepsilon\right), & |\rho_w| &\leq 1 \\
z_{i,a} &\sim iid\left(0, \sigma^2_z\right)
\end{aligned}$$

Back

Autocovariance

- Use autocovariance to estimate ex-ante and ex-post conditions importance
- Estimate autocovariance for two groups of firms:
  - Constrained: When a firm has potential credit equal to zero at age a-j
  - Unconstrained: Firms that have potential credit available at age a-j

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$

$$+ \underbrace{\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

◂ Back ▸ Derivation

Derivation of autocovariance formula (1/2)

Write stochastic processes in MA representation:

$$u_{i,t} = \rho_u^{t+1} u_{i,-1} + \sum_{k=0}^{a} \rho_u^k \theta_i$$

$$v_{i,a} = \rho_v^{a+1} v_{i,-1}$$

$$w_{i,a} = \sum_{k=0}^{a} \rho_w^k \varepsilon_{i,a-k} = \sum_{k=0}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} \quad 0 \leq j \leq a$$

So the level of log employment of firm i at age a is:

$$\ln n_{i,a} = \rho_u^{a+1} u_{i,-1} + \sum^{a} \rho_u^k \theta_i + \rho_v^{a+1} v_{i,-1} + \sum_{i=1}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{i=1}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} + z_{i,a}$$

◂ Back

Derivation of autocovariance formula (2/2)

Then the autocovariance of log employment at age a and $a-j$ for $j \geq 0$ is:

$$\begin{aligned}
\mathrm{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] &= \left(\sum_{k=0}^{a} \rho_u^k\right) \sigma_\theta^2 \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \rho_u^{a+1}\sigma_{\tilde{u}}^2 \rho_u^{a-j+1} + \rho_v^{a+1}\sigma_{\tilde{v}}^2 \rho_v^{a-j+1} \\
&\quad + \mathrm{Cov}\left[\rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}, \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}\right] + \mathbf{1}_{\{j=0\}}\sigma_z^2 \\
&= \sigma_\theta^2 \left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right) + \sigma_{\tilde{u}}^2 \rho_u^{2(a+1)-j} + \sigma_{\tilde{v}}^2 \rho_v^{2(a+1)-j} + \sigma_\varepsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \mathbf{1}_{\{j=0\}}\sigma_z^2
\end{aligned}$$

◂ Back

Ex-ante contribution

$$\frac{\text{Ex-ante variance}}{\text{Total variance}}(\log n_{i,a}) = \frac{\left(\sum_{k=0}^{a}\rho_u^k\right)^2\sigma_\theta^2 + \rho_u^{2(a+1)}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)}\sigma_{\hat{v}}^2}{\left(\sum_{k=0}^{a}\rho_u^k\right)^2\sigma_\theta^2 + \rho_u^{2(a+1)}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)}\sigma_{\hat{v}}^2 + \sigma_\epsilon^2\sum_{k=0}^{a}\rho_w^{2k} + \sigma_z^2}$$

◂ Back

Simple Model results

The elasticity of capital with respect to the shock ϵ is decreasing on capital and increasing on the productivity of the firm

$$\frac{\Delta g_{cons}}{\Delta \epsilon}\Big|_{\epsilon \approx 0} = (1 + q_t \xi)(z\theta_i k_{t,1}^{\alpha-1}) = \frac{(1 + q_t \xi)}{\alpha} mpk_i$$

◂ Back

Households

A representative household solves the following recursive maximisation problem

$$V(k) = \max_{c,l,k'} \{U(c,l) + \beta \mathbb{E} V(k')\}$$

subject to:

$$k' + c = (1+r)k + \omega l + D,$$

First order conditions are standard, pinning down interest rate and wages in steady state:

$$(1+r) = \frac{1}{\beta}$$

$$\omega = \psi C$$

◂ Back

Production and labor

Firm's total profits are given by

$$\pi_t = y_t - \omega_t l_t$$

The firm's optimal labor decision is a static choice which can be found through the firm's first order condition

$$l(k_t, \varphi_t; \omega_t) = \left(\frac{\upsilon \varphi_t}{\omega_t} k_t^{\alpha} \right)^{\frac{1}{1-\upsilon}}$$

Back

Firm problem after production

- Expected value of the firm after production is:

$$V_t^1(x_t, \varphi_t) = \pi_d x_t + (1 - \pi_d)\, V_t^2(x_t, \varphi_t)$$

where $x_t \equiv \pi(k_t, \varphi_t) + (1 - \delta)k_t - b_t$ is current cash-on-hand

- Surviving firms face the following optimisation problem:

$$V^2(x_t, \varphi_t) = \max_{k_{t+1}, b_{t+1}} \left\{ D_t + \mathbb{E}_t \left[\Lambda_{t+1|t} V^1 \left(x_{t+1}, \varphi_{t+1} \right) | \varphi \right] \right\}$$

s.t.

$$D_t \equiv x_t + q_t b_{t+1} - k_{t+1} \geq 0$$

$$b_{t+1} \leq \xi x_t$$

Back

Model - Entry and exit

- Fixed measure of potential entrants π_d
- Enter with zero debt and average capital of $\mu_{ke}\bar{k}^*_{i,t}$
- Draw productivity from stationary distribution of φ
- Entry takes place at the end of the period, start operating in the next period, given $(x_{i,t,0}, \varphi_{i,t,0})$

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