Bank of Japan's ETF Purchase Program and Equity Risk Premium: A CAPM Interpretation ¹

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 $^{^{1}}$ The views expressed here are those of the author and do not necessarily represent the views of BIS.

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 - $\star\,$ The amount purchased exceeded 5% of the total market value.
- However, it is still unclear whether and why the BOJ's ETF purchase program has effects on equity risk premia.
- This paper: we quantify the effects of the BOJ's ETF purchase program on a systematic risk of Japanese stocks.
 - $\star\,$ 1. Estimate the impact of the BOJ's purchase on risk factors.
 - \star 2. Quantify the effect of changes in risk factors on risk premia.
- ► A particular focus is on a counter-cyclical nature of the policy.







- Effects of BOJ's ETF purchase program: Shirota (2018), Barbon & Gianinazzi (2019), Harada & Okimoto (2021), Adachi et al. (2021), Maeda et al. (2021), Charoenwong et al. (2021)
- Effects of asset purchase program (esp. flow effects): D'amico & King (2013), Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2011), Bernanke (2020)
- Systematic Risk: Solnik (1974), Harvey and Siddique (2000), Ang et al. (2006)

Counter-cyclical Nature of BOJ's ETF Purchases and Flow effects of the ETF Purchase Program

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We focus on the global investors' point of view instead of domestic investors.

Counter-cyclical Nature of BOJ's ETF Purchases

We estimate the ETF purchasing probability with a probit model where the explanatory variable is the market index return in the morning session.

$$Pr\left(D_{t}=1|R_{t}^{M},X_{t}\right)=\Phi\left(\gamma_{0}+\gamma_{1}R_{t}^{M}+\gamma_{2}X_{t}+\varepsilon_{t}\right),\qquad(1)$$

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ETF purchase probability

The estimated probability implies its counter-cyclicality.

Identification Issue on Policy Effects

The BOJ's counter-cyclical rule makes it difficult to identify the policy effects due to the endogeneity problem. The BOJ's counter-cyclical rule makes it difficult to identify the policy effects due to the endogeneity problem.

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The BOJ purchases ETFs tracking different stock indexes,

- * **TOPIX**: All listed stocks with market value weights.
- * Nikkei225: Only 225 stocks with price weights.
- Given the BOJ's policy changes on purchasing share across ETFs, we exploit the cross-sectional variations.

Panel Data for BOJ's Purchases

We construct a panel dataset for the BOJ's *indirect* purchases of stock *i* in time *t* (relative to *i*'s market value) by:

$$ETF_{it} = \frac{w_{it}^T ETF_t^T + w_{it}^N ETF_t^N}{Mkt_{it}} = \frac{ETF_t^T}{Mkt_t} + \frac{w_{it}^N}{w_{it}^T} \cdot \frac{ETF_t^N}{Mkt_t}$$

- * w_{it}^{T} , w_{it}^{N} : Stock *i*'s weight in TOPIX and Nikkei225
- ★ ETF^T_t, ETF^N_t: The BOJ's purchases of ETFs tracking TOPIX and Nikkei225

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- * w_{it}^{T} , w_{it}^{N} : Stock *i*'s weight in TOPIX and Nikkei225
- * \mathbf{ETF}_t^T , \mathbf{ETF}_t^N : The BOJ's purchases of ETFs tracking TOPIX and Nikkei225
- ▶ Due to the differences in w_{it}^N / w_{it}^T across stocks, changes in ETF_t^N induce cross-sectional variations in ETF_{it} .

Purchasing Amount by ETF Types



- The BOJ started by purchasing TOPIX and Nikkei225 equally and decreased the share of Nikkei225 ETFs over time.
 - $\star\,$ The purchasing amount of JPX400 is very small.

Flow effects

Table: Effect of the BOJ's ETF Put	rchases on Daily Stock Returns
------------------------------------	--------------------------------

	(1)	(2)	(3)
	r _{it}	r _{it}	r _{it}
ETF	35.98***	52.98***	58.68***
	(0.815)	(4.637)	(4.960)
Nikkei_TPX			-0.509*** (0.0614)
Individual Stock Fixed Effect	YES	YES	YES
Time Fixed Effect	No	YES	YES
Ν	1251307	1251307	1250501

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- We investigate the policy effects on (1) market beta, (2) downside/upside beta, and (3) coskewness.
- ► For panel regression, we use the four 4-year time-windows:
 - 1. Aug.2006-Jul.2010: Before the program
 - 2. Aug.2010-Jul.2014: Introduction of the program (Dec.2010)
 - Aug.2014-Jul.2018: Expansions to 3 tril. yen (Oct.2014) and to 6 tril. yen (Jul.2016)
 - Aug.2018-Dec.2021: Reductions in Nikkei225's purchasing share (Jul.2018 and Mar.2021)

BOJ's ETF Purchases and Market Beta

Estimation of Market Beta

Following a standard CAPM, a market beta for stock *i* in time *t*, *Beta_{it}*, is estimated by (for each $t \in \{1, 2, 3, 4\}$):

$$\frac{R_{i\tau}}{\Delta F X_{\tau}} - R_{\tau}^{f} = \alpha_{it} + \hat{Beta}_{it} \times \left(R_{\tau}^{M} - R_{\tau}^{f}\right) + \varepsilon_{i\tau} \text{ where } \tau \in t.$$

where

- ► $R_{i\tau}/\Delta F X_{\tau}$: Weekly returns of stock *i* on the USD basis for $\tau \in t$
- ► $R_{i\tau}$: Weekly returns of stock *i* on the Japanese Yen basis
- ΔFX_{τ} : Weekly changes in the Yen-Dollar exchange rate.
- R_{τ}^{f} : Weekly returns of 6-month U.S. treasury.
- ► R_{τ}^{M} : Weekly returns of the FTSE ALL-World Index
- α_{it} : CAPM alpha at t (constant term).

BOJ's ETF Purchases and Market Beta

Estimation of Policy Effects on Market Beta

Using the estimated beta, we estimate the effects of the ETF purchases on market beta by a panel regression analysis:

$$Beta_{it} = FE_i + T_t + \phi_1 ETF_{it} + \phi_2 X_{it} + \varepsilon_{it}$$
, where $t \in \{1, ..., 4\}$

where

- Beta_{it} : estimated market beta for stock i in period t
- ETF_{it} : sum of the BOJ's purchases of stock i divided by market value of stock i in period t.
- ► *FE_i* : time-invariant fixed effect for stock *i*.
- T_t : time dummy.
- \blacktriangleright X_{it} : vector of control variables.

Effects of the BOJ's ETF Purchases on Market Beta											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Beta	Beta	Beta (2y)	Beta (2y)	Beta (6w)	Beta ^{3f}	Beta ^{3f}	Beta ^{4f}	Beta ^{4f}	Beta ^{FX}	Beta ^{FX}
ETF	-0.045***	-0.041***	-0.042**	-0.046**	-0.042**	-0.047***	-0.046***	-0.049***	-0.048***	-0.048***	-0.050***
	(0.005)	(0.005)	(0.021)	(0.018)	(0.020)	(0.005)	(0.005)	(0.002)	(0.001)	(0.003)	(0.003)
log(P/E ratio)		0.024***		0.030***	0.029***		0.034***		0.030***		0.023***
		(0.009)		(0.011)	(0.007)		(0.007)		(0.008)		(0.006)
log(book-to-market)		-0.058***		-0.101***	-0.064***		-0.098***		-0.088***		-0.080***
		(0.004)		(0.023)	(0.008)		(0.013)		(0.011)		(0.012)
log(size)		0.001		0.018	0.018		-0.028		-0.031		-0.025
		(0.022)		(0.033)	(0.015)		(0.025)		(0.024)		(0.023)
NikkeiDummy		0.067***		0.043***	0.025		0.067***		0.088***		0.100***
		(0.017)		(0.012)	(0.037)		(0.012)		(0.015)		(0.013)
Individual stock FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Ν	7859	7107	15458	14518	11180	7867	7122	7871	7123	7901	7127
R^2	0.398	0.412	0.233	0.370	0.218	0.137	0.163	0.170	0.185	0.024	0.042

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 - \star It explains various asset returns, too (Lettau et al. 2014)
- BOJ purchases ETFs only when downside risks materialize.
- Thus, separately estimating the policy effects on upside and downside risks may give more precise policy effects.

BOJ's ETF Purchases and Coskewness

Coskewness is a systematic exposure to market volatility (Harvey and Siddique 2000) estimated by:

$$\frac{R_{i\tau}}{\Delta F X_{\tau}} - R_{\tau}^{f} = \alpha_{it} + \hat{Beta}_{it} \times \left(R_{\tau}^{M} - R_{\tau}^{f}\right) + Co\hat{Skew}_{it} \times \left(R_{\tau}^{M} - R_{\tau}^{f}\right)^{2} + \varepsilon_{i\tau}$$

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- Investors require higher risk premia for stocks with negative coskewness (i.e., more exposure to market volatility).
 - ★ Coskewness can resolve the low-beta anomaly, i.e., low-beta stocks offer high risk-adjusted returns (Schneider 2020).

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 - Coskewness can resolve the low-beta anomaly, i.e., low-beta stocks offer high risk-adjusted returns (Schneider 2020).
- The BOJ's purchases may affect coskewness as it purchases ETFs only when the market return is *very* negative.

BOJ's ETF Purchases and Downside Beta & Coskewness

Estimation Results of Policy Effects

	(1)	(2)	(3)	(4)	(5)	(6)
	Beta _{down}	Beta _{down}	Betaup	Betaup	CoSkew	CoSkew
ETF	-0.032**	-0.027**	-0.026**	-0.023**	0.001***	0.001***
	(0.013)	(0.012)	(0.011)	(0.011)	(0.000)	(0.000)
log(P/E ratio)		0.039***		-0.012***		-0.002***
		(0.014)		(0.004)		(0.001)
log(book-to-market)		-0.057***		-0.037***		-0.000
,		(0.010)		(0.001)		(0.001)
log(size)		-0.047***		0.003		0.004***
,		(0.016)		(0.009)		(0.001)
NikkeiDummy		0.100*		0.036**		-0.002
		(0.060)		(0.018)		(0.003)
Individual stock FE	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	YES
Ν	7865	7124	7857	7130	7866	7142
R^2	0.223	0.246	0.369	0.440	0.063	0.081

The BOJ's purchases have significantly: (i) reduced both upside and downside beta, and (ii) increased coskewness.

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- Question: How much does the reduction in systematic risk affect equity risk premia and stock prices?
- We answer this question by:
 - 1. estimating the size of compensation for the systematic risk measures in the Japanese stock market, and
 - 2. doing a back-of-envelope calculation to quantify the policy effects on risk premia and stock prices.

Panel Regression for Cross-section of Stock Returns

Dependent variable: Excess returns on stock i, rit

	(1)	(2)	(3)	(4)	(5)
Beta	0.073**		0.068**		
	(0.031)		(0.029)		
- 26					
Beta ⁵⁷		0.069**			
		(0.027)			
CoSkew			0.006***		0 177
COSKEW			-0.900		(0.250)
			(0.050)		(0.259)
Betadown				0.086***	0.076***
down				(0.008)	(0.013)
				()	()
Beta _{up}				-0.048***	-0.039*
				(0.013)	(0.024)
	0.050***	0.001***	0.050***	0.000***	0.000***
log(book-to-market)	0.059***	0.061***	0.059***	0.060***	0.060***
	(0.008)	(0.008)	(0.008)	(0.008)	(0.009)
log(size)	-0.171***	-0.169***	-0.166***	-0.165***	-0.165***
	(0.020)	(0.020)	(0.019)	(0.019)	(0.019)
	. ,	. ,	. ,	. ,	
lagged stock return	-0.187***	-0.188***	-0.185***	-0.180***	-0.181***
	(0.045)	(0.046)	(0.046)	(0.045)	(0.045)
	2/50	2/50	2/50	2/50	2/50
Individual stock FE	YES	YES	YES	YES	YES
I Ime FE	YES	YES	YES	YES	YES
N D ²	6749	6749	6749	6749	6749
K-	0.362	0.362	0.367	0.366	0.366

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Back-of-Envelope Calculation for Policy Effects Overview

To quantify the policy effects, we examine "what if" scenario that the BOJ did not purchase ETFs at all.

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Back-of-Envelope Calculation for Policy Effects

- To quantify the policy effects, we examine "what if" scenario that the BOJ did not purchase ETFs at all.
- ► ΔRP_{it}: The etimated decline in risk premia from those in the counterfactual case without the BOJ's ETF purchases.
- Our estimation results so far imply that:

$$\Delta RP_{it} = \underbrace{\underbrace{0.041 \times ETF_{it}}_{ETF \ purchase \Rightarrow Beta} \times 0.073}_{Beta \Rightarrow Risk \ premia}$$

Back-of-Envelope Calculation for Policy Effects Overview (cont'd)

Similarly, when we take the effects of the program on beta & coskewness into consideration:

$$\Delta RP_{it} = \underbrace{\underbrace{0.041 \times ETF_{it}}_{\textit{ETF purchase} \Rightarrow \textit{Beta}}_{\textit{Beta} \Rightarrow \textit{Risk premia}} \times 0.068 + \underbrace{\underbrace{(-0.001) \times \textit{ETF}_{it}}_{\textit{ETF purchase} \Rightarrow \textit{Coskewness}}_{\textit{Coskewness} \Rightarrow \textit{Risk premia}} \times (-0.906) .$$

• ΔRP_{it} is transformed into stock price changes, $\Delta P_{it}/P_{it}$, by:

$$\frac{\Delta P_{it}}{P_{it}} = -PER_{it} \times \Delta RP_{it}$$

 By taking the weighted average, we can calculate the policy effects on TOPIX and Nikkei225.

Back-of-Envelope Calculation for Policy Effects

Quantitative Results

Stock index	T	OPIX	Nikkei 225		
	$\Delta RP(\%)$	$\Delta P/P$ (%)	$\Delta RP(\%)$	$\Delta P/P$ (%)	
Panel (A): O	ne-factor bet	ta			
(ii)	-0.14	3.10	-0.35	8.99	
(iii)	-0.50	10.56	-0.99	26.08	
(iv)	-0.37	9.49	-0.44	12.96	
Panel (B): One-factor beta and coskewness					
(ii)	-0.18	4.07	-0.45	11.80	
(iii)	-0.65	13.87	-1.29	34.23	
(iv)	-0.49	12.46	-0.58	17.01	

- The BOJ's purchases boosted TOPIX and Nikkei225 by 10.56 and 26.08 percentage points at the peak.
- By considering policy effects though coskewness, estimated policy effects become 1.2 times larger.

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- We construct unique panel data to utilize the cross-sectional variations in stock purchases by the BOJ.
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- …leading to an economically significant decline in risk premia.

Thank you.

Appendix A: BOJ ETF Purchasing Program

Date	Total size	Purchasing ETFs and their share.
(1) 10/28/2010	The maximum outstanding amount	ETFs tracking Tokyo Stock Price Index (TOPIX)
	purchased: about 0.45 tril. yen	and the Nikkei 225 Stock Average are purchased so
		that the bank's purchase would roughly be propor-
(2) 3/14/2011	The "maximum" amount is in-	tionate to the total market value of that ETF issued. Unchanged
	creased to about 0.9 tril. yen	
(3) 8/4/2011	Up to about 1.4 tril. yen	Unchanged
(4) 4/27/2012	Up to about 1.6 tril. yen	Unchanged
(5) 10/30/2012	Up to 2.1 tril. yen	Unchanged
(6) 4/4/2013	At an annual pace of 1 tril. yen.	Introduction of the QQE
(7) 10/31/2014	3.0 tril. yen/year	Unchanged
(8) 11/19/2014	Unchanged	JPX400 ETFs is included. (The decision was made
		on 10/31/2014)
(9) 3/15/2016	3.3 tril. yen/year	Of which 0.3 tril. yen/year for purchasing ETFs com-
(4/1/2016)		posed of stocks issued by firms that are proactively
		investing in physical and human capital.
(10) 7/29/2016	6.0 tril. yen/year	Unchanged
(11) 9/21/2016	Unchanged	3.0 tril. yen/year for TOPIX, Nikkei225 and JPX400
		(proportional to the market value of that ETF); 2.7
		tril. yen/year for TOPIX; 0.3 tril. yen/year for ETFs
(12) 7/31/2018	Unchanged	to support corporate investment 1.5 tril. yen/year for TOPIX, Nikkei225 and JPX400
(8/6/2018)		ETFs; 4.2 tril. yen/year for TOPIX ETFs; 0.3 tril.
		yen/year for ETFs to support corporate investment
(13) 3/16/2020	12 tril. yen/year as a temporary	Unchanged
(14) 3/19/2021	measure for COVID-19 12 tril. yen/year even after	The Bank only purchases ETFs tracking TOPIX.

Appendix B: BOJ ETF Purchasing Amount

Variable	Mean	Std. Dev.	Min.	Max.	
ETF_BOJ					
(ii) Aug. 2010–Jul. 2014	0.42	0.58	0	5.83	1935
(iii) Aug. 2014–Jul. 2018	1.75	1.40	0	14.17	2143
(iv) Aug. 2018–Dec. 2021	1.78	0.64	0.05	3.74	2172
∆ETF_BOJ					
$(i) \rightarrow (ii)$	0.46	0.59	0	5.85	1755
(ii) $ ightarrow$ (iii)	1.47	0.86	-0.31	8.52	1935
(iii) $ ightarrow$ (iv)	0.05	1.04	-10.85	2.14	2143

Note: ETF_BOJ indicates the cross-sectional average of the total purchased amount of each stock in each period shown as the ratio to its market value (measured as a percentage) and Δ ETF_BOJ shows its change from the previous period. For details of the calculation, see Appendix.