On the Combination of Biased Members

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Purpose of This Paper

- I consider the information transmission problem within the organization by using a cheap talk model
- Particularly, I focus on how to combine biased subordinates to elicit truthful information from them
- Key element: uncertainty about the size of subordinates' biases

Krishna and Morgan (2001)

- Cheap talk model with two senders
- Biases are common knowledge
- The receiver prefers the senders with biases in the opposite directions (heterogeneous senders) to those with biases in the same direction (homogeneous senders)

Results of This Paper

- *The directions* of senders' biases are common knowledge, but not *their sizes*
- The receiver prefers the homogeneous senders to heterogeneous senders
- In the case of homogeneous senders, the effect of one's false report might be accelerated by another false report
$\Rightarrow$ This anxiety reduces the sender's incentive to send a false report

Baseline Model

- Baseline model: a variety of KM
- Main differences from KM
    - States: binary
    - Senders' biases: private information
    - Number of Senders: any finite number is allowed

Players, States, and Actions

- Player $\mathbf{0}$: Boss (Receiver)
- Player $\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}$: Subordinates (Senders)
- $\boldsymbol{t} \in \{\mathbf{0}, \mathbf{1}\}$: state w/ equal probs
- $\boldsymbol{a} \in \mathbb{R}$: action chosen by Boss

Payoffs

- Player i's payoff: $-(t + b_i - a)^2$
- I assume $b_0 = 0$ for normalization
  $\Rightarrow$ Boss's best response = Boss's belief over $t = 1$
- $F_i(b_i)$: distribution function of b_i $(i \neq 0)$
- $t, b_1, b_2, \ldots, b_n$: mutually independent

Information & Timing

- t is observed only by Subordinates
- To transmit information about t, Subordinate i announces a cheap talk message $m_i \in M_i$ ($\# M_i \geq 2$)
- I consider a sequential information transmission protocol, to exclude a fragile fully revealing equilibrium
- $b_0 = 0$: common knowledge
- b_i ($i \neq 0$): Subordinate i's private information
- F_i ($i \neq 0$): common knowledge

Bias Distribution

- F: baseline distribution function satisfying
  1. Continuity
  2. Dispersion: $F\left(\frac{1}{4}\right) > 0$ and $F\left(\frac{1}{2}\right) < 1$
  3. Non-negativity: $supp\ F \subseteq [0, 1]$
- I assume there are the following 2 kinds of Subordinates
  1. i: *upward biased* if $F_i = F$
  2. i: *downward biased* if $F_i(b) = 1 - F(-b)$
     ($f_i(b) = f(-b)$ if density function exists)
- I assume $F_1 = F$ wlog

Organization Mode

- Organization mode (n, k):
  - n active Subordinates
  - k upward biased Subordinates among n
  - $(n - k)$ downward biased Subordinates among n
- (n, n): completely homogeneous mode
- I only consider $k \geq \frac{n}{2}$ wlog

Threshold Strategy Equilibrium

- I focus on the class of Subordinate's (history and order independent) strategies that are characterized by his threshold (b_+ or b_-) and 2 messages (m^0 and m^1)
- For upward biased Subordinate i,
  - $m_i = m^1$ if $t = 1$: truthful message
  - $m_i = m^0$ if $t = 0$ and $b_i \leq b_+$: truthful message
  - $m_i = m^1$ if $t = 0$ and $b_i > b_+$: false message
- For downward biased Subordinate i,
  - $m_i = m^0$ if $t = 0$: truthful message
  - $m_i = m^1$ if $t = 1$ and $|b_i| \leq b_-$: truthful message
  - $m_i = m^0$ if $t = 1$ and $|b_i| > b_-$: false message

Threshold Strategy Equilibrium (cont'd)

- Credible messages

$$\begin{cases} \boldsymbol{m} = \boldsymbol{m}^0 & \text{for upward biased Subordinate} \\ \boldsymbol{m} = \boldsymbol{m}^1 & \text{for downward biased Subordinate} \end{cases}$$

- Dubious messages

$$\begin{cases} \boldsymbol{m} = \boldsymbol{m}^1 & \text{for upward biased Subordinate} \\ \boldsymbol{m} = \boldsymbol{m}^0 & \text{for downward biased Subordinate} \end{cases}$$

- Boss's best response: $\boldsymbol{a}\left(\tilde{\boldsymbol{k}}, \tilde{\ell}\right)$
  - $\tilde{\boldsymbol{k}}$: # of dubious messages sent by upward biased Subordinates
  - $\tilde{\ell}$: # of dubious messages sent by downward biased Subordinates

Preliminary Results

- I can show that there is no fully revealing equilibrium
- I can show that any PBE is essentially outcome-equivalent to some threshold strategy equilibrium as long as $\#M_i$ is finite

Boss's Best Response

$$a^{(n,k)}(\tilde{k},\tilde{\ell}) = \begin{cases} 1 & \text{if } \tilde{k} = k, \tilde{\ell} < n-k \\ \hat{a}^{(n,k)}(b_+^{(n,k)}, b_-^{(n,k)}) & \text{if } \tilde{k} = k, \tilde{\ell} = n-k \\ 0 & \text{if } \tilde{k} < k, \tilde{\ell} = n-k \end{cases}$$

where

$$\hat{a}^{(n,k)}(b_+^{(n,k)}, b_-^{(n,k)}) = \frac{\left(1 - F\left(b_-^{(n,k)}\right)\right)^{n-k}}{\left(1 - F\left(b_-^{(n,k)}\right)\right)^{n-k} + \left(1 - F\left(b_+^{(n,k)}\right)\right)^{k}}$$

Subordinates' Incentive Conditions

- The incentive for Subordinate matters only when he is pivotal, i.e., all the other Subordinates send their dubious messages

$$b_{+}^{(n,k)} = \frac{\hat{a}^{(n,k)}(b_{+}^{(n,k)}, b_{-}^{(n,k)})}{2}$$

$$b_{-}^{(n,k)} = \frac{1 - \hat{a}^{(n,k)}(b_{+}^{(n,k)}, b_{-}^{(n,k)})}{2}$$

Boss's Trade-Off

- More sensitive Boss's response to messages disciplines Subordinates more to send a truthful message

$$\hat{a}^{(n,k+1)}(b_+, b_-) > \hat{a}^{(n,k)}(b_+, b_-)$$
$$\Rightarrow \begin{cases} b_+^{(n,k+1)} > b_+^{(n,k)} \\ b_-^{(n,k+1)} < b_-^{(n,k)} \end{cases}$$

- Increase in the number of upward biased Subordinates makes upward biased Subordinates more disciplined, but downward biased Subordinates less disciplined

Equilibrium Conditions

$$\begin{cases} b_+^{(n,k)}\left(1-F\left(b_+^{(n,k)}\right)\right)^k = b_-^{(n,k)}\left(1-F\left(b_-^{(n,k)}\right)\right)^{n-k} \\ b_+^{(n,k)} + b_-^{(n,k)} = \frac{1}{2} \end{cases}$$

- There may be multiple threshold strategy equilibria
- I focus on the Boss's best equilibrium

Preliminary Propositions

- Proposition 1: The existence of threshold strategy equilibria
- Proposition 2: In completely homogeneous mode, Boss can receive the largest payoff when no Subordinates babble

Main Result: Completely Homogeneous Mode Is the Best for Boss

Proposition 3

For any $n \geq 2$, *any* k *such that* $n - 1 \geq k \geq \frac{n}{2}$, *and any* $b_{+}^{(n,k)}$, *there exists* $b_{+}^{(n,n)}$ *such that*

- $b_{+}^{(n,n)} > b_{+}^{(n,k)}$
- $EU_{0}^{(n,n)}\left(b_{+}^{(n,n)}\right) > EU_{0}^{(n,k)}\left(b_{+}^{(n,k)}\right)$

- Boss can receive the largest payoff in the completely homogeneous mode

Robustness

I can extend the previous results to the following environments:

- heterogeneous baseline distributions
- bias support for upward biased Subordinates is slightly overlapping with one for downward biased Subordinates
- simultaneous information transmission protocol
- biases are common knowledge among Subordinates

Conclusion

- I consider how to combine biased subordinates to elicit truthful information from them
- The key element is uncertainty about the sizes of subordinates' biases
- I show completely homogeneous subordinates are most desirable for Boss
- This is because, in the case of completely homogeneous subordinates, the effect of one's false report might be accelerated by another false report and this anxiety reduces an incentive to send a false report

Future Research

- Extension to the more general environments, especially one with general state space
- This would clarify the underlying logic of the results and the tension between KM's and my logic

Related Literature: Cheap Talk

- Uncertain biases:
  - One sender: Morgan and Stocken (2003), Dimitrakas and Sarafidis (2005), Li and Madarász (2008)
  - One sender in dynamic situations: Sobel (1985), Benabou and Laroque (1992), Morris(2001)
  - Two Senders: Li (2008, 2010), Rantakari (2014, 2021), Shimizu (2016), Karakoç (2021)

Related Literature: Organizational Economics

- Homogeneity/Heterogeneity between principal and agent:
  - Separation of decision and implementation: Blanes i Vidal and Möller (2007), Bester and Krähmer (2008), Landier et al (2009), Marino et al (2010), Van deb Steen (2010b), Ishihara and Miura (2021), Itoh and Morita (2023)
  - Information acquisition: Szalay (2005), Hori (2008), Che and Kartik (2009), Van den Steen (2010a), Omiya et al (2017), de Bettigniesand and Zábojnìk (2019)
- Homogeneity/Heterogeneity between multiple agents: Prasad and Tomaino (2020), Prasad and Tanase (2021), Rantakari (2014, 2021)

Equilibrium Condition (cont'd)

$b_+^{(n,k)}$ is the solution of $G^{(n,k)}(b) = 0$ where

$$G^{(n,k)}(b) = b\left(1 - F(b)\right)^k - \left(\frac{1}{2} - b\right)\left(1 - F\left(\frac{1}{2} - b\right)\right)^{n-k}$$

- The uniqueness of the solution is no longer guaranteed
- If $\exists b_+^{(n,k)} \in \left(0, \frac{1}{4}\right)$, $\exists b_+^{(n,n-k)} = \frac{1}{2} - b_+^{(n,k)} \in \left(\frac{1}{4}, \frac{1}{2}\right)$

PR1 Proof PR2 Proof LM1 Proof PR3 pt1 Proof PR3 pt2

Boss's Equilibrium Expected Payoff

$$EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$$
$$= -\frac{1}{2}\left(1 - F\left(b_+^{(n,k)}\right)\right)^k \left(a^{(n,k)}(k, n-k)\right)^2$$
$$- \frac{1}{2}\left(1 - F\left(b_-^{(n,k)}\right)\right)^{n-k} \left(1 - a^{(n,k)}(k, n-k)\right)^2$$

Boss's Equilibrium Expected Payoff (cont'd)

- By using the equilibrium condition, it can be rewritten as
  - $EU_0^{(n,k)} = -b_+^{(n,k)} \left(1 - F\left(b_+^{(n,k)}\right)\right)^k$
  - $EU_0^{(n,k)} = -\left(\frac{1}{2} - b_+^{(n,k)}\right)\left(1 - F\left(\frac{1}{2} - b_+^{(n,k)}\right)\right)^{n-k}$
- It then follows that
  - If $b_+^{(n,k)} \in \left(0, \frac{1}{4}\right)$, $\exists b_+^{(n,n-k)} = \frac{1}{2} - b_+^{(n,k)} \in \left(\frac{1}{4}, \frac{1}{2}\right)$ such that $EU_0^{(n,n-k)}\left(b_+^{(n,n-k)}\right) = EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$
  - $EU_0^{(n,n)} = b_+^{(n,n)} - \frac{1}{2}$

◂ Proof PR2 ◂ LM1 ◂ Proof LM1 ◂ Proof PR3

Proposition 1: Existence of Equilibrium

Proposition 1

For any $n \geq 2$, *any* k *such that* $n \geq k \geq \frac{n}{2}$, *there exists* $b_+^{(n,k)}$.
Moreover,

- $0 < b_+^{(n,k)} < \frac{1}{2}$
- $\frac{1}{4} < b_+^{(n,n)} < \frac{1}{2}$

Proof:

- $G^{(n,k)}(0) < 0$
- $G^{(n,k)}(b) > 0 \ \forall b \geq \frac{1}{2}$
- $G^{(n,n)}(b) < 0 \ \forall b \leq \frac{1}{4}$

Equilibrium Condition

Proposition 2: Comparison among Completely Homogeneous Modes

Proposition 2

For any $\boldsymbol{n} \geq \boldsymbol{2}$ *and any* $\boldsymbol{b}_+^{(\boldsymbol{n}-1,\boldsymbol{n}-1)}$, *there exists* $\boldsymbol{b}_+^{(\boldsymbol{n},\boldsymbol{n})}$ *such that*

- $\boldsymbol{b}_+^{(\boldsymbol{n},\boldsymbol{n})} > \boldsymbol{b}_+^{(\boldsymbol{n}-1,\boldsymbol{n}-1)}$
- $\boldsymbol{EU}_0^{(\boldsymbol{n},\boldsymbol{n})}\left(\boldsymbol{b}_+^{(\boldsymbol{n},\boldsymbol{n})}\right) > \boldsymbol{EU}_0^{(\boldsymbol{n}-1,\boldsymbol{n}-1)}\left(\boldsymbol{b}_+^{(\boldsymbol{n}-1,\boldsymbol{n}-1)}\right)$

- In any completely homogeneous mode, Subordinates are most disciplined and Boss receives the largest payoff Boss listens to all Subordinates

Corollary 1: Efficiency Loss of Completely Homogeneous Modes Vanishes in the Limit

Corollary 1

There exists a strictly increasing sequence $\{b_+^{(n,n)}\}_{n\geq 2}$ *such that*

- $\lim_{n\to\infty} b_+^{(n,n)} = \frac{1}{2}$
- $\lim_{n\to\infty} EU_0^{(n,n)}\left(b_+^{(n,n)}\right) = 0$

Proof of Proposition 2

- $G^{(n,n)}\left(b_+^{(n-1,n-1)}\right) < 0$
  $\Rightarrow \exists b_+^{(n,n)} > b_+^{(n-1,n-1)}$
  $\Rightarrow EU_0^{(n,n)}\left(b_+^{(n,n)}\right) > EU_0^{(n-1,n-1)}\left(b_+^{(n-1,n-1)}\right)$

Equilibrium Condition Boss's Payoff

Proposition 3: Comparison between Completely Homogeneous Mode and Any Other Mode

Proposition 3

For any $n \geq 2$, any k such that $n-1 \geq k \geq \frac{n}{2}$, and any $b_+^{(n,k)}$, there exists $b_+^{(n,n)}$ such that

- $b_+^{(n,n)} > b_+^{(n,k)}$
- $EU_0^{(n,n)}\left(b_+^{(n,n)}\right) > EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$

- Boss receives the largest payoff in the completely homogeneous mode

Lemma 1

Lemma 1

For any $n \geq 2$*, any* k *such that* $n - 1 \geq k \geq 1$*, and any* $b_+^{(n,k)} \in \left[\frac{1}{4}, \frac{1}{2}\right)$*, there exists* $b_+^{(n,n)}$ *such that*
$$EU_0^{(n,n)}\left(b_+^{(n,n)}\right) > EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$$

- Proved by verifying $G^{(n,n)}\left(EU_0^{(n,k)}\left(b_+^{(n,k)}\right) + \frac{1}{2}\right) < 0$

Boss's Payoff

Proof of Lemma 1

- Notation:
  - $x := b_{+}^{(n,k)}$
  - $y := 1 - F(x)$
  - $z := 1 - F\left(\frac{1}{2} - x\right)$
- $G^{(n,k)}\left(b_{+}^{(n,k)}\right) = 0 \Leftrightarrow xy^{k} = \left(\frac{1}{2} - x\right) z^{n-k}$
  - $z < 1 \Rightarrow \frac{1}{2} - xy^{k} > x$,
  - $y \leq z \Rightarrow x \geq \frac{y^{n-k}}{2\left(y^{n-k}+y^{k}\right)}$.
- $EU_{0}^{(n,k)}\left(b_{+}^{(n,k)}\right) = -xy^{k}$

Equilibrium Condition
Boss's Payoff

Proof of Lemma 1 (cont'd)

$$
\begin{aligned}
& G^{(n,n)}\left(EU_0^{(n,k)}\left(b_+^{(n,k)}\right)+\frac{1}{2}\right) \\
&= \left(\frac{1}{2}-xy^k\right)\left(1-F\left(\frac{1}{2}-xy^k\right)\right)^n - xy^k \\
&\leq \left(\frac{1}{2}-xy^k\right)y^n - xy^k \\
&\leq \frac{1}{2}y^n - y^k\left(1+y^n\right)\frac{y^{n-k}}{2\left(y^{n-k}+y^k\right)} \\
&= -\frac{y^n\left(1-y^{n-k}\right)\left(1-y^k\right)}{2\left(y^{n-k}+y^k\right)} < 0
\end{aligned}
$$

Proof of Proposition 3

- $G^{(n,n)}\left(b_+^{(n,k)}\right) < 0$
  $\Rightarrow \exists \bar{b}_+^{(n,n)} > b_+^{(n,k)}$
- If $b_+^{(n,k)} \in \left[\frac{1}{4}, \frac{1}{2}\right)$, Lemma 1 guarantees $\exists \hat{b}_+^{(n,n)}$ such that $EU_0^{(n,n)}\left(\hat{b}_+^{(n,n)}\right) > EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$

Equilibrium Condition

Proof of Proposition 3 (cont'd)

- If $b_+^{(n,k)} \in \left(0, \frac{1}{4}\right)$, $\exists b_+^{(n,n-k)} = \frac{1}{2} - b_+^{(n,k)}$ such that $EU_0^{(n,n-k)}\left(b_+^{(n,n-k)}\right) = EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$
  $\Rightarrow$ Lemma 1 guarantees $\exists \hat{b}_+^{(n,n)}$ such that $EU_0^{(n,n)}\left(\hat{b}_+^{(n,n)}\right) > EU_0^{(n,k)}\left(b_+^{(n,k)}\right)$
- Choose $\max\left\{\bar{b}_+^{(n,n)}, \hat{b}_+^{(n,n)}\right\}$

Equilibrium Condition Boss's Payoff

Example: Asymmetric Distribution

- $n = 2$
- $b_i \sim U[0, w_i]$ for $i = 1, 2+$
- $b_{2-} \sim U[-w_{2-}, 0]$
- $w_i \in \left(\frac{1}{2}, 1\right)$ for $i = 1, 2+, 2-$
- $\exists \hat{w}_{2-} \in \left(\frac{1}{2}, w_{2+}\right)$ such that
  - $EU_0^{(+,+)} > EU_0^{(+,-)}$ if $w_{2-} > \hat{w}_{2-}$
  - $EU_0^{(+,+)} < EU_0^{(+,-)}$ if $w_{2-} < \hat{w}_{2-}$
- As w_2 deceases, downward biased Subordinate 2 has more tendency to send a truthful message, while upward biased Subordinate 1 becomes less disciplined

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