Competing to Commit: Markets with Rational Inattention

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Motivation

Standard models of competition assume that consumers are perfectly informed about all payoff-relevant variables.

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What if we drop costless information processing?

How does market structure or competition interact with consumers having limited attention?

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Interpretation:

- Relevant markets: offers difficult to evaluate
  (health insurance, life insurance, complex loans, etc)

The Model

Agents

- Two identical firms $i \in I = \{1, 2\}$.
  - Homogeneous product. $MC = 0$.
  - Random quality: $\mathbf{q} \sim \lambda$ (finite support on $\mathbb{R}_+$)
- One RI consumer.
  - Unitary demand.
  - Valuation = quality

Strategies

Firms:

- $\sigma_i \in \Delta(\mathbb{R}_+)^Q$ *behavior strategy* of firm $i \in I$.
- $\sigma = (\sigma_1, \sigma_2)$.

Consumer: *Recommendation strategy*

- $\beta_i(q, x_1, x_2) \in [0, 1]$ conditional probability of buying from i.
- $\beta = (\beta_1, \beta_2)$ strategy of the consumer.

game structure

Payoffs

Let $\mu \in \Delta(Q \times R^2_+)$ be consistent with λ and σ.

- Firms: $\mathbb{E}_\mu \left[\mathbf{x}_i \cdot \beta_i(\mathbf{q}, \mathbf{x})\right]$.
- Consumer: $\mathbb{E}_\mu \left[\sum_i (\mathbf{q} - \mathbf{x}_i) \cdot \beta_i(\mathbf{q}, \mathbf{x})\right] - k \cdot I(\mu, \beta)$.

$k > 0$: unitary cost of info processing.

$I(\mu, \beta)$: mutual information. formula

Analysis

Solution concept

BNE yields a great multiplicity of equilibria. (Ravid, 2020)

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Refinement: Robustness to vanishing perturbations (RVP).

- β is RVP if it can be justified on and off-path.
- Extends credible best response (Ravid, 2020) to multi-firm setting.
- Weaker version of trembling-hand. (Selten, 1975)

Implication of RVP

Lemma 1

Let β *be a RVP best response to* μ. *Then, for every* $q \in Q$ *and* $x_1, x_2 \geq 0$

$$\beta_i(q, x_1, x_2) = \frac{\pi_i \cdot e^{\frac{q - x_i}{k}}}{\sum_{j=1,2} \pi_j \cdot e^{\frac{q - x_j}{k}} + 1 - \pi_1 - \pi_2}$$

where $\pi_i = \mathbb{E}_\mu[\beta_i] \in [0, 1]$ *for each* $i \in I$.

- π_i: consumer's trade engagement level with firm i.

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- β_i describes the **endogenous** demand firm i faces.
- Finite number of equilibrium outcomes.
  (No trade, monopolist, competitive)

Existence and uniqueness

Competitive Trading Equilibrium $:=$ Both firms trade w.p. > 0.
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(a) *A competitive trading equilibrium exists iff $k < k^t$.*

(b) *If a competitive trading equilibrium exists it is unique.*

(c) *Equilibrium trade occurs w.p. 1 iff $k \leq k^e$.*

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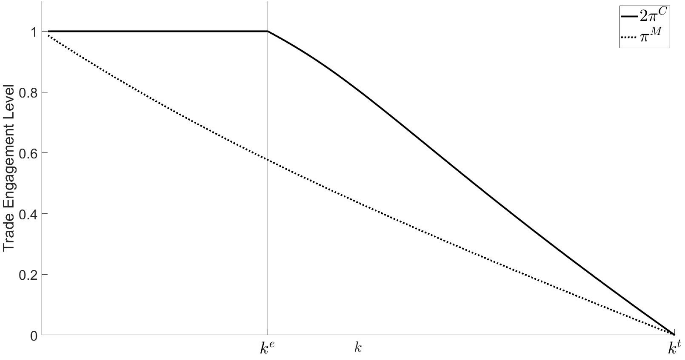
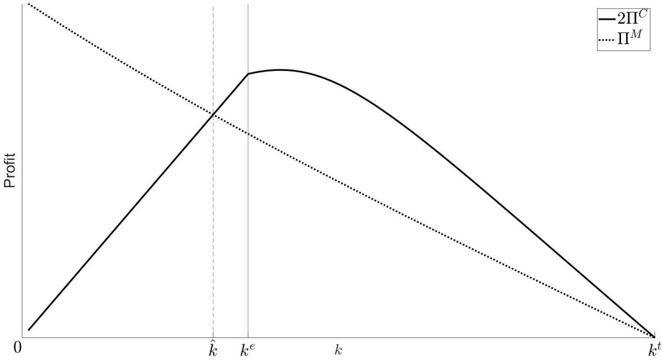
Remark:
With monopoly k^t is the same & k^e does not exist.

Overall trade probability



Competition alleviates commitment issue: Trading surplus ↑.

Producer surplus



Recall: homogeneous products! **Key:** endogeneity of demand.

Main Result

$\Pi^m(k) :=$ equilibrium expected industry profits. $m \in \{M, C\}$.

Theorem 2

There exists $\hat{k} \in (0, k^t)$ *such that*

$$\Pi^M(k) < \Pi^C(k), \ \forall k \in (\hat{k}, k^t).$$

Intuition:

For high k, expansion in demand $>$ negative effect on prices.

Conclusion

- Market structure affects attention allocation.
- Competition acts as a commitment device for the firms not to overcharge the consumer.
- Competition shifts the demand curve up.
- Profits can be higher under competition than under collusion.

“Attention is the rarest and purest form of generosity.”

Simone Weil, 1909-1943

THANK YOU!

EXTENSIONS

More than two firms

Let N be the number of active firms.

- Overall trade probability increases with N.
- The region with efficient trade expands ($\bar{k}(N)$ increasing).

Proposition 1

Let $N > M \geq 2$. There exists $\hat{k} \in (\bar{k}(N), k^*)$ such that $\Pi^C(N) > \Pi^C(M)$ for all $k \in (\hat{k}, k^*)$.

Consumer Surplus

Consumer's Payoff:

$$\mathbb{E}_\mu \left[\sum_i (\mathbf{q} - \mathbf{x}_i) \cdot \beta_i(\mathbf{q}, \mathbf{x}) \right] - k \cdot I(\mu, \beta)$$

- If average prices are lower, consumer surplus is higher under competition.
- Prices are strictly lower for low ($k < \bar{k}$) and high k.

Random Marginal Cost

- Suppose quality is known, but consumer is uncertain about the firms' marginal costs:
- Marginal cost $c \sim \tilde{\lambda}$ with finite support $\tilde{\lambda} \in \Delta[0, q]$.
- Same qualitative results hold.
- Key force: Rational inattention about endogenous variable.

Beyond Entropic Costs

Assume information processing cost proportional to:

$$C(\mu, \beta) = f(\mathbb{E}_\mu[\beta_1], \mathbb{E}_\mu[\beta_2]) - \mathbb{E}_\mu[f(\beta_1, \beta_2)]$$

for some strictly concave $f : [0,1]^2 \to \mathbb{R}$

We provide conditions on f, under which the total trade engagement level is higher under competition than under collusion.

If $f(p_1, p_2) = -\left(\frac{1}{2}p_1^2 + \frac{1}{2}p_2^2 + \theta p_1 p_2\right)$ for $\theta \in (0,1)$, profits are higher under competition, whenever $Var(q)$ is small.

APPENDIX

Consumer's best response

Lemma 2

Let β be a RVP best response to μ. Then, for every $q \in Q$ and $x_1, x_2 \geq 0$

$$\beta^i(q, x_1, x_2) = \frac{\pi^i \cdot e^{\frac{q - x_i}{k}}}{\sum_{j=1,2} \pi^j \cdot e^{\frac{q - x_j}{k}} + 1 - \pi^1 - \pi^2}$$

where $\pi^i = \mathbb{E}_\mu \left[\beta^i\right] \in [0, 1]$ for each $i \in I$.

$\Longrightarrow$ Consumer worse-off compared to costless information.

- Why? Under RVP, consumer treats ex-ante perfectly homogeneous goods as if they were differentiated.

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Example

Let $\lambda = \delta_q$ and fix $\alpha \in [0, 1]$.

- $\beta^i_\alpha(q, x_1, x_2) = \frac{1}{2}\mathbf{1}(x_i = \alpha \cdot q)$
- $\sigma^\alpha_i(\cdot|q) = \delta_{\alpha \cdot q}$

$\implies (\mu_\alpha, \sigma^\alpha, \beta_\alpha)$ is a BNE.

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Entropic costs

$$I(\beta, \mu) := H(\mathbb{E}_\mu[\beta]) - \mathbb{E}_\mu[H(\beta)],$$

where

$$H(\beta) = -\beta_1 \log(\beta_1) - \beta_2 \log(\beta_2) - (1 - \beta_1 - \beta_2) \log(1 - \beta_1 - \beta_2).$$

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Comparison with the Literature

Limited attention & competition studied in Behavioral IO.

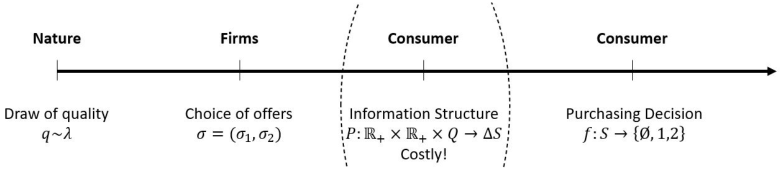
- Search cost models (Diamond71)
- Captive consumer models (Varian80)
- Others – miscellaneous: Discrete RI, framing, etc.

Our approach: Costly info processing à la Sims03 (entropy).

Differences:

1) Move away from 0-1 attention paradigm.
2) **Attention and Demand are linked.**

Game structure



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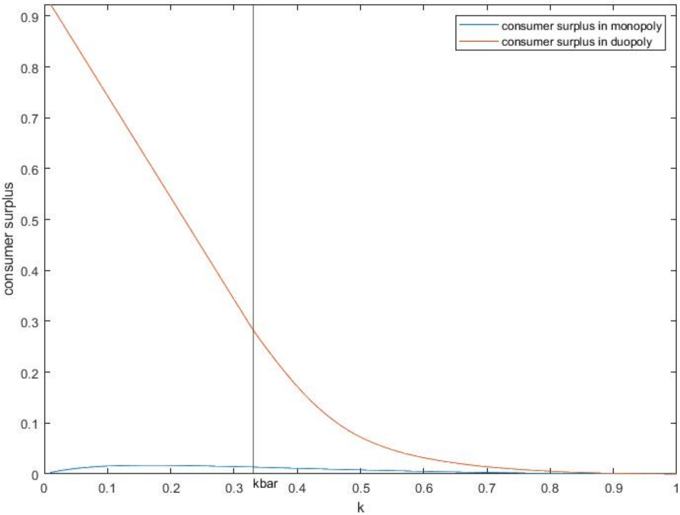
Consumer surplus



Prices per quality-valuation