Information Design with Costly State Verification

Lily Ling Yang
University of Mannheim

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Motivating example

- Platform makes product recommendation
- Given recommendation, consumers can always look for information outside platform
- **Platform can provide more information so that consumers don't look further**

Amazon's Choice

Related literature

1. Bayesian persuasion with private information: Kolotilin et al. (2017), Guo and Shmaya (2019), Kolotilin et al. (2023)
2. Information design with costly information acquisition: **Bizzotto et al. (2020)**, **Matyskova and Montes (2023)**

Setup

- One Sender and one Receiver: platform and consumer
- State $\theta \in [0,1]$ following $F(\theta)$
- Receiver chooses $a \in \{0,1\}$
  - $a = 1,\ u_S = 1, u_R = \theta - R$
  - $a = 0,\ u_S = 0, u_R = 0$
- An information structure is (M, G)
  - A set of message M
  - $G : [0,1] \to \Delta M$
  - Message as action recommendation

State verification

Receiver can probabilistically learn the state perfectly

- Receiver chooses $e \in \{0, 1\}$
  - $e = 1 : s = \theta$ w.p. q, and $s = \phi$ w.p. $1 - q$
  - $e = 0 : s = \phi$
- q is state independent: $E(\theta | s = \phi, e = 1) = E(\theta | s = \phi, e = 0) = E(\theta)$
  - Receiver chooses the same action when receiving a null signal as without verification
- Cost is c, and $C =: c/q$

Timeline

1. Sender chooses (M, G)
2. Nature draws θ according to F, and message m is sent to Receiver according to $G(\theta)$
3. Receiver observes m and makes verification decision $e \in \{0, 1\}$
4. Receiver observes signal s and takes action $a \in \{0, 1\}$

Information design

Disclosure affects Receiver in three ways:

1. Changing action choice without private information: $E(\theta)$
2. Changing verification choice: not only $E(\theta)$, but entire posterior distribution
3. Changing action choice with private information: simplified by state verification (no effect when $s = \theta$, no updating when $s = \phi$)

Four messages

- State verification perfectly reveals θ, so no need to track optimal action when $s = \theta$
- Optimal information structure sends four messages
  - Binary action recommendation: action chosen when $s = \phi$ (default action a_ϕ)
  - Binary verification recommendation: whether to verify state

Three messages

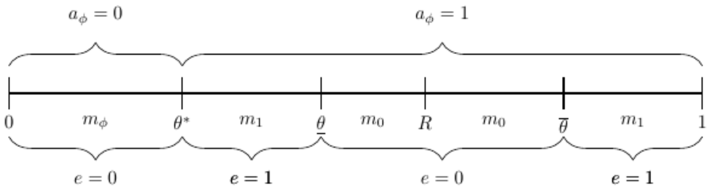
- Optimal information structure does not recommend $a_\phi = 0$ and $e = 1$
  - $a_\phi = 0$ and $e = 1$: $a = 1$ only if "good news" is found
  - Truth revealing instead
- Optimal information structure sends three messages
  1. $m_\phi : a_\phi = 0, e = 0$ (equivalent to truth revealing)
  2. $m_0 : a_\phi = 1, e = 0$ (Sender's most preferred message: $a = 1$ is always chosen)
  3. $m_1 : a_\phi = 1, e = 1$ ($a = 1$ unless "bad news" is found)

Main result

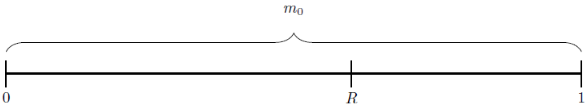
Theorem

The optimal information structure sends three messages, and has

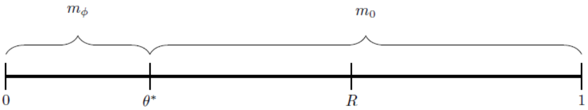
1. *a cutoff structure for action recommendation, and*
2. *a negative assortative structure for verification recommendation for the same action recommendation.*



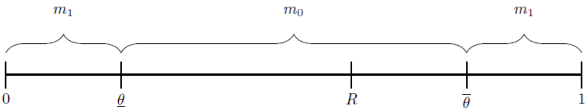
Four types of optimal information structures



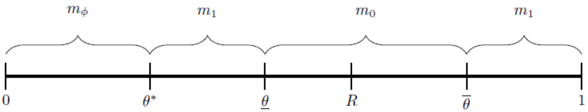
(1) No-disclosure mechanism



(2) Cut-off mechanism



(3) Negative assortative mechanism



(4) Three-message mechanism

Sketch of proof

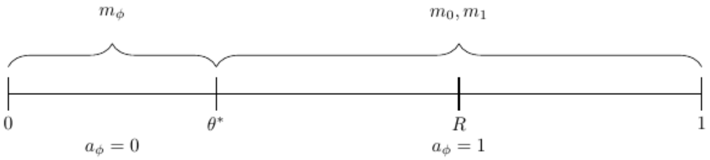
The information design problem can be reduced to

$$\max_{x_\phi(\theta),x_0(\theta),x_1(\theta)\in[0,1]} \int_0^R \left[x_0(\theta) + x_1(\theta)(1-q)\right] f(\theta)\,d\theta$$
$$+\int_R^1 (x_0(\theta)+x_1(\theta))\, f(\theta)\,d\theta$$

s.t.

$$\int_0^1 (\theta - R)\, x_0(\theta)\, f(\theta)\,d\theta \geq 0, \qquad \text{(A0)}$$
$$\int_0^1 (\theta - R)\, x_1(\theta)\, f(\theta)\,d\theta \geq 0, \qquad \text{(A1)}$$
$$C\int_0^1 x_0(\theta)\, f(\theta)\,d\theta + \int_0^R (\theta - R)\, x_0(\theta)\, f(\theta)\,d\theta \geq 0, \qquad \text{(C0)}$$
$$x_\phi(\theta) + x_0(\theta) + x_1(\theta) = 1.$$

- Suppose θ sends m_ϕ and θ' sends m_0 or m_1 s.t. $\theta > \theta'$
  - Interchanging the messages is a profitable deviation



- The information design problem can reduced to

$$\max_{x_0(\theta)\in[0,1],\theta^*\in[0,R]} \int_{\theta^*}^{R} \left[x_0(\theta) + (1 - x_0(\theta))(1-q)\right] f(\theta)\, d\theta + \int_{R}^{1} f(\theta)\, d\theta$$

s.t.

$$\int_{\theta^*}^{1} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0, \quad \text{(A0)}$$

$$\int_{\theta^*}^{1} (\theta - R)(1 - x_0(\theta))\, f(\theta)\, d\theta \geq 0, \quad \text{(A1)}$$

$$C\int_{\theta^*}^{1} x_0(\theta)\, f(\theta)\, d\theta + \int_{\theta^*}^{R} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0, \quad \text{(C0)}$$

- When $\Pr(m_1) > 0$, $(C0)$ is binding: given any feasible information structure that sends m_1, can we further relax $(C0)$?

- Persuasion constraint $(A0)$:

$$\int_{\theta^*}^{1} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0$$

$$\Leftrightarrow$$

$$\underbrace{\int_{R}^{1} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta}_{\text{resource}} \geq \underbrace{\int_{\theta^*}^{R} (R - \theta)\, x_0(\theta)\, f(\theta)\, d\theta}_{\text{cost}}$$

- $\theta > R$ raises credibility resource: $(\theta - R)\, f(\theta)$
- $\theta < R$ costs credibility resource: $(R - \theta)\, f(\theta)$
- Same for $(A1)$

- Verification cost constraint $(C0)$:

$$C\int_{\theta^*}^{1} x_0(\theta) f(\theta)\, d\theta + \int_{\theta^*}^{R} (\theta - R) x_0(\theta) f(\theta)\, d\theta \geq 0$$

$$\Leftrightarrow$$

$$\underbrace{C\int_{R}^{1} x_0(\theta) f(\theta)\, d\theta}_{\text{cost}} \geq \underbrace{\int_{\theta^*}^{R} (R - \theta - C) x_0(\theta) f(\theta)\, d\theta}_{\text{benefit}}$$

- $\theta > R$ raises verification cost: $Cf(\theta)$
- $\theta < R$ increases verification benefit: $(R - \theta - C) f(\theta)$

For $\theta > R$

- Sending m_0 raises credibility resource by $(\theta - R)\, f(\theta)$ and verification cost by $Cf(\theta)$
- Cost-to-resource ratio from sending m_0 is

$$\gamma(\theta) =: \frac{C}{\theta - R}$$

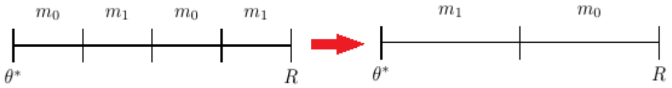
  - $\gamma(\theta)$ is decreasing in θ
  - Low θ induces higher verification cost when raising same amount of credibility resource

For $\theta < R$

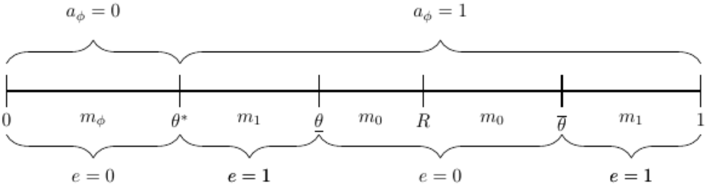
- Sending m_0 costs credibility resource by $(R - \theta) f(\theta)$ and increases verification benefit by $(R - \theta - C) f(\theta)$
- Benefit-to-cost ratio from sending m_0 is

$$\gamma(\theta) =: \frac{R - \theta - c}{R - \theta} = 1 + \frac{c}{\theta - R}$$

  - $\gamma(\theta)$ is decreasing in θ
  - High θ induces lower verification benefit when costing same amount of credibility resource



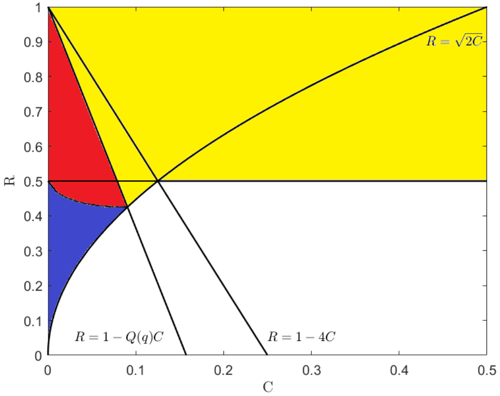
Optimal information structure



1. $\Pr\left(a = 1|\theta\right)$ is increasing in θ
2. $\Pr\left(a = 1|m_\phi\right) < \Pr\left(a = 1|m_1\right) < \Pr\left(a = 1|m_0\right)$
3. $E\left(\theta|m_\phi\right) < E\left(\theta|m_1\right) \leq E\left(\theta|m_0\right)$

A uniform example

Suppose $F(\theta) = \theta$ and $q = 0.8$



White: no disclosure; yellow: cutoff; blue: negative assortative; red: 3 message

THANK YOU!