Voting to Persuade

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Introduction

- Advisory committees are often involved in making important decisions
  - Examples: Federal Advisory Council, Investor Advisory Committee, FDA advisory committees, etc.
- Advisory committees are different from decision-making committees
  - Decision rules in advisory committees are endogenous
  - Cheap talk

Overview of the main results

In a continuous signal model, we provide

1. Necessary and sufficient condition for successful *information transmission*
2. Necessary and sufficient condition for full *information aggregation* when the committee size is large

The conditions are closely related to the *unanimity rule*

Setup

- Players: $\mathcal{N}$-person committee and DM
- State of the world: $\theta \in \{y, n\}$
- Prior belief: $\Pr(\theta = y) = p$
- DM's action: $D \in \{Y, N\}$
- Committee members' signals: s_i is independently distributed on $(a, b) \subseteq \mathbb{R}$ according to continuous distributions $F(.)$ in state y and $G(.)$ in state n

Setup

- Payoff tables:

C

	y	n
Y	$1/2$	$-1/2$
N	0	0

DM

	y	n
Y	$1-\alpha$	$-\alpha$
N	0	0

  - DM is more conservative: $\frac{1}{2} < \alpha$
- Voting strategy: $m_i : (a, b) \rightarrow [0, 1]$
- Decision rule: $d : \{Y, N\}^{\mathcal{N}} \rightarrow \{Y, N\}$

Timeline

1. Committee members observe their private signals and then vote simultaneously
2. The DM observes vote share and vote identity (full transparency)
3. The DM chooses between policy change Y and status quo N

Literature review

- Advisory committee
  - Wolinsky (2002), Levit and Malenko (2011), Battaglini (2017), Gradwohl and Feddersen (2018)
- Decision-making committee
  - Feddersen and Pesendorfer (1998), Duggan and Martinelli (2001), Martinelli (2002)

Assumptions on information structure

Assumption 1 (MLRP) $\frac{f(s)}{g(s)}$ is strictly increasing in s.

Assumption 2 $\lim_{s\downarrow a} \frac{f(s)}{g(s)} < \frac{1-p}{p} < \lim_{s\uparrow b} \frac{f(s)}{g(s)}$.

Assumption 3 (Increasing hazard ratio property, IHRP) $\frac{h_F(s)}{h_G(s)}$ is strictly increasing in s, where $h_F(s) := \frac{f(s)}{1-F(s)}$ and $h_G(s) := \frac{g(s)}{1-G(s)}$.

Increasing hazard ratio property

- First introduced: Kalashnikov and Rachev (1986)
- Decision-making committees: Duggan and Martinelli (2001)
- Information cascade: Herrera and Hörner (2011, 2013)
- Most distributions commonly used in economics satisfy IHRP
  - e.g., normal distributions, power distributions, gamma distributions, chi distributions, chi-squared distributions

Terminologies

Definition

A decision rule d is a k-rule if there exists $k \in \{1, 2, ..., \mathcal{N}\}$ s.t. $d(v) = Y$ iff $|v| \geq k$.

Definition

A decision rule d is a *weighted voting rule* if there exists $(w_1, w_2, ...w_{\mathcal{N}}) \in \mathbb{R}_+^{\mathcal{N}}$ and $Q \in \mathbb{R}_+$ such that $d(v) = Y$ iff $\sum_{i=1}^{\mathcal{N}} w_i \mathbf{1}_{\{v_i = Y\}} \geq Q$, where $\mathbf{1}$ is the indicator function.

Equilibrium characterization

Proposition

It is without loss of generality to assume that an equilibrium (m, d) of our model is such that

1. *m_i is a cutoff strategy or a partisan strategy;*
2. *The DM's decision rule d is a weighted voting rule.*

- In a symmetric equilibrium, the decision rule is a k-rule
- In an asymmetric equilibrium, the decision rule is a weighted voting rule
- Proof: Additivity of log-likelihood ratio

k-rules

- Given a k-rule, the equilibrium cutoff s^* of a symmetric equilibrium is given by

$$\underbrace{\frac{p}{1-p}}_{\text{prior}} \times \underbrace{\left(\frac{1-F\left(s^*\right)}{1-G\left(s^*\right)}\right)^{k-1}}_{\text{votes for } Y} \times \underbrace{\left(\frac{F\left(s^*\right)}{G\left(s^*\right)}\right)^{\mathcal{N}-k}}_{\text{votes for } N} \times \underbrace{\frac{f\left(s^*\right)}{g\left(s^*\right)}}_{\text{own signal}} = 1$$

k-rules

- The DM is willing to follow the k-rule iff

$$\frac{p}{1-p}\left(\frac{1-F\left(s^{*}\right)}{1-G\left(s^{*}\right)}\right)^{k-1}\left(\frac{F\left(s^{*}\right)}{G\left(s^{*}\right)}\right)^{\mathcal{N}-k+1}<\frac{\alpha}{1-\alpha}$$

and

$$\frac{\alpha}{1-\alpha}\leq\frac{p}{1-p}\left(\frac{1-F\left(s^{*}\right)}{1-G\left(s^{*}\right)}\right)^{k}\left(\frac{F\left(s^{*}\right)}{G\left(s^{*}\right)}\right)^{\mathcal{N}-k}$$

which become

$$\frac{g\left(s^{*}\right)}{f\left(s^{*}\right)}\frac{F\left(s^{*}\right)}{G\left(s^{*}\right)}<\frac{\alpha}{1-\alpha}\leq\frac{g\left(s^{*}\right)}{f\left(s^{*}\right)}\frac{1-F\left(s^{*}\right)}{1-G\left(s^{*}\right)}$$

k-rules

- The likelihood ratio of k yay votes and $\mathcal{N}-k$ nay votes is

$$\frac{p}{1-p}\left(\frac{1-F\left(s^{*}\right)}{1-G\left(s^{*}\right)}\right)^{k}\left(\frac{F\left(s^{*}\right)}{G\left(s^{*}\right)}\right)^{\mathcal{N}-k}=\underbrace{\frac{g\left(s^{*}\right)}{f\left(s^{*}\right)}}_{\text{a signal } s^{-}}\times\underbrace{\frac{1-F\left(s^{*}\right)}{1-G\left(s^{*}\right)}}_{\text{a vote for } Y}$$

- When k increases, the equilibrium cutoff s^{*} is lower
  1. The likelihood ratio of the signal s^{-}, $\frac{g(s^{*})}{f(s^{*})}$, is higher
  2. The likelihood ratio of the yay vote, $\frac{1-F(s^{*})}{1-G(s^{*})}$, is lower
- By IHRP, the first effect dominates

k-rules

Definition

Let $\boldsymbol{\alpha}(k, \mathcal{N})$ be the unique solution to

$$\frac{\alpha}{1-\alpha} = \frac{g(s^*(k,\mathcal{N}))}{f(s^*(k,\mathcal{N}))} \frac{1-F(s^*(k,\mathcal{N}))}{1-G(s^*(k,\mathcal{N}))}.$$

k-rules

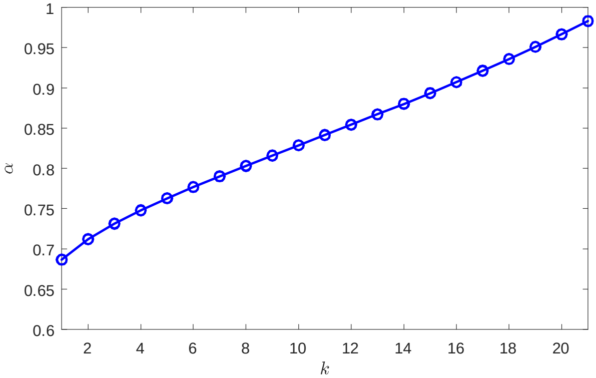


Figure: $\boldsymbol{\alpha}\left(k, \mathcal{N}\right)$

k-rules

Corollary

For all $k' > k$, there exists an informative equilibrium with k'-rule if there exists an informative equilibrium with k-rule.

- An informative equilibrium with the unanimity rule exists for the largest range of parameter
- The unanimity rule is the most "robust" decision rule

k-rules

- So far consider only k-rules and symmetric equilibria
- True also if include asymmetric equilibria with other decision rules

Condition for information transmission

Proposition

There exists an informative equilibrium if and only if $\alpha \leq \boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$.

- The existence of informative equilibrium implies the existence of a informative equilibrium with the unanimity rule
- IHRP is important

Condition for information transmission

- Our result recovers the intuitive idea that the unanimity is the most persuasive
- If DM cannot be persuaded by unanimity, she can never be persuaded
- Not true for the discrete model

Condition for information aggregation

- What about information aggregation?
- The unanimity rule may not aggregate information, but all other q-rules do (Feddersen and Pesendorfer 1998; Duggan and Martinelli 2001)

Condition for information aggregation

Proposition

There exists a sequence of equilibria along which the probabilities of the DM choosing Y in state y and N in state n approach 1 as $\mathcal{N} \to \infty$ if and only if $\alpha < \lim_{\mathcal{N}\to\infty} \boldsymbol{\alpha}(\mathcal{N},\mathcal{N})$.

k-rules

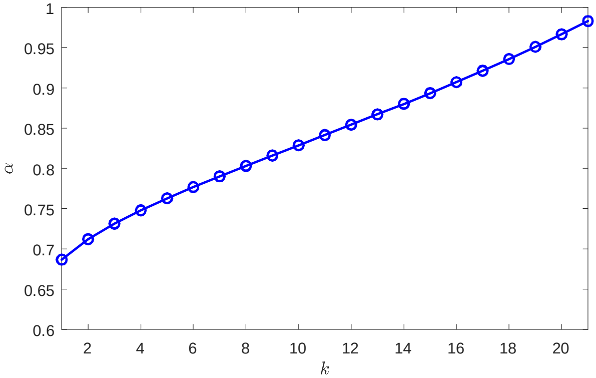
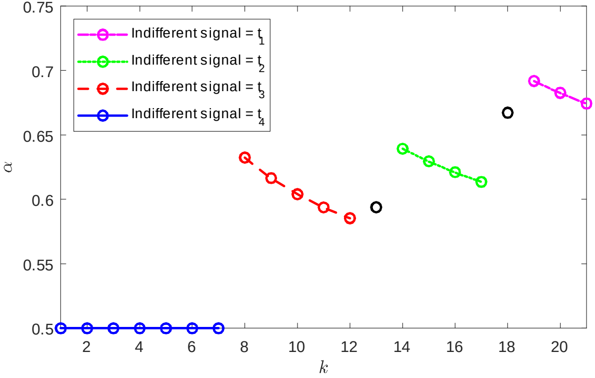


Figure: $\boldsymbol{\alpha}\left(k, \mathcal{N}\right)$

Discrete signals

- $\alpha(k, \mathcal{N})$ is not increasing



Conclusion

- We provide necessary and sufficient conditions for information transmission and aggregation in a model of advisory committees
- Intuition: If DM cannot be persuaded by unanimity, she can never be persuaded
- Our results does not hold for discrete models