Voting to Persuade

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August 31, 2023
Advisory committees are often involved in making important decisions

Examples: Federal Advisory Council, Investor Advisory Committee, FDA advisory committees, etc.

Advisory committees are different from decision-making committees

- Decision rules in advisory committees are endogenous
- Cheap talk
Overview of the main results

In a continuous signal model, we provide

1. Necessary and sufficient condition for successful *information transmission*

2. Necessary and sufficient condition for full *information aggregation* when the committee size is large

The conditions are closely related to the *unanimity rule*
Players: $\mathcal{N}$-person committee and DM

State of the world: $\theta \in \{y, n\}$

Prior belief: $\Pr(\theta = y) = p$

DM’s action: $D \in \{Y, N\}$

Committee members’ signals: $s_i$ is independently distributed on $(a, b) \subseteq \mathbb{R}$ according to continuous distributions $F(\cdot)$ in state $y$ and $G(\cdot)$ in state $n$. 
Setup

Payoff tables:

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<th>DM</th>
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<td>N</td>
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DM is more conservative: $\frac{1}{2} < \alpha$

Voting strategy: $m_i : (a, b) \rightarrow [0, 1]$

Decision rule: $d : \{ Y, N \}^N \rightarrow \{ Y, N \}$
Committee members observe their private signals and then vote simultaneously.

The DM observes vote share and vote identity (full transparency).

The DM chooses between policy change $Y$ and status quo $N$. 
Literature review

- Advisory committee

- Decision-making committee
Assumptions on information structure

Assumption 1 (MLRP) \( \frac{f(s)}{g(s)} \) is strictly increasing in \( s \).

Assumption 2 \( \lim_{s \downarrow a} \frac{f(s)}{g(s)} < \frac{1-p}{p} < \lim_{s \uparrow b} \frac{f(s)}{g(s)} \).

Assumption 3 (Increasing hazard ratio property, IHRP) \( \frac{h_F(s)}{h_G(s)} \) is strictly increasing in \( s \), where \( h_F(s) := \frac{f(s)}{1-F(s)} \) and \( h_G(s) := \frac{g(s)}{1-G(s)} \).
Increasing hazard ratio property

- First introduced: Kalashnikov and Rachev (1986)
- Decision-making committees: Duggan and Martinelli (2001)
- Information cascade: Herrera and Hörner (2011, 2013)
- Most distributions commonly used in economics satisfy IHRP
  - e.g., normal distributions, power distributions, gamma distributions, chi distributions, chi-squared distributions
**Definition**

A decision rule $d$ is a *k-rule* if there exists $k \in \{1, 2, ..., N\}$ s.t. $d(v) = Y$ iff $|v| \geq k$.

**Definition**

A decision rule $d$ is a *weighted voting rule* if there exists $(w_1, w_2, ..., w_N) \in \mathbb{R}_+^N$ and $Q \in \mathbb{R}_+$ such that $d(v) = Y$ iff $\sum_{i=1}^{N} w_i \mathbf{1}_{\{v_i = Y\}} \geq Q$, where $\mathbf{1}$ is the indicator function.
Proposition

It is without loss of generality to assume that an equilibrium \((m, d)\) of our model is such that

1. \(m_i\) is a cutoff strategy or a partisan strategy;
2. The DM’s decision rule \(d\) is a weighted voting rule.

- In a symmetric equilibrium, the decision rule is a \(k\)-rule
- In an asymmetric equilibrium, the decision rule is a weighted voting rule
- Proof: Additivity of log-likelihood ratio
Given a $k$-rule, the equilibrium cutoff $s^*$ of a symmetric equilibrium is given by

$$\frac{p}{1-p} \times \left( \frac{1 - F(s^*)}{1 - G(s^*)} \right)^{k-1} \times \left( \frac{F(s^*)}{G(s^*)} \right)^{N-k} \times \frac{f(s^*)}{g(s^*)} = 1$$
The DM is willing to follow the $k$-rule iff
\[
\frac{p}{1-p} \left( \frac{1 - F(s^*)}{1 - G(s^*)} \right)^{k-1} \left( \frac{F(s^*)}{G(s^*)} \right)^{N-k+1} < \frac{\alpha}{1-\alpha}
\]
and
\[
\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p} \left( \frac{1 - F(s^*)}{1 - G(s^*)} \right)^k \left( \frac{F(s^*)}{G(s^*)} \right)^{N-k}
\]
which become
\[
\frac{g(s^*)}{f(s^*)} \frac{F(s^*)}{G(s^*)} < \frac{\alpha}{1-\alpha} \leq \frac{g(s^*)}{f(s^*)} \frac{1 - F(s^*)}{1 - G(s^*)}
\]
- The likelihood ratio of $k$ yay votes and $N - k$ nay votes is

$$\frac{p}{1 - p} \left( \frac{1 - F(s^*)}{1 - G(s^*)} \right)^k \left( \frac{F(s^*)}{G(s^*)} \right)^{N-k} = \frac{g(s^*)}{f(s^*)} \times \frac{1 - F(s^*)}{1 - G(s^*)}$$

- When $k$ increases, the equilibrium cutoff $s^*$ is lower
  1. The likelihood ratio of the signal $s^-$, $\frac{g(s^*)}{f(s^*)}$, is higher
  2. The likelihood ratio of the yay vote, $\frac{1 - F(s^*)}{1 - G(s^*)}$, is lower

- By IHRP, the first effect dominates
Definition

Let $\alpha(k,\mathcal{N})$ be the unique solution to

$$\frac{\alpha}{1-\alpha} = \frac{g(s^*(k,\mathcal{N}))}{f(s^*(k,\mathcal{N}))} \frac{1-F(s^*(k,\mathcal{N}))}{1-G(s^*(k,\mathcal{N}))}.$$
Figure: $\alpha (k, \mathcal{N})$
Corollary

For all \( k' > k \), there exists an informative equilibrium with \( k' \)-rule if there exists an informative equilibrium with \( k \)-rule.

- An informative equilibrium with the unanimity rule exists for the largest range of parameter
- The unanimity rule is the most “robust” decision rule
So far consider only $k$-rules and symmetric equilibria

True also if include asymmetric equilibria with other decision rules
Condition for information transmission

Proposition

There exists an informative equilibrium if and only if $\alpha \leq \alpha (N^*, N^*)$.

- The existence of informative equilibrium implies the existence of a informative equilibrium with the unanimity rule
- IHRP is important
Our result recovers the intuitive idea that the unanimity is the most persuasive.
If DM cannot be persuaded by unanimity, she can never be persuaded.
Not true for the discrete model.
What about information aggregation?

The unanimity rule may not aggregate information, but all other $q$-rules do (Feddersen and Pesendorfer 1998; Duggan and Martinelli 2001)
Condition for information aggregation

Proposition

There exists a sequence of equilibria along which the probabilities of the DM choosing \( Y \) in state \( y \) and \( N \) in state \( n \) approach 1 as \( N \to \infty \) if and only if

\[
\alpha < \lim_{N \to \infty} \alpha(N, N').
\]

Wong, Yang and Zhao

Voting to Persuade
Figure: $\alpha (k, N)$
\[ \alpha (k, \mathcal{N}) \text{ is not increasing} \]
We provide necessary and sufficient conditions for information transmission and aggregation in a model of advisory committees.

Intuition: If DM cannot be persuaded by unanimity, she can never be persuaded.

Our results does not hold for discrete models.