Welfare Analysis of Changing Notches: Evidence from Bolsa Família*

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Abstract

We develop a framework to bound the welfare impacts of arbitrarily large reforms to notches using two sufficient statistics: the number of households bunching at the old notch who move toward the new notch and the number of households who "jump" down to the new notch. Our bounds hold in a wide class of models, highlighting a new way to use reduced-form bunching evidence for welfare analysis without strong assumptions on the economic environment. We estimate these two statistics using a difference-in-difference strategy for a reform to the anti-poverty program Bolsa Família, finding that the reform's MVPF is between 0.90-1.12.

Keywords: bunching, jumping, notches, MVPF, bounds, sufficient statistics

JEL: H31, H53, I38, O12

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1 Introduction

Throughout both developed and developing countries, tax and transfer schedules often feature notches wherein incremental changes in behavior lead to discrete changes in benefits or tax liabilities (Slemrod (2010); Kleven (2016)). For example, Saving and Viard (2021) document numerous notches in the United States tax code including the well-known Medicaid eligibility notch (Yelowitz, 1995), Kleven and Waseem (2013) document notches of up to 4 percentage points in average personal income tax rates in Pakistan, and Bachas and Soto (2021) document 10 percentage point jumps in the average corporate tax rate in Costa Rica. Given this prevalence, it is important for economists and policymakers to understand the welfare impacts of policy reforms that change the structure of notches in tax and transfer systems.

Typically the behavioral and welfare impacts of notches are analyzed using the standard bunching approach whereby one estimates the mass of agents bunching around a notch and then translates this bunching mass into a structural parameter(s) (Kleven, 2016). One can then use these structural parameters to gauge the welfare impacts of previous or proposed policy changes, although in this literature they are more commonly used to identify alternative schedules that will improve welfare. However, the chief limitation of this approach is that translating a bunching mass into a structural parameter typically relies heavily on modeling assumptions. For example, assumptions on household preferences, the types of behavioral responses available to households, and the degree of optimization frictions can all greatly impact the mapping between a bunching mass and a structural parameter.[1] Consequently, Kleven (2016) states "although bunching provides compelling non-parametric evidence of a behavioral response, moving from observed bunching to a structural parameter that can be used to predict the effects of policy changes is difficult."

This paper develops a method that uses changes in bunching to form bounds for the welfare impacts of notch reforms without making strong modeling assumptions about the household optimization problem. In particular, we show how to bound the welfare impact of notch reforms in transfer schedules using two sufficient statistics: (1) the number of households bunching at the original notch that move towards the new notch as a result of the reform, and (2) the number of households that "jump" down to bunch at the new notch as a result of the reform. Importantly, our bounds require only minimal structure on the household problem: households can have arbitrary preference heterogeneity, households can have any number of choice variables (and, therefore, can respond to the reform in various ways), and households may face optimization frictions such as limited choice sets and/or adjustment costs. Thus, our approach overcomes an important limitation of the standard bunching approach by using reduced-form bunching evidence to inform welfare analysis without making strong assumptions about the household optimization problem. However, our approach can only be used to answer a narrower set of questions; thus, our approach should be viewed as *complementary* to the standard bunching approach.

We then apply our method to bound the welfare impact of a reform that expanded a notch in one

1. This is not always the case: in some applications, structural parameters appear robust to model augmentations, e.g., Best et al. (2020).

of the world's largest cash transfer programs, Bolsa Família (BF). BF, a means-tested anti-poverty program in Brazil, is a particularly interesting program to study for two reasons: (1) BF is one of the few cash transfer programs for which eligibility is based on self-reported household income (Fruttero, Leichsenring and Paiva, 2020), and (2) the benefit schedule features a pronounced eligibility threshold - a notch - wherein households are eligible for benefits only if they report an income below this threshold. Given the large portion of Brazil's workforce in the informal sector, these two features generate substantial scope for income misreporting. We study a 2014 reform to BF in which the government increased both the eligibility threshold and the amount of money given to beneficiary households. Using longitudinal administrative data, we find strong evidence of changes in bunching in response to this reform. Plugging our empirical estimates into our sufficient statistics framework, we find that the marginal value of public funds (MVPF) is between 0.90 and 1.12, implying that the reform was welfare improving if the government values giving R\$0.90 to BF recipients more than spending R\$1 on their next best alternative and welfare decreasing if the government values giving R\$1.12 to BF recipients less than spending R\$1 on their next best alternative. Because BF has high coverage of very poor households, we argue that it is highly likely that the government is willing to spend R\$1 to get R\$0.90 to BF recipients. Thus, despite strong evidence of behavioral responses to the reform, our findings indicate that these responses are unlikely to be large enough to generate efficiency costs that outweigh the equity benefits of increased program generosity. Thus, our findings highlight the importance of having theoretical frameworks that translate behavioral responses to social programs into welfare impacts, a point recently emphasized in Gerard and Gonzaga (2021).

The paper begins by building intuition in a simple, static model in which a government provides a constant transfer only to households who report an income below a certain threshold; hence, the transfer schedule features a notch. Households are endowed with an income and choose how much income to report to the government subject to misreporting costs. We derive bounds on the MVPF of changing the notch, comprising both an increase in the benefit level and eligibility threshold. The MVPF is the ratio of households' willingness-to-pay (WTP) for the reform relative to the government's budgetary cost of the reform (Hendren and Sprung-Keyser, 2020). We show that aggregate WTP can be bounded using two empirical objects: (1) the number of "bunching households" who bunch at the old notch and move towards the new notch, and (2) the number of "jumping households" who jump down to bunch at the new notch. To show this, we first note that for households who do not jump or bunch, their WTP is simply equal to the amount of additional money they receive. However, for the bunching and jumping households, their WTP differs from their increase in benefits; this is because we cannot simply appeal to the envelope condition given we consider arbitrary discrete reforms. Instead, we use revealed preference arguments to bound the amount that each bunching and jumping household is willing to pay. We can therefore bound aggregate WTP and, in turn, the MVPF provided we can observe the number of bunching and jumping households. Because the welfare impact of a reform is equal to the MVPF multiplied by a normative welfare weight, our bounds on the MVPF enable us to make welfare statements about the impacts of changing a notch using estimable, reduced-form objects.

While our results are initially stated in the context of a simple, static misreporting model to build intuition, we show that our welfare bounds are highly robust to the household problem. In particular, our bounds hold in models with any sort of behavioral response margin (e.g., households can respond to the reform via labor supply responses instead of misreporting responses), arbitrary preference heterogeneity, adjustment costs, limited choice sets (e.g., households face labor supply frictions), and (non-extreme) misperceptions of the benefit schedule. Moreover, we discuss how our bounds can be augmented to allow for dynamic decision making with uncertainty and for more complex policy environments. We therefore refer to the number of jumping and bunching households as sufficient statistics to bound the welfare impact of a notch change. We view this generality as a key contribution of our framework as not only does it allow for our framework to be adapted to analyze notch reforms in other transfer programs, but it also highlights a new way to use reduced-form bunching evidence to inform welfare analysis without making strong assumptions on the household problem.

We then turn our attention to the welfare impact of the June 2014 Bolsa Família reform. Prior to June 2014, single-adult households with no children were eligible for a monthly unconditional benefit of R$70 if and only if they reported an income below the extreme-poverty threshold of R$70 per-capita per-month.[2] In June 2014, both the eligibility threshold and benefit were raised by 10%: the eligibility threshold increased from R$70 per-capita, per-month to R$77 per-capita, per-month, while the unconditional benefit increased from R$70 per-month to R$77 per-month. Two adult households and households with children also faced a reform in June 2014; however, their benefit schedules are more complex, rendering our identification assumptions (discussed below) slightly less credible for these households. Thus, we restrict our main empirical analysis to single-adult households with no children.[3]

We have access to administrative data spanning December 2011 to September 2016 from Cadastro Único, which is the Brazilian government's national registry used to determine eligibility for all federal social welfare programs. Using this data, we seek to estimate the number of original bunching households who move towards the new notch and the number of households jumping down to the new notch as a result of the reform. The number of bunching households is simply equal to the reduction in the bunching mass at the old notch, while the number of jumping households is equal to the increase in the bunching mass at the new notch less the reduction in the bunching mass at the old notch. The key identification challenge is to understand how the number of households reporting incomes at the old and new notch would have evolved in the post-reform period had the reform not occurred.

At a high level, our identification strategy is based on the following insight: the incentives to report an income at and above the original notch are affected by the reform, but the incentives to report an income below the original notch are unaffected by the reform. Thus, we discretize the reported income distribution into bins and use the number of households reporting incomes in bins below the original notch as controls for the number of households reporting incomes in bins at and above the original notch. We show that in the pre-reform period, the relationships between our treatment and control bins are well approximated by (low-order) polynomials. We therefore employ a generalized difference-

2. BF also includes a conditional variable benefit for households with children. To receive this conditional benefit, households must make specified health and education investments in their children and have a reported per-capita income below the "poverty threshold" (which is higher than the extreme-poverty threshold).

3. Single-adult households with no children make up 6.3% of BF beneficiaries in June 2014. We also repeat our empirical analysis for two-adult households with no children (who make up 21.1% of BF beneficiaries) and find very similar welfare bounds.

in-difference strategy; this strategy requires that there exist stable relationships between treatment and control groups that can be well approximated by low order polynomials (see, for example, Wolfers (2006) and Mora and Reggio (2013)). The key identification assumption is that these relationships between treatment and control groups would persist in the post-reform period in absence of the reform. We perform numerous placebo exercises to support the validity of this identification assumption.

Using this generalized difference-in-difference strategy, we find that the number of households reporting incomes at the new notch increased by approximately 49,000 and the number of households reporting incomes at the old notch decreased by approximately 27,000 as a result of the reform. Hence, the reform induced 27,000 bunching households to move toward the new threshold and an additional 22,000 households to jump down to the new threshold. This translates into a lower bound for the MVPF of the reform of approximately 0.9 and an upper bound for the MVPF of approximately 1.12. In terms of welfare implications, as long as the government values giving R$0.90 (in a non-distortionary manner) to the BF households more than spending R$1 on their next best alternative, the reform was welfare improving. We argue that this is likely the case given that BF has high coverage of households in extreme poverty and given that the households misreporting to get the benefit typically fall in the poorest half of the population (Bastagli (2008) and Lindert et al. (2007)). A back-of-the-envelope calculation suggests that the welfare effect of spending R$1 on the BF reform is at least as high as the welfare effect of spending R$1.50 on a non-distortionary universal transfer. Hence, our empirical findings contribute to the debate around targeted vs. universal transfers in developing settings (Hanna and Olken, 2018): even in a setting with a highly pronounced notch and substantial scope for misreporting, the efficiency cost generated by behavioral responses is simply not large enough to outweigh the equity gain associated with the increased generosity of benefits targeted to poor households (relative to a universal transfer).

Relationship to the literature: First, we contribute to the large literature that explores bunching at notches and kinks. Typically papers in this literature use reduced-form bunching evidence to pin down structural parameters of interest (e.g., the elasticity of taxable income) by making assumptions about the household optimization problem (Kleven, 2016). These parameters can then be used to inform the welfare impacts of previous or proposed policy changes, although, in this literature, they are more commonly used to identify alternative schedules that would improve welfare (see, for example, Best et al. (2015) or Bachas and Soto (2021)). However, the chief limitation of this approach is that translating a bunching mass into a structural parameter typically relies heavily on the modeling assumptions of the household problem (Kleven, 2016). Conversely, our framework uses reduced-form bunching evidence in a sufficient statistics approach, i.e., without making substantive parametric assumptions on the household optimization problem. Hence, our approach is complementary to standard bunching approaches in the sense that they have opposing strengths and weaknesses: our framework avoids making strong assumptions on the households' optimization problem, but our framework can only be used to answer a narrower set of questions than the standard bunching approach.

Furthermore, our empirical strategy differs from standard bunching analyses. We focus on estimating *changes* in bunching that result from a reform, whereas standard bunching analyses focus on estimating static bunching at a given notch or kink. As such, our approach has stronger data requirements - we require longitudinal data whereas the standard bunching approach only requires cross-sectional data. Using a difference-in-difference strategy, we use portions of the distribution unimpacted by the reform to control for underlying time trends in portions of the distribution which are impacted by the reform. Conversely, the standard bunching approach uses portions of the distribution unimpacted by the notch combined with smoothness assumptions on the underlying counterfactual distribution to identify bunching induced by the notch.[4] However, because our distribution of reported incomes is extremely non-smooth (we have extreme bunching at numbers equal to 0 mod 50, substantial bunching at numbers which were notches many years earlier, and less extreme bunching at numbers equal to 0 mod 10) applying standard bunching techniques to our setting is infeasible. While bunching estimation methods have been augmented to deal with "round-number" bunching and bunching at "reference-points" (e.g., Kleven and Waseem (2013) and Best and Kleven (2017)) along with frictions (Anagol et al. (2022)), we believe that the pervasiveness and the variability of round-number and reference-point bunching in our setting will make it too difficult to precisely identify the counterfactual distributions around our notches.

Our paper also contributes to the sufficient statistics approach for welfare analysis (Chetty, 2009). One broad contribution to this literature is showing how to apply a sufficient statistics framework to settings with large, discrete policy reforms when the envelope condition cannot be applied. Kleven (2021) discusses how to do approximate welfare analysis using an expanded set of sufficient statistics when reforms are large; however, he argues that these additional statistics are difficult to estimate. We overcome the need to estimate a large set of complex parameters by focusing on welfare bounds. This paper is also related to Lockwood (2020) who investigates the welfare impacts of changing the top marginal tax rate in a piecewise linear tax system that features a notch, arguing that the sufficient statistic approach for the welfare analysis of tax systems needs to be augmented to include a correction term which captures the change in bunching at the notch in response to a tax reform. Lockwood (2020) considers a model with infinitesimal reforms where households have one choice variable (labor supply), one dimension of heterogeneity (labor productivity), and quasi-linear utility. In contrast, our theoretical framework shows that we can use two sufficient statistics to bound the welfare impacts of reforms to transfer programs with notches of any size while allowing for arbitrary heterogeneity in household primitives/preferences, for households to respond to reforms on many different margins (e.g., labor supply and misreporting), and for limited choice sets and adjustment costs. Our framework also allows us to understand welfare impacts of changes in the location of notches, which turns out to be theoretically more complex.

Finally, we contribute to the small but growing literature on estimating the welfare impacts of cash transfer programs in developing countries (e.g., Bergolo and Cruces (2021); Bergstrom and Dodds (2021*b*), Hanna and Olken (2018)). Existing papers on cash transfers in developing settings typically

4. Because we use unimpacted regions of the reported income distribution to form a counterfactual for the impacted region, our method is conceptually closer to the bunching literature than it is to papers such as Cengiz et al. (2019) who use the income distributions in unimpacted *geographical* areas to form counterfactuals for the income distribution in impacted *geographical* areas. We cannot apply this sort of methodology in our context because the BF reform was a national reform implemented uniformly across Brazil in June 2014.

estimate impacts (e.g., behavioral responses), but often lack theoretical frameworks to infer associated welfare implications (Gerard and Gonzaga, 2021). By developing a theoretical framework to translate behavioral responses to notch reforms into welfare impacts, we show that despite strong evidence of behavioral responses to the BF reform, such responses are unlikely to be large enough to generate efficiency losses that outweigh the equity benefits.[5] Notably, because we are analyzing a reform to a program with a pronounced eligibility notch based on self-reported income in a high-informality setting, one may ex-ante expect the efficiency costs of the reform to be very large. The fact that we do not find sizable efficiency costs provides evidence against the commonly held belief that targeting transfer programs based on self-reported incomes will have substantial efficiency costs in high informality settings. Therefore, we conjecture that in settings with similar or lower levels of informality and similar government administrative capacity, using self-reported incomes to determine eligibility could be a viable targeting strategy for other anti-poverty programs.[6] Thus our findings also contribute to the literature on optimal targeting of cash transfers in developing countries and may have implications for the future design of cash transfer programs.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical framework for welfare analysis of changing notches, Section 3 discusses the Bolsa Família program and our data, Section 4 discusses our strategy to empirically identify the number of jumping and bunching households for the June 2014 reform, Section 5 presents our results and robustness analysis, and Section 6 concludes.

2 Theoretical Framework for Welfare Analysis of Changing Notches

In this section we devise a sufficient statistics framework to bound the welfare impacts of changing a notch in a transfer program. While we will apply this framework to the June 2014 BF reform, we believe this framework can be used more generally to analyze reforms to notches in other transfer programs. To begin, we derive bounds in a simple, static misreporting model. This simple setup is useful not only for building intuition but also because misreporting responses are likely common in response to the BF reform. We show that we can bound the welfare impact of a notch change using two empirical objects: (1) the number of households bunching at the old notch who move towards the new notch as a result of the reform, and (2) the number of households who jump down to the new notch as a result of the reform. We then show that these bounds hold in a much more general model that places minimal structure on the household problem, thereby arguing that our two empirical objects are "sufficient statistics" to bound the change in welfare from a notch change.

5. Similarly, Bergolo and Cruces (2021) estimate sizable equity benefits relative to efficiency costs for a cash transfer program in Uruguay that bases eligibility, in part, on reported incomes. They estimate that the MVPF is 0.61, which is in line with cash transfer programs in the United States for which Hendren and Sprung-Keyser (2020) estimate an average MVPF of 0.74.

6. As informality decreases so that incomes are more verifiable (e.g., we move to more developed settings), we conjecture that targeting based on self-reported incomes will generate even smaller behavioral responses.

2.1 Baseline Model Set-Up

To begin, we consider a static world in which households choose how much income to report, $\hat{y}$, subject to a policy $\mathbf{p} = \{b, \tau\}$ where b denotes the level of the benefit and τ denotes the eligibility threshold s.t. those reporting $\hat{y} \leq \tau$ receive b and those reporting $\hat{y} > \tau$ receive nothing.[7] Households have two dimensions of heterogeneity: (1) endowed income y distributed according to CDF $F(y)$, and (2) aversion to misreporting governed by $\mu \in \{1, 2\}$ with probability mass function $\pi(\mu)$. Type $\mu = 1$ households are "truth-telling" households who never misreport, whereas type $\mu = 2$ households are willing to misreport their income. For simplicity, we assume there is no fixed cost of reporting an income and that all households know the policy $\mathbf{p}$. We also assume households have quasi-linear utility in consumption. Type $\mu = 1$ households therefore have utility under policy $\mathbf{p}$ given by:

$$U^*(y, \mu = 1; \mathbf{p}) \equiv y + b\mathbb{1}\left(y \leq \tau\right) \tag{1}$$

Utility for type $\mu = 2$ households under policy $\mathbf{p}$ is equal to:

$$\begin{aligned} U^*(y, \mu = 2; \mathbf{p}) \equiv \max_{\hat{y}} \;& c - v\left(y - \hat{y}\right) \\ \text{s.t. } \;& c = y + b\mathbb{1}\left(\hat{y} \leq \tau\right) \end{aligned} \tag{2}$$

where $v\left(y - \hat{y}\right)$ captures the disutility of reporting $\hat{y}$ when true income is y. We assume $v(y - \hat{y}) = 0$ if $\hat{y} \geq y$ (so that only under reporting is costly) along with $v' > 0$ when $y > \hat{y}$ (so that the cost of misreporting is increasing in the discrepancy between true and reported income). Optimal reported incomes, $\hat{y}^*$, for type $\mu = 2$ households are characterized as follows (see Appendix A.1 for a formal derivation):

$$\hat{y}^*(y, \mu = 2; \mathbf{p}) = \begin{cases} y & \text{if } y \leq \tau \\ \tau & \text{if } y \in (\tau, y^c(\mathbf{p})] \\ y & \text{if } y > y^c(\mathbf{p}) \end{cases} \tag{3}$$

where $y^c(\mathbf{p})$ is the income level for which households are indifferent between misreporting at τ and reporting truthfully, implicitly defined by $y^c(\mathbf{p}) + b - v\left(y^c(\mathbf{p}) - \tau\right) = y^c(\mathbf{p})$. In words, type $\mu = 2$ individuals with $y < \tau$ report truthfully and get the benefit, those with $y \in (\tau, y^c(\mathbf{p})]$ get the benefit by misreporting and bunching at the notch, and those with $y > y^c(\mathbf{p})$ report their income truthfully and do not get the benefit.

Next, we define $G(x; \mathbf{p})$ as the number of households reporting an income less than or equal to x under policy $\mathbf{p}$:

$$G(x; \mathbf{p}) = \sum_{\mu \in \{1,2\}} \int_Y \mathbb{1}(\hat{y}^*(y, \mu; \mathbf{p}) \leq x) dF(y|\mu)\pi(\mu)$$

7. For sake of parsimony, our simple framework abstracts from a number of complexities. For instance, we abstract from the presence of other tax and transfer programs. We also assume that all households reporting below the threshold τ receive the benefit with certainty. We discuss in Section 2.3 how to incorporate these more complex policy environments into our framework, and in Section 5.2 we discuss how these additional complexities may impact our empirical results.

Hence, $G(\tau; \mathbf{p})$ captures the number of households locating under the eligibility threshold τ. Finally, we assume that total social welfare under policy $\mathbf{p}$, $\mathcal{W}(\mathbf{p})$, is given by a weighted sum of household utilities less the budgetary cost of the policy multiplied by the shadow value of public funds λ, which captures the welfare gain of spending a dollar on the government's next best alternative. Hence, we assume that spending \$1 on the program comes at the cost of spending \$1 less on some other program which decreases welfare by a constant value λ. Welfare under policy $\mathbf{p}$ is therefore given by:

$$\mathcal{W}(\mathbf{p}) = \sum_{\mu \in \{1,2\}} \int_Y \phi(y, \mu) U^*(y, \mu; \mathbf{p}) dF(y|\mu)\pi(\mu) - \lambda b G(\tau; \mathbf{p}) \tag{4}$$

where $\phi(y, \mu)$ denotes the government's welfare weight on a household with income y and type μ.

2.2 Welfare Effect of the Reform in the Baseline Model

Our goal is to evaluate the welfare impact of a reform from policy $\mathbf{p} = \{b, \tau\}$ to $\mathbf{p}' = \{b', \tau'\}$, defining $\{\Delta b, \Delta\tau\} = \mathbf{p}' - \mathbf{p}$. For ease of exposition, let us assume $\Delta b > 0$ and $\Delta\tau > 0$ (i.e., we increase the level and location of the notch). Note that we are *not* restricting to infinitesimal reforms; the bounds we derive allow for arbitrary, discrete reforms to b and τ. We discuss welfare bounds for infinitesimal reforms in Appendix A.4. Our goal is to derive bounds for $\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})$ in terms of empirically observable objects (we discuss the advantages of focusing on bounding the welfare impact as opposed to characterizing the exact impact in Section 2.3). Hendren and Sprung-Keyser (2020) show that the welfare impacts of any policy reform can be expressed in terms of a *normative* welfare weight along with a *positive* sufficient statistic called the marginal value of public funds (MVPF), which captures households' willingness-to-pay (WTP) for the reform relative to the total budgetary cost of the reform. Our goal then is to derive bounds for the MVPF.

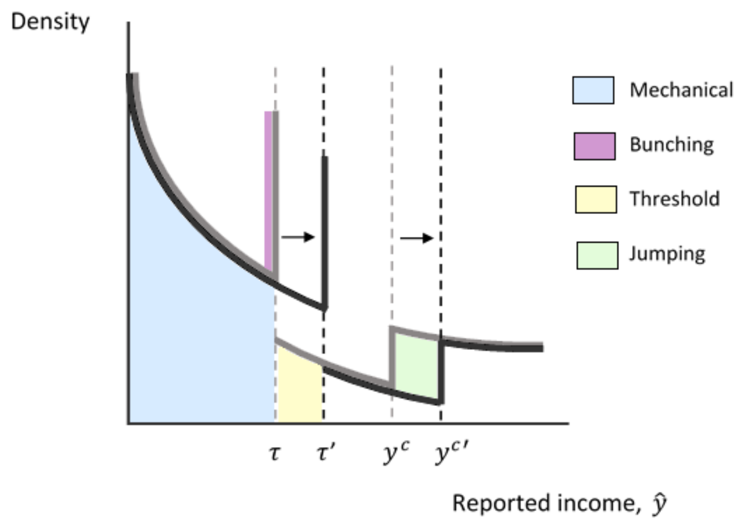
Because the utility function is quasi-linear, a household with income y and misreporting type μ has a WTP for the reform implicitly defined by:

$$U^*(y, \mu; \mathbf{p}) = U^*(y, \mu; \mathbf{p}') - WTP$$

Equivalently, WTP is simply equal to the compensating variation. We now heuristically derive bounds on the WTP for all households impacted by the reform. Let us split the set of households impacted by the reform into four groups: the mechanical households, the bunching households, the threshold households, and the jumping households. Figure 1 depicts these four groups for (hypothetical) reported income distributions under policies $\mathbf{p}$ and $\mathbf{p}'$.

The mechanical households are the households who report at or below τ under policy $\mathbf{p}$ and who do not change their behavior in response to the reform. The number of mechanical households is given by $M = G(\tau; \mathbf{p}')$ (notably, M is not equal to the mass reporting at or below τ under policy $\mathbf{p}$ as this mass includes bunching households who do update their behavior as a result of the reform). Because the reform increases benefits for mechanical households by Δb, their WTP for the reform is simply Δb.

The bunching households are the households who misreport and bunch at the original threshold τ



Note: This figure shows a hypothetical density of reported incomes under the initial policy $\mathbf{p} = \{b, \tau\}$ (solid grey curve) and how this density changes as a result of a reform that increases the policy to $\mathbf{p}' = \{b', \tau'\}$ (solid black curve). Note, the vertical solid grey line at τ and the vertical solid black line at τ' represent the bunching households under the initial policy and new policy, respectively. This figure also depicts which households are classified as mechanical households, bunching households, threshold households, and jumping households.

FIGURE 1: A HYPOTHETICAL DENSITY OF REPORTED INCOMES UNDER $\mathbf{p}$ AND $\mathbf{p}'$

under policy $\mathbf{p}$ and move with the threshold as it is increased (i.e., they report between $(\tau, \tau']$ under policy $\mathbf{p}'$).[8] Thus, the number of bunching households is equal to the reduction in households locating at or below τ *as a result of the reform*: $B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$. The bunching households receive an increase in benefits equal to Δb. Moreover, they experience a reduction in their misreporting costs as they move from τ to τ'.[9] Hence, the bunching households have a WTP of at least Δb. Moreover, by revealed preference arguments, the most the bunching households can value this reduction in misreporting costs is b dollars. If they value this reduction by more than b dollars, it would not have been optimal for these households to bunch at τ under policy $\mathbf{p}$.[10] Thus, the bunching households' WTP for the reform is in $[\Delta b, \Delta b + b] = [\Delta b, b']$.

The threshold households are the households who report in $(\tau, \tau']$ under policy $\mathbf{p}$; we denote the number of threshold households by $T = G(\tau'; \mathbf{p}) - G(\tau; \mathbf{p})$. These households will not update their behavior in response to the reform because all households reporting above τ given policy $\mathbf{p}$ are reporting truthfully regardless of their type μ. Under $\mathbf{p}'$ the optimal choice for these households is to continue

8. Not all bunching households will move all the way to τ': bunchers with $y \in (\tau, \tau')$ will report $\hat{y} = y$ under $\mathbf{p}'$.

9. Even for an infinitesimal reform, the utility gain bunching households experience from moving with the notch has a first-order impact on welfare. This is because the envelope theorem cannot be applied for these individuals: in order to argue that the derivative of indirect utility with respect to the policy is equal to the derivative of utility with respect to the policy evaluated at optimal decisions, utility must be differentiable with respect to the policy given *any* fixed choices (see Theorem 2 of Milgrom and Segal (2002)). However, utility is actually discontinuous as a function of the parameter τ holding decisions fixed: for example, individuals reporting an income of τ see a discrete drop in consumption (and hence utility) if τ is reduced by any amount. See Appendix A.4 for more details.

10. Suppose bunching households have a WTP for the relaxation in their misreporting costs greater than b: $v(y - \tau) - v(y - \tau') > b$. This implies $v(y - \tau) > b$. However if $v(y - \tau) > b$, bunching households would have preferred to report truthfully above τ over misreporting at τ under policy $\mathbf{p}$.

reporting truthfully as doing so allows them to receive b' and not incur any misreporting costs. These households go from receiving no benefits to receiving b' in benefits. Hence their WTP is equal to b'.

Finally, the jumping households are the households who report above τ' given policy $\mathbf{p}$ but who report at τ' given policy $\mathbf{p}'$. In particular, households who were previously close-to-indifferent between misreporting at the threshold and truthfully reporting above the threshold but opted for the latter, i.e., type $\mu = 2$ households with $y \in (y^c(\mathbf{p}), y^c(\mathbf{p}')]$, will now jump and misreport to the new threshold τ'.[11] Notably, we describe the behavioral response of these households as a "jump" because these households experience a discontinuous change in their optimal reported income as we move from $\mathbf{p}$ to $\mathbf{p}'$. The number of jumping households, J, is given by $J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$, which is equivalent to the increase in households reporting at or below the new threshold *as a result of the reform* (note, the number of jumping households is not equal to the mass of households locating at τ' given policy $\mathbf{p}'$ as some of the households locating at τ' may be original bunching households who moved with the notch). By revealed preference, jumping households' utility is improved by changing their behavior; hence, their WTP is weakly positive. Moreover, after the reform, jumping households get b' more dollars but incur misreporting costs. Therefore jumping households' WTP cannot exceed b' as misreporting costs are weakly positive. Hence the WTP of jumping households is in $[0, b']$.

Table 1 summarizes the bounds on the WTP for each of our four groups along with the number of households falling into each group and the cost that each group imposes on the government.

Table 1: WTP and Cost to the Government of the Reform

Group	Number of households	Utility under $\mathbf{p}$ (per household)	Utility under $\mathbf{p}'$ (per household)	WTP (per household)	Cost to Govt. (per household)
Mechanical	$M = G(\tau; \mathbf{p}')$	$y + b$	$y + b'$	Δb	Δb
Bunching	$B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$	$y + b - v(y - \tau)$	$y + b' - v(y - \hat{y}^*(\mathbf{p}'))$	$\in [\Delta b, b']$	Δb
Threshold	$T = G(\tau'; \mathbf{p}) - G(\tau; \mathbf{p})$	y	$y + b'$	b'	b'
Jumping	$J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$	y	$y + b' - v(y - \tau')$	$\in [0, b']$	b'

Note: This table shows utility under both policy regimes $\mathbf{p}$ and $\mathbf{p}'$ along with willingness-to-pay (WTP) and the cost to the government for all the households impacted by the reform from policy $\mathbf{p}$ to $\mathbf{p}'$. Note, for bunching households, $\hat{y}^*(\mathbf{p}') \in (\tau, \tau']$.

This brings us to Lemma 1 which bounds the total WTP for the reform:

Lemma 1. *If individuals solve Problem (2), the total WTP of the reform from* $\mathbf{p}$ *to* $\mathbf{p}'$ *with* $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta \tau\} > 0$ *can be bounded as follows:*

$$\Delta b(M + B) + b'T \leq \textit{Total WTP} \leq \Delta b M + b'(B + T + J)$$

where M, B, T *and* J *denote the mass of mechanical, bunching, threshold and jumping households defined in Table 1.*

11. For changes in τ s.t. $y^c(\mathbf{p}) < \tau'$, only $\mu = 2$ households with $y \in (\tau', y^c(\mathbf{p}')]$ jump and misreport to the new notch. See Appendix A.5 for Figure 1 under this scenario.

Proof. See Appendix A.2. □

Moreover, using the cost per-household to the government given in Table 1, we can express the total budgetary cost of the reform as follows:

$$\text{Total Cost} \equiv b'G(\tau'; \mathbf{p}') - bG(\tau; \mathbf{p}) = \Delta b(M + B) + b'(T + J) \tag{5}$$

Hence, we can construct bounds for the MVPF of the reform:

Proposition 1. *If individuals solve Problem (2), the marginal value of public funds of the reform from* $\mathbf{p}$ *to* $\mathbf{p}'$ *with* $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta\tau\} > 0$ *can be bounded as follows:*

$$MVPF \geq MVPF_L \equiv \frac{\Delta b(M + B) + b'T}{\Delta b(M + B) + b'(T + J)} = 1 - b'\frac{J}{\textit{Total Cost}}$$

$$MVPF \leq MVPF_U \equiv \frac{\Delta bM + b'(B + T + J)}{\Delta b(M + B) + b'(T + J)} = 1 + b\frac{B}{\textit{Total Cost}}$$

Proof. This follows directly from Lemma 1 and Equation (5). □

The lower bound for the MVPF captures the fact that the lower bound for the WTP of the jumping households is 0 while the cost they impose on the government is b' each (whereas the lower bound for the WTP of bunching households is exactly equal to the cost they impose on the government). Hence, if all jumping households value the reform at their lower bound, $b'J/(\text{Total Cost})$ of each dollar of spending is "wasted". Meanwhile, the upper bound for the MVPF captures the fact that the upper bound for the WTP of the bunching households is b' while the cost they impose on the government is only $b' - b$ (whereas the upper bound for the WTP of jumping households is exactly equal to the cost they impose on the government). Hence, if bunching households all value the reform at their upper bound, the government "gains" $bB/(\text{Total Cost})$ for each dollar of spending. We can then use these bounds on the MVPF to construct bounds on the money metric welfare gain relative to the budgetary cost of the reform using Proposition 2:[12]

Proposition 2. *If individuals solve Problem (2) and social welfare is given by Equation (4), then the money metric welfare gain relative to the budgetary cost of the reform from* $\mathbf{p}$ *to* $\mathbf{p}'$ *with* $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta\tau\} > 0$ *can be bounded as follows:*

$$\omega_L MVPF_L - 1 \leq \frac{\frac{1}{\lambda}\left[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})\right]}{b'G(\tau'; \mathbf{p}') - bG(\tau; \mathbf{p})} \leq \omega_U MVPF_U - 1$$

where ω_L (ω_U) *captures the weighted average money-metric welfare gain from giving a dollar to mechanical, bunching, threshold, and jumping households, where the weights are determined by the relative size of each group's lower bound (upper bound) WTP for the reform.*

12. Note, that the welfare change is expressed in dollar units (as opposed to welfare units) as we divide through by the shadow value of public funds, λ.

Proof. We do not provide a separate proof for Proposition 2; we only provide a proof for Proposition 3, which nests Proposition 2 (see Appendix A.3). □

Proposition 2 bounds the increase in total welfare from spending \$1 on the reform. In particular, $MVPF_L$ captures our lower bound on the total WTP when we spend \$1 on the reform, while ω_L denotes the incidence-weighted welfare gain, measured in dollars, of splitting \$1 among the mechanical, bunching, threshold, and jumping households. In other words, ω_L is the average welfare weight for mechanical households multiplied by the (lower bound) share of WTP that goes to mechanical households plus the average welfare weight for threshold households multiplied by the (lower bound) share of WTP that goes to threshold households, etc.[13] Subtracting the budgetary cost of \$1 from $\omega_L MVPF_L$ gives a lower bound for the total welfare gain from spending \$1 on the reform. Symmetric logic explains why $\omega_U MVPF_U - 1$ is an upper bound for the increase in total welfare of spending \$1 on the reform. Intuitively, these welfare bounds will be informative when either the lower bound is close to 1 and the government's welfare weight on the affected group of households is high (in this case the policy reform is welfare increasing) or when the upper bound is close to 1 and the government's welfare weight on the affected group of households is low (in this case the policy reform is welfare decreasing). Because Proposition 1 tells us that $MVPF_L$ and $MVPF_U$ are functions of two objects, the number of bunching households and the number of jumping households, Proposition 2 implies that the number of bunchers and jumpers are the empirical objects needed to construct bounds for the welfare impact of the reform.[14]

2.3 Robustness to Model Specification

To highlight the robustness of our welfare bounds, we now show that we actually require very little structure on preferences or behavioral responses of households. Suppose households have several decisions variables denoted by the vector x within a choice set X. Household decisions are made conditional on primitives denoted by the vector $\theta \in \Theta$ and the policy $\mathbf{p}$. Households get the benefit b if their reported income $\hat{y}$, which is a function of decision variables and primitives, is below τ. Household income, denoted y, is also potentially a function of decision variables. For example, we could consider a model where households have two working adults, x is a vector representing two labor supply decisions l_1 and l_2 along with a choice of reported income $\hat{y}$, θ is a vector representing two productivities n_1 and n_2 along with an aversion to misreporting μ, and household income is given by $y = n_1 l_1 + n_2 l_2$. In such a model households can respond to reforms via changing their labor supply as well as misreporting. More generally, we consider the following household problem:

$$\begin{aligned} U^*(\theta; \mathbf{p}) = & \max_{x \in X} \; u\left(c, x; \theta\right) \\ & \text{s.t. } c = y(x, \theta) + b\mathbb{1}(\hat{y}(x, \theta) \leq \tau) \end{aligned} \tag{6}$$

13. These incidence-weighted welfare weights are identical to those used in Hendren and Sprung-Keyser (2020). We provide a more detailed example in Appendix A.6 showing how these incidence-weighted welfare weights interact with the MVPF to make welfare statements.

14. Technically, we also need to estimate the total cost of the reform; however, Equation (5) shows that the total cost is a function solely of $G(\tau'; \mathbf{p}')$ and $G(\tau; \mathbf{p})$, which are components needed to construct B and J.

where c denotes consumption. We assume total welfare is given by a weighted sum of utilities, with welfare weights given by $\phi(\theta)$:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta) U^*(\theta; \mathbf{p}) dF(\theta) - \lambda b G(\tau; \mathbf{p}) \tag{7}$$

where λ represents the shadow value of public funds and $G(\tau; \mathbf{p}) = \int_{\theta: \hat{y}(\theta, \mathbf{p}) \leq \tau} dF(\theta)$ represents the number of households receiving the benefit under policy $\mathbf{p}$. More generally, we define $G(z; \mathbf{p}) = \int_{\theta: \hat{y}(\theta, \mathbf{p}) \leq z} dF(\theta)$. This setup allows us to substantially generalize Proposition 1 and Proposition 2:

Proposition 3. *Suppose households solve Problem (6) and welfare is given by Equation (7). Define:*

$$MVPF_L \equiv 1 - \frac{b' \left[G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})\right]}{b' G(\tau'; \mathbf{p}') - b G(\tau; \mathbf{p})} = 1 - b' \frac{J}{\textit{Total Cost}} \tag{8}$$

$$MVPF_U \equiv 1 + \frac{b[G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')]}{b' G(\tau'; \mathbf{p}') - b G(\tau; \mathbf{p})} = 1 + b \frac{B}{\textit{Total Cost}} \tag{9}$$

Then as long as $\tau' > \tau$ and $b' G(\tau'; \mathbf{p}') - b G(\tau; \mathbf{p}) > 0$ we have:

$$\omega_L MVPF_L - 1 \leq \frac{\frac{1}{\lambda} \left[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})\right]}{b' G(\tau'; \mathbf{p}') - b G(\tau; \mathbf{p})} \leq \omega_U MVPF_U - 1 \tag{10}$$

where ω_L (ω_U) captures the weighted average money-metric welfare gain from giving a dollar to mechanical, bunching, threshold, and jumping households, where the weights are determined by the relative size of each group's lower bound (upper bound) WTP for the reform.

Proof. See Appendix A.3. □

Proposition 3 highlights that we can bound the MVPF and, consequently, bound the welfare impacts of the reform using empirically observable objects while only putting limited structure on the household problem.[15] In particular, to bound the MVPF we simply need to estimate how the number of people locating below the new notch changes as a result of the reform, $J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$, and how the number of people locating below the old notch changes as a result of the reform, $B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$. Note, it is important to distinguish between the mathematical objects $B = G(\tau, \mathbf{p}) - G(\tau, \mathbf{p}')$ and $J = G(\tau', \mathbf{p}') - G(\tau', \mathbf{p})$ and the number of "bunching" and "jumping" households discussed in Section 2.1. In the context of our simple baseline model, B equals the number of households bunching at the original notch and J equals the number of households jumping down to bunch at the new notch. However, if households solve the more general household problem (6), then B and J no longer cleanly correspond to the number of bunching and jumping households. For instance, consider a labor supply model in

15. Assuming $\tau' > \tau$ in Proposition 3 is WLOG - we are just arbitrarily labeling policy $\mathbf{p}'$ as the one with larger τ'. Moreover, in Proposition 3 we also assume that the policy reform leads to a net budgetary cost so that $b' G(\tau'; \mathbf{p}') - b G(\tau; \mathbf{p}) > 0$; this is also WLOG as if the policy reform leads to a net budgetary revenue gain, then both inequalities in Equation (10) are simply reversed.

which households can only work full-time, half-time, or not at all. In this case $B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$ is just the change in the number of households reporting at or below the original notch as a result of the reform and does not correspond to a reduction in the bunching mass at the original notch as almost all households cannot precisely bunch. Similarly, $J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$ simply captures the change in the number of households reporting below the new notch as a result of the reform and does not correspond to the number of households jumping down to bunch at the new notch as almost all households cannot precisely bunch. Nonetheless, for ease of exposition, we will continue to refer to B and J as the number of bunching and jumping households, respectively.

Problem (6) can encompass a variety of important realisms: (1) households may respond by changing labor supply instead of misreporting their income (or respond on a variety of dimensions), (2) households may face limited choice sets (e.g., restrictions on labor supply), (3) households may face reporting costs (e.g., hassle or time costs) and thereby face a decision of whether to report/update their income on the registry[16], (4) households may have a wide range of heterogeneity in their utility functions (e.g., households may have varying preferences over the labor/leisure trade-off or varying preferences to locate at round numbers). The key intuition for why Proposition 3 is so general comes from revealed preference arguments. We derive a lower bound for the MVPF of a policy shift from policy $\mathbf{p}$ to policy $\mathbf{p}'$ by calculating how welfare would change if no one re-optimized (by revealed preference, behavioral responses can only improve welfare). Similarly we derive an upper bound for the MVPF of a policy shift from policy $\mathbf{p}$ to policy $\mathbf{p}'$ by starting from policy $\mathbf{p}'$ and thinking about how welfare would change as we move to policy $\mathbf{p}$ if no one re-optimized. Hence, our bounds are robust to the types of behavioral responses available to households along with the restrictions on household choice sets precisely because behavioral responses are held fixed when formulating our MVPF bounds. We view this robustness as perhaps the most important aspect of our theory: we can construct bounds for the MVPF under relatively few assumptions.

But of course there are *some* implicit restrictions encoded in the assumed household problem (Problem (6)) used to prove Proposition 3. Perhaps most importantly, Proposition 3 requires that households correctly perceive the benefit schedule and the reform. The proof to Proposition 3 uses the fact that household re-optimization *improves* utility; if misperceptions are extreme for many households this may not be the case. However, it is straight-forward to extend Proposition 3 when households misperceive the schedule if we are willing to assume that, on average, behavioral responses to the reform from $\mathbf{p}$ to $\mathbf{p}'$ improve welfare (i.e., perceptions are not so extreme that households, on average, harm themselves by responding to the reform; see Appendix A.7). For example, if some proportion of households are entirely unaware of the reform while the rest of the population is perfectly aware of the reform, our bounds hold as unaware households will not respond to the reform whereas those who are aware of the reform improve their utility via their behavioral response.

Moreover, Problem (6) is also a static problem. We augment Proposition 3 in Appendix A.8 to show that we can bound the discounted welfare impact of the policy over time relative to the discounted

16. To capture adjustment/updating costs, one could suppose $x = \hat{y}_t$ and θ consists of current income y_t, aversion to misreporting μ, and prior reported income $\hat{y}_{t-1}$; households incur an adjustment cost k if their reported income today differs from prior reported income.

total budgetary cost in a general dynamic model allowing for income dynamics, savings, and stochastic shocks. In this case, the relevant bounds for the MVPF are constructed using the discounted sum of the expected number of jumping households over time and the discounted sum of the expected number of bunching households over time. Proposition 3 also requires that there are no externalities from household decisions so that decisions of one household do not directly impact the utility of any other household. Our bounds can be generalized to allow for externalities by augmenting the upper and lower bounds for the MVPF with an additional term measuring the WTP for these externalities relative to the total cost; however, measuring WTP for externalities is likely difficult in practice.

Additionally, Proposition 3 can be augmented to allow for more complex policy environments. For instance, Proposition 3 holds even if households only receive the benefit with some probability if they report below the notch (see Appendix A.9). The intuition being that this probability appears in both the numerator and denominator of the MVPF bounds (as it impacts both households' WTP for the reform as well as the total cost of the reform) and thus cancels out.[17] Moreover, while Proposition 3 assumes that there is no underlying tax and transfer system beyond the benefit b given to those with a reported income $\hat{y} \leq \tau$, it can easily be extended to account for more complex underlying tax and transfer schedules; in this case, the total budgetary cost of the reform must include the impacts that behavioral responses have on other programs that impact the government's budget (i.e., we need to calculate the fiscal externalities associated with the reform).[18]

Lastly, we discuss the advantages of bounding the welfare impacts (as opposed to exactly characterizing the welfare impacts) of a notch reform. First, because we cannot appeal to envelope conditions in our setting, exactly characterizing welfare impacts requires one to take a stance on which margins households are responding, the functional form of their utility function, the sorts of frictions they face, etc. In contrast, bounding welfare impacts requires very little structure on the household optimization problem. In fact, we prove in Appendix A.10 that the bounds in Proposition 3 are as tight as possible without making additional assumptions on primitives. Second, our bounds on the welfare impact of a notch change are expressed in terms of estimable reduced-form objects, whereas exactly characterizing welfare impacts would require one to estimate a potentially large number of structural parameters.

17. If the probabilities of receiving the benefit vary by groups, the MVPF bounds must be augmented to account for differing probabilities (we omit a proof for brevity, the logic is similar to Appendix A.9). Letting q_M, q_B, q_T, q_J denote the average probability of receiving the benefit for mechanical, bunching, threshold, and jumping households we have:

$$MVPF \geq MVPF_L \equiv \frac{q_M \Delta bM + q_B \Delta bB + q_T b'T}{q_M \Delta bM + q_B \Delta bB + q_T b'T + q_J b'J} = 1 - q_J b' \frac{J}{\text{Total Cost}}$$

$$MVPF \leq MVPF_U \equiv \frac{q_M \Delta bM + q_B b'B + q_T b'T + q_J b'J}{q_M \Delta bM + q_B \Delta bB + q_T b'T + q_J b'J} = 1 + q_B b \frac{B}{\text{Total Cost}}$$

18. In particular, the total cost of the reform would equal $b'G(\tau'; \mathbf{p}') + R(\mathbf{p}') - bG(\tau; \mathbf{p}) - R(\mathbf{p})$ where $R(\mathbf{p})$ denotes net government spending under policy $\mathbf{p}$ (exclusive of spending on BF), and $\Delta R = R(\mathbf{p}') - R(\mathbf{p})$ captures the fiscal externalities of the reform. In this case, the lower and upper bounds for the MVPF are given by:

$$MVPF_L \equiv 1 - \frac{b'\left[G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})\right] - \Delta R}{b'G(\tau'; \mathbf{p}') - bG(\tau; \mathbf{p}) + \Delta R} = 1 - b' \frac{J}{\text{Total Cost}} - \frac{\Delta R}{\text{Total Cost}}$$

$$MVPF_U \equiv 1 + \frac{b[G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')] + \Delta R}{b'G(\tau'; \mathbf{p}') - bG(\tau; \mathbf{p}) - \Delta R} = 1 + b \frac{B}{\text{Total Cost}} - \frac{\Delta R}{\text{Total Cost}}$$

Thus, in addition to B and J, we need to observe ΔR to assess the welfare impact of the reform.

3 The Bolsa Família Program

Next, we turn to our empirical application: bounding the MVPF of a notch reform to the Brazilian Bolsa Família program. This section will discuss the Bolsa Família program, the June 2014 reform, as well as our data; Section 4 will dicuss how we identify the sufficient statistics discussed in Section 2.

3.1 Program Details

The Brazilian anti-poverty program Bolsa Família (BF), which has been recently supplanted by Auxílio Brasil, provided cash transfers to poor households based on their reported, monthly per-capita income. BF was implemented in October 2003 and was administered by the social development ministry (Ministério do Desenvolvimento Social, or MDS). BF was one of the world's largest cash transfer programs with around 14 million households receiving benefits in 2014 (Gazola Hellmann, 2015). Applicants reported their information, including household income, expenditures, assets, socioeconomic characteristics, and demographic characteristics to interviewers at Cadastro Único agencies, which were program offices spread across Brazil's 5,570 municipalities. Beneficiaries of the BF program were required to update their information once every 2 years to maintain their benefits.

Eligibility for the BF program was based on reported household per-capita income. Household per-capita income was calculated in five steps. First, the applicant was asked to report labor income for the last month as well as average monthly labor income over the past year for each member in the household (the applicant must present a government-issued ID for herself and for each family member thus making it difficult to register fictitious family members). Second, the applicant was asked to report average monthly income received from five additional sources for each household member (see Appendix B.1, which shows the questionnaire used to calculate monthly income). Third, for each individual, a computer calculated the minimum between the average monthly labor income over the past year and last month's labor income. Fourth, a computer summed this minimum monthly labor income along with income from the five additional sources to get a measure of total monthly income for each individual. Finally, a computer summed this individual total monthly income across all household members and divided it by the number of household members.

Households were classified as eligible if their reported per-capita income fell below one of two thresholds. First, households reporting a per-capita income below the "extreme-poverty threshold" were eligible for an unconditional benefit (referred to as the "basic benefit"). Second, households with children reporting a per-capita income below the higher "poverty threshold" were eligiblie for a conditional benefit (referred to as the "variable benefit") provided that they made health and education investments in their children. Notably, not all households reporting incomes below these thresholds received the benefit. This is because there was a quota (cap) on the number of beneficiaries per municipality. Prior to 2009, these quotas were based on the predicted number of households below the poverty threshold in each municipality. Post 2009, these quotas were based on the predicted number of households below the poverty threshold scaled by 1.18 (Gerard, Naritomi and Silva, 2021). Hence, if quotas were binding in a given municipality, some eligible households may not have received benefits.[19]

Finally, the MDS had several enforcement mechanisms to prevent income misreporting. First, during the interview, the income questions came at the end of the questionnaire so that questions on expenditures and assets could help the interviewer asses the veracity of reported income (Bastagli, 2008). Second, during the interview, the applicant was reminded of her responsibility to provide true statements under penalty of losing the right to be eligible for government programs (Gazola Hellmann, 2015). Third, the ministry conducted audits, which could be triggered by citizens' complaints and cross-checks of registry data with other datasets such as administrative data on formal employment, deaths, or automobile purchases (Gazola Hellmann, 2015). However, despite these attempts, the large informal sector in the Brazilian economy generated substantial scope for income misreporting.

3.2 The Transfer Schedule and the June 2014 Reform

Between July 2009 and June 2014, the extreme-poverty threshold was R$70 per-capita, per-month and the poverty threshold was R$140 per-capita, per-month.[20] The basic (unconditional) benefit was equal to R$70 per-month, while the variable (conditional) benefits were based on the number and ages of the children in the household (see Appendix B.2 for more information on the variable benefits). In June 2014, the government increased both the benefits and thresholds by 10%. Thus, the extreme-poverty threshold was raised from R$70 to R$77 per-capita, per-month, the poverty threshold was raised from R$140 to R$154 per-capita, per-month, the basic benefit was raised from R$70 to R$77 per-month, and the variable benefits were also increased by 10%. This reform was announced on national television by the president in April 2014.

Our main empirical analysis will focus on single individual households (households with one adult and no children). These households are not eligible for the variable benefits as they do not have children. Thus, prior to June 2014, these households were eligible for R$70 per-month if their reported income was less than or equal to R$70 per-month and 0 otherwise. After June 2014, these households were eligible for R$77 per-month if their reported income was less than or equal to R$77 per-month and 0 otherwise. Thus, the benefit schedule for these households has a single notch which increased both in level and location as a result of the June 2014 reform; see Figure 2.

19. Recall that in Section 2.3 we discussed how our framework can be augmented when only a fraction of eligible households receive benefits. In Section 5.2, we will discuss why this feature of the program is unlikely to impact our MVPF bounds.

20. In 2003, the extreme-poverty threshold and the poverty threshold were set to equal one-fourth and one-half of the monthly minimum wage of R$200, respectively. These thresholds were periodically adjusted for inflation; the time betwen adjustments was *ad hoc* and not linked to the minimum wage. Prior to June 2014, the last readjustment was in July 2009 (Gazola Hellmann, 2015).

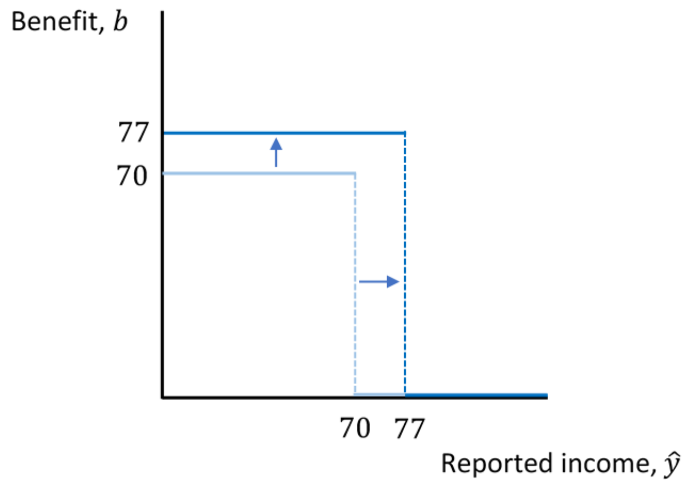


Figure 2: June 2014 Reform for Single Individual Households

We focus on single individual households because in February 2013, the government instituted a guaranteed minimum income of R$70 per-capita for all households which was subsequently raised to R$77 in the June 2014 reform. However, because the basic benefit is equal to the guaranteed minimum income, the benefit schedule for single individual households is not impacted by this minimum. In contrast, for all other households, this minimum creates a kink in the benefit schedule below the extreme poverty threshold (the location of this kink will vary based on household composition); moreover, the location of this kink changed with the 2014 reform. For example, prior to the reform, households with two adults and no children had a kink at the reported per-capita income level of R$35 which increased to R$38.5 post June 2014.[21] This is problematic because, as will be discussed in Section 4, one of our identification assumptions is that the reported income distribution below the extreme-poverty threshold is unaffected by the June 2014 reform; this is not necessarily true for households with more than one member because the kink created by the guaranteed minimum income changes concurrently with the notch. Nonetheless, we will discuss the impacts of the June 2014 reform on households with more than one individual in Section 5.5.

Finally, in June 2016, there was another reform to the BF program where both the benefit and the threshold were further increased. This reform, like the June 2014 reform, affected households of all compositions. As discussed next, our data ends in September 2016; thus, we do not have sufficient data beyond June 2016 to analyze the June 2016 reform.

3.3 Data Sources and Sample Description

We have access to the Cadastro Único household registry, which is used to determine the eligibility of households for BF as well as all other targeted federal social programs (Veras Soares, 2011). Many

21. Prior to June 2014, a two adult household with a reported per-capita income less than R$35, $\hat{y} < 35$, will receive an additional monthly benefit equal to $2(70 - \hat{y}) - 70$. E.g., a two member household reporting a per-capita income of R$20 will receive R$70 in the basic benefit and an additional benefit of R$30.

of these other programs have eligibility criteria above the BF thresholds which explains the large number of ineligible applicants in the registry (see Table 2).[22] These other programs do not change concurrently with the BF reform that we analyze and are discussed in more detail in Appendix B.3.

Our final dataset is constructed by appending eight extractions of the registry: one in December of each year from 2011 until 2015, one in April 2015, one in August 2015, and one in September 2016.[23] Each extraction contains the latest information for all households on the registry at the time of the extraction date. For instance, if a household updated its information in August 2011 and September 2013, its information will appear as of August 2011 in the 2011 and 2012 extractions and as of September 2013 in the 2013, 2014, 2015, and 2016 extractions. Table 2 presents summary statistics on household per-capita income as of June 2014 for all households in the registry as well as for all single adult households; Table 2 also presents statistics for beneficiary households who receive BF payments.

Table 2: Summary Statistics as of June, 2014

Variables	Mean	Median
Per Capita Income (single individual households)	330.51	200.00
	(324.30)	
Per Capita Income (all households)	151.17	76.66
	(184.97)	
Per Capital Income (single individual beneficiary households)	39.73	40.00
	(39.72)	
Per Capital Income (beneficiary households)	56.20	47.00
	(53.24)	
Observations (single individual households)	4,232,528	
Observations (all households)	28,932,001	
Observations (single individual beneficiary households)	823,708	
Observations (beneficiary households)	13,124,018	

Note: This table shows summary statistics for households in the Cadastro Único database as of June, 2014. Per-capita income denotes household, monthly, per-capita income and is measured in Brazilian reais. The PPP conversion from US dollars to Brazilian reais was 1.813 in 2014 (OECD, `https://data.oecd.org/conversion/purchasing-power-parities-ppp.htm`).

4 Empirical Strategy

We now discuss our empirical strategy to estimate the number of bunching and jumping households for the June 2014 BF reform. For simplicity, we focus on estimating the MVPF bounds in June 2016. Assuming that all behavioral responses to the June 2014 reform have occurred by June 2016 (which is reasonable given households are required to update their information every two years), the

22. Moreover, the government aims to register all families with per-capita incomes below half the minimum wage (or total incomes below three times the minimum wage) (Veras Soares, 2011). The minimum wage was R$724 per-month in 2014; thus half the minimum wage was R$362, which is substantially higher than the BF threshold.

23. Every time the MDS analyzes the Cadastro Único data, it creates one of these extractions. Therefore, the frequency of the extractions is a result of previous data analyses by the ministry. Appendix B.4 contains a figure depicting the timeline of the data extractions.

MVPF in June 2016 is a reasonable approximation for the long-term MVPF of this reform.[24] From the theoretical framework developed in Section 2, in order to estimate MVPF bounds in June 2016, we need to form estimates of B and J. More fundamentally, estimating these objects requires understanding the distribution of reported incomes under policy $\mathbf{p}$, $G(x;\mathbf{p})$, as well as the distribution of reported incomes under policy $\mathbf{p}'$, $G(x;\mathbf{p}')$ *holding the distribution of primitives in the population fixed.*

Ideally, we could identify $G(x;\mathbf{p})$ and $G(x;\mathbf{p}')$ from experimental variation. For example, if the implementation of the 2014 BF reform was randomly staggered across identical areas, we could estimate the income distributions in June 2016 for areas that experienced the reform vs. the identical regions which did not yet experience the reform. However, the BF reform was a national reform implemented uniformly across Brazil in June 2014: hence, we can observe $G(x;\mathbf{p}')$ under the primitive distribution in June 2016, but we can only observe $G(x;\mathbf{p})$ under the primitive distribution *prior to June 2014*. Put another way, we are going to need a control group to control for time trends in the underlying primitive distribution. This motivates augmenting our model from Section 2 to include a simple notion of time.

Returning to our general model from Section 2.3, let us suppose that the distribution of primitives, $F(\theta)$, is varying over time, which we denote by $F_t(\theta)$. Hence, we can also define the number of households reporting an income under z over time given a particular policy $\mathbf{p}$ as:

$$G_t(x;\mathbf{p}) = \int_{\theta:\hat{y}(\theta,\mathbf{p})\leq x} dF_t(\theta)$$

Let us denote June 2014 as $t = 0$. Returning to the specifics of the June 2014 BF reform, for single individual households, the reform changed the policy from $\mathbf{p} = \{b,\tau\} = \{70,70\}$ to $\mathbf{p}' = \{b',\tau'\} = \{77,77\}$. Plugging these values into Equations (8) and (9), our bounds on the MVPF in period t are given by:

$$MVPF_{L,t} \equiv 1 - \frac{77 \times [G_t(77;\mathbf{p}') - G_t(77;\mathbf{p})]}{77 \times G_t(77;\mathbf{p}') - 70 \times G_t(70;\mathbf{p})} = 1 - 77 \times \frac{J_t}{\text{Total Cost}_t} \tag{11}$$

$$MVPF_{U,t} \equiv 1 + \frac{70 \times [G_t(70;\mathbf{p}) - G_t(70;\mathbf{p}')]}{77 \times G_t(77;\mathbf{p}') - 70 \times G_t(70;\mathbf{p})} = 1 + 70 \times \frac{B_t}{\text{Total Cost}_t} \tag{12}$$

Thus, to estimate the MVPF in a particular time period $t \geq 0$, we need to estimate the number of jumping and bunching households at time period t, which, in turn, requires us to estimate the number of households locating below the old and new notch under both $\mathbf{p}$ and $\mathbf{p}'$ at time t: $G_t(70;\mathbf{p})$, $G_t(70;\mathbf{p}')$, $G_t(77;\mathbf{p})$, and $G_t(77;\mathbf{p}')$.[25] Our fundamental identification challenge is that $G_t(70;\mathbf{p})$ and $G_t(77;\mathbf{p})$ are not observed at time $t \geq 0$ under policy $\mathbf{p}$: we only observe the income distribution

24. We also calculate MVPF bounds for each month between June 2014 and June 2016 along with the cumulative MVPF for all months post-reform; the latter is computed using a discounted sum of jumping and bunching households over the entire time horizon a la Appendix A.8. Note the cumulative MVPF bounds are very similar to the June 2016 MVPF bounds given the assumption that all behavioral responses are observed by June 2016 (i.e., the number of bunching and jumping households post-June 2016 are equal to the numbers for June 2016). This is, however, an untestable assumption as there is another reform in June 2016 meaning we cannot estimate the impact of the June 2014 reform beyond June 2016. See Appendix C.1 for associated results and discussion.

25. Recall that the total cost can be estimated from the components necessary to estimate the bunchers and jumpers as $\text{Total Cost}_t = 77G_t(77;\mathbf{p}') - 70G_t(70;\mathbf{p})$.

under policy $\mathbf{p}$ *prior* to June 2014 (i.e., $t < 0$). For example, given that the distribution of household primitives may be changing over time, $G_t(77; \mathbf{p}') - G_{-1}(77; \mathbf{p})$ will conflate the true effect of interest, $G_t(77; \mathbf{p}') - G_t(77; \mathbf{p})$, with underlying time variation, $G_t(77; \mathbf{p}) - G_{-1}(77; \mathbf{p})$. Thus, we require a control group to estimate how the income distribution would have evolved from the period before the reform ($t = -1$ corresponding to May 2014) to time $t \geq 0$ *in absence of the reform*: $G_t(x; \mathbf{p}) - G_{-1}(x; \mathbf{p})$. In short, our identification strategy is going to rely on using regions of the reported income distribution that *were not* impacted by the reform as a control group to identify underlying time trends in the portions of the reported income distribution that *were* impacted by the reform. This brings us to our two identification assumptions.

4.1 Identification Assumption 1

Our identification strategy relies on first finding a region of the reported income distribution which was not impacted by the reform. In the context of the baseline model in Section 2.1, the number of single individual households reporting an income strictly below R$70 should be unchanged by the reform. The intuition for this is that anyone who has a true income below R$70 always reports truthfully both pre- and post-reform and anyone who has a true income above R$70 prefers to misreport at the threshold rather than misreport to an income level below the threshold (because misreporting costs are increasing in the distance between true and reported incomes). While in this baseline model bunching should occur precisely at R$70, in reality bunching is typically more diffuse due to small optimization errors and/or frictions (Kleven, 2016). Thus, our first identification assumption is that the number of people reporting incomes at or below R$63 is unaffected by the reform (i.e., we assume that R$63 is sufficiently far below R$70 such that there are no "bunchers" at or below R$63):[26]

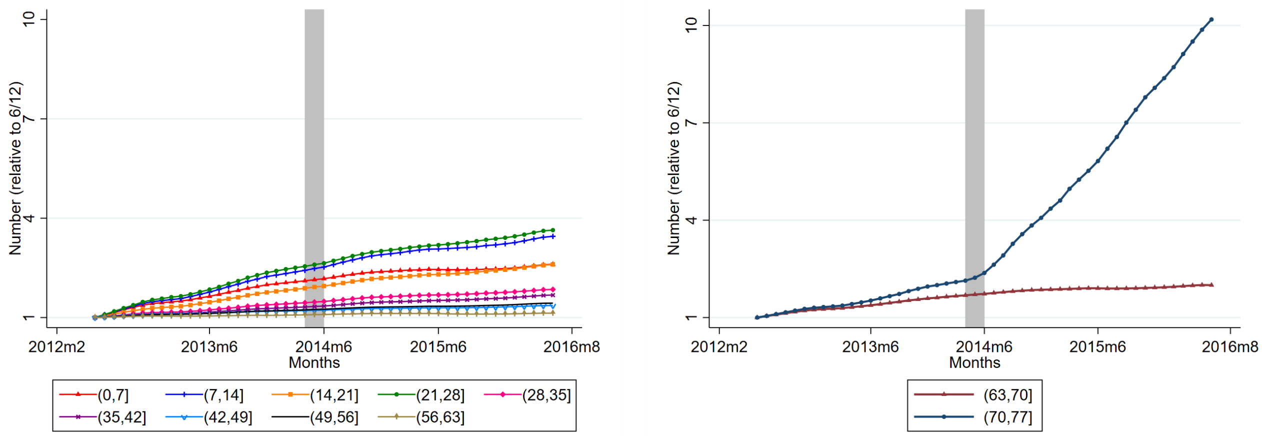
Identification Assumption 1. *The number of households reporting incomes below R$63 is unaffected by the reform:* $G_t(x; \mathbf{p}) = G_t(x; \mathbf{p}') \ \forall x \leq 63,\ t.$

To provide suggestive evidence that Assumption 1 is reasonable, Figure 3a plots the number of single individual households reporting in income bins of size 7 from R$0 to R$63 between June 2012 and June 2016 (with the numbers in each bin normalized to 1 in June 2012). Figure 3a shows that it is not obvious any of the bins below R$63 were impacted by the reform.

Identification Assumption 1 implies that J_t and B_t can be expressed solely in terms of how the number of households reporting incomes in bins R$$(63, 70]$ and R$$(70, 77]$ changed as a result of the reform:

$$\begin{aligned} B_t &= -\left[G_t(70; \mathbf{p}') - G_t(70; \mathbf{p})\right] \\ &= -\left(\left[G_t(70; \mathbf{p}') - G_t(63; \mathbf{p}')\right] - \left[G_t(70; \mathbf{p}) - G_t(63; \mathbf{p})\right]\right) \end{aligned} \tag{13}$$

26. Implicitly, Assumption 1 requires that households do not change their reported income in response to receiving a higher benefit, i.e., there are no income effects. While this is the case in the baseline misreporting model of Section 2.1, it may not be the case in the more general model of Section 2.3 as households may reduce their labor supply and, in turn, their reported income in response to a higher benefit. We discuss in Appendix C.2 why we believe income effects are unlikely to be impacting the reported income distribution in our setting.



(a) Number Reporting in Bins Below R$63 (b) Number Reporting in R$(63,70], R$(70,77]

Note: Panel (a) shows the number of single individual households with reported incomes in each seven increment bin below R$63 between June 2012 and June 2016. Panel (b) shows the number of single individual households with reported incomes in R$(63,70] and R$(70,77]. The number in each bin is normalized to 1 in June 2012. For example, there were approximately 10 times as many single individual households with reported incomes in R$(70, 77] in June 2016 compared to June 2012. The timing of the reform (from the announcement in April 2014 to the enactment in June 2014) is indicated by the gray, shaded region.

Figure 3: Number of Single Individual Households Reporting Incomes in Bins $\leq$ R$77

$$\begin{aligned} J_t =& G_t(77;\mathbf{p}') - G_t(77;\mathbf{p}) \\ =& [G_t(77;\mathbf{p}') - G_t(70;\mathbf{p}')] - [G_t(77;\mathbf{p}) - G_t(70;\mathbf{p})] + \\ & [G_t(70;\mathbf{p}') - G_t(63;\mathbf{p}')] - [G_t(70;\mathbf{p}) - G_t(63;\mathbf{p})] \end{aligned} \tag{14}$$

In particular, B_t is equal to the reduction in households locating in R$(63,70] at time t as a result of the reform (i.e., the reduction in households bunching at the old notch) while J_t is equal to the increase in households locating in R$(70,77] less the reduction in households locating in R$(63,70] at time t as a result of the reform (i.e., the increase in households bunching at the new notch less the reduction in households bunching at the old notch).

Figure 3b shows how the number of households reporting incomes in bins R$(63, 70] and R$(70, 77] evolved in a four year window around the reform from June 2012 to June 2016. In contrast to Figure 3a, Figure 3b shows a clear departure in trend for the bin R$(70, 77] commensurate with reform. This provides highly suggestive evidence that the BF reform induced a substantial behavioral response that increased the number of households bunching at the new threshold. Note that the number of individuals reporting incomes in R$(70, 77] is increasing over time prior to the reform - this is likely due to a combination of factors including population growth and (nominal) income dynamics.[27] In particular, Brazil was experiencing slowing and then negative per-capita GDP growth from 2012-2016 with a large drop in 2015 along with relatively high inflation (World Bank Data). However, the slope of the line for bin R$(70, 77] in Figure 3b clearly changes at the time of the reform, leading to a sharp

27. Similarly, the number of households on the Cadastro Único registry is growing over this time period - see Figure 16 in Appendix C.5. This too is likely due to a variety of factors including population growth, a struggling Brazilian economy, and increased awareness/understanding of the BF program over time.

increase in the number of individuals locating in this bin. The fact that this reform induced a slope shift rather than a level shift may reflect households learning about the threshold shift over time.[28]

The scale of Figure 3b makes it difficult to determine how the number of households in R$(63,70] changed as a result of the reform; Figure 13 in Appendix C.3 suggests that the number of households in R$(63,70] may have declined as a result of the reform although it is difficult to assess due to underlying time trends (Figure 14 in Appendix C.4 shows a much starker downward trend break for two adult households). Hence, we need a way to control for underlying time trends in the numbers reporting in R$(63, 70] and R$(70, 77] to estimate how these quantities would have evolved over time *in absence of the reform*. This brings us to our second identification assumption.

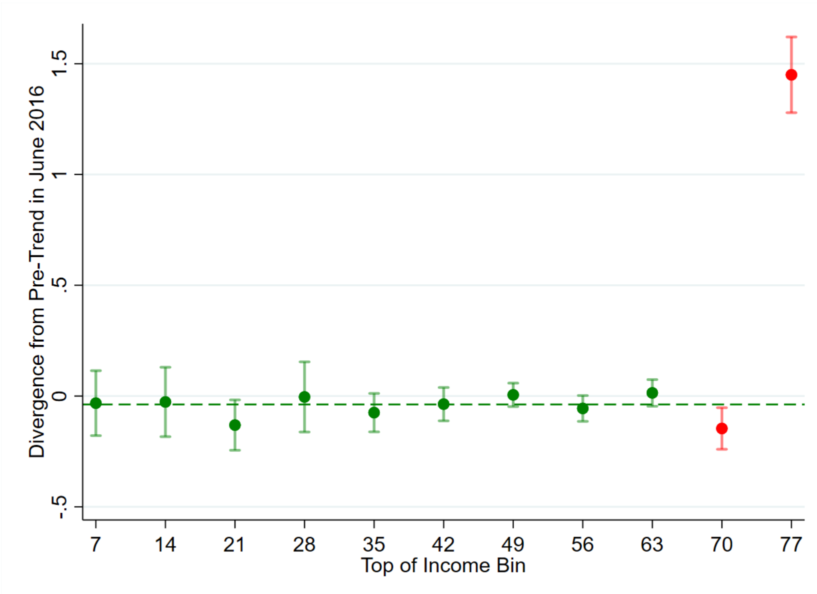
4.2 Identification Assumption 2

At a high level, our second identification assumption is going to relate how the numbers reporting in bins below R$63 evolve over time to how the numbers reporting in our two bins of interest, R$(63, 70] and R$(70, 77], would have evolved over time in absence of the reform. In this sense, bins below R$63 can be viewed as our "control bins" while R$(70,77] and R$(63,70] can be viewed as our "treatment bins". Building towards our second identification assumption and our main empirical specification, consider the following regression, where $N_{(x-7,x],t}$ denotes the number of individuals reporting in income bin R$$(x-7, x]$ in month t:

$$\underbrace{\log\left(N_{(x-7,x],t}\right)}_{\text{log \# in } (x-7,x]} = \underbrace{\sum_{k=0}^{K} \alpha_{k,x} t^k}_{\text{bin-specific polynomial}} + \underbrace{\beta_{1,x} post_t + \beta_{2,x} post_t \times t}_{\text{post-reform deviation from polynomial}} + \epsilon_{xt} \tag{15}$$

where $post_t$ takes value 1 if month t is after the reform and 0 otherwise (i.e., $post_t = 1$ if $t \geq$June 2014 and 0 otherwise). Regression (15) simply estimates a break from the (bin-specific) polynomial time trend in the post-reform period for the (log) number of people reporting in R$$(x-7, x]$. Running Regression (15) separately for each bin (using cubic bin-specific polynomials), Figure 4 plots $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}\bar{t}$ for each $x \in \{7, 14, ..., 77\}$ where $\bar{t}$ represents the final month in our analysis period (June 2016). In other words, Figure 4 plots how the log number of people reporting in each seven increment bin below R$77 deviated from its bin-specific cubic time trend in the post-reform period:

28. Alternatively, households may wait to update simply because they are only required to update every two years and/or to avoid suspicion of misreporting that could result from updating immediately after the reform.



Note: This figure plots how the log number of people reporting in each seven increment bin deviated in June 2016 from a bin-specific, cubic time trend estimated via Regression (15) using monthly reported income data from June 2012 to June 2016 along with 95% confidence intervals using robust standard errors. For each $x \in \{7, 14, ..., 77\}$, we estimate the deviation from the bin-specific time trend in June, 2016 as $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}\bar{t}$, where $\bar{t}$ represents the final month in our analysis period (June 2016). The green horizontal line plots the average deviation (-0.038) from trend for bins with $x \leq R\$63$.

FIGURE 4: DEVIATION FROM CUBIC TIME TREND IN POST-REFORM PERIOD BY INCOME BIN

Figure 4 shows that the number of people reporting in income bins below R$63 saw very minor (and mostly statistically insignificant) trend breaks in the post-reform period. The dashed green line in Figure 4 indicates an average trend break of -0.038 across these bins, indicating that the number of people reporting incomes in bins $\{R\$(0, 7], R\$(7, 14], ..., R\$(56, 63]\}$ was, on average, approximately 3.8% lower in June, 2016 relative to polynomial trend. However, under Identification Assumption 1, any post-reform trend break for these bins must be due to underlying time variation unrelated to the reform. Loosely speaking, our second identification assumption is that our two treatment bins, R$(63, 70] and R$(70, 77], would have seen the same deviation post-reform as the control bins had the reform not happened, i.e., these two bins also would have seen a 3.8% reduction in the number of people in June, 2016 (relative to their bin-specific polynomial time trends) had the reform not happened. However, as can be seen in Figure 4, the number of people reporting incomes in R$(63, 70] saw a reduction of around 13.6% (a 0.146 log-point decrease) while the number of people reporting incomes in R$(70, 77] saw an increase of 326.3% (a 1.45 log-point increase) as of June 2016. Formally, our second identification assumption is:

Identification Assumption 2. *In absence of the reform, the (log) number of people reporting in each bin evolves according to:*

$$\log\left(N_{(x-7,x],t}\right) = h(t) + \sum_{k=0}^{K} \alpha_{k,x} t^k + \nu_{x,t} \quad \textit{for } x \in \{7, 14, ..., 77\}$$

Under Identification Assumption 2, the difference between the (log) number of households in any two 7-increment bins below R\$77 is, *in absence of the reform*, governed by a stable polynomial plus a random error term. Combining Assumptions 1 and 2, we can use our control bins (i.e., bins $\leq$R\$63) to identify $h(t)$ in the post-reform period.[29] In other words, we can use our control bins to identify the expected deviation from bin-specific polynomials in the post-reform period if the reform did not occur. Any deviation observed above and beyond $h(t)$ in our two treatment bins is then attributed to the reform. This brings us to our main empirical specification: a generalized difference-in-difference specification where we allow for flexible pre-treatment dynamics between treatment and control bins:

$$\log\left(N_{(x-7,x],t}\right) = \delta_t + \sum_{k=0}^{K} \alpha_{k,x} t^k + [\beta_{1,x} post_t + \beta_{2,x} post_t \times t] \times treat_x + \epsilon_{xt} \text{ for } x \in \{7, 14, ..., 77\} \tag{16}$$

where δ_t represents a set of month fixed-effects; $treat_x$ takes value 1 if $x \in \{70, 77\}$ and 0 otherwise; and $\sum_{k=0}^{K} \alpha_{k,x} t^k$ captures polynomial time trends for each bin (indexed by x) which predate the reform and are assumed to persist into the post-reform period had the reform not occurred. Under our two identification assumptions, the causal impact of the reform on the (log) number of households locating in R\$$(x-7, x]$ in month t, denoted $\Delta \log\left(N_{(x-7,x],t}\right)$, is equal to $\hat{\beta}_{1,x} + \hat{\beta}_{2,x} t$ for $x \in \{70, 77\}$.

Before we present the results from Equation (16) in Section 5 (using various polynomial degrees K), let us discuss the statistical and economic interpretations of Identification Assumption 2. From a statistical perspective, Identification Assumption 2 is simply a generalization of the standard difference-in-difference assumption (this generalization is employed in, for example, Wolfers (2006) and is discussed more generally in Mora and Reggio (2013)). Setting $K = 0$ results in a standard difference-in-difference estimator in which the control groups and treatment groups are assumed to have "parallel trends" pre-reform that would have persisted in the post-reform period had the reform not occurred. Larger values of K require progressively weaker parallel assumptions. For instance, setting $K = 1$ requires the "parallel growths" assumption which asserts that the growth rates in the treatment and control groups over time are the same (i.e., that second and higher differences between treatment and control groups are constant over time).[30] Setting $K > 1$ results in what Mora and Reggio (2013) refer to as the "parallel-K" assumption which asserts that the $K + 1$ and higher differences between the treatment and control groups are constant over time.

But what is the economic meaning of Identification Assumption 2? Recall that in light of Identification Assumption 1, our fundamental identification challenge is that we do not know how the number of households locating in the two treatment bins, R\$$(63, 70]$ and R\$$(70, 77]$, would have evolved over time in absence of the reform due to changes in the underlying distribution of household primitives (e.g.,

29. We use the distribution below R\$63 to control for underlying time trends consistent with Kleven (2016), who suggests using data below the notch to estimate bunching when extensive margin responses are present. In theory, we could also use the distribution *sufficiently far* above R\$77 to control for underlying time trends as this region is also unimpacted by the reform. Kleven and Waseem (2013) discuss how to identify an unimpacted region of the density sufficiently far above a notch by equating the excess mass below the notch with the missing mass above the notch. However, our setting likely features extensive margin responses: some of the "excess" households locating below R\$77 post-reform may have simply not reported an income (and thus not be on the registry) in absence of the reform. Thus, we cannot equate the excess mass below R\$77 with the missing mass above R\$77.

30. A number of papers discuss the extended version of differences-in-differences under the "parallel growths" assumption (e.g., Mora and Reggio (2013), Bilinski and Hatfield (2019), and Rambachan and Roth (2020)).

endowed incomes, labor productivities, preferences over misreporting etc.) By making Identification Assumption 2, we assume that the underyling distribution of primitives is evolving such that the differences over time between the number of people reporting in each income bin are well approximated by polynomials which would have persisted in absence of the reform.[31] Note, every difference-in-difference study implicitly requires an assumption like Identification Assumption 2 that ensures the relationships between treatment and control groups are stable over time (in absence of reforms). Identification Assumption 2 is not fully testable in the same way that the standard differences-in-differences assumption (that parallel pre-trends persist post treatment) is not fully testable. However, as in the case of the standard "parallel trends" assumption, we can gauge whether Identification Assumption 2 seems sensible by looking at pre-reform data. Appendix C.6 shows how the differences $\log\left(N_{(70,77]}\right) - \log\left(N_{(x-7,x]}\right)$ and $\log\left(N_{(63,70]}\right) - \log\left(N_{(x-7,x]}\right)$ evolved in the pre-reform period for $x \in \{7, 14, ..., 63\}$. These differences follow stable, low order polynomial relationships in the pre-period. While this of course does not imply that these relationships would have persisted into the post-reform period (just as parallel pre-trends do not imply that the trends would continue on a parallel path post treatment), it is at least suggestive that stable relationships exists between the various income bins. Moreover, because we have several control bins, we can *partially* test the validity of our two identification assumptions. In particular, we can run placebo tests where we pretend that some of our control bins are impacted by the reform and use the remaining control bins to estimate these placebo "treatment effects". Results from these placebo tests are presented at the end of Section 5.

4.3 Could We Use a Bunching Estimator to Estimate B and J?

We use a difference-in-difference strategy to estimate the reduction in bunching at the old notch and the increase in bunching at the new notch as a result of the reform. But one may wonder whether we could have used standard bunching techniques (developed in Kleven and Waseem (2013) and Saez (2010)) to back out the changes in these masses. Of course, even using standard bunching techniques, one must account for time variation by estimating how bunching would have changed in absence of the reform. For instance, one could make standard smoothness assumptions to estimate a counterfactual income distribution that would ensue in absence of the notch both pre- and post-reform. By assuming that the relationship between the counterfactual and observed reported income distributions in the pre-reform period would have stayed the same in the post-reform period in absence of the reform, one could estimate how bunching would have changed over time due to time trends; taking the observed difference in bunching minus the difference in bunching due to time trends would yield an estimate of the change in bunching due to the reform. A similar strategy is employed in Carril (2022). However, using these sorts of standard bunching methods is unlikely to be feasible in our setting. As seen in Figure 20 in Appendix C.7, the reported income distribution in Brazil is

31. One criticism of the standard bunching approach (see Blomquist et al. (2021)) is that smoothness assumptions used to identify counterfactual distributions around notches/kinks are implicitly assumptions on the shape of underlying primitive distributions. Similarly, Identification Assumption 2 is implicitly making assumptions about how primitives are evolving over time. However, in contrast to the standard bunching approach, we can leverage the longitudinal nature of our data to explore the validity of these implicit assumptions by investigating whether the relationships between bins are stable in the pre-reform period and by doing placebo tests in the post-reform period.

extremely non-smooth. For instance, in June 2014, in addition to substantial bunching at the notch of R\$70, there is extreme bunching at numbers equal to 0 mod 50, less severe bunching at numbers equal to 0 mod 10, and substantial bunching at R\$60 (R\$60 was a previous BF eligibility threshold implemented in 2006 while the notch of R\$70 was implemented in 2009). While bunching estimators have been augmented to deal with "round-number" bunching and other "reference-point" bunching (e.g., Kleven and Waseem (2013) and Best and Kleven (2017)), we believe that the pervasiveness and the variability of round-number and reference-point bunching in our setting will make it too difficult to precisely identify the counterfactual distributions around our notches. Thus, our empirical strategy highlights an alternative way to estimate changes in bunching when smoothness assumptions cannot be used to estimate counterfactual distributions.

5 Results

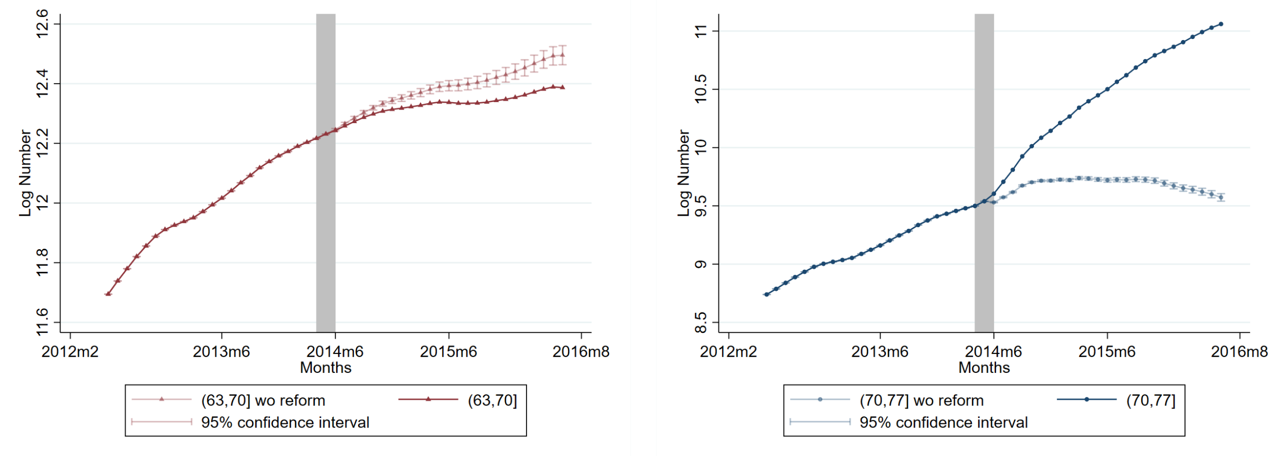
In this section, we present results from our generalized difference-in-difference specification, Equation (16), and, in turn, calculate the number of bunching and jumping households along with our bounds on the MVPF. Based on these bounds, we then discuss the welfare implications of the reform. We then discuss robustness of our results.

5.1 Difference-in-Difference Results

First, we present results from estimating Equation (16) setting $K = 3$, which assumes the difference in the number of households reporting in any two income bins below R\$77 approximately follows a cubic polynomial over time (in absence of the reform). Once we have estimated Equation (16), we can recover the causal impact of the reform on the log number of households in R$\$(x-7, x]$ for $x \in \{70, 77\}$ in any given month t, denoted $\Delta \log\left(N_{(x-7,x],t}\right) = \hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$. Figure 5 plots the evolution of $\log\left(N_{(63,70],t}\right)$ and $\log\left(N_{(70,77],t}\right)$ over time along with the estimated counterfactual path if the reform did not occur: $\log\left(N_{(63,70],t}\right) - \Delta\log\left(N_{(63,70],t}\right)$ and $\log\left(N_{(70,77],t}\right) - \Delta\log\left(N_{(70,77],t}\right)$. The difference between the actual path and the counterfactual path yields the causal impact of the reform under our two identification assumptions.

Consistent with the theoretical model from Section 2.1, Figure 5 shows that the reform led to a large, significant increase of about 1.5 log points in the number of households reporting in bin R$\$(70, 77]$ and a significant decrease of about 0.1 log points in the number of households reporting in bin R$\$(63, 70]$.[32] Translating these log changes into levels, the reform led to an increase of approximately

32. All standard errors for our analysis are clustered at the bin level (using STATA's default small-number-of-clusters bias adjustment) as this is the level of "treatment assignment", see Abadie et al. (2017). Thus, we have 9 control clusters and 2 treatment clusters. However, Imbens and Kolesár (2016) show that these standard errors are modestly underestimated when the number of clusters is small (they find that for samples with 10 clusters, 95% confidence intervals only have a 91% coverage rate). Hence, our standard errors may be modestly overstating the true statistical confidence we have in our estimates. An alternative approach is to use non-clustered wild bootstrapped p-values (note, wild *cluster* bootstrapped p-values lead to severe bias in difference-in-difference settings with a small number of treated clusters, see Roodman et al. (2019)). In our setting, these p-values are smaller than those generated from our cluster-robust standard errors. Finally, Ferman and Pinto (2019) suggest another alternative for difference-in-difference settings with a small



(a) $N_{(63,70]}$: Actual & Counterfactual (b) $N_{(70,77]}$: Actual & Counterfactual

Note: This figure shows the log number of single individual households reporting incomes in R\$$(63, 70]$ and R\$$(70, 77]$ from June 2012 to June 2016 along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual log number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 5: Causal Impact of the Reform Estimated From Equation (16)

49,000 households locating in R\$$(70, 77]$ and a decrease of approximately 27,000 households locating in R\$$(63, 70]$.[33] Plugging these numbers into Equations (13) and (14) and denoting June 2016 as $\bar{t}$, we get that $B_{\bar{t}} \approx 27,000$ and $J_{\bar{t}} \approx 22,000$. Thus, of the additional 49,000 households bunching at the new notch, 27,000 would have bunched at the old notch had the reform not happened, while 22,000 would have located above the new notch had the reform not happened (i.e., 22,000 households jump into the program as a result of the reform).[34]

We repeat this exercise under the alternative assumptions that the differences in the (log) numbers in any two bins follow quadratic, quartic, or quintic polynomials over time, i.e., we re-estimate Equation (16) for $K = 2, 4, 5$.[35][36] Figures of the counterfactual paths for our two treatment bins under these alternative assumptions are shown in Appendix C.11. Table 3 presents the estimated impact of the reform on the numbers locating in R\$(63,70] and R\$(70,77], $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$, along with the estimated number of bunching households $B_{\bar{t}}$, estimated number of jumping households $J_{\bar{t}}$, and lower

number of treated clusters; unfortunately, their results rely on asymptotic theory in the number of control clusters, which we believe is inappropriate for our setting given that we only have 9 control clusters.

33. In Figure 5b, we estimate that the number of households in R\$$(70, 77]$ would have slightly declined over the first six months of 2016 had the reform not occurred (this results from the relative concavity between the R\$$(70, 77]$ bin and our control bins in the pre-reform period). This does not significantly impact our results: we show in Appendix C.8 that our main results are very similar even if we make the conservative assumption that R\$$(70, 77]$ would have (counterfactually) grown at the maximum rate observed for any control bin over the post-reform period.

34. Technically, some of these 22,000 jumping households may have responded on the extensive margin, i.e., in absence of the reform, they may have simply not have reported an income to the registry. We provide descriptive evidence to suggest that a number of these jumping households are likely responding on the extensive margin - see Appendix C.9.

35. We do not estimate Equation (16) for $K = 0$ or $K = 1$ as pre-trends between treatment and control bins are often not constant or linear (see Figures 17 and 18 in Appendix C.6).

36. Our results are also robust to estimating Equation (16) with more granular income bins, see Appendix C.10.

and upper bounds on the MVPF in June 2016. All of these quantities are stable across different values of K.

Table 3: Impacts of Reform and MVPF Bounds in June 2016 Estimated From Equation (16)

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-26,279	51,759	26,279	25,480	0.88	1.11
	(6,163)	(2,160)	(6,163)	(6,659)	(0.03)	(0.03)
Cubic, $K = 3$	-27,452	49,247	27,452	21,794	0.90	1.12
	(4,357)	(234)	(4,357)	(4,592)	(0.02)	(0.02)
Quartic, $K = 4$	-29,338	50,873	29,338	21,535	0.90	1.13
	(6,257)	(1,345)	(6,257)	(6,503)	(0.03)	(0.03)
Quintic, $K = 5$	-29,240	50,559	29,240	21,318	0.90	1.13
	(5,912)	(1,184)	(5,912)	(6,141)	(0.03)	(0.03)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R$(63,70] and R$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated using Equation (16) with various polynomial degrees $K \in \{2, 3, 4, 5\}$. Columns (3) and (4) show the estimated number of bunching and jumping households for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations (13) and (14). Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed via the delta method from the clustered standard errors estimated in Equation (16).

5.2 Discussion

Columns (5) and (6) of Table 3 show that our lower bound on the MVPF of the June 2014 BF reform is approximately 0.90 whereas our upper bound on the MVPF is approximately 1.12. Notably, these bounds are somewhat tightly centered around 1. This is because the number of mechanical households (households locating below R$70 both with and without the reform) is much larger than the number of bunching or jumping households. In particular, for June 2016, we estimate there to be around 1.88 million mechanical households compared to 22,000 jumping households and 27,000 bunching households.

What can we say about the welfare implications of an MVPF bounded between 0.90 and 1.12? We can be sure that the reform was welfare improving as long as the government values giving R$0.90 to BF households in a non-distortionary manner more than spending R$1 on their next best alternative. On the other hand, the reform was welfare decreasing if the government values spending R$1 on their next best alternative more than giving R$1.12 to BF households in a non-distortionary manner.[37]

But what is the government's best alternative use of funds? In general, this is a difficult question to

37. Technically, the reform is welfare improving as long as the government values spending R$1 on their next best alternative less than splitting R$0.90 (in a non-distortionary manner) among the mechanical, threshold, bunching, and jumping households, where the split is determined by the lower bounds on each group's WTP for the reform. Similarly, the reform is welfare decreasing if the government values spending R$1 on their next best alternative more than splitting R$1.12 (in a non-distortionary manner) among the mechanical, threshold, bunching, and jumping households, where the split is determined by the upper bounds on each group's WTP for the reform.

answer. Lindert et al. (2007) show that BF performs better (in terms of targeting performance) than other anti-poverty programs in both Brazil and other countries in Latin America; this suggests that there is not a better alternative anti-poverty program that the government could finance. Instead, for expositional simplicity we assume that the next best use of funds is a non-distortionary UBI. Under this assumption, we can perform a conservative back-of-the-envelope calculation to argue that the 2014 BF reform was almost certainly welfare improving. Suppose the government is utilitarian and that the true income distribution is log-normal with a mean per-capita income of R$615/month and a standard deviation of R$844/month (this is taken from the World Bank's PovcalNet database for Brazil in 2016). Conservatively, we assume that the BF household per-capita income distribution matches the distribution for the bottom 50% of income earners in Brazil.[38] Moreover, we assume utility of consumption is CRRA: $u(c) = c^{1-\gamma}/(1-\gamma)$. Based on these assumptions, the government prefers giving R$0.90 (in a non-distortionary manner) to BF households more than spending R$1 on a non-distortionary UBI as long as $\gamma > 0.14$. While estimates for γ vary widely, the vast majority of estimates are bigger than 0.14 (Chetty (2006), Outreville (2014), Bergstrom and Dodds (2023)). Alternatively, if $\gamma = 1$ (i.e., $u(c) = \log(c)$), giving R$0.90 (in a non-distortionary manner) to BF households yields equivalent welfare gains to spending R$1.50 on a non-distortionary UBI. Thus, if the next best alternative policy is a non-distortionary UBI and the government is utilitarian, the BF reform was almost certainly welfare improving. This finding contributes to the debate around targeted vs. universal transfers in developing settings (Hanna and Olken, 2018): we find that even in a setting with a highly pronounced notch and substantial scope for misreporting, the efficiency cost generated by behavioral responses is simply not large enough to outweigh the equity gain associated with increased benefit generosity targeted to poor households (relative to a universal transfer).

Finally, when calculating our bounds for the MVPF, we have abstracted from a couple of complexities in the policy environment. First, we have ignored the impact that behavioral responses to the BF reform had on other components of the government's budget. There is evidence to suggest that some beneficiaries partially substitute formal sector employment for informal sector employment to become eligible for the BF benefit (as informal income is less easily verified making misreporting less costly; see De Brauw et al. (2015)). While it is easier to evade taxes in the informal sector, it is unlikely that this behavioral response will affect income tax revenues as BF beneficiaries are almost certainly sufficiently poor so as to be exempt from income taxation.[39] However, such behavioral responses may lead to reductions in payroll tax revenues (as argued by Bergolo and Cruces (2021) in the context of a cash transfer program in Uruguay). A reduction in payroll tax revenue would increase the total cost of the reform leading to a reduction in both our lower and upper bounds for the MVPF (see footnote 18). However, there is an offsetting effect: Gerard, Naritomi and Silva (2021) recently showed that a reform to BF in 2009, which led to an increase in transfers, also led to a rise in formal sector employment of

38. Bastagli (2008) finds that of all transfers paid in 2004, 91% went to households in the bottom 50% of the income distribution. Lindert et al. (2007) suggests even better targeting performance, finding that 94% of all benefits go to the bottom 40%. Notably, transfers are not evenly distributed across the bottom 50% of households: the poorest households within this group receive more (e.g., Lindert et al. (2007) find that the bottom 20% of the income distribution receive 73% of BF transfers). Thus, we believe the assumption that the income distribution of BF households is the same as the income distribution for the bottom 50% of income earners in Brazil is highly conservative.

39. For example, in 2015, individuals earning less than R$1903.98 per-month were exempt from income taxation.

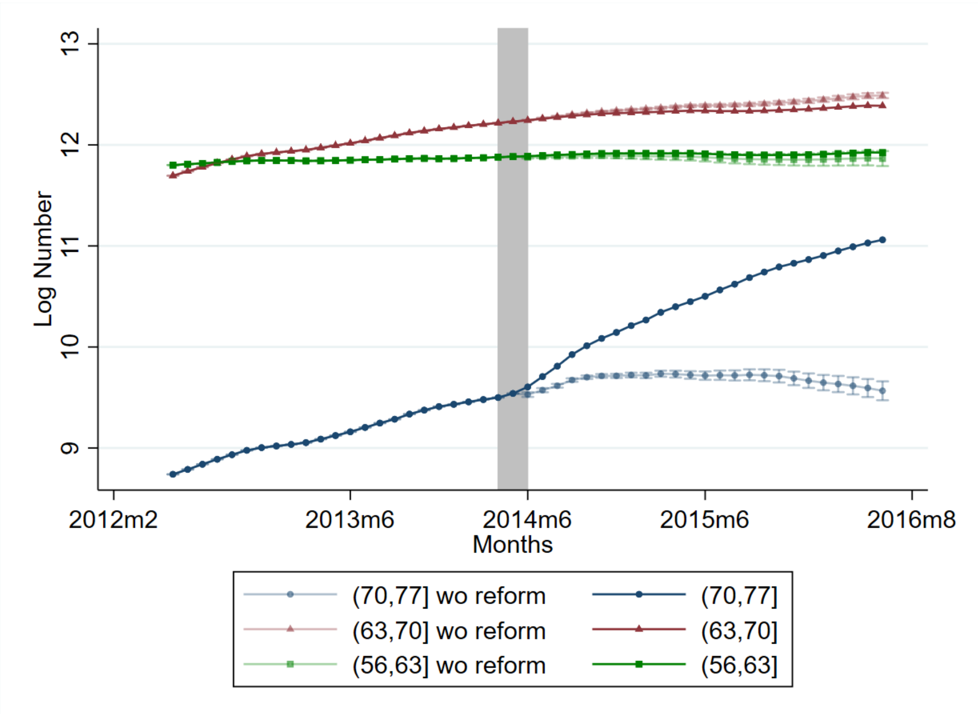
non-beneficiaries. They show that this offsetting effect dominates, generating a net positive impact on overall formal sector employment and tax revenue. Thus, we conjecture that the June 2014 BF reform may have had a small, positive impact on formal sector employment, generating a modest positive fiscal externality.[40] A positive fiscal externality would raise our estimates for both the lower and upper bound of the MVPF, and would, thus, only reinforce our finding that the 2014 reform was welfare improving relative to a non-distortionary UBI.

Second, as mentioned in Section 3, not all eligible households receive the BF benefit (due to municipality-level quotas on the number of beneficiaries). We discussed an extension in Section 2.3 that showed that our MVPF bounds are robust to eligible households only receiving the benefit with some probability. However, this extension assumed that the probability of receiving the benefit does not vary across households (conditional on reporting an income below the threshold); this allows us to cancel out the probability in both the numerator and denominator of the MVPF. Fortunately, this appears to be the case for the BF reform: using data from the Cadastro Único registries on which eligible households are BF beneficiaries, we see that in June 2016, 77.6% of those reporting at or below R$77 are beneficiaries with this percentage mostly constant across all eligible bins, suggesting that the probability of receiving the benefit is roughly constant across groups of households (see Figure 29 in Appendix C.12). This percentage is very similar to Gerard, Naritomi and Silva (2021) who find that in 2010, 79% of households reporting below the extreme poverty threshold are BF beneficiaries.

5.3 Robustness to Identification Assumption 1

We now explore robustness of our main results. First, Identification Assumption 1 may not hold if optimization frictions are large. For example, consider households who, in response to the reform, jump below the threshold by changing their labor supply but face large labor market frictions. These households may not be able to perfectly jump and bunch at the new notch, but may instead jump to an income level below R$63. Hence, large labor market frictions may imply that the distribution below R$63 is impacted by the reform. To test for this possibility (or equivalently, to allow for larger frictions), we relax Identification Assumption 1 by instead assuming that the number of people reporting in bins ≤R$56 are unaffected by the reform, allowing for the possibility that the number reporting in bin R$$(56, 63]$ is affected by the reform. We augment Equation (16) by setting $treat_x = 1$ if $x \in \{63, 70, 77\}$ and 0 otherwise, i.e., we include R$$(56, 63]$ as a treatment bin. Figure 6 plots the actual and counterfactual paths for $\log\left(N_{(56,63],t}\right), \log\left(N_{(63,70],t}\right)$, and $\log\left(N_{(70,77],t}\right)$. It does not appear that the reform had a significant impact on the number reporting in R$$(56, 63]$ and the impact of the reform on our two original treatment bins is robust to including R$(56,63] as a treatment bin. Table 9 in Appendix C.13 presents results for this alternative specification.

40. As discussed in Appendix B.3, most federal social programs in Brazil have substantially higher income thresholds than BF. Hence, the June 2014 BF reform would not have induced fiscal externalities on other social programs as long as the jumping households who changed their reported income in response to the reform did not have inordinately large jumps (i.e., in absence of the reform, jumping households would have reported below the thresholds of these other programs).



Note: This figure shows the log number of single individual households reporting incomes in R\$$(56, 63]$, R\$$(63, 70]$, and R\$$(70, 77]$ from June 2012 to June 2016 along with the counterfactual paths had the reform not happened. The counterfactual paths are estimated using Equation (16) (where we set $treat_x = 1$ if $x \in \{63, 70, 77\}$, i.e., we have three treatment bins: R\$$(56, 63]$, R\$$(63, 70]$, and R\$$(70, 77]$). We assume the difference in the (log) number reporting in any two bins follows a cubic time trend in absence of the reform (i.e., we set $K = 3$ in Equation (16)). Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 6: Counterfactual Paths for Three Treatment Bins Using Equation (16)

5.4 Robustness to Identification Assumption 2

Identification Assumption 2 essentially boils down to assuming that the differences over time between the (log) number of people reporting in each income bin are well approximated by polynomials which would have persisted in absence of the reform. Identification Assumption 2 therefore implies that if, for example, bin R\$(35,42] experiences a 1% deviation from its bin-specific polynomial trend, bin R\$(63,70] would also experience a 1% deviation from its bin-specific polynomial trend in absence of the reform. However, one may believe that, for example, a structural change in the economy which leads to a 1% deviation from the bin-specific polynomial time trend for bin R\$(35,42] would lead to a 2% deviation for bin R\$(63,70]. As such, we relax Identification Assumption 2 to instead assume that the log number reporting in each bin evolves according to:

$$\log\left(N_{(x-7,x],t}\right) = \gamma_x h(t) + \sum_{k=0}^{K} \alpha_{k,x} t^k + \nu_{x,t} \quad \text{for } x \in \{7, 14, ..., 77\}$$

Thus, we now assume that deviations from underlying bin-specific polynomials (in logs or, equivalently, percentage terms) are not necessarily the same across all bins. This relaxation of Identification Assumption 2 allows for a structural change in the economy which creates a 1% deviation from bin-specific polynomial trend for R$(35,42] to create a $(\gamma_{70}/\gamma_{42})$% deviation from bin-specific polynomial trend for bin R$(63,70]. Under this relaxed identification assumption (which nests Identification Assumption 2), we estimate the following non-linear least squares regression:

$$\log\left(N_{(x-7,x],t}\right) = \gamma_x \delta_t + \sum_{k=0}^{K} \alpha_{k,x} t^k + [\beta_{1,x} post_t + \beta_{2,x} post_t \times t] \times treat_x + \epsilon_{xt} \text{ for } x \in \{7, ..., 77\} \quad (17)$$

Essentially, running Regression (17) will estimate a common set of month dummies, δ_t, as well as bin-specific factors, γ_x, which multiply these common month dummies for each bin x. We show in Appendix C.14 that our results are very robust to this relaxation of Identification Assumption 2.

5.5 Robustness to Household Composition

Our main analysis is restricted to single individual households due to the fact that the schedule for households with more than one individual features a notch at R$70 and a kink below R$70 (e.g., this kink is at R$35 for two adult households with no children; see Section 3). Consequently, the 2014 BF reform led to both a change in the notch *and* a change in the kink for households with more than one individual. Thus, for these households, it is not necessarily reasonable to make Identification Assumption 1 as we might expect the reform to affect the number of people reporting in bins around the kink. However, many studies have shown that behavioral responses to kinks are typically small; kinks generally induce substantially less bunching than do notches (Kleven, 2016). Hence, we estimate MVPF bounds for two adult households without children by simply ignoring the presence of the kink and using all income bins below R$63 as control groups just as in our main analysis, noting that our identification assumptions are imperfect for these households. For two adult households without children we see the same pattern of behavioral responses to the reform and estimate lower bounds for the MVPF between 0.88 and 0.96 and upper bounds for the MVPF between 1.08 and 1.12; see Appendix C.4.[41]

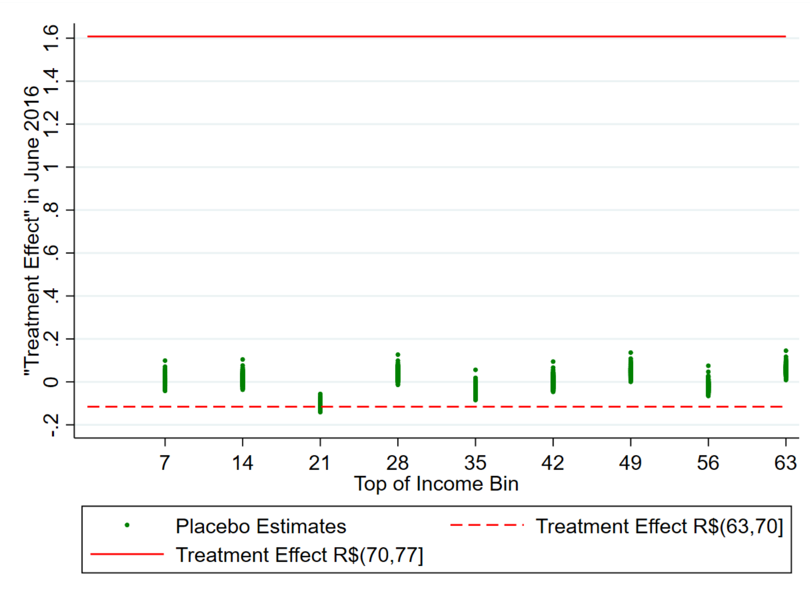
5.6 Placebo Tests

We can *partially* test the validity of our two identification assumptions via placebo tests. Together, Identification Assumptions 1 and 2 imply that all of the income bins below R$63 should evolve approximately according to a common time trend plus a bin-specific polynomial throughout the entire analysis period. To test this, we re-estimate Equation (16) but only include bins below R$63 (i.e., $x \in \{7, 14, ..., 63\}$) and randomly assign some of these bins to be "treatment" bins (i.e., $treat_x = 1$). If both of our identification assumptions are valid, the "treatment effects" from these placebo regressions

41. In Appendix C.15, we show strong suggestive evidence of a behavioral response to the change in the basic benefit notch for households with children, but we do not attempt to bound the MVPF for these households. This is because households with children may also receive the "variable benefit". Both the level and location of the notch associated with the variable benefit also changed with the June 2014 reform. Thus, the WTP of households with children needs to account for changes in the variable benefit schedule in addition to changes in the basic benefit schedule. This exercise is beyond the scope of the current paper.

should be close to zero, i.e., the post-reform deviation of the "treated" bins relative to the post-reform deviation of the "control" bins should be close to 0. There are a total of $\sum_{i=1}^{8}\binom{9}{i}= 510$ different ways to assign our nine bins below R$63 "treatment" status.

Figure 7 plots the results from all 510 such regressions, showing the estimated "treatment effects" in June 2016: $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}\bar{t}$ for $x \in \{7, .., 63\}$, where $\bar{t}$ represents June 2016. Each bin gets assigned $treat_x = 1$ 255 times meaning we have 255 "treatment effect" estimates for each bin. Figure 7 shows a fairly tight clustering of these treatment effects around zero: the mean "treatment effect" (in absolute value) across all bins is around 4% (0.04 log points). Hence, the "treatment effects" are relatively small for bins below R$63.[42] This suggests that Identification Assumptions 1 and 2 are reasonable.



Note: This figure shows the placebo "treatment effects" (in logs) for each income bin R$$(x-7, x]$ obtained from estimating Regression (16) with only bins $x \in \{7, 14, ..., 63\}$ and every possible assignment of $treat_x \in \{0, 1\}$ for each $x \in \{7, 14, ..., 63\}$. There are $510 = \sum_{i=1}^{8}\binom{9}{i}$ such regressions and we plot the values of $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}\bar{t}$ for each bin R$$(x-7, x]$ whenever bin R$$(x-7, x]$ is assigned to be a treatment bin ($\bar{t}$ represents June, 2016). We use bin-specific cubic polynomials in each regression, i.e., $K = 3$. The solid red line shows the treatment effect for R$(70,77] and the dashed red line shows the treatment effect for R$(63,70] when $K = 3$.

FIGURE 7: PLACEBO ANALYSIS: CALCULATING "TREATMENT EFFECTS" FOR EACH CONTROL BIN

Finally, we can also use these placebo tests as an alternative gauge of the statistical significance of our estimated treatment effects for income bins R$(70,77] and R$(63,70] from Regression (16). Under Identification Assumptions 1 and 2, the post-reform deviations from bin-specific trend plotted in Figure 7 are due to randomness. Hence, the distribution of "treatment effects" in Figure 7 can be used to construct p-values for the estimated treatment effects for income bins R$(70,77] and R$(63,70] from Regression (16) in a manner akin to randomization inference.[43] In Figure 7, the solid red line

42. We find a negative "average treatment effect" of -0.105 log points for R$$(14, 21]$. However, R$$(14, 21]$ seems implausibly far away from the notch to be impacted by the reform, especially given that other bins around R$(14,21] do not appear to be impacted by the reform. Hence, we interpret this negative "average treatment effect" simply as random variation.

43. See Imbens and Rubin (2021) or Young (2018) for discussions of randomization inference.

shows the treatment effect of about 1.6 log points for R$(70,77] while the dashed red line shows the treatment effect of about -0.12 log points for R$(63,70]. The treatment effect for R$(70,77] is an order of magnitude larger than any of the "treatment effects" from the placebo regressions, giving us high certainty that the number of individuals reporting incomes in R$(70,77] was impacted by the reform. Similarly, the treatment effect for R$(63,70] is larger (in absolute magnitude) than 97.8% of the placebo "treatment effects", implying a p-value of 0.022 against the null hypothesis that R$(63,70] was not impacted by the reform.

6 Conclusion

This paper develops a sufficient statistics framework to bound the welfare impacts of reforms featuring notches. We then estimate these sufficient statistics using longitudinal administrative data in the context of the 2014 reform to one of the world's largest cash transfer programs, Bolsa Família. Despite finding strong evidence of behavioral responses to the BF reform, we find that the corresponding efficiency costs are small relative to the equity benefits of increased transfers to poor households: a back-of-the-envelope calculation suggests that the welfare effect of spending R$1 on the reform is at least as high as the welfare effect of spending R$1.50 on a non-distortionary universal transfer. Because the Bolsa Família eligibility threshold is based on self-reported income, this result provides some evidence against the commonly held belief that targeting transfer programs based on self-reported incomes will generate substantial efficiency costs in high informality settings. Thus, our findings have important implications for the future design of cash transfer programs, especially if developing countries are to increasingly use self-reported income to determine eligibility for social programs.

Moreover, we believe that our sufficient statistics framework highlights a new manner in which reduced-form evidence on jumping and bunching can be used to inform policy. Given the ubiquity of notches, we hope that the methods developed in this paper will be useful for analyzing reforms in a variety other contexts such as Medicaid, income-dependent tax credits, or firm tax schedules. Finally, the bounding techniques developed in this paper may be helpful for bounding welfare impacts of large reforms which do not necessarily feature notches.

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A For Online Publication: Theory Appendix

A.1 Optimal Reported Incomes in Baseline Model

Type $\mu = 2$ households with $y \leq \tau$ get the following utility dependent on their choice of reported income $\hat{y}$:

$$\begin{cases} y + b - v(y - \hat{y}) & \text{if } \hat{y} < y \\ y + b & \text{if } \hat{y} \in [y, \tau] \\ y & \text{if } \hat{y} > \tau \end{cases}$$

Given that $v(y - \hat{y}) = 0$ for $\hat{y} \geq y$ and $v'(\cdot) > 0$, we know that $v(y - \hat{y}) \geq 0$. Thus, households with $y \leq \tau$ prefer to report $\hat{y} \in [y, \tau]$ (and are indifferent between reporting income levels in this range as only under-reporting is costly). We assume they break this indifference by reporting at y. Anyone with $y > \tau$ gets the following utility dependent on their choice of reported income $\hat{y}$:

$$\begin{cases} y + b - v(y - \hat{y}) & \text{if } \hat{y} \leq \tau \\ y - v(y - \hat{y}) & \text{if } \hat{y} \in (\tau, y) \\ y & \text{if } \hat{y} \geq y \end{cases}$$

Clearly, reporting $\hat{y} \in (\tau, y)$ is always dominated by reporting $\hat{y} \geq y$ as $v(y - \hat{y}) \geq 0$. Moreover, reporting $\hat{y} = \tau$ always dominates reporting $\hat{y} < \tau$ because $v' > 0$. By definition, those with $y = y^c(\mathbf{p})$ are indifferent between misreporting at the threshold τ and truthfully reporting (we break their indifference by assuming they will misreport at τ), where $y^c(\mathbf{p})$ solves $y^c(\mathbf{p}) + b - v(y^c(\mathbf{p}) - \tau) = y^c(\mathbf{p})$. The existence of a unique $y^c(\mathbf{p})$ follows from $v' > 0$. Those with $\tau < y \leq y^c(\mathbf{p})$ prefer misreporting with $\hat{y} = \tau$ over truthfully reporting and those with $y > y^c(\mathbf{p})$ prefer truthfully reporting $\hat{y} = y$ by the fact that $v' > 0$.

A.2 Proof of Lemma 1

Let us begin by calculating the WTP for the mechanical households, which are the households for which $\hat{y}^*(y, \mu; \mathbf{p}) \leq \tau$ and $\hat{y}^*(y, \mu; \mathbf{p}') \leq \tau$. For type $\mu = 1$ households, these are just households for whom $y \leq \tau$. For type $\mu = 2$ households, we appeal to the optimal choice function in Equation (3), finding that $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau \iff y \leq \tau$ and $y \leq \tau \implies \hat{y}^*(y, \mu = 2; \mathbf{p}) \leq \tau$. Thus, the number of mechanical households is just the set which report an income below τ under policy $\mathbf{p}'$: $M = G(\tau, \mathbf{p}')$. Moreover, $y \leq \tau$ implies $\hat{y}^*(y, \mu; \mathbf{p}) = y$ and $\hat{y}^*(y, \mu; \mathbf{p}') = y$. Hence, WTP for mechanical households is defined by:

$$y + b = y + b' - WTP \implies WTP = b' - b = \Delta b$$

Next, let us discuss the WTP for bunching households. Bunching households are those with $\hat{y}^*(y, \mu; \mathbf{p}) = \tau$ and $\hat{y}^*(y, \mu; \mathbf{p}') > \tau$. From Equation (3), one can immediately see that only households with $y \in (\tau, y^c(\mathbf{p})]$ and $\mu = 2$ satisfy these criterion. Equation (3) also implies that $\hat{y}^*(y, \mu = 2; \mathbf{p}) \leq \tau \iff y \leq y^c(\mathbf{p})$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau \iff y \leq \tau$ so that the number of bunching individuals is given by $B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$. In words, only bunching and mechanical households report incomes $\leq \tau$ under policy $\mathbf{p}$; under policy $\mathbf{p}'$ bunching households strictly increase their reported income while mechanical households do not. Based on their optimal choices from Equation (3), bunching individuals have utility $y + b - v(y - \tau)$ under policy $\mathbf{p}$. Also based on Equation (3), bunching households with $y \in (\tau, \tau']$ will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$, getting utility $y + b'$ under policy $\mathbf{p}'$. Those with $y \in (\tau', y^c(\mathbf{p})]$ will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = \tau'$, getting utility $y + b' - v(y - \tau')$ under policy $\mathbf{p}'$.[44] Hence, we know that the WTP for bunching households satisfies:

$$y + b - v(y - \tau) = y + b' - WTP - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$$

where $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$ if $y \in (\tau, \tau']$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') = \tau'$ if $y \in (\tau', y^c(\mathbf{p})]$. Equivalently:

$$WTP = b' - b + v(y - \tau) - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$$

Given that $v(y - \tau) - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) > 0$ (as $v' > 0$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') > \tau$ for bunching households), we know $WTP \geq b' - b = \Delta b$. Moreover, because $y + b - v(y - \tau) \geq y$ for bunching households (or, equivalently, $v(y - \tau) \leq b$), we know that $WTP \leq b' - b + b - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) = b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$. And because $v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq 0$, we therefore know that $WTP \leq b'$ for bunching households.

Next, we turn to threshold households: those with $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$. There are $T = G(\tau'; \mathbf{p}) - G(\tau; \mathbf{p})$ such households. Equation (3) implies that anyone with $\mu = 2$ and $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$ must have $y = \hat{y}^*(y, \mu; \mathbf{p})$ and clearly anyone with $\mu = 1$ with $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$ must also have $y = \hat{y}^*(y, \mu; \mathbf{p})$. Equation (3) also then implies that all threshold households report $\hat{y}^*(y, \mu; \mathbf{p}') = y \in (\tau, \tau']$ as well. All of these households therefore get utility y under policy $\mathbf{p}$ and receive utility $y + b'$ under policy $\mathbf{p}'$. Hence, their WTP is given by:

$$y = y + b' - WTP \implies WTP = b'$$

Finally, there are jumping households for whom $\hat{y}^*(y, \mu; \mathbf{p}) > \tau'$ and $\hat{y}^*(y, \mu; \mathbf{p}') \leq \tau'$. Clearly, these must be type $\mu = 2$ households. By Equation (3), $\hat{y}^*(y, \mu; \mathbf{p}) \leq \tau' \implies \hat{y}^*(y, \mu; \mathbf{p}') \leq \tau'$ so that the number of jumping households is given by the increase in households reporting at or below τ' as a result of the reform: $J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$. Next, note that by Equation (3), if someone has $\hat{y}^*(y, \mu; \mathbf{p}) > \tau'$ then they must report $\hat{y}^*(y, \mu; \mathbf{p}) = y$ yielding utility y under policy $\mathbf{p}$. Under policy $\mathbf{p}'$ these households have $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau'$ yielding utility $y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$. Thus, WTP for these individuals is given by:

$$y = y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) - WTP$$

Because $y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq y$ by revealed preference, $WTP \geq 0$. And because $v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq 0$, $WTP \leq b'$ for jumping individuals.

44. Note, if $y^c(\mathbf{p}) \leq \tau'$, i.e., the change in τ is large, all bunching households will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$.

Putting this all together we get:

$$\Delta b(M+B) + b'T \leq \text{Total WTP} \leq \Delta bM + b'(B+T+J)$$

A.3 Proof of Proposition 3

Proof. We start with proving the lower bound for $\frac{\frac{1}{\lambda}[\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p})]}{[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})]}$. First, welfare under policy $\mathbf{p}$ is given by:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta) u\left(y\left[x^*(\theta,\mathbf{p}),\theta\right] + b\mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right) dF(\theta) - \lambda bG(\tau;\mathbf{p}) \tag{18}$$

Next, note that by revealed preference, we have the following for any $x \in X$:

$$u\left(y\left[x^*\left(\theta,\mathbf{p}\right),\theta\right] + b\mathbb{1}\left[\hat{y}(x^*\left(\theta,\mathbf{p}\right),\theta) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right) \geq u\left(y\left[x,\theta\right] + b\mathbb{1}\left[\hat{y}(x,\theta) \leq \tau\right], x;\theta\right) \tag{19}$$

Put simply, optimal decisions conditional on any given θ under $\mathbf{p}$, $x^*(\theta,\mathbf{p})$, yield weakly higher utility than any other set of decisions x that one could make. Using Equations (18) and (19), we can bound welfare under policy $\mathbf{p}' = \{b',\tau'\}$ by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}
\mathcal{W}(\mathbf{p}') &= \int_{\Theta} \phi(\theta) u\left(y\left[x^*(\theta,\mathbf{p}'),\theta\right] + b'\mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p}');\theta\right) dF(\theta) - \lambda b'G(\tau';\mathbf{p}') \\
&\geq \int_{\Theta} \phi(\theta) u\left(y\left[x^*(\theta,\mathbf{p}),\theta\right] + b'\mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p});\theta\right) dF(\theta) - \lambda b'G(\tau';\mathbf{p}')
\end{aligned} \tag{20}$$

So as to slightly reduce some cumbersome notation, let us define:

$$y^*(\theta,\mathbf{p}) \equiv y\left[x^*\left(\theta,\mathbf{p}\right),\theta\right]$$

$$\hat{y}^*(\theta,\mathbf{p}) \equiv \hat{y}[x^*\left(\theta,\mathbf{p}\right),\theta]$$

Thus, for the reform from $\mathbf{p} = \{b,\tau\}$ to $\mathbf{p}' = \{b',\tau'\}$ with $\mathbf{p}'-\mathbf{p} = \{\Delta b,\Delta\tau\}$ and $\Delta\tau > 0$ we can combine Equations (18) and (20) as well as split up the domain Θ into sets of households who receive the benefit under policy $\mathbf{p}$ and those who do not receive the benefit under policy $\mathbf{p}$ to yield:

$$\begin{aligned}
\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq &\int_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p})+b', x^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p})+b, x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) \\
&+ \int_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p})+b', x^*\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}), x^*\left(\theta,\mathbf{p}\right);\theta\right]\right\} dF(\theta) \\
&- \lambda[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})]
\end{aligned} \tag{21}$$

Note, the change in utility for those with $\hat{y}^*(\theta, \mathbf{p}) > \tau'$ is zero as we move from $\mathbf{p}$ to $\mathbf{p}'$, holding decisions fixed, as they do not receive a transfer. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^* \leq \tau$ under policy $\mathbf{p}$:

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\frac{1}{b'-b}\left\{u\left[y^*(\theta,\mathbf{p})+b', x^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p})+b, x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta)}{G(\tau,\mathbf{p})} \tag{22}$$

Note that in Equation (22), we divide by $\frac{1}{b'-b}$, which simply renormalizes by the additional amount of money given to households with $\hat{y}^*(\theta, \mathbf{p}) \leq \tau$ due to the reform; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}$ as the average welfare gain from giving these households an extra \$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau, \tau']$ under policy $\mathbf{p}$:[45]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{b'}\left\{u\left[y^*(\theta,\mathbf{p})+b', x^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p}), x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta)}{G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})} \tag{23}$$

Again, note that in Equation (23) we have divided by $\frac{1}{b'}$, which simply renormalizes by the amount of money given to households with $\hat{y}^*(\theta, \mathbf{p}) \in (\tau, \tau']$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the average welfare gain from giving these households an extra \$1. We can rewrite Equation (21) using Equations (22) and (23) as follows:

$$\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq \eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta b G(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right] - \lambda\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right] \tag{24}$$

Next, let us define the aggregate welfare weight, η_L, which equals the weighted average welfare weight of giving a dollar to all households, where the weights are determined by the lower bound of WTP for the reform:

$$\eta_L = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta b G(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{\Delta b G(\tau;\mathbf{p}) + b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]} \tag{25}$$

Then, dividing Equation (24) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\left[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})\right]}{\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} &\geq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta b G(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\lambda\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} - 1 \\
&= \frac{\eta_L}{\lambda}\frac{\Delta b G(\tau;\mathbf{p}) + b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} - 1 \\
&= \omega_L MVPF_L - 1
\end{aligned} \tag{26}$$

where $\omega_L = \eta_L/\lambda$ and $MVPF_L$ is given by:

$$MVPF_L = \frac{\Delta b G(\tau;\mathbf{p}) + b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} = 1 - \frac{b'\left[G(\tau';\mathbf{p}') - G(\tau';\mathbf{p})\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \tag{27}$$

45. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})$.

Next, we prove the upper bound for $\frac{\frac{1}{\lambda}[\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p})]}{[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})]}$. We use identical revealed preference logic to bound welfare under policy $\mathbf{p} = \{b, \tau\}$ by evaluating utility under policy $\mathbf{p}$, but holding household decisions constant at their values under policy $\mathbf{p}'$:

$$\begin{aligned}\mathcal{W}(\mathbf{p}) &= \int_{\Theta} \phi(\theta) u\left(y^*(\theta,\mathbf{p}) + b\mathbb{1}\left[\hat{y}^*(\theta,\mathbf{p}) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right) dF(\theta) - \lambda b G(\tau;\mathbf{p}) \\ &\geq \int_{\Theta} \phi(\theta) u\left(y^*(\theta,\mathbf{p}') + b\mathbb{1}\left[\hat{y}^*(\theta,\mathbf{p}') \leq \tau\right], x^*(\theta,\mathbf{p}');\theta\right) dF(\theta) - \lambda b G(\tau;\mathbf{p})\end{aligned} \tag{28}$$

Hence, for the reform from $\mathbf{p} = \{b, \tau\}$ to $\mathbf{p}' = \{b', \tau'\}$ with $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta\tau\}$ and $\Delta\tau > 0$:

$$\begin{aligned}\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \leq &\int_{\theta:\hat{y}^*(\theta,\mathbf{p}')\leq\tau} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p}') + b', x^*(\theta,\mathbf{p}');\theta\right] - u\left[y^*(\theta,\mathbf{p}') + b, x^*(\theta,\mathbf{p}');\theta\right]\right\} dF(\theta) \\ &+ \int_{\theta:\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p}') + b', x^*\left(\theta,\mathbf{p}'\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}'), x^*\left(\theta,\mathbf{p}'\right);\theta\right]\right\} dF(\theta) \\ &- \lambda[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})]\end{aligned} \tag{29}$$

Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^* \leq \tau$ under policy $\mathbf{p}'$:

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p}')\leq\tau} \phi(\theta)\frac{1}{b'-b}\left\{u\left[y^*(\theta,\mathbf{p}') + b', x^*(\theta,\mathbf{p}');\theta\right] - u\left[y^*(\theta,\mathbf{p}') + b, x^*(\theta,\mathbf{p}');\theta\right]\right\} dF(\theta)}{G(\tau,\mathbf{p}')} \tag{30}$$

Note that in Equation (30), we divide by $\frac{1}{b'-b}$, which simply renormalizes by the amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}') \leq \tau$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}$ as capturing the average welfare gain from giving these households an extra \$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau, \tau']$ under policy $\mathbf{p}'$:[46]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{b'}\left\{u\left[y^*(\theta,\mathbf{p}') + b', x^*(\theta,\mathbf{p}');\theta\right] - u\left[y^*(\theta,\mathbf{p}'), x^*(\theta,\mathbf{p}');\theta\right]\right\} dF(\theta)}{G(\tau';\mathbf{p}') - G(\tau,\mathbf{p}')} \tag{31}$$

Again, note that in Equation (31) we have divided by $\frac{1}{b'}$, which simply renormalizes by the amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}') \in (\tau,\tau']$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}$ as capturing the average welfare gain from giving these households an extra \$1. We can rewrite Equation (29) using Equations (30) and (31) as follows:

$$\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \leq \eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta b G(\tau;\mathbf{p}') + \eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}} b'\left[G(\tau';\mathbf{p}') - G(\tau;\mathbf{p}')\right] - \lambda\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right] \tag{32}$$

Next, let us define the aggregate welfare weight, η_U, which equals the weighted average welfare

46. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}') - G(\tau,\mathbf{p}')$.

weight of giving a dollar to all households, where the weights are determined by the upper bound of WTP for the reform:

$$\eta_U = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta bG(\tau;\mathbf{p}') + \eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}b'[G(\tau';\mathbf{p}') - G(\tau,\mathbf{p}')]}{\Delta bG(\tau;\mathbf{p}') + b'[G(\tau';\mathbf{p}') - G(\tau,\mathbf{p}')]}$$

Then, dividing Equation (32) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\left[\mathcal{W}\left(\mathbf{p}'\right) - \mathcal{W}(\mathbf{p})\right]}{\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} &\leq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta bG(\tau;\mathbf{p}') + \eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}b'\left[G(\tau';\mathbf{p}') - G(\tau;\mathbf{p}')\right]}{\lambda\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} - 1 \\
&= \frac{\eta_U}{\lambda}\frac{\Delta bG(\tau;\mathbf{p}') + b'\left[G(\tau';\mathbf{p}') - G(\tau;\mathbf{p}')\right]}{\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} - 1 \\
&= \omega_U MVPF_U - 1
\end{aligned} \tag{33}$$

where $\omega_U = \eta_U/\lambda$ and $MVPF_U$ is given by:

$$MVPF_U = \frac{\Delta bG(\tau;\mathbf{p}') + b'[G(\tau';\mathbf{p}') - G(\tau,\mathbf{p}')]}{b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)} = 1 + \frac{b\left[G(\tau;\mathbf{p}) - G(\tau;\mathbf{p}')\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \tag{34}$$

□

A.4 Welfare Impacts of Infinitesimal Reforms to Notches

In this Appendix, we show how to construct bounds for welfare impacts of infinitesimal reforms in the context of the model from Section 2.1. First, let us start by explicitly writing out the expression for total welfare under policy $\mathbf{p}$ in Equation (4) given the optimal decision rule specified in Equation (3):

$$\begin{aligned}
\mathcal{W}(\mathbf{p}) = &\int_0^\tau \phi(y,1)[y+b]dF(y|\mu=1)\pi(1) + \int_\tau^\infty \phi(y,1)ydF(y|\mu=1)\pi(1) \\
&+ \int_0^\tau \phi(y,2)[y+b]dF(y|\mu=2)\pi(2) + \int_\tau^{y^c(\mathbf{p})} \phi(y,2)[y+b-v(y-\tau)]dF(y|\mu=2)\pi(2) \\
&+ \int_{y^c(\mathbf{p})}^\infty \phi(y,2)ydF(y|\mu=2)\pi(2) - \lambda bG(\tau;\mathbf{p})
\end{aligned} \tag{35}$$

We are interested in understanding the object:

$$\frac{d\mathcal{W}(\mathbf{p})}{d\mathbf{p}} \cdot d\mathbf{p} = \frac{\partial\mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial\mathcal{W}(\mathbf{p})}{\partial\tau}d\tau$$

for some small $db, d\tau$ (this is just a first order Taylor series approximation of welfare around $\mathbf{p} = [b,\tau]$).

Differentiating Equation (35), we get:

$$\begin{aligned}
&\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\
&= db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu) dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\
&+ db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) + d\tau \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)v'(y-\tau)dF(y|\mu=2)\pi(2) \\
&- \lambda \frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau
\end{aligned} \tag{36}$$

Note, in computing Equation (36), we have used Leibniz integral rule multiple times (along with the facts that $y^c(\mathbf{p}) + b - v(y^c(\mathbf{p}) - \tau) = y^c(\mathbf{p})$ and $v(0) = 0$). For instance:

$$\begin{aligned}
&\frac{\partial}{\partial \tau}\left[\int_0^{\tau} \phi(y,1)[y+b]dF(y|\mu=1)\pi(1) + \int_{\tau}^{\infty} \phi(y,1)ydF(y|\mu=1)\pi(1)\right] \\
&= [y+b]\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau - y\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau = b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau
\end{aligned} \tag{37}$$

The Leibniz terms for the type $\mu = 2$ individuals all cancel out as well. For instance:

$$\phi(\tau,2)[\tau+b]f(\tau|\mu=2)\pi(2) = \phi(\tau,2)[\tau+b-v(\tau-\tau)]f(\tau|\mu=2)\pi(2) \tag{38}$$

and

$$\begin{aligned}
&\left[\phi(y^c(\mathbf{p}),2)[y^c(\mathbf{p})+b-v(y^c(\mathbf{p})-\tau)]f(y^c(\mathbf{p})|\mu=2)\pi(2)\right]\frac{\partial y^c(\mathbf{p})}{\partial \tau} \\
&= \left[\phi(y^c(\mathbf{p}),2)y^c(\mathbf{p})f(y^c(\mathbf{p})|\mu=2)\pi(2)\right]\frac{\partial y^c(\mathbf{p})}{\partial \tau}
\end{aligned} \tag{39}$$

Looking at Equation (36), it becomes clear that to recover the exact welfare impact, we need to understand $v'(\cdot)$, which is difficult because, to the best of our knowledge, $v'(\cdot)$ is not related to any observable behavioral effects. Nonetheless, we can bound the infinitesimal welfare impact from below by noting that $v'(\cdot) > 0$ so that:

$$\begin{aligned}
&\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\
&\geq db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu) dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\
&+ db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) - \lambda \frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau
\end{aligned} \tag{40}$$

Moreover, we can bound the infinitesimal welfare impact from above by noting that $v(y^c(\mathbf{p}) - \tau) = b$ (by the indifference condition for $y^c(\mathbf{p})$) so that $\int_{\tau}^{y^c(\mathbf{p})} v'(y-\tau)dy = b$ (by the fundamental theorem of calculus and the fact that $v(0) = 0$):

$$\int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)v'(y-\tau)dF(y|\mu=2)\pi(2) \leq \int_{\tau}^{y^c(\mathbf{p})} \phi(\bar{y},2)v'(y-\tau)f(\bar{y}|\mu=2)dy\pi(2) = b\phi(\bar{y},2)f(\bar{y}|\mu=2)\pi(2)$$

where $\bar{y}$ is the value of $y \in [\tau, y^c(\mathbf{p})]$ that maximizes $\phi(y,2)f(y|\mu=2)$. Conceptually, we're bounding the welfare impact of moving the threshold for the bunching households by b times $\phi(\bar{y},2)f(\bar{y}|\mu = 2)\pi(2)$, which is a sort of maximium density weighted welfare weight that we put on bunching individuals. Hence, we have that:

$$\begin{aligned}
&\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\
&\leq db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\
&+ db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) + d\tau\phi(\bar{y},2)bf(\bar{y}|\mu=2)\pi(2) \\
&- \lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau
\end{aligned} \tag{41}$$

Next, let us discuss the budgetary impacts of the reform. First, note that:

$$bG(\tau;\mathbf{p}) = b\left[\int_0^{\tau} dF(y|\mu=1)\pi(1) + \int_0^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)\right]$$

Again using Leibniz integral rule:

$$\begin{aligned}
\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau &= \left[\int_0^{\tau} dF(y|\mu=1)\pi(1) + \int_0^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)\right]db \\
+ b\left\{f(\tau|\mu=1)\pi(1)d\tau + \left[\frac{\partial y^c(\mathbf{p})}{\partial b}db + \frac{\partial y^c(\mathbf{p})}{\partial \tau}d\tau\right]f(y^c(\mathbf{p})|\mu=2)\pi(2)\right\}
\end{aligned} \tag{42}$$

Note, for an infinitesimal reform, jumping households have a WTP of 0 as the only households who jump are those that are indifferent between locating at the notch and reporting truthfully. However, jumping households still have a first order impact on social welfare through their effect on the government's budget as each jumping household costs the government b' dollars (despite the fact that the mass of jumpers is measure 0 for an infinitesimal reform; see Bergstrom and Dodds (2021*a*) for further discussion).

To simplify expressions, let us note that in the infinitesimal case, the number of mechanical households, M, equals $\sum_{\mu=1,2}\int_0^{\tau} dF(y|\mu)\pi(\mu)$, the number of threshold households, T, equals $f(\tau|\mu=1)\pi(1)d\tau$, the number of bunching households, B, is equal to $\int_{\tau}^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)$, and the number of jumping households, J, is equal to $\left[\frac{\partial y^c(\mathbf{p})}{\partial b}db + \frac{\partial y^c(\mathbf{p})}{\partial \tau}d\tau\right]f(y^c(\mathbf{p})|\mu=2)\pi(2)$. Thus:

$$\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau = dbM + bT + dbB + bJ \tag{43}$$

Assuming that money spent on the transfer goes up as a result of the reform (so that $\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau > 0$):

$$\begin{aligned}
&\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau}{\lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau} \\
&\geq \left\{db\sum_{\mu=1,2}\int_0^{\tau}\phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau\right. \\
&\left.+\, db\int_{\tau}^{y^c(\mathbf{p})}\phi(y,2)dF(y|\mu=2)\pi(2)\right\}\Big/\left\{dbM + bT + dbB + bJ\right\} - 1
\end{aligned} \tag{44}$$

Next, let us convert the above equation into an expression involving a lower bound for the MVPF by defining the incidence-weighted welfare weight:

$$\omega_L \equiv \frac{db\sum_{\mu=1,2}\int_0^{\tau}\phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau + db\int_{\tau}^{y^c(\mathbf{p})}\phi(y,2)dF(y|\mu=2)\pi(2)}{db\sum_{\mu=1,2}\int_0^{\tau}dF(y|\mu)\pi(\mu) + bf(\tau|\mu=1)\pi(1)d\tau + db\int_{\tau}^{y^c(\mathbf{p})}dF(y|\mu=2)\pi(2)} \tag{45}$$

Next, note that:

$$db\sum_{\mu=1,2}\int_0^{\tau}dF(y|\mu)\pi(\mu) + bf(\tau|\mu=1)\pi(1)d\tau + db\int_{\tau}^{y^c(\mathbf{p})}dF(y|\mu=2)\pi(2) = dbM + bT + dbB \tag{46}$$

Plugging Equations (43), (45), and (46) into Equation (44) we get:

$$\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau}{\lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau} \geq \omega_L\frac{dbM + bT + dbB}{dbM + bT + dbB + bJ} - 1 \equiv \omega_L MVPF_L - 1 \tag{47}$$

where $MVPF_L = \frac{dbM+bT+dbB}{dbM+bT+dbB+bJ}$. Note that the lower bound for the MVPF in the infinitesimal case is the same as in the non-infinitesimal case discussed in Section 2.1.

As far as the upper bound for the infinitesimal welfare change, let us define:

$$\omega_U \equiv \frac{db\sum_{\mu=1,2}\int_0^{\tau}\phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau + db\int_{\tau}^{y^c(\mathbf{p})}\phi(y,2)dF(y|\mu=2)\pi(2) + d\tau\phi(\bar{y},2)bf(\bar{y})\pi(2)}{db\sum_{\mu=1,2}\int_0^{\tau}dF(y|\mu)\pi(\mu) + bf(\tau|\mu=1)\pi(1)d\tau + db\int_{\tau}^{y^c(\mathbf{p})}dF(y|\mu=2)\pi(2) + d\tau bf(\bar{y}|\mu=2)\pi(2)} \tag{48}$$

Plugging in Equation (48) into Equation (41), substituting the above expressions for M, T, B, J and dividing by $\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau > 0$, we get:

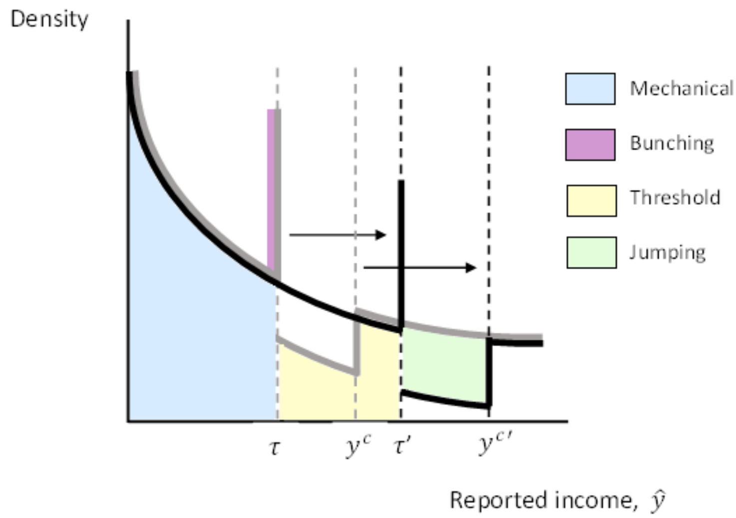
$$\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau}{\lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau} \leq \omega_U\frac{dbM + bT + dbB + d\tau bf(\bar{y}|\mu=2)\pi(2)}{dbM + bT + dbB + bJ} - 1 \equiv \omega_U MVPF_U - 1 \tag{49}$$

where $MVPF_U = \frac{dbM+bT+dbB+d\tau bf(\bar{y}|\mu=2)\pi(2)}{dbM+dbB+bT+bJ}$. There are two key differences for the upper bound

MVPF in the infinitesimal case relative to the bound given in Proposition 3. First, note that by the envelope theorem, the welfare impact for jumping individuals is zero, so they do not enter the numerator of the MVPF (whereas for large reforms, jumping individuals may have a positive WTP). As such, the upper bound for the MVPF may be below one in the infinitesimal case. Second, in the infinitesimal case, the upper bound for the MVPF contains the object $d\tau b f(\bar{y}|\mu = 2)\pi(2)$, which captures the welfare impact of increasing the threshold τ for bunching individuals (due to decreasing their costs of misreporting). Note that this object is different than the bound we use in the non-infinitesimal case: $b \times B$. It is worthwhile to mention that $b \times B$ *also* is a bound for the welfare impact of increasing the threshold for bunching individuals in the infinitesimal case, but is vacuous because $b \times B \not\to 0$ as $db, d\tau \to 0$, so that using $b \times B$ in the infinitesimal case merely allows us to say the welfare impact of an infinitesimal change (which is also infinitesimal for small $db, d\tau$) is smaller than a constant. Similarly, $d\tau b f(\bar{y}|\mu = 2)\pi(2)$ is also a bound for the welfare impact of increasing the threshold for bunching individuals in the non-infinitesimal case, but we do not use this bound for three reasons. First, in order to gauge $d\tau b f(\bar{y}|\mu = 2)\pi(2)$, we need an estimate of $f(\bar{y})$ which requires one to estimate the density of true (rather than reported) incomes along with the fraction of households who are willing to misreport in the first place, $\pi(2)$. Second, because $\bar{y}$ is the value of $y \in [\tau, y^c(\mathbf{p})]$ that maximizes $\phi(y, 2)f(y|\mu = 2)$, we also need to know welfare weights $\phi(y, \mu)$, meaning that $f(\bar{y}|\mu = 2)$ is not a purely positive object. Third, the economic interpretation of $d\tau b f(\bar{y}|\mu = 2)\pi(2)$ is far less clear than the economic interpretation of $b \times B$.

A.5 Figure 1 when $\tau' > y^c(\mathbf{p})$

Figure 8 redraws Figure 1 in the main text under the scenario where $\tau' > y^c(\mathbf{p})$.



Note: This figure shows a hypothetical density of reported incomes under the initial policy $\mathbf{p} = \{b, \tau\}$ (solid grey curve) and how this density changes as a result of a reform that increases the policy to $\mathbf{p}' = \{b', \tau'\}$ (solid black curve). Note, the vertical solid grey line at τ and the vertical solid black line at τ' represent the bunching households under the initial policy and new policy, respectively. This figure also depicts which households are classified as mechanical households, bunching households, threshold households, and jumping households.

FIGURE 8: A HYPOTHETICAL DENSITY OF REPORTED INCOMES UNDER $\mathbf{p}$ AND $\mathbf{p}'$

A.6 Incidence-Weighted Welfare Weights and MVPF Bounds

It may not be immediately clear from Proposition 1 how exactly the MVPF can be combined with incidence-weighted welfare weights to make welfare statements. To illustrate this point, we work through a simple example. Suppose in the setup from Section 2.1 that the government places a welfare weight of 1 on the "honest" individuals with $\mu = 1$ and places a welfare weight of 0 on the "dishonest" individuals with $\mu = 2$. That is to say that the government's welfare function is given by:

$$\mathcal{W}(\mathbf{p}) = \sum_{\mu \in \{1,2\}} \int_Y \mathbb{1}[\mu = 1] U^*(y, \mu; \mathbf{p}) dF(y|\mu)\pi(\mu) - \lambda b G(\tau; \mathbf{p}) \tag{50}$$

Let us now explain the incidence-weighted welfare weights from Proposition 2 in this case. Let us begin with ω_L, the lower bound incidence-weighted welfare weight. Let us denote the fraction of mechanical households that are type $\mu = 1$ as $P_{M,1}$. Similarly, let $P_{B,1}$, $P_{T,1}$, $P_{J,1}$ denote the fraction of bunching, threshold, and jumping households that are type $\mu = 1$ (note $P_{B,1} = 0$ and $P_{J,1} = 0$ because type $\mu = 1$ households do not change their reported income in response to a policy change). The incidence-weighted welfare weight is as follows (this can be derived from the more general formulas in Appendix A.3, see Equations (22), (23), and (25) or, alternatively, see Equation 1 and related discussion from Hendren and Sprung-Keyser (2020)):

$$\begin{aligned}\omega_L &= \frac{1}{\lambda} \frac{[\Delta b M P_{M,1} + \Delta b B P_{B,1} + b'TP_{T,1}] \times 1 + [\Delta b M(1 - P_{M,1}) + \Delta b B(1 - P_{B,1}) + b'T(1 - P_{T,1})] \times 0}{\Delta b(M + B) + b'T} \\ &= \frac{1}{\lambda} \frac{\Delta b M P_{M,1} + b'TP_{T,1}}{\Delta b(M + B) + b'T}\end{aligned} \tag{51}$$

Hence, the lower bound welfare weight ω_L captures the fact that $P_{M,1}$ of the total WTP for mechanical households goes to $\mu = 1$ households we care about whereas $(1 - P_{M,1})$ of this WTP is "wasted" on $\mu = 2$ households that the government does not care about. Likewise, for the threshold households, $(1 - P_{T,1})$ of the total WTP for these households is "wasted" on the type $\mu = 2$ households that the government does not care about. Finally, all of the bunching and jumping households are type $\mu = 2$, so all of their WTP is "wasted" in this example. When we use ω_L to translate the lower bound $MVPF_L$ into a welfare statement, we find that:

$$\omega_L \times MVPF_L = \frac{1}{\lambda} \frac{\Delta b M P_{M,1} + b' T P_{T,1}}{\Delta b(M+B) + b'T} \frac{\Delta b(M+B) + b'T}{\Delta b(M+B) + b'(T+J)} = \frac{1}{\lambda} \frac{\Delta b M P_{M,1} + b' T P_{T,1}}{\Delta b(M+B) + b'(T+J)}$$

In words, to determine the lower bound for the welfare effect of the reform (relative to the government cost), the government takes the lower bound $MVPF_L$ and scales it by the fraction of the total WTP that goes to type $\mu = 1$ households. The same logic holds for how we use the upper bound incidence-weighted welfare weight ω_U to translate the upper bound $MVPF_U$ into an upper bound for the total welfare impact of the reform.

A.7 Incorporating Misperceptions of the Schedule

We now assume that households do not necessarily understand how the policy impacts their consumption. Households solve the following problem:

$$\begin{aligned} &\max_{x \in X} \; u\left(c, x; \theta\right) \\ &\text{s.t. } \; c = f(y(x,\theta), \hat{y}(x,\theta), \mathbf{p}, \theta) \end{aligned} \tag{52}$$

In words, households make decisions under the assumption that their consumption is some function of their true income y, their reported income $\hat{y}$, the $\mathbf{p}$, and state variables θ. For instance, this framework allows for households to misperceive the threshold τ or the benefit level b (e.g., $f(y(x,\theta), \hat{y}(x,\theta), \mathbf{p}, \theta) = y(x,\theta) + (b + \theta_1)\mathbb{1}[\hat{y}(x,\theta) \le (\tau + \theta_2)]$, for some values of θ_1, θ_2). Total welfare is still given by:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta) u\left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + b\mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \le \tau\right], x^*(\theta,\mathbf{p}); \theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p}) \tag{53}$$

In order for Proposition 3 to hold under the more general model with misperceptions (Problem (52)), we need to make two additional assumptions. We need to assume that when the policy changes from $\mathbf{p}$ to $\mathbf{p}'$, household behavioral re-optimization improves welfare, *on average*. In other words, misperceptions of the policy reform cannot be so severe that households make themselves worse off (on average) by responding to the reform. Mathematically, we require that:

$$\begin{aligned}\mathcal{W}(\mathbf{p}') &= \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}'),\theta\right) + b' \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}') \\ &\geq \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + b' \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}')\end{aligned} \tag{54}$$

Note, that the previous inequality holds by revealed preference if households correctly perceive the schedule (i.e., behavioral re-optimization can only improve utility). If households misperceive the schedule, we simply need to assume that behavioral responses improve welfare on average. Correspondingly, our second assumption is that if, hypothetically, the policy were to change from $\mathbf{p}'$ to $\mathbf{p}$, household behavioral re-optimization would also improve welfare, *on average*. Mathematically, this amounts to assuming:

$$\begin{aligned}\mathcal{W}(\mathbf{p}) &= \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + b \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p}) \\ &\geq \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}'),\theta\right) + b \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p})\end{aligned} \tag{55}$$

If we are willing to make these two assumptions, the rest of the proof to Proposition 3 goes through, so that we can bound the welfare impact of changing notches if individuals misperceive the schedule. Hence, we can state:

Proposition 4. *Suppose households solve Problem (52), welfare is given by Equation (53) and $\tau' > \tau$. If we assume:*

$$\mathcal{W}(\mathbf{p}') \geq \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + b' \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}')$$

and

$$\mathcal{W}(\mathbf{p}) \geq \int\limits_{\Theta} \phi(\theta) u \left[y\left(x^*(\theta,\mathbf{p}'),\theta\right) + b \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p})$$

Then as long as $b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p}) > 0$ have:

$$\omega_L MVPF_L - 1 \leq \frac{\frac{1}{\lambda}\left[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \leq \omega_U MVPF_U - 1$$

where $MVPF_L$ is given by Equation (8), $MVPF_U$ is given by Equation (9), and ω_L (ω_U) captures the weighted average money-metric welfare gain from giving a dollar to mechanical, bunching, threshold, and jumping households, where the weights are determined by the relative size of each group's lower bound (upper bound) WTP for the reform.

A.8 Discounted Welfare Impact of Reform

Suppose households have several decisions variables at time t denoted by the vector x_t (within a potentially limited choice set X_t). Household decisions are made conditional on state variables denoted by the vector $\theta_t \in \Theta_t$ and the policy $\mathbf{p}$. Households get the benefit b if their reported income $\hat{y}_t$, which is a function of decision variables x_t, is below τ. Household income, denoted y_t, is also potentially a function of decisions x_t.[47] Households in period t solve the following problem:

$$\begin{aligned} V(\theta_t) &= \max_{x_t \in X_t} \; u\left(c_t, x_t; \theta_t\right) + \beta \mathbb{E}_{\theta_{t+1}}[V(\theta_{t+1}(\theta_t, x_t))] \\ \text{s.t. } \; c_t &= y_t(x_t, \theta_t) + b\mathbb{1}(\hat{y}_t(x_t, \theta_t) \leq \tau) \end{aligned} \tag{56}$$

where c_t denotes consumption in period t, β is a discount factor, and $\mathbb{E}_{\theta_{t+1}}[V(\theta_{t+1}(\theta_t, x_t))]$ represents the expected value of starting period $t+1$ with state variables θ_{t+1}, noting that the state variables tomorrow, θ_{t+1}, may be impacted by current state variables θ_t and current decisions x_t along with random variation. Equivalently, we can write out cumulative individual utility over T time periods from the perspective of time period 0 as:

$$\sum_{t=0}^{T} \beta^t \mathbb{E}_{\theta_t}[u\left(c_t, x_t; \theta_t(\theta_0, \{x_t\}_{t-1})\right]$$

where $\mathbb{E}_{\theta_t}$ represents the expectation over θ_t from the perspective of time period 0 (taking into account the impact of all conditional decisions $\{x_t\}_{t-1}$ between time 0 and time $t-1$ on the underlying expectations).

So as to slightly reduce some cumbersome notation, let us define:

$$y_t^*(\theta_t, \mathbf{p}) \equiv y_t\left(x_t^*\left(\theta_t, \mathbf{p}\right), \theta_t\right)$$

$$\hat{y}_t^*(\theta_t, \mathbf{p}) \equiv \hat{y}_t(x_t^*\left(\theta_t, \mathbf{p}), \theta_t\right)$$

Using this notation, we again assume total discounted welfare is given by a weighted discounted sum of utilities, with welfare weights given by $\phi(\theta_0)$, less the total discounted budgetary cost of the policy multiplied by a shadow value of public funds λ:

$$\begin{aligned} &\sum_{t=0}^{T} \beta^t \mathcal{W}_t(\mathbf{p}) = \\ &\int\limits_{\Theta_0} \phi(\theta_0) \left[\sum_{t=0}^{T} \beta^t \mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t, \mathbf{p}) + b\mathbb{1}\left[\hat{y}_t^*(\theta_t, \mathbf{p}) \leq \tau\right], x_t^*(\theta_t, \mathbf{p}); \theta_t\right)] dF(\theta_0) \right] - \lambda \sum_{t=0}^{T} \beta^t b G_t(\tau; \mathbf{p}) \end{aligned} \tag{57}$$

where we have dropped the arguments of $\theta_t(\theta_0, \{x_t^*(\theta_t, \mathbf{p})\}_{t-1})$ for brevity, β represents the governments

47. For example, in a dynamic misreporting model, $x_t = \hat{y}_t$ and θ_t could include current income y_t, aversion to misreporting μ, prior reported income $\hat{y}_{t-1}$, and a parameter governing expected future income growth. Households may also make savings decisions if assets are a state variable in θ_t, current savings is included in x_t, and c_t represents post-transfer income.

discount rate, λ represents the shadow value of public funds in time $t = 0$ (so that the shadow value of public funds in future periods equals $\beta^t \lambda$), and:

$$G_t(\tau; \mathbf{p}) = \int\limits_{\Theta_0} \mathbb{E}_{\theta_t}[\mathbb{1}(\hat{y}_t^*(\theta_t, \mathbf{p}) \leq \tau)] dF(\theta_0)$$

represents the *expected* number of households receiving the benefit under policy $\mathbf{p}$ in period t. More generally, we define:

$$G_t(z; \mathbf{p}) = \int\limits_{\Theta_0} \mathbb{E}_{\theta_t}[\mathbb{1}(\hat{y}_t^*(\theta_t, \mathbf{p}) \leq z)] dF(\theta_0)$$

as the expected number of households with a reported income below z under policy $\mathbf{p}$ at time t.

This setup allows us to bound the cumulative welfare impacts of a policy reform over T time periods:

Proposition 5. *Suppose households solve Problem (56), total welfare is given by Equation (57), and $\tau' > \tau$. Defining:*

$$MVPF_{L,T} \equiv 1 - \frac{\sum\limits_{t=0}^{T} \beta^t b' \left[G_t(\tau'; \mathbf{p}') - G_t(\tau'; \mathbf{p})\right]}{\sum\limits_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 - \frac{b' \sum\limits_{t=0}^{T} \beta^t J_t}{\sum\limits_{t=0}^{T} \beta^t \, Total\ Cost_t}$$

$$MVPF_{U,T} \equiv 1 + \frac{b \sum\limits_{t=0}^{T} \beta^t [G_t(\tau; \mathbf{p}) - G_t(\tau; \mathbf{p}')]}{\sum\limits_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 + \frac{b \sum\limits_{t=0}^{T} \beta^t B_t}{\sum\limits_{t=0}^{T} \beta^t \, Total\ Cost_t}$$

Then as long as $\sum_t \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})] > 0$ have:

$$\omega_{L,T} MVPF_{L,T} - 1 \leq \frac{\frac{1}{\lambda} \sum\limits_{t=0}^{T} \beta^t [\mathcal{W}_t(\mathbf{p}') - \mathcal{W}_t(\mathbf{p})]}{\sum\limits_{t=0}^{T} \beta^t \left[b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})\right]} \leq \omega_{U,T} MVPF_{U,T} - 1$$

where $\omega_{L,T}$ ($\omega_{U,T}$) captures the discounted weighted average money-metric welfare gain from giving a dollar to mechanical, bunching, threshold, and jumping households, where the weights are determined by the relative size of each group's lower bound (upper bound) discounted WTP for the reform.

Proof. We start with proving the lower bound for $\frac{\frac{1}{\lambda} \sum_{t=0}^{T} \beta^t [\mathcal{W}_t(\mathbf{p}') - \mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]}$. First, note that by revealed preference, we have the following:

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\right],x_t^*(\theta_t,\mathbf{p});\theta_t\right)]\\
&\equiv\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t\left(x_t^*\left(\theta_t,\mathbf{p}\right),\theta_t\right)+b\mathbb{1}\left[\hat{y}_t(x_t^*\left(\theta_t,\mathbf{p}),\theta_t\right)\leq\tau\right],x_t^*(\theta_t,\mathbf{p});\theta_t\right)]\\
&\geq\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t\left(x_t,\theta_t\right)+b\mathbb{1}\left[\hat{y}_t(x_t,\theta_t\right]\leq\tau\right),x_t;\theta_t]
\end{aligned}$$

Put simply, optimal decisions conditional on any given θ_t under $\mathbf{p}$, $x^*(\theta_t,\mathbf{p})$, yields weakly higher utility than any other set of decisions x_t that one could make. This yields the following bound on welfare under policy $\mathbf{p}'=\{b',\tau'\}$, which ensues by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\mathcal{W}_t(\mathbf{p}')=\\
&\int_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b'\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau'\right],x_t^*(\theta_t,\mathbf{p}');\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau';\mathbf{p}')\\
&\geq\int_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b'\mathbb{1}\left[\hat{y}_t(x_t^*\left(\theta_t,\mathbf{p}),\theta_t\right)\leq\tau'\right],x_t^*(\theta_t,\mathbf{p});\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau';\mathbf{p}')
\end{aligned}$$

Hence, for the reform from $\mathbf{p}=\{b,\tau\}$ to $\mathbf{p}'=\{b',\tau'\}$ with $\mathbf{p}'-\mathbf{p}=\{\Delta b,\Delta\tau\}$ and $\Delta\tau>0$ (noting that we have written $\mathbb{E}_{\theta_t}$ as an integral over θ_t conditional on θ_0):

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\left[\mathcal{W}_t\left(\mathbf{p}'\right)-\mathcal{W}_t(\mathbf{p})\right]\geq\\
&\sum_{t=0}^{T}\beta^t\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',x_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p})+b,x_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\
&+\sum_{t=0}^{T}\beta^t\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',x_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}),x_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\
&-\lambda\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]
\end{aligned} \tag{58}$$

Next, define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}$ as the government's average expected welfare weight on the households who optimally report incomes $\leq\tau$ under policy $\mathbf{p}$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}$ captures the average expected welfare gain from giving these households an extra \$1:

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}} = \frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau} \phi(\theta_0)\frac{1}{b'-b}\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',x_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p})+b,x_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau,\mathbf{p})}$$

And define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average expected welfare weight of giving a dollar to the households who optimally report incomes $\in(\tau,\tau']$ under policy $\mathbf{p}$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1:[48]

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']} \phi(\theta_0)\frac{1}{b'}\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',x_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}),x_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})}$$

Next, let us define an aggregate discounted welfare weight, $\eta_{L,t}$, which equals the weighted average discounted expected welfare weight of giving a dollar to all households, where the weights are determined by the (discounted) lower bound of expected WTP for the reform:

$$\eta_{L,T} = \frac{\sum\limits_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}\Delta bG_t(\tau;\mathbf{p})+\sum\limits_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]}{\sum\limits_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]]}$$

Then, dividing Equation (58) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is >0):

$$\begin{aligned}
\frac{\sum\limits_{t=0}^{T}\beta^t\frac{1}{\lambda}\left[\mathcal{W}_t\left(\mathbf{p}'\right)-\mathcal{W}_t(\mathbf{p})\right]}{\sum\limits_{t=0}^{T}\beta^t\left[b'G_t\left(\tau';\mathbf{p}'\right)-bG_t\left(\tau;\mathbf{p}\right)\right]} &\geq \frac{\sum\limits_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}\Delta bG_t(\tau;\mathbf{p})+\sum\limits_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}b'\left[G_t(\tau';\mathbf{p})-G_t(\tau;\mathbf{p})\right]}{\sum\limits_{t=0}^{T}\beta^t\lambda\left[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})\right]}-1\\
&= \frac{\eta_{L,T}}{\lambda}\frac{\sum\limits_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'\left[G_t(\tau';\mathbf{p})-G_t(\tau;\mathbf{p})\right]]}{\sum\limits_{t=0}^{T}\beta^t\left[b'G_t\left(\tau';\mathbf{p}'\right)-bG_t\left(\tau;\mathbf{p}\right)\right]}-1\\
&= \omega_{L,T}MVPF_{L,T}-1
\end{aligned}$$

where $\omega_{L,T}=\eta_{L,T}/\lambda$ and $MVPF_{L,T}$ is given by:

$$MVPF_{L,T} = \frac{\sum\limits_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]]}{\sum\limits_{t=0}^{T}\beta^t[b'G_t\left(\tau';\mathbf{p}'\right)-bG_t\left(\tau;\mathbf{p}\right)]} = 1-\frac{\sum\limits_{t=0}^{T}\beta^tb'\left[G_t(\tau';\mathbf{p}')-G_t(\tau';\mathbf{p})\right]}{\sum\limits_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]}$$

48. We have used $\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']} dF(\theta_t|\theta_0)dF(\theta_0)=G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})$.

Next, we prove the upper bound for $\frac{\frac{1}{\lambda}\sum_{t=0}^{T}\beta^t[\mathcal{W}_t(\mathbf{p}')-\mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]}$. We use identical revealed preference logic to bound welfare under policy $\mathbf{p}=\{b,\tau\}$ by evaluating utility under policy $\mathbf{p}$, but holding household decisions constant at their values under policy $\mathbf{p}'$:

$$\begin{aligned}\sum_{t=0}^{T}\beta^t\mathcal{W}_t(\mathbf{p}) &= \int\limits_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\right],x_t^*(\theta_t,\mathbf{p});\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau;\mathbf{p})\\ &\geq \int\limits_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p}')+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\right],x_t^*(\theta_t,\mathbf{p}');\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau;\mathbf{p})\end{aligned}$$

Hence, for the reform from $\mathbf{p}=\{b,\tau\}$ to $\mathbf{p}'=\{b',\tau'\}$ with $\mathbf{p}'-\mathbf{p}=\{\Delta b,\Delta\tau\}$ and $\Delta\tau>0$ (again noting that we have written $\mathbb{E}_{\theta_t}$ as an integral over θ_t conditional on θ_0):

$$\begin{aligned}&\sum_{t=0}^{T}\beta^t\left[\mathcal{W}_t\left(\mathbf{p}'\right)-\mathcal{W}_t(\mathbf{p})\right]\leq\\ &\sum_{t=0}^{T}\beta^t\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',x_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}')+b,x_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\ &+\sum_{t=0}^{T}\beta^t\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',x_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}'),x_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\ &-\lambda\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]\end{aligned} \tag{59}$$

Next, define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}}$ as the government's average expected welfare weight on the households who optimally report incomes $\leq\tau$ under policy $\mathbf{p}'$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}}$ captures the average expected welfare gain from giving these households an extra \$1:

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}}=\frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau}\phi(\theta_0)\frac{1}{b'-b}\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',x_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}')+b,x_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau,\mathbf{p}')}$$

And define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}}$ as the government's average expected welfare weight of giving a dollar to the households who optimally report incomes $\in(\tau,\tau']$ under policy $\mathbf{p}'$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1:[49]

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}}=\frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}\phi(\theta_0)\frac{1}{b'}\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',x_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}'),x_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau';\mathbf{p}')-G_t(\tau,\mathbf{p}')}$$

49. We have again used $\int_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}dF(\theta_t|\theta_0)dF(\theta_0)=G_t(\tau';\mathbf{p}')-G_t(\tau,\mathbf{p}')$.

Next, let us define an aggregate discounted welfare weight, $\eta_{U,t}$, which equals the weighted average discounted expected welfare weight of giving a dollar to all households, where the weights are determined by the (discounted) upper bound of expected WTP for the reform:

$$\eta_{U,T} = \frac{\sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}} \Delta b G_t(\tau;\mathbf{p}') + \sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}} b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]]}$$

Then, dividing Equation (59) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\sum_{t=0}^{T} \beta^t [\mathcal{W}_t(\mathbf{p}') - \mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} &\leq \frac{\sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}} \Delta b G_t(\tau;\mathbf{p}') + \sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}} b' [G_t(\tau';\mathbf{p}') - G_t(\tau;\mathbf{p}')]}{\lambda \sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} - 1 \\
&= \frac{\eta_{U,T}}{\lambda} \frac{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b' [G_t(\tau';\mathbf{p}') - G_t(\tau;\mathbf{p}')]]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} - 1 \\
&= \omega_{U,T} MVPF_{U,T} - 1
\end{aligned}$$

where $\omega_{U,T} = \eta_{U,T}/\lambda$ and $MVPF_{U,T}$ is given by:

$$MVPF_{U,T} = \frac{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} = 1 + \frac{\sum_{t=0}^{T} \beta^t b [G_t(\tau;\mathbf{p}) - G_t(\tau;\mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]}$$

□

A.9 Proof of Proposition 3 when Eligible Households Do Not Receive Benefit With Certainty

We now prove that our bounds still hold in a model where households reporting below the threshold receive the benefit with some probability q. Those reporting above the threshold do not receive the benefit. Under this more complex policy environment, the household problem is as follows:

$$\begin{aligned}
U^*(\theta,\mathbf{p}) = \max_{x\in X} \mathbb{E}_\alpha [u(c,x;\theta)] \\
s.t.\ c = y(x,\theta) + \alpha b \mathbb{1}(\hat{y}(x,\theta) \leq \tau)
\end{aligned}$$

where α denotes a Bernoulli random variable that takes value 1 with probability q and takes value 0 with probability $1-q$. We again consider the impact on welfare from moving from policy $\mathbf{p} = \{b,\tau\}$ to $\mathbf{p}' = \{b',\tau'\}$ with $\{\Delta b, \Delta\tau\} = \{b'-b, \tau'-\tau\}$ and $\Delta\tau > 0$. We start with proving the lower bound on the welfare impact is unchanged. This proof simply requires some minor adjustments to the steps used to prove the lower bound in Appendix A.3. First, welfare under policy $\mathbf{p}$ is given by:

$$\mathcal{W}(\mathbf{p}) = \int\limits_{\Theta} \phi(\theta)\mathbb{E}_\alpha \left\{u\left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + \alpha b \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) - \lambda b q G(\tau;\mathbf{p}) \quad (60)$$

Next, note that by revealed preference, we have the following for any $x \in X$:

$$\mathbb{E}_\alpha \left\{u\left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + \alpha b \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], x^*(\theta,\mathbf{p});\theta\right]\right\} \geq \mathbb{E}_\alpha \left\{u[y\left(x,\theta\right) + \alpha b \mathbb{1}\left[\hat{y}(x,\theta) \leq \tau\right], x;\theta]\right\} \quad (61)$$

This yields the following bound on welfare under policy $\mathbf{p}' = \{b', \tau'\}$, which ensues by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}\mathcal{W}(\mathbf{p}') &= \int\limits_{\Theta} \phi(\theta)\mathbb{E}_\alpha \left\{u\left[y\left(x^*(\theta,\mathbf{p}'),\theta\right) + \alpha b' \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p}');\theta\right]\right\} dF(\theta) - \lambda b' q G(\tau';\mathbf{p}') \\ &\geq \int\limits_{\Theta} \phi(\theta)\mathbb{E}_\alpha \left\{u\left[y\left(x^*(\theta,\mathbf{p}),\theta\right) + \alpha b' \mathbb{1}\left[\hat{y}\left(x^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) - \lambda b' q G(\tau';\mathbf{p}')\end{aligned} \quad (62)$$

So as to slightly reduce some cumbersome notation, let us define:

$$y^*(\theta,\mathbf{p}) \equiv y\left(x^*\left(\theta,\mathbf{p}\right),\theta\right)$$

$$\hat{y}^*(\theta,\mathbf{p}) \equiv \hat{y}(x^*\left(\theta,\mathbf{p}\right),\theta)$$

Thus, for the reform from $\mathbf{p}$ to $\mathbf{p}'$ we get:

$$\begin{aligned}\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq &\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\mathbb{E}_\alpha \left\{u\left[y^*(\theta,\mathbf{p}) + \alpha b', x^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p}) + \alpha b, x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) \\ &+ \int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\mathbb{E}_\alpha \left\{u\left[y^*(\theta,\mathbf{p}) + \alpha b', x^*\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}), x^*\left(\theta,\mathbf{p}\right);\theta\right]\right\} dF(\theta) \\ &- \lambda q[b' G(\tau';\mathbf{p}') - b G(\tau;\mathbf{p})]\end{aligned} \quad (63)$$

Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^* \leq \tau$ under policy $\mathbf{p}$:

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\frac{1}{q(b'-b)}\mathbb{E}_\alpha \left\{u\left[y^*(\theta,\mathbf{p}) + \alpha b', x^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p}) + \alpha b, x^*(\theta,\mathbf{p});\theta\right]\right\} dF(\theta)}{G(\tau,\mathbf{p})} \quad (64)$$

Note that in Equation (64), we divide by $\frac{1}{q(b'-b)}$, which simply renormalizes by the expected additional amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}') \leq \tau$ as a result of the reform; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}$ as capturing the average expected welfare gain from giving these households an extra \$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau, \tau']$ under policy $\mathbf{p}$:[50]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{qb'}\mathbb{E}_\alpha\left\{u\left[y^*(\theta,\mathbf{p})+\alpha b', x^*\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}), x^*\left(\theta,\mathbf{p}\right);\theta\right]\right\}dF(\theta)}{G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})} \tag{65}$$

Again, note that in Equation (65), we divide by $\frac{1}{qb'}$, which simply renormalizes by the expected amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}) \in (\tau, \tau']$; hence, $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1 with probability. We can rewrite Equation (63) using Equations (64) and (65) as follows:

$$\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq \eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right] - \lambda q\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right] \tag{66}$$

Next, let us define the aggregate welfare weight, η_L, which equals the weighted average welfare weight of giving a dollar to all households, where the weights are determined by the lower bound of WTP for the reform:

$$\eta_L = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}qb'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{q\Delta bG(\tau;\mathbf{p}) + qb'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]} \tag{67}$$

Then, dividing Equation (66) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\left[\mathcal{W}\left(\mathbf{p}'\right) - \mathcal{W}(\mathbf{p})\right]}{q\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} &\geq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\lambda q\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} - 1 \\
&= \frac{\eta_L}{\lambda}\frac{q\Delta bG(\tau;\mathbf{p}) + qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{q\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} - 1 \\
&= \frac{\eta_L}{\lambda}\frac{\Delta bG(\tau;\mathbf{p}) + b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} - 1 \\
&= \omega_L MVPF_L - 1
\end{aligned} \tag{68}$$

where $\omega_L = \eta_L/\lambda$ and $MVPF_L$ is given by:

$$MVPF_L = \frac{\Delta bG(\tau;\mathbf{p}) + b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)} = 1 - \frac{b'\left[G(\tau';\mathbf{p}') - G(\tau';\mathbf{p})\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \tag{69}$$

The upper bound can be proved in a analogous manner by adjusting the upper bound portion of the proof in Appendix A.3 for the fact that the benefit is only received with some probability q.

50. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})$.

A.10 Proposition 3 Cannot Be Improved

Proposition 6. *Without further assumptions on primitives, the bounds in Proposition 3 cannot be improved.*

Proof. To show that one cannot construct tighter bounds than Proposition 3 without additional structure on the household's problem, we provide examples for which these bounds are attained. In particular, we create examples for which (1) bunching households have a WTP of Δb and jumping households have a WTP of 0 as well as (2) bunching and jumping households both have a WTP of b'.[51]

Example 1: Consider our baseline, misreporting model in Section 2.1 with $v(0) = 0$, $v' > 0$. Suppose that $\Delta b, \Delta\tau > 0$. Consider a distribution for the misreporting types ($\mu = 2$) with a mass point at $y = \tau$, no density on $(\tau, y^{c'})$, and another mass point at $y = y^{c'}$, where $y^{c'}$ solves $b' = v(y^{c'} - \tau')$. All bunching househols therefore have $y = \tau$. Thus, the utility change for bunching households is equal to:

$$\tau + b' - [\tau + b - v(\tau - \tau)] = \Delta b$$

Hence, the WTP for bunching households is Δb. Because there are no individuals with incomes $\in (\tau, y^{c'})$, all jumping households have $y = y^{c'}$. Hence, all the jumping households report truthfully (and do not get the benefit) under the original policy $\mathbf{p}$. We break their indifference by assuming that they jump to the threshold τ' under the new policy $\mathbf{p}'$. The change in utility for jumpers is therefore given by:

$$y^{c'} + b' - v(y^{c'} - \tau') - y^{c'} = b' - b' = 0$$

Thus, each jumping household's WTP is equal to 0. Hence, the total WTP for the reform equals $(M + B)\Delta b + Tb' = MVPF_{LB}$.

Example 2: Consider our baseline, misreporting model in Section 2.1 with $v(0) = 0$, $v' > 0$. Suppose that $\Delta b, \Delta\tau > 0$. Let y^c solve $b = v(y^c - \tau)$ and $y^{c'}$ solve $b' = v(y^{c'} - \tau')$. Finally, suppose that $\tau' - \tau$ is large enough so that $\tau' > y^c$. Consider a distribution for the misreporting types ($\mu = 2$) with no density on $[\tau, y^c)$, a mass point at $y = y^c$, and no density on $(y^c, y^{c'}]$. For the $\mu = 2$ individuals with $y = y^c$, they are indifferent between bunching at τ and reporting truthfully under policy $\mathbf{p}$. In the former case, they are bunching individuals (and there are no jumping individuals) and in the latter case they are jumping individuals (and there are no bunching individuals). Regardless, the utility gain for these households is equal to:

$$y^c + b' - [y^c + b - v(y^c - \tau)] = b'$$

Thus the total WTP for the reform will equal $M\Delta b + (B + T + J)b' = MVPF_{UB}$.

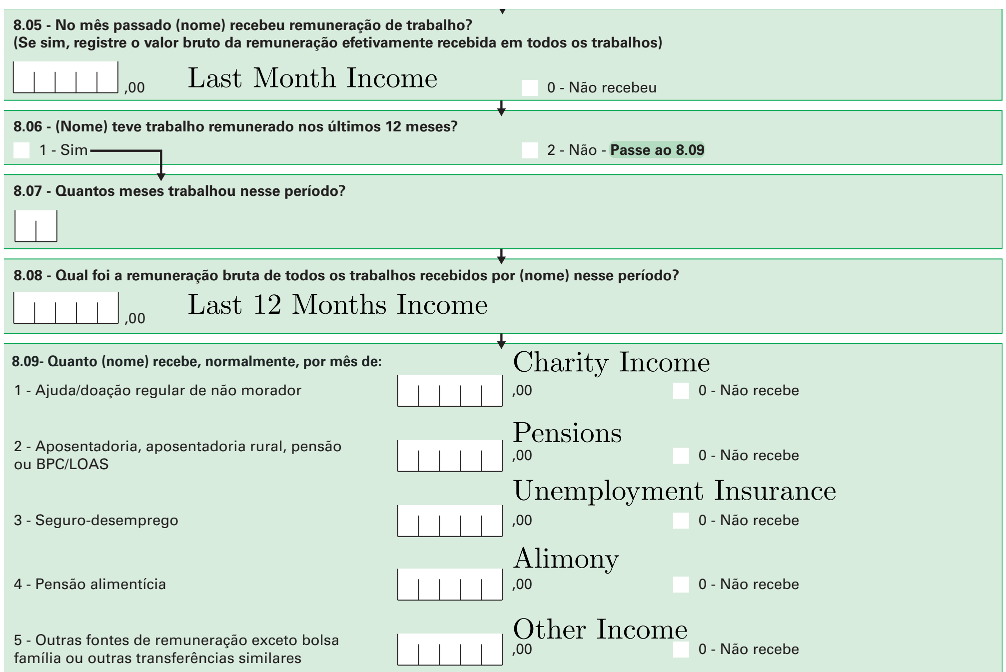
51. Note that our examples require mass points of the income distribution. But one can approximate our example scenarios arbitrarily well with smooth income distributions; hence, we can get arbitrarily close to the cases when either (1) all bunching households have a WTP of Δb and all jumping households have a WTP of 0 or (2) all bunching and jumping households both have a WTP of b'.

□

B For Online Publication: Bolsa Família Program Appendix

B.1 Bolsa Família Questionnaire

Figure 9 shows the entries on the questionnaire used to calculate each individual's total monthly income (household per-capita income will be calculated via summing total individual monthly incomes across all members of a household divided by the number of members in the household). Questions 8.05-8.08 relate to determining last month's labor income and the labor income over the last 12 months. The computer will then calculate an individual's minimum monthly labor income via taking the minimum between the individual's labor income last month and the individual's average monthly labor income over the last 12 months. Question 8.09 relates to determining the average monthly income from five other income sources: charity, pensions, unemployment insurance, alimony, and other. An individual's total monthly income is then equal to their monthly income from these five sources plus their minimum monthly labor income.



8.05 - No mês passado (nome) recebeu remuneração de trabalho?
(Se sim, registre o valor bruto da remuneração efetivamente recebida em todos os trabalhos)

,00 Last Month Income — 0 - Não recebeu

8.06 - (Nome) teve trabalho remunerado nos últimos 12 meses?

1 - Sim — 2 - Não - **Passe ao 8.09**

8.07 - Quantos meses trabalhou nesse período?

8.08 - Qual foi a remuneração bruta de todos os trabalhos recebidos por (nome) nesse período?

,00 Last 12 Months Income

8.09- Quanto (nome) recebe, normalmente, por mês de:

1 - Ajuda/doação regular de não morador	Charity Income	,00	0 - Não recebe
2 - Aposentadoria, aposentadoria rural, pensão ou BPC/LOAS	Pensions	,00	0 - Não recebe
3 - Seguro-desemprego	Unemployment Insurance	,00	0 - Não recebe
4 - Pensão alimentícia	Alimony	,00	0 - Não recebe
5 - Outras fontes de remuneração exceto bolsa família ou outras transferências similares	Other Income	,00	0 - Não recebe

Note: The figure depicts the income categories reported by applicants for each member of the household. Each category has been translated into English in the figure. This is a print out of the screen seen by interviewers on their computers when filling in applicants' information.

Figure 9: Income Questionnaire

B.2 Bolsa Família Schedule for Households with Children

At the beginning of our dataset (December 2011), the BF program has two eligibility thresholds in the per-capita monthly income distribution for households with children: the extreme-poverty line (R$70) and the poverty line (R$140). Households with per-capita income below the extreme-poverty line are eligible for the constant basic benefit (R$70 per-month), a variable benefit proportional to the number of family members between 0 and 15 years old (R$32 per-child, per-month), and a teenager benefit proportional to the number of members aged 16 or 17 years old (R$38 per-teenager, per-month). Households with per-capita income between the extreme-poverty and poverty thresholds are only eligible for the variable and teenager benefits. Households with per-capita income above the second threshold are not eligible for any BF cash transfers. Moreover, the total variable benefit was capped at R$160 (5 children per household) and the total teenager benefit was capped at R$76 (two teenagers per household).[52]

The reform this paper studies, which occurred in June 2014, increased the extreme poverty threshold from R$70 to R$77 and the poverty threshold from R$140 to R$154. The basic benefit was raised from R$70 to R$77, the benefit per child from R$32 to R$35, and the benefit per teenager from R$38 to R$42. Note that the thresholds are based on per-capita income but the benefits are denominated in raw amounts. This reform was announced on national television by the president in April 2014.

Table 4 summarizes these aspects of the schedule before (first column) and after (second column) the reform for households with children.

Table 4: Summary of Bolsa Família Schedule for Families with Children Before and After June 2014 Reform

	Before	After
Extreme-Poverty Threshold	70	77
Poverty Threshold	140	154
Basic Benefit (for those in extreme-poverty)	70	77
Variable Benefit Per Child 15 or Younger (max 5) (for those in poverty)	32	35
Teenager Benefit Per Teen 16-18 (max 2) (for those in poverty)	38	42

Note: The first two rows correspond to the extreme-poverty and poverty thresholds, respectively. These are measured in monthly, per-capita income. I.e., before the reform a household is below the extreme-poverty threshold if their monthly, per-capita income is below R$70. The third, fourth, and fifth rows display the benefits given to households; these are denoted in monthly amounts. I.e., before the reform a household below the extreme-poverty threshold receives R$70 per-month in the basic benefit.

Between December 2011 and February 2013 there were three other reforms to the BF program, which successively instituted a guaranteed minimum income of R$70 per-capita (along with an as-

52. Households with children need to fulfill three additional conditions to receive the variable benefit and/or teenager benefit: (1) children must maintain a minimum of 85% school attendance between ages 6 and 15 and 75% school attendance between 16 and 17; (2) households must keep track of their children's vaccines; and (3) parents must maintain at least 85% attendance in a social-education program if the household has violated child labor laws in the past. All conditionalities were held constant during the analysis period.

sociated negative income tax) for several groups of households. This guaranteed minimum income was instituted in June 2012 for households with children below 6 years of age, in November 2012 for households with children below 15 years of age, and in February 2013 for all remaining households. These reforms thus created a kink (which varies with household composition) in the benefit schedule as a function of reported, per-capita household income for households with children as well as for two adults households with no children. For example, if a two adult household without kids prior to June 2014 has a reported per-capita income of R$20, they get R$70 from the basic benefit and then get an additional R$30 to bring them up to the guaranteed minimum per-capita income of R$70. Mathematically, prior to June 2014 two adult households without kids face a benefit schedule as a function of reported per-capita income, $\hat{y}$, equal to:

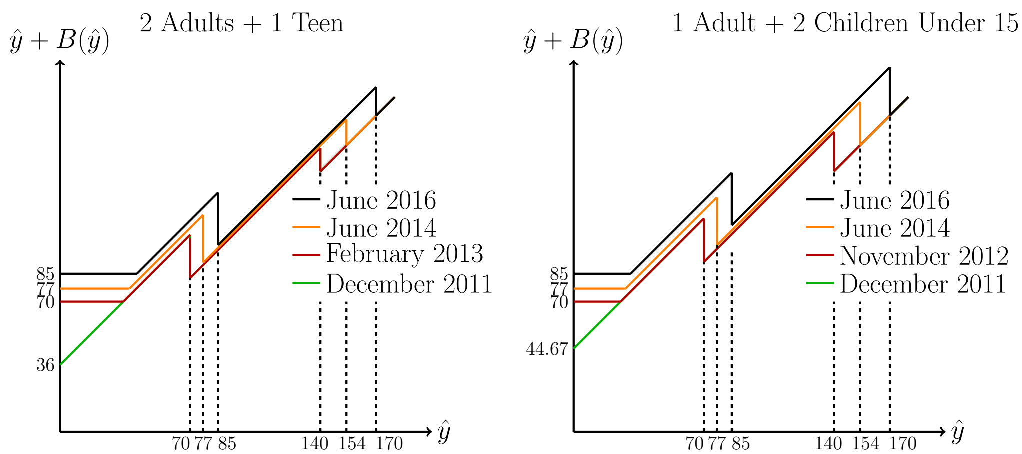
$$B(\hat{y}) = 70\mathbb{1}(\hat{y} \leq 70) + \max\{0, 70 - 2 \times \hat{y}\}$$

This benefit schedule therefore has a kink at R$35. The kink then changed slightly with the June 2014 reform as the guaranteed minimum income was raised to R$77 per-capita. For example, after June 2014 two adult households without kids face a benefit schedule with a kink at R$38.5:

$$B(\hat{y}) = 77\mathbb{1}(\hat{y} \leq 77) + \max\{0, 77 - 2 \times \hat{y}\}$$

Thus, the June 2014 reform potentially impacted the reported income distribution around the kink. Because the kink is located below the first BF notch of R$70, Identification Assumption 1 is less likely to hold for these households as households with reported incomes under R$63 may respond to this changing kink. Finally, a reform in June 2016 further increased the extreme-poverty threshold to R$85 (per-capita, per-month), the poverty threshold to R$170 (per-capita, per-month), the basic benefit to R$85 (per-month), the variable benefit R$39 (per-child, per-month), and the teenager benefit to R$46 (per-teenager, per-month).

For purposes of illustration, Figure 10 plots how the benefit schedules for two particular household compositions varied over time.



Note: $\hat{y}$ denotes the reported, per-capita, monthly household income. $B(\hat{y})$ denotes the monthly, per-capita benefits a household receives if they report $\hat{y}$. These benefits will also depend on household composition. For example, a household with 2 adults and 1 teenager reporting $\hat{y} = 0$ in December 2011 will receive R$70 in the basic benefit and R$38 in the teenager benefit. Thus $\hat{y} + B(\hat{y}) = (70 + 38)/3 = 36$.

Figure 10: Bolsa Família Schedule Reforms For Two Example Household Compositions

B.3 Other Social Security Programs Based on the Cadastro Único

This appendix describes other programs that set their eligibility based on information from the Cadastro Único database.

Benefício de Pretação Continuada (BPC): This benefit targets the elderly (above 65 years of age) and disabled. It gives a minimum wage to all households with per-capita income up to a quarter of the minimum wage. Table 5 reports the minimum wage and BPC threshold across all years of the analysis. The Brazilian Social Security System administers its own exam to define eligibility for this program.

Table 5: Minimum Wage and BPC Eligibility Thresholds

Year	Minimum Wage	BPC Threshold
2011	545.00	136.25
2012	622.00	155.50
2013	678.00	169.50
2014	724.00	181.00
2015	788.00	197.00
2016	880.00	220.00

Carteira do Idoso: This "Elderly Card" guarantees to all individuals 60 years of age or older and with income up to two times the minimum wage at least a 50% discount on any interstate trip by road, rail, or waterway.

Créditos Instalação do Programa Nacional de Reforma Agrária: Households with per-capita income up to three times the minimum wage and that are living in camping grounds get points in a system that selects beneficiaries to be settled through the Brazilian land reform.

Facultativo de Baixa Renda: This is an option to contribute to social security at a lower rate (5% of the minimum wage). The individual cannot have any income and household income must be below two times the minimum wage.

Identidade Jovem (ID Jovem): Discounts for cultural events and trips by road, rail, or waterway for individuals between 15 and 29 years of age living in a household with up to two times the minimum wage.

Isenção de taxas de inscrição em concursos públicos: Since 2008, households with per-capita income up to half of the minimum wage or total income of up to three times the minimum wage are exempt from public tender registration payment.

Política Nacional Assistência Técnica Rural — PNATER Brasil Sem Miséria: Technical assistance for households working on activities for their own consumption in rural areas.

Programa Água para Todos — Programa Nacional de Universalização do Acesso e Uso da Água: Since July 2011, the government has installed cisterns to ensure access to clean water for all Brazilians, with priority going to those who satisfy the criteria for BF program.

Bolsa Estiagem: This is a benefit of at least R$80 per month to households with total income up to two times the minimum wage that live in areas hit by natural disasters.

Programa Bolsa Verde — Programa de Apoio à Conservação Ambiental: Since October 2011, this program transfers R$300 every 3 months to households in extreme poverty (first threshold of BF) and that follow the requirements for using natural resources.

Programa Cisternas: This program aims to provide cisterns to low-income families registered in the Cadastro Único.

Programa de Erradicação do Trabalho Infantil: This program transfers benefits similar to the BF (R$25 and R$40 per child per month in municipalities with less and more than 250,000 inhabitants, respectively) to households whose incomes are above the BF threshold with working children (up to 16 years of age) conditional on these children attending school 85% of the time instead of working.

Programa de Fomento às Atividades Produtivas Rurais: Since 2012, the government has made a one-time transfer of around R$2,400 to families that are eligible for the BF program and work on agricultural activities or belong to native or traditional communities.

Programa Minha Casa Minha Vida: Households with total monthly income up to R$1,416.67 have access to a subsidized credit line to purchase a house.

Programa Nacional de Crédito Fundiário: Households with total monthly income up to R$2,500 have access to a subsidized credit line to purchase land for production.

Serviços Socioassistenciais: MDS offers social services to poor individuals who have suffered any type of violence or neglect.

Sistema de Seleção Unificada — Sisu/Lei de Cotas: Since 2016, all federal universities in the country reserve some seats for students coming from families with per-capita income up to 1.5 times the minimum wage.

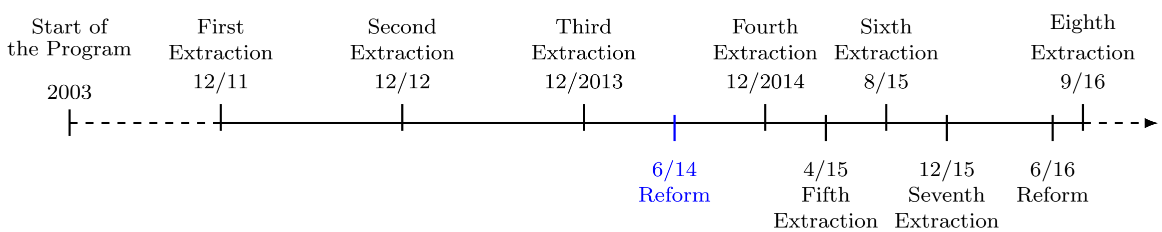
Tarifa Social de Energia Elétrica: Households with monthly per-capita income of up to a half the minimum wage have access to a discounted electricity price.

Tefone Popular — Acesso Individual Classe Especial: The government offers a landline with lower prices for individuals registered in the Cadastro Único database.

Distribuição de Conversores de TV Digital: Since the September of 2015, MDS has offered digital converters to beneficiaries of the program, which helps them transition from open TV to the new system.

B.4 Bolsa Família Program and Data Extraction Timeline

Figure 11 presents the data extraction timeline relative to when the BF program started and the June 2014 reform. As can be seen, the program started in 2003, the extractions we have span the months between December 2011 to September 2016, and the reform we study occurred in June 2014. Note, there was also another reform in June 2016.



Note: The figure describes the timeline of the program and the data extractions. BF started in 2003, and the reform we study occurred in June 2014. The final dataset is constructed from 8 extractions from December 2011 until September of 2016. Each extraction contains the most recent information on each household as of the extraction date. Note, there was another reform in June 2016.

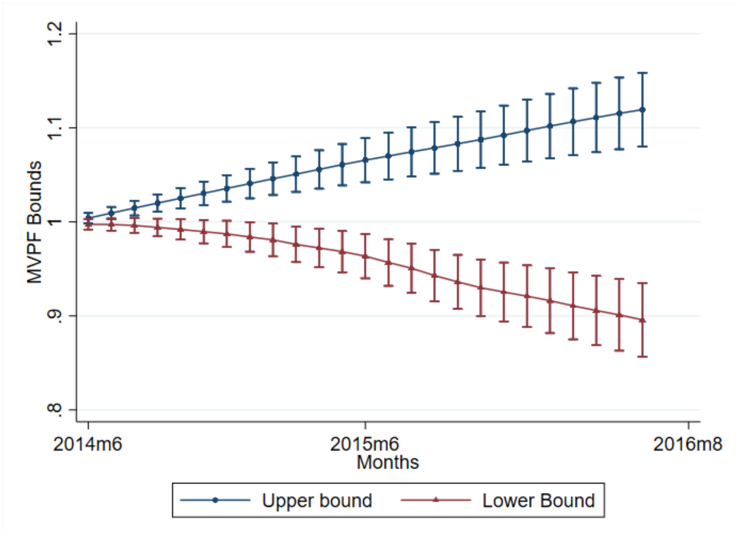
FIGURE 11: TIMELINE

C For Online Publication: Empirical Appendix

C.1 Calculating the MVPF Each Month in the Post-Reform Period

In this appendix, we calculate upper and lower bounds for the MVPF for each month from June, 2014 to June, 2016. To do so, we estimate the number of bunching and jumping households for each month and plug these estimates into Equations (11) and (12). We use Regression (16) with a polynomial of degree $K = 3$ to infer the number of bunching and jumping households in each month. Results are presented in Figure 12 below.

As can be seen, the MVPF bounds are initially tightly centered around 1 at the beginning of the reform. This is because the number of bunching and jumping households are initially small (as seen in Figure 5). As the number of bunching and jumping households grow overtime, our lower bound increases and our upper bounds decreases. One might be worried that these bounds will continue to grow as we go past June 2016. While we cannot test this with our data (as mentioned in Section 3, there is another reform to BF in June 2016 and our data only goes out to September 2016), we suspect that because households are required to update their information every two years to remain eligible for BF, most behavioral responses to the reform should be observed within the first two years post-reform.



Note: This figure calculates the upper and lower bounds for the MVPF for each month from June 2014 to June 2016. We calculate the number of bunching and jumping households for each month in the post-reform period from Regression (16) (with polynomial of degree $K = 3$) and plug these numbers into Equations (11) and (12). Confidence intervals are constructed via the delta method from the clustered standard errors estimated in Regression (16).

FIGURE 12: MVPF BOUNDS FOR EACH MONTH IN THE POST-REFORM PERIOD

Related to the above analysis, we can also calculate bounds for the MVPF for all periods post-reform. As shown in Appendix A.8, to do this, we need to observe the number of jumpers and bunchers for every period post-reform (i.e., for all months beyond June 2014). We show in Appendix A.8 that the lower and upper bounds for the cumulative MVPF over T time periods are given by:

$$MVPF_{L,T} \equiv 1 - \frac{\sum_{t=0}^{T} \beta^t b' \left[G_t(\tau'; \mathbf{p}') - G_t(\tau'; \mathbf{p})\right]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 - \frac{b' \sum_{t=0}^{T} \beta^t J_t}{\sum_{t=0}^{T} \beta^t \text{Total Cost}_t}$$

$$MVPF_{U,T} \equiv 1 + \frac{b \sum_{t=0}^{T} \beta^t [G_t(\tau; \mathbf{p}) - G_t(\tau; \mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 + \frac{b \sum_{t=0}^{T} \beta^t B_t}{\sum_{t=0}^{T} \beta^t \text{Total Cost}_t}$$

where β denotes the discount rate. The limitation here is that we can't estimate the number of jumpers and bunchers beyond June 2016. Thus, we make the assumption that the number of jumpers and bunchers beyond June 2016 are equal to the number of jumpers and bunchers in June 2016 (we also assume the total cost for periods beyond June 2016 is equal to the total cost for June 2016). Under this assumption, we evaluate the MVPF bounds setting $T = \infty$ and using an annual discount rate $\beta = 0.98$. We estimate the number of bunchers and jumpers using Regression (16) for polynomial degrees $K = \{2, 3, 4, 5\}$. Results are presented in Table 6. The cumulative MVPF bounds in Table 6 are close to the monthly MVPF bounds for June 2016 presented in Table 3. This is not surprising given our assumption that the number of bunchers, jumpers, and total cost post-June 2016 are equal to the number of bunchers, jumpers, and total cost in June 2016 so that our lower bound of the cumulative MVPF is given by:

$$MVPF_{L,\infty} = 1 - \frac{b' \sum_{t=0}^{24} \beta^{t/12} J_t + b' \sum_{t=25}^{\infty} \beta^{t/12} J_{24}}{\sum_{t=0}^{24} \beta^{t/12} \text{Total Cost}_t + \sum_{t=25}^{\infty} \beta^{t/12} \text{Total Cost}_{24}} \approx 1 - \frac{J_{24}}{\text{Total Cost}_{24}}$$

where $t = 0$ denotes June 2014, and $t = 24$ denotes June 2016 (we divide t by 12 in $\beta^{t/12}$ because β is an annual discount rate).

Table 6: MVPF Bounds for All Months Post-Reform

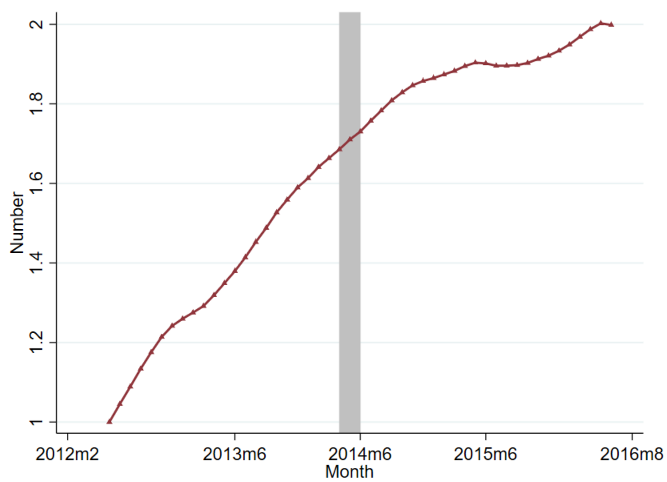
Polynomial Degree, K	$MVPF_{L,\infty}$	$MVPF_{U,\infty}$
Quadratic, $K = 2$	0.90	1.10
	(0.02)	(0.02)
Cubic, $K = 3$	0.91	1.10
	(0.02)	(0.02)
Quartic, $K = 4$	0.91	1.11
	(0.02)	(0.02)
Quintic, $K = 5$	0.91	1.11
	(0.03)	(0.03)

Note: This table presents the cumulative MVPF bounds of the reform (i.e., across all months after the June 2014 reform). We calculate the number of bunching and jumping households for each month in the post-reform period from Regression (16) for various polynomial degress and we assume the number of bunching and jumping households post-June 2016 are equal to the numbers in June 2016. Standard errors are presented in parentheses and are constructed via the delta method from the clustered standard errors estimated in Regression (16).

C.2 Discussion on the Presence of Income Effects

Identification Assumption 1 relies on households not changing their reported incomes in response to a higher benefit, i.e., households do not experience income effects. If households did experience income effects, then households reporting below R$63 under policy $\mathbf{p}$ would change their reported income under policy $\mathbf{p}'$ (all else equal) as their benefit rises from b to b'. While their are no income effects in the baseline model of Section 2.1, this may not be the case in the more general model of Section 2.3 as households may reduce their labor supply and, in turn, their reported income in response to a higher benefit. However, we believe that our placebo tests in Section 5.6 provide suggestive evidence that income effects are not present in our empirical application. The placebo tests shown in Figure 7 suggest that the trends in the number of individuals reporting in income bins below R$63 all evolve very similarly over time. This could result from either (1) income effects being negligible or (2) income effects impacting each reported income bin in the same way so that their trends remain similar post-reform. However, (2) is inconsistent with the fact that the income distribution is highly non-smooth (see Figure 19). The reasoning is that if income effects were affecting the distribution, we would expect to see some relatively constant *proportion* of individuals in each income bin move to a neighboring bin; however, this would lead to substantially different trends in the post-reform period due to differing numbers of people in each bin to start with. As an example, note that there are relatively few individuals in the income bin R$(42,49] (see Figure 19) but there are a comparatively large number of individuals in the income bin R$(49,56]. If income effects lead some households to reduce their income, then we might expect some proportion of households initially in R$(49,56] to move down to R$(42,49]. But because the number of households in R$(42,49] is so small relative to the number in R$(49,56], even a small proportion of households in R$(49,56] moving down to R$(42,49] would have a large impact on the post-reform trend for the R$(42,49] bin. Hence, because the trends in the number of individuals reporting in income bins below R$63 all evolve very similarly over time, we believe that income effects are unlikely to be impacting the reported income distribution in our setting (consistent with our hypothesis that most of the observed responses are reporting, rather than real, responses). While we cannot rule out some sort of non-standard, non-monotonic income effects affecting the distribution, this seems unlikely.

C.3 Number of Households Reporting an Income in R$(63,70]



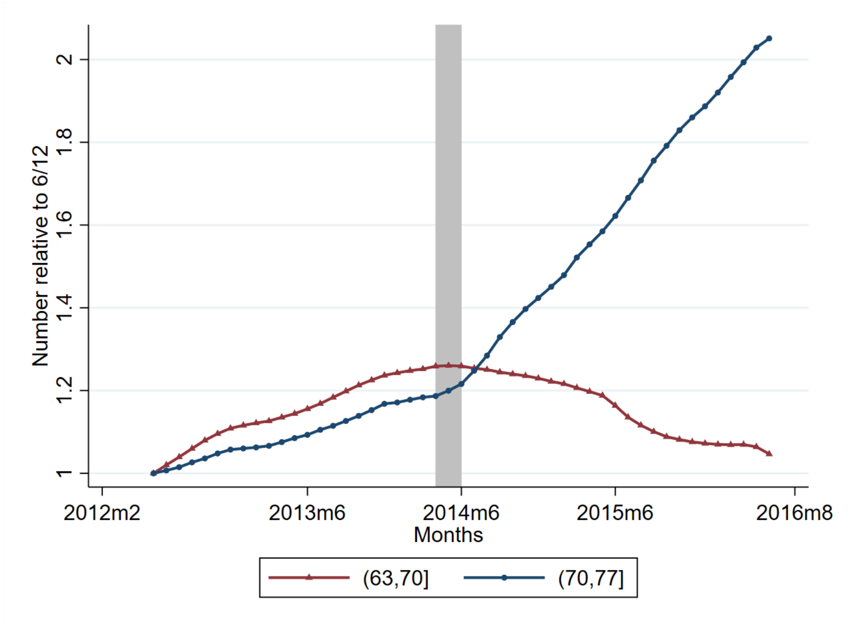
Note: This figure shows the number of single individual households with reported incomes in R$(63,70]. The number in each bin is normalized to 1 in June 2012. The timing of the reform (from the announcement in April 2014 to the enactment in June 2014) is indicated by the gray, shaded region.

Figure 13: Number of Households Reporting an Income in R$(63,70]

C.4 Results for Two Adult Households with No Children

In this appendix, we present results for households with two adults and no children. As discussed in Section 3.2 and Appendix B.2, the guaranteed minimum income, which was instituted in February 2013 for two adult households without kids, generates a kink in the benefit schedule that occurs at a per-capita household income of R$35. Hence, after February 2013 and prior to June 2014, two adult families in extreme poverty (i.e., with household incomes below R$70) get the basic benefit of R$70 plus additional funds to bring their per-capita income up to R$70. For example, a two adult family with a combined household income of R$40 gets R$70 *plus* an additional R$30 (giving them a total income of R$140) to reach the guaranteed minimum per-capita income of R$70. This generates a kink in the benefit schedule for two adult families at the per-capita household income of R$35 after February 2013 and prior to June 2014.

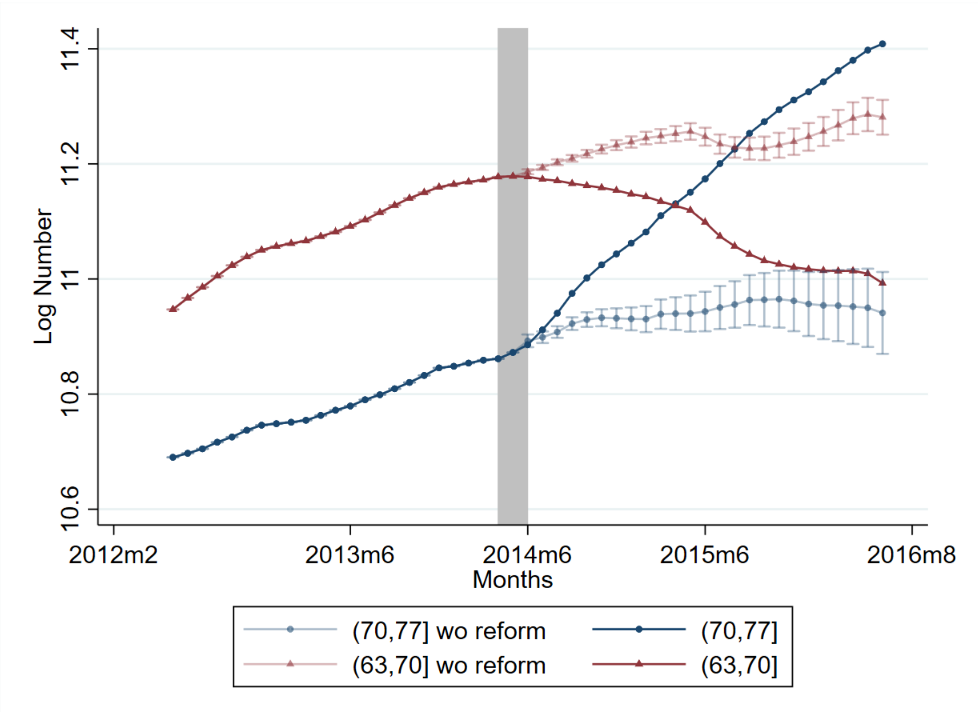
The June 2014 reform not only changed the extreme poverty threshold and the basic benefit both from R$70 to R$77 but it also changed the guaranteed minimum income from R$70 per-capita to R$77 per-capita. Hence, the June 2014 reform changed the location and level of both the notch and the kink in the benefit schedule (for example, the location of the notch moved from R$70 to R$77 per-capita while the location of the kink moved from R$35 to R$38.5 per-capita). Identification Assumption 1 is now harder to justify given that the June 2014 reform changed incentives for households to locate around R$35 by changing the kink in the benefit schedule from R$35 to R$38.5. However, many studies have shown that behavioral responses to kinks are typically very small; kinks generally induce substantially less bunching than do notches (Kleven, 2016). Hence, we simply ignore the presence of the kink and use all seven-increment income bins below R$63 as control groups just as in our main analysis. Figure 14 shows the raw data for how the number of two adult households reporting incomes in bins R$$(63, 70]$ and R$$(70, 77]$ evolved in a four year window around the reform from June 2012 to June 2016. As in the corresponding figure for one adult households (Figure 3b), Figure 14 depicts a clear trend departure for bin R$$(70, 77]$ commensurate with the reform, providing highly suggestive evidence that the BF reform induced a substantial behavioral response that increased the number of households bunching at the new threshold. Figure 14 also depicts a clear decrease in the number of two adult households reporting incomes in R$$(63, 70]$, providing highly suggestive evidence that the number of households reporting at/just below the old threshold decreased as a result of the reform.



Note: This figure shows the number of two adult households with no children that report incomes in the intervals R$(63, 70] and R$(70, 77] for each month between June 2012 to June 2016. The number in each bin is normalized to 1 in June 2012. The timing of the reform (from the announcement in April 2014 to the enactment in June 2014) is indicated by the gray, shaded region.

Figure 14: Number of Two Adult Households Reporting an Income in R$(63,70] or R$(70,77]

Figure 15 shows the analogue of Figure 5 for two adult households with no kids. There is a clear increase in the number of households locating in R$(70, 77] and a clear decrease in the fraction locating between R$(63, 70] just as for single individual households.



Note: This figure shows the log number of two adult households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The sample is restricted to two adult households without children. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 15: Counterfactual Paths for Treatment Bins, Two Adult Households with No Children

Results from estimating Equation (16) for various polynomial degrees K can be found in Table 7. The MVPF bounds are roughly similar in magnitude as for the single individual households discussed in Section 5 but are a bit more sensitive to the degree of polynomial used.[53]

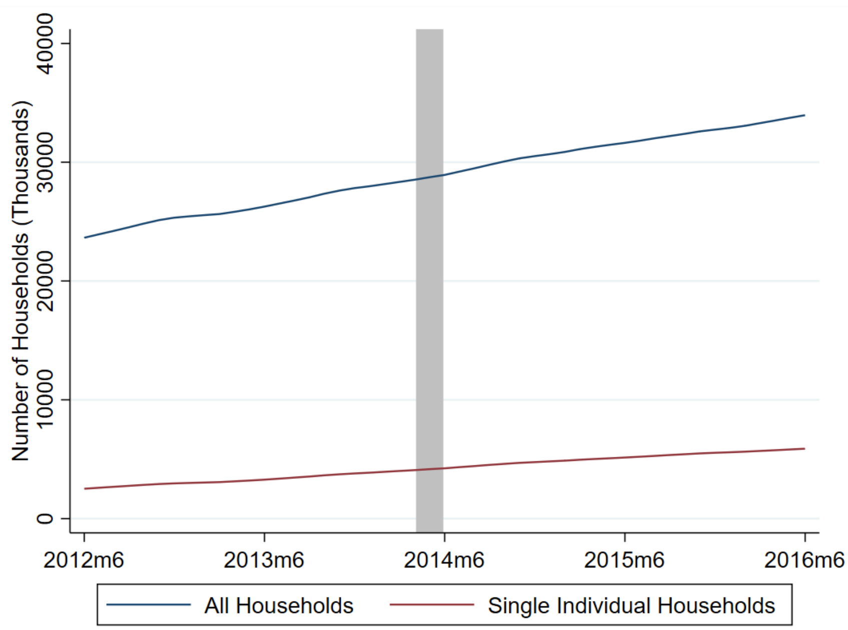
53. Note these bounds are for the MVPF associated with changing the location and level of the notch only, i.e., we ignore any welfare impacts of changing the level and location of the kink.

Table 7: Impacts of Reform and Efficiency Loss as of June, 2016 Estimated from Equation (16), Two Adult Households with No Children

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-19,897	37,055	19,897	17,157	0.88	1.12
	(1,795)	(3,592)	(1,795)	(4,292)	(0.03)	(0.01)
Cubic, $K = 3$	-19,862	33,645	19,862	13,783	0.91	1.12
	(1,221)	(2,048)	(1,221)	(2,536)	(0.02)	(0.01)
Quartic, $K = 4$	-14,347	21,043	14,347	6,696	0.96	1.08
	(2,100)	(5,011)	(2,100)	(5,610)	(0.04)	(0.01)
Quintic, $K = 5$	-14,157	25,714	14,157	11,557	0.93	1.08
	(1,883)	(2,625)	(1,883)	(3,467)	(0.02)	(0.01)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of two adult households reporting incomes in bins R$(63,70] and R$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from Equation (16) with various polynomial degrees $K \in \{2, 3, 4, 5\}$, restricting the sample to two adult households without children. Columns (3) and (4) show the estimated number of bunching and jumping households for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations (13) and (14). Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation (16).

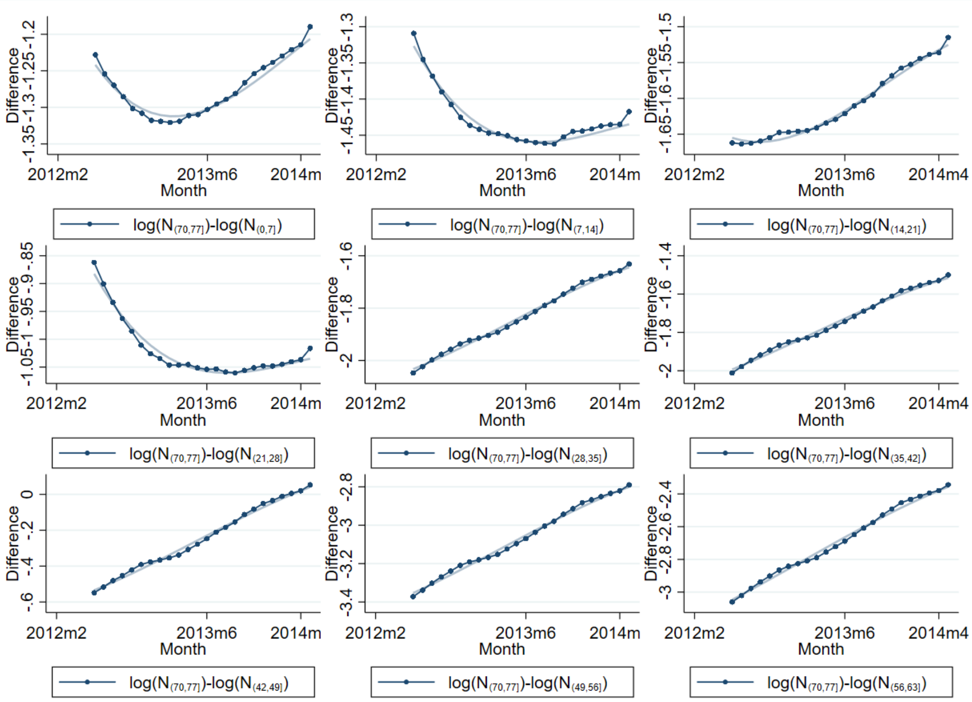
C.5 Number of Households on the Cadastro Único Registry Over Time



Note: This figure shows the raw number of households on the Cadastro Único Registry over time. The timing of the reform is indicated by the gray, shaded region.

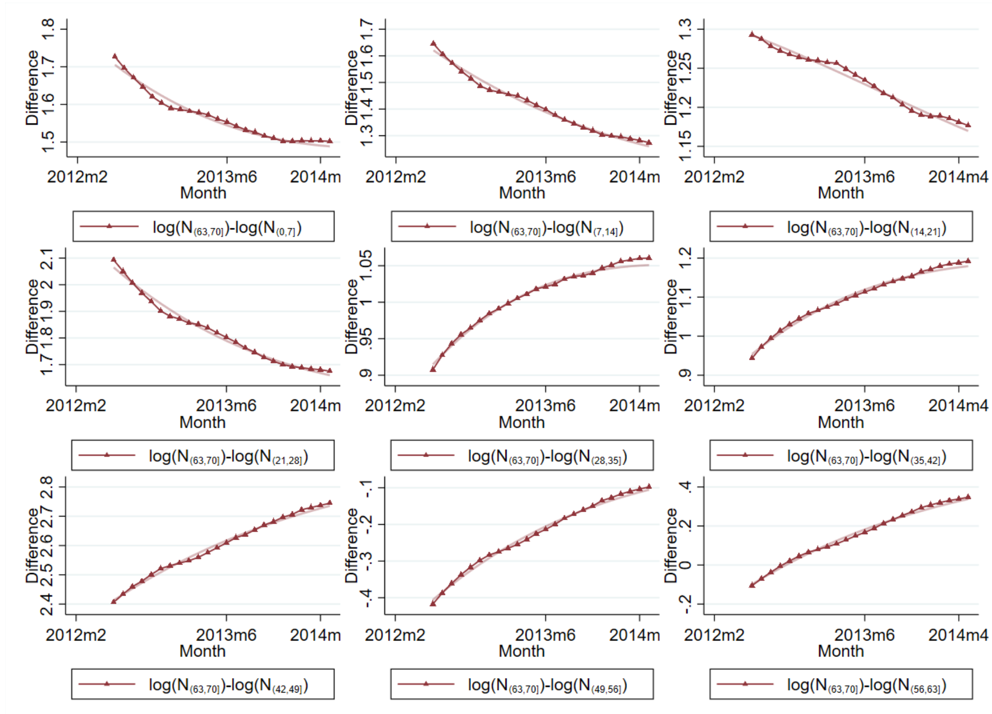
Figure 16: Number of Households on the Cadastro Único Registry

C.6 Pre-Reform Differences



Note: This figure shows how the difference between the log number of people reporting incomes in R\$(70,77], denoted $\log\left(N_{(70,77]}\right)$, and the log number of people reporting in R\$$(x-7,x]$, denoted $\log\left(N_{(x-7,x]}\right)$, varied prior to the reform in June, 2014. Each plot also includes the cubic trend estimated from Equation (16).

FIGURE 17: PRE-REFORM DIFFERENCES BETWEEN $N_{(70,77]}$ AND $N_{(x-7,x]}$ FOR $x \in \{7, 14, ..., 63\}$

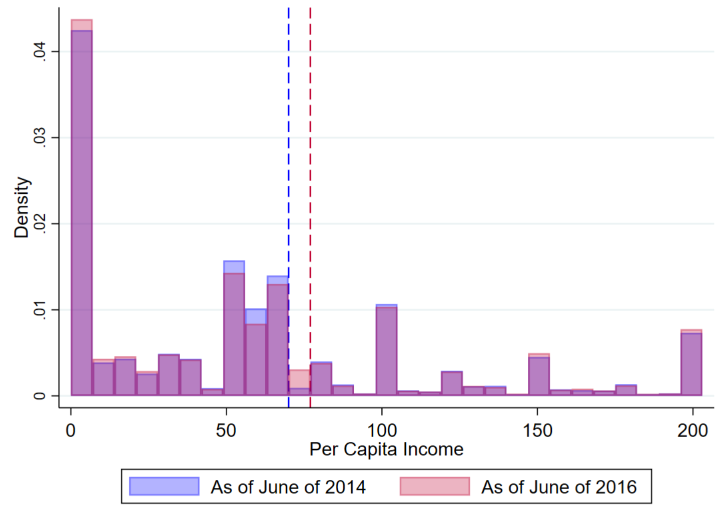


Note: This figure shows how the difference between the log number of people reporting incomes in R\$(63,70], denoted $\log\left(N_{(63,70]}\right)$, and the log number of people reporting in R\$$(x-7,x]$, denoted $\log\left(N_{(x-7,x]}\right)$, varied prior to the reform in June, 2014. Each plot also includes the cubic trend estimated from Equation (16).

FIGURE 18: PRE-REFORM DIFFERENCES BETWEEN $N_{(63,70]}$ AND $N_{(x-7,x]}$ FOR $x \in \{7, 14, ..., 63\}$

C.7 Histograms of Reported Income Distribution

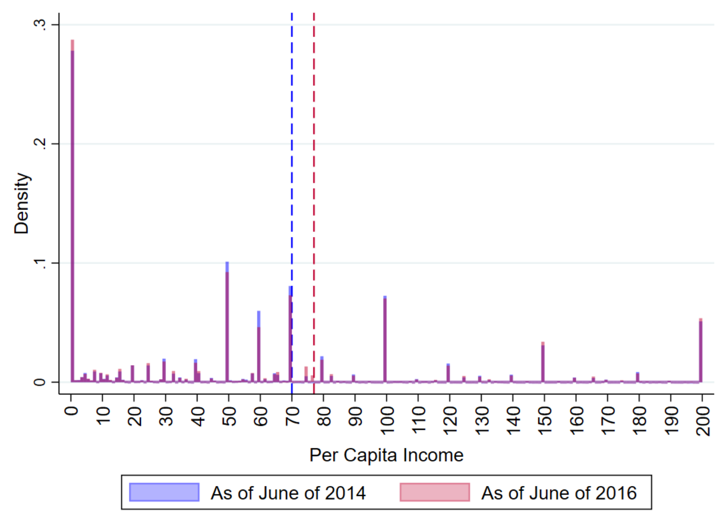
Figure 19 plots the distribution of reported incomes in June 2014 and June 2016 for single individual households split into seven increment bins. While these histograms show the pre- and post-reform distributions, they *should not* be used to make inferences about the causal impact of the reform due to significant underlying time trends in the reported income distribution.



Note: This figure shows the number of single individual households that report incomes in seven increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme poverty threshold before June 2014 is shown with a dashed blue line and the extreme poverty threshold after June 2014 is shown with a dashed red line.

Figure 19: Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform

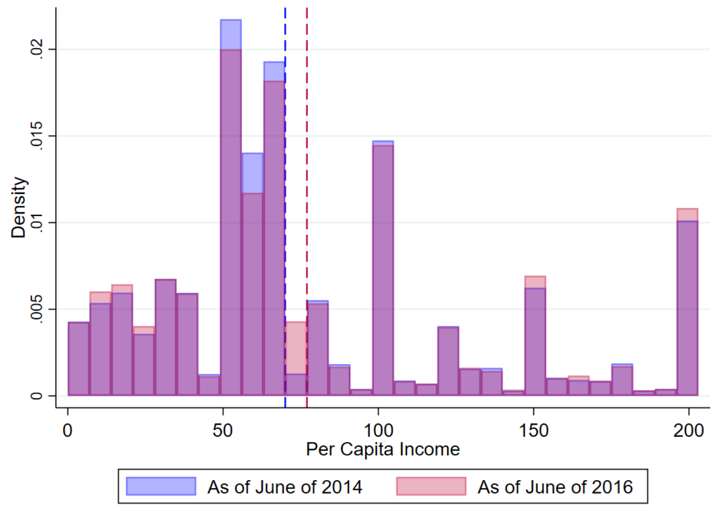
Figure 20 shows a more granular view of the same reported income distributions for single individual households split into one increment bins. There is substantial bunching at 0 mod 50 numbers and a lesser degree of bunching at 0 mod 10 numbers:



Note: This figure shows the number of single individual households that report incomes in one increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme poverty threshold before June 2014 is shown with a dashed blue line and the extreme poverty threshold after June 2014 is shown with a dashed red line.

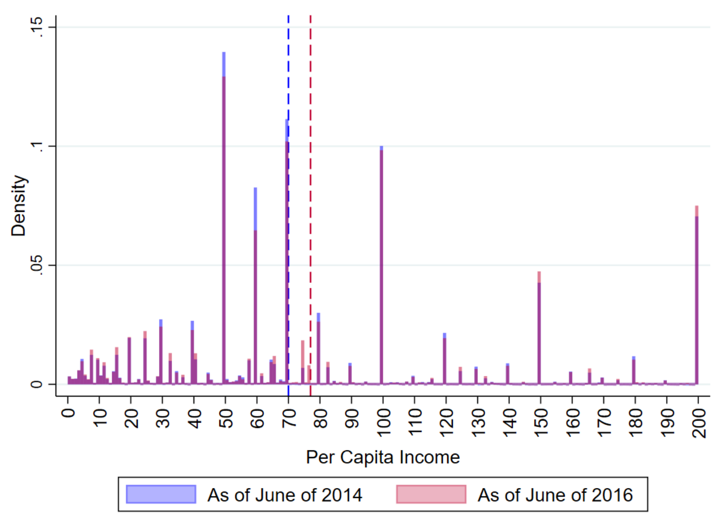
Figure 20: Granular Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform

Finally, for better visualization of the reported income distribution for per-capita incomes> 0, we repeat Figures 19 and 20 restricting to households with strictly positive per-capita incomes:



Note: This figure shows the number of single individual households that report incomes in seven increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme poverty threshold before June 2014 is shown with a dashed blue line and the extreme poverty threshold after June 2014 is shown with a dashed red line.

Figure 21: Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform (strictly positive incomes only)



Note: This figure shows the number of single individual households that report incomes in one increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme poverty threshold before June 2014 is shown with a dashed blue line and the extreme poverty threshold after June 2014 is shown with a dashed red line.

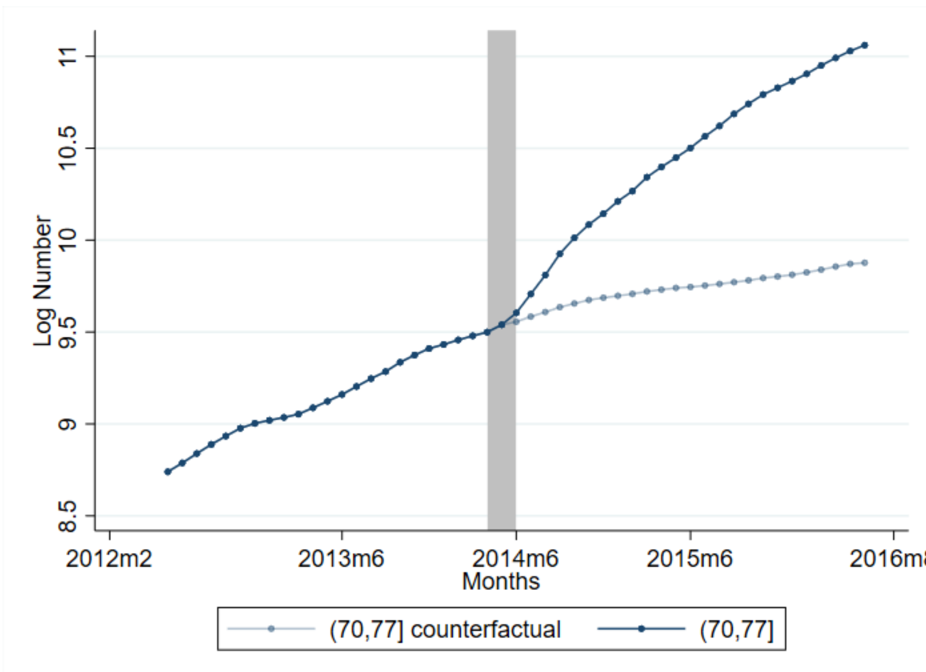
Figure 22: Granular Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform (strictly positive incomes only)

C.8 Robustness: Conservative Assumption for Counterfactual Growth of R$(70,77]

Ignoring any relationships in the pre-reform period between the R$(70,77] bin and our control bins under R$63, we calculate how the R$(70,77] income bin would have grown counterfactually if it evolved according to the maximum growth of all control bins. Therefore, we calculate the following series:

$$\log(N_{(70,77],t})_{max,\mathbf{p}} = \log(N_{(70,77],0}) + \sum_{s=1}^{t} \max_{x\in\{7,\ldots,63\}} \left[\log(N_{(x-7,x],s}) - \log(N_{(x-7,x],s-1})\right] \tag{70}$$

for $t > 0$ where $t = 0$ denotes May 2014 and $t > 0$ denotes each month in the post-reform period. The figure below plots this counterfactual series.



Note: This figure shows the number of single individual households that report incomes in R$(70,77] along with a conservative counterfactual series for R$(70,77] in absence of the reform computed via Equation (70). The timing of the reform is indicated by the gray, shaded region.

Figure 23: Actual vs. Counterfactual Growth of R$(70,77] if R$(70,77] Grew at Rate of Fastest Growing Control Bin Each Month

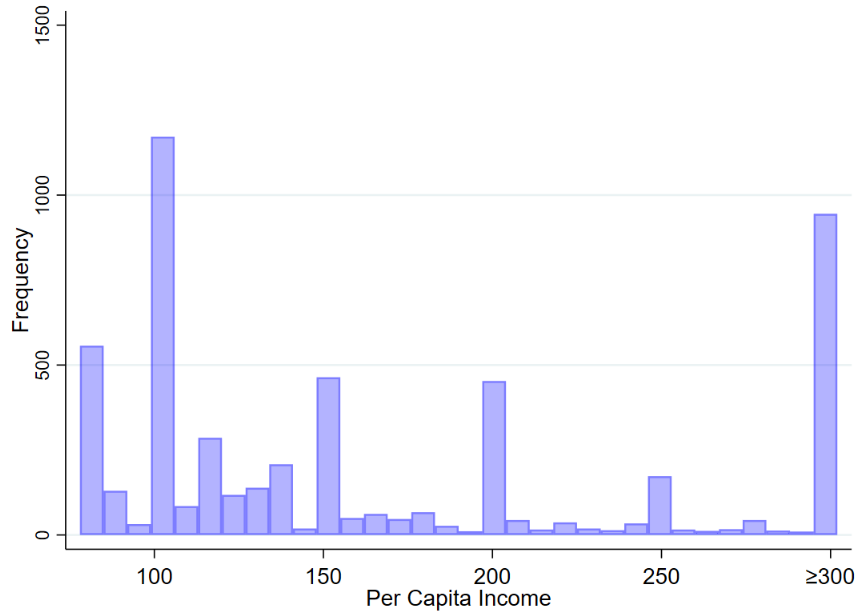
Using this series, we calculate a conservative estimate for the change in the number of households locating in R$(70,77] as a result of the reform to be 44,140 (recall that this number is $\approx 49,000$ for the cubic polynomial specification in Table 3). Using our estimate of 27,452 bunchers from the cubic specification in Table 3, this estimate would put the number of jumpers at 16,688, generating a lower MVPF bound of 0.921 (the upper bound MVPF is unchanged). Thus, even if counterfactually the R$(70,77] bin would have grown at the rate of the fastest growing control group in each month, we would still estimate a sizable number of jumping households and our MVPF bounds would be very similar.

C.9 *Investigating Where the Jumpers are Coming From*

In this appendix, we exploit the panel nature of our data to provide descriptive evidence on where the jumpers are coming from. First, for households reporting in R$(70,77] in June 2016, we investigate what their last reported income was prior to the reform (i.e., prior to June 2014). We focus on households reporting above R$77 prior to June 2014 and households not reporting any income prior to June 2014 so as to shed light on the extent to which our ≈ 22,000 jumpers were responding to the reform on the intensive margin (i.e., jumping down from higher reported incomes) vs. extensive margin (i.e., entering the program). We can also shed light on the possible size of the intensive margin responses by looking at the range of incomes these households were reporting prior to the reform.

Figure 24 shows the (frequency) distribution of reported incomes prior to reform (June 2014) for those who report in R$(70,77] in June 2016 (restricting to those reporting above R$77 prior to the reform). As can be seen, these households were reporting a range of incomes above R$77 prior to the reform with many reporting incomes at or above R$300. In total, 5,321 households locating in R$(70,77] in June 2016 reported an income above R$77 prior to reform. Interestingly, 27,844 households locating in R$(70,77] in June 2016 were not in the registry prior to the reform. While this evidence is purely descriptive, it does suggestive that of the ≈ 22,000 jumping households we identify in Table 3, that many of them were likely extensive margin households (i.e., in absence of the reform, many of the 22,000 jumping households would've simply not reported an income in June 2016). One may wonder why $5,321 + 27,844 = 33,165 > 22,000$. It's worth reiterating that this analysis is descriptive - we are not determining counterfactual reported incomes in June 2016 had the reform not occurred. Some of these 33,165 households would have located in R$(63,70] in June 2016 had the reform not occurred (and are thus part of B rather than J).

Finally, we think it is worth noting why we only present descriptive evidence on where the jumpers are coming from. While we are able to precisely and robustly identify a gain of 22,000 jumping households to the R$(70, 77] income bin, identifying where these jumpers coming from involves estimating very small decreases across many bins above R$77; ultimately, our identification strategy doesn't have the power to estimate these very small treatment effects.

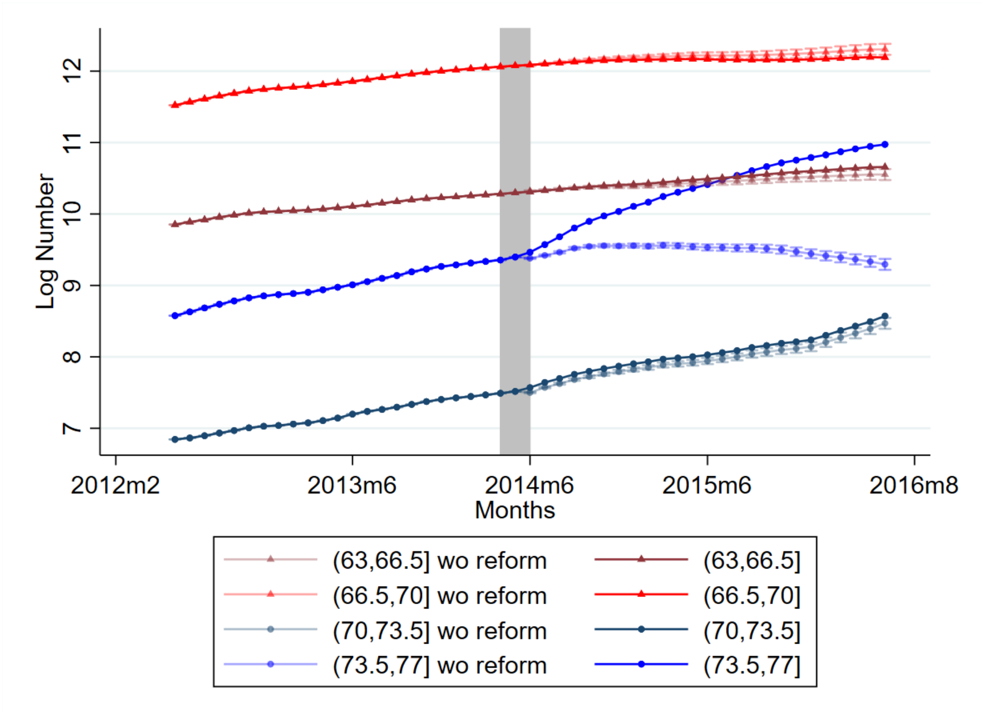


Note: This figure plots the last reported income prior to June 2014 for single individual households who report an income in R$(70,77] in June 2016, restricting to households who were reporting above R$77 in June 2014.

Figure 24: Reported Income prior to June 2014 for those Reporting in R$(70,77] in June 2016

C.10 Results with Smaller Bin Sizes

This Appendix contains results using a bin size of 3.5 rather than a bin size of 7 as in the main text. We estimate Equation (16) for all $x \in \{3.5, 7, 10.5, ..., 77\}$. Estimated counterfactuals for bins R$(63,66.5], R$(66.5,70], R$(70,73.5], and R$(73.5,77] are shown in Figure 25. The estimated treatment effects are large for R$(66.5,70] and R$(73.5,77] but small for R$(63,66.5] and R$(70,73.5] suggesting that bunching is relatively precise. The estimated effects and corresponding lower and upper bounds for the MVPF are shown in Table 8. The estimated MVPF bounds are relatively similar to our main results in Table 3.



Note: This figure shows the log number of single individual households reporting incomes in R\$(63,66.5], R\$(66.5,70], R\$(70,73.5], and R\$(73.5,77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{66.5, 70, 73.5, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

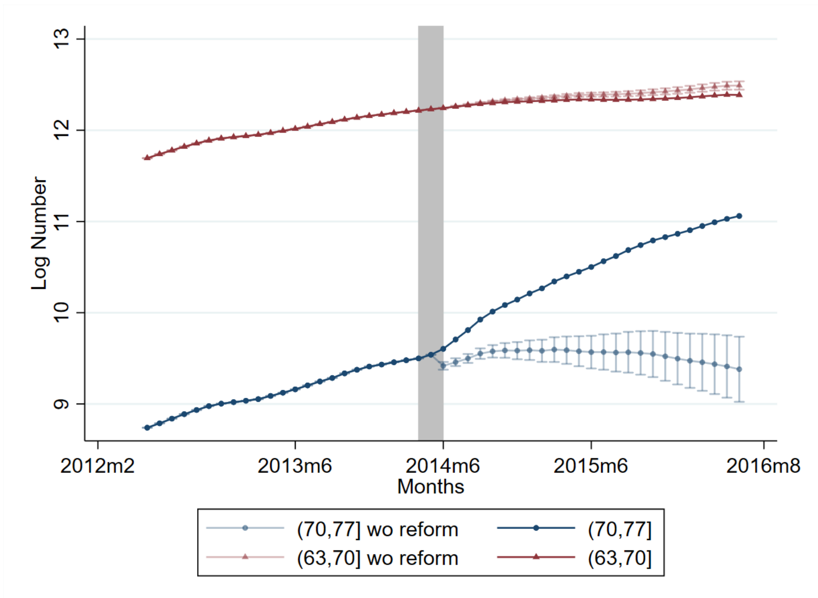
FIGURE 25: ACTUAL AND COUNTERFACTUAL PATHS FOR TREATMENT BINS USING EQUATION (16) WITH SMALLER BIN SIZES

Table 8: IMPACTS OF REFORM AND MVPF BOUNDS IN JUNE 2016 ESTIMATED FROM EQUATION (16) USING SMALLER BIN SIZES

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,66.5],\bar{t}}$	$\Delta N_{(66.5,70],\bar{t}}$	$\Delta N_{(70,73.5],\bar{t}}$	$\Delta N_{(73.5,77],\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	5,362	-20,258	563	49,646	0.84	1.06
	(1,831)	(4,331)	(517)	(1,884)	(0.02)	(0.02)
Cubic, $K = 3$	4,270	-23,606	514	47,450	0.87	1.08
	(1,501)	(8,659)	(187)	(427)	(0.04)	(0.05)
Quartic, $K = 4$	-8,170	-10,852	3,421	48,206	0.85	1.08
	(3,253)	(6,265)	(194)	(1,177)	(0.03)	(0.03)
Quintic, $K = 5$	-5,393	-12,304	3,338	47,911	0.85	1.07
	(1,139)	(6,133)	(141)	(1,884)	(0.03)	(0.03)

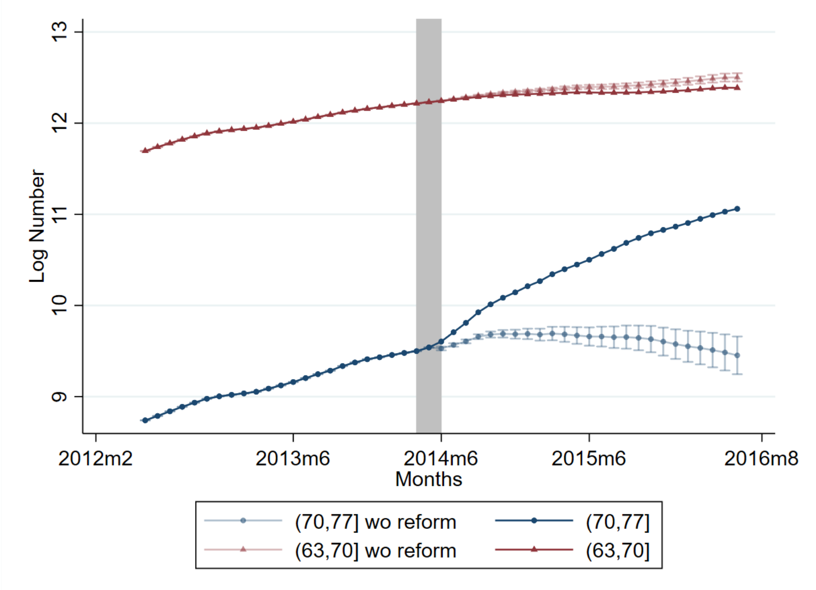
Note: Columns (1), (2), (3), and (4) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R\$(63,66.5], R\$(66.5,70], R\$(70,73.5], and R\$(73.5,77] for June 2016. Estimates are calculated using Equation (16) with various polynomial degrees $K \in \{2, 3, 4, 5\}$. Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed from the delta method using the clustered standard errors estimated in Equation (16).

C.11 Actual and Counterfactual Paths Estimated from Equation (16) with Different Polynomial Degrees



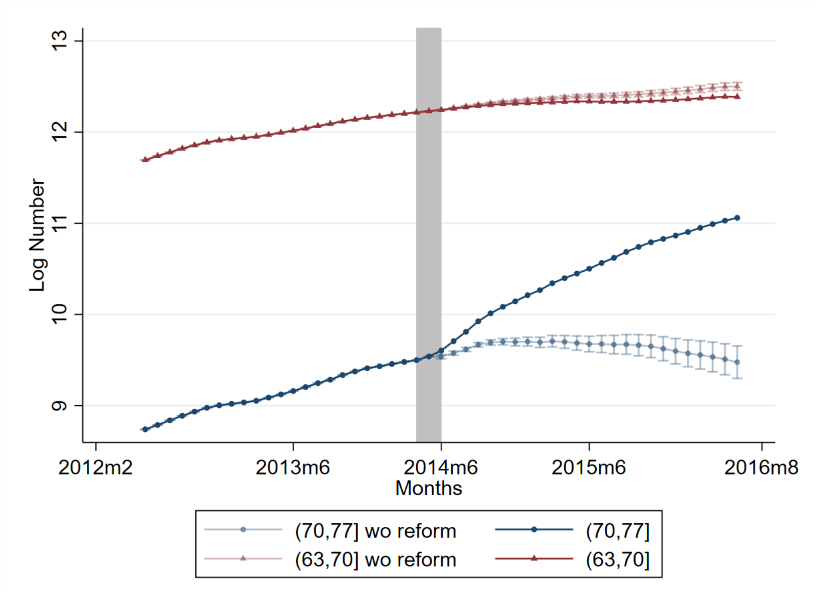
Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 2$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

FIGURE 26: ACTUAL AND COUNTERFACTUAL PATHS FOR TREATMENT BINS, $K = 2$



Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 4$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

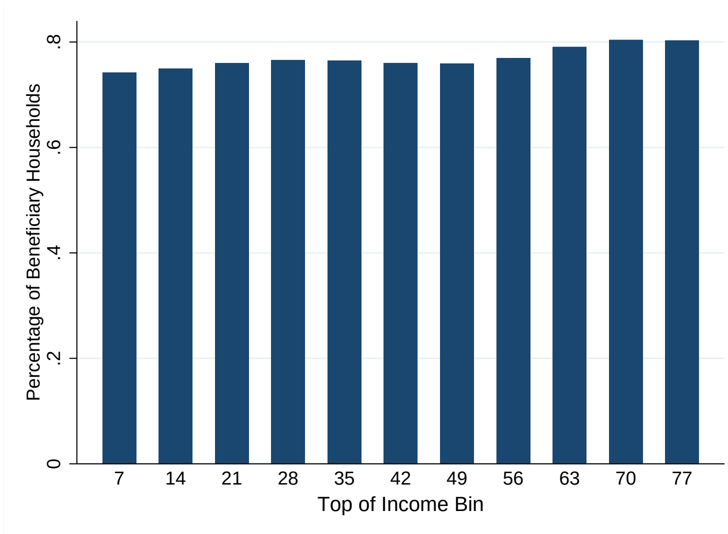
Figure 27: Actual and Counterfactual Paths for Treatment Bins, $K = 4$



Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (16) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 5$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 28: Actual and Counterfactual Paths for Treatment Bins, $K = 5$

C.12 Fraction of Households Receiving the Benefit at Different Reported Incomes



Note: This figure shows the fraction of households that receive the benefit in each seven increment reported income bin in June, 2016.

Figure 29: Fraction of Households of Receiving Benefit at Different Reported Incomes

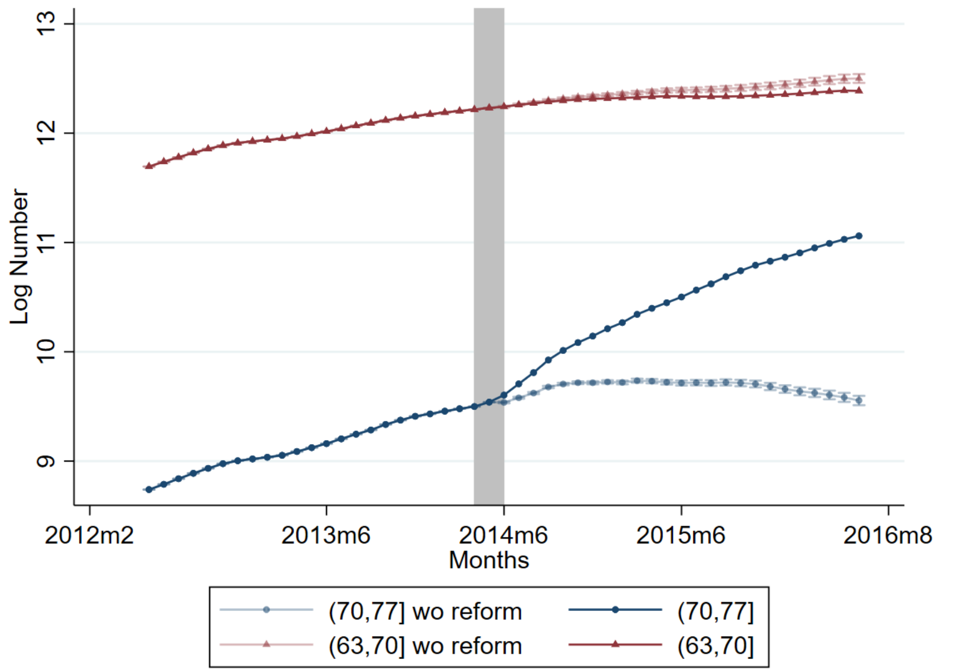
C.13 Results Using R$(56,63] as a Treatment Bin

Table 9: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation (16) Using R$(56,63] as Treatment Bin

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Polynomial Degree, K	$\Delta N_{(56,63],\bar{t}}$	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	13,173	-23,591	51,879	10,418	41,461	0.82	1.04
	(5,309)	(6,397)	(2,144)	(9,607)	(9,995)	(0.04)	(0.04)
Cubic, $K = 3$	8,579	-25,719	49,340	17,140	32,199	0.85	1.07
	(5,381)	(3,580)	(682)	(7,006)	(7,119)	(0.03)	(0.03)
Quartic, $K = 4$	9,825	-27,331	50,968	17,507	33,462	0.85	1.07
	(8,922)	(6,275)	(1,338)	(11,592)	(11,759)	(0.05)	(0.05)
Quintic, $K = 5$	12,489	-26,669	50,683	14,180	36,503	0.83	1.06
	(10,645)	(6,397)	(1,175)	(12,709)	(12,841)	(0.05)	(0.06)

Note: Columns (1) (2), and (3) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R$(56,63], R$(63,70], and R$(70,77] for June 2016: $\Delta N_{(56,63],\bar{t}}$, $\Delta N_{(63,70],\bar{t}}$, and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated using Equation (16) with various polynomial degrees $K \in \{2, 3, 4, 5\}$. Columns (4) and (5) show the estimated number of bunching and jumping households for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations (13) and (14). Columns (6) and (7) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation (16).

C.14 Results Estimated from Nonlinear Least Squares Equation (17)



Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation (17) where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

FIGURE 30: COUNTERFACTUAL PATHS FOR TREATMENT BINS FROM EQUATION (17)

Table 10: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation (17)

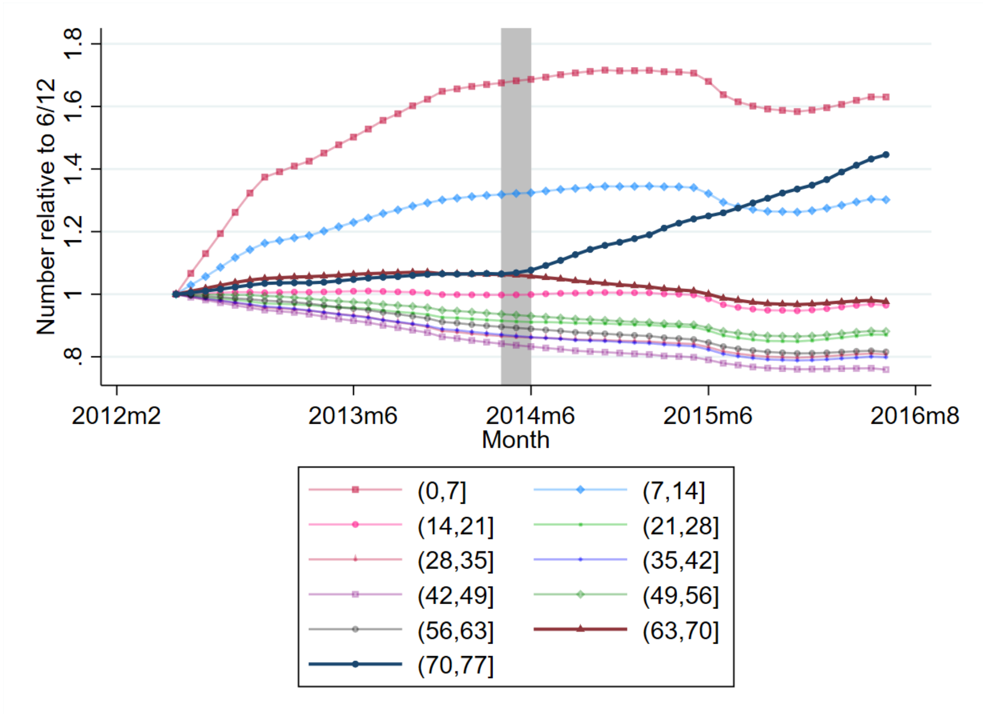
	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-29,038	50,724	29,038	21,686	0.90	1.13
	(13, 711)	(635)	(13, 711)	(13, 723)	(0.06)	(0.07)
Cubic, $K = 3$	-28,834	49,505	28,834	20,671	0.90	1.13
	(5,485)	(309)	(5, 485)	(5, 532)	(0.02)	(0.03)
Quartic, $K = 4$	-23,825	50,824	23,825	26,999	0.87	1.10
	(9,358)	(485)	(9, 358)	(9, 429)	(0.04)	(0.04)
Quintic, $K = 5$	-24,571	51,228	24,571	26,658	0.87	1.11
	(8,311)	(426)	(8,311)	(8,374)	(0.04)	(0.04)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R\$(63,70] and R\$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from the nonlinear least squares regression given by Equation (17) with various polynomial degrees $J \in \{2, 3, 4, 5\}$. Columns (3) and (4) show the estimated number of bunching and jumping households for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations (13) and (14). Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation (16).

C.15 Results for Households with Kids

The benefit schedule for families with children is substantially more complex than the benefit schedule for households without children, as discussed in detail Appendix B.2. Prior to June 2014, in addition to the guaranteed minimum income and the basic benefit, there was also a variable per-child benefit for households below the poverty threshold of R\$140 per-capita. The June 2014 reform led to changes in the levels and locations of the guaranteed minimum income kink, the basic benefit notch, and the variable benefit notch. For example, the poverty threshold was raised from R\$140 per-capita to R\$154 per-capita and the levels of the variable benefits were also increased by around 10% (see Appendix B.2 for more details).

Estimating the MVPF of the reform for households with children will require estimating the WTP for the all of the different changes to the BF schedule. Estimating the WTP for the change to the variable benefit schedule would be particularly difficult given that this benefit is made conditional on investments in children. For example, suppose parents are under-investing in their children's education and the reform increases school attendance. Then we would need to estimate the childrens' WTP for the increased education they receive as a result of the reform to the variable benefit schedule. Thus, we leave calculating the MVPF of the reform for households with children to future work. We do, however, show strong evidence that households with kids respond to the reform. In particular, Figure 31 shows *prima facie* evidence that the reform increased the number of households reporting incomes in R\$$(70, 77]$.



Note: This figure shows the number of households with kids that report incomes in the various bins. The number in each bin is normalized to 1 in June, 2012. The timing of the reform is indicated by the gray, shaded region.

Figure 31: Number of Households with Kids Reporting Incomes in Various Bins

C.16 Results for Households with Constant Composition

We also consider the possibility that some of the estimated behavioral impact of the reform for single individual households may be coming from households misreporting their family composition. For example, a two adult household can receive greater benefits if they report to be two separate one adult households as benefits are paid out per-household as opposed to per-capita. For example, a household with two adults and no children that has a combined per-capita income of R$60 is eligible for R$70 in transfers pre-reform and R$77 in transfers post-reform. However, if this household were to report that they were actually two single individual households with incomes of R$60, they would *each* be eligible for R$70 in transfers pre-reform and R$77 in transfers post-reform. Hence, the reform may have increased incentives to misreport family composition as well as income.[54] From a theoretical perspective, Proposition 3 holds even if misreporting responses occur on the family composition margin. However, such a behavioral response may affect the validity of our identification strategy (for example, it may no longer be reasonable to assume that the distribution below R$63 is unaffected by the reform if these new "single individual" households enter at income levels well below the threshold).

Thus, in this appendix, we present results for our sample of single individual households restricted

54. However, the ability of households to misreport the number of members is limited as individuals must provide government issued IDs for all family members to be on the registry. Moreover, household composition is arguably more verifiable than income for many households given the large informal sector in Brazil. Thus, we suspect that households are more likely to misreport income than family composition.

to those who do not change their reported family composition throughout the analysis period (June 2012 to June 2016). We drop any household that reports a composition change over the analysis period. Moreover, we drop any household who enters the registry post June-2014 as we cannot tell whether these new entrants are truly new households or are pre-existing households (with multiple adults) pretending to be separate households so as to increase the amount of benefits they receive. In other words, we restrict our sample to single individual households who were (a) on the registry prior to June 2014, and (b) do not report a change in composition over the analysis period. This reduces our sample from 1,938,653 single individual households with incomes below R$77 in June 2016 to 1,039,573 single individual households with incomes below R$77 in June 2016. Table 11 presents results for this exercise. We find very similar estimates for the MVPF lower bound and slightly smaller estimates for the MVPF upper bound. Note that the estimated numbers of jumpers and bunchers are smaller than in Table 3 due to the smaller sample size.

Table 11: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation (16), Constant Composition Households

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-4,714	19,019	4,714	14,305	0.88	1.04
	(3,071)	(1,106)	(3,071)	(3,404)	(0.03)	(0.02)
Cubic, $K = 3$	-6,847	17,713	6,847	10,866	0.91	1.05
	(2,345)	(619)	(2,345)	(2,524)	(0.02)	(0.02)
Quartic, $K = 4$	-862	13,722	862	12,860	0.89	1.01
	(3,483)	(1,696)	(3,483)	(3,984)	(0.03)	(0.03)
Quintic, $K = 5$	-1,586	14,679	1,586	13,094	0.89	1.01
	(3,302)	(1,188)	(3,302)	(3,661)	(0.03)	(0.03)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R$(63,70] and R$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from Equation (16) with various polynomial degrees $K \in \{2, 3, 4, 5\}$, restricting the sample to single individual households who do not change their reported household composition throughout the analysis period. Columns (3) and (4) show the estimated number of bunching and jumping households for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations (13) and (14). Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations (11) and (12). Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation (16).