Voting to Persuade*

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Abstract

We consider a model of collective persuasion, in which members of an advisory committee with private continuous signals vote on a policy change. A decision maker then decides whether to adopt the change upon observing each vote. Information transmission between the committee and the decision maker is possible if and only if there exists an informative equilibrium in which the decision maker only adopts the policy change after a unanimous vote. Similarly, full information aggregation is achievable if and only if such an equilibrium exists when the size of the committee is large enough. We further discuss why our continuous-signal model produces results different from discrete-signal models.

1 Introduction

In this paper we study a model of information transmission between an advisory committee and a decision maker (DM) who cannot commit to a decision rule. Each member of the committee receives a private continuous signal about an unknown binary state of the world and then votes

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over two policy options, to maintain the status quo or to adopt a policy change. The DM makes the decision upon observing all votes of the committee. Both the committee and the DM want to match the decision with the state, but the DM is more conservative towards making the policy change. Therefore, unless persuaded by the committee's votes to do otherwise, the DM would choose to maintain the status quo. We study the conditions under which the DM can be persuaded to adopt the policy change, i.e., information is successfully transmitted between the two parties in equilibrium.

Previous versions of this problem have been studied by Wolinsky (2002), Levit and Malenko (2011), Battaglini (2017), and Gradwohl and Feddersen (2018), who assume that the committee members receive *discrete* signals about the underlying state. A key insight of this literature is that information transmission is impossible if the conflict of interest between the committee and the DM is large enough, regardless of the committee size. Our contribution consists in studying the effect of an alternative assumption on the structure of signals, i.e., the signals of the members are *continuous*. Different from the discrete-signal models, we are able to derive the exact condition under which information transmission is possible, i.e., an equilibrium with information transmission exists *if and only if* the DM can be persuaded to adopt the policy change in an equilibrium where she adopts the unanimity rule. Since we can pinpoint the most conservative DM that can be persuaded in equilibrium with the unanimity rule, we check the existence of informative equilibria by simply looking at the level of conservativeness of the DM.

Our finding also suggests that when the decision rule is endogenous, the unanimity rule could be the only decision rule that allows information transmission when the DM is very conservative. Our result thus offers a new rationale, from the perspective of information transmission, why the unanimity rule is so prevailing in decision-making processes, even though it has shown to have poor property in aggregating information in models with exogenous decision rule (Feddersen and Pesendorfer 1998).

In deriving our main result, we also find that when restricting the decision rule of the DM to k-rules, i.e., the DM requires at least k affirmative votes to adopt the policy change, information transmission is more likely to happen in equilibrium where the DM chooses a higher k-rule. Even though this result is seemingly very intuitive, it does not hold in discrete-signal models (Battaglini

2017; Gradwohl and Feddersen 2018). In Section 4, we explore the difference between the two models and offer an explanation why this is the case.

When information can be transmitted, another concern is whether information of the committee members can be fully aggregated, i.e, whether the probability of making a mistake in the DM's decision vanishes when the size of the committee becomes large. The problem of information aggregation when the decision rule is exogenously given has been studied by Feddersen and Pesendorfer (1997, 1998), Duggan and Martinelli (2001) and Martinelli (2002). In a discrete-signal model, Feddersen and Pesendorfer (1998) show that information is not fully aggregated under the unanimity rule. Duggan and Martinelli (2001) and Martinelli (2002) consider continuous-signal models and prove that the unanimity rule can also lead to full information aggregation if the likelihood ratio of each committee member's signal is unbounded. In our model, the decision rule is endogenous. When the likelihood ratio is unbounded, given our characterization of equilibria with information transmission, full information aggregation follows directly from Duggan and Martinelli (2001). When the likelihood ratio is bounded, we show that full information aggregation is achievable as long as information transmission is possible in equilibrium, a result similar to what Battaglini (2017) derives in a discrete-signal model of persuasion.

2 Model

A committee of $\mathcal{N}$ homogeneous members advises a decision maker (DM) on the choice of two policy options, status quo N (or nay) and alternative Y (or yay). Each member i (he) receives a private signal s_i about the state $\theta \in \{y, n\}$ of the world, then votes simultaneously over the two options. The DM (she) makes the final decision $D \in \{Y, N\}$ after observing each member's vote.

Payoffs. The payoffs of the committee members and the DM depend on the DM's decision D and the state θ. We normalize the payoffs of both parties under $D = N$ to 0 in both states. Their payoffs under $D = Y$ vary with θ: For the committee members, the payoff is $-1/2$ if $\theta = n$ and is $1/2$ if $\theta = y$; for the DM, the payoff is $-\alpha$ if $\theta = n$ and is $1 - \alpha$ if $\theta = y$. The parameter $\alpha \in (1/2, 1)$ measures the conflict of interest between the DM and the committee members. With complete information about the state, all players have the same preference, i.e., they all strictly prefer Y in state y and N in state n. With incomplete information, the DM's expected payoff of choosing Y is lower than that of the committee members. Let $p \in (0,1)$ be the common prior probability of the state being y. We assume that the optimal uninformed decision of the DM is N, i.e., $\alpha > p$.[1]

Information. Each member i receives a private signal s_i that is identically and independently distributed on (a,b) conditional on the true state, with distribution F and continuous density f if $\theta = y$ and distribution G and continuous density g if $\theta = n$, where $a, b \in \mathbb{R} \cup \{-\infty, +\infty\}$.[2]

Strategies. A *voting strategy* of member i is a (measurable) function $m_i : (a,b) \rightarrow [0,1]$ that maps his signal into the probability of voting for Y. We say that a voting strategy m_i is *partisan* if $\Pr(m_i(s_i) = 1) = 1$ or $\Pr(m_i(s_i) = 0) = 1$. Otherwise, m_i is *nonpartisan.* A voting strategy m_i is *increasing* if $m_i(s_i) \geq m_i(s_i')$ for all $s_i \geq s_i'$. A voting strategy m_i is a *cutoff strategy* if there exists $s_i^* \in (a,b)$ such that $m_i(s_i) = I$ for all $s_i > s_i^*$ and $m_i(s_i) = J$ for all $s_i < s_i^*$, where $I, J \in \{0,1\}$ and $I \neq J$. Denote the strategy profile of the committee by $m := (m_1, m_2, \ldots, m_{\mathcal{N}})$.

A *decision rule* of the DM is a function $d : \{Y,N\}^{\mathcal{N}} \rightarrow \{Y,N\}$ that maps a vote profile $v := (v_1, \ldots, v_{\mathcal{N}}) \in \{Y,N\}^{\mathcal{N}}$ of the committee, with v_i being the vote of member i, into one of the two options. We assume that the DM uses a pure strategy. Allowing mixed strategies for the DM would not affect our main results.[3] We introduce here two types of decision rules that are important for our analysis. Let $|v|$ denote the number of yay votes in v. A decision rule d is a k-rule if there exists a threshold $k \in \{1, 2, ..., \mathcal{N}\}$ such that for all $v \in \{Y,N\}^{\mathcal{N}}$, $d(v) = Y$ if and only if $|v| \geq k$. A decision rule d is a *weighted voting rule* if there exists a weight profile $w = (w_1, w_2, ...w_{\mathcal{N}}) \in \mathbb{R}_+^{\mathcal{N}}$ and a quota $Q \in \mathbb{R}_+$ such that $d(v) = Y$ if and only if $\sum_{i=1}^{\mathcal{N}} w_i \mathbf{1}_{\{v_i = Y\}} \geq Q$, where $\mathbf{1}$ is the indicator function. A k-rule corresponds to a weighted voting rule in which $w_1 = ... = w_{\mathcal{N}} = 1$ and $Q = k$.

Equilibrium. We use perfect Bayesian equilibrium (PBE) as the solution concept. A PBE consists of a voting strategy profile m, a decision rule d, and a system of belief $\mu : \{Y,N\}^{\mathcal{N}} \rightarrow [0,1]$ that specifies the DM's posterior belief of the state being y for each vote profile. We assume that the DM always chooses Y when indifferent. An equilibrium is *informative* if the

[1] This assumption is inessential for our results. If $\alpha < p$, the DM will choose Y instead of N in an uninformative equilibrium. Our characterizations for informative equilibria remain valid.

[2] Given a function of s_i, if its limit at $s_i = a, b$ exists in $\mathbb{R} \cup \{-\infty, +\infty\}$, we take it as the value at $s_i = a, b$. For example, if $\lim_{s_i \rightarrow b} f(s_i)/g(s_i) = \infty$, we write $f(b)/g(b) = \infty$.

[3] The detailed proofs are available in the working paper version of this paper.

DM chooses both options with positive probabilities in equilibrium. We say that a DM *can be persuaded* if there exists an informative equilibrium. Two equilibria with strategy profiles (m, d) and (m', d') are *outcome-equivalent* if for all signal profiles $(s_1, s_2, .., s_{\mathcal{N}}) \in (a, b)^{\mathcal{N}}$, $\Pr(d(v) = Y|(s_1, s_2, .., s_{\mathcal{N}}); m) = \Pr(d'(v) = Y|(s_1, s_2, .., s_{\mathcal{N}}); m')$.

For the purpose of our analysis, we impose the following assumptions on F and G. Let $h_F(s) := f(s)/(1-F(s))$ and $h_G(s) := g(s)/(1-G(s))$ be the hazard functions of distributions F and G, respectively. Define the hazard ratio at signal s as $h_F(s)/h_G(s)$.

Assumption 1 (MLRP) *F and G satisfy the strict monotone likelihood ratio property (MLRP), i.e., $f(s)/g(s)$ is strictly increasing in s.*

Assumption 2 *The likelihood ratio of the members' signals satisfies $\frac{f(a)}{g(a)} < \frac{1-p}{p} < \frac{f(b)}{g(b)}$.*

Assumption 3 (IHRP) *F and G satisfy the strict increasing hazard ratio property (IHRP), i.e., $h_F(s)/h_G(s)$ is strictly increasing in s.*

Assumption 1 is standard in the literature and ensures that a higher signal is more indicative of the state being y. It is well known that MLRP implies that (i) $(1 - F(s))/(1 - G(s))$ and $F(s)/G(s)$ are strictly increasing in s, (ii) $F(s)/G(s) < f(s)/g(s) < (1 - F(s))/(1 - G(s))$ for all $s \in (a, b)$, and (iii) $F(s) < G(s)$ for all $s \in (a, b)$. See Appendix B of Krishna (2009) for the proofs.

Assumption 2 is employed by Duggan and Martinelli (2001). It ensures that a committee member who behaves "naively", i.e., as if his vote alone determines the outcome, will vote for N (Y) after receiving a signal that is low (high) enough.

Assumption 3 is a regularity assumption that has been studied by Duggan and Martinelli (2001) in the context of a decision-making committee. It was also shown to be an important condition in observational learning models (Herrera and Hörner 2011, 2013). Most but not all distributions commonly used in economics and political science satisfy IHRP.[4] For example, if both F and G are normal distributions that satisfy MLRP, then they satisfy IHRP.

[4] See Herrera and Hörner (2011) for a discussion and a list of distributions that satisfy IHRP. A notable case that fails IHRP is the exponential distribution, whose hazard ratio is a constant (Duggan and Martinelli 2001; Herrera and Hörner 2011). It is thus a knife edge case.

3 Equilibrium analysis

In this section, we first establish, for each $k \in \{1, .., \mathcal{N}\}$, the necessary and sufficient condition for the existence of an informative equilibrium with the corresponding k-rule (Proposition 2). Then, we show that there exists an informative equilibrium if and only if there exists an informative equilibrium with the unanimity rule (Proposition 3). By considering the asymptotic version of the latter condition, we derive the necessary and sufficient condition for full information aggregation in a large committee (Proposition 4).

We begin our analysis by showing that it is without loss to focus on equilibria in which the committee members use either cutoff strategies or partisan strategies and the DM uses a weighted voting rule. This greatly reduces the space of equilibrium strategies we need to consider.

Proposition 1 *For any equilibrium, there exists an outcome-equivalent equilibrium, in which (i) each member's voting strategy is either an increasing cutoff strategy or a partisan strategy, and (ii) the DM's decision rule d is a weighted voting rule.*

In this model, there always exists an uninformative equilibrium in which the DM maintains the status quo and ignores the committee's votes. We focus instead on informative equilibria in the rest of the paper.

3.1 Informative equilibria

We first consider symmetric informative equilibria in which the DM's decision rule is a k-rule. In such an equilibrium, the committee members must use a cutoff strategy. A committee member with the cutoff signal s^* must be indifferent between voting for Y and N conditional on being pivotal, i.e., $|v_{-i}| = k - 1$. Thus, the cutoff signal s^* solves the equation

$$\frac{p}{1-p}\left(\frac{1-F(s)}{1-G(s)}\right)^{k-1}\left(\frac{F(s)}{G(s)}\right)^{\mathcal{N}-k}\frac{f(s)}{g(s)} = 1. \tag{1}$$

To understand (1), note that $p/(1-p)$ is the prior likelihood ratio of the state (state y versus state n), $(1-F(s))/(1-G(s))$ is the likelihood ratio of a yay vote, $F(s)/G(s)$ is the likelihood ratio of a nay vote, and $f(s)/g(s)$ is the likelihood ratio of signal s. By Bayes' rule, the product of these terms on the left-hand side of (1) is the posterior likelihood ratio of the state conditional on a committee member receiving a signal s, knowing that the other members cast $k-1$ yay votes and $\mathcal{N}-k$ nay votes. For a member with the cutoff signal s^*, this posterior likelihood ratio must equal 1, so that he is indifferent between N and Y.

By MLRP, the left-hand side of (1) is strictly increasing in s. Combined with Assumption 2, this implies that (1) has a unique solution. Denote the unique solution by $s(k,\mathcal{N})$. In a symmetric equilibrium with k-rule, the DM optimally chooses Y if $|v| \geq k$, and N if $|v| < k$. Given the voting cutoff $s^* = s(k,\mathcal{N})$, this implies that

$$\frac{p}{1-p}\left(\frac{1-F(s^*)}{1-G(s^*)}\right)^{k-1}\left(\frac{F(s^*)}{G(s^*)}\right)^{\mathcal{N}-k+1} < \frac{\alpha}{1-\alpha} \leq \frac{p}{1-p}\left(\frac{1-F(s^*)}{1-G(s^*)}\right)^{k}\left(\frac{F(s^*)}{G(s^*)}\right)^{\mathcal{N}-k}, \quad (2)$$

where the lower bound is the posterior likelihood ratio of the state when $|v| = k-1$, the upper bound is the posterior likelihood ratio when $|v| = k$, and the term $\alpha/(1-\alpha)$ is the posterior likelihood ratio that makes the DM indifferent between Y and N. Since $s^* = s(k,\mathcal{N})$ solves equation (1), condition (2) can be reformulated as

$$\frac{F(s^*)}{G(s^*)}\frac{g(s^*)}{f(s^*)} < \frac{\alpha}{1-\alpha} \leq \frac{(1-F(s^*))}{(1-G(s^*))}\frac{g(s^*)}{f(s^*)}. \quad (3)$$

By MLRP, $F(s^*)/G(s^*) < f(s^*)/g(s^*)$, so the left inequality of (3) always holds for $\alpha > 1/2$. Therefore, for a given k, the right inequality in (3) is necessary and sufficient for the existence of a *symmetric* informative equilibrium with the corresponding k-rule. The following proposition shows that this condition applies more generally without the symmetry restriction.

Proposition 2 *For each $k \in \{1,2,...,\mathcal{N}\}$, an informative equilibrium with k-rule exists if and only if $\alpha \leq \boldsymbol{\alpha}(k,\mathcal{N})$, where $\boldsymbol{\alpha}(k,\mathcal{N})$ is the unique solution to*

$$\frac{\alpha}{1-\alpha} = \frac{h_G(s(k,\mathcal{N}))}{h_F(s(k,\mathcal{N}))}.$$

The idea behind the proof of Proposition 2 is that in an asymmetric informative equilibrium with k-rule, among all the vote profiles with k yay votes, some induce a lower posterior belief of the state being y than others. IHRP ensures that at least one of these vote profiles is less indicative of the state being y than k yay votes under the symmetric informative equilibrium with k-rule. As a result, $\alpha < \boldsymbol{\alpha}(k, \mathcal{N})$. Thus, the value $\boldsymbol{\alpha}(k, \mathcal{N})$ represents the most conservative DM that may adopt k-rule in *any* informative equilibrium when the committee size is $\mathcal{N}$.

Next, we consider the effect of varying k. When the DM adopts a higher k rule, the members cast yay votes more often in the symmetric informative equilibrium. This means that $s(k, \mathcal{N})$ is strictly decreasing in k. IHRP then implies that $\boldsymbol{\alpha}(k, \mathcal{N})$ is strictly increasing in k. It then follows from Proposition 2 that a higher k-rule can sustain information transmission for a larger set of α.

Corollary 1 *For all $k' > k$, there exists an informative equilibrium with k'-rule if there exists an informative equilibrium with k-rule, but the converse is in general not true.*

Corollary 1 thus rationalizes the pressure for a higher level of consensus in many institutions. To understand Corollary 1, note that, for k-rules, when the threshold k increases, there are two opposite effects on $\boldsymbol{\alpha}(k, \mathcal{N})$. One is a direct *consensus* effect. An increase in k means that the DM asks for a higher consensus level among the members to choose Y. Fixing the voting strategies of the committee members, a higher consensus level is more indicative of the state being y. The other is an indirect *strategic* effect. Because the members cast yay votes more often, each yay vote is now less indicative of the state being y. IHRP implies that the consensus effect dominates the strategic effect. As a result, $\boldsymbol{\alpha}(k, \mathcal{N})$ is strictly increasing in k.

Corollary 1 implies that the unanimity rule can sustain information transmission for a larger set of α than any other k-rules. Our next result shows that the latter fact is in fact true not only for k-rules, but for all decision rules.

Proposition 3 *There exists an informative equilibrium if and only if $\alpha \leq \boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$.*

Proposition 3 provides a simple way to check, for a committee of any size, if information transmission is possible. The basic idea underlying this result is that in an equilibrium with asymmetric voting, IHRP ensures that among all the vote profiles that induce the DM to choose Y, we can find one that leads to a lower posterior belief of the state being y than a unanimous vote in the symmetric informative equilibrium with the unanimity rule.

Proposition 3 also provides a tight upper bound for the degree of conflict of interest between the DM and the committee members that allows information transmission. The upper bound $\boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$ is achievable only by the unanimity rule. Gradwohl and Feddersen (2018) also derive an upper bound in a binary-signal model, but their upper bound is not achievable by the unanimity rule. In a general discrete-signal Poisson game, Battaglini (2017) proves the existence of an upper bound, but does not explicitly characterize it. In Section 4, we discuss the differences between the continuous-signal model and discrete-signal models in more detail.

Proposition 3 also indicates that the existence of an informative equilibrium depends on the size of the committee. Because $\boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$ strictly increases with $\mathcal{N}$, for all $\mathcal{N}' > \mathcal{N}$, if an informative equilibrium exists for a size-$\mathcal{N}$ committee, an informative equilibrium exists for a size-$\mathcal{N}'$ committee, but the converse is in general not true. For large committee, we have

Corollary 2 *There exists an informative equilibrium when the committee size is sufficiently large if and only if $\alpha < \bar{\boldsymbol{\alpha}}$, where*

$$\bar{\boldsymbol{\alpha}} := \lim_{\mathcal{N} \to \infty} \boldsymbol{\alpha}(\mathcal{N}, \mathcal{N}) = \frac{g(a)/f(a)}{1 + g(a)/f(a)}. \tag{4}$$

The corollary follows from the fact that $s(\mathcal{N}, \mathcal{N})$ converges to a as $\mathcal{N}$ goes to infinity. From (4), we can see that if the committee members' signals induce an unbounded likelihood ratio at a, i.e., $g(a)/f(a) = \infty$, then $\bar{\boldsymbol{\alpha}} = 1$, which means that any DM can be persuaded by a committee that is sufficiently large. If $g(a)/f(a) < \infty$, then $\bar{\boldsymbol{\alpha}} < 1$. As a result, a DM with $\alpha \in [\bar{\boldsymbol{\alpha}}, 1)$ can never be persuaded no matter how large the committee is.

3.2 Information aggregation

Corollary 2 provides the exact condition under which an informative equilibrium exists when the size of the committee is large. However, the existence of an informative equilibrium does not guarantee full aggregation of the committee members' private information in the DM's decision in equilibrium. Indeed, if we restrict to the informative equilibrium with the unanimity rule, when $g(a)/f(a) < \infty$, full information aggregation is not achievable as the size of the committee becomes infinitely large (Duggan and Martinelli 2001, Theorem 4).[5] However, if we consider informative equilibria with non-unanimity rules, full information aggregation is always achievable as long as informative equilibria exist for a sufficiently large committee.

Proposition 4 *There exists a sequence of equilibria along which the probabilities of the DM choosing Y in state y and N in state n approach 1 as $\mathcal{N} \to \infty$ if and only if $\alpha < \bar{\boldsymbol{\alpha}}$.*

Proposition 4 states that the condition $\alpha < \bar{\boldsymbol{\alpha}}$ is *necessary and sufficient* for full information aggregation. The necessity follows directly from Corollary 2, since uninformative equilibria can never aggregate information. To establish the sufficiency, for $\alpha < \bar{\boldsymbol{\alpha}}$, we show in the proof that for any $\alpha < \bar{\boldsymbol{\alpha}}$, we can find $q \in (0,1)$ such that $\alpha < \lim_{\mathcal{N}\to\infty} \boldsymbol{\alpha}(q\mathcal{N}, \mathcal{N}) < \bar{\boldsymbol{\alpha}}$. By Proposition 2, an informative equilibrium with $q\mathcal{N}$-rule exists when $\mathcal{N}$ is large enough. By Theorem 5 of Duggan and Martinelli (2001), full information aggregation is achieved as $\mathcal{N} \to \infty$. Thus, even though the mistake probabilities do not go to 0 under the unanimity rule, when $\alpha < \bar{\boldsymbol{\alpha}}$, we can always find a sequence of k-rules that are close enough to the unanimity rule, along which the mistake probabilities converge to 0 as $\mathcal{N} \to \infty$.

4 Discrete signals

In this section, we study an alternative model in which the committee members receive discrete signals instead of continuous signals and illustrate why our characterizations apply to the continuous-signal model but not to the discrete-signal model.[6]

Suppose each member i receives a private signal $s_i \in \{t_1, t_2, ...t_M\}$, where $M \geq 2$ is the number of possible signal realizations. Let $r_F(t_m)$ and $r_G(t_m)$ be the probabilities that $s_i = t_m$ when the state is y and n, respectively. Define the hazard functions as $h_F(t_m) := r_F(t_m) / \sum_{l=m}^{M} r_F(t_l)$ and $h_G(t_m) := r_G(t_m) / \sum_{l=m}^{M} r_G(t_l)$. The assumptions of MLRP and IHRP become:

Assumption 4 (MLRP) *F and G satisfy the strict monotone likelihood ratio property (MLRP), i.e., $r_F(t_m) / r_G(t_m)$ is strictly increasing in m.*

[5] Following Battaglini (2017), we say that *full information aggregation is achievable* if there exists a sequence of equilibria such the probability that the DM makes mistakes converges to 0 as the committee size goes to infinity.

[6] Battaglini (2017) adopts a very similar information structure in a Poisson voting game. However, he does not investigate how the existence condition of the symmetric informative equilibrium with k-rule changes with k, which is mainly discussed in this section.

Assumption 5 (IHRP) *F and G satisfy the strict increasing hazard ratio property (IHRP), i.e., $h_F(t_m)/h_G(t_m)$ is strictly increasing in m.*

We focus on symmetric voting. In this case, the equilibrium decision rule in an informative equilibrium must be a k-rule. Define $\boldsymbol{\alpha}_M(k,\mathcal{N})$ as the solution to $\frac{\alpha}{1-\alpha} = \frac{\Pr(|v|=k|\theta=y)}{\Pr(|v|=k|\theta=n)}$. Thus, $\boldsymbol{\alpha}_M(k,\mathcal{N})$ is the upper bound of α such that a symmetric informative equilibrium with k-rule exists.

In proving Corollary 1, we show that $\boldsymbol{\alpha}(k,\mathcal{N})$ is strictly increasing in k. This is, however, not true for $\boldsymbol{\alpha}_M(k,\mathcal{N})$. As a result, Propositions 3 and 4 do not apply to the discrete-signal model. To see that, consider a symmetric informative equilibrium with k-rule. Suppose the committee members are indifferent after receiving t_m. Then, the posterior likelihood ratio given $|v|=k$ is given by

$$\frac{\Pr(|v|=k|\theta=y)}{\Pr(|v|=k|\theta=n)} = \underbrace{\frac{r_G(t_m)}{r_F(t_m)}}_{\text{anti-signal } t_m^-} \times \underbrace{\frac{r_F(t_m)\,\rho_m(k,\mathcal{N}) + \sum_{l=m+1}^{M} r_F(t_l)}{r_G(t_m)\,\rho_m(k,\mathcal{N}) + \sum_{l=m+1}^{M} r_G(t_l)}}_{\text{a yay vote}}, \tag{5}$$

where $\rho_m(k,\mathcal{N})$ is the probability that a member votes for Y after receiving the indifferent signal t_m. Given signal t_m, consider a hypothetical signal t_m^- such that $r_F(t_m^-)/r_G(t_m^-) = r_G(t_m)/r_F(t_m)$, that is, the signal t_m^- cancels the signal t_m exactly. We call t_m^- the *anti-signal* of signal t_m. As indicated in (5), the the posterior likelihood ratio of vote profile v with $|v|=k$ is equal to the product of the likelihood ratios of the anti-signal t_m^- and a yay vote.

As k increases, intuitively the committee members vote for Y more often in equilibrium. The increase in the probability of a yay vote could be associated with an increase in $\rho_m(k,\mathcal{N})$ with the same indifferent signal or a lower indifferent signal.[7] These two changes potentially have

[7] For illustrative purposes, we focus on situations where the committee members are indifferent between Y and N after receiving some signal. When the committee members are never indifferent, (5) does not apply and the behavior of $\boldsymbol{\alpha}_M(k,\mathcal{N})$ is less regular. However, suppose the committee members strictly prefer to vote for Y after receiving signal t_m but strictly prefer to vote for N after receiving signal t_{m-1}. Then, the posterior likelihood ratio is bounded below by the inverse of the hazard ratio, i.e.,

$$\frac{\Pr(|v|=k|\theta=y)}{\Pr(|v|=k|\theta=n)} \geq \frac{r_G(t_m)}{r_F(t_m)} \frac{\sum_{l=m}^{M} r_F(t_l)}{\sum_{l=m}^{M} r_G(t_l)} = \frac{h_G(t_m)}{h_F(t_m)}.$$

By IHRP, $h_G(t_m)/h_F(t_m)$ is strictly decreasing in m. This suggests that, as k increases and the cutoff signal t_m decreases, this lower bound rises and $\boldsymbol{\alpha}_M(k,\mathcal{N})$ could exhibit an upward trend as in the continuous-signal case, even when the committee members are not indifferent at any signal (see Figure 1).

opposing effects on the likelihood ratio $\frac{\Pr(|v|=k|\theta=y)}{\Pr(|v|=k|\theta=n)}$. When the indifferent signal t_m is unchanged as k increases, $\rho_m(k,\mathcal{N})$ increases. By MLRP, the likelihood ratio of a yay vote in (5) decreases, so the value of $\boldsymbol{\alpha}_M(k,\mathcal{N})$ *decreases*. If the indifferent signal t_m decreases, the likelihood ratio of the anti-signal t_m^- in (5) increases according to MLRP while the likelihood ratio of a yay vote decreases. The overall effect on $\boldsymbol{\alpha}_M(k,\mathcal{N})$ is ambiguous.

In contrast, when the signals are continuous, as k increases, the indifferent/cutoff signal always decreases and IHRP makes sure that the increase in the likelihood ratio of the anti-signal dominates the decrease in the likelihood ratio of a yay vote, resulting in an increase in $\boldsymbol{\alpha}(k,\mathcal{N})$. The discrete analogue of IHRP, however, is insufficient to determine the overall effect on $\boldsymbol{\alpha}_M(k,\mathcal{N})$.

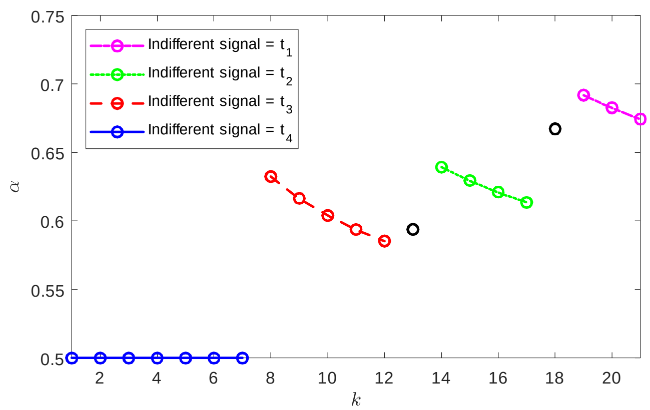

Figure 1: The function $\boldsymbol{\alpha}_M(k,\mathcal{N})$.

Parameters: $p = 1/2$, $\mathcal{N} = 21$, $r_F = (1/8, 3/16, 1/4, 7/16)$, $r_F = (1/4, 1/4, 1/4, 1/4)$.

Figure 1 illustrates these two effects graphically. Notice first that since $\boldsymbol{\alpha}_M(21,21) < \boldsymbol{\alpha}_M(19,21)$, a symmetric informative equilibrium with other k-rules could exist even when the symmetric informative equilibrium with the unanimity rule does not. Consider next $8 \le k \le 12$. For these values of k, the committee members mix at signal t_3 and, as a result, only the effect of decreasing $\rho_m(k,21)$ is present and $\boldsymbol{\alpha}_M(k,21)$ decreases with k. However, if we compare $k = 12$, $k = 17$, and $k = 21$, we have $\boldsymbol{\alpha}_M(12,21) < \boldsymbol{\alpha}_M(17,21) < \boldsymbol{\alpha}_M(21,21)$. Since the committee members are mixing at different signals, both effects are present. In this comparison, the effect of the decreasing indifferent signal dominates, and $\boldsymbol{\alpha}_M(k,21)$ increases with k, similar to the continuous-signal case. If we consider a sequence of signal structures converging to a continuous-signal structure that satisfies Assumptions 1–3, $\boldsymbol{\alpha}_M(k, \mathcal{N})$ would be increasing in k in the limit.

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Appendix

Proof of Proposition 2. We have already shown in the main text that a symmetric informative equilibrium with k-rule exists if and only if $\alpha \leq \boldsymbol{\alpha}(k, \mathcal{N})$. To complete the proof, we only need to show that if an asymmetric informative equilibrium with k-rule exists, then a symmetric informative equilibrium with k-rule exists.

By Proposition 1, it is without loss to assume that each member's voting strategy is either an increasing cutoff strategy or a partisan strategy. If a member uses a partisan strategy, it effectively changes the committee size to $\mathcal{N} - 1$ and the decision rule to either $(k-1)$-rule or k-rule. Thus, it is without loss to assume that all committee members use increasing cutoff strategies in equilibrium. (If $k \leq 0$ or $k > \mathcal{N}$, then the equilibrium is not informative.)

Consider an asymmetric informative equilibrium (s^*, d), where d is a k-rule. If for all $j \in \{1, 2, .., \mathcal{N}\}$, $s_j^* \leq s(k, \mathcal{N})$, then by MLRP, for all $v \in \{Y, N\}^{\mathcal{N}}$ such that $|v| = k$,

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p} \frac{\Pr(v|\theta = y)}{\Pr(v|\theta = n)} \leq \frac{p}{1-p} \left(\frac{1 - F(s(k, \mathcal{N}))}{1 - G(s(k, \mathcal{N}))}\right)^k \left(\frac{F(s(k, \mathcal{N}))}{G(s(k, \mathcal{N}))}\right)^{\mathcal{N}-k}.$$

Thus, a symmetric informative equilibrium with k-rule exists.

Suppose that for some member i, $s_i^* > s(k, \mathcal{N})$. Since for every $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $|v_{-i}| = k - 1$, the profile (Y, v_{-i}) induces the DM to choose Y in equilibrium, we have, for all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $|v_{-i}| = k - 1$,

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p} \frac{\Pr(v_{-i}|\theta = y)(1 - F(s_i^*))}{\Pr(v_{-i}|\theta = n)(1 - G(s_i^*))}.$$

The optimality of cutoff s_i^* implies that there exists $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $|v_{-i}| = k - 1$ and

$$\frac{p}{1-p} \frac{\Pr(v_{-i}|\theta = y)}{\Pr(v_{-i}|\theta = n)} \leq \frac{g(s_i^*)}{f(s_i^*)}.$$

Therefore,

$$\frac{\alpha}{1-\alpha} \leq \frac{g(s_i^*)(1 - F(s_i^*))}{f(s_i^*)(1 - G(s_i^*))} < \frac{g(s(k, \mathcal{N}))(1 - F(s(k, \mathcal{N})))}{f(s(k, \mathcal{N}))(1 - G(s(k, \mathcal{N})))},$$

where the last inequality follows from IHRP and the fact that $s_i^* > s(k, \mathcal{N})$. This implies that a symmetric informative equilibrium with k-rule exists. ■

Proof of Corollary 1. As discussed in the main text, we only need to show that $s(k,\mathcal{N})$ is strictly decreasing in k. For all $s \in (a,b)$, we have

$$\begin{aligned}\frac{(1-F(s))^{k-1}F(s)^{\mathcal{N}-k}}{(1-G(s))^{k-1}G(s)^{\mathcal{N}-k}} &= \frac{(1-F(s))^{k}F(s)^{\mathcal{N}-(k+1)}}{(1-G(s))^{k}G(s)^{\mathcal{N}-(k+1)}}\frac{(1-G(s))F(s)}{(1-F(s))G(s)} \\ &< \frac{(1-F(s))^{k}F(s)^{\mathcal{N}-(k+1)}}{(1-G(s))^{k}G(s)^{\mathcal{N}-(k+1)}},\end{aligned}$$

where the inequality follows from MLRP, so the left-hand side of (1) is strictly increasing in k. Also by MLRP, the left-hand side of (1) is strictly increasing in s. Thus, to satisfy (1), it must be the case that $s(k+1,\mathcal{N}) < s(k,\mathcal{N})$. ■

Proof of Proposition 3. To prove this proposition, we only need to show that the existence of an asymmetric informative equilibrium implies the existence of an informative symmetric equilibrium with the unanimity rule. Consider an asymmetric informative equilibrium. By Proposition 1, it is without loss to assume that every committee member uses either an increasing cutoff strategy or a partisan strategy. Consider first that no committee member uses a partisan strategy. The equilibrium in this case is characterized by the pair (s^*,d) alone, where $s^* \in (a,b)^{\mathcal{N}}$ is a cutoff profile and d is a weighted voting rule.

In an asymmetric equilibrium (s^*,d), if for all $j \in \{1,2,..,\mathcal{N}\}$, $s_j^* \leq s(\mathcal{N},\mathcal{N})$, then by MLRP, for all $v \in \{Y,N\}^{\mathcal{N}}$, we have

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p}\frac{\Pr(v|\theta=y)}{\Pr(v|\theta=n)} \leq \frac{p}{1-p}\left(\frac{1-F(s(\mathcal{N},\mathcal{N}))}{1-G(s(\mathcal{N},\mathcal{N}))}\right)^{\mathcal{N}}.$$

Thus, a symmetric informative equilibrium with the unanimity rule exists.

Suppose that there exists a member i with $s_i^* > s(\mathcal{N},\mathcal{N})$. In equilibrium, since for every $v_{-i} \in \boldsymbol{piv}_i$, the profile (Y,v_{-i}) induces the DM to choose Y, we have that for all $v_{-i} \in \boldsymbol{piv}_i$,

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p}\frac{\Pr(v_{-i}|\theta=y)(1-F(s_i^*))}{\Pr(v_{-i}|\theta=n)(1-G(s_i^*))}.$$

The optimality of the cutoff s_i^* implies that there exists $v_{-i} \in \boldsymbol{piv}_i$,

$$\frac{p}{1-p}\frac{\Pr(v_{-i}|\theta=y)}{\Pr(v_{-i}|\theta=n)} \leq \frac{g(s_i^*)}{f(s_i^*)}.$$

Therefore,

$$\frac{\alpha}{1-\alpha} \leq \frac{g(s_i^*)(1-F(s_i^*))}{f(s_i^*)(1-G(s_i^*))} < \frac{g\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\left(1-F(s\left(\mathcal{N},\mathcal{N}\right))\right)}{f\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\left(1-G\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\right)},$$

where the last inequality follows from IHRP and the fact that $s_i^* > s\left(\mathcal{N},\mathcal{N}\right)$. This implies that a symmetric informative equilibrium with the unanimity rule exists.

Finally, suppose in equilibrium some committee members use partisan strategies. For these members, their votes do not depend on the signals received. The other members and the DM behave as if the partisan voters are absent. Hence, dropping the partisan members out of the committee could generate the same equilibrium outcome. This means that the existence of partisan committee members effectively reduces the committee size. Thus, to complete the proof, we only need to show that $\boldsymbol{\alpha}(\mathcal{N},\mathcal{N})$ is strictly increasing in $\mathcal{N}$.

From the definition of $s\left(\mathcal{N},\mathcal{N}\right)$, we have

$$\frac{p}{1-p}\left(\frac{1-F\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}{1-G\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}\right)^{\mathcal{N}-1} = \frac{g\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}{f\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}. \tag{6}$$

MLRP implies that $s\left(\mathcal{N},\mathcal{N}\right)$ is strictly decreasing in $\mathcal{N}$. By IHRP, $\boldsymbol{\alpha}\left(\mathcal{N},\mathcal{N}\right)$ is strictly increasing in $\mathcal{N}$. ■

Proof of Corollary 2. We first show $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N},\mathcal{N}\right) = a$. Since $s\left(\mathcal{N},\mathcal{N}\right)$ is strictly decreasing in $\mathcal{N}$, $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N},\mathcal{N}\right)$ exists. Let $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N},\mathcal{N}\right) := \underline{s}$. If $\underline{s} > a$, then $\lim_{\mathcal{N}\to\infty}\frac{g(s(\mathcal{N},\mathcal{N}))}{f(s(\mathcal{N},\mathcal{N}))} = \frac{g(\underline{s})}{f(\underline{s})} < \frac{g(a)}{f(a)} \leq \infty$, and

$$\lim_{\mathcal{N}\to\infty}\left(\frac{1-F\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}{1-G\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}\right)^{\mathcal{N}-1} = \lim_{\mathcal{N}\to\infty}\left(\frac{1-F\left(\underline{s}\right)}{1-G\left(\underline{s}\right)}\right)^{\mathcal{N}-1} = \infty,$$

which violates (6). Therefore, $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N},\mathcal{N}\right) = a$. Then, we have,

$$\lim_{\mathcal{N}\to\infty}\frac{h_G\left(s\left(\mathcal{N},\mathcal{N}\right)\right)}{h_F\left(s\left(\mathcal{N},\mathcal{N}\right)\right)} = \lim_{s\to a}\frac{h_G\left(s\right)}{h_F\left(s\right)} = \lim_{s\to a}\frac{g\left(s\right)}{f\left(s\right)}\frac{1-F\left(s\right)}{1-G\left(s\right)} = \frac{g\left(a\right)}{f\left(a\right)},$$

This implies $\boldsymbol{\alpha}(\mathcal{N},\mathcal{N}) \to \bar{\boldsymbol{\alpha}} := \frac{g(a)}{f(a)} / \left(1 + \frac{g(a)}{f(a)}\right)$ as $\mathcal{N} \to \infty$. ■

Proof of Proposition 4. As discussed in the main text, we only need to show that

$$\lim_{q\to 1}\lim_{\mathcal{N}\to\infty} \boldsymbol{\alpha}(q\mathcal{N},\mathcal{N}) = \bar{\boldsymbol{\alpha}},$$

which would imply that we can always find $q \in (0,1)$ such that $\alpha < \lim_{\mathcal{N}\to\infty} \boldsymbol{\alpha}(q\mathcal{N},\mathcal{N})$ whenever $\alpha < \bar{\boldsymbol{\alpha}}$. To simplify the discussion, we follow Duggan and Martinelli (2001) and consider only combinations of q and $\mathcal{N}$ so that $q\mathcal{N}$ is an integer. Let $s^{\infty}(q) := \lim_{\mathcal{N}\to\infty} s(q\mathcal{N},\mathcal{N})$. Since the hazard ratio $\frac{h_G(s)}{h_F(s)}$ is continuous in s, it suffices to show that $\lim_{q\to 1} s^{\infty}(q) = a$. For all $q \in (0,1)$ and $s \in (a,b)$, define

$$L(q,s) := \left(\frac{1-F(s)}{1-G(s)}\right)^{q}\left(\frac{F(s)}{G(s)}\right)^{1-q}.$$

Duggan and Martinelli (2001) show that for all $q \in (0,1)$, $s^{\infty}(q)$ is the solution to $L(q,s) = 1$. By MLRP, for all $q \in (0,1)$, $L(q,s)$ is strictly increasing in q and s. For all $q \in (0,1)$, $s^{\infty}(q)$ is strictly decreasing in q. Thus, $\lim_{q\to 1} s^{\infty}(q)$ exists. Suppose $\lim_{q\to 1} s^{\infty}(q) > a$. Then,

$$\lim_{q\to 1} L(q, s^{\infty}(q)) = \frac{1-F(\lim_{q\to 1} s^{\infty}(q))}{1-G(\lim_{q\to 1} s^{\infty}(q))} > 1,$$

which is a contradiction.

For any $\alpha < \bar{\boldsymbol{\alpha}}$, the proof above implies that we can find a sufficiently large $q \in (0,1)$ and an $\hat{\mathcal{N}}$ such that for any $\mathcal{N} > \hat{\mathcal{N}}$, $\boldsymbol{\alpha}(q\mathcal{N},\mathcal{N}) > \alpha$. By Proposition 2, for any $\mathcal{N} > \hat{\mathcal{N}}$, an informative equilibrium with $q\mathcal{N}$-rule exists. By Theorem 5 of Duggan and Martinelli (2001), full information aggregation is achieved in the limit as $\mathcal{N} \to \infty$. ■