

Inequality and Asset Prices during Sudden Stops [†]

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Abstract

This paper studies the cross-sectional dimension of Fisher’s debt-deflation mechanism that triggers endogenous financial crises of the Sudden Stop type. Analyzing microdata from Mexico, we show that this mechanism’s cross-sectional dimension has macroeconomic implications that operate via two opposing effects, making the role of inequality during crises ambiguous. We propose a small open economy, asset-pricing model with heterogeneous-agents and aggregate risk to measure the effects of inequality on the severity of financial crises. In contrast to the representative-agent framework, the model with heterogeneity generates persistent current account reversals with smaller drops in asset prices and larger drops in consumption driven by the leveraged households. Moreover, calibrating the heterogeneous-agents model to an advanced economy where the dividend risk, which drives wealth inequality, is one-half of that in the benchmark emerging-markets model, we find that larger debt positions are supported and Sudden Stop crises are less severe, as observed in the data.

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1 Introduction

In the past 40 years, 58 financial crises of the Sudden Stop type have occurred in both emerging and developed economies, each characterized by episodes of a large reversal in the current account deficit.¹ The occurrence of these crises has led to a vast literature that studies Sudden Stops using models with financial frictions but assuming a representative-agent framework. In this paper, we argue that inequality in wealth and leverage across households plays an important role in determining the aggregate effects of a financial crisis.² Specifically, an economy's aggregate exposure to tighter financial conditions depends on the share of financially vulnerable households, defined as those that end up constrained when the crisis happens. Sudden Stops are characterized by large declines in asset prices, which affect households differently depending on their balance sheet. For example, microdata evidence from Mexico (an open economy commonly used to study Sudden Stops) shows that during the 2009 crisis, households in the top decile of leverage decreased their expenditures 6.2 percent, while households in the bottom decile increased their expenditures 5.4 percent. Moreover, the asset holdings of wealthy households with leverage in the bottom eight deciles increased 59.4 percent, while wealthy households in the top two deciles of leverage fire-sold and decreased their assets the most during the crisis. Hence, studying only aggregate dynamics misses the fact that financial crises do not affect all households in the same way and that inequality has aggregate implications.

This paper addresses this issue by examining the cross-sectional dimension of the debt-deflation mechanism introduced by Fisher (1933). This mechanism works as follows. After a *negative* aggregate shock that tightens the financial conditions of the economy, financially constrained agents sell part of their collateralizable assets, which puts downward pressure on asset prices.³ As asset prices drop, (possibly

¹See Bianchi and Mendoza (2020) for a recent survey and review of the stylized facts of Sudden Stops.

²Figure 6 shows descriptive evidence that emerging economies are more unequal than advanced economies, and that Sudden Stop episodes are more severe in more unequal economies.

³Commonly studied negative shocks in small open economy models are an increase in the international interest rate, a decrease in total factor productivity, a drop in the terms of trade, or an ad-hoc tightening of the financial conditions of the economy. In this paper, the financial tightening shock will be an increase in the international interest rate.

more) financially constrained agents have to sell a larger asset position, which causes feedback that puts additional downward pressure on asset prices, and this behavior, in turn, further tightens aggregate financial conditions. This paper posits that the cross-sectional dimension of the debt-deflation mechanism matters for macro dynamics of Sudden Stops via two opposing effects: (1) a crisis-damping effect that weakens the debt-deflation mechanism because unconstrained wealthy households can buy the depressed assets fire-sold by financially constrained households and (2) a crisis-amplifying effect that strengthens the debt-deflation mechanism because of financially vulnerable households that become credit constrained as asset prices fall. As aggregate financial conditions tighten, such households also have to sell assets, increasing the downward pressure on asset prices. Because these two cross-sectional effects constitute opposing forces, the role of the cross section and inequality during crises is quantitatively ambiguous. Hence, this paper conducts a quantitative investigation of the degree to which the severity of Sudden Stop crises is affected by inequality in an economy.

To shed light on the empirical relevance of these issues, we examine a panel household survey for Mexico that provides evidence of the damping and amplifying cross-sectional effects. Moreover, we test – and reject – the individual complete-market hypothesis. These results support our decision to use a heterogeneous-agents framework to study financial crises and cross-sectional dynamics in households’ consumption and portfolio choice.

Then the paper conducts a quantitative analysis of the effect of wealth inequality on Sudden Stops. To this end, we propose a small open economy, asset-pricing Bewley model with debt and assets, an endogenous occasionally-binding loan-to-value (LtV) collateral constraint, and aggregate risk. At the individual level, markets are incomplete, and households face both idiosyncratic labor and dividend income risk. The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth tradeoff*: more asset holdings relax the individual collateral constraint and allow for better consumption smoothing (reducing consumption volatility), but more asset holdings also increase the dividend risk exposure, which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This tradeoff makes high-dividend,

asset-rich households deleverage faster than low-dividend households, producing an empirically plausible leverage ratio distribution with wealthy unconstrained and well diversified households that face nondegenerate portfolio choices.

In a version of the model calibrated to an emerging economy (Mexico), the quantitative analysis shows that the damping effect dominates and asset prices drop less in heterogeneous-agents economies. In contrast to the representative-agent framework, the model generates persistent current account reversals with larger drops in consumption driven by the most leveraged households, consistent with the data. Moreover, calibrating the model to an advanced economy where the dividend risk variance is one-half of that in the benchmark emerging-markets model, we find that the average net foreign debt position is 8.5 percent larger and that, during a crisis, consumption drops 1.1 percentage points less and asset prices drop 0.4 percentage point less. Hence, the model predicts that in economies with lower dividend return variance, income inequality is lower, the economy supports larger debt positions, and Sudden Stop crises are less severe, as observed in the data.

After reviewing the literature in Section 2, in Section 3 we describe the empirical evidence that supports the cross-sectional effects of the debt-deflation mechanism. The proposed model is described in Section 4. Section 5 describes the cross-sectional effects through the lens of the model. Section 6 presents the quantitative analysis, and Section 7 concludes.

2 Related Literature

This paper contributes to three strands in the economics literature. In the first strand, Sudden Stop crises have been studied using representative-agent models with financial frictions. For instance, [Mendoza \(2010\)](#) studies Sudden Stops in a standard representative firm-agent real business cycle model augmented with a debt-deflation mechanism. He introduces an LtV collateral constraint that generates a pecuniary externality, reflecting that agents do not internalize how their decisions today affect the equilibrium Tobin's Q price of capital that tightens or loosens the debt capacity. In a related paper, [Mendoza and Smith \(2006\)](#) study the debt-deflation mechanism in a small open economy with a representative-agent that trades domestic equity with

a foreign investor. In their model, the combination of a collateral constraint and equity trading costs can produce realistic Sudden Stops. Our paper complements both studies, yet it differs fundamentally from them because we study the cross-sectional dimension of the debt-deflation mechanism. To this end, we introduce market incompleteness at the individual level and study how the distribution of households along bonds, assets, and individual productivities affects asset prices, portfolio choices, and consumption dynamics during crises.

A second strand of the literature focuses on asset prices in closed economies with individual incomplete markets. [Aiyagari and Gertler \(1991\)](#) study asset prices, particularly the equity premium puzzle (see [Mehra and Prescott \(1985\)](#)), in a closed economy with two assets (bonds and stocks), adjustment costs, and individual labor income risk. The authors conclude that the difference in relative adjustment costs between assets and the need to trade assets for consumption smoothing – introduced by the individual market incompleteness – can generate a spread between the return on bonds and stocks. [Heaton and Lucas \(1996\)](#), who study an economy with two types of agents, income risk, adjustment costs, short-sales constraints, and debt constraints, find that the adjustment costs can generate higher equity premiums. Studying the excess volatility in asset prices that an LtV constraint causes, [Aiyagari and Gertler \(1999\)](#) explain price volatility in a model with limited heterogeneity. There are only two representative-agents in their environment: a household and a trader. When the trader is constrained, the multiplier in the collateral constraint is active for the whole population of traders. This behavior translates into higher volatility in asset prices. More recently, [Storesletten, Telmer, and Yaron \(2007\)](#) show that in a life-cycle model, the effects of idiosyncratic labor risk are quantitatively significant if the idiosyncratic risk becomes more volatile during economic contractions. They further demonstrate that idiosyncratic risk inhibits inter-generational risk sharing, imposing a disproportionate share of aggregate risk on the wealthy middle-aged cohorts, who demand an equity premium for their exposure to this risk. In their setting, the young cohorts do not hold equity to avoid the counter-cyclical volatility risk. Our paper differs from but complements this strand of the literature because, although we study a small open economy, the risky domestic asset is closed to foreign investors and behaves as it would in a closed economy. This framework allows analyzing the dis-

tributional effects of an endogenous occasionally-binding constraint that introduces a pecuniary externality. We introduce an LtV collateral constraint that, together with the uninsurable dividend risk, generates an *asset-wealth tradeoff* from holding additional assets. This tradeoff generates unconstrained wealthy households that endogenously have a diversified portfolio. Moreover, we show that in our setting, the equity premium can be decomposed into a constraint effect, a risk effect, a trading cost effect that is expected to be close to zero, and a short-sales effect. The trading cost effect will only be non-zero because of the combination of the collateral constraint and the trading cost function. Hence, most risk compensation proceeds from the LtV constraint and individual risk.

A third strand studies the macroeconomy, accounting for individual heterogeneity, a line of inquiry begun with the pioneering work of [Krusell and Smith \(1997\)](#), who developed quantitative tools to analyze economies in which the market clearing price is a function of the distribution of agents (and not only of the mean aggregate state) with individual incomplete markets and aggregate risk. [Mendoza, Quadrini, and Ríos-Rull \(2009\)](#) examine how global imbalances can be precipitated by the integration of economies that have different financial market development. They study the transition path after an unexpected integration of economies and analyze the global balance sheet and equilibrium interest rates. In a related paper, [Kaplan and Violante \(2014\)](#) study households with access to two types of assets that differ in their liquidity. [Guerrieri and Lorenzoni \(2017\)](#) study the transition path in a closed economy that experienced an unexpected tightening in the exogenous debt limit. Also, [Huo and Ríos-Rull \(2016\)](#) examine the effect of asset prices in a closed economy without aggregate risk and study the transition after an unexpected shock in financial conditions. In contrast, we study the general equilibrium in a small open economy with aggregate risk and individual labor and dividend productivities. This setup, augmented with an individual LtV collateral constraint, allows us to analyze the cross-sectional dimension of the debt-deflation mechanism and the pecuniary externality that it generates. Finally, in a series of empirical papers that study the relationship between income inequality and crises, [Bordo and Meissner \(2012\)](#) and [Morelli and Atkinson \(2015\)](#) examine the predictive power of rising income inequality for financial crises without finding conclusive evidence. One exception is [Kumhof, Rancière, and Winant](#)

(2015), who propose a model to study the effect of changes in the top income distribution on household leverage and crises. Lastly, [Guntin, Ottonello, and Perez \(2020\)](#) use micro-data to assess individual consumption changes in episodes of large aggregate consumption adjustments. The authors argue that, consistent with the permanent income hypothesis, households with high income and liquid assets adjust their consumption severely during such episodes. The present paper complements but differs fundamentally from these papers, because it studies a model with ex-ante homogeneous agents with ex-post heterogeneity and uses this heterogeneous-agents framework to investigate Sudden Stops and the cross-sectional dynamics in the consumption and portfolio choice of households. Moreover, we document the importance of leverage and not only the liquidity of assets. In particular, we find that during a Sudden Stop, households with high leverage adjust their consumption the most.

3 The Cross-Sectional Effects in the Data

This section first describes the data used to show that the cross-sectional effects of the debt-deflation mechanism are empirically relevant. Then, sorting households according to their net wealth and leverage ratio, we obtain the changes in their individual asset values and consumption during the 2009 Sudden Stop crisis. The results show that households in the top decile of wealth and top decile of leverage ratio fire-sold their assets the most, while the low-leveraged households increased their asset holdings.

3.1 Description of the Data

We use data from the Mexican Family Life Survey (MxFLS) for the three available waves: 2002, 2005, and 2009. The MxFLS is a longitudinal household survey that collected information from a representative sample of approximately 8,400 households in 150 localities throughout Mexico. The survey covers information on expenditures, income, assets, and liabilities.⁴ The MxFLS is representative at the national, urban-

⁴To the best of our knowledge, this survey is the only publicly available data source that covers information about households' stock of assets and liabilities.

rural, and regional levels.⁵ The sample selection criterion we used corresponds to households that answered the survey in all three waves. The resulting subsample corresponds to 78 percent of households in 2005.

Table 1 shows the mean net wealth, the portfolio decomposition, and the leverage ratio in 2005, ordered by deciles of the net wealth distribution. The leverage ratio is defined as the household’s total debt over the sum of the household’s assets. As the second and third rows show, Mexican households’ wealth is mostly in physical assets (real estate and other durable goods). Although the proportion of debt decreases as households amass higher net wealth, as we can see from the last two rows of the table, there are leveraged and non-leveraged households in each of the deciles. The next subsection will analyze the asset and consumption dynamics for households grouped by their level of leverage ratio and net wealth.

Table 1: Mean Net Wealth and Its Composition by deciles in 2005

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net wealth	-507	761	2,564	5,368	9,184	14,451	20,524	29,512	45,067	204,855
Assets										
Real estate (%)	-103.6	24.2	46.9	69.6	76.9	80.9	82.5	82.8	82.1	75.1
Other (%)	-68.5	88.3	49.5	30.7	23.4	19.8	15.8	14.2	14.2	9.3
Financial (%)	-10.7	9.7	12	7.5	4.5	4.9	3.4	5.3	6.3	16.8
Debt (%)	282.8	-22.2	-8.3	-7.7	-4.9	-5.6	-1.7	-2.3	-2.6	-1.2
Leverage ratio										
Mean	0.75	0.05	0	0.01	0.01	0.03	0	0	0	0
p90	1.69	0.38	0.17	0.16	0.12	0.09	0.04	0.05	0.06	0.04
p10	0	0	0	0	0	0	0	0	0	0

Note: Ordered by deciles of net wealth in 2005 dollars. Source: MxFLS.

3.2 Stylized Facts: Differentiated Individual Effects

Mexico, like almost any other open economy, experienced a severe Sudden Stop crisis in 2009. Aggregate data show a current account reversal of 1.5 percentage points relative to GDP, a 7 percent drop in per capita consumption, and house prices 4 percent below the pre-crisis trend in 2010 (for an overview of the aggregate time series, see Appendix A). Moreover, the MxFLS survey shows that from 2005 to 2009, the sum of households’ gross asset values dropped 1 percent. At the household level,

⁵For a detailed description of the survey, see [Rubalcava and Teruel \(2006\)](#) and [Rubalcava and Teruel \(2013\)](#).

however, the crisis had different effects depending on the composition of households' balance sheets.

Supporting evidence of the cross-sectional effects. The damping cross-sectional effect comes from the unconstrained wealthy households that can buy the depressed assets fire-sold by the financially constrained households during a crisis. Table 2 shows the median change in the real estate owned by households sorted according to their net wealth and leverage ratio in 2009.⁶ Wealthy households correspond to the top decile of net wealth, and the financially constrained households correspond to the top decile of the leverage ratio.⁷ As shown in the table, the real estate held by wealthy unconstrained households (top-right cell) increased by 59.4 percent, while the rest of households experienced drops in their asset holdings. Hence, this evidence supports the damping effects coming from the cross-sectional dimension: wealthy unconstrained agents take advantage of the depressed prices and increase their asset positions.

Assuming that there was no creation or destruction of real estate, it must be the case that because the assets held by the unconstrained wealthy agents increased, they were necessarily buying assets from someone else. Hence, other households were selling their assets. Since the amplifying effect comes from households that are close to becoming financially constrained, once the mechanism is triggered, they end up financially constrained and strengthen the downward pressure on asset prices. The magnitude of the numbers in the table suggests that the wealthy financially constrained – households in the top deciles according to net wealth and to the leverage ratio – fire-sold their assets the most putting downward pressure on their prices. Furthermore, the wealthy financially vulnerable – households in the top decile according to net wealth and in the ninth decile according to the leverage ratio – also ended up fire-selling their assets as financial conditions tightened. Hence, this evidence supports the amplifying effects coming from the cross-sectional dimension: financially vulnerable agents end up constrained and decrease their asset positions, increasing downward pressure on asset prices.

⁶The survey data correspond to the value of real estate. To obtain the quantity change, we deflated the value change with an aggregate house price index.

⁷In the calibration of the model, we will set the leverage limit equal to the leverage of the 90th percentile.

Table 2: Median Percent Real Estate Change, 2005–09

Leverage Ratio	Net Wealth	
	I–IX (Non-Wealthy)	X (Wealthy)
I–VIII	-1.1	59.4
IX	-1.9	-15.0
X	-1.4	-36.5

Note: Ordered by deciles in 2009. Source: MxFLS.

Additionally, in Table 3, we show the median change in the consumption of households according to their leverage ratio in 2005. During the crisis, households that in 2005 were in the top decile of leverage decreased their consumption 6.2 percent. These households were the most affected by the crisis because right before the crisis happened, they were the most exposed to changes in the financial conditions of the economy. In contrast to the declines in consumption of high-leveraged households, the ones in the first decile of leverage, which mostly had no debt and were net savers, increased their consumption 5.4 percent. Households that were moderately leveraged – second to ninth deciles – increased their consumption by less than the non-leveraged households, supporting a potential snowball effect: as financial conditions tightened because financially constrained agents fire-sold their assets, financially vulnerable households ended up constrained. Moreover, these dynamics are different during normal years. In the first column of the table, we can see that households in the second to ninth deciles of leverage are the ones that slightly decreased their expenditures, while households in the top and bottom deciles of leverage increased their expenditures 3.8 percent and 1.9 percent, respectively.

3.3 Stylized Facts: Heterogeneous Consumption Dynamics

In this subsection, we give evidence that households have heterogeneous consumption dynamics and that the modeling choice of a heterogeneous-agents framework is supported by the data. Following [Jappelli and Pistaferri \(2017\)](#), we perform a test of the complete-market hypothesis for Mexico. Under complete markets, changes in individual consumption depend only on aggregate fluctuations common to all individuals.

Table 3: Median Percent Consumption Change

Leverage Ratio	Normal Times	Crisis Times
	2002–05	2005–09
I	1.9	5.4
II–IX	-1.2	3.6
X	3.8	-6.2

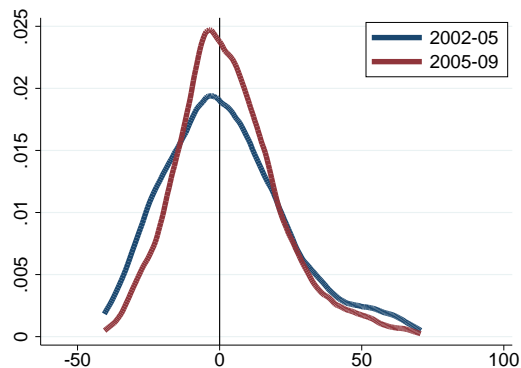
Note: Ordered by deciles in 2002 (first column) and 2005 (second column). Source: MxFLS.

To perform the test, we estimate the following regression:

$$\Delta \log c_t^i = \beta \Delta \log C_t + \delta \Delta \log y_t^i + u_t^i, \quad (1)$$

where c_t^i is the household i consumption, C_t is the aggregate consumption in year t , and y_t^i is the household i income in year t . We reject at the 1 percent significance level the joint test of $\beta = 1$ and $\delta = 0$. The point estimates with standard errors in parentheses are $\beta = 0.41$ (0.16) and $\delta = 0.04$ (0.006), which are similar to the evidence from Thailand presented in [Townsend \(1995\)](#). Moreover, as we can see in [Figure 1](#), changes in consumption vary across households in both normal and crisis years. However, during the crisis, there is a larger negative mass and a more concentrated distribution.

Figure 1: Household Distribution of Annualized Percent Change in Consumption



Note: The distributions are truncated at the top and bottom 1%. Source: MxFLS.

Additionally, [Table 4](#) and [Figure 2](#) show how households' leverage ratio distribu-

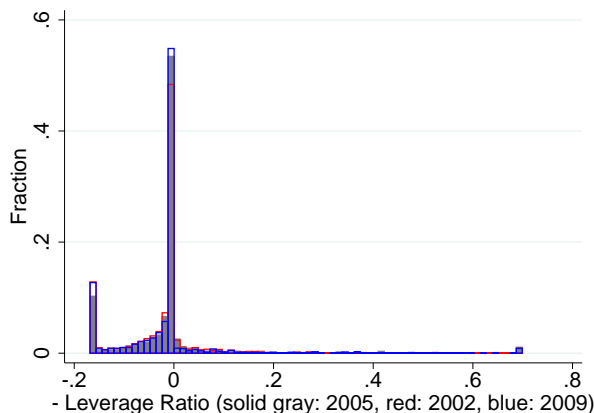
tion changed before and during the crisis. Between 2002 and 2005, before the crisis, the share of net saver households increased 1.7 percentage points, and the share of financially constrained households decreased 2.3 percentage points. Then, between 2005 and 2009, as the crisis unfolds and aggregate liquidity is reduced, we see a large decline of 5 percentage points in the share of net saver households. Such a decline could be the consequence of households having to use part of their savings to smooth consumption during the crisis. Also, in the same period, the share of financially constrained households increased 1.7 percentage points as a result of tightening financial conditions.

Table 4: Distribution of Households (percent)

	2002	2005	2009
Savers (leverage ratio ≤ 0)	12.5	14.2	9.2
Indebted unconstrained (leverage ratio $\in (0, 0.168)$)	75.2	75.8	79.1
Financially constrained (leverage ratio ≥ 0.168)	12.3	10.0	11.7

Note: The leverage ratio level considered to be the threshold between financially constrained and indebted unconstrained households is 0.168 and corresponds to the 90th percentile of its distribution in 2005. Source: MxFLS.

Figure 2: Negative Leverage Ratio Histogram

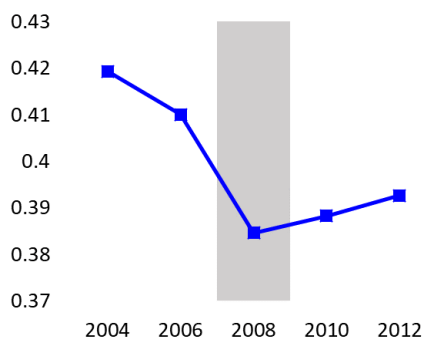


Note: The leverage ratio corresponds to the total debt over the total assets of the household.

Positive values of the negative leverage ratio correspond to households with net savings, and negative values correspond to households with net debts. The distribution is truncated at -0.168, which is the 90th percentile of the leverage ratio distribution in 2005, and at the top 1%. Source: MxFLS.

Finally, we complement the evidence from the MxFLS with that from the Income and Expenditure Household Survey (ENIGH). This survey is cross-sectional and is done every two years. In Figure 3, we show the Gini coefficient for consumption, and we can see that during the crisis, consumption inequality decreased more than the pre-crisis trend. This evidence is in line with the higher concentration documented in Figure 1.

Figure 3: Consumption Gini Coefficient



Note: The gray area corresponds to the crisis. Source: ENIGH.

Having documented stylized facts about households' cross-section, we describe the proposed model that accounts for households' balance sheet heterogeneity in the next section.

4 Model

4.1 Environment

The proposed framework is a Bewley model of a small open economy with international bonds, domestic equity, and an endogenous occasionally binding constraint. Time is discrete and infinite: $t = 0, \dots, \infty$. The economy is populated by a unit measure of households. There are two financial assets: a one-period risk-free international bond that households can trade with the rest of the world and a risky domestic asset (land) that is tradable only between households and is subject to a trading

cost.⁸ Borrowing is subject to an LtV collateral constraint by which households' international debt cannot exceed a fraction of the market value of their assets – i.e., the domestic asset is collateralizable.⁹ Regarding the financial market's structure in the economy, markets are incomplete at the aggregate and individual levels. With respect to aggregate risk, the economy is subject to an aggregate shock that determines the international interest rate. Concerning individual risk, households face non-insurable idiosyncratic labor income risk and dividend income risk. The latter risk means that households buy ex-ante identical shares of the risky domestic asset but get ex-post heterogeneity in the return. Evidence of a similar individual return on wealth is documented by [Fagereng et al. \(2020\)](#), and related individual capital income risk has been used by [Angeletos \(2007\)](#), [Mendoza, Quadrini, and Rios-Rull \(2009\)](#), [Benhabib, Bisin, and Zhu \(2011\)](#), and [Hubmer, Krusell, and Smith Jr \(2020\)](#). The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth tradeoff*: more asset holdings relax the collateral constraint and allow for better consumption smoothing (reducing consumption volatility), but more asset holdings also increase the dividend risk exposure, which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This *asset-wealth tradeoff* will be studied in Section 5.1.

⁸The assumption of only domestic trading could be relaxed to allow foreign ownership up to a certain percentage of the shares in the economy. With an exogenous stochastic foreign demand for domestic shares, asset prices could become more volatile.

⁹The micro-foundations of the collateral constraint are similar to the ones presented by [Bianchi and Mendoza \(2018\)](#) extended for an economy with non-insurable idiosyncratic risk. Specifically, the LtV constraint can be derived from an incentive compatibility constraint that arises due to a limited enforcement problem, in an economy where debt contracts are signed with competitive creditors, and households can switch to another creditor at any given point in time. At the beginning of the period, credit and asset markets open, production happens, and households choose b_{t+1}^i with price R_t^{-1} and a_{t+1}^i with price q_t . Then markets close, and households decide to divert resources from the credit and default. Local competitive financial intermediaries monitor costlessly who diverts resources and seize a fraction κ of the household asset holdings, which are $q_t a_{t+1}^i$. After defaulting, the household regains access to credit markets instantaneously and repurchases the assets that investors sell in open markets at a price q_t . In this environment, a household that borrows $-R_t^{-1} b_{t+1}^i$ and engages in diversion activities gains $-R_t^{-1} b_{t+1}^i$ and loses $\kappa q_t a_{t+1}^i$. Hence, households repay if and only if $-R_t^{-1} b_{t+1}^i \leq \kappa q_t a_{t+1}^i$.

4.2 Households

There is a continuum unit measure of households. Each household $i \in [0, 1]$ maximizes

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^i) \right], \quad (2)$$

where c_t^i is the consumption of household i , $\beta \in (0, 1)$ is the common discount factor, and the utility function, $u(\cdot)$, has a common constant relative risk aversion (CRRA) form. Households have access to the international bond market and the domestic asset market. However, since debt markets are imperfect, only secured debt is available, and households' domestic assets serve as collateral. At the beginning of the period, each household holds b_t^i risk-free international bonds and a_t^i shares of the risky domestic asset that has an endogenous price q_t and pays a dividend d_t^i . The household receives labor endowment income w_t^i and uses funds to buy consumption goods c_t^i , bonds to carry for the next period at an exogenous price equal to the inverse of the gross international rate R_t , and asset holdings to carry for the next period facing a quadratic trading cost of the form $\Phi(a_{t+1}^i, a_t^i) = \frac{\phi}{2}(a_{t+1}^i - a_t^i)^2$. This cost reflects that trading the domestic asset requires a higher level of financial knowledge relative to the bond market and that physical assets are relatively less liquid than bonds. The household's budget constraint is

$$c_t^i + R_t^{-1}b_{t+1}^i + q_t(a_{t+1}^i + \Phi(a_{t+1}^i, a_t^i)) = w_t^i + a_t^i(q_t + d_t^i) + b_t^i. \quad (3)$$

Households face an LtV constraint that limits their ability to leverage foreign debt on domestic asset holdings. Next-period debt (negative bonds) cannot exceed a constant fraction κ of the market value of asset holdings. The collateral constraint is

$$R_t^{-1}b_{t+1}^i \geq -\kappa q_t a_{t+1}^i. \quad (4)$$

In addition, there is a short-sales constraint on the asset $a_{t+1}^i \geq 0$.¹⁰ Note that the

¹⁰The short-sales constraint is needed to ensure that the state space of asset holdings is compact and that the LtV constraint is not irrelevant. If unlimited short selling of assets were possible, households could always undo the effect of Equation 4.

portfolio choice problem is well defined, given the combination of the trading costs in the asset market and the LtV debt constraint.

Lastly, the income of households is composed of an idiosyncratic and an aggregate part, as in [Benhabib, Bisin, and Zhu \(2015\)](#). The individual wage takes the form $w_t^i = \epsilon_t^{i,w} w$, and the individual rate of return $d_t^i = \epsilon_t^{i,d} d$, where $\{\epsilon_t^{i,w}, \epsilon_t^{i,d}\}$ correspond to the idiosyncratic risk components, which will be specified in the next subsection, and $\{w, d\}$ correspond to the aggregate, exogenous, and constant components.¹¹

4.3 Exogenous Stochastic Processes

The economy is exposed to only one aggregate shock. The process for the international interest rate is $R_t = \epsilon_t^R \bar{R}$ and $\log(\epsilon_t^R) = \rho_R \log(\epsilon_{t-1}^R) + \eta_t^R$, with $\eta_t^R \sim \mathcal{N}(0, \sigma_R^2)$. Regarding the individual shocks, the individual wage takes the form $w_t^i = \epsilon_t^{i,w} \bar{w}$ and $\log(\epsilon_t^{i,w}) = \rho_w \log(\epsilon_{t-1}^{i,w}) + \eta_t^{i,w}$, with $\eta_t^{i,w} \sim \mathcal{N}(0, \sigma_w^2)$, and the individual dividend takes the form $d_t^i = \epsilon_t^{i,d} \bar{d}$ and $\log(\epsilon_t^{i,d}) = \rho_d \log(\epsilon_{t-1}^{i,d}) + \eta_t^{i,d}$, with $\eta_t^{i,d} \sim \mathcal{N}(0, \sigma_d^2)$. Note that the idiosyncratic labor and dividend risk that households face does not have aggregate implications on the returns.¹²

$$\int_0^1 d_t^i di = \int_0^1 \epsilon_t^{i,d} d di = d \quad \text{and} \quad \int_0^1 w_t^i di = \int_0^1 \epsilon_t^{i,w} w di = w.$$

4.4 Closing the Domestic Asset Market

The domestic asset is in constant positive net supply equal to \bar{K} , and in equilibrium, it is equal to the total asset holdings (demand) of households. Hence, market clearing in the asset market requires $\int_0^1 a_t^i di = \bar{K}$ for every t .

¹¹The structure of the income endowments is similar to that of an economy in which households supply one unit of labor inelastically, and production is done with a competitive constant-returns-to-scale production function that demands only aggregate labor and pays competitive wages w to each household. Additionally, households have an “ Ak ” production function that uses their individual assets to produce, and households obtain dividends d from such production. In the end, households supply effective units of labor and assets so returns are multiplied by the idiosyncratic shocks.

¹²However, as noted in [Hubmer, Krusell, and Smith Jr \(2020\)](#), the idiosyncratic dividend risk will impact the aggregate endowment, which will be a function of households’ distribution of assets and dividend returns.

4.5 Recursive Formulation

To characterize the problem of the agents and the equilibrium in recursive form, we start by defining the states of the economy. Households are heterogeneous in their current holding of bonds, assets, idiosyncratic labor, and dividend productivity. The individual states are $(b, a, \epsilon^w, \epsilon^d)$. We need to keep track of both the individual bonds and assets, given the asset trading costs and the imperfect debt market. Let $\Omega(b, a, \epsilon^w, \epsilon^d)$ be the endogenous distribution of households according to their bonds, assets, and individual productivities. Regarding aggregate states, to forecast asset prices, households need to know the distribution of wealth. Hence, the aggregate states correspond to the endogenous distribution Ω , and the exogenous shock to the international interest rate ϵ^R . Letting the superscript \prime correspond to the variables in the next period, we determine that the recursive problem of a household becomes

$$\begin{aligned}
 v(\underbrace{b, a, \epsilon^w, \epsilon^d}_{\text{Ind. states}}, \underbrace{\Omega, \epsilon^R}_{\text{Agg. states}}) &= \max_{\{c, b', a' \geq 0\}} u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, \Omega', \epsilon^{R'})] \quad s.t. \\
 c + R(\epsilon^R)^{-1}b' + q(\Omega, \epsilon^R)(a' + \Phi(a', a)) &= \epsilon^w w + a(q(\Omega, \epsilon^R) + \epsilon^d d) + b, \quad \text{multiplier } \lambda(\cdot), \\
 R(\epsilon^R)^{-1}b' &\geq -\kappa q(\Omega, \epsilon^R)a', \quad \text{multiplier } \mu(\cdot), \\
 \Phi(a', a) &= \frac{\phi}{2}(a' - a)^2, \\
 \Omega' &= H^\Omega(\Omega, \epsilon^R), \tag{5}
 \end{aligned}$$

where $H^\Omega(\cdot)$ corresponds to the aggregate law of motion of the distribution of households.

Now we can define a recursive competitive equilibrium. Let the individual bond and asset holdings be elements $(b, a) \in [\underline{b}, \bar{b}] \times [0, \bar{a}] \equiv \mathcal{S}$, and let the individual productivities be elements $(\epsilon^w, \epsilon^d) \in \{\epsilon_1^w, \dots, \epsilon_{N_w}^w\} \times \{\epsilon_1^d, \dots, \epsilon_{N_d}^d\} \equiv \mathcal{E}^I$. In addition, let \mathcal{M} be the set of probability measures of the set $\mathcal{S} \times \mathcal{E}^I$, and let the aggregate shocks be elements $\epsilon^R \in \{\epsilon_1^R, \dots, \epsilon_{N_R}^R\} \equiv \mathcal{E}^A$. Finally, let the function $\pi(\epsilon'|\epsilon)$ be the exogenous Markov transition probability that the next-period shock takes the value ϵ' conditional on the shock in the current period being ϵ , where $\epsilon = (\epsilon^w, \epsilon^d, \epsilon^R) \in \mathcal{E}^I \times \mathcal{E}^A$.

Definition 1. *A recursive competitive equilibrium in this economy is given by a value function $v : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$; policy functions for the household $c : \mathcal{S} \times \mathcal{E}^I \times$*

$\mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, $b' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, and $a' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$; a domestic asset-pricing function $q : \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$; and an aggregate law of motion $H^\Omega : \mathcal{M} \times \mathcal{E}^A \rightarrow \mathcal{M}$ such that

1. Given the asset-pricing function and the aggregate law of motion, the value function v satisfies the household's Bellman equation 5, and c , a' , and b' are the associated policy functions.

2. For all $\Omega \in \mathcal{M}$ and all $\epsilon^R \in \mathcal{E}^A$, the asset market clears:

$$\int_{\mathcal{S} \times \mathcal{E}^I} a \, d\Omega = \int_{\mathcal{S} \times \mathcal{E}^I} a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega = \bar{K}.$$

3. For all $\Omega \in \mathcal{M}$ and $\epsilon^R \in \mathcal{E}^A$, the aggregate resource constraint is satisfied:

$$\begin{aligned} & \int_{\mathcal{S} \times \mathcal{E}^I} c(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega + R(\epsilon^R)^{-1} \int_{\mathcal{S} \times \mathcal{E}^I} b'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega \\ & + q(\Omega, \epsilon^R) \int_{\mathcal{S} \times \mathcal{E}^I} \Phi(a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R), a) \, d\Omega = w + \int_{\mathcal{S} \times \mathcal{E}^I} a \epsilon^d \, d\Omega + \int_{\mathcal{S} \times \mathcal{E}^I} b \, d\Omega. \end{aligned}$$

4. The aggregate law of motion is generated by the exogenous Markov process π and the policy functions b' and a' as described below:

Let $(\epsilon^w, \epsilon^d) = \epsilon^I$ and $\epsilon^R = \epsilon^A$ and define the transition function $Q_{\Omega, \epsilon^A} : \mathcal{S} \times \mathcal{E}^I \times \mathcal{B}(\mathcal{S}) \times \mathcal{B}(\mathcal{E}^I) \rightarrow [0, 1]$, where $\mathcal{B}(\cdot)$ is the corresponding Borel set, by

$$Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathcal{S}, \mathcal{E}^I) = \begin{cases} \sum_{\epsilon^{I'} \in \mathcal{E}^I, \epsilon^{A'} \in \mathcal{E}^A} \pi(\epsilon^{I'}, \epsilon^{A'} | \epsilon^I, \epsilon^A), & \text{if } (b'(b, a, \epsilon^I, \Omega, \epsilon^A), a'(b, a, \epsilon^I, \Omega, \epsilon^A)) \in \mathcal{S}. \\ 0, & \text{otherwise.} \end{cases}$$

Then, for any $\mathcal{S} \in \mathcal{B}(\mathcal{S})$ and any $\mathcal{E}^I \in \mathcal{B}(\mathcal{E}^I)$ the aggregate law of motion is given by

$$\Omega'(\mathcal{S}, \mathcal{E}^I) = (H^\Omega(\Omega, \epsilon^A))(\mathcal{S}, \mathcal{E}^I) = \int_{\mathcal{S} \times \mathcal{E}^I} Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathcal{S}, \mathcal{E}^I) \, d\Omega.$$

5 The Cross-Sectional Effects in the Model

In this section, we study the cross-sectional effects on the credit and equity channel of the economy.

5.1 Market Incompleteness and Risk Exposure

Households are exposed to two sources of non-insurable idiosyncratic risk that have different equilibrium implications. Note that the standard Bewley non-insurable persistent labor income risk, ϵ^w , together with the constant aggregate labor income endowment assumption implies a fixed labor risk exposure, which means that the exposure to labor earnings risk is independent of households' decisions. In contrast, the idiosyncratic persistent dividend productivity, ϵ^d , allows households to change future risk exposure by changing the next-period holdings of the asset.

This varying dividend risk exposure, combined with the LtV collateral constraint, generates an *asset-wealth tradeoff*. To see this point, first, note that when households are in an adverse state, they can smooth consumption in two ways – by lowering their bond holdings b' (if they are already negative, this fact means borrow more) or by reducing their asset holdings a' . Given the financial frictions in the debt market (see Equation 4), to have credit capacity and hence borrow, the household needs first to buy domestic assets. Note that although the current dividend return is given since the current asset holdings are fixed in the current period (they are an individual state variable), the household chooses how much future exposure to have by choosing the next-period asset holdings a' . Because the flow income of the household is given by $FI(a, \epsilon^w, \epsilon^d) = \epsilon^w w + a \epsilon^d d$, with independent idiosyncratic risks its variance is $\mathbb{V}[FI(a, \epsilon^w, \epsilon^d)] = w^2 \sigma_{\epsilon^w}^2 + a^2 d^2 \sigma_{\epsilon^d}^2$, which is a convex function with respect to asset holdings. This convexity translates into more income volatility for asset-rich households. This property of the flow income generates the following tradeoff from getting more assets:

1. Households get higher debt capacity that allows higher smoothing and reduces consumption volatility since $R(\cdot)^{-1}b'(\cdot) \geq -\kappa q(\cdot)a'(\cdot)$, incentivizing lower precautionary savings.

2. Households get higher future income risk that increases consumption volatility, incentivizing higher precautionary savings.

In equilibrium, indebted asset-poor households increase their debts as they increase their assets, and for households with high dividend returns, when they become asset-rich, they start deleveraging (precautionary saving motives kick in), and some end up being savers due to the increasing income risk.¹³ This behavior generates unconstrained wealthy households, which endogenously have a diversified portfolio: asset-rich households end up holding both positive international bonds and domestic assets.

Similar tradeoffs have been studied in the literature but through different mechanisms. [Mendoza, Quadrini, and Rios-Rull \(2009\)](#) find that an individual investment shock (similar to an individual dividend shock) makes agents lower their debt positions as they increase their net wealth. The outcome for asset-rich households is the same but for different reasons. Because we introduce the shock with persistence (there is an independently and identically distributed, or iid shock), households with a negative dividend shock want to lower their bond position (or increase debts if the position is negative) as the asset position increases. Moreover, in our paper, introducing the LtV constraint and the individual nontrivial portfolio choice problem makes asset-poor households increase their debts as they increase their assets. In another study, [Benhabib, Bisin, and Zhu \(2011\)](#) show that idiosyncratic capital returns determine the properties of the right tail of the wealth distribution in a Bewley economy. Their theoretical result is in line with the *asset-wealth tradeoff* described earlier, since asset-rich households that get a positive dividend shock will increase their net wealth by two sources – by buying more assets and by increasing their bond position (or decreasing their debt if the bond position is negative). Hence, the share of wealthy households and the wealth inequality increase. However, again, the combination of the dividend risk with the LtV constraint allows the model to generate an empirically plausible distribution of constrained households, financially vulnerable households that hold debt, and households with positive bond positions (savers).

¹³See the Online Appendix for a graphical analysis of the policy functions done for the calibrated stationary model of Section 6.2.

5.2 Financial Premiums

In this subsection, we study the effects that households' balance sheet heterogeneity introduces. Specifically, we analyze the cross-sectional dimension of the debt-deflation mechanism in terms of the external financing premium and equity premium at the individual and aggregate levels. For simplicity, we omit the state variables and reintroduce the superscript i to identify household-specific variables. Let λ^i , μ^i , and ψ^i be the multipliers on the budget constraint, the collateral constraint, and the short-sales constraint, respectively, and let $\tilde{\mu}^i = \frac{\mu^i}{\lambda^i}$ and $\tilde{\psi}^i = \frac{\psi^i}{\lambda^i}$.

Similar to the analysis done by [Mendoza and Smith \(2006\)](#) but for an economy with heterogeneous-agents, from the first-order conditions of household i 's problem, we obtain an Euler equation for individual bonds.

$$\lambda^i R^{-1} - \mu^i R^{-1} = \beta \mathbb{E}[\lambda^{i'}] \quad \Rightarrow$$

$$0 < 1 - \tilde{\mu}^i = \beta R \mathbb{E} \left[\frac{\lambda^{i'}}{\lambda^i} \right] \leq 1 \quad \text{since } \lambda^i > 0, \mu^i \geq 0 \text{ and } \tilde{\mu}^i = \frac{\mu^i}{\lambda^i} \in [0, 1).$$

Let the individual expected effective interest rate be the inverse of the individual stochastic discount factor $\mathbb{E}[R^{i,eff}] = \mathbb{E}[SDF^i]^{-1} = \mathbb{E} \left[\beta \frac{\lambda^{i'}}{\lambda^i} \right]^{-1}$. Then, from the previous Euler equation, we get an individual expected external financing premium on debt:

$$\mathbb{E}[R^{i,eff}] - R = R \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i} \geq 0. \quad (6)$$

This individual premium reflects the fact that when the constraint binds ($\tilde{\mu}^i > 0$), the household would want to borrow more than what the collateral constraint allows. Also, note that the individual premium is increasing on $\tilde{\mu}^i$, which means that as the constraint tightens, the household would be willing to pay an interest rate higher than R for more debt.

Similarly, from the first-order conditions of household i 's problem, we obtain the Euler equation for individual assets:

$$q(\lambda^i(1 + \Phi_1^i) - \kappa\mu^i) - \psi^i = \beta \mathbb{E}[\lambda^{i'}(q' + d^{i'} - q'\Phi_2^{i'})],$$

where Φ_j^i corresponds to the partial derivative with respect to argument j . Let

$\tilde{d}^{i,'} = d^{i'} - q'\Phi_2^{i'}$ and the individual return on the asset be $\tilde{R}^{i,q} = \left(\frac{q'+\tilde{d}^{i,'}}{q}\right)$. Then, from the aforementioned Euler equation, we get an individual expected equity premium:

$$\mathbb{E}[\tilde{R}^{i,q}] - R = \frac{R \left((1 - \kappa)\tilde{\mu}^i - \text{COV}[SDF^i, \tilde{R}^{i,q}] + \Phi_1^i - \tilde{\psi}^i \right)}{1 - \tilde{\mu}^i}. \quad (7)$$

As in [Mendoza and Smith \(2006\)](#) but at an individual level, in Equation 7, we see a direct positive effect on the individual equity premium coming from the collateral constraint: as $\tilde{\mu}^i$ increases, the individual equity premium increases by an additive term that multiplies $R(1 - \kappa)$ and by a multiplicative factor $(1/(1 - \tilde{\mu}^i))$ that affects the premiums. Also, there is a positive risk effect coming from the covariance term that will become more negative due to the precautionary savings.¹⁴ Lastly, there is an ambiguous effect coming from the marginal trading costs. This last effect is expected to be negative for financially constrained households because, when $\tilde{\mu}^i > 0$, the household will sell assets to smooth consumption and $a^{i'} < a^i \Rightarrow \Phi_1^i < 0$. When the constraint binds, a larger equity premium reflects that buying an extra unit of the asset provides an additional benefit since this additional unit also relaxes the constraint. However, this additional benefit is imperfect since κ fraction of the assets is pledgeable as collateral.

The aggregate expected equity rate of return, $\mathbb{E}[R^q]$, can be obtained by first integrating the individual expected asset returns over all households:

¹⁴This risk effect also includes the next period's marginal trading cost effect that is expected to increase the precautionary motives. The intuition for this finding is the following. Note that the household that, next period, gets a high dividend return will buy more shares. Hence, $a^{i''} > a^{i'} \Rightarrow \Phi_2^{i'} < 0 \Rightarrow \tilde{d}^{i,'} > d^{i'}$ – that is, effectively, the individual dividend risk increases because of the trading costs.

$$\begin{aligned}
\int_0^1 \mathbb{E}[\tilde{R}^{i,q}] di &= \int_0^1 \left(\mathbb{E} \left[\frac{q' + \tilde{d}^{i'}}{q} \right] \right) di = \mathbb{E} \left[\int_0^1 \left(\frac{q' + \tilde{d}^{i'}}{q} \right) di \right] = \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \int_0^1 \tilde{d}^{i'} di \right] = \\
&= \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \int_0^1 d^{i'} - q' \Phi_2^{i'} di \right] = \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \underbrace{\int_0^1 d^{i'} di}_{=d'} + \frac{1}{q} \int_0^1 q' \phi(a^{i''} - a^{i'}) di \right] = \\
&= \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} d' + \frac{q' \phi}{q} \left(\underbrace{\int_0^1 a^{i''} di}_{=\bar{K}} - \underbrace{\int_0^1 a^{i'} di}_{=\bar{K}} \right) \right] = \mathbb{E} \left[\frac{q' + d'}{q} \right] \equiv \mathbb{E}[R^q].
\end{aligned}$$

Then we use the expected returns derived in Equation 7 to obtain a decomposition of the aggregate expected equity premium. Assuming that fraction $\bar{I} \in [0, 1]$ of households are credit constrained and, without loss of generality, sorting constrained households from 0 to \bar{I} , we obtain the following result:

$$\begin{aligned}
\mathbb{E}[R^q] - R &= \underbrace{R(1 - \kappa) \int_0^{\bar{I}} \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i} di}_{\text{Constraint Effect: } +\bar{I} \text{ and } +\tilde{\mu}} - \underbrace{R \int_0^1 \frac{\text{COV}[SDF^i, \tilde{R}^{i,q}]}{1 - \tilde{\mu}^i} di}_{\text{Risk Effect: "+"}} \\
&+ \underbrace{R \int_0^1 \frac{\Phi_1^i}{1 - \tilde{\mu}^i} di}_{\text{Trading Cost Effect: } \approx 0} - \underbrace{\frac{R}{q} \int_0^1 \frac{\tilde{\psi}^i}{1 - \tilde{\mu}^i} di}_{\text{Short-Sales Effect: "-"}}. \tag{8}
\end{aligned}$$

Equation 8 shows that aggregate excess returns can be decomposed into four effects: first, a positive direct effect coming from the measure of constrained households and from how “strong” the constraint binds: and, second, the risk effect coming from the covariance between the individual stochastic discount factor and the individual return on equity (note that the integral becomes a weighted average of the covariances, with larger weights on constrained households since $\tilde{\mu}^i > 0 \Rightarrow 1/(1 - \tilde{\mu}^i) > 1$). Since

constrained households are expected to have more negative covariances because of the increased individual consumption volatility and the precautionary savings behavior, we expect a positive risk effect. Third, there is the trading cost effect – again, the weighted average puts more weight on constrained households, and since $\int_0^1 \Phi_1^i di = 0$, we can expect the aggregate effect to be close to zero and decreasing with respect to ϕ . This trading cost effect comes from the interaction of the collateral constraint and the trading cost function, since if there are no constrained households, this term becomes zero. Fourth, we observe a short-sales effect that decreases the equity premium, since households with a binding short-sales constraint increase the marginal gain of additional asset holdings, and has no effect on the marginal benefit of saving in assets.

Finally, the debt-deflation cross-sectional effects in the risk premiums are:

1. Damping effect: having more unconstrained wealthy households reduces the equity premium by having a smaller risk effect, since they are better able to smooth consumption.
2. Amplifying effect: having more financially vulnerable households increases the equity premium because of a larger constraint effect (larger \bar{I}) and by having a larger risk effect, since these constrained households have more consumption volatility.

Note that the precautionary behavior introduced by the *asset-wealth tradeoff*, under empirically suitable high persistence of the dividend risk, generates unconstrained households. Hence, in the stationary equilibrium, the measure of financially constrained households is $\bar{I} < 1$. Intuitively, when households get a high individual dividend return, they accumulate more assets. Since the individual risk is sufficiently persistent, this persistence gives households enough time to become asset-rich, and the dividend risk exposure is high enough that the precautionary savings motive makes households deleverage and become unconstrained. In the next section, we use the model as a measurement device to quantitatively study the cross-sectional effects of a Sudden Stop episode.

6 Quantitative Analysis

This section presents the quantitative results of the model. Because of the computational intensity of the solution method, we calibrate the parameters using the stationary model without aggregate risk.¹⁵ To calibrate the model, we use data for Mexico. Table 6.1 shows the calibrated parameters.

6.1 Calibration

Regarding the set of parameters that are calibrated outside of the model, we set the households' risk aversion, ν , equal to 2, which is a value common in the literature, and the collateral debt fraction, κ , equal to 0.168, which is the 90th percentile of the leverage ratio distribution in 2005. Lastly, the net asset supply is normalized at 1. Then we calibrate by simulation the discount factor $\beta = 0.90$ to match the average net foreign asset position relative to GDP for Mexico, equal to -35 percent, and we also calibrate the trading cost parameter $\phi = 2.7$ to obtain an average transaction cost of 5 percent, which is consistent with the estimates from [Aiyagari and Gertler \(1999\)](#).

To estimate the exogenous earning process, we apply the methodology described in [Krueger, Mitman, and Perri \(2016\)](#) using Mexican data.¹⁶ First, we estimate a Mincer log-earnings equation with time fixed effects:

$$\log(Y_{a,t}^i) = \beta' X_{a,t}^i + D_t + y_{a,t}^i, \quad (9)$$

where each observation corresponds to an individual i , with quarterly age a and in quarter t . $Y_{a,t}^i$ corresponds to the annual income of the person, and the vector of controls $X_{a,t}^i$ includes a cubic polynomial on age, dummy variables for the education

¹⁵Since the economy has an endogenous occasionally-binding constraint, the household's policy functions are expected to be highly nonlinear, and a global solution method is needed. We use the *FiPIt* algorithm proposed by [Mendoza and Villalvazo \(2020\)](#) to solve the household's problem combined, with the stochastic-simulation approach by [Maliar, Maliar, and Valli \(2010\)](#) and [Krusell and Smith \(1997\)](#) to solve the aggregate uncertainty problem.

¹⁶There is a vast literature on the estimation of labor income risk (see [Meghir and Pistaferri \(2004\)](#), [Storesletten, Telmer, and Yaron \(2004\)](#), [Guvenen \(2007\)](#), and [Heathcote, Storesletten, and Violante \(2010\)](#)).

Table 5: Parameters

Parameter	Value	Source or Target
Calibrated outside of the model		
ν Risk aversion	2	Common in the literature
κ Debt fraction of collateral	0.168	90th percentile lev. ratio in 2005
\bar{K} Net asset supply	1	Normalization
Calibrated by simulation		
β Discount factor	0.90	Average NFA/GDP ratio of -35%
ϕ Trading cost	2.7	Average transaction cost of 5%
Individual labor income risk		
w Average wage	0.072	See Section 6.1
ρ_w Autocorrelation	0.906	
σ_w Std. dev. (%)	19.8	
Individual dividend income risk		
d Average dividend yield	0.0445	See Section 6.1
ρ_d Autocorrelation	0.905	
σ_d Std. dev. (%)	61.8	
Aggregate interest rate risk		
R Average interest rate	1.054	See Section 6.1
ρ_R Autocorrelation	0.905	
σ_R Std. dev. (%)	1.9	

level, and a dummy variable that identifies whether the worker is in the informal sector. Finally, D_t corresponds to the time fixed effects dummy variables. After running the regression, we obtain the residuals $y_{a,t}^i$ and assume the income risk follows a stationary process with a persistent and transitory component. The stationarity assumption allows us to drop the time dimension, and the income risk model becomes

$$\begin{aligned}
y_a^i &= z_a^i + \epsilon_a^i, \\
z_a^i &= \rho_w z_{a-1}^i + \eta_a^{i,w}, \\
\eta_a^{i,w} &\sim (0, \sigma_w^2), \quad z_0^i \sim (0, \sigma_{z_0}^2), \quad \epsilon_a^i \sim (0, \sigma_\epsilon^2).
\end{aligned} \tag{10}$$

Now the objective is to estimate the vector of parameters $\theta = (\rho_w, \sigma_w^2, \sigma_{z_0}^2, \sigma_\epsilon^2)$. These parameters are identified with the following theoretical moments:

$$\begin{aligned}\rho_w &= \frac{\text{COV}[y_a^i, y_{a-2}^i]}{\text{COV}[y_{a-1}^i, y_{a-2}^i]}, \\ \sigma_\epsilon^2 &= \text{V}[y_{a-1}^i] - \rho^{-1} \text{COV}[y_a^i, y_{a-1}^i], \\ \sigma_w^2 &= \text{V}[y_{a-1}^i] - \text{COV}[y_a^i, y_{a-2}^i] - \sigma_\epsilon^2, \\ \sigma_{z_0}^2 &= \text{V}[y_0^i] - \sigma_\epsilon^2.\end{aligned}\tag{11}$$

We use data from the National Survey of Employment and Occupation (ENOE) to do an over-identified GMM estimation with an identity weighting matrix.¹⁷ The ENOE survey is a quarterly household rotating panel with a representative sample of 120,000 households that started in 2005:Q1. Every household is interviewed for five consecutive quarters, and, each quarter, 20 percent of the sample is replaced. Consistent with the standard practice in the literature, our sample selection criteria are male individuals with ages between 20 and 60 and with positive earnings. Table 6 shows the estimated parameters and compares them with the literature’s estimation done for the U.S.

Table 6: Annual Income Process Estimates

	Mexico Benchmark	Mexico Formal Employment	U.S. (a)	U.S. (b)	U.S. (c)
ρ_w	0.906	0.922	0.999	0.988	0.970
σ_w^2	0.039	0.038	0.017	0.015	0.038

Note: The results for Mexico correspond to data from the ENOE survey from 2005:Q1 to 2014:Q4. The estimates are annualized following [Krueger, Mitman, and Perri \(2016\)](#). Column (a) corresponds to [Storesletten, Telmer, and Yaron \(2004\)](#), (b) to [Guvenen \(2009\)](#), and (c) to [Krueger, Mitman, and Perri \(2016\)](#).

We find that the estimated persistence of the income risk process is smaller, and the variance is larger, for Mexico compared with the U.S. A reason for this difference

¹⁷Note that to just-identify the parameters, we need data only for ages $(a, a - 1, a - 2)$. Since we are using data for 160 quarterly ages, the system is over-identified.

could come from the informal market structure that is common in emerging economies (Leyva and Urrutia (2020)). The Mexican labor market is characterized by a high informality rate – more than 50 percent informal employment. Since the informal sector is relatively more flexible than the formal sector, it could create a less permanent effect of idiosyncratic shocks. Moreover, Gomes, Iachan, and Santos (2020) find that informality is associated with more volatile earnings. Finally, the combination of a large informal sector and the lack of unemployment insurance could also cause a higher income risk.¹⁸ To explore this reason, in the second column, we show the results from the estimation done with a subsample of only formal employment. As expected, the difference narrows, although the change is small. Given that we do not explore specific heterogeneity in the labor markets in the model, we still use as a benchmark the results from the first column that include all the employment. Lastly, the discrete labor income risk process is approximated using a symmetric two-state Markov chain that employs a simple persistence rule following Mendoza (2010). The discretized risk takes the values $\epsilon^w \in \{\epsilon_L^w = 0.80, \epsilon_H^w = 1.20\}$, and the probability that the next-period realization of the shock is the same as that of the current period is $Pr[\epsilon^{w'} = \epsilon_j^w | \epsilon^w = \epsilon_j^w] = 0.953$ for $j \in \{L, H\}$.

The dividend income risk plays a key role in the decision rules of households and drives the *asset-wealth tradeoff* discussed in Section 5.1. However, a proper estimation of this process is infeasible due to the lack of available data in most economies.¹⁹ Because of the restrictions of the available data for Mexico, we take the following estimation strategy. We jointly estimate the three parameters that characterize the dividend income risk (d, ρ_d, σ_d) to match the leverage ratio distribution of households in 2005. Specifically, we focus on three distribution statistics: the measure of savers who have financial assets (negative leverage ratio), indebted households that have positive debts but are not close to their debt limit, and financially constrained households that have a leverage ratio above 0.168 (the 90th percentile). The matched distribution is shown

¹⁸Bosch and Esteban-Pretel (2015) study the consequences for the labor market of implementing an unemployment benefit system in economies with large informal sectors and find that an unemployment benefit could increase the formality rate.

¹⁹One exception is the work by Fagereng et al. (2020), who estimate the wealth risk using administrative data from Norway and find that there is high heterogeneity in the wealth returns and that these differences are highly persistent.

in Table 7 and the calibrated parameters are ($d = 0.0445, \rho_d = 0.905, \sigma_d = 0.618$). Similarly to the labor risk, the discrete dividend risk process is approximated using a symmetric two-state Markov chain that employs a simple persistence rule. Hence, the discretized risk takes the values $\epsilon^d \in \{\epsilon_L^d = 0.38, \epsilon_H^d = 1.62\}$, and the probability that the next-period realization of the shock is the same as that of the current period is $Pr[\epsilon^{d'} = \epsilon_j^d | \epsilon^d = \epsilon_j^d] = 0.9525$ for $j \in \{L, H\}$. These estimates imply that the effective dividend yield ($\epsilon^d d$) households will face can take the following two values in percent: $\{1.7, 7.2\}$. Lastly, the aggregate wage level, w , is set equal to $4d\bar{K}$ such that the average household has a total flow income that corresponds to four-fifths labor income and one-fifth dividend income.

Table 7: Leverage Ratio Distribution of Households (percent)

	Data	Model
Savers (lev. ratio ≤ 0)	14.2	14.3
Indebted unconstrained (lev. ratio $\in (0, 0.168)$)	75.8	75.7
Financially constrained (lev. ratio ≥ 0.168)	10.0	10.0

Note: The leverage ratio level considered to be the threshold between financially constrained and indebted unconstrained households is 0.168 and corresponds to the 90th percentile of its distribution in 2005. Source: MxFLS.

The last exogenous process that needs to be estimated corresponds to the international interest rate. This process was estimated using data from [Meza \(2018\)](#), and the parameters are ($R = 1.054, \rho_R = 0.905, \sigma_R = 0.019$). Similarly, the interest rate process is approximated using a symmetric two-state Markov chain that employs a simple persistence rule. Hence, the discretized interest rate takes the values $R \in \{R_H = 1.073, R_L = 1.035\}$, and the probability that the next-period realization of the interest rate is the same as that of the current period is $Pr[R' = R_j | R = R_j] = 0.9525$ for $j \in \{L, H\}$. These values are common in the literature of small open economies and are close to the estimates obtained in studies of the Mexican economy (see [Mendoza \(2010\)](#) and [Bianchi \(2016\)](#), among others).

6.2 Stationary Model

In this subsection, we analyze the stationary equilibrium for an economy in which the interest rate is constant at its steady state value of 5.4 percent – i.e., a Bewley economy without aggregate risk. Because of the *asset-wealth tradeoff* described in Section 5.1, the stationary model does a good job of capturing the wealth inequality, as seen in Table 8.

Table 8: Nontargeted Inequality Measure

	Data	Model
Net wealth Gini coefficient	0.733	0.592

Note: Data corresponds to 2005. Source: MxFLS.

Moreover, in Table 9, we show the average net wealth, assets, and debts by deciles relative to the median level of each variable for simulated data and observed data in 2005. As we can see in the top and medium rows, the net wealth and assets distributions generated by the model are very close to the ones obtained from the MxFLS in 2005 – with the exception of the top deciles. Regarding the total debt, the only decile that is significantly different is the bottom decile. One possible reason for this difference is that we do not allow households to default in the model, and households cannot hold more debt than the collateral limit – in contrast to the observed data, where households in the bottom decile have negative net wealth. However, for the rest of the deciles, the model does a good job of capturing the inequality in terms of net wealth, total assets, and debt.

With respect to the aggregate equity premium, in Table 10, we show its level and decomposition. The model generates a high equity premium that is close to the data (first column). As expected, the risk component contributes the most to the equity premium, about 89 percent, while the other 11 percent corresponds to the constraint effect. Note that the calibration was done to capture the measure of constrained households in 2005, equal to 10 percent (see Table 7). Hence, even if only these households have an active debt constraint, there is a significant contribution to the aggregate equity premium.

Finally, notice that the debt-deflation mechanism affects a household’s consump-

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net wealth										
Data	-0.1	0.1	0.3	0.6	1	1.6	2.2	3.2	4.9	22.3
Model	0.1	0.2	0.4	0.7	1	1.5	2.1	2.9	4.5	9.5
Assets										
Data	0.1	0.1	0.3	0.6	1	1.6	2.2	3.1	4.8	21.5
Model	0.1	0.2	0.3	0.6	1	1.5	2.2	3.4	5.9	18.3
Debt										
Data	2.9	0.4	0.5	0.9	1	1.8	0.8	1.5	2.6	5.2
Model	0	0.1	0.2	0.5	1	1.6	2.3	3.3	5	9.2

Note: Deciles ordered by net wealth. Source: MxFLS.

Table 10: Decomposition of the Equity Premium (percent)

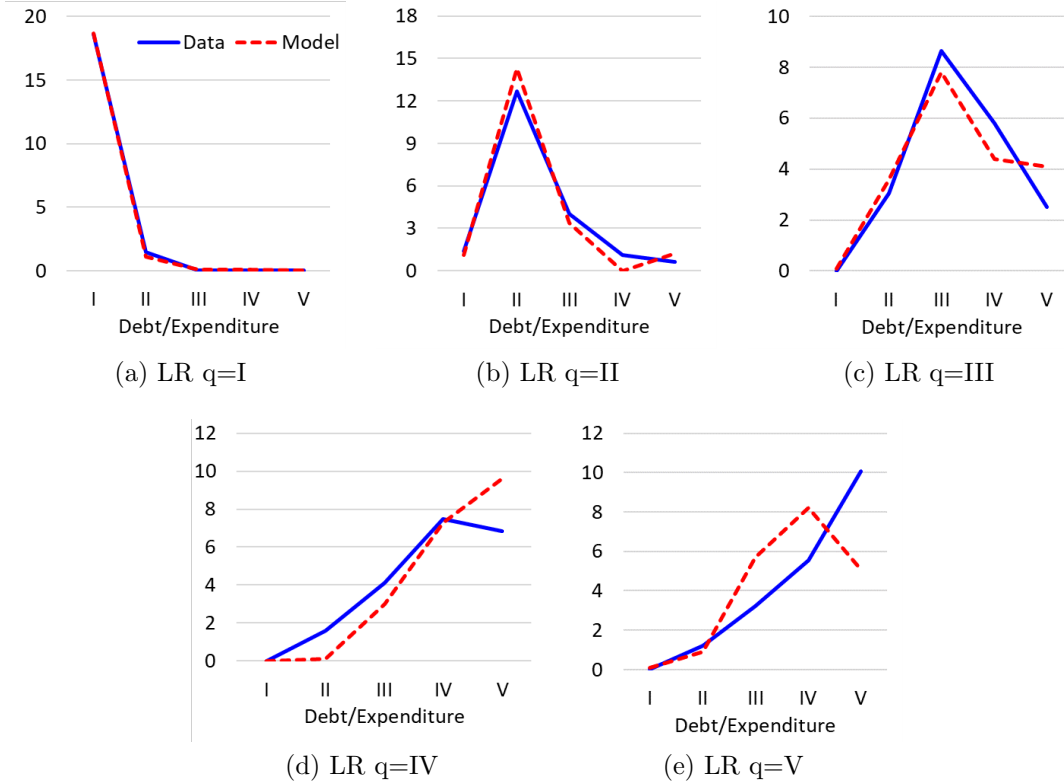
	Data	Model
Equity premium	6.5	5.2
Constraint effect	-	0.6
Risk effect	-	4.8
Trading cost effect	-	0.02
Short-sales effect	-	-0.02

Note: Data correspond to Mexico in 2005.

Source: [Damodaran \(2013\)](#)

tion when two things happen. First, the household must be highly leveraged, so when the collateral constraint tightens, the household is close to (or at) the binding region and needs to adjust its asset holdings. Second, the household must have a large debt-to-expenditure ratio, so when it has to deleverage, there is a significant effect on its consumption. As a model validation exercise, Figure 4 shows how well the model replicates the distribution of households with respect to the joint leverage ratio and debt-to-expenditure ratio. In overall terms, the model does a good job of replicating the joint distribution, with a slight underestimation of the measure of households in the top quintile for the leverage ratio and debt-to-expenditure ratio.

Figure 4: Joint Leverage Ratio and Debt-to-Expenditure Ratio Distribution



Note: Solid blue lines correspond to the simulated distribution of the stationary model. Dashed red lines correspond to the distribution for Mexican households in 2005. Source: MxFLS.

6.3 Aggregate Risk Model

To solve the aggregate risk model, we adapt the *nontrivial* market clearing algorithm proposed by [Krusell and Smith \(1997\)](#) to a small open economy framework. Specifically, we use the current aggregate net foreign asset position, $B \equiv \int_0^1 b^i di$, and the current interest rate, $R - 1$, to forecast the next period's net foreign asset position, B' . Additionally, to forecast the domestic asset price, q , we also use last period's asset price, q_{-1} . This algorithm is computationally intensive since the current market clearing asset price depends on the whole distribution of asset holdings and not only on the aggregate holdings (which are constant). Hence, to obtain a simulated time series, each period we use the aggregate law of motion to forecast the next period's aggregate net foreign asset position and the next period's asset price. With these

forecasts, we then solve a fixed-point problem for every period, which gives as a solution the current equilibrium market clearing price.²⁰ The solution of the aggregate law of motion is as follows:

$$\begin{aligned} B' &= -0.020 + 0.798 B + 0.129 (R - 1), & R^2 &= 0.99, \\ q &= 0.507 + 0.194 B - 0.261 (R - 1) + 0.083 q_{-1}, & R^2 &= 0.93. \end{aligned} \quad (12)$$

6.3.1 Simulation and Event Study of Sudden Stops

Using the solution to the aggregate law of motions, we simulate a panel of 1,000 households for 6,000 periods and drop the first 1,000 periods. Table 11 columns (1) and (3) report long-run moments of the main macro aggregates from both the benchmark model with heterogeneous-agents and a representative-agent version without idiosyncratic risk and a lower leverage limit, κ , that matches the same average leverage ratio of 0.122 obtained in the model with heterogeneity. Regarding the mean of the variables, the current account as a percentage of GDP is zero for both models. In the heterogeneous-agents model, average consumption is 4.5 percent higher, the net foreign asset position relative to GDP is 4.7 percentage points larger, and the asset price is 26 percent higher. Since households do not need to self-insure against idiosyncratic shocks in the representative-agent model, there are less precautionary savings and lower demand pressure for the domestic asset. This equilibrium effect lowers the average asset price, tightening the aggregate financial conditions and lowering both average consumption and total debt.

Regarding the standard deviations, consumption volatility is about three times as volatile, and asset price is about one-half as volatile, in the benchmark heterogeneous-agents economy compared with the representative-agent economy. This result comes from the larger consumption adjustments that high-leveraged households have to make when they get hit by a negative shock.

To construct the event study of the simulated Sudden Stops, we average across all the identified crisis periods. Figure 5 shows the percent deviations from the steady state, where the crisis period corresponds to $t = 0$. The average of the simulated crisis episodes in the heterogeneous-agents economy corresponds to the solid lines, and the

²⁰See Appendix B for a description of the solution algorithm.

Table 11: Business Cycle Statistics

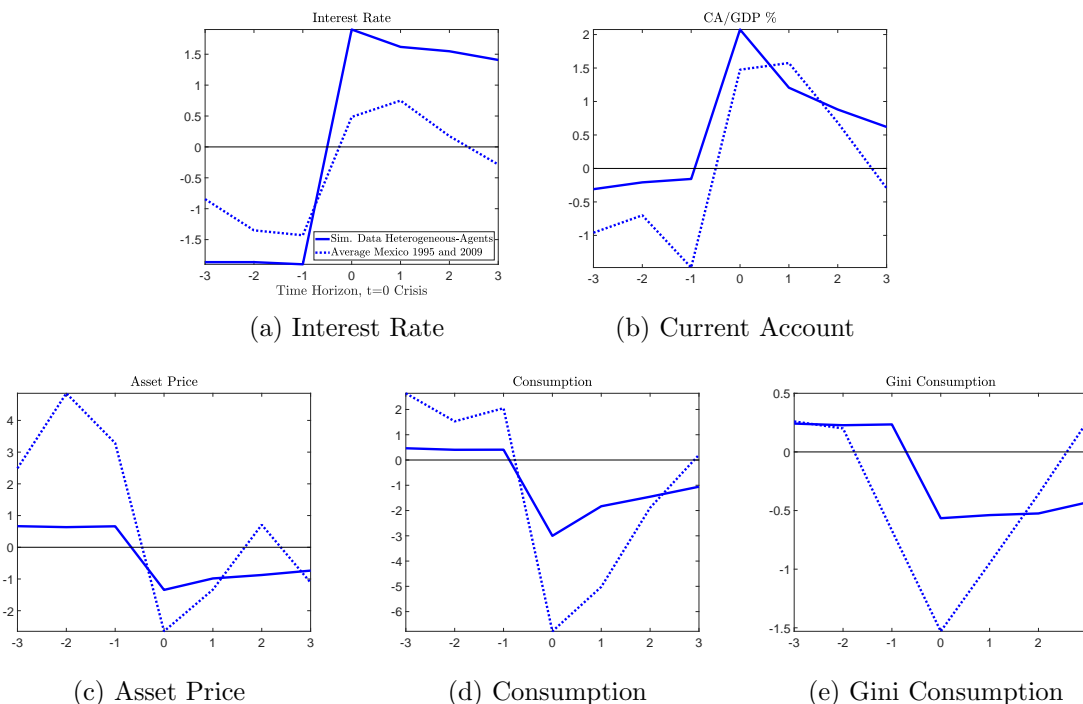
	(1)	(2)	(3)
	Het. Agents Benchmark Eme. Eco.	Het. Agents ($\sigma^d/2$) Adv. Eco.	Rep. Agent Same Mean Lev. Ratio
Mean			
CA/GDP (%)	0.00	0.00	0.00
Consumption	0.23	0.22	0.22
NFA/GDP (%)	-29.12	-31.64	-24.43
Leverage ratio	0.122	0.157	0.122
Asset price	0.53	0.52	0.42
Standard deviation (%)			
CA/GDP	0.75	0.29	0.10
Consumption	1.35	0.97	0.43
NFA/GDP	4.44	1.02	0.09
Leverage ratio	1.95	0.67	0.00
Asset price	0.80	0.66	1.50

Note: The representative-agent calibration has a lower leverage limit, κ , that matches the same average leverage ratio of the heterogeneous-agents model of 0.122.

average of the data for Mexico around 1995 and 2009 Sudden Stops corresponds to the dashed line.

Figure 5(a) shows that the Sudden Stops occur when there is an interest rate increase. This result is expected since the interest rate is the only source of aggregate uncertainty in this economy. However, note that not all the interest rate increases cause a crisis. Specifically, the long-run probability of a Sudden Stop in the simulated benchmark economy is 2.16 percent. In 5(b), we can see that a crisis episode is preceded by periods with the current account below the long-run average. Then, when the crisis happens ($t = 0$), there is a sharp reversal in the current account, which means that international capital stops flowing into the economy. Consistent with the data, the crisis is persistent and takes more than three years for the international capital to flow back into the economy. Regarding the asset price drop, in 5(c), we can see that the simulated price is 1.3 percent below the steady state, which is below

Figure 5: Event Study of a Sudden Stop



Note: Solid lines correspond to the simulated data using the heterogeneous-agents model calibrated to the Mexican economy, and dotted lines correspond to the average of the Mexican data around the 1995 and 2009 Sudden Stops. Panels (a), (b), and (e) correspond to the level difference from the long-run mean. Panels (c) and (d) correspond to percentage point deviations from the long-run average. Mexican data in panel (e) is bi-annual.

the asset price index for Mexico, and in 5(d), we can see that the model is able to generate a large and persistent aggregate consumption drop. Finally, 5(e) shows that the model is able to capture a decline in consumption inequality during the crisis – as measured by the Gini coefficient – consistent with the data.

Regarding the differentiated individual effects during a Sudden Stop, in Tables 12 and 13, we show the dynamics of the asset holdings and consumption according to the leverage ratio and wealth of households, as we similarly did for the results presented in Section 3.2. We can see that the model does a good job of capturing the damping effect coming from the wealthy unconstrained households that buy assets during a crisis and relieve the downward pressure on the price. In particular, these households increased their asset holdings by 6.4 percent during the crises. Moreover, in line with

the empirical evidence on the amplifying effect, the financially constrained wealthy households were the ones that fire-sold their assets the most during the crisis and decreased their asset holdings by 13.6 percent. Although in the model households in decile IX of the leverage ratio do not sell their assets, we can see that they increase in a smaller amount than those of households in deciles I through VIII. Hence, the model is able to capture both cross-sectional effects.

Table 12: Median Asset Holdings Change in a Crisis (percent)

Leverage Ratio	Net Wealth	
	I-IX (Non-Wealthy)	X (Wealthy)
I-VIII	-0.6	6.4
IX	2.2	3.4
X	1.8	-13.6

Note: Ordered in the period of the crisis.

In Table 13, we see that, in line with the empirical evidence, households with lower leverage ratios decrease their consumption the least. Thus, the model captures the heterogeneous consumption dynamics coming from the different leverage ratio levels and that crises do not affect every household in the same way.

Table 13: Median Consumption Change (percent)

Leverage Ratio	Pre-Crisis Period	Crisis Period
I	0.0	-1.9
II-IX	0.2	-2.7
X	0.5	-2.5

Note: Ordered in the period previous to the crisis.

Lastly, in Table 14, we show percent deviations from the steady state of the current account as a percentage of GDP, consumption, and the asset price for Mexico and different simulated economies. Columns (1) and (2) show the observed deviations in 1995 and 2009 for Mexico, respectively. In column (3), we show the benchmark heterogeneous-agents model calibrated to an emerging economy (Mexico). We can see that in the benchmark calibration, the asset price drop is smaller than the

consumption drop, consistent with the data. Finally, in column (5), we show the representative-agent version of the model in which there is no idiosyncratic risk and the leverage ratio limit, κ , is reduced to match the average leverage obtained in the heterogeneous-agents economy. Comparing columns (3) and (5), we can see that in the heterogeneous-agents economy, the damping effect dominates and asset prices drop less. However, there is a larger adjustment in aggregate consumption, driven mainly by the leveraged households (see Table 13).

Table 14: Comparison of Dynamics during Sudden Stops

	(1)	(2)	(3)	(4)	(5)
	Mexico	Mexico	Het. Agents	Het. Agents	Rep. Agent
	1995	2009	Benchmark	$(\sigma^d/2)$	Same Mean
			Eme. Eco.	Adv. Eco.	Lev. Ratio
CA/GDP (p.p.)	2.6	0.4	2.1	0.9	0.3
Consumption (%)	-8.3	-5.3	-3.0	-1.9	-1.3
Asset price (%)	-3.7	-1.8	-1.3	-0.9	-3.0

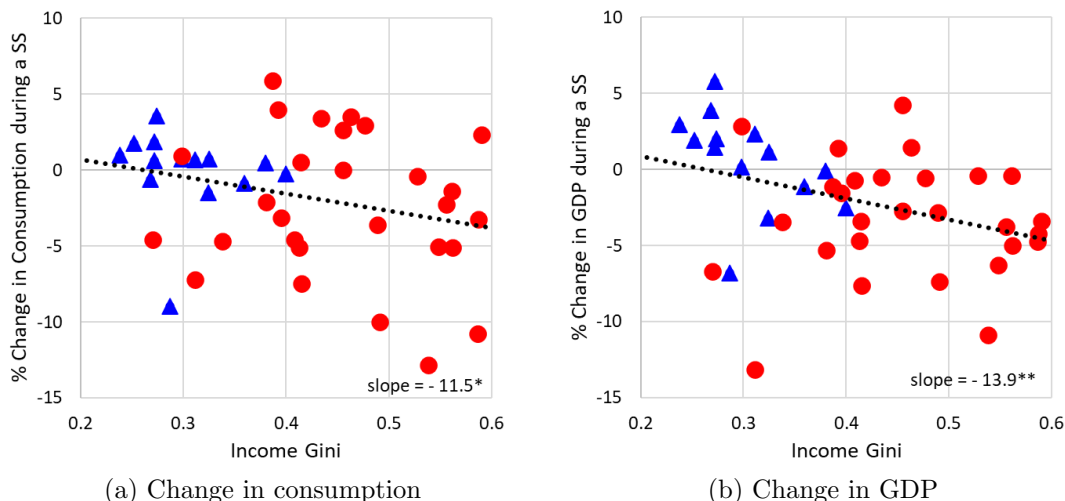
Note: Sudden Stop episodes are defined as the periods when the current account as a percentage of GDP is two standard deviations above its mean.

6.3.2 Effect of a Lower Variance in the Dividend Risk

In this subsection, we compare the severity of Sudden Stops in economies with different degrees of inequality. Figure 6 shows descriptive evidence that crises are more severe in more unequal economies. The figure shows a scatterplot with the percentage change in consumption (panel (a)) and in GDP (panel (b)) during Sudden Stops for different economies (advanced in triangles and emerging in circles) charted against their income Gini index. This evidence suggests that emerging economies are more unequal and that there is a negative correlation between both variables.

To quantitatively assess the effects of lower income inequality, we calibrate the model to an advanced economy where the dividend risk is one-half of that in the benchmark emerging-markets model. The results, summarized in Table 11 column (2) and Table 14 column (4), show that in the version of the model calibrated to an advanced economy, the average net foreign debt position is 8.5 percent larger, consumption drops 1.1 percentage points less, and asset prices drop 0.4 percentage

Figure 6: Severity of Sudden Stops and Inequality



Note: Triangle (circle) markers correspond to advanced (emerging) economies. Dates of Sudden Stop episodes come from [Bianchi and Mendoza \(2020\)](#). Gini index measures income inequality; larger numbers mean larger inequality (income instead of wealth is used because of the availability in a larger sample of countries). ** $p < 0.05$, * $p < 0.1$. Source: Own calculations with data from the World Bank.

point less. Hence, the model predicts that in economies with less dividend return inequality, the economy supports larger debt positions, and Sudden Stop crises are less severe, as observed in the data.

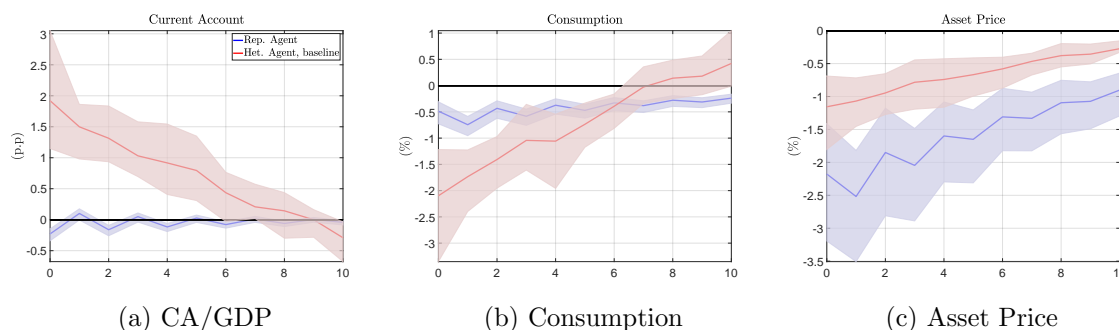
6.3.3 Impulse Response to an Interest Rate Shock

Lastly, this subsection looks at the impulse response functions after an interest rate shock two of standard deviations. We compare the model with heterogeneity for different initial distributions and the representative-agent model. In the baseline model with heterogeneity, the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. With respect to the same initial condition model, the responses are obtained by conditioning the economy to start with a perfect equality distribution where all households hold the long-run average level of individual assets and the long-run average level of individual bonds. Finally, the representative-agent model results are obtained by conditioning the economy to start at the long-run mean

bond position. All three simulations also start at the long-run mean interest rate.

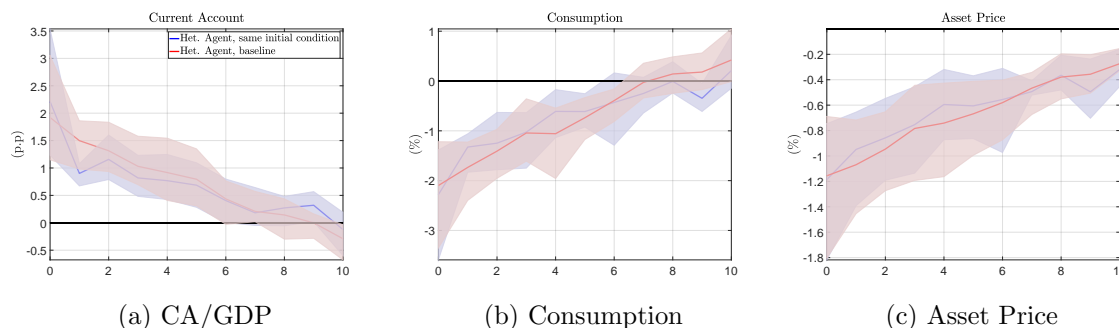
In line with the results from the previous subsection, Figure 7(a) shows that the model with heterogeneity generates persistent current account reversals, which are 2 percentage points larger than in the representative-agent model, which produces an isolating response in the current account around zero. In panels (b) and (c), we see that the response of the model with heterogeneity is about four times larger for consumption, and about half as large for asset prices, compared with the representative-agent economy, respectively. Lastly, in Figure 8, comparing both blue and red lines, we see that the effect of starting with a different initial distribution is minimal. However, in line with the results of the previous subsection, under a perfect equality initial condition, the responses are slightly smaller after impact. This set of results suggests that while inequality coming from the economy’s fundamental processes matters for the severity and propagation of aggregate shocks, the effect coming from starting with different individual distributions is minimal.

Figure 7: Impulse Responses to an Interest Rate Shock – Baseline Heterogeneous and Representative-Agent Economies



Note: Impulse response functions after an interest rate shock of two standard deviations. In the baseline model with heterogeneity (red line), the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. In the representative-agent model (blue line), results are obtained by conditioning the economy to start at the long-run mean bond position. Both simulations also start at the long-run mean interest rate. Bands represent 68% credible intervals, and solid lines are averages over 10 simulations.

Figure 8: Impulse Responses to an Interest Rate Shock – Baseline and Perfect Equality Initial Condition Heterogeneous-Agents Economies



Note: Impulse response functions after an interest rate shock of two standard deviations. In the baseline model with heterogeneity (red line), the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. In the same initial condition model (blue line), the responses are obtained by conditioning the economy to start with a perfect equality distribution where all the agents have the long-run average level of individual assets and the long run average level of individual bonds. Both simulations also start at the long-run mean interest rate. Bands represent 68% credible intervals, and solid lines are averages over 10 simulations.

7 Conclusion

This paper studies the cross-sectional dimension of the debt-deflation mechanism that triggers endogenous financial crises of the Sudden Stop type. This dimension is relevant for the macroeconomy for two reasons. First, there is a damping effect on the deflation of asset prices coming from the unconstrained wealthy households that buy depressed assets, relieving the downward pressure on asset prices. Second, there is an amplifying effect on the asset price deflation coming from the financially vulnerable households that fire-sale assets, generating a stronger downward pressure on asset prices. Because these two cross-sectional effects move asset prices in opposite directions, the inequality role during crises is quantitatively ambiguous. Hence, this paper examines how the frequency and severity of Sudden Stop crises are affected by inequality in an economy.

Using panel data for Mexican households, we document microdata evidence that supports both effects. Specifically, the 2009 crisis had different effects on households depending on the composition of their balance sheets. The real estate holdings of low-leveraged wealthy households increased 59.4 percent during the crisis, while wealthy

households with high leverage fire-sold and decreased their assets the most during the crisis. Additionally, in terms of the consumption dynamics, households in the top decile of leverage decreased their expenditures 6.2 percent during the crisis, while households in the bottom decile increased their expenditures 5.4 percent. These heterogeneous asset and consumption dynamics highlight the importance of the opposing forces that are missed when financial crises are studied under a representative-agent framework. For this reason, we proposed a model to quantify a Sudden Stop's effect on asset prices and consumption, accounting for the household's heterogeneity in its balance sheet.

Using the proposed asset-pricing Bewley model of a small open economy, we find that in a version of the model calibrated to an emerging economy (Mexico), the model can explain Sudden Stops' key stylized facts and generate persistent current account crises. Regarding the cross-sectional forces, the damping effect dominates, and asset prices drop less during Sudden Stop episodes in heterogeneous-agents economies. In contrast to the representative-agent framework, the model produces an empirically plausible leverage ratio distribution and generates persistent current account reversals with larger drops in consumption driven by the most leveraged households, consistent with the data. Moreover, calibrating the model to an advanced economy where the dividend risk is one-half of that in the benchmark emerging-markets model, we find that the average net foreign debt position is 8.5 percent larger, consumption drops 1.1 percentage points less, and asset prices drop 0.4 percentage point less. Additionally, an impulse response analysis, which compares the effects of an interest rate shock, suggests that while inequality coming from the economy's fundamental processes matters for the severity and propagation of aggregate shocks, the effect coming from starting with different individual distributions is minimal. Hence, the model predicts that larger debt positions are supported in economies with less dividend return inequality, and Sudden Stop crises are less severe in such economies, as observed in the data.

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Online Appendix to “Inequality and Asset Prices during Sudden Stops”

Sergio Villalvazo

This Online Appendix consists of the following sections:

- A. The 2009 Mexican Sudden Stop at the Aggregate Level
- B. Solution Algorithm
- C. Nonlinearities in the Stationary Model
- D. Event Study of an Economy with Lower Variance in the Dividend Risk

A The 2009 Mexican Sudden Stop at the Aggregate Level

A Sudden Stop is a fast and large outflow of international capital ([Calvo, Izquierdo, and Talvi \(2006\)](#)). Hence, these types of episodes are characterized by large current account (CA) movements.²¹ In this appendix, we use aggregate data to show the Sudden Stop that the Mexican economy experienced in 2009.

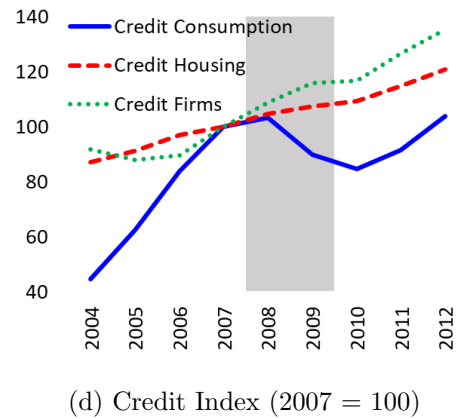
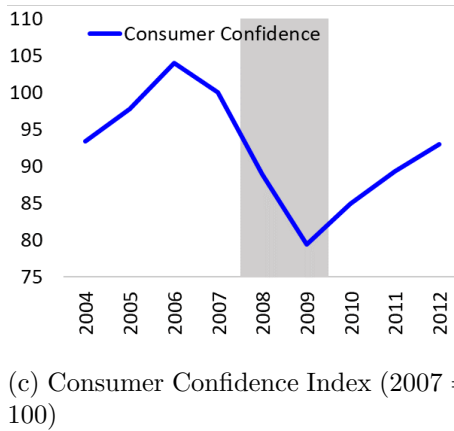
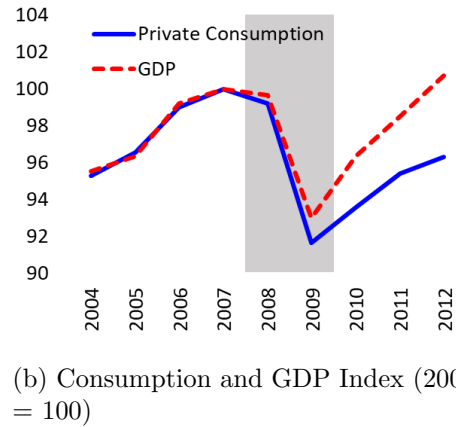
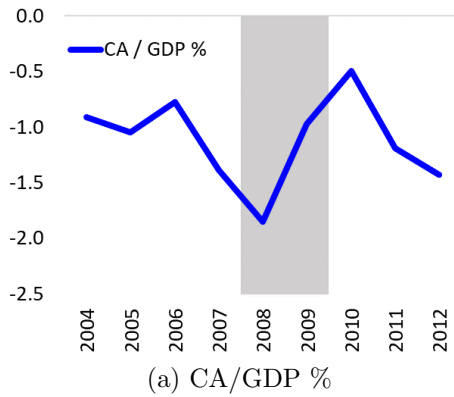
In Figure [A-1](#), we can see that the CA deficit reversed around 1.5 percentage points of GDP. Also, GDP and consumption declined, and there was a drop in consumer confidence and a decline in consumption credit, while firm and housing credit was not affected.

On the prices side, in Figure [A-2](#), we see that there was a large decline in the stock market, house prices decelerated and remained constant for about four years after the crisis burst, the J.P. Morgan EMBI+ spread that measures the Mexican sovereign bond risk increased about 2 percentage points, and there was a large depreciation of the Mexican peso against the dollar.

The aggregate dynamics shown in this Appendix are not particular to Mexico. See [Bianchi and Mendoza \(2020\)](#) for a recent survey of Sudden Stop episodes among both advanced and emerging economies.

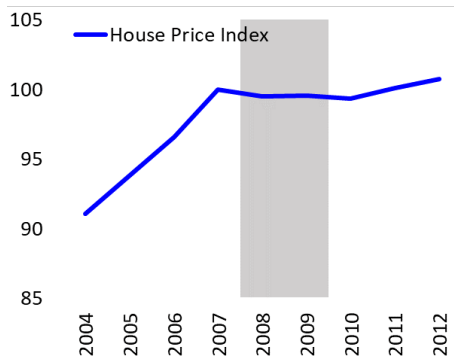
²¹Some Sudden Stop episodes have even registered CA reversals, meaning that the economy transitions from having a negative CA (foreign capital entering the economy) to having positive CA surpluses (capital leaving the economy).

Figure A-1: Quantities and Consumption Determinants

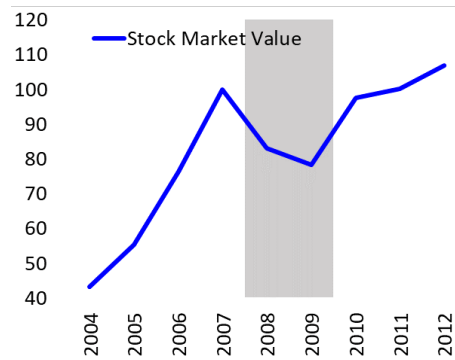


Note: The gray area corresponds to the crisis. Source: INEGI, World Bank, Banxico.

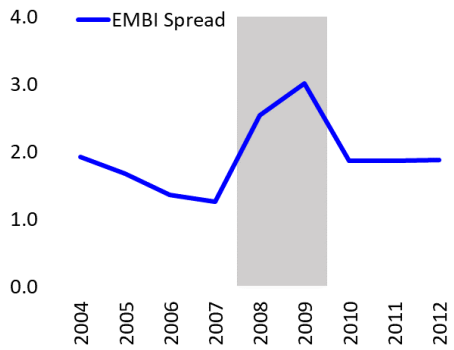
Figure A-2: Asset Prices



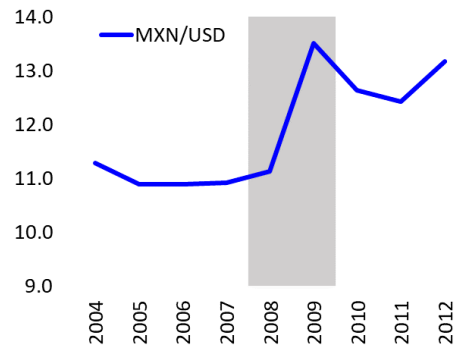
(a) House Price Index (2007 = 100)



(b) Stock Market Value Index (2007 = 100)



(c) J.P. Morgan EMBI Spread for Mexico in %



(d) Mexican Peso Exchange Rate for USD

Note: The gray area corresponds to the crisis. Source: Sociedad Hipotecaria Federal, Moody's Analytics, INEGI, World Bank.

B Solution Algorithm

In this appendix, we describe the solution method. Building from [Krusell and Smith \(1997\)](#), we adapt their *nontrivial* market clearing algorithm to a small open economy framework. In particular, instead of solving problem 5, we solve

$$\begin{aligned}
 \tilde{v}(b, a, \epsilon^w, \epsilon^d, B, \epsilon^R, q) &= \max_{\{c, b', a' \geq 0\}} u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, B', \epsilon^{R'})] \quad s.t. \\
 c + R(\epsilon^R)^{-1}b' + q(a' + \Phi(a', a)) &= \epsilon^w w + a(q + \epsilon^d d) + b, \\
 R(\epsilon^R)^{-1}b' &\geq -\kappa q a', \\
 \Phi(a', a) &= \frac{\phi}{2}(a' - a)^2, \\
 B' &= \gamma_B^0 + \gamma_B^1 B + \gamma_B^2 (R - 1), \\
 q &= \gamma_q^0 + \gamma_q^1 B + \gamma_q^2 (R - 1) + \gamma_q^3 q_{-1}, \tag{A.1}
 \end{aligned}$$

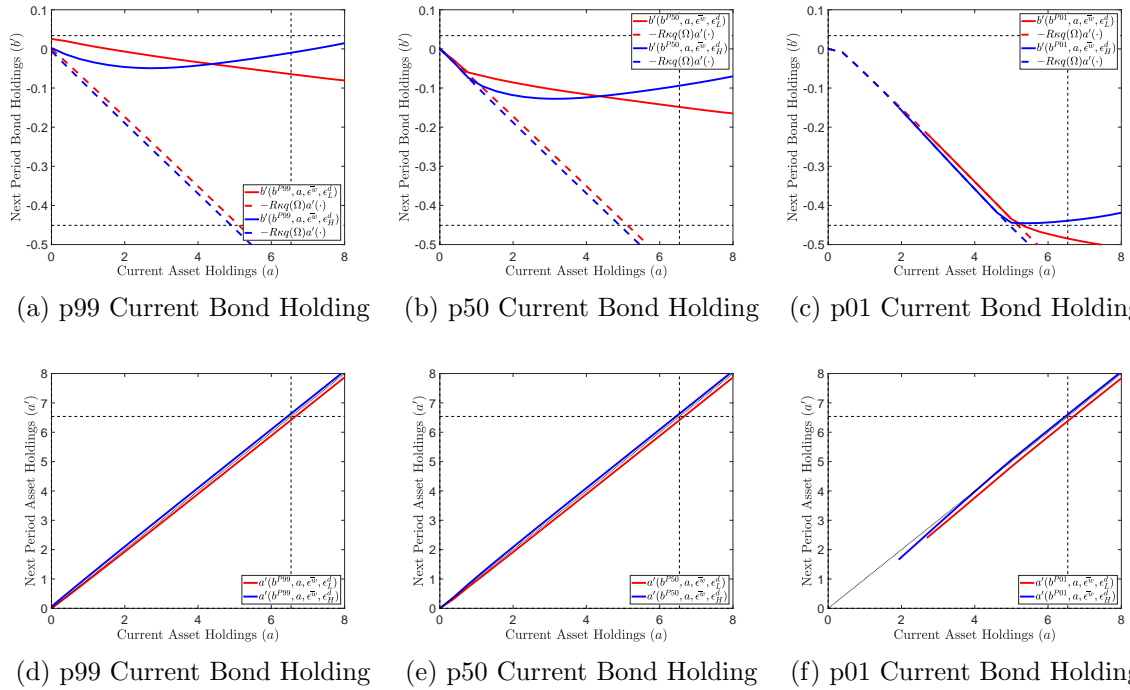
where we replaced the full household distribution Ω with the aggregate bond position $B = \int b d\Omega$ and market clearing in the asset holdings is achieved using a fixed-point iteration on q such that $\bar{K} = \int a'(\cdot) d\Omega$. Then the solution algorithm follows the simulation method described in [Krusell and Smith \(1997\)](#).

C Nonlinearities in the Stationary Model

To better understand the mechanism and the *asset-wealth tradeoff*, Figures [A-3](#) through [A-6](#) show the policy functions and the nonlinearities generated in the model. In the upper row of Figure [A-3](#), the solid lines correspond to the bond policy for the high–(low–) dividend shock in blue (red) and the average labor income shock as a function of the current asset holdings for three different values of the current bond holding $b^\#$. Additionally, the dashed lines represent the corresponding debt limits, and the black dashed lines correspond to the bottom 1 and top 99 percentiles of bond and asset holdings obtained from the model’s simulated cross-section. The figure shows that for low–dividend shocks (red lines), a household lowers its bond holdings (or gets more debt) as it increases its asset holdings. This effect is stronger for constrained house-

holds, as shown in panels (b) and (c). As described in Section 5.1, the *asset-wealth tradeoff* generates the convex form of the bond policy for high-dividend shocks (blue lines). For asset-poor households, as they increase their assets, they also lower their bond holdings (or get more debt if the holdings are negative), and there is a certain level for which the dividend risk exposure overcomes the benefit from more debt capacity that makes households increase their bond holdings. Regarding the bottom row of the figure, we can see the asset policy function that is highly linear and behaves as expected: for high-dividend shocks, households accumulate more assets, and for low-dividend shocks, households decumulate assets.

Figure A-3: Stationary Bond and Asset Policies as a Function of Current Asset Holdings

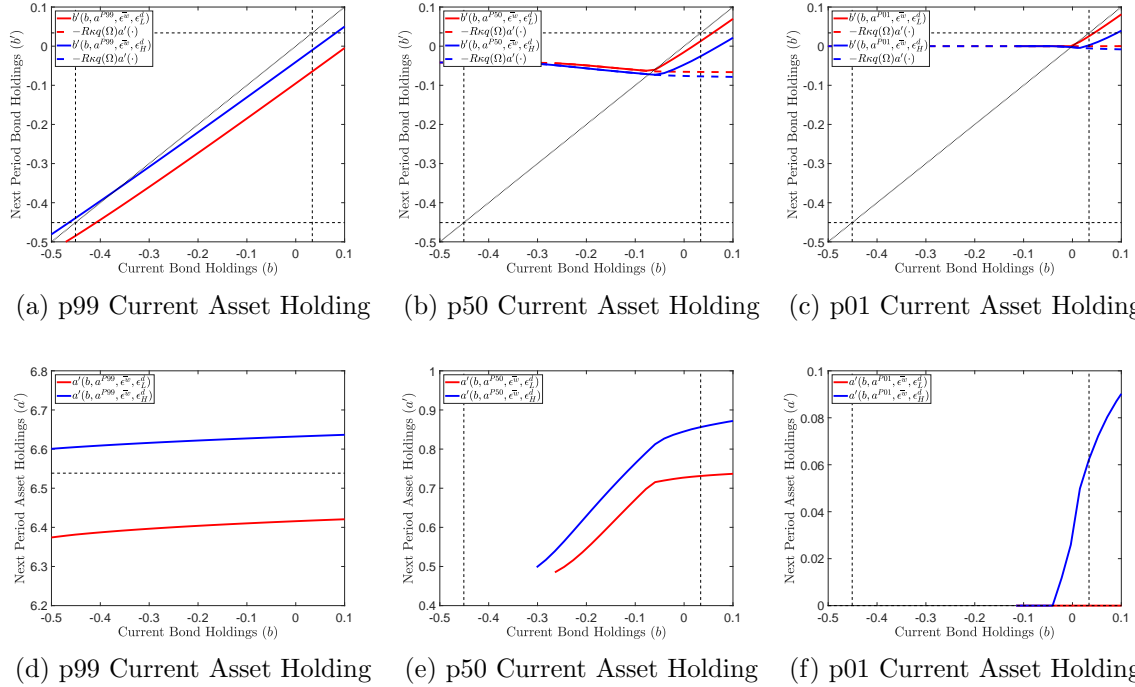


Note: For a current bond holding $b^\#$ and mean labor shock $\bar{\epsilon}^w$, the upper (lower) row corresponds to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high- (low-) dividend shock, and the dashed blue (red) line corresponds to the debt limit with the high- (low-) dividend shock. Dashed black lines correspond to the bottom 1% and top 99% of bond and asset holdings obtained from the model's simulated cross-section. Dotted black lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

Moreover, in Figure A-4, we show similar bond and asset policies but now as a function of the current bond holdings. In the upper row, we can see the standard bond policies under a binding debt limit. Panel (a) shows the policy for a high-asset holder. Here we can see that the debt limit is not binding for the states within the 1st and 99th percentiles. However, as we move to lower asset holdings, in panels (b) and (c), we can see that the LtV becomes binding when households accumulate enough debt. With respect to the cross-sectional fire-sales in the model, in the bottom row of the figure, we can see that households accumulate less assets as they increase their debt holdings. However, this relation is highly strengthened (households incur fire-sales) when the debt limit becomes binding, which can be seen using panels (b) and (e) and also panels (c) and (f). There are strong declines in asset holdings (panels (e) and (f)) in the states where bond holdings reach the debt limit (panels (b) and (c)).

Additionally, in Figure A-5, we show the difference in the bond policy function for a high- and a low-dividend shock in panel (a) and a labor income shock in panel (b). We can see a positive and increasing difference in the next-period bond holdings between the high- and low-dividend productivities as we move to higher current asset holdings (Figure A-5(a)). This result means that when the idiosyncratic dividend realization is high, the household optimally chooses larger bond holdings for the next period. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. In contrast, in Figure A-5(b), we can see that the difference in the bond policy function between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout all the feasible state-space. Similarly, in Figure A-6, we show the difference in the asset policy function for a high- and a low-dividend shock in panel (a) and a labor income shock in panel (b). We can see a positive and increasing difference in the next-period asset holdings between the high- and low-dividend productivities as we move to higher current asset holdings (Figure A-6(a)). However, for high enough asset values, this positive difference becomes relatively constant. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. Finally, similarly to the bond policy function, in Figure A-6(b), we can see that the next-period asset holdings difference between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout

Figure A-4: Stationary Bond and Asset Policies as a Function of Current Bond Holdings

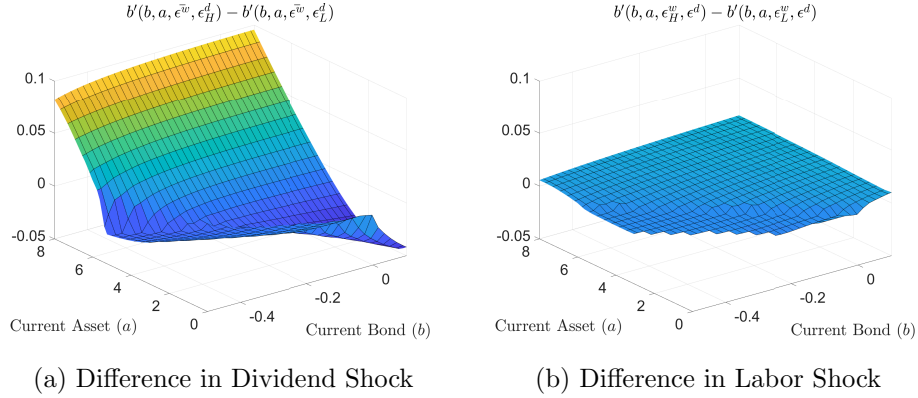


Note: For a current bond holding $b^\#$ and mean labor shock $\bar{\epsilon}^w$, the upper (lower) row corresponds to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high- (low-) dividend shock, and the dashed blue (red) line corresponds to the debt limit with the high- (low-) dividend shock. Dashed black lines correspond to the bottom 1% and top 99% of bond and asset holdings obtained from the model's simulated cross-section. Dotted black lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

all the feasible state-space.

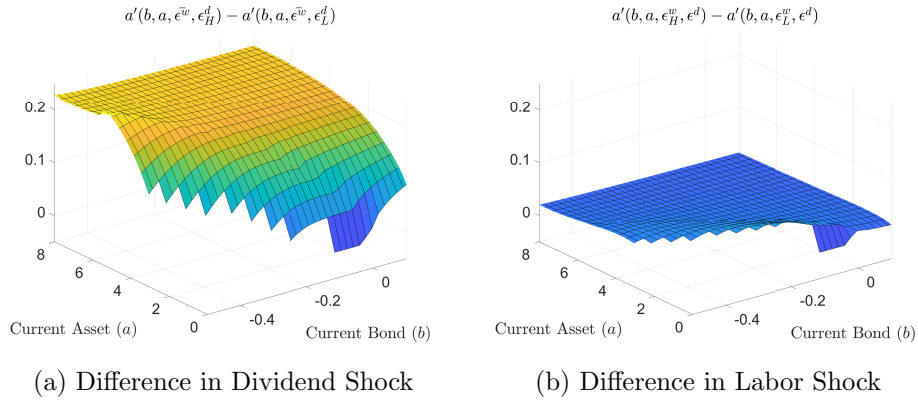
In summary, we used the stationary model to show the cross-sectional behavior of households. We can see that households with high-dividend shocks will accumulate more assets, and, while they are still asset poor, they decumulate bonds. Once they become asset rich, because of the *asset-wealth tradeoff*, they start accumulating more bonds (Figure A-3). This behavior generates wealthy unconstrained households that drive the damping cross-sectional effect. Moreover, we also show that households decumulate assets as they increase their debts, and that this relation strengthens (households incur fire-sales) when the debt limit is reached, driving the strength of the

Figure A-5: Effect of Non-insurable Individual Shocks in the Bond Policy



Note: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

Figure A-6: Effect of Non-insurable Individual Shocks in the Asset Policy



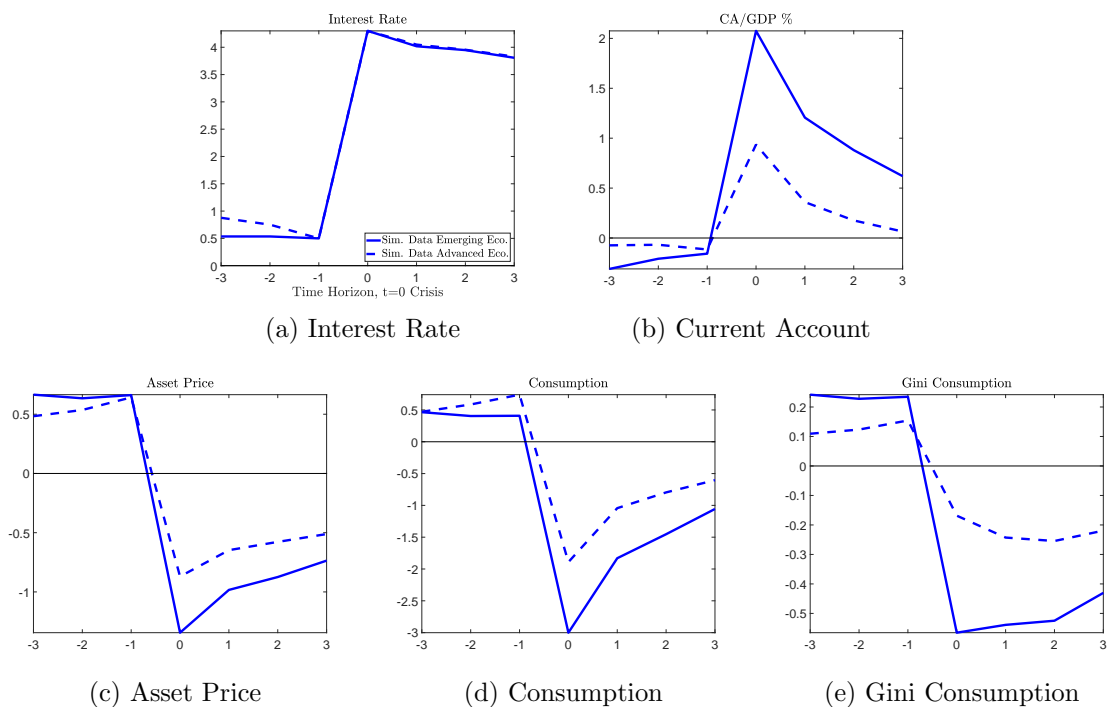
Note: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

amplifying effect (Figure A-4). Note that the representative-agent model would miss both effects. First, since there are no individual shocks, every household will behave in the same way. Hence, they want to either sell or buy more assets. Second, in that model, the average debt constraint multiplier will be the same as the individual debt multiplier, while in the heterogeneous-agents model, although fewer households could be constrained (calibrated to be only 10 percent), they could have a larger multiplier given the individual states. Finally, we used the stationary solution for simplicity and to avoid the extra aggregate states that would be needed in the aggregate risk model.

D Event Study of an Economy with Lower Variance in the Dividend Risk

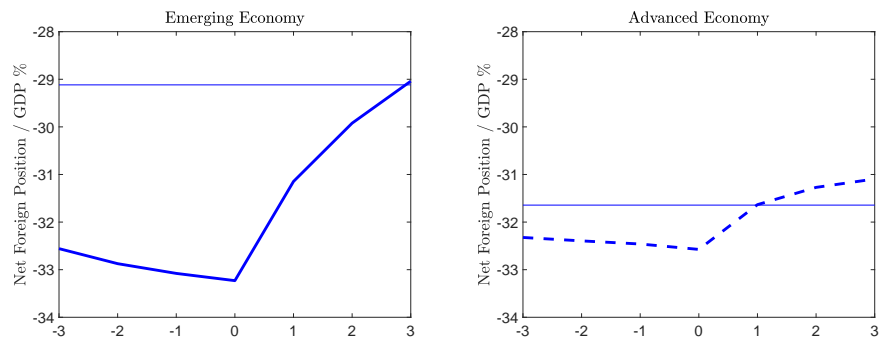
In Figures A-7 and A-8, we show the event study analysis for the same history of individual and aggregate shocks for the two calibrations: (1) the baseline emerging economy (in solid lines) and (2) the advanced economy with the same calibration but with half the variance in the dividend risk (in dashed lines).

Figure A-7: Event Study of a Sudden Stop in Simulated Economies



Note: Solid lines correspond to the simulated data using the heterogeneous-agents model calibrated to an emerging economy (Mexico), and dashed lines to the heterogeneous-agents model calibrated to an advanced economy that has one-half the variance in the dividend risk. Panels (a), (b), and (e) correspond to the level difference from the long-run mean. Panels (c) and (d) correspond to percentage point deviations from the long-run average.

Figure A-8: Net Foreign Asset Position Event Study of a Sudden Stop in Simulated Economies



(a) Emerging Economy

(b) Advanced Economy

Note: Solid blue horizontal lines correspond to the long-run averages.