Verifiability and effective persuasion

Noémie Cabau¹ Ming Li²

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¹Budapest University of Technology and Economics

²Concordia University, CIRANO, and CIREQ

Cabau and Li-Verifiability

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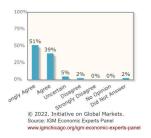
Carbon Tax

DECEMBER 20, 2011

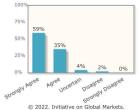
ADD TO POLL

A tax on the carbon content of fuels would be a less expensive way to reduce carbon-dioxide emissions than would a collection of policies such as "corporate average fuel economy" requirements for automobiles.

Responses



Responses weighted by each expert's confidence



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Recent Polls



Stablecoins May 25th, 2022



Stablecoins May 25th, 2022



Energy Sanctions May 11th, 2022



Energy Sanctions May 11th, 2022



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Ranked-Choice Voting April 22nd, 2022

| Introduction and Motivation | | |
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Introduction and Motivation

- Experts often fail to convince decision makers to select the right action appropriate for the situation.
 - medical experts on COVID-related health measures
 - economists on carbon tax
- Experts' interests are not perfectly aligned with the decision maker
 - experts consider what is optimal from their perspective, but decision maker has to consider other tradeoffs
 - experts may have biases: conflict of interest, strongly held opinion, etc.

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Introduction and Motivation

Some casual empiricism:

- Judicial trials are very good at finding the facts; organizations' internal investigations in response to accusations usually less convincing.
- Using history to make an argument is easier than using forecasts to do so.

Observation:

- Information is not always fully verifiable.
 - Existence of accounting error is verifiable, but absence of one is not;
 - Possession of a qualification is verifiable, but lack of one is not.
- Verifiability matters to whether persuasion is effective.

When is effective persuasion possible and when is it not?

• Punchline: nature of verifiability matters.

Related literature

- Communication of hard information: Milgrom (1981), Milgrom and Roberts (1986), Grossman (1981);
- Multiple experts: Bhattacharya and Mukherjee (2013), Wolinsky (2002);
- Partial verifiability: Wolinsky (2002, 2003), Dziuda (2011);
- Information aggregation: Austen-Smith and Banks (1996), McLennan (1998).

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Model setup

We follow closely the setup of Wolinsky (2002).

- Decision maker chooses action from $a \in \{L, H\}$.
- Experts $i \in \{1, 2, \dots, n\}$ each observes a signal $s_i \in \{0, 1\}$.
- Signal $s_i = 1$ is more favorable news for H than $s_i = 0$.
- State of the world: $S := \sum_j s_j$.

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Preferences

There is a conflict of interest between experts and decision maker.

• *DM*'s payoff *V*:

$$\tilde{V}(s|a) = \begin{cases} V(S) & \text{if } a = H \\ 0 & \text{if } a = L \end{cases}$$

where V is increasing.

• All experts have the same preferences, U, given by:

$$\tilde{U}(s|a) = \begin{cases} U(S) & \text{if } a = H \\ 0 & \text{if } a = L \end{cases}$$

where U is increasing.

- V(S-1) > U(S) for any S ∈ {1,..., n}: panel has a conservative bias (think reopening)-less eager to take action H.
 U(n) ≥ 0 ≥ V(0): accomment in automa states
- U(n) > 0 > V(0): agreement in extreme states.

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Strategies

- Experts simultaneously report $r_i \in \{0, 1\}$;
- Decision maker chooses action H with probability $\rho(r_1, r_2, ..., r_n) \in [0, 1];$
- Expert's strategy $y_i(0)$ and $y_i(1)$: Probability of telling the truth when signal is 0 or 1, respectively.

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Verifiability

- **Definition.** A signal s is verifiable if the expert can prove with hard evidence that he has received signal s; signal s' is unverifiable if the expert cannot prove he has received s' (or equivalently, has not received $s \neq s'$).
- Possible verifiability scenarios:
 - Neither 0 nor 1 is verifiable \Rightarrow Cheap-Talk;
 - Signal 1 is verifiable but 0 is not, verifiability opposite to experts' bias (organization scandals/accounting error);
 - Signal 0 is verifiable but 1 is not, verifiability towards experts' bias (employee competence);
 - Both signals are verifiable \Rightarrow Mandatory disclosure (judicial trials).

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Benchmark: Cheap talk

• Key observation: when an expert considers their strategy, they consider only the instances in which they are pivotal – changing a decision from *L* to *H* or vice versa.

Lemma 1. In the cheap-talk scenario, whenever pivotal, experts always tell the truth when their signal is 0: $y_i(0) = 1$, ignoring perverse equilibria.

• Intuition: in the pivotal instances, the DM is "indifferent" between H and L, but given that an expert is more conservative, they always prefer to implement L. Hence, they will not report 0 as 1.

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Some unpleasant pivotal math

If *i*'s report is pivotal given other experts' report r_{-i} , it must be that

$$\sum_{k=0}^{n} Pr_{DM}(S = k | r_{-i}, \mathbf{r}_{\mathbf{i}} = \mathbf{0}, y) V(k) \le 0$$
$$\le \sum_{k=0}^{n} Pr_{DM}(S = k | r_{-i}, \mathbf{r}_{\mathbf{i}} = \mathbf{1}, y) V(k).$$

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Some unpleasant pivotal math

However, for expert *i* with signal $s_i = 0$, payoff from reporting 1 is

$$\begin{split} \mathbb{E}_{S}[U(S)|r_{i} &= 1, r_{-i}, y] \\ &= \sum_{k=0}^{n-1} Pr_{i}(S_{-i} = k|r_{-i}, y_{-i})U(k), \\ &= \sum_{k=0}^{n-1} Pr_{DM}(S = k|r_{i} = 0, \tilde{y}_{i} = 1, r_{-i}, y_{-i})U(k), \\ &< \sum_{k=0}^{n-1} Pr_{DM}(S = k|r_{i} = 0, \tilde{y}_{i} = 1, r_{-i}, y_{-i})V(k), \\ &\leq \sum_{k=0}^{n} Pr_{DM}(S = k|r_{i} = 0, y_{i}, r_{-i}, y_{-i})V(k) \leq 0. \end{split}$$

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Some unpleasant pivotal math

- First equality: DM infers that s_i = 0 if Expert i always tells the truth, so DM's posterior is the same as that of i;
- Strict inequality: Assumption that experts are more conservative than the DM;
- First weak inequality: Roughly speaking, if Expert *i* always reports the truth about 0, DM's inference about state is lower when receiving report r_i = 0 versus when Expert *i* sometimes potentially reports 0 as 1;
- Last weak inequality: Expert i is pivotal.

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Unpleasant pivotal math: Some details

Let

$$\omega_{DM}(k) := \Pr_{DM}(S_{-i} = k | r_{-i}, y_{-i}),$$

Under $\tilde{y}_i = 1$,

$$Pr_{DM}(S=k|r_i=0,r_{-i},y_{-i},\tilde{y}_i)=\omega(k).$$

Under y_i ,

$$Pr_{DM}(S = k | r_i = 0, r_{-i}, y) = \begin{cases} \lambda_i \omega(0), & x = 0; \\ \lambda_i \omega(k) + (1 - \lambda_i) \omega(k - 1), & 1 \le x < n; \\ (1 - \lambda_i) \omega(n - 1) & x = n, \end{cases}$$

where

$$\lambda_i := \Pr_{DM}(s_i = 0 | r_i = 0, y_i) = \frac{(1 - q)y_i}{(1 - q)y_i + q(1 - y_i)}.$$

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Unpleasant pivotal math: Some details

Therefore,

$$\sum_{k=0}^{n-1} \Pr_{DM}(S = k | r_i = 0, r_{-i}, y_{-i}, \tilde{y}_i) V(k) - \sum_{k=0}^{n} \Pr_{DM}(S = k | r_i = 0, r_{-i}, y)$$

$$= \sum_{k \in \{0, \dots, n-1\}} \omega(k) V(k) - \lambda_i \omega(0) V(0)$$

$$- \sum_{k \in \{1, \dots, n-1\}} [\lambda_i \omega(k) + (1 - \lambda_i) \omega(k - 1)] V(k) - (1 - \lambda_i) \omega(n - 1) V(n)$$

$$= (1 - \lambda_i) \sum_{k \in \{0, \dots, n-1\}} \omega(k) V(k) - (1 - \lambda_i) \sum_{k \in \{1, \dots, n\}} \omega(k - 1) V(k)$$

$$= - (1 - \lambda_i) \sum_{k \in \{0, \dots, n-1\}} \omega(k) [V(k + 1) - V(k)] \le 0.$$

Last inequality: V is weakly increasing in k, for all $k \in \{0, ..., n-1\}$.

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Two corollaries

Corollary 1. If neither signals are verifiable, there is no informative equilibrium. DM takes the ex ante optimal action.

Corollary 2. (Wolinsky 2002) If 1 is verifiable but 0 is not, there is no informative equilibrium. DM takes the ex ante optimal action.

Intuition for both corollaries: given that 0 is always reported as 0, it will sway the decision towards L if equilibrium is informative. Thus, whenever expert is pivotal, they would lie about signal 1.

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Conditional on Expert *i* being pivotal, when $s_i = 1$,

$$\begin{split} \mathbb{E}_{S}[U(S)|r_{i} &= 1, r_{-i}, y] \\ &= \sum_{k=0}^{n-1} \Pr(S_{-i} = k | r_{-i}, y_{-i}) U(k+1) \\ &= \sum_{k=0}^{n-1} \Pr_{DM}(S = k+1 | r_{i} = 1, r_{-i}, y) U(k+1) \\ &< \sum_{k=0}^{n-1} \Pr_{DM}(S = k+1 | r_{i} = 1, r_{-i}, y) V(k) \\ &\leq \sum_{k=0}^{n} \Pr_{DM}(S = k | r_{i} = 0, r_{-i}, y) V(k) \leq 0 \end{split}$$

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Result

Proposition. If signal 0 is verifiable but 1 is not, there exists a fully-revealing equilibrium.

Intuition: if an expert lies, they will switch the decision from L to H – bad pivotal math.

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Asymmetric equilibria

An example. Suppose that n = 3, q = 0.35, $n_{DM} = 1$, and $n_E = 3$. The payoff from action *H* is

$$V(S) = S - 0.995$$

for DM and

$$U(S) = \begin{cases} -2 & \text{if } S \in \{0, 1, 2\} \\ 0.5 & \text{if } S = 3. \end{cases}$$

Expert suffers a big loss from action H taken when they do not think it is appropriate.

Asymmetric equilibrium: $(y_1, y_2, y_3) = (1, 0.7, 0)$ and the decision rule:

$$\rho(r) = \begin{cases} 0 & \text{if } r \in \{(0,0,0), (0,0,1), (0,1,0), (0,1,1)\} \\ 1 & \text{if } r \in \{(1,0,0), (1,1,0), (1,0,1), (1,1,1)\}. \end{cases}$$

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Welfare

- DM: more information is always better. So full information equilibria are Pareto efficient.
- Experts: welfare comparison depends on comparison between "type-1" error and "type-2" error.

Example above. Experts 2 and 3 are not pivotal. When S = 0, H is not taken in the above equilibrium or the truthful equilibrium. When S = 3, H is always taken. However when S = 1, 2, H is taken less frequently, compared with the truthful equilibrium, which is a gain for the experts.

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Asymmetric equilibria

Lemma

Suppose that $s_i = 0$ is verifiable. If (ρ^*, y^*) is an equilibrium and $y_i^* \in (0, 1)$ for some $i \in E$, then the strategy profile (ρ', y') where $y_i' = 0$ and $y_j' = y_j^*$ for all $j \neq i$, and

$$\rho'(r) = \rho^*(r) \qquad \forall r \in \{0, 1\}^n \tag{1}$$

is also an equilibrium. Furthermore, the two equilibria give the experts the same expected payoff.

Intuition.

Proposition

We may focus on equilibria where some experts always babble (bad cop) and some always tell the truth (good cop).

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Mixed panel of experts

Proposition

Let C denote the number of conservatives experts, and L denote the number of liberal experts. (i) If no signal is verifiable, then no equilibrium is influential. (ii) Suppose that $s_i = 1$ is verifiable. If

$$\sum_{k=0}^{C} \binom{C}{k} q^{k} (1-q)^{C-k} V(k) \leq 0 \leq \sum_{k=0}^{C} \binom{C}{k} q^{k} (1-q)^{C-k} V(L+k),$$

then an influential equilibrium exists, where all liberal experts reveal their signals and all conservatives babble. (iii) Suppose that $s_i = 0$ is verifiable. A similar statement to (ii) follows under an analogous condition.

• Intuition. Same as above.

Experts-DM vs. Condorcet

- Differences between Experts-DM and Condorcet:
 - Decision rules are endogenous in ED, whereas it is exogenous in Condorcet.
 - Partial verifiability \Rightarrow "vote" is restricted in ED, whereas it is not restricted in Condorcet.
- Consequences:
 - DM does not just count "votes"-in particular, not in asymmetric equilibria.
 - McLennan's (1998, APSR) insightful result does not apply.

Model

Experts-DM vs. Condorcet

- McLennan (1998) shows that even though "sincere voting" is not necessarily an equilibrium, voters with common interests can achieve their ex ante optimal outcome through voting. So voters do not need commitment power.
- We have counterexample in which commitment benefits experts. ("Bayesian persuasion-ish"). Intuition: experts want to commit to misreporting "0" in a way that is not ex post incentive compatible.

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Experts-DM vs. Condorcet: Example

In the Example above, y = (1, 0.7, 0) is worse for the experts than $\hat{y} = (0.7, 0.7, 0)$, but the latter is not an equilibrium.

Table: Expected gain from $\hat{y} = \sum_{s \in \{0,1\}^n} \Pr(s) \times \Delta \pi(y|s) = 0.10595.$

| Signal profile | Pr(H s,.) | | $- \pi(\hat{y} s) - \pi(y s)$ |
|----------------|------------------|---|---|
| Signal prome | ŷ | у | $\mathcal{H}(\mathbf{y} \mathbf{S}) \to \mathcal{H}(\mathbf{y} \mathbf{S})$ |
| s = (0, 0, 0) | 0.3 ² | 0 | -0.18 |
| s = (0, 0, 1) | 0.3 ² | 0 | -0.18 |
| s = (0, 1, 0) | 0.3 | 0 | -0.6 |
| s = (1, 0, 0) | 0.3 | 1 | +1.4 |
| s = (0, 1, 1) | 0.3 | 0 | -0.6 |
| s = (1, 0, 1) | 0.3 | 1 | +1.4 |
| s = (1, 1, 0) | 1 | 1 | 0 |
| s = (1, 1, 1) | 1 | 1 | |

Conclusion

- Verifiability matters to effectiveness of persuasion.
- Verifiability of evidence in experts' favour: good for everyone.
- Verifiability of evidence against experts: difficulty of persuasion.