IAMs and CO_2 Emissions – An Analytic Discussion

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Introduction

Most widespread IAMs ("workhorse models")

- Nordhaus et al.'s (1994,2000,2013,2017,2023) DICE model
- Golosov et al. (2014)

Attractiveness: Low complexity, easy to use

Characteristics:

- DICE: numeric all way
- Golosov et al. (2014): analytic SCC, numeric emission simulations

This paper: Analytic discussion of emissions in

- DICE
- Golosov et al. (2014)
- Traeger's (2023) Analytic Climate Economy - ACE

short-cut

"Literature" and Motivation

Motivation for present study:

- How bad are emissions and climate change going to be under BAU and policy?
- Understand emission drivers in the most widespread IAMs DICE and Golosov et al. (2014)
- Merge strengths and move beyond in ACE
- while keeping some analytic tractability and enabling transparency & insight

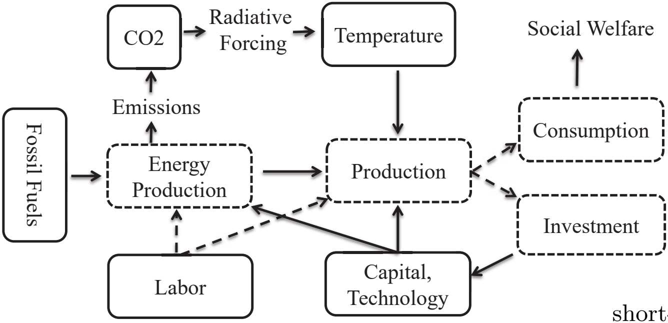
Note: should not and cannot be replaced more serious IAMs ("real workhorses")

- WITCH, REMIND, FUND, Detailed Energy Models,...

but gets a little closer while focusing on analytic insight.

Integrated Assessment Models (IAMs)

- Joint representation of climate system & economy
- Integrates cause and effect of climate change
- Matches stylized market and climatic observations



short-cut

The Model

General model

- utility $u(c_t)$.
- Production vectors energy, capital, labor, exog. A_t:
  $Y_t = F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t)$
- Capital: $K_{t+1} = (1-\delta)K_t + Y_t - C_t$.
- Emissions: Fossil-fuel energy sources emit: $\sum_{i=1}^{I^d} E_{i,t}$
  (LUCF, Non-CO_2 exog.)
- Carbon cycle or Impulse Responds (Joos et al. (2013))
- Standard radiative forcing equation
- Arbitrary temperature model
- Resources: $\boldsymbol{R}_{t+1} = \boldsymbol{R}_t - \boldsymbol{E}_t^d$
- Damages: $D(T_{1,t})$

The Model

General model (in black). In green: For analytic SCC.

- utility $u(c_t)$. (log-utility)
- Production vectors energy, capital, labor, exog. A_t:
  $Y_t = F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t)$
  with $F(\boldsymbol{A}_t, \boldsymbol{N}_t, \gamma \boldsymbol{K}_t, \boldsymbol{E}_t) = \gamma^{\kappa} F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t) \; \forall \gamma \in \mathbb{R}_+$.
- Capital: $K_{t+1} = (1-\delta)K_t + Y_t - C_t$.
  $K_{t+1} = (Y_t - C_t)\left[\frac{1+g_{k,t}}{\delta+g_{k,t}}\right]$ with $g_{k,t}$ exogenous growth approx
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  (LUCF, Non-CO_2 exog.)
- Carbon cycle or Impulse Responds (Joos et al. (2013))
- Standard radiative forcing equation
- Arbitrary (ACE's non-linear-) temperature model
- Resources: $\boldsymbol{R}_{t+1} = \boldsymbol{R}_t - \boldsymbol{E}_t^d$
- Damages: $D(T_{1,t})$ ($D(T_{1,t}) = 1 - \exp(-\xi_0 \exp[\xi_1 T_{1,t}] + \xi_0)$)

Definitions

Let the sequence of value functions $V_t(K_t, \boldsymbol{T}_t, \boldsymbol{M}_t, \boldsymbol{R}_t)$, $t \in \mathbb{N}$ solve the DP problem.

(capital, temperature layers, carbon reservoirs, resources, **bold**=vectors)

Optimal carbon tax (Damage from emitting a ton):

$$SCC_t = \frac{\beta \frac{\partial V_{t+1}(K_{t+1}, \boldsymbol{T}_{t+1}, \boldsymbol{M}_{t+1}, \boldsymbol{R}_{t+1})}{\partial M_{1,t+1}}}{u'(C_t)}$$

Hotelling rent (intertemporal fossil fuel scarcity):

$$HOT_{i,t} = \frac{\beta \frac{\partial V_{t+1}(\cdot)}{\partial R_{i,t+1}}}{u'(c_t)}$$

Total social cost of a (CO_2-content-measured) unit of foss fuel i

$$\Gamma_{i,t} = HOT_{i,t} + SCC_t$$

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$$\Gamma_{i,t} = HOT_{i,t} + SCC_t \quad \text{and} \quad \widetilde{\Gamma}_{i,t} = \frac{\Gamma_{i,t}}{Y_t^{net}}.$$

Convenient normalization: per unit of net output

Emission levels - A general statement

Proposition

Optimal emissions from a dirty resource satisfy

$$E_{i,t}^{*} = \frac{\sigma_{Y,E_i}(\boldsymbol{A}_t, \boldsymbol{N}_t^{*}, \boldsymbol{K}_t^{*}, \boldsymbol{E}_t^{*}) Y_t^{net}}{HOT_{i,t} + SCC_t} = \frac{\sigma_{Y,E_i}(\cdot)}{\tilde{\Gamma}_{i,t}} \quad (1)$$

where $\sigma_{Y,E_i}(\cdot) = \frac{\partial F(\cdot)}{\partial E_i} \frac{E_i}{Y}$ *is the production elasticity of the resource and stars denote the optimal allocation.*

Comments:

- The “proposition” is a simple FOC statement.
  Insights derive from application to different settings and evaluating the elasticty
- In general, equation 1 is an implicit equation

Illustration: A simple Cobb-Douglas economy

DICE almost and Golosov et al. (2014) satisfy:

$$Y_t = F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t) = A_t K_t^{\kappa} N_t^{\eta}\, G\left(\boldsymbol{A}_t^E, \boldsymbol{N}_t^E, \boldsymbol{E}_t\right). \qquad (2)$$

First: The simple Cobb-Douglas climate economy

$$G(E_t) = E_t^{\nu} \quad \text{with } \kappa + \eta + \nu = 1.$$

Here E_t denotes the aggregate fossil-based energy input (measuring it in terms of CO_2 content, so = emissions)

$$E_t^* = \frac{\nu Y_t^{net}}{HOT_{R,t} + SCC_t} = \frac{\nu}{\tilde{\Gamma}_t}.$$

Emissions increase in energy share = production elasticity & decrease in SCC and Hotelling rent (per unit of output).

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Policy Response - related general statement

Response of fossil use/emissions to a *change in policy*:

- Given a marginal (exogenous) change of $\widetilde{SCC}$, denote
- resulting rate of change of an *endogenous* variable x by $\widehat{x}$ $\left(= \frac{dx}{d\widetilde{SCC}}\frac{1}{x}\right)$.

Proposition

A relative change in the social cost of carbon per unit of output $\widehat{\widetilde{SCC}}$ results in the relative emission change

$$\widehat{E}_{i,t} = -\widehat{\widetilde{SCC}} + \widehat{\sigma}_{Y,E_i}(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t) + \gamma_{i,t}(\widehat{\widetilde{SCC}} - \widehat{HOT}_{i,t})$$

where $\gamma_{i,t} = \frac{HOT_{i,t}}{\Gamma_{i,t}}$ *denotes the Hotelling share of the total social cost.*

Policy Response: General

Interpretation of

$$\widehat{E}_{i,t} = -\widehat{SCC} + \widehat{\sigma}_{Y,E_i}(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t) + \gamma_{i,t}(\widehat{SCC} - \widehat{HOT}_{i,t})$$

where $\gamma_{i,t} = \frac{HOT_{i,t}}{\Gamma_{i,t}}$.

- $\widehat{SCC}$: Primary policy push
- $\widehat{\sigma}_{Y,E_i}(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t)$: restructuring of economy in response to SCC change
- $\gamma_{i,t}(\widehat{SCC} - \widehat{HOT}_{i,t})$: Hotelling crowd-out

Policy Response: Simple Cobb Douglas

Assumption for (most of) this talk: Absence of Hotelling rent.

Back to simple Cobb-Douglas with aggregate fossil fuel:

$$E_t^* = \frac{\nu Y_t^{net}}{HOT_t + SCC_t} = \frac{\nu}{\tilde{\Gamma}_t}.$$

Policy response: As elasticity ν constant ($\hat{\nu} = 0$) we have:

$$\widehat{E}_t = -\widehat{SCC}. \quad (3)$$

10% SCC increase (e.g. damage) ⇒ Emissions fall by 10%.

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DICE

If taking BAU emissions as given, DICE satisfies:

$$Y_t = F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t) = A_t K_t^{\kappa} N_t^{\eta} \, G\left(\boldsymbol{A}_t^E, \boldsymbol{N}_t^E, \boldsymbol{E}_t\right). \tag{4}$$

Fairly complicated $G(\cdot)$ with lots of parameters and equations

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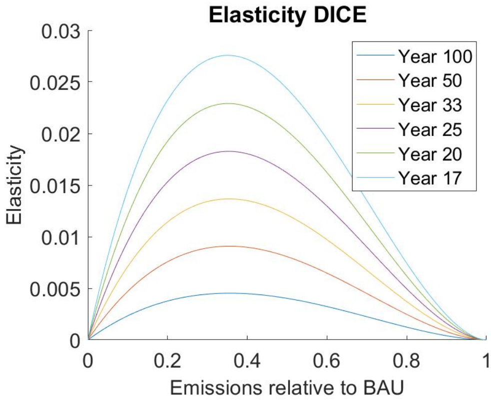
DICE's fossil fuel (emission) elasticity of production $\sigma_{Y,E}$.

Observations: Falls

- for high emissions (finite BAU exists)
- for low emissions (decarb possible)
- over time



Policy Response: DICE

DICE's emissions response:

$$\widehat{E}_{i,t} = -\widehat{SCC} + \widehat{\sigma}_{Y,E_i}(\cdot).$$

Initially:

- Low abatement: $\hat{\sigma}_{Y,E_i}(\cdot) > 0$ counteracting policy

"Later":

- High abatement: $\hat{\sigma}_{Y,E_i}(\cdot) < 0$ reinforcing policy

"Restructuring" of economy in response to SCC

Note:

- Elasticity, so rate change
- final percent of abatement cheap because tiny quantity

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DICE: Abatement Rate

DICE optimizes abatement rate

$$\mu_t \equiv 1 - \frac{E_t}{E_t^{BAU}}$$

rather than emissions directly. Optimal abatement rate is:

$$\mu_t = \left(\frac{\Gamma_t}{p_t^{back}[1 - D_t(T_{1,t})]} \right)^{\frac{1}{\theta_2 - 1}} \approx \sqrt{\frac{\Gamma_t}{p_t^{back}[1 - D_t(T_{1,t})]}}. \quad (5)$$

Observations:

- increases with square root of social cost (=carbon tax in absence of Hotelling)

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Golosov et al. (2014)

Golosov et. al (2014)'s production uses CES energy composite:

$$Y_t = A_t K_t^\kappa N_t^\eta \, G\left(\boldsymbol{A}_t^E, \boldsymbol{N}_t^E, \boldsymbol{E}_t\right) = A_t K_t^\kappa N_t^\eta \, E(\cdot)^\nu. \tag{6}$$

with energy composite

$$E_t(\cdot) = \left(a_{oil} E_{oil,t}^s + a_{coal} \underbrace{\left(A_{coal,t} N_{coal,t}\right)}_{=E_{coal}}^s + a_{ren} \underbrace{\left(A_{ren,t} N_{ren,t}\right)}_{=E_{ren}}^s \right)^{\frac{1}{s}}$$

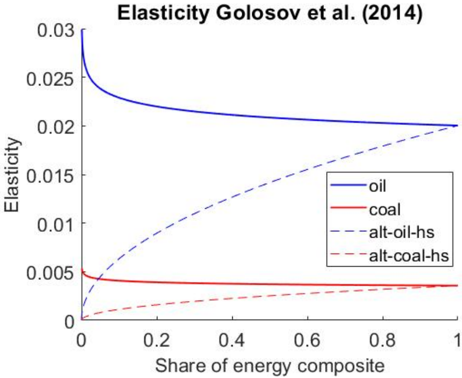
- distinguish primary energy: oil, coal, renewable
- Leontjev production of coal & renewable using labor
- coal: Only extraction costs, no scarcity rent (Hotelling)
- oil : No extraction costs, only Hotelling rent
- no capital in energy sectors.

Golosov et al. (2014): Production elasticity

The production elasticity perspective:
Structural comparison to other model structures like DICE

Production elasticities:

$$\sigma_{Y,E_i} = \nu a_i \left(\frac{E_{i,t}}{E_t}\right)^s$$



Solid: Golosov et al.'s calibration of interfuel substitutability of 0.95.
Dashed: Elasticity of 2 (hypothetical scenarios mentioned by authors)
Horizontal axis: share oil or coal relative to energy composite, i.e., $\frac{E_i,t}{E_t}$.

Golosov et al. (2014): Coal

Findings coal use:

- BAU 5-fold (40-fold) increase by 2100 (2200)
- Still increases slightly in optimal scenario

Explanation: $s = -0.05$. No Hotelling rent. Coal emissions:

$$E_{coal} = \left(\frac{\nu \ a_{coal}}{\frac{1-a-\nu}{N_{0,t} A_{coal,t}} + \tilde{\Gamma}_{coal,t}} \right)^{\frac{1}{1-s}} E_t^{-\frac{s}{1-s}}$$

$$\stackrel{BAU}{\approx} \left(N_{0,t} A_{coal,t} \frac{\nu \ a_{coal}}{1-a-\nu} \right) E_t^{0.05} \sim A_{coal,t}.$$

$A_{coal,t}$ grows at 2% annually

- BAU: $\rightarrow$ coal use grows 2% (explains above 5 and 40-fold).
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Golosov et al. (2014): Oil

Findings oil use:

- falls strongly over time in both BAU and optimal
- optimal and BAU almost coincide

Explanation: Focusing on difference to coal use:

$$\frac{E_{oil,t}}{E_{coal,t}} = \left(\frac{a_{oil}}{a_{coal}} \; \frac{\frac{\omega_t}{A_{coal,t}} + SCC_t}{HOT_{oil,t} + SCC_t} \right)^{\frac{1}{1-s}} .$$

- instead of resource increasing *technological progress*
- now have resource decreasing *Hotelling rent*
- Why policy not responsive? Hotelling & *next slide*

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ACE: Analytic Climate Economy (Traeger 2023)

Base Model (analytic solution for SCC):

- Much improved climate system and impulse response
- General production system comprising earlier models

Here: Combine "best of DICE and Golosov et al. (2014)"
& add some structure to energy use

- Primary resource $e_{i,t}$
  - coal, oil, gas, bio, renewable
  - production includes capital
  - saturation possible
- Electricity sector
- Final goods: Transport, industry, other
- each final sectors uses specific energy composite
- with sector-specific interfuel substitutability

ACE: Calibration

IEA energy data, BP prices, PWT, GCP, and other.
Interfuel elasticity of substitution “from literature”:

- Transport: Lowest, 0.5 (how soon above unity?)
- Industry: ≈1 (above/below?)
- Other: ≈ 1.2
- Electricity between primary energy inputs: ≈ 2

Fitting CES-consumption, Cobb-Douglas final sectors, which use CES energy composite

- fits data well, but a lot of degrees of freedom
- decentralized calibration based on quantity & prices
- least-square quantities only fit gives similar result

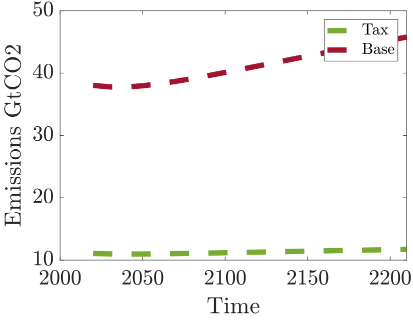
For presentation: Skip analytics and preview time paths

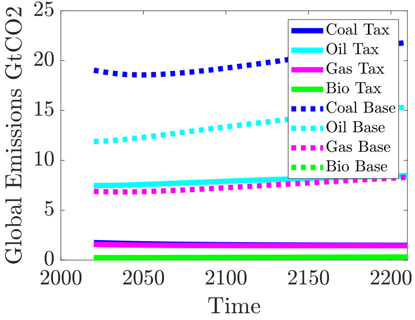
ACE: Preliminary Previews

Scenario:

- high damages (Howard Sterner (2017), Pindyck (2020))
- no demand increase
- renewable efficiency increase

Emission (with & without optimal tax), overall and by source





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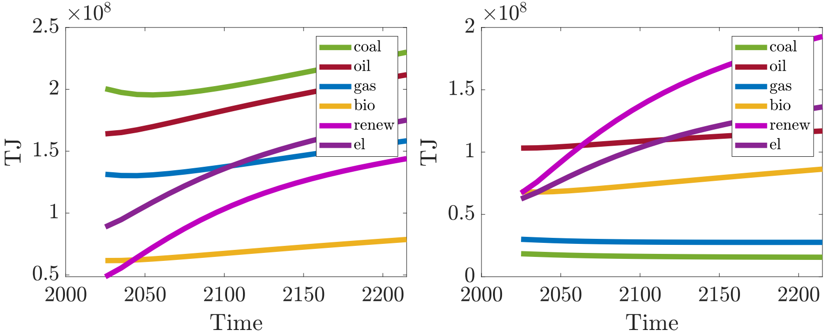
Energy use: without & with tax



Conclusions

- Analytic structural comparison of emissions in widespread "simple IAMs" (focus: DICE, Golosov et al. (2014), ACE)
- Emission response as "SCC + econ restructuring + Hotelling crowd-out" perspective
- Explain Golosov et al. (2014)'s surprising simulation results in simple analytic formulas
- Simple abatement rate formula for DICE
  drivers are backstop price & cost convexity
- Use ACE to combine & extend features of DICE and Golosov et al. (2014) with different sectors and sector-specific substitutabilities
- Appendix: Non-constant elasticities of interfuel substitution