Framing and Ambiguity

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Motivation

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- **Theoretical perspective:** bounded rational agent is not able to integrate all payoff-relevant details coherently
- **What if framing is unobservable?**

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- Difficulty of the exam is unknown
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- *What data is consistent with framing of ambiguity?*
- *Can the frames be identified from the behavior?*
- *How to connect the frames to become more consistent in choices?*

Model

Setup

Decision maker (DM) chooses from menus of Anscombe-Aumann acts

- X—arbitrary set of prizes
- $\triangle X$— probability distributions with finite support on X
- S—finite set of states

- f,g,h,...—Anscombe-Aumann acts $S \to \triangle X$
- p,q,r,...—constant acts $p(s) = p \ \forall s \in S$

- A,B,...—non-empty compact sets of acts (menus) with finite set of potential prizes $\{x \in X | \exists f \in A \ \exists s \in S : f(s)(x) > 0\}$

Choice Correspondence

Primitive: a subset of acts that could be chosen from a menu

- Choice correspondence $\varnothing \neq c(A) \subseteq A$
- Ways to observe choice correspondence:
  - Repeated observations of choices from each menu
  - Choices of a group of agents (population interpretation)

WARP Is Relaxed

(A1) Framed Uncertainty:

- $\boldsymbol{\alpha}$: $c(A \cup B) \cap A \subseteq c(A)$
- **C**-$\boldsymbol{\beta}$: for constant acts $c(A \cup B) \cap A \neq \varnothing \implies c(A) \subseteq c(A \cup B)$
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Recall that *WARP* is equivalent to [α and β]

- *WARP* holds for (menus of) constant acts
- Aizerman's Property relaxes β

"Standard" Axioms

(A2) C-Independence: $c(\lambda A + (1-\lambda)p) = \lambda c(A) + (1-\lambda)p$

(A3) Strict Monotonicity: $g(s) \notin c(\{f(s), g(s)\})\ \forall s \implies g \notin c(\{f, g\})$

(A4) Continuity: $\{(A, f) \mid f \in c(A)\}$ is closed

(A5) C-Non-Degeneracy: $\exists p, q: \quad \{p\} = c(\{p, q\})$

A New Axiom: No Hedging by Constant Acts

(A6) No C-Hedging: $f, p \in A \implies c(A) \subseteq c(A \cup \{\lambda f + (1-\lambda)p\})$

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If *WARP* holds, follows from *C-Independence* and *Strict Monotonicity*

Ambiguity Aversion Robust to the Framing

(A7) Indirect Ambiguity Aversion: for $h \in A$

$$h \notin c(A \cup \{f\}) \text{ and } h \notin c(A \cup \{g\}) \implies h \notin c(A \cup \{\lambda f + (1-\lambda)g\})$$

Representation

Framed ambiguity representation $(U, \mathcal{A})$

- $U: \triangle X \to \mathbb{R}$ —vNM expected utility
- $\mathcal{A}$ —non-empty closed family of non-empty compact convex sets of beliefs

$$c(B) = \bigcup_{P \in \mathcal{A}} \arg\max_{f \in B} W_P(f)$$

where

$$W_P(f) = \min_{\mu \in P} \sum_{s \in S} \mu(s) U(f(s))$$

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Theorem 1. *A choice correspondence $c(\cdot)$ has a framed ambiguity representation if and only if axioms 1–7 hold.*

Proposition. *Axioms 1–7 are independent.*

Identification

Definition: $P = \bigcap_{Q \in \mathcal{C}} Q \neq \varnothing$ is a *coherent intersection* of sets of beliefs in a closed (in Hausdorff metric) family $\mathcal{C}$ if for any linear subspace $\mathbb{T}$ of $\mathbb{R}^S$,

$$proj_{\mathbb{T}} \left(\bigcap_{Q \in \mathcal{C}} P' \right) = \bigcap_{Q \in \mathcal{C}} proj_{\mathbb{T}}(Q).$$

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Theorem 2. *Let $c(\cdot)$ satisfy axioms 1–7. Then there is a unique minimum family of frames $\mathcal{A}$, a unique maximum family of frames $\mathcal{B}$ and VNM expected utility function U such that:*

(i) $(U', \mathcal{A}')$ represents $c(\cdot)$ if and only if $U' \approx U$, and $\mathcal{A} \subseteq \mathcal{A}' \subseteq \mathcal{B}$;

(ii) $P \in \mathcal{B}$ if and only if P is a coherent intersection of some $\mathcal{C} \subseteq \mathcal{A}$.

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Corollary. *If all frames $P \in \mathcal{A}$ are singletons, then $\mathcal{A}$ is unique.*

Comparative Statics

Consistency of Choices

Definition: DM 1 is *more consistent* than DM 2 if for all menus A
$|c_2(A)| = 1 \implies |c_1(A)| = 1$.

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Alternative characterization:

Proposition. *Let* $c_1(\cdot)$ *and* $c_2(\cdot)$ *satisfy axioms 1–7. Then DM 1 is more consistent than DM 2 if and only if* $c_1(\{f, g\}) \subseteq c_2(\{f, g\})$ *for all acts* f, g.

A Convex Combination of Sets of Beliefs

Crès, Gilboa, and Vieille (2011): let λ be convex weights: $\lambda_i \geq 0$, $\sum_{i=1}^{N} \lambda_i = 1$;

$$P \;=\; \sum_{i=1}^{N} \lambda_i P_i \;\equiv\; \left\{ \mu \in \triangle S \;\middle|\; \exists \mu_i \in P_i : \; \mu = \sum_{i=1}^{N} \lambda_i \mu_i \right\}$$

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The associated maxmin utility function: $W_P(f) \ = \ \displaystyle\sum_{i=1}^{N} \lambda_i W_{P_i}(f)$

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Observation. DM $(U, \sum_{i=1}^{N} \lambda P_i)$ is more consistent than DM $(U, \{P_1, ..., P_N\})$

A Convex Union of Sets of Beliefs

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$$P \; = \; \mathit{conv}\left(\bigcup_{Q\in\mathcal{C}} Q\right), \text{ where } \mathcal{C} \text{ is non-empty and closed}$$

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Observation. DM $\left(U, conv\left(\bigcup_{Q\in\mathcal{C}} Q\right)\right)$ is more consistent than DM $(U, \mathcal{A})$ if $\mathcal{C} \subseteq \mathcal{A}$

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This project:

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Observation. DM $\left(U, \bigcap_{Q \in \mathcal{C}} Q\right)$ is more consistent than DM $(U, \mathcal{A})$ if $\mathcal{C} \subseteq \mathcal{A}$, and the intersection $\bigcap_{Q \in \mathcal{C}} Q$ is coherent

Definition: Given a compact collection of frames $\mathcal{A}$, its closure with respect to operations of convex combination, convex union and coherent intersection is the minumum compact collection of frames $\Gamma(\mathcal{A})$ such that:

(i) $\forall \ \{P_1, ..., P_N\} \subseteq \Gamma(\mathcal{A}) \ \forall \lambda \quad \sum_{i=1}^{N} \lambda_i P_i \in \Gamma(\mathcal{A})$;

(ii) $\forall \mathcal{C} \subseteq \Gamma(\mathcal{A})$ if $\mathcal{C}$ non-empty, closed, then *conv* $\left(\bigcup_{P \in \mathcal{C}}\right) \in \Gamma(\mathcal{A})$;

(iii) $\forall \mathcal{C} \subseteq \Gamma(\mathcal{A})$ if $\bigcap_{P \in \mathcal{C}} P$ is coherent, then $\bigcap_{P \in \mathcal{C}} P \in \Gamma(\mathcal{A})$.

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Remark: $\Gamma(\mathcal{A})$ is well-defined.

Characterization of Comparative Consistency

Theorem 3. *Consider decision makers 1 and 2 represented by models* $(U_1, \mathcal{A}_1)$ *and* $(U_2, \mathcal{A}_2)$*, where* $\mathcal{A}_2$ *is finite. Then the following statements are equivalent:*

(i) *DM 1 is more consistent than DM 2;*

(ii) $U_1 \approx U_2$*, and* $\mathcal{A}_1 \subseteq \Gamma(\mathcal{A}_2)$*;*

(iii) $U_1 \approx U_2$*, and any* $P \in \mathcal{A}_1$ *is a coherent intersection of convex unions of convex combinations of frames in* $\mathcal{A}_2$*.*

An Application to Aggregation of Preferences

- A group of (maxmin) ambiguity averse agents agree on utilities but disagree on beliefs
- Want to aggregate their preferences into a (maxmin) representative

Definition. A preference relation $\succeq$ satisfies *Unanimity* with respect to $\{\succeq_i\}_{i=1}^N$ if $[f \succeq_i g \ \forall i = 1, ..., N]$ implies $f \succeq g$.

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Corollary. *Let $\{\succeq_i\}_{i=1}^N$ and $\succeq$ admit maxmin representations with the same utility index and different sets of beliefs $\{P_i\}_{i=1}^N$ and P. Then $\succeq$ satisfies Unanimity with respect to $\{\succeq_i\}_{i=1}^N$ if and only if P is a coherent closure of convex unions of convex combinations of $\{P_1, ..., P_N\}$.*

Identification of Frames from Preference Relation

Definition. Framed ambiguity model $(U, \mathcal{A})$ represents $\succeq$ if $(U, \mathcal{A})$ represents c such that $f \succeq g$ iff $f \in c(\{f, g\})$.

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Corollary. *Framed ambiguity models* $(U, \mathcal{A})$ *and* $(U', \mathcal{A}')$ *with* $|\mathcal{A}|, |\mathcal{A}'| < \infty$ *represent* $\succeq$ *if and only if* $U \approx U'$ *and* $\Gamma(\mathcal{A}) = \Gamma(\mathcal{A}')$.

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Remark. There is $\succeq$ such that its framed ambiguity representations do not admit a minimum family of frames.

Proposition. *Let $(U, \mathcal{A})$ represents $c_2(\cdot)$, and either Condition 1 or Condition 2 holds for $\mathcal{A}$. Then the following statements are equivalent:*

(i) $c_1(\cdot)$ *satisfies WARP and Continuity, and*

$$f \in c_1(\{f, p\}) \Longleftrightarrow \exists \; decomposition \begin{cases} \lambda f + (1-\lambda)q = \sum_{i=1}^{k} \sigma_i f_i \\ \\ \forall i \;\; f_i \in c_2(\{f_i, \lambda p + (1-\lambda)q\}) \end{cases}$$

(ii) $c_1(\cdot)$ *is represented by the maxmin model* $\left(U, \bigcap_{Q \in \mathcal{A}} Q\right)$.

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Condition 1: $\mathcal{A}$ is finite, $\bigcap_{Q \in \mathcal{A}} Q \neq \varnothing$, and each $P \in \mathcal{A}$ is polyhedral.

Condition 2: $\mathcal{A}$ is finite, and $\bigcap_{P \in \mathcal{A}} ri(P)$ is non-empty.

Literature and Conclusion

Related Literature

- Gilboa and Schmeidler (1989), Salant and Rubinstein (2008)
- Lu (2014), Kopylov (2021), Chandrasekher, Frick, Iijima, and Yaouang (2022), Stoye (2011)
- Lehrer and Teper (2011), Heller (2012)
- Bourgeois-Gironde and Giraud (2009), Ahn and Ergin (2010), Caplin and Martin (2020)
- Ok, Ortoleva, and Riella (2012), Galaabaatar and Karni (2013), Hara, Ok, and Riella (2019)
- Crès, Gilboa, and Vieille (2011), Hill (2011)

Conclusion

- ▷ A model of framing under Knightian Uncertainty is developed
- ▷ The analyst identifies the minimum set of frames from the choice
- ▷ The agent becomes less susceptible to framing by combining frames in cautious, optimistic way, or by linear combination

Supplementary Slides

WARP Is Relaxed

Recall that utility representation $\approx$ *WARP* $=$ conditions $\alpha + \beta$:

- $\boldsymbol{\alpha}$: $c(A \cup B) \cap A \subseteq c(A)$
- $\boldsymbol{\beta}$: $c(A \cup B) \cap A \neq \varnothing \implies c(A) \subseteq c(A \cup B)$

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Weaker then $\alpha + \beta =$ WARP. For example, CAN HAVE:

- $c(\{f, g\}) = \{f, g\}$;
- $c(\{f, h\}) = \{f, h\}$;
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- $c(\{f, g, h\}) = \{f, g\}$.

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Weaker then $\alpha + \beta =$ WARP. For example, CAN HAVE:

- $c(\{f, g\}) = \{f, g\}$;
- $c(\{f, h\}) = \{f, h\}$;
- $c(\{g, h\}) = \{g, h\}$;
- $c(\{f, g, h\}) = \{f, g\}$.

h is not chosen from $\{f, g, h\}$, although g and f are chosen, and h "is as good as g and f in pairwise comparisons."

WARP Is Relaxed

(A1) Framed Uncertainty:

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But, CANNOT HAVE:

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Remark. $cl(\{P(B)|B \text{ is maximal for interior } f\})$ is the *minimum* family of frames that must be part of any representation of c.

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$$c(B) \;=\; \bigcup_{P \in \mathcal{A}} \arg\max_{f \in B} W_P(f)$$

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Define $f \succeq g$ iff $\exists A : g \in A, f \in c(A)$. Under α, $f \succeq g$ iff $f \in c(\{f, g\})$

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Proposition 2

Let $(U, \mathcal{A})$ represents $c(\cdot)$. Then $|\mathcal{A}| = 1$ is equivalent to $c(\cdot)$ satisfying any of the following properties: β, WARP, Revealed Preference Rationality, γ, Normality, Ambiguity Aversion, Pairwise No-C-Hedging.

Comparative Decisiveness

Definition: DM 1 is *more decisive* than DM 2 if $c_1 \subseteq c_2$.

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Proposition

Let $c_1(\cdot)$ *and* $c_2(\cdot)$ *be represented by* $(U_1, \mathcal{A}_1)$ *and* $(U_2, \mathcal{A}_2)$*. Then DM 1 is more decisive than DM 2 if and only if* $U_1 \approx U_2$*, and* $\mathcal{A}_1$ *is a subset of the maximum family of frames representing* $c_2(\cdot)$*.*

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Unanimity $\iff$ convex combinations + convex unions + coherent intersections

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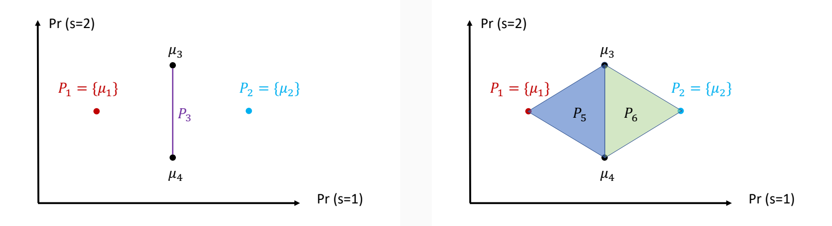
Unanimity $\Longleftarrow$ *EUA* $\iff$ convex combinations + convex unions

- Hill (2012) imposes "Weak Independence" axiom that connects aggregation rules for different preferences' profiles

Unanimity + *WI* $\iff$ convex combinations + convex unions

Non-Existence of a Minimum Family of Frames Representing $\succeq$



- $\mathcal{A} = \{P_1, P_2, P_3\}$ (left plot)
- $\mathcal{A}' = \{P_1, P_2, P_5, P_6\}$ (right plot)
- $P_5 = conv(P_1 \cup P_3)$, $P_6 = conv(P_2, P_3)$, hence $\mathcal{A}' \in \Gamma(\mathcal{A})$
- $P_3 = P_5 \cap P_6$, and the intersection is coherent, hence $\mathcal{A} \in \Gamma(\mathcal{A}')$

Proposition. *Let* $(U, \mathcal{A})$ *represents* $c_2(\cdot)$, *and* $(V, \{P\})$ *represents* $c_1(\cdot)$. *Then the following statements are equivalent:*

(i) *If* $f_i \in c_2(\{f_i, p\})$ *for all* $i = 1, .., k$, *then* $\sum_i^k \sigma_i f_i \in c_1\Big(\Big\{ \sum_i^k \sigma_i f_i, p \Big\}\Big)$ *for all convex weights* σ.

(ii) V *is a positive affine transformation of* U, *and* $P \subseteq \bigcap_{Q \in \mathcal{A}} Q \neq \varnothing$.