# Optimal Fiscal Policy under Preference Heterogeneity

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# The Problem

- How should marginal taxes and transfers change over the business cycle and with government spending?
- Reexamine some of the long-standing questions from the Ramsey literature in a dynamic Mirrlees framework with hererogenity in productivity, IES and risk aversion.

Lucas and Stokey 83, Karantounias 18, Werning 07

What is the role of heterogenity in the intertemporal elasticity of substitution (IES) and risk aversion (RA) in the optimal tax design?

# Motivation

Substantial body of evidence on hetereogeneity in IES:

across income levels: Blundell-Browning-Meghir (1994)

stockholders vs non-stockholders: Vissing-Jorgensen (2002)

Barsky, Juster, Kimball, and Shapiro (1997)

Thinking about dynamic fiscal policy requires a framework with realistic movements in asset prices (especially risk-free rate), because it directly affects the cost of debt.

# Preview of Results

- If low income individuals have either higher RA or lower IES, they face lower marginal taxes and higher transfers in bad times
- Subsequent dynamics differs. In the example solved,
  - If low income individuals have higher RA, the changes are persistent
  - If low income individuals have lower IES, the changes are anti-persistent
  - If both, subsequent effects are zero
- Preliminary quantitative results: If IES of low income individuals is 0.1 and IES of high income individuals is 0.3,
  - 1% increase in TFP leads to 2% increase in the marginal tax rate
  - 1% increase in govt spending leads to 0.2% decrease in the marginal tax rate

# Outline

The model

Three examples

differences in RA

differences in IES

differences in both IES and RA

Log-linearization around the steady state

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Time is discrete and infinite

Aggregate shocks  $s_t \in S$ . Probability of  $s^t = (s_0, s_1, \dots, s_t)$  is  $\pi_t(s^t)$ . Time zero shock  $s_0$  known.

Aggregate shocks determine aggregate productivity Z<sub>t</sub>(s<sup>t</sup>) and government spending G<sub>t</sub>(s<sup>t</sup>).

Two types of agents, A and B. Differ in their productivity, IES and risk aversion.

• type A agents have low productivity, normalized to  $Z_t(s^t)$ .

• type *B* agents have high productivity  $\theta Z_t(s^t)$ , where  $\theta > 1$ .

Fraction of A types is  $\lambda$ . Fraction of B types is  $1 - \lambda$ .

Preferences

• Preferences of type 
$$i \in \{A, B\}$$
 agents:

$$V_t^i(s^t) = \left[ (1 - \beta) U\left(c_t^i(s^t), n_t^i(s^t)\right)^{1 - \rho^i} + \beta \mu_t^i\left(V_{t+1}^i(s^{t+1})\right)^{1 - \rho^i} \right]^{\frac{1}{1 - \rho^i}}$$

- ▶  $c_t^i(s^t) \ge 0$  is consumption,  $n_t^i(s^t) \ge 0$  are hours worked
- U(c, n) is a period utility function, increasing in c, decreasing in n, concave, twice differentiable
- $\mu^i$  is the certainty equivalent of a risky continuation utility,

$$\mu_t^i(V) = [\mathbb{E}_t(V^{1-\gamma^i})]^{\frac{1}{1-\gamma^i}}$$

- relative risk aversion is  $\gamma_i$
- intertemporal elasticity of substitution of utilities is  $1/\rho_i$

incentive compatibility

- Agent's type is either  $(1, \rho^A, \gamma^A)$  or  $(\theta, \rho^B, \gamma^B)$ .
- Income and consumption is publicly observable. Type and hours worked are private information.
- I focus on a situation where the incentive constraints for type B bind.
- The utility that type B agent gets from choosing consumption and income of type A is

$$\hat{V}_{t}^{B}(s^{t}) = \left[ (1-\beta)U\left(c_{t}^{A}(s^{t}), \frac{n_{t}^{A}(s^{t})}{\theta}\right)^{1-\rho^{B}} + \beta\mu_{t}^{B}\left(\hat{V}_{t+1}^{B}(s^{t+1})\right)^{1-\rho^{B}} \right]^{\frac{1}{1-\rho^{B}}}$$

Incentive compatibility requires

$$V_0^B(s_0) \ge \hat{V}_0^B(s_0).$$
(1)

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The social planner maximizes a weighted average of the types' utilities

$$\max \alpha V_0^A(s_0) + V_0^B(s_0)$$

subject to the aggregate resource constraint

$$\lambda c_t^A(s^t) + (1-\lambda)c_t^B(s^t) + G_t(s^t) \le Z_t(s^t) \left[\lambda n_t^A(s^t) + (1-\lambda)\theta n_t^B(s^t)\right].$$

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and the incentive constraint (1).

The relative Pareto weight of type A is α > 0 (assumed sufficiently large)

# Why state-dependent marginal tax rates on low types?

Optimal tax formulas:

$$\tau_t^A(s^t) = \frac{\kappa \left(\nu_t(s^t) - \phi_t(s^t)\right)}{\alpha_t(s^t) - \kappa \phi_t(s^t)}$$

where

κ is the Lagrange multiplier on IC
 φ<sub>t</sub> = (ût<sup>A</sup>)<sup>-ρ<sup>B</sup></sup>/(ut<sup>A</sup>)<sup>-ρ<sup>A</sup></sup> U<sub>n</sub>(ct<sup>A</sup>,nt<sup>A</sup>/θ)/U<sub>n</sub> to represent the informational rent from higher productivity,
 ν<sub>t</sub> = (ût<sup>A</sup>)<sup>-ρ<sup>B</sup></sup>/(ut<sup>A</sup>)<sup>-ρ<sup>A</sup></sup> U<sub>c</sub>(ct<sup>A</sup>,nt<sup>A</sup>/θ)/U<sub>c</sub>(ct<sup>A</sup>,nt<sup>A</sup>/θ) represents the effects from non-separability in the utility function,

*α<sub>t</sub>* represents implicit Pareto weight on type *A*, and has initial value of *α*.

# A Benchmark Result

What if there is no hetereogenity in preferences?

No changes in the implicit Pareto weight over time. If, in addition, there are no changes in the informational rent, then marginal taxes are constant over time and state:

#### Proposition

Suppose that  $\rho^{A}=\rho^{B}=\rho$  and  $\gamma^{A}=\gamma^{B}=\gamma,$  and that

$$U(c,n) = \left[c^{1-\rho} - (1-\rho)\frac{n^{1+\eta}}{1+\eta}\right]^{\frac{1}{1-\rho}}.$$

Then the marginal tax rates are constant over time and states.

# A Benchmark Result

The result differs markedly from what one would obtain in the Ramsey environment, where Epstein-Zin preferences matter (Karantounias 2018).

In Ramsey, the government manipulates interest rates and the marginal cost of issuing debt by varying the tax rates.

In Mirrlees, the presence of nonlinear taxes means that the marginal cost of issuing debt is equalized across states.

# Three examples

► Type *B* agents have zero aversion toward utility risk ( $\gamma^B = 0$ ) and infinite elasticity of intertemporal substitution ( $\rho^B = 0$ ):

$$V^{B} = (1 - \beta) \mathbb{E} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}^{B}(s^{t}), n_{t}^{B}(s^{t})\right).$$

preferences of type A agents:

- 1. infinite risk aversion ( $\gamma^B = \infty$ )
- 2. zero intertemporal elasticity of substitution ( $\rho^A = \infty$ )
- both infinite risk aversion and zero intertemporal elasticity of substitution. (γ<sup>B</sup> = ρ<sup>A</sup> = ∞)

Period utility is

$$U(c,n) = \ln c - \frac{n^{1+\eta}}{1+\eta}.$$

#### Three examples

Let κ and β<sup>t</sup>π(s<sup>t</sup>)α<sub>t</sub>(s<sup>t</sup>) be the Lagrange multiplier on IC and on the promise keeping constraint on type A (after a history s<sup>t</sup>)

lnterpretation of  $\alpha_t$ : implicit Pareto weights of type A:

$$\alpha_t(s^t) = (1+\kappa) \frac{\lambda}{1-\lambda} \frac{U^B_{ct}(s^t)}{U^A_{ct}(s^t)} + \kappa.$$

The optimal marginal tax rate on type A reduces in all three examples to

$$\tau_t^A(s^t) = \frac{\kappa(1-\phi)}{\alpha_t(s^t) - \kappa\phi}, \quad \phi = \theta^{-1-\eta}.$$
 (2)

- ▶ Intuition? Higher  $\alpha_t \rightarrow$  relatively higher consumption, of type A agent  $\rightarrow$  decreases hours worked (positive income effect)  $\rightarrow$  relax the incentive constraint  $\rightarrow$  less distortion needed.
- ► The three examples differ in the behavior of  $\alpha_t(s^t)$ .

## Example 1: differences in risk aversion

Type A agents have infinite intertemporal elasticity of substitution (ρ<sup>A</sup> = 0), and infinite aversion (γ<sup>A</sup> = ∞)

#### Preferences are

$$V_t^A(s^t) = (1 - \beta) U\left(c_t^A(s^t), n_t^A(s^t)\right) + \beta \min_{s_{t+1}} \left\{ V_{t+1}^A(s^{t+1}) \right\}.$$

► This requires that  $V_{t+1}^A(s^t, s_{t+1})$  is independent of  $s_{t+1}$ , and so  $V_t^A(s^{t-1}) = (1 - \beta)U\left(c_t^A(s^t), n_t^A(s^t)\right) + \beta V_{t+1}^A(s^t) \quad \forall s^t.$ 

# Example 1: differences in risk aversion

• Key result:  $\alpha_t$  it follows a random walk:

$$\alpha_t(s^t) = \sum_{s_{t+1}} \pi(s_{t+1}|s^t) \alpha_{t+1}(s^{t+1}).$$

- Type A agents are fully insured against fluctuations in states, and their implicit Pareto weight fluctuates across states.
- On the other hand, type A agents are no different in their IES, and their Pareto weights are, in expectation, constant.
- Two implications:
  - 1. marginal tax rates are, on average, increasing over time
  - 2. inverse marginal tax rates are follow a random walk

# Example 2: differences in IES

Type A agents have zero intertemporal elasticity of substitution (ρ<sup>A</sup> = ∞) and zero aversion toward utility risk (γ<sup>A</sup> = 0):

$$V_t^A(s^t) = \min\left\{ U\left(c_t^A(s^t), n_t^A(s^t)\right), \sum_{s_{t+1}} \pi(s_{t+1}|s^t) V_{t+1}^A(s^{t+1}) \right\}$$

• Key result: changes in  $\alpha_t$  must off-set over time:

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t \alpha_t(s^t) = \alpha \quad \forall s^{\infty} \in S^{\infty}.$$

- The opposite of what was in Example 1: an increase in α<sub>t</sub>(s<sup>t</sup>) will be followed by a decrease.
- Marginal tax rates inherit the off-setting property.

# Example 3: differences in both IES and risk aversion

Type A agents have both zero IES (ρ<sup>A</sup> = ∞) and infinite risk aversion (γ<sup>A</sup> = ∞):

$$V_t^A(s^t) = \min\left\{ U\left(c_t^A(s^t), n_t^A(s^t)\right), \min\left\{V_{t+1}^A(s^{t+1})\right\} \right\},\$$

- This is equivalent to requiring a constant period utility V<sup>A</sup> across time and states.
- The solution is now "static" in that α<sub>t</sub>, as well as the allocations, are only a function of the current shocks Z<sub>t</sub>, G<sub>t</sub>:

$$\alpha_t(s^t) = \bar{\alpha} \left[ Z_t(s^t), G_t(s^t) \right]$$

- Reminiscent of Ramsey under complete markets.
- It follows that τ<sup>A</sup><sub>t</sub> is also a function of Z<sub>t</sub>, G<sub>t</sub> only. No history dependence in taxes!

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#### Numerical Example

- 6 periods, equal shares (λ = 0.5), type B agents are 40 percent more productive (θ = 1.4), and the Frisch elasticity of labor is 0.5 (η = 2).
- The Pareto weight α is set so that the initial period marginal tax rate is 0.4.
- Aggregate productivity is  $Z(s_0) = 1$  in the first period, while  $Z_t(s^t)$  is an iid random variable taking three possible values 0.9 (recession), 1.0, and 1.1 (expansion) with equal probability

Government consumption set to zero

Aggregate Productivity Shock: Marginal Tax Rates



Figure: Marginal tax rates on type A agents. Infinite RA, zero IES, or a combination of both refers to preferences of type A agents.

# Transfers

- Transfers are not unique. Under complete markets, Ricardian equivalence applies. Time and state variations in transfers will be offset by asset trades.
- We determine transfers by assuming incomplete markets for type A agents: type-A agents are hand-to-mouth:

$$T_t^A(s^t) = c_t^A(s^t) - z_t(s^t)n_t^A(s^t)$$

- This assumption will not change the optimal allocation, because the government can still issue state contingent claims and trade them with type B agents.
- Full market incompleteness would change the solution.

# Aggregate Productivity Shock: Transfers



Figure: Transfers to type A agents as a fraction of income. Infinite RA, zero IES, or a combination of both refers to preferences of type A agents.

# Log-Linearization around Steady State

- Study the role of preference parameters for the response of the optimal tax and transfer system  $(\tau_t^A, T_t^A)$ , and the optimal allocations, to a small change in  $Z_t$  or  $G_t$ .
- Solve for the steady-state and log-linearize around it.
- No need to worry about IC: It holds in steady state by construction, and continues to hold (up to the log-linear approximation)
- Simplifying assumptions:

$$ho^A=\gamma^A$$
 and  $ho^B=\gamma^B$ 

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$$U^{i}(c,n) = \left[c^{1-\rho^{i}} - (1-\rho^{i})\frac{n^{1+\eta}}{1+\eta}\right]^{\frac{1}{1-\rho^{i}}}$$

 Assumption 1 simplifies the dynamics of the responses similarly to example 3.

#### Log-Linearization around Steady State First-order conditions

marginal tax rates (optimal formulas):

$$\tau_t^A = \frac{\kappa \left( (c_t^A)^{\rho^A - \rho^B} - \phi \right)}{\alpha - \kappa \phi}$$
$$\tau_t^B = 0$$

where

$$au_t^A = 1 - rac{(n_t^A)^\eta (c_t^A)^{
ho^A}}{Z_t}, \quad au_t^B = 1 - rac{(n_t^B)^\eta (c_t^B)^{
ho^B}}{\theta Z_t}$$

efficient allocation of consumption:

$$lpha \cdot (c_t^A)^{-
ho^A} - \kappa \cdot (c_t^A)^{-
ho^B} = rac{\lambda}{1-\lambda} (1+\kappa) \cdot (c_t^B)^{-
ho^B}$$

plus IC and RC

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# Log-Linearization around Steady State Log-linearization

Let

$$dx_t = \ln x_t - \ln x$$

be a percentage deviation of a variable x<sub>t</sub> from steady state.
relative response of consumption:

$$dc_t^B = \left(1 + rac{
ho^A - 
ho^B}{
ho^B} rac{lpha}{lpha - \kappa \phi} rac{1}{1 - au^A}
ight) dc_t^A$$

- If A type has low IES (high ρ<sup>A</sup>), his consumption responds relatively less.
- marginal tax rates:

$$d au_t^A = rac{(c^A)^{
ho^A - 
ho^B}}{(c^A)^{
ho^A - 
ho^B} - \phi} (
ho^A - 
ho^B) dc_t^A$$
  
 $d au_t^B = 0.$ 

► If A type has low IES (high p<sup>A</sup>), marginal tax rate is positively correlated with his consumption

#### Log-Linearization around Steady State Log-linearization

Solving for the response of consumption,

$$dc_t^A = \frac{dz_t \left(1 + \frac{1}{\eta}\right) - (1 - \chi)dg_t}{\chi \left[h_c + (1 - h_c) \left(1 + F_1\right)\right] + \frac{1}{\eta} \left[h_y \left(\rho^A + F_2\right) + (1 - h_y)\rho^B \left(1 + F_1\right)\right]}$$

where  $F_1$  and  $F_2$  are positive coefficients, and  $h_c$  and  $h_y$  are steady-state consumption and income shares of type A.

- Consumption of type A responds positively to TFP shock and negatively to government spending shock
- If ρ<sup>A</sup> > ρ<sup>B</sup>, marginal tax rate on type A also responds positively to TFP shock and negatively to government spending shock.

#### Log-Linearization around Steady State Quantification

Set λ = 0.6 and θ = 2.0 (loosely approximating college vs non-college division)

• Set 
$$\eta = 2$$
 (Frisch = 0.5)

- Choose G to match steady-state government consumption to output ratio of 0.15.
- Fix  $1/\rho^B = 0.3$  (Guevenen, 2009) and vary  $1/\rho^A$  from 0.3 to 0.067
- For each exercise, I choose the Pareto weight α so that the steady-state marginal tax rate τ<sup>A</sup> equals 0.4

# Response to 1 % increase in TFP



Figure: Response of the marginal tax rate  $\tau_t^A$  and transfers to low types  $T_t^A$  to a 1 % increase in  $Z_t$ .

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# Response to 1 % increase in Govt Spending



Figure: Response of the marginal tax rate  $\tau_t^A$  and transfers to low types  $T_t^A$  to a 1 % increase in  $G_t$ .

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# Response to 1 % increase in TFP: First vs Second Best



Figure: Response of transfers to low types  $T_t^A$  to a 1 % increase in  $Z_t$ .

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# Conclusions

 Preference heterogeneity important for procyclical marginal tax rates and countercyclical transfers.

Higher risk aversion and lower IES for type A agents have similar effects on the current marginal taxes, but an opposite effect on future dynamics.

 Optimal tax systems in general depend on the history of aggregate shocks.

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