

WHAT DRIVES VIOLATIONS OF THE INDEPENDENCE AXIOM? THE ROLE OF DECISION CONFIDENCE

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Caltech

CONFIDENCE AND DECISION-MAKING UNDER RISK

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- However, decision confidence is a dimension neglected by standard economic models
 - Economically relevant dimension if it can be used to **predict** behavior
- **Question 1:** Are individuals more likely to choose certain or risky options when they are **not confident** about what to choose?

INFORMING ECONOMIC MODELS: COMMON RATIO EFFECT

Option A: 100% chance of \$7 vs. **Option B:** 50% chance of \$20

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 - Under Expected Utility (EU), choose either options A and C, or B and D
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EMPIRICAL RELEVANCE OF THE CRE

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- **Reverse Common Ratio Effect (RCRE):** choose Option B and Option C
- **Question 3:** Which non-EU behavior is more empirically relevant? Can decision confidence provide a rationale for why?

INTERPRETING DECISION CONFIDENCE

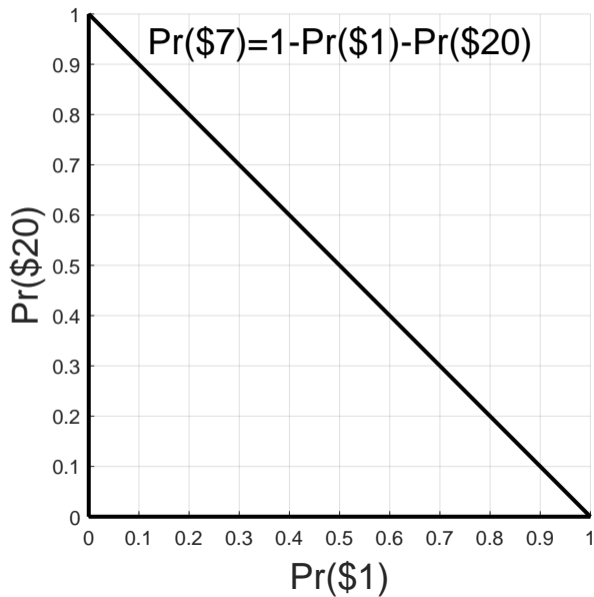
- This paper documents **positive** correlation between **non-EU** behavior and **low** decision confidence
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- This paper documents **positive** correlation between **non-EU** behavior and **low** decision confidence
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- Hard question whose answer depends on what decision confidence captures
- A possible answer: (lack of) confidence captures **preference imprecision**
- Structural model that involves the estimation of a **set of utility functions** (Kobayashi and Lucia, 2023)

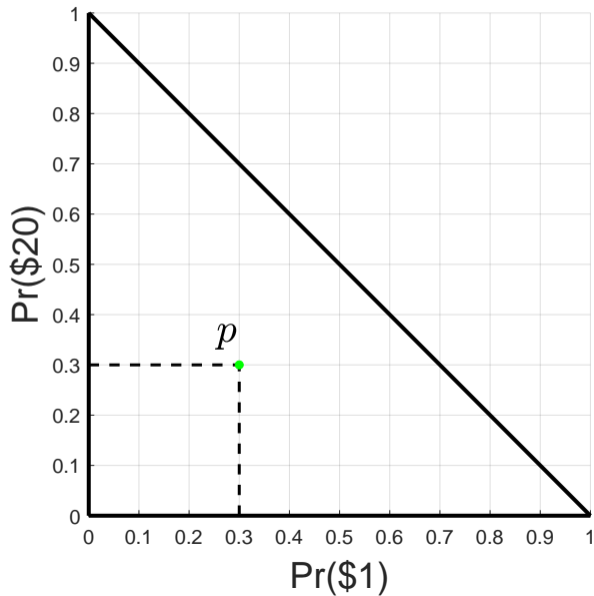
- Run an online experiment in which subjects
 1. Make choices between lotteries
 2. Express how **confident** they feel about their choices
- Test CRE and RCRE using a wide range of lotteries
- Recruit 300 subjects from Prolific
 - Between subjects, variation in lotteries

EXPERIMENTAL DESIGN

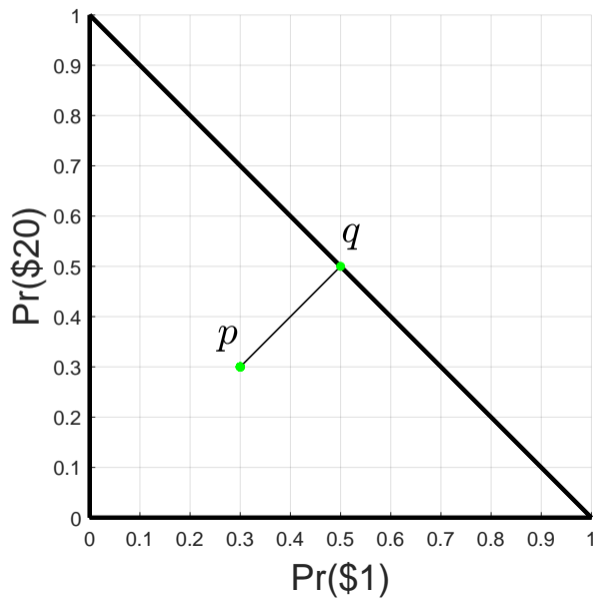
MARSCHAK-MACHINA TRIANGLE: LOTTERIES WITH 3 PRIZES (\$1, \$7, \$20)



LOTTERIES

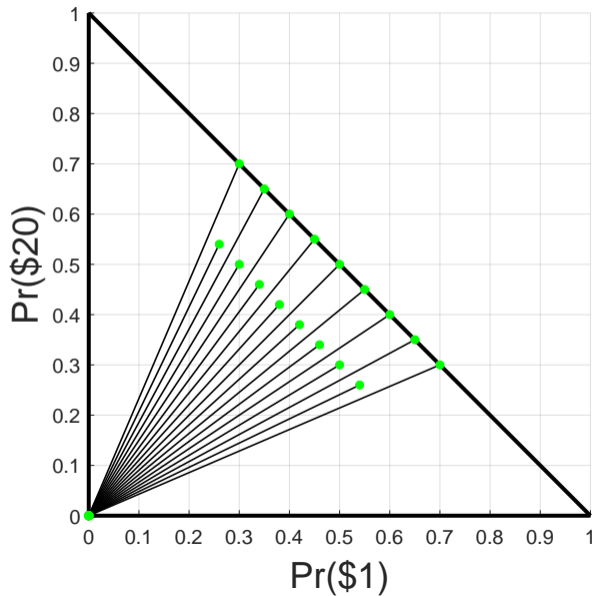


BINARY CHOICE TASKS



- Two types of binary choice tasks to test the CRE and the RCRE:
Unmixed: certain prize (\$7) vs. risky lottery

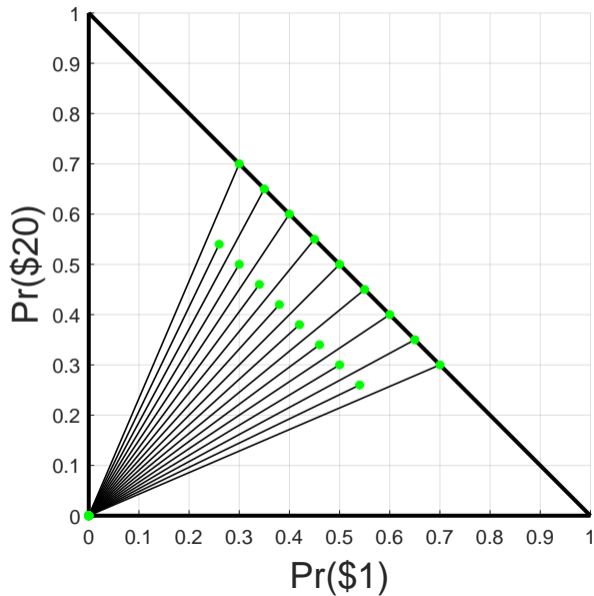
UNMIXED CHOICE TASKS



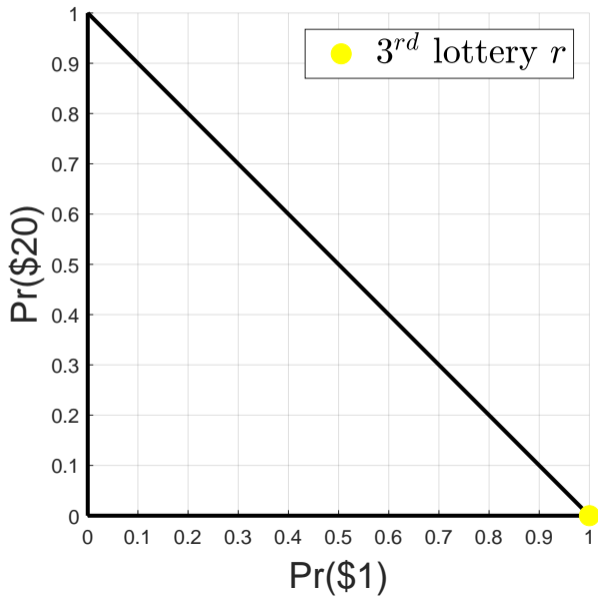
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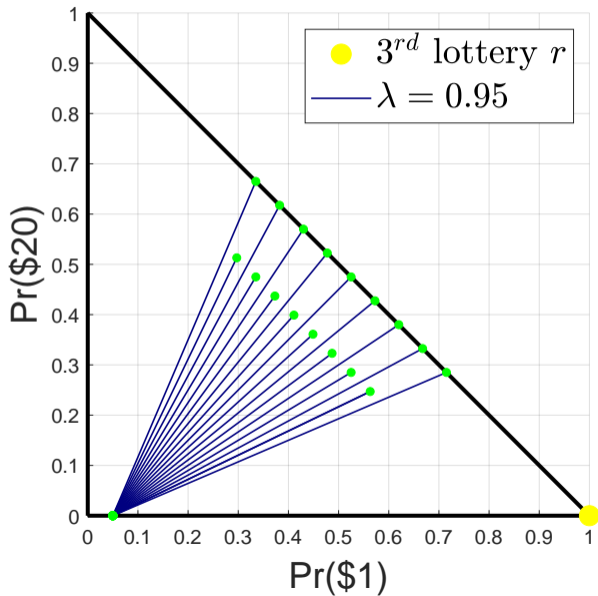
START FROM UNMIXED CHOICE TASKS



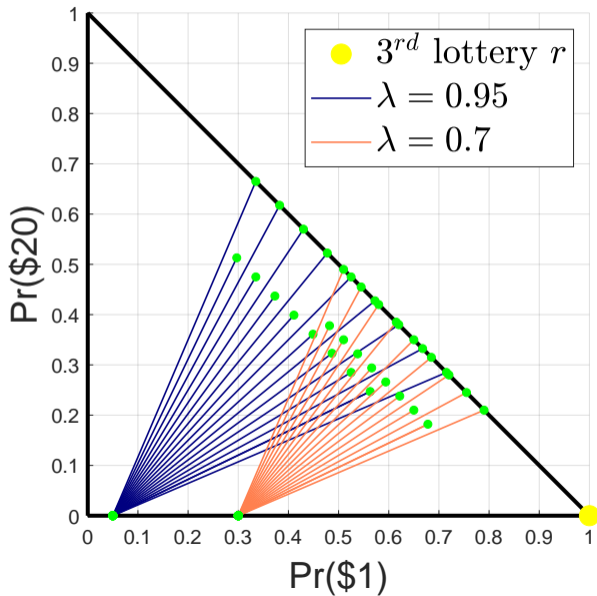
MIXED EXAMPLE



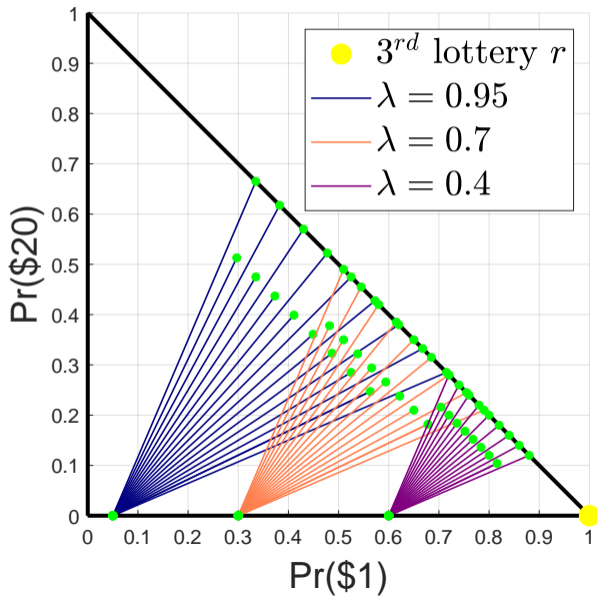
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- Report confidence for each decision (0-100 scale, unincentivized)

Pair 1 of 74

Lottery Ticket A

0% chance of \$1
100% chance of \$7
0% chance of \$20

Lottery Ticket B

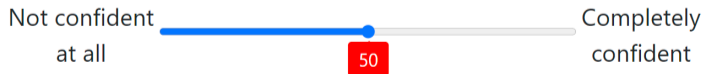
45% chance of \$1
0% chance of \$7
55% chance of \$20

- **Question 1:** which lottery ticket do you **prefer**?

Lottery ticket A

Lottery ticket B

- **Question 2:** you chose lottery ticket **A**. On a scale from 0 to 100, how **confident** do you feel about this choice? **The higher the number, the more confident you are about this choice.**



Next

- Two types of binary choice tasks to test the CRE and the RCRE:
 1. **Unmixed**: certain prize (\$7) vs. risky lottery
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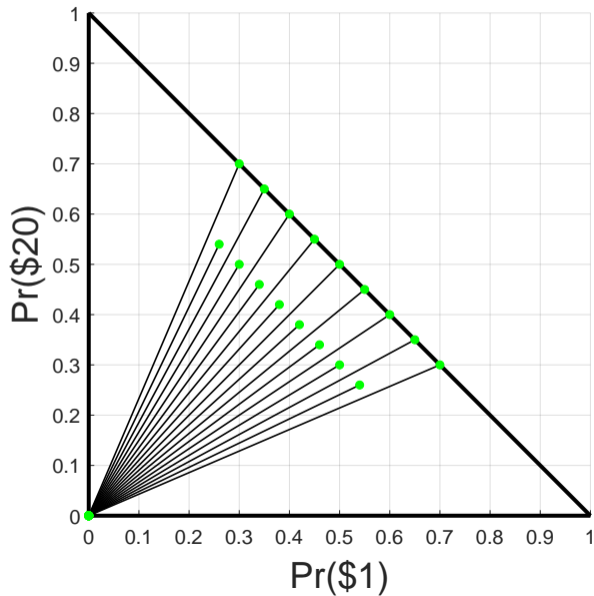
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- Main Outcomes:
 - Proportion of riskier choices in unmixed and mixed (study CRE and RCRE)
 - Correlation with decision confidence

MAIN RESULTS

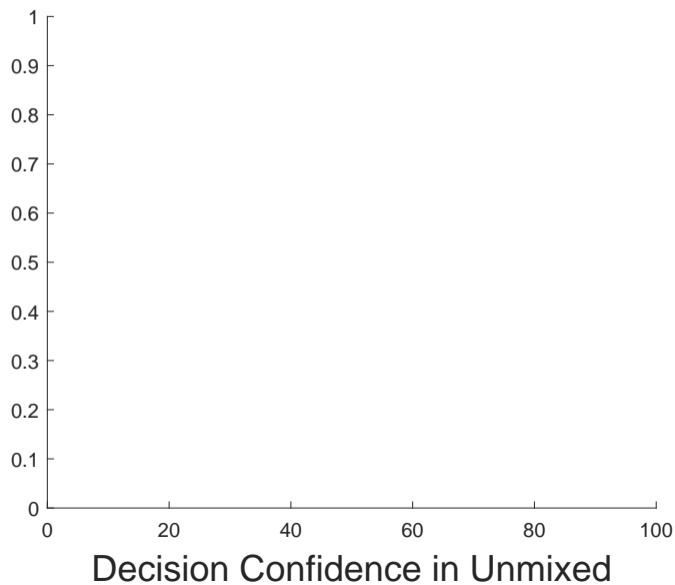
QUESTION 1

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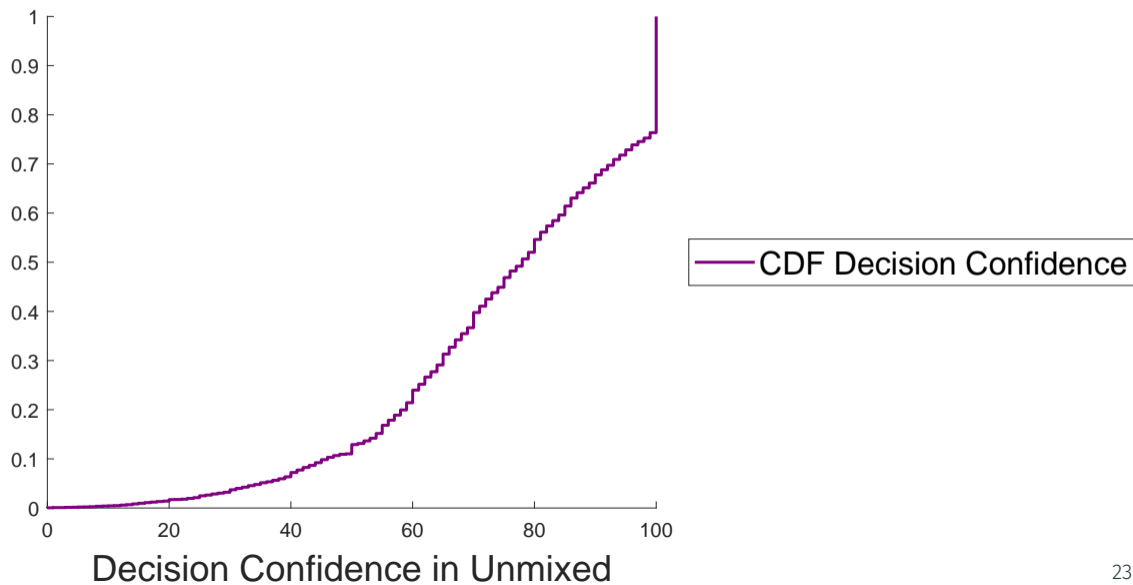
UNMIXED CHOICE TASKS



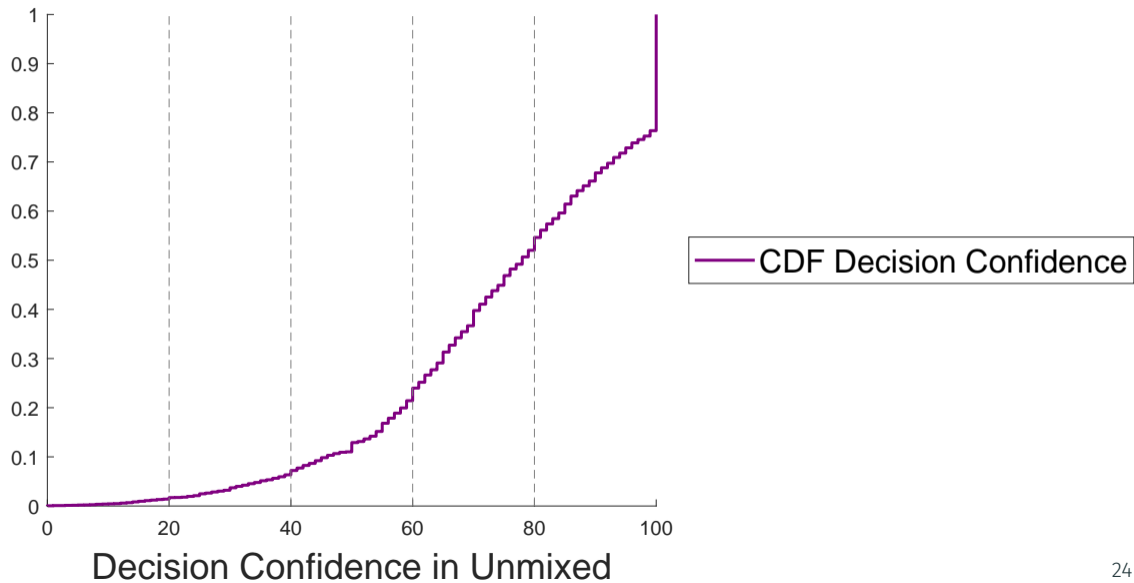
BEHAVIOR AT DIFFERENT CONFIDENCE LEVELS



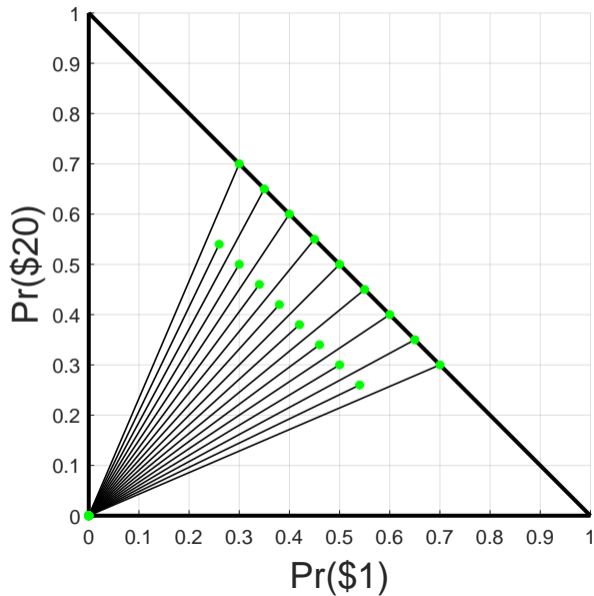
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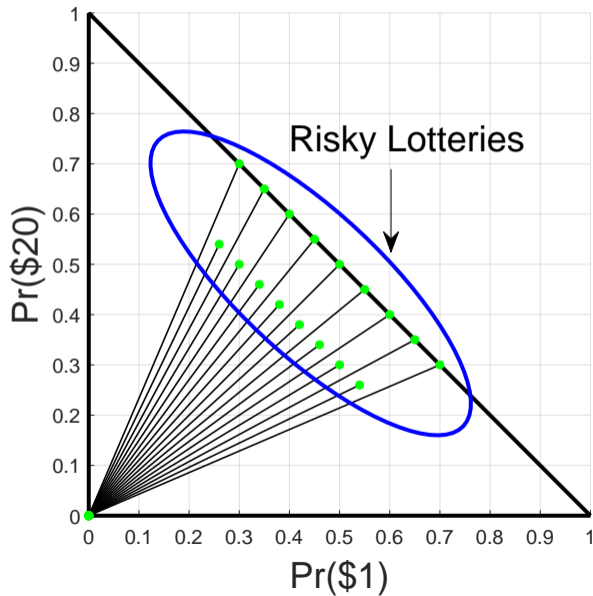
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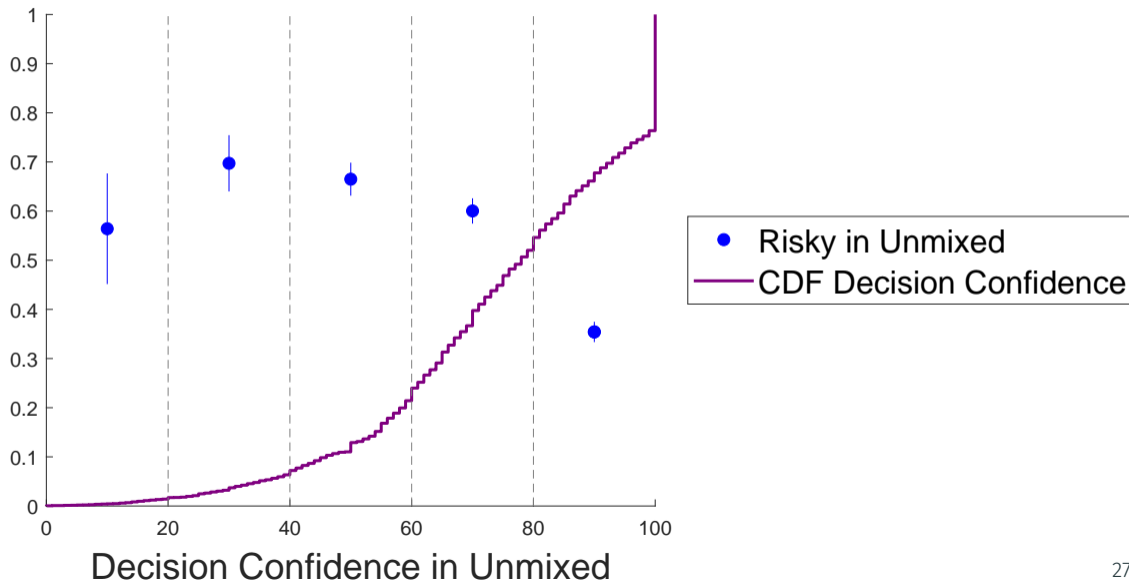
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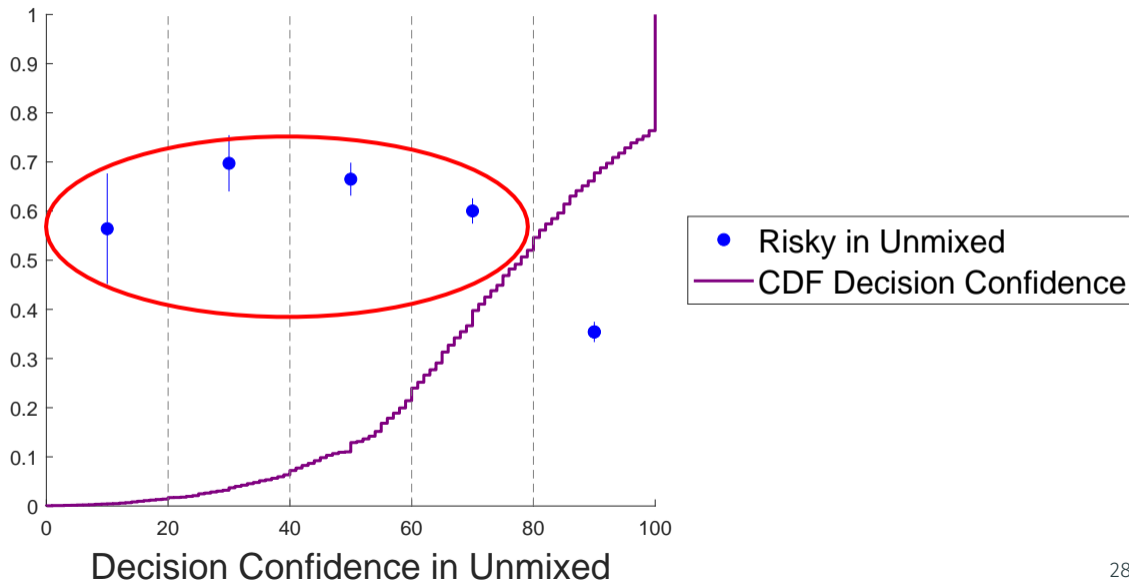
HOW OFTEN DO INDIVIDUALS CHOOSE RISKY LOTTERIES?



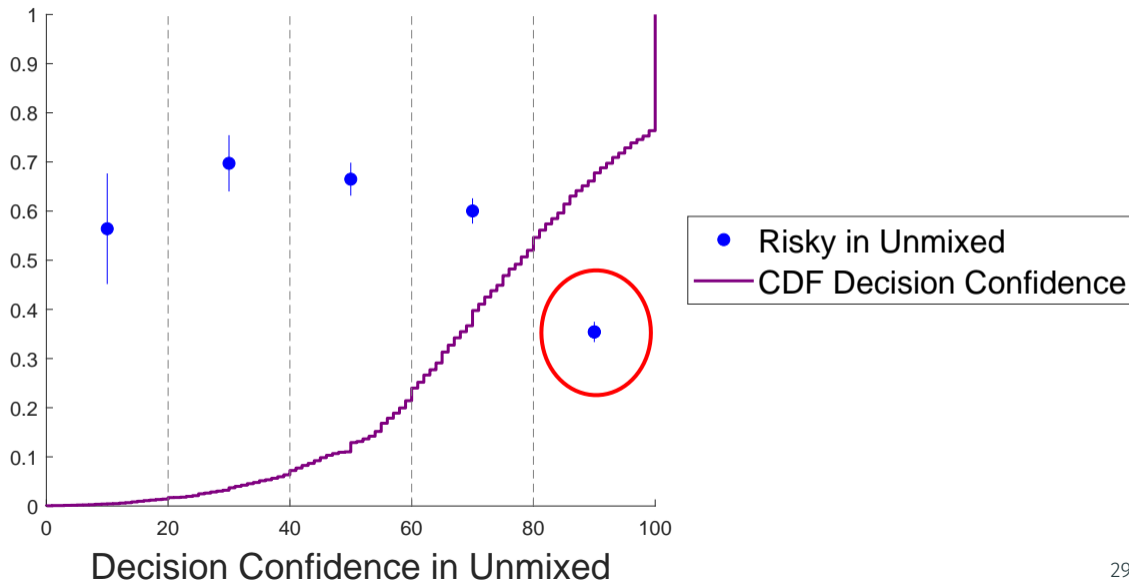
PROPORTION OF CHOICES FOR RISKY LOTTERIES IN UNMIXED



MORE LIKELY TO CHOOSE RISKY LOTTERIES AT LOW CONFIDENCE



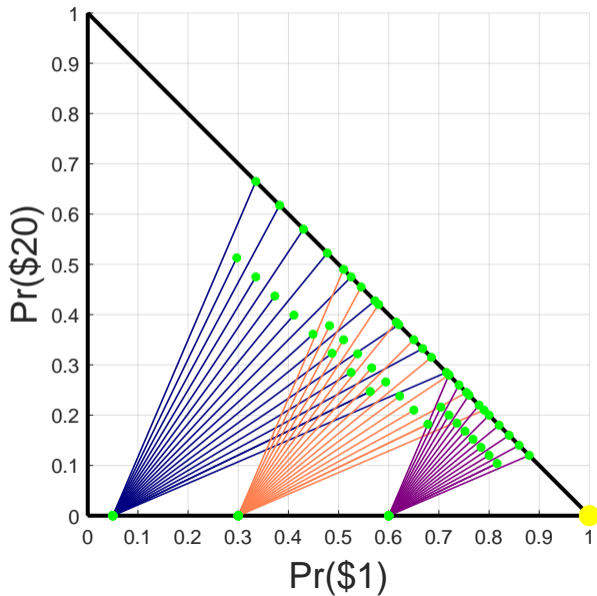
MORE LIKELY TO CHOOSE THE CERTAIN PRIZE AT HIGH CONFIDENCE



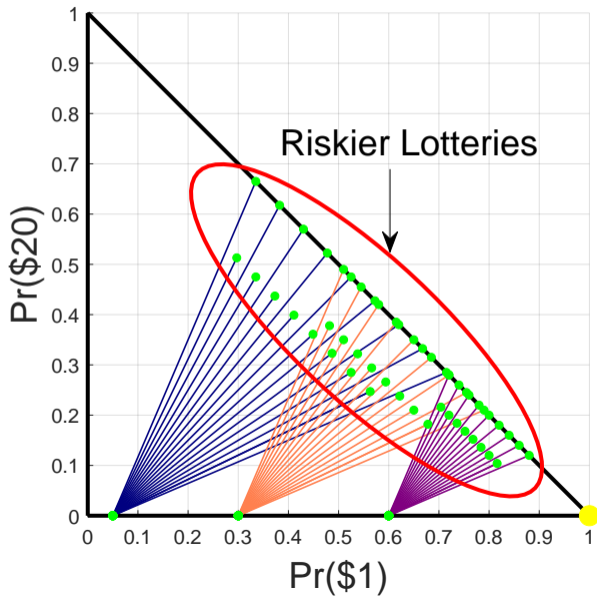
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- ⇒ Individuals are more likely to choose **risky over certain** options when they are not confident about their choices

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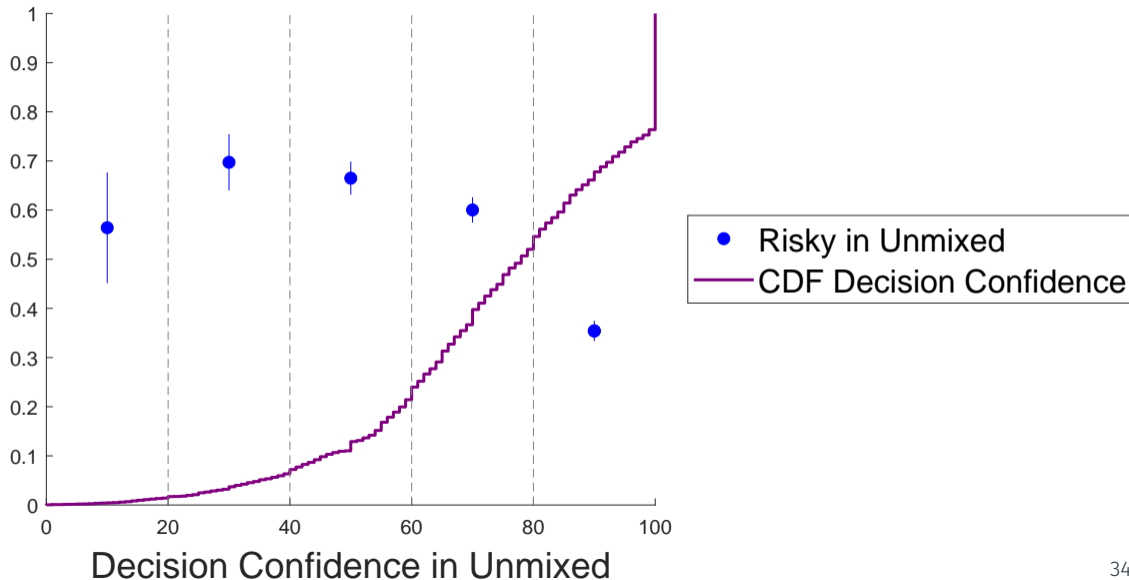
MIXED EXAMPLE



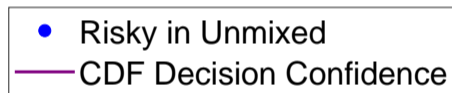
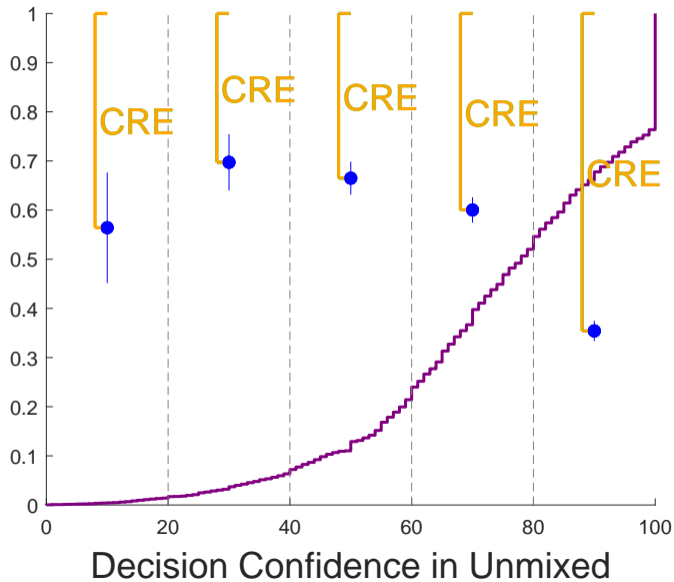
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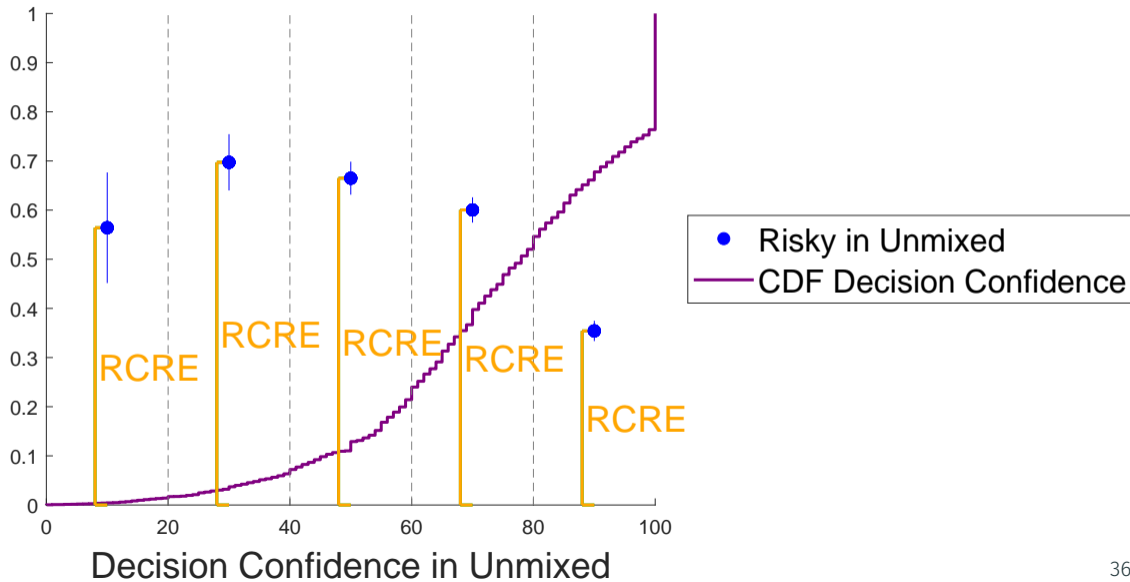
2 POSSIBLE NON-EU BEHAVIORS: CRE AND RCRE



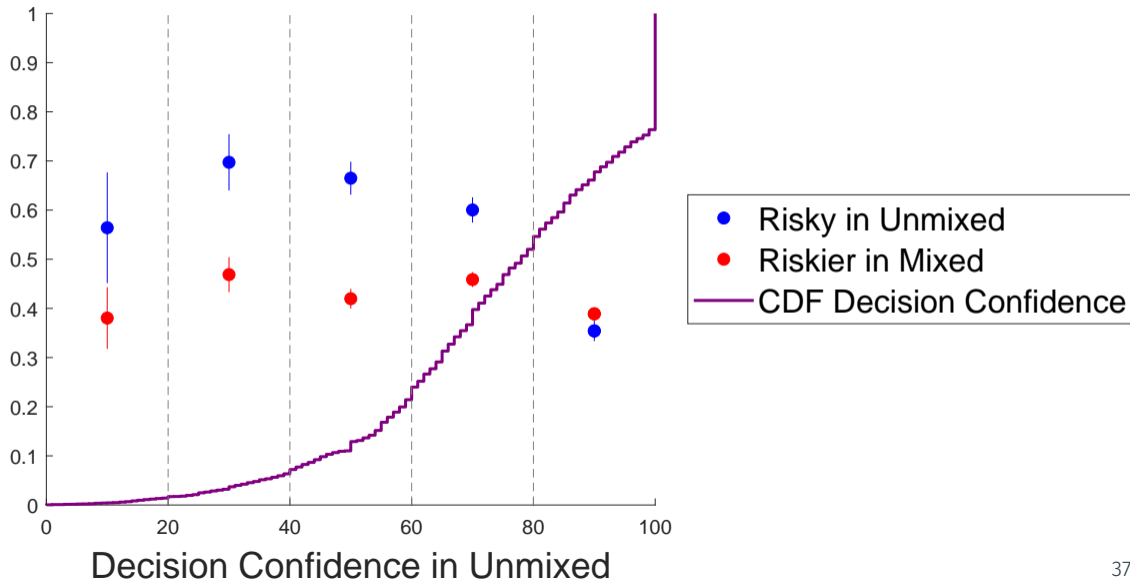
CRE: MORE RISK SEEKING IN MIXED CHOICE TASKS



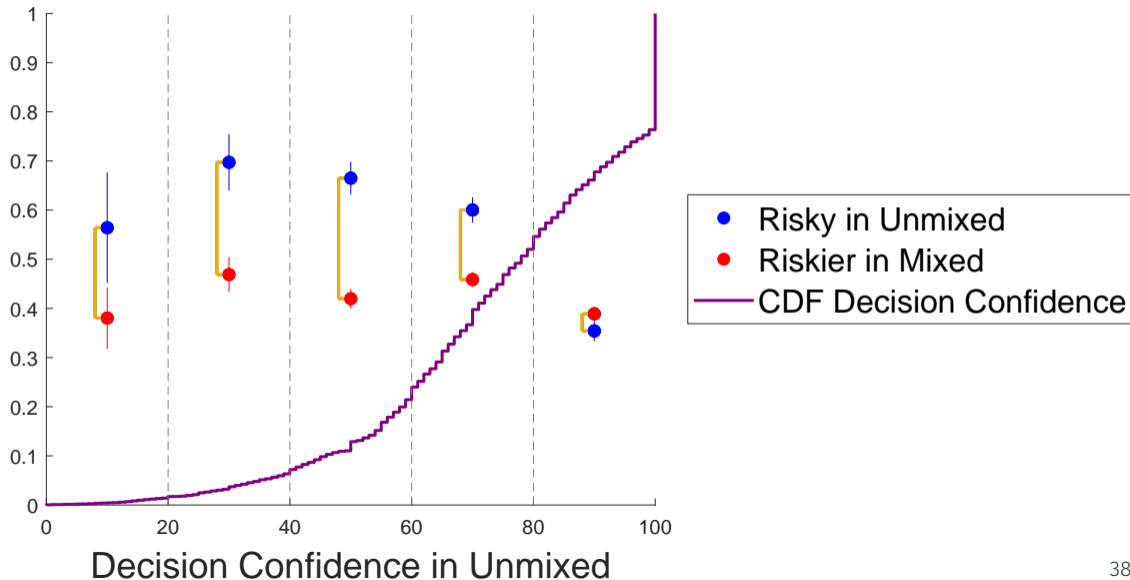
RCRE: LESS RISK SEEKING IN MIXED CHOICE TASKS



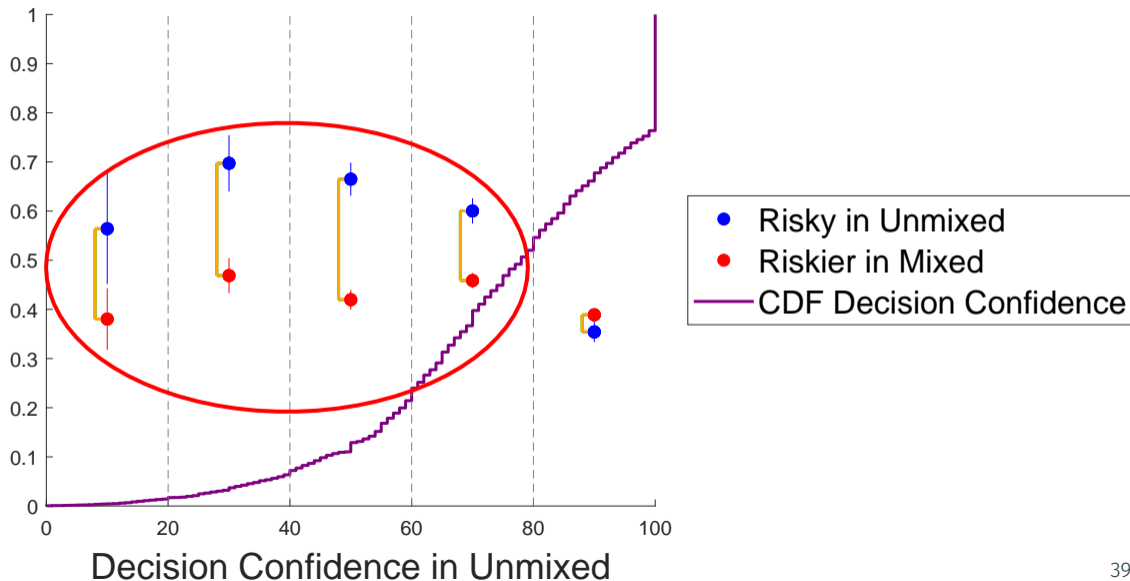
PROPORTION OF CHOICES FOR RISKIER LOTTERIES IN MIXED



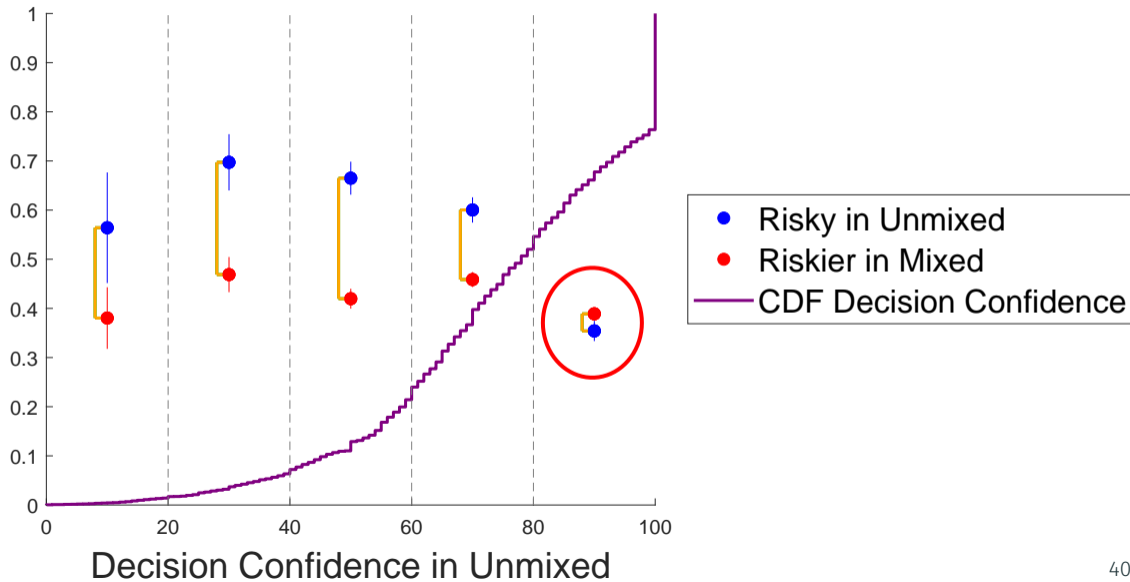
PROXY FOR NON-EU BEHAVIOR



NON-EU BEHAVIOR MORE LIKELY AT LOW CONFIDENCE

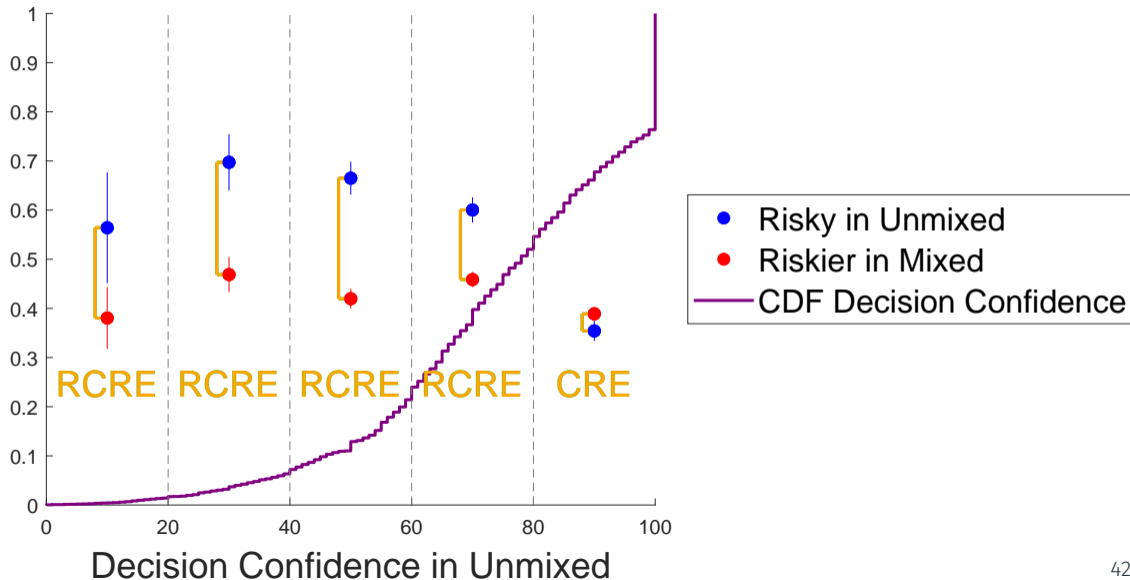


NON-EU BEHAVIOR LESS LIKELY AT HIGH CONFIDENCE



- **Question 2:** Does non-EU behavior correlate with the lack of decision confidence?
- ⇒ Non-EU behavior is **more likely** to emerge when subjects express **low confidence** in their choices
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RCRE MOST RELEVANT NON-EU BEHAVIOR

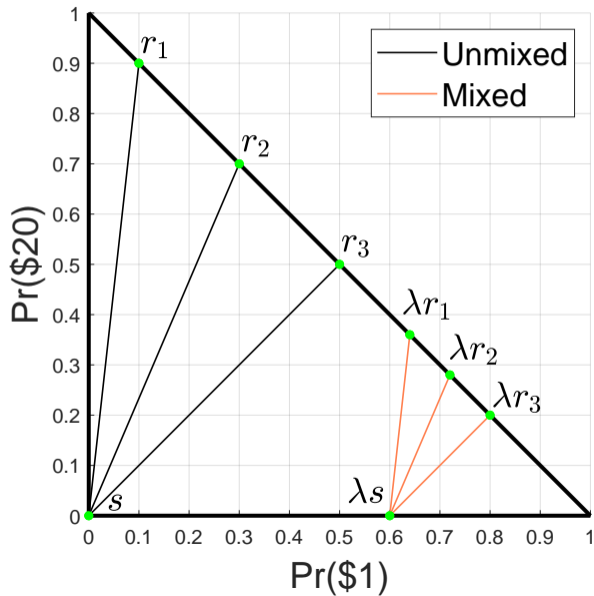


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⇒ The **RCRE** is the most relevant behavioral deviation from EU. Studying behavior in situations of low confidence can explain its prevalence

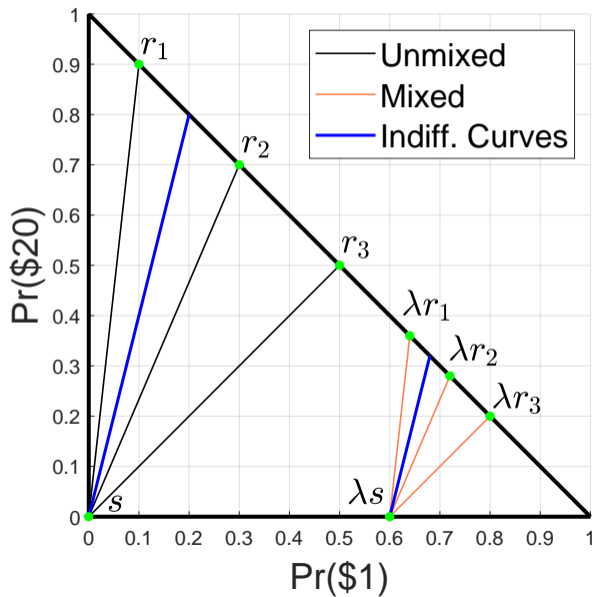
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- ⇒ These findings suggest a possible rationale for the **RCRE**, which is the most frequent EU violation
- **Question 4:** How should we interpret the documented correlation between non-EU behavior and low confidence?
 - Introduce structural model developed by Kobayashi and Lucia (2023)

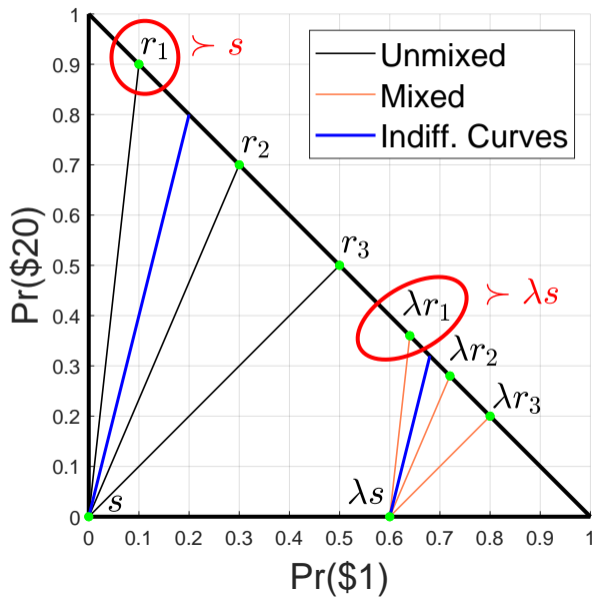
MULTI-UTILITY MODEL - EXAMPLE



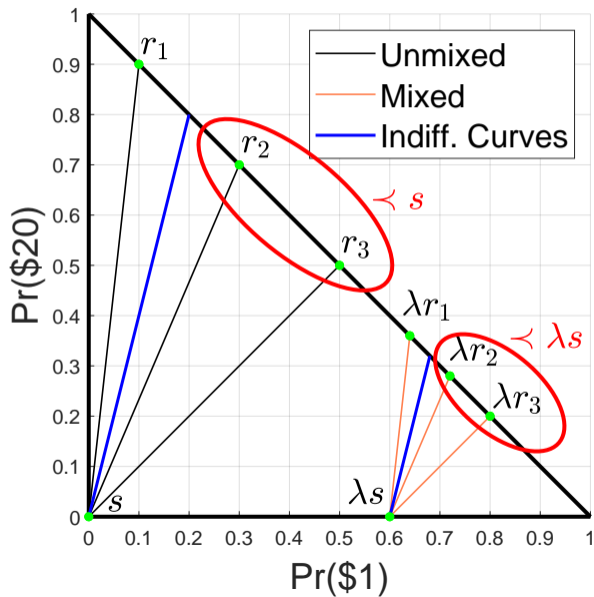
EU BENCHMARK: ONE UTILITY



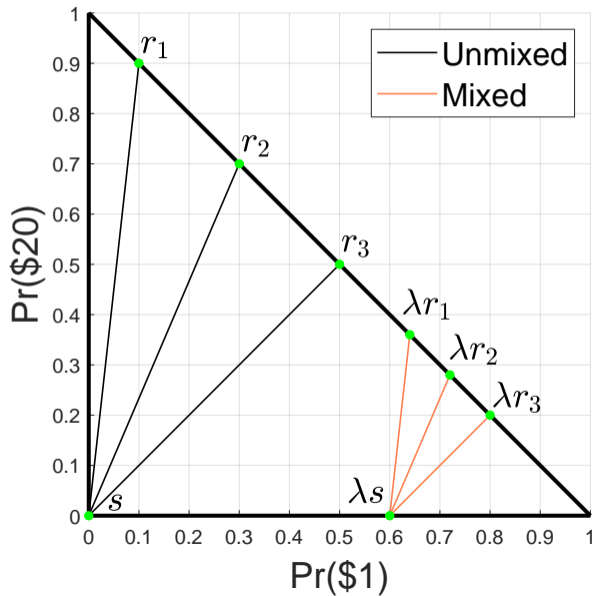
LOTTERIES PREFERRED TO s AND λs



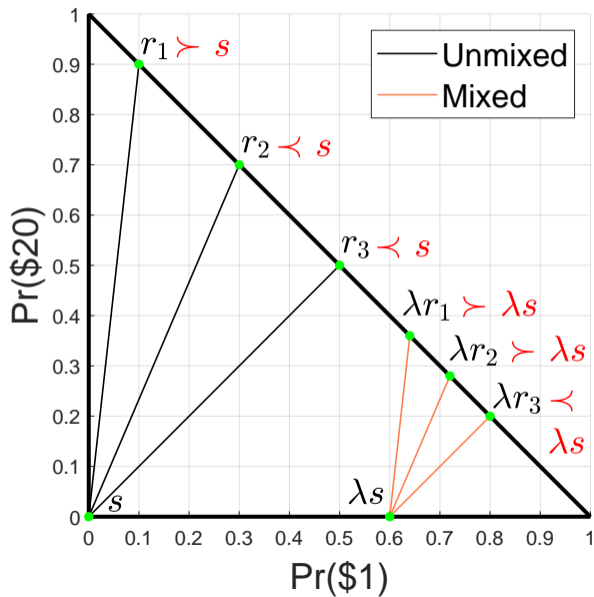
LOTTERIES PREFERRED TO r AND λr



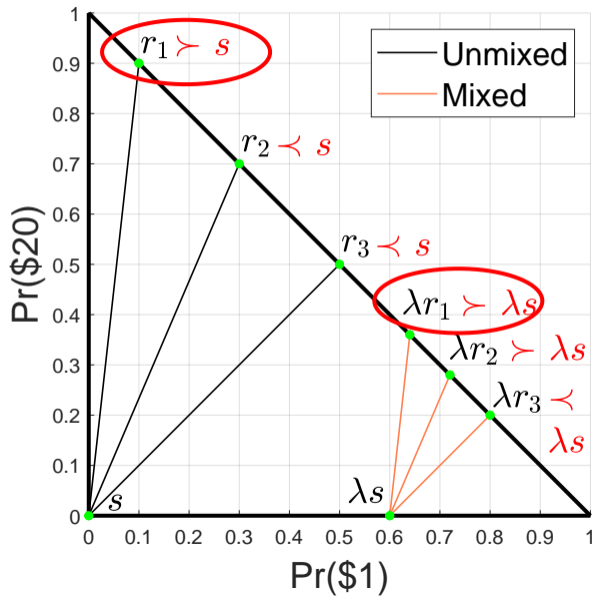
MULTI-UTILITY MODEL - EXAMPLE



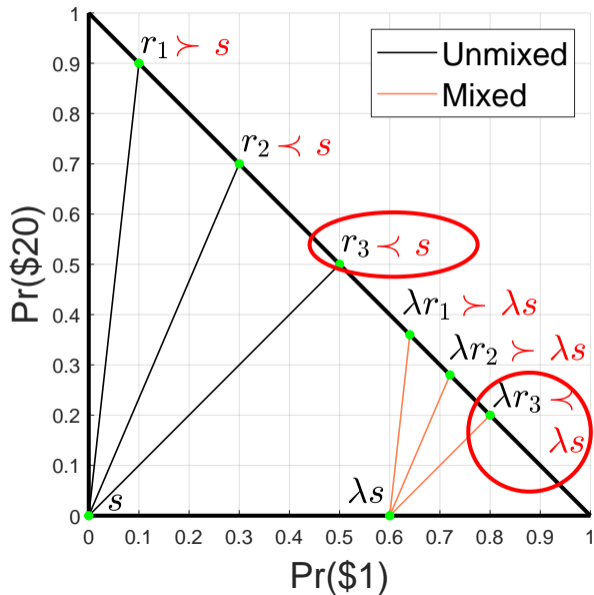
EXAMPLE OF CHOICE PATTERNS



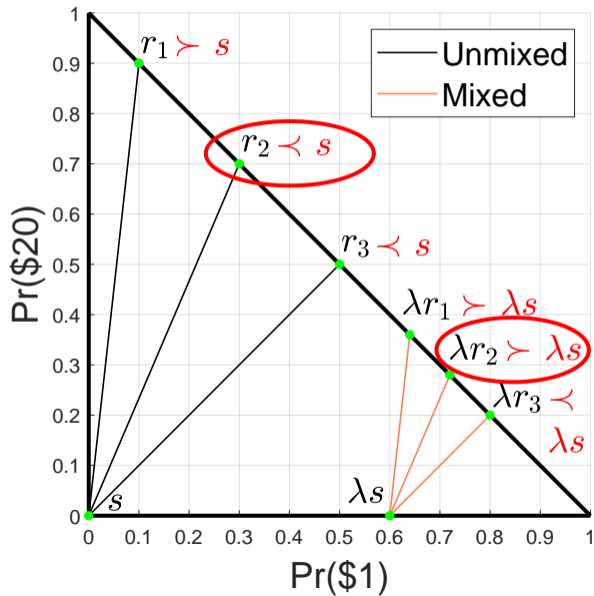
EU PATTERN: CHOOSE ALWAYS RISKIER



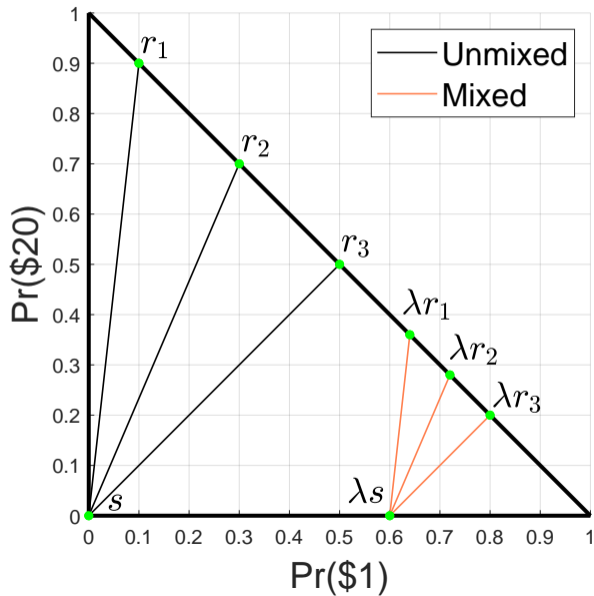
EU PATTERN: CHOOSE ALWAYS SAFER



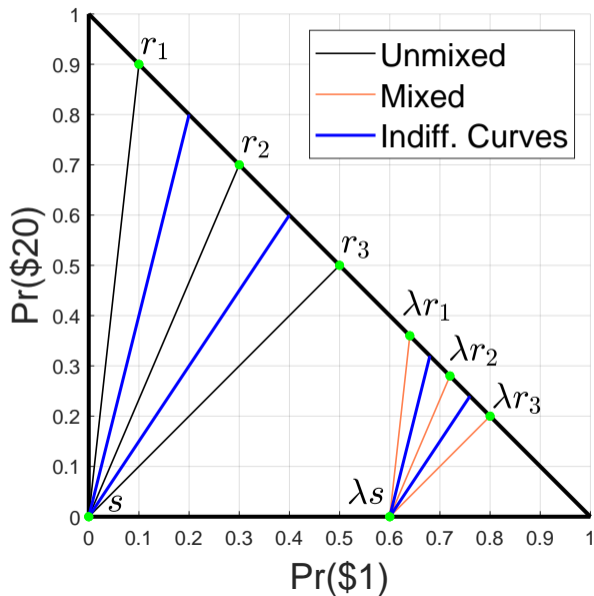
NON-EU PATTERN: COMMON RATIO EFFECT



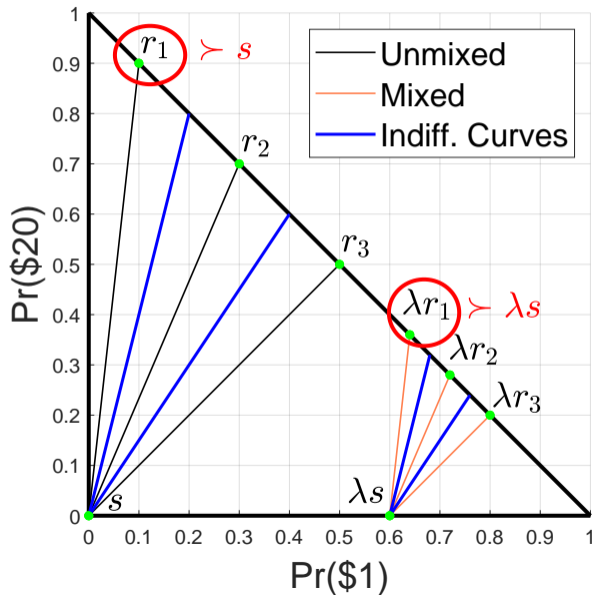
SEPARATE EU AND NON-EU CHOICES USING MULTIPLE UTILITIES



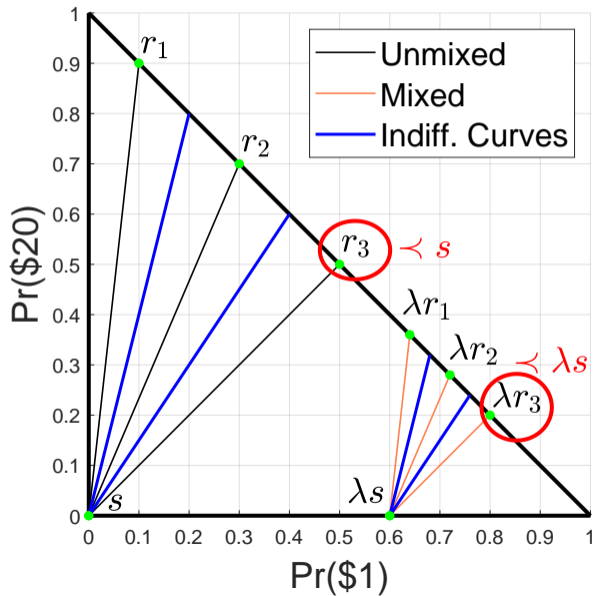
MULTI-UTILITY MODEL: TWO UTILITIES



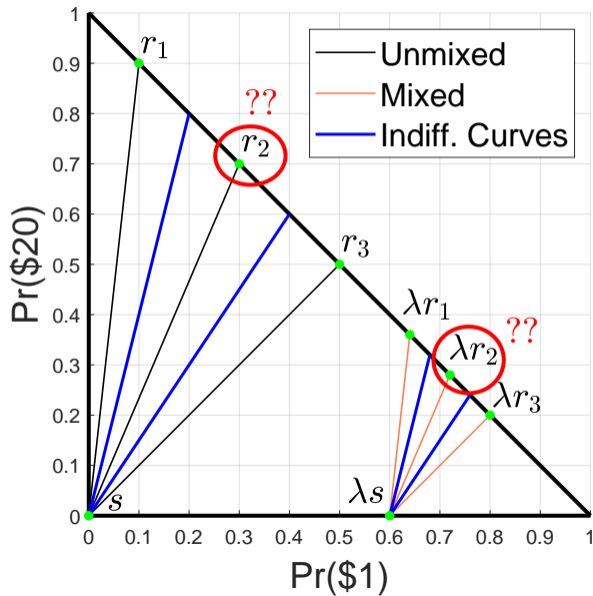
LOTTERIES UNAMBIGUOUSLY PREFERRED TO s AND λs



LOTTERIES UNAMBIGUOUSLY PREFERRED TO r AND λr



AMBIGUOUS RANKINGS

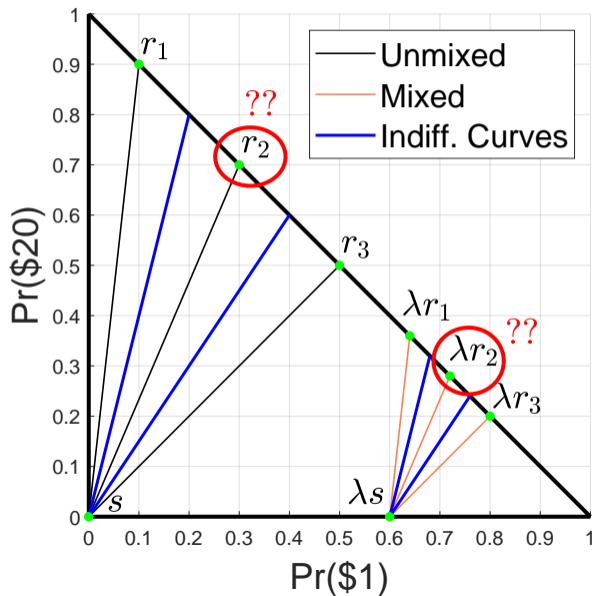


- Represent utilities $v = [v_1, v_7, v_{20}]$ as vectors, where v_x is the utility of prize $\$x$
- Estimate set of **normalized** utilities \mathcal{W} , with $v_1 = 0$ and $v_{20} = 1$ for all $v \in \mathcal{W}$

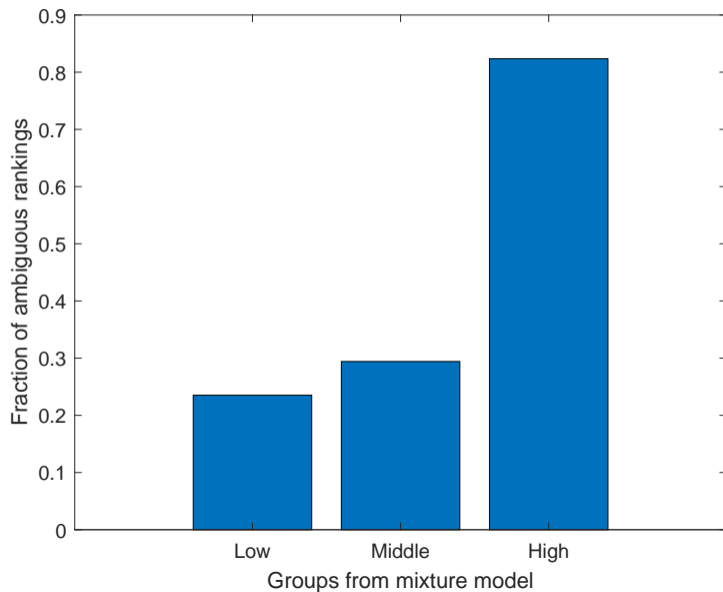
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- Estimate set of **normalized** utilities \mathcal{W} , with $v_1 = 0$ and $v_{20} = 1$ for all $v \in \mathcal{W}$
- $\mathcal{W} \leftrightarrow [\underline{v}_7, \bar{v}_7] \subseteq [0, 1]$ range of utilities of $\$7$
- Estimation idea:
 - **EU holds** $\Rightarrow \underline{v}_7 = \bar{v}_7$
 - **EU fails** $\Rightarrow \underline{v}_7 < \bar{v}_7$ (preference imprecision)

- **Mixture model** with 3 groups (number determined through model selection)
- Estimate one range of utilities $[\underline{v}_7, \bar{v}_7]$ for each group
- Show the implications of the estimated ranges of utilities in the dataset

FRACTION OF AMBIGUOUS RANKINGS. 1/3 IN THIS EXAMPLE



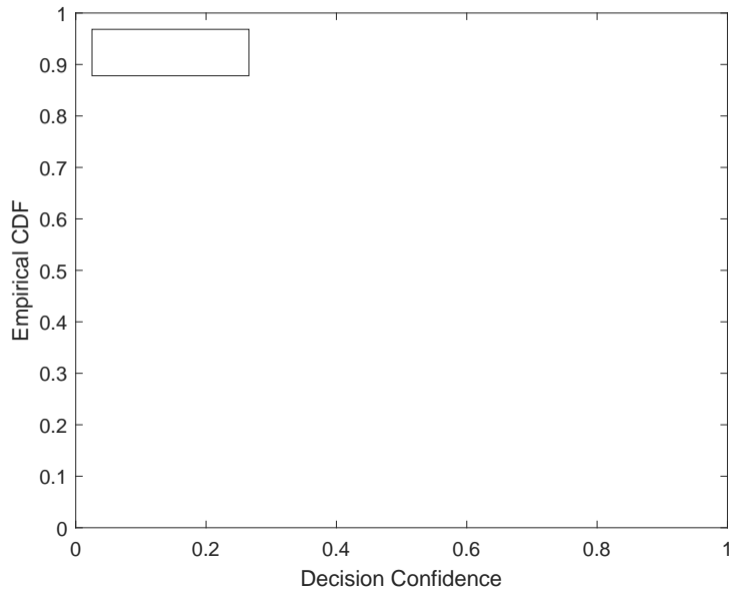
PERCENTAGES OF AMBIGUOUS RANKINGS - MIXTURE MODEL WITH 3 GROUPS



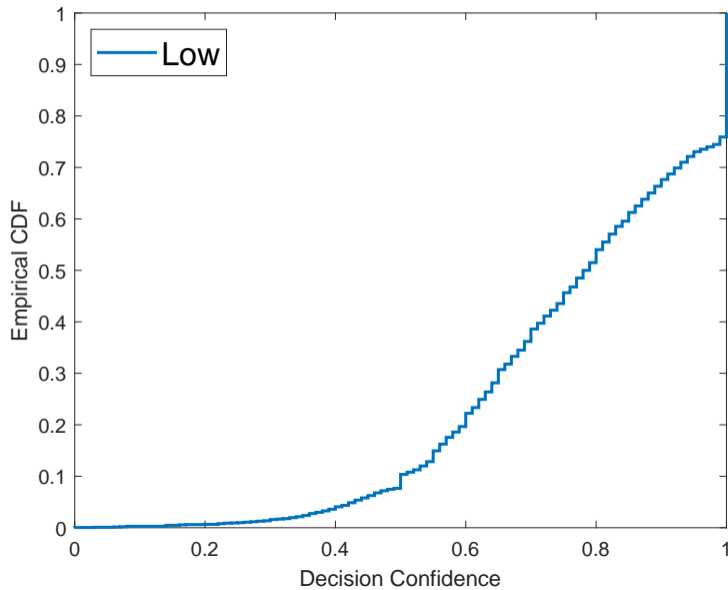
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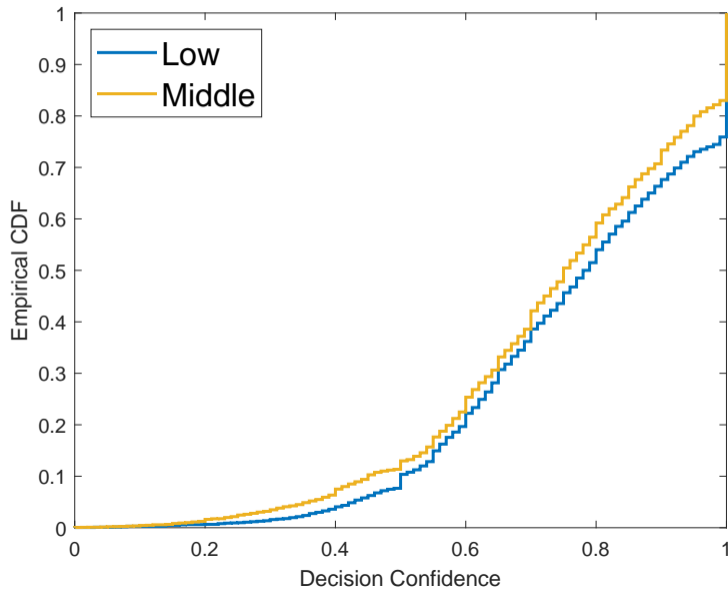
CONFIDENCE DISTRIBUTION IN THE 3 GROUPS



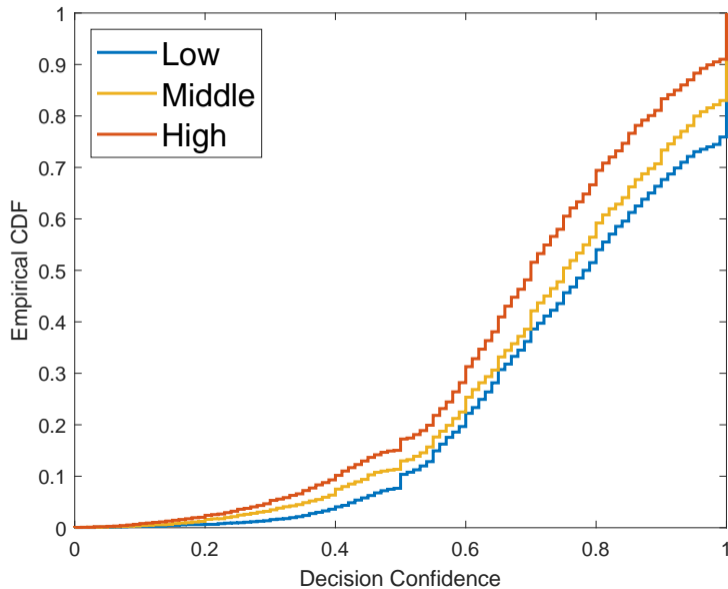
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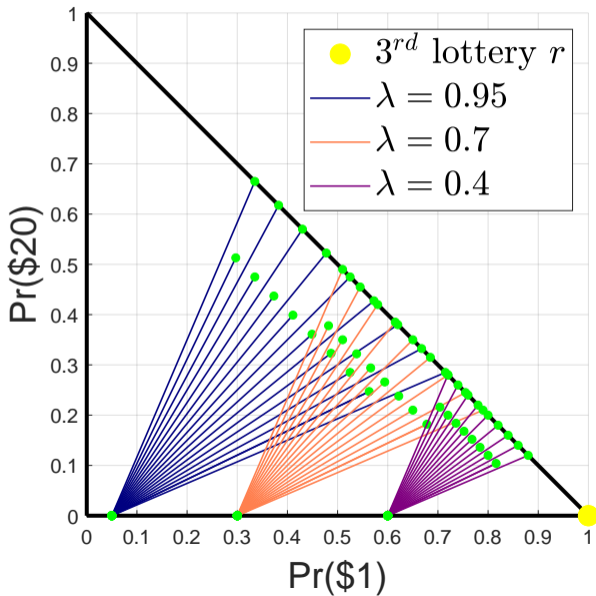
⇒ Lack of decision confidence is a proxy for **preference imprecision**

CONCLUSION

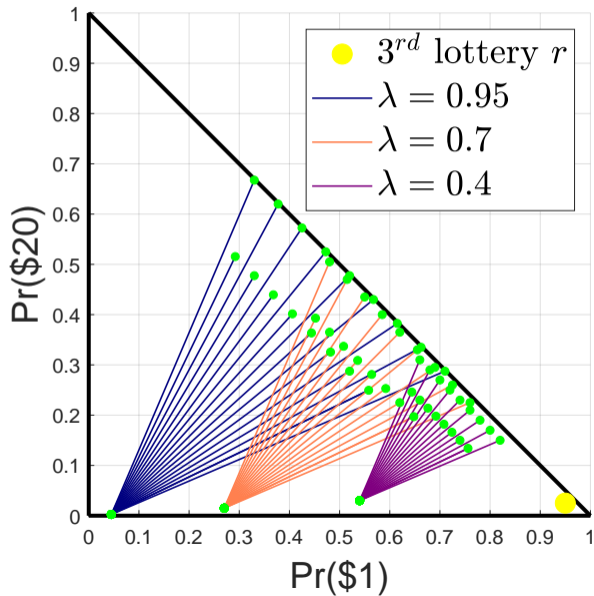
1. Individuals are **more likely to choose risky** over certain options when **not confident** about their choices
 2. When individuals choose without being confident, they are more likely to violate EU
- ⇒ These findings suggest a possible rationale for the **RCRE**, which is the most frequent EU violation
3. The correlation between confidence and non-EU violations is consistent with theories of **preference imprecision**
- ⇒ Individuals with higher preference imprecision tend to report lower confidence levels

THE END

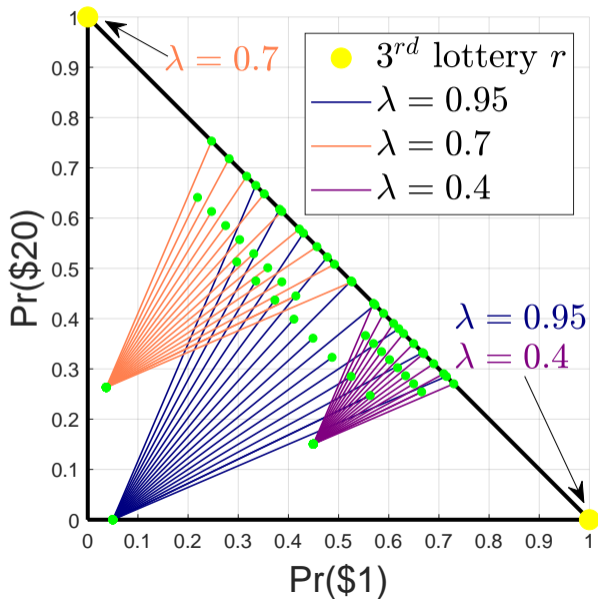
“WORST” MIXED CHOICE TASKS



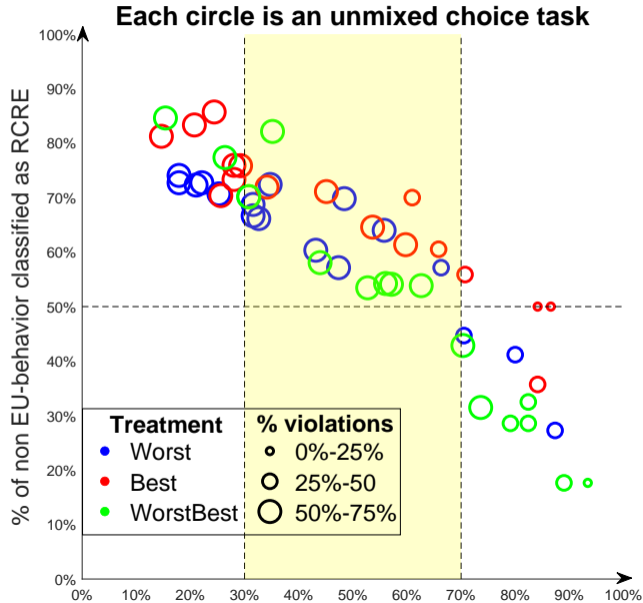
"BAD" MIXED CHOICE TASKS



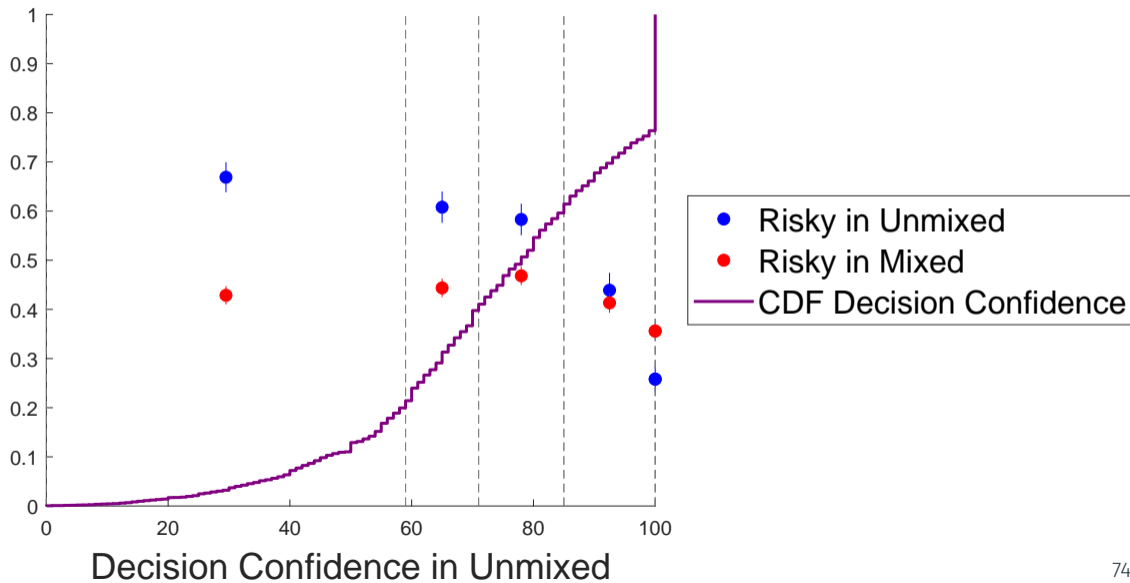
“WORSTBEST” MIXED CHOICE TASKS



COMPARE CRE AND RCRE ACCOUNTING FOR NOISE



EQUAL SIZE PARTITION



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