## Persuasion in Random Networks

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How and when to exploit the network rather than public communication? Is the existence of network detrimental to information?



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- *Is the existence of network detrimental to information?* 
  - $\rightarrow\,$  Existence of vulnerable network might be the lesser of two evils

## Literature

Bayesian Persuasion

- Seminal works: Kamenica and Gentzkow (2011)
- Heterogenous unconnected receivers: Innocenti (2021)
  - $\rightarrow$  *Introduce*: networks
- Homogenous connected receivers:
  - With voting quota: Kerman and Tenev (2021)
  - ▶ For general games: Galperti and Perego (2019)
  - → *Introduce*: polarization & random networks

Information design

- link with Bayesian persuasion: Bergemann and Morris (2019)
  - $\rightarrow$  *Introduce*: endogeneity of publicness of signals
- link with network: Egorov and Sonin (2020), Candogan (2019)

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- Classical Bayesian persuasion:
  - A sender wants to induce receivers to take some *favorable* action
  - N receivers want to match a payoff-relevant state
  - $\rightarrow$  Sender commits to signal structure conditional rate of success (and correlation)

### Setup "Classical" Unique Strategy

- SoW  $\omega \in \{0,1\}$ , common prior  $\Pr(\omega = 1) = \mu$
- Sender sets:

	$\omega = 0$	$\omega = 1$
s=0	1-p	1-q
s=1	р	q

• Receivers' posterior after s = 1:  $\beta(1) = \Pr(\omega = 1 | s = 1) = \frac{q\mu}{q\mu + (1-\mu)\rho}$  $\rightarrow$  Assume  $a^*(\beta(s)) = 1 \Leftrightarrow \beta(s) \ge t$ 

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- Key insight: p, q such that  $\beta(1) = t \Rightarrow q^* = 1; p^* = \frac{\mu(1-t)}{t(1-\mu)} =: \alpha$ 
  - $\Rightarrow$  Persuasion payoff:  $V = \mu + (1 \mu)\alpha$

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  - Two groups (denoted A and B) defined by their priors
  - Persuade one group without dissuading the other

### Setup Heterogenous Agents: "Hard News" Strategy

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- Key insight: p, q such that  $\beta_B(1) = t \Rightarrow q^* = 1; p^* = \frac{\mu_B(1-t)}{t(1-\mu_B)} =: \alpha_B$ 
  - $\Rightarrow$  Persuasion payoff:  $V = \mu + (1 \mu)\alpha_{B}$



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- $\rightarrow$  Add a communication network
  - ► Exogenous and random (→ degree distribution)
  - Exogenous communication of signals -i observes his and his neighbor's signals.

#### An easy example

Assume:

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- $\delta_A(d_i = 1) = 1$  and  $\delta_B(d_i = 2) = 1$



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$$V_{MM} = \mu + (1 - \mu) \left[ p^2 + \frac{a}{2} 2p(1 - p) \right] \approx 13/16$$



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- Costs decrease with within-B degree homogeneity
- Costs do not increase with connectivity
- Benefits increase with *a* and costs increase with  $\alpha_B$  and  $\mu$
- $\rightarrow$  Seggregation and extreme belief make the network more vulnerable

### Setup Heterogenous Agents: "Soft News" Strategy

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  - $\Rightarrow$  Persuasion payoff:  $V = \mathbf{a} + \mathbf{b}[\mu + (1 \mu)\alpha_B]\mathbf{q}^*$

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Generalization of results require considering more strategies. Most important:

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- Informative signals only to  $B \rightarrow$  optimal with "general" SN strategy and > 2 signals?
  - Move towards optimization over space of posteriors?
  - How to link posteriors space of agents with different degrees?

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