Sentiment, Mispricing and Excess Volatility in Presence of Institutional Investors

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Sentiment trading in highly institutionalized markets

Q: Effect of sentiment on asset prices in highly institutionalized markets?

- Sentiment: Investors' irrational optimism/pessimism
- Institutionalization: Participation of financial institutions in asset markets

- What about sentiment/institutions?
  - Sentiment-driven trading can differ substantially from rational paradigm
    - Deviations can be long lasting, even in frictionless markets
    - Economically relevant price impact $\Rightarrow$ deviation of prices from fundamentals
    - More or less temporarily, depending on limits to arbitrage
  - Institutions' portfolios can also differ from standard prescription
    - Even if fully rational and sophisticated
    - Possibly, in response to shareholders' investment constraints
    - Benchmarking concerns, risk-limiting provisions, etc.

▷ Do institutions help correct or worsen sentiment-induced AP distortions?

Motivation: Why care?

- (Retail) investor sentiment important to explain asset pricing puzzles
  - e.g., IVOL anomaly, momentum and value effects, etc.
  - Beliefs biases in portfolio decisions are significant and long lasting
    - Giglio et al. (2021)
- Institutions arguably single most important investor class in today's markets
  - e.g., 13F institutions manage 68% of the US stock market
    - Koijen & Yogo (2019)
  - Even higher percentage of daily equity trading volume (think, e.g., of HFTs)
  - Subject to economically significant investment constraints
    - De Vault et al. (2019), Cao et al. (2017)
    - Possibly driven by agency frictions, asymmetric info, etc.
  - ▷ Our focus: Relative performance concerns w.r.t. a benchmark index

Economy

- Continuous-time pure-exchange economy
- Finite (typically, short) investment horizon T
- Financial markets:
  - 1 **riskfree** asset (cash) in zero net supply
    - ⋆ Payoff normalized to 1 at T
  - 1 **risky** asset (a stock market portfolio) in unit supply
    - ⋆ Pays dividend D_T (only) at T
    - ⋆ Cash flow "news" D_t arrive continuously over $t \in [0, T]$ according to

$$dD_t = D_t(\mu dt + \sigma dB_t), \tag{1}$$

      where

      ▷ μ, σ are constant, and

      ▷ dB_t is a Wiener process under the actual probability $\mathbb{P}$
  - No frictions: no leverage/short-sale constraints, taxes, etc.

Preferences

- Log preferences over final wealth: for agent k,

$$J_k(W_t^k) \quad = \quad \max_{\theta^k} \; E_t^k[u_k(W_T^k)],$$

- Two types of agents: $k \in \{R, I\}$, with

$$u_k(W_T^k) = \begin{cases} \log W_T^k, & k = \text{Retail investors}, \\ (1 - \upsilon + \upsilon Y_T) \log W_T^k, & k = \text{Institutional investors}. \end{cases}$$

- I investors:
  - have otherwise standard preferences, except that
  - they have relative performance concerns $(1 - \upsilon + \upsilon Y_T)$, $\upsilon \in [0, 1)$
  - w.r.t. a benchmark index Y
  - ▷ Basak & Pavlova (2013)'s reduced-form approach to benchmarking concerns

- Utility max problem subject to standard self-financing constraint

Beliefs

- R-investors potentially feature "sentiment"
  - Belief that the mean growth rate of D is equal to $\mu^R = \mu + \sigma^2 \delta^R$
  - δ^R reflects degree of optimism / bullish sentiment about the market
    - ⋆ with $\delta^R < 0$ reflecting pessimism / bear sentiment
  - Their "perceived" dynamics for the dividend process is:

$$dD_t = D_t \left((\mu + \sigma^2 \delta^R) dt + \sigma dB_t^R \right),$$

where $dB_t^R = dB_t - \sigma \delta^R dt$ is the increment of a Wiener process under $\mathbb{P}^R$

- I-investors are fully rational (no sentiment)
  - i..e, they see the actual dividend dynamics (1)

Equilibrium Characterization

- Equilibrium prices and allocations depend on:
  - The share of aggregate wealth in I-investors' hands:

$$\varpi_t^I \triangleq \frac{W_t^I}{W_t^I + W_t^R}$$

  - The strength of institutions' benchmarking concerns:

$$q_t \triangleq \frac{\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}, \qquad q_t \in [0,1),$$

  - The relative strength of R's sentiment over I's benchmarking concerns

$$\delta^R - q_t$$

- **Goal**: compare equilibrium in this economy w.r.t. equilibria in

  STD: A standard rational non-institutionalized economy: $\delta^R = 0, \upsilon = 0$

  BP: A rational institutionalized economy: $\delta^R = 0$ (Basak & Pavlova, 2013)

  SENT: A non-institutionalized economy: $\upsilon = 0$

Equilibrium price-dividend ratio and market price of risk

- Proposition: The equilibrium price-dividend ratio and market price of risk are:

$$S_t/D_t = \overline{(S/D)}_t \frac{1}{\varpi_t^I(1-\gamma(T-t)q_t) + (1-\varpi_t^I)(1-\gamma(\delta^R(T-t)))},$$

$$\kappa_t = \bar{\kappa}\left(1 - \frac{\varpi_t^I(1-\gamma(T-t))q_t + (1-\varpi_t^I)(1-\gamma(\delta^R(T-t)))\delta^R}{\varpi_t^I(1-\gamma(T-t)q_t) + (1-\varpi_t^I)(1-\gamma(\delta^R(T-t)))}\right),$$

  where

  - $\gamma(x) < 1, \gamma'(x) > 0$
  - $\overline{(S/D)}_t \triangleq (S_t/D_t)|_{\delta^R=0,\upsilon=0}$
  - $\bar{\kappa} \triangleq \kappa_t|_{\delta^R=0,\upsilon=0} = \sigma$

- Both bullish sentiment and benchmarking concerns have similar
  - **boosting** effect on prices
  - **depressing** effect on the market price of risk

Equilibrium portfolio allocations and return volatility

- Proposition: The equilibrium portfolio weights in the stock are:

$$\theta_t^R = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}} \delta^R,$$

$$\theta_t^I = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}} q_t,$$

where the equilibrium stock return volatility, $\sigma_{S,t}$, is:

$$\sigma_{S,t} = \bar{\sigma}_S \left(1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi_t^I)\big(\gamma(\delta^R(T-t)) - \gamma(T-t)q_t\big)(\delta^R - q_t)}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)} \right),$$

$$\bar{\sigma}_S \triangleq \sigma_{S,t}|_{\delta^R=0,\upsilon=0} = \sigma$$

- Portfolio allocations in the stock are the sum of:
  - a **mean-variance** (MV, standard) component $\kappa_t/\sigma_{S,t}$
  - a **non-standard feature-specific** component:
    - $\sigma/\sigma_{S,t}\delta^R$: sentiment-driven demand wedge
    - $\sigma/\sigma_{S,t}q_t$: Institution's hedging demand

Excess volatility decomposition

- Lemma: The equilibrium stock return volatility can be decomposed as:

$$\sigma_{S,t} \quad = \quad \varepsilon^{D}_{S,t}\sigma + \varepsilon^{q}_{S,t}\sigma_{q,t} + \varepsilon^{\varpi^I}_{S,t}\sigma_{\varpi^I,t},$$

where $\varepsilon^{x}_{S,t} = \frac{\partial S_t}{\partial x_t} \times \frac{x_t}{S_t}$, $\sigma_{q,t} = (1 - q_t)\sigma$, $\sigma_{\varpi^I,t} = -(1 - \varpi^I_t)(\delta^R - q_t)\sigma$, and

$$\varepsilon^{D}_{S,t} \quad = \quad 1,$$

$$\varepsilon^{q}_{S,t} \quad = \quad \frac{\varpi^I_t \gamma(T-t) q_t}{\varpi^I_t(1-\gamma(T-t)q_t) + (1-\varpi^I_t)(1-\gamma(\delta^R(T-t)))} > 0,$$

$$\varepsilon^{\varpi^I}_{S,t} \quad = \quad \frac{\gamma(T-t)q_t - \gamma(\delta^R(T-t))}{\varpi^I_t(1-\gamma(T-t)q_t) + (1-\varpi^I_t)(1-\gamma(\delta^R(T-t)))}\varpi^I_t.$$

- The excess volatility ratio (w.r.t. STD case) is:

$$EVR_t \quad \triangleq \quad \sigma_{S,t}/\bar{\sigma}_S - 1 = \Psi_{q,t} + \Psi_{\varpi^I,t},$$

where:

Benchmarking channel	$\Psi_{q,t} =$	$\varepsilon^{q}_{S,t}(1 - q_t) > 0,$
Relative-wealth channel	$\Psi_{\varpi^I,t} =$	$-\varepsilon^{\varpi^I}_{S,t}(1 - \varpi^I_t)(\delta^R - q_t).$

Excess volatility decomposition: Interpretation

- Benchmarking concerns amplify the effect of CF news on returns
  - By creating a positive feedback from prices to stock demand
  - $\Psi_{q,t}$ increases with δ^R

  $\Rightarrow$ Institutions' amplification of EV greater when trading with bullish R-investors

- Relative-wealth effects on EV intensify with portfolio heterogeneity
  - No relative-wealth channel in markets with no investor heterogeneity

$$\Psi_{\varpi^I,t} \cong \frac{\sigma^2(T-t)}{1-(\varpi_t^I q_t + (1-\varpi_t^I)\delta^R)\sigma^2(T-t)}\varpi_t^I(1-\varpi_t^I)(\delta^R - q_t)^2$$

  - $\Psi_{\varpi^I,t}$ increases with both q_t and δ^R
  - As long as not $q_t \approx \delta^R$!

- Both channels affected by q and δ^R
  - Complex interaction effects

Relative wealth channel: Simple illustration

Homogeneous portfolios

- Assume: $S_t = 1 \to S_{t+1} = 1.2$
  - i.e., $r_{t,t+1} = 0.2$

	ϖ_t^k	W_t^k	θ_t^k	$1-\theta_t^k$	Stock Demand$_t$ (\$)	W_{t+1}^k	ϖ_t^k	θ_{t+1}^k	$1-\theta_{t+1}^k$	Stock Demand$_{t+1}$ (\$)
R	0.50	0.50	1.00	0.00	0.50	0.60	0.50	1.00	0.00	0.60
I	0.50	0.50	1.00	0.00	0.50	0.60	0.50	1.00	0.00	0.60
	1.00	1.00			1.00	1.20	1			1.20

- Aggregate stock demand (in \$) grows by 0.2

Relative wealth channel: Simple illustration

Heterogeneous portfolios

- Assume: $S_t = 1 \rightarrow S_{t+1} = 1.2$
  - i.e., $r_{t,t+1} = 0.2$

	ϖ_t^k	W_t^k	θ_t^k	$1-\theta_t^k$	Stock Demand$_t$ (\$)	W_{t+1}^k	ϖ_t^k	θ_{t+1}^k	$1-\theta_{t+1}^k$	Stock Demand$_{t+1}$ (\$)
R	0.50	0.50	1.15	-0.15	0.58	0.62	0.51	1.15	-0.15	0.71
I	0.50	0.50	0.85	0.15	0.43	0.59	0.49	0.85	0.15	0.50
	1.00	1.00			1.00	1.20	1			1.21

- Aggregate stock demand (in \$) grows by 0.21

Relative wealth channel: Simple illustration

Even more heterogeneous portfolios

- Assume: $S_t = 1 \rightarrow S_{t+1} = 1.2$
  - i.e., $r_{t,t+1} = 0.2$

	ϖ_t^k	W_t^k	θ_t^k	$1-\theta_t^k$	Stock Demand$_t$ (\$)	W_{t+1}^k	ϖ_t^k	θ_{t+1}^k	$1-\theta_{t+1}^k$	Stock Demand$_{t+1}$ (\$)
R	0.50	0.50	1.30	-0.30	0.65	0.63	0.53	1.30	-0.30	0.82
I	0.50	0.50	0.70	0.30	0.35	0.57	0.47	0.70	0.30	0.40
	1.00	1.00			1.00	1.20	1.00			1.22

- Aggregate stock demand (in \$) grows by 0.22

Comparison of BP and SENT reference economies

- Benchmarking concerns and bullish sentiment can be observationally similar
  - But not equivalent
- Lemma: At any given horizon $T - t$, the degree of optimism

$$\delta^R = \check{\delta}_t^R = \frac{1}{\sigma^2(T-t)} \log \frac{1}{1-\gamma(T-t)q_t} > 0.$$

  leads to identical stock price-dividend ratios across the BP and SENT cases

- Return volatility and MPR, however, are always higher under the BP case
  - Not only the benchmarking channel is 0 in the SENT case
  - But also, for low enough ϖ_t^R, $\Psi^{SE}_{\varpi^I,t} < \Psi^{BP}_{\varpi^I,t}$

General (GE) case: Sentiment meets Institutions

- Interaction of sentiment and benchmarking concerns important to assess
  1. Effect of sentiment on excess vol in institutionalized markets
  2. Effect of institutionalization on prices in sentiment-driven economy
- Link to relevant empirical problems
  1. Can irrational noise be empirically associated to excess volatility?
     - Answer key to interpretation of Volatility Ratio (VR) tests
  2. Will continuation of recent institutionalization trends worsen mispricing?
     - As conjectured by DeVault et al. (2019)

1. Excess volatility

- Proposition: $\exists' \hat{\delta}^R(D_t, \varpi^I_t, T-t) > \check{\delta}^R_t > 0$ such that:

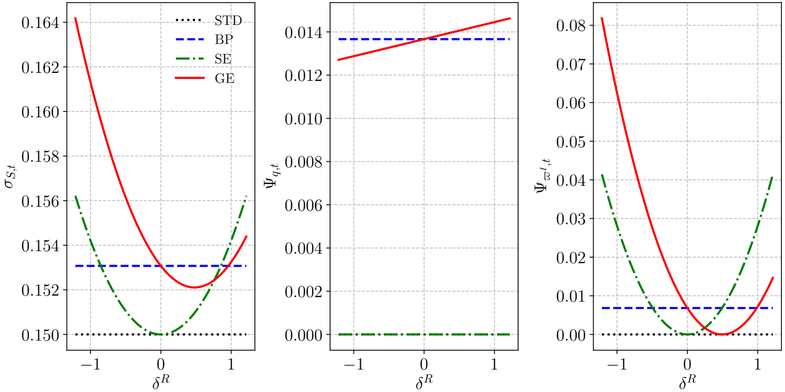
$$\frac{\partial \sigma_{S,t}}{\partial \delta^R} \begin{cases} > 0, & \delta^R > \hat{\delta}^R(D_t, \varpi^I_t, T-t) \\ = 0, & \delta^R = \hat{\delta}^R(D_t, \varpi^I_t, T-t) \\ < 0, & \delta^R < \hat{\delta}^R(D_t, \varpi^I_t, T-t) \end{cases} .$$

- In particular, for $0 < \delta^R < \check{\delta}^R_t$, higher optimism
  - *reduces* the stock return volatility
  - across all wealth distributions ϖ^I_t
  - relative to a rational institutionalized (BP) economy

∴ Link between irrationality and excess vol "broken" in institutionalized markets

  - Key prediction of noise/sentiment risk model
    - ★ De Long et al. (1990), Dumas et al. (2009)
  - Important to associate excess vol in VR tests to irrationality/mispricing
    - ★ e.g., Shiller (1979, 1981), Giglio & Kelly (2018)

1. Excess volatility: Illustration

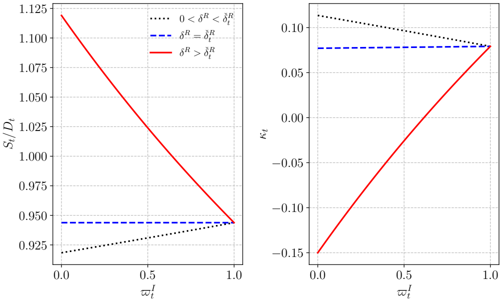


- Effect arises purely from relative-wealth channel
  - As sentiment rises and tends to offset I's demand for the stock
- Asymmetric effect of sentiment on (excess) vol

1. Effect of institutionalization

- Lemma: In presence of both R- and I-investors,

$$\frac{\partial (S/D)_t}{\partial \varpi^I_t} \gtreqless 0 \Leftrightarrow \delta^R \lesseqgtr \check{\delta}^R_t$$



- DeVault et al. (2019)'s conjecture holds only for low levels of sentiment
- For high enough δ^R, institutions always push prices closer to fundamentals

Dynamics: Interaction of cash flow and wealth effects

- So far, effects are mostly static
- As the economy unfolds, CF shocks endogenously change ϖ_t^I
- Lemma: In presence of both R- and I-investors,

$$\frac{\partial(\varpi_t^I)}{\partial D_t}\begin{cases} > 0, & \delta^R < q_t \\ = 0, & \delta^R = q_t \\ < 0, & \delta^R > q_t \end{cases} .$$

- Positive CF news *decrease* the I-investors' share of aggregate wealth if:
  - R-investors are sufficiently bullish
  - The market is already in a low state D_t
  - Benchmarking weight υ in I's preference is sufficiently low

prices and MPR

Dynamics of relative-wealth effect: Simple illustration

Heterogeneous (fixed) portfolios

- Assume: $S_t = 1 \rightarrow S_{t+1} = 1.2$
  - i.e., $r_{t,t+1} = 0.2$

	ϖ_t^k	W_t^k	θ_t^k	$1-\theta_t^k$	Stock Demand$_t$ (\$)	W_{t+1}^k	ϖ_t^k	θ_{t+1}^k	$1-\theta_{t+1}^k$	Stock Demand$_{t+1}$ (\$)
R	0.50	0.50	1.30	-0.30	0.65	0.63	0.53	1.30	-0.30	0.82
I	0.50	0.50	0.70	0.30	0.35	0.57	0.47	0.70	0.30	0.40
	1.00	1.00			1.00	1.20	1.00			1.22

- Aggregate stock demand (in \$) grows by 0.22

Dynamics of relative-wealth effect: Simple illustration

Less heterogeneous next-period portfolios

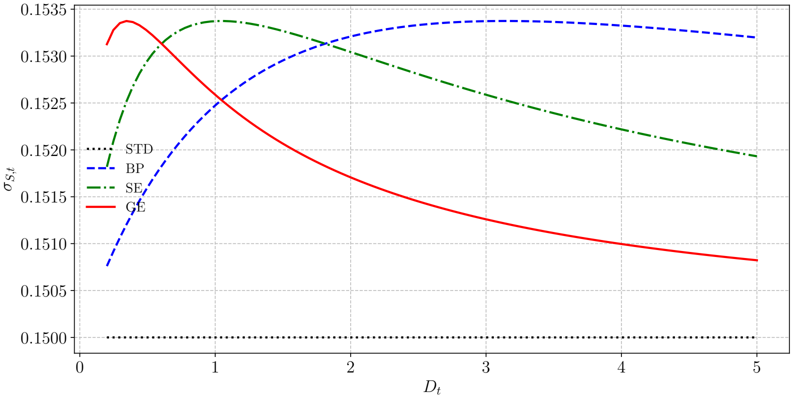
- Assume: $S_t = 1 \rightarrow S_{t+1} = 1.2$
  - i.e., $r_{t,t+1} = 0.2$

	ϖ_t^k	W_t^k	θ_t^k	$1-\theta_t^k$	Stock Demand$_t$ (\$)	W_{t+1}^k	ϖ_t^k	θ_{t+1}^k	$1-\theta_{t+1}^k$	Stock Demand$_{t+1}$ (\$)
R	0.50	0.50	1.30	-0.30	0.65	0.63	0.53	1.10	-0.10	0.69
I	0.50	0.50	0.70	0.30	0.35	0.57	0.47	0.90	0.10	0.51
	1.00	1.00			1.00	1.20	1.00			1.21

- Aggregate stock demand (in \$) grows by 0.21

Dynamics: Effect on volatility

$\delta^R = 1, \mu = 0, \sigma = 0.15, t = 1, T = 5, D_0 = 1, \upsilon = 0.5$



- For $\delta^R \geq 1$, I's trading with R- (instead of rational) investors lead to:
  - Counter-cyclical pattern in return volatility
  - Lower excess vol than in both BP and SENT cases

Dynamics: Practical Implications

1. In presence of high sentiment:
   - institutions can have a stronger depressing effect on volatility
   - than equivalently rational but non-institutional investors
   - despite their benchmarking concerns
2. In highly institutionalized markets:
   - sentiment need not create "excess volatility"
   - but substantially reduce it

∴ Importance of distinguishing degree of institutionalization of markets
   - e.g., if degree is high:
     - ⋆ High P/D ratio + Low excess vol ⇒ High sentiment
     - ⋆ High P/D ratio + High excess vol ⇒ Low sentiment

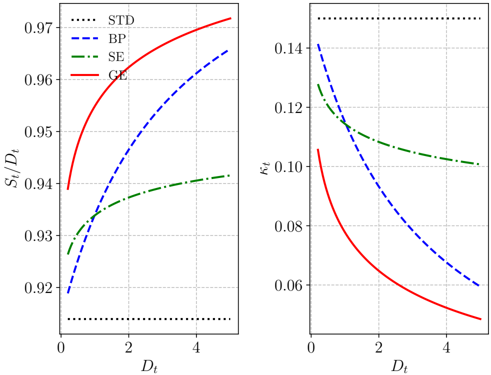
Conclusions

- Complex joint effect of sentiment and benchmarking concerns on volatility
  - Can be very different from the addition of the individual effects
  - Relative-wealth channel can attenuate transmission of shocks to prices
- Rich implications for the levels and dynamics of volatility
  - Excess vol can decrease with the level of investor optimism
  - Can fall below the levels prevailing in pure rational and sentiment driven cases
  - Lead to countercyclical patterns in vol
    - Consistent with empirical evidence
- Role of institutionalization of markets as correcting force is ambiguous
  - Institutions worsen overpricing created by low-to-moderate optimism
  - But help correct the severe mispricing created by "exuberant" beliefs
  - Benchmark-related pressure on stock prices is positive but bounded

Dynamics: Prices and MPR

$\mu = 0, \sigma = 0.15, t = 1, T = 5, D_0 = 1, \upsilon = 0.5, \delta^R = \check{\delta}_0^R = 0.486$

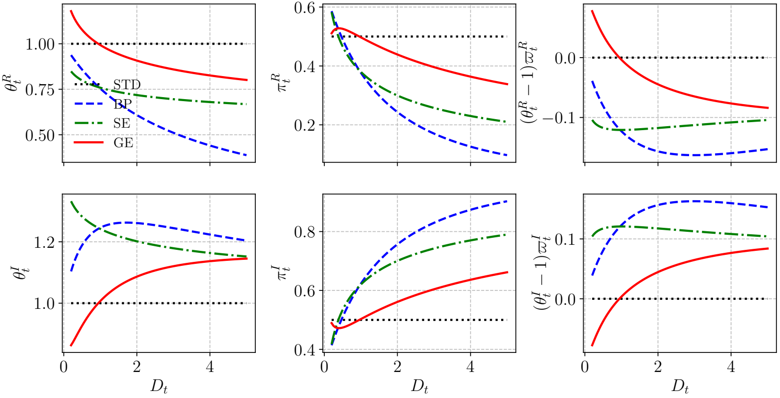
vol fig



- After positive shocks $dB_t > 0$ in the BP (SENT) economy
  - The $I(R)$-investor becomes wealthier
  - Stock prices increase and the MPR falls (cyclical / counter-cyclical patterns)
- Effects in GE economy combine ($\approx$ linearly) those in the BP and SENT cases

Dynamics: Associated portfolios

- Opposite trading patterns for I and R investors across BP and SENT cases



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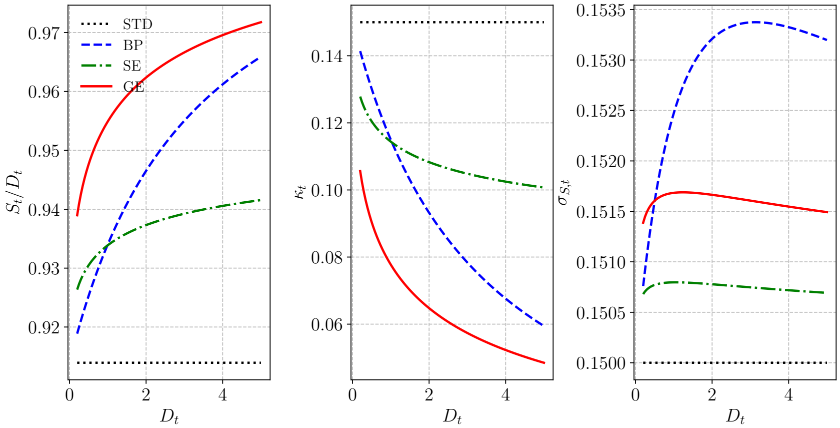
Dynamics: Prices and MPR

$\delta^R = \check{\delta}_0^R = 0.486, \mu = 0, \sigma = 0.15, t = 1, T = 5, D_0 = 1, \upsilon = 0.5$



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Reference economy 1: BP case

- Deviations of prices, MPR and volatility from STD case a function of $\varpi_t^I q_t$
  - "Benchmarked wealth"

$$(S/D)_t^{BP} \triangleq (S_t/D_t)|_{\delta^R=0} = \overline{(S/D)}_t \frac{1}{1-\gamma(T-t)\varpi_t^I q_t} \geq \overline{(S/D)}_t,$$

$$\kappa_t^{BP} \triangleq \kappa_t|_{\delta^R=0} = \bar{\kappa}\left(1 - \frac{(1-\gamma(T-t))\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}\right) \leq \bar{\kappa},$$

$$\sigma_{S,t}^{BP} \triangleq \sigma_{S,t}|_{\delta^R=0} = \bar{\sigma}_S\left(1 + \gamma(T-t)\frac{\varpi_t^I q_t(1-\varpi_t^I q_t)}{1-\gamma(T-t)\varpi_t^I q_t}\right) \geq \bar{\sigma}_S.$$

- Excess vol is: $EVR_t^{BP} = \Psi_{q,t}^{BP} + \Psi_{\varpi^I,t}^{BP}$, with:

$$\Psi_{q,t}^{BP} = \frac{\gamma(T-t)\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}(1-q_t) > 0,$$

$$\Psi_{\varpi^I,t}^{BP} = \frac{\gamma(T-t)\varpi_t^I(1-\varpi_t^I)q_t^2}{1-\gamma(T-t)\varpi_t^I q_t} > 0.$$

- In rational markets:
  - Greater institutionalization ϖ_t^I increases $\Psi_{q,t}^{BP}$
  - Benchmarking-induced EV always amplified by relative wealth channel

Reference economy 2: SENT case

- Bullish (bearish) investors over(under)weigh the stock in their portfolios
- Pushing stock prices up (down) and the MPR down (up) accordingly

$$(S/D)_t^{SE} \triangleq (S_t/D_t)|_{\upsilon=0} = \overline{(S/D)}_t \frac{1}{1-\varpi_t^R \gamma(\delta^R(T-t))},$$

$$\kappa_t^{SE} \triangleq \kappa_t|_{\upsilon=0} = \bar{\kappa}\left(1 - \frac{\varpi_t^R(1-\gamma(\delta^R(T-t)))\delta^R}{1-\varpi_t^R\gamma(\delta^R(T-t))}\right).$$

  - Deviations from STD equilibrium values increase monotonically with $|\delta^R|$

- **Both** bullish and bearish sentiment symmetrically increase return (excess) vol

$$\sigma_{S,t}^{SE} \triangleq \sigma_{S,t}|_{\upsilon=0} = \bar{\sigma}_S\left(1 + \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R\right),$$

$$EVR_t^{SE} \triangleq EVR_t|_{\upsilon=0} = \Psi_{\varpi^I,t}^{SE} = \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R.$$

  - Basis of excess volatility ratio tests of market irrationality
  - e.g., Shiller 1979, 1981; Giglio and Kelly 2018.

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