IMPLICIT CARBON PRICES
Making do with the taxes we have*

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Abstract

Climate and fiscal policy interact closely. The former imposes explicit prices for carbon emissions, while the latter affects emissions implicitly. We study the correspondence between explicit and implicit carbon pricing of a Ramsey-optimal fiscal policy in a neoclassical growth model of climate change. Our central result is that any arbitrary sequence of explicit carbon prices can be achieved implicitly through a blend of conventional taxes (e.g., consumption, energy, and income taxes), when lump-sum transfers are available. In a Ramsey setting, policy balances these taxes' traditional revenue-raising role with the Pigouvian role of fixing the climate externality. We characterize the Ramsey and Pigouvian components of optimal tax rates. We show that explicit carbon pricing is implicitly implementable through a mix of conventional taxes also in this framework. We extend these findings to scenarios compatible with net-zero emissions, adding carbon capture technologies and a cap on cumulative emissions.

Keywords: Fiscal Policy; Optimal Taxation; Tax Equivalence; Implicit carbon prices.
JEL: E6, E62, H21, Q54, Q4.

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1 Introduction

The direct way to reduce carbon emissions and fight climate change is to price emissions, via a carbon tax or a cap-and-trade system. Explicit policy measures, however, currently price about half of covered global emissions at less than US$10/tCO2e (World Bank, 2020) with the global average lying below US$2/tCO2e (Nordhaus, 2021). Such explicit carbon prices form only part of the effective carbon prices faced by emitters. Implicit carbon prices, i.e. the equivalent carbon price embodied in indirect policy measures, must be added to any explicit carbon pricing (Hoeller and Wallin, 1991; OECD, 2013). Effective carbon prices have been estimated empirically for over a decade, e.g., by the OECD's Effective Carbon Prices project, primarily via energy taxes and various demand elasticities.

Governments across the globe rely heavily on consumption, energy, and income taxes for several reasons, including raising revenue to fund public expenditure, addressing market distortions, and correcting primary income and wealth distributions. However, little is known about the implicit carbon prices imposed by these taxes. In this paper we take a first step towards understanding these effects by studying the correspondence between explicit and implicit carbon pricing in a neoclassical growth model of climate change.

The paper's central finding is an equivalence result: any competitive equilibrium involving an arbitrary explicit carbon price can also be implemented using only the implicit carbon prices embodied in other taxes and government revenue handed back in the usual lump-sum manner. This alternative fiscal policy combines energy, consumption, income taxes, and a renewable energy subsidy but no carbon taxes.

Our chief finding demonstrates, somewhat surprisingly, that the existing tax system is "complete", in the sense that it encompasses a sufficient number of independent taxes capable of influencing all relevant wedges, including those governing global carbon emissions, despite the absence of carbon markets and market incompleteness.

Governments rely on consumption and income taxes for achieving policy goals other than fighting climate change. We, therefore, also study policy in a second-best setting, in which the government must finance spending using distortionary taxation. Optimal tax rates are the sum of two elements: a revenue-raising "Ramsey" component and a carbon pricing "Pigouvian" one. Again, we show that the socially optimal allocation can be implemented by relying only on implicit carbon pricing and traditional taxes performing the Ramsey and Pigouvian tasks.

We illustrate some of our results with a simple quantitative example that we calibrate to match the optimal carbon tax in Golosov et al. (2014). The optimal carbon tax starts at \$56 per ton of carbon. We show that a fiscal package starting with an energy tax of 48%, a renewable subsidy of 92%, a consumption tax of 41%, a labor income tax of 41%, and a capital income tax of 1.7% is equivalent to such a carbon tax. Interestingly, the optimal policy mix resembles real-world policies for which governments have chosen alternatives to carbon taxes, such as renewable energy subsidies and fuel consumption taxes, as part of their climate policy strategies. Our policy mix for pricing carbon implicitly entails more instruments, as it also considers the distortions of energy taxes on energy markets, e.g., the intertemporal depletion decision of a scarce fossil resource.

The paper's findings apply the "principle of targeting", a concept in public finance which posits that the most effective approach to address a distortion is through a tax directly impacting the relevant margin. When a direct tax is unavailable, indirect taxes represent a second-best policy alternative. In the context of this economy, indirect taxes emerge as the first-best solution; however, it necessitates the implementation of several taxes to replicate the effects achieved by a single tax. Therefore, a version of the principle of targeting creates the need for multiple tax instruments to achieve the optimal outcome that a singular direct tax can provide. Yet, in illustrating the equivalence between explicit and implicit pricing schemes, we highlight the interactive effects between climate and broader fiscal policy.

Related literature. The paper contributes to the growing literature which applies findings from public finance on Ramsey taxation in general equilibrium (e.g., Chari and Kehoe (1999), Chamley (1986), and others) to taxation in a growing economy with a climate externality. It resembles recent applications of Schmitt (2014), Belfiori (2017), and Barrage (2018), who consider the problems of distortionary revenue-raising and fighting climate change jointly. Our paper is arguably closest related to Barrage (2020), who asks how carbon should be taxed as a part of fiscal policy and considers how carbon taxes should be adjusted to account for the inefficiency of existing taxes. Unlike those contributions, we focus on the case when the explicit carbon price is zero. A further contribution upon Barrage (2020) is that we characterize the optimal tax rates by decomposing them into the Ramsey and the Pigouvian components, showing the additive nature of both parts in an application of the general case proven in Sandmo (1975).

Auerbach (2018) extensively examines the equivalence of tax systems and their implications for policy design. While the existence of alternative ways for decentralizing an optimal allocation is well understood, the specific tax equivalence concerning an optimal carbon tax has yet to be explored. By applying the concept of tax equivalences to a carbon tax, the paper sheds light on the relevant, underlying economic margins and provides insights into the climate policies implied by taxes traditionally considered unrelated to climate.

This paper also relates to the literature examining the various politico-economic barriers to carbon pricing and policymakers' search for efficient policy alternatives. Politicians increasingly face popular opposition, like the Yellow Vest movement in France or the Dutch farmers' protests, when trying to introduce explicit climate policies. Carattini et al. (2018) analyze public opposition causing policymakers to avoid carbon taxes and instead prioritize alternative measures such as renewable energy subsidies or fuel consumption taxes. Sallee (2011) and Yokoyama et al. (2000) assess the efficiency of indirect taxes on fossil fuel consumption in the United States and Japan relative to direct taxation.

The paper is organized as follows. Section 2 sets up the basic model. Section 3 solves the social planning problem. Section 4 proposes a market economy with taxes. Section 5 demonstrates the equivalence between different policy mixes and characterizes optimal tax rates. Section 6 presents the results of the quantitative exercise. Section 7 extends our model to the cases of negative emission technologies and limits on cumulative emissions. Section 8 concludes, and the appendix presents all mathematical proofs and technical details.

2 Model

Consider the following global economy. Time is discrete and infinite, $t \in \{0, ..., \infty\}$. The economy is populated by a unit mass continuum of identical individuals. There is a single consumption good that is produced using capital, labor, and energy. There are four production units, the final good producers and the energy sectors indexed by $i = \{0, 1, 2, 3\}$. Capital, labor, and productivity in each sector are denoted K_{it}, N_{it}, and A_{it}. All production functions exhibit constant returns to scale and satisfy the Inada conditions. The production function for the final consumption good is given by

$$\tilde{F}(A_{0t}, N_{0t}, K_{0t}, E_{0t}) \tag{1}$$

where E_{0t} is an energy composite only used in the final good sector. There are three energy sources: an exhaustible resource (E_{1t}); an exhaustible but abundant resource (E_{2t}); and a clean energy source (E_{3t}). These sources can be thought of as oil/natural gas, coal, and renewables, respectively. The energy composite, E_{0t}, is defined as

$$E_{0t} = [\kappa_1 E_{1t}^{\rho} + \kappa_2 E_{2t}^{\rho} + \kappa_3 E_{3t}^{\rho}]^{1/\rho} \tag{2}$$

where $\sum_{i=1}^{3} \kappa_i = 1$, and the parameter ρ represents the elasticity of substitution between the energy components.

The energy components are produced according to sector specific technologies. Oil is exhaustible resource R_t and is costless to extract. At each point in time, oil use equals total oil extraction

$$R_{t+1} = R_t - E_{1t} \tag{3}$$

The economy starts with an initial stock of oil, R_0. Coal is also finite, but it is an abundant resource. Hence, there are no scarcity rents associated with coal use. Coal and renewable energy are produced using capital and labor according to the function

$$E_{it} = F_i(A_{it}, N_{it}, K_{it}) \quad \text{for i = 2, 3.} \tag{4}$$

Oil and coal use increases the stock of carbon in the atmosphere, S_t. We allow for separate carbon stocks with varying carbon dynamics, S_{jt}, with $S_t = \sum_j S_{jt}$ and j arbitrary. Carbon in container S_j evolves according to

$$S_{jt+1} = (1 - \gamma_j) S_{jt} + \varphi_j (E_{1t+1} + \phi E_{2t+1}) \tag{5}$$

where $\gamma_j \in [0, 1]$ is the rate of carbon dissipation and $\varphi_j \in [0, 1]$ the share of emissions entering container j with $\sum_j \varphi_j = 1$. The economy starts with a stock of carbon S_{j0}. The parameter ϕ captures the relative carbon intensity of coal and oil use with coal being typically more polluting. The stock of carbon in the atmosphere generates a climate externality that takes the form of an output loss. Thus, total output is given by

$$Y_t = F(S_t, A_{0t}, N_{0t}, K_{0t}, E_{0t}) = [1 - x(S_t)]\tilde{F}(A_{0t}, N_{0t}, K_{0t}, E_{0t}) \tag{6}$$

The damage function x is increasing, convex and twice differentiable with $\lim_{S \to \bar{S}} x'(S) = 0$, where $\bar{S}$ represents a lower bound on the atmospheric CO2 concentration. The amount of labor is exogenously given and can vary over time.

Individuals consume, work, and invest in capital. Capital depreciates fully within one period and the economy starts with a given stock, K_0. The feasibility constraint in this economy are given by

$$C_t + K_{t+1} + G_t = Y_t \tag{7}$$

for every period t, where $\{G_t\}_{t=0}^{\infty}$ is some exogenously given stream of government spending, together with

$$K_t = \sum_{i=0}^{3} K_{it} \tag{8}$$

$$\overline{N}_t \geq N_t = \sum_{i=0}^{3} N_{it} \tag{9}$$

for every period t, where $\overline{N}_t$ is the economy's labor endowment. Individuals derive utility from consumption and leisure, $\overline{N}_t - N_t$, and discount the future with the discount factor $\beta \in (0,1)$. Over time, individuals care about the value

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) - v(N_t)] \tag{10}$$

Utility functions u and v are increasing, concave, and twice differentiable with $lim_{C \to 0} u'(C) = \infty$ and $lim_{N \to 0} v'(N) = 0$.

3 Optimal Allocation

The *socially optimal allocation* is the path of consumption, labor, energy, capital, and carbon, $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0, i=0,1,2,3}^{\infty}$, that maximizes the welfare function (10) subject to the resource constraint (7), the carbon cycle (5) and the initial conditions K_0, R_0, and S_{j0}.

At an interior solution, two intertemporal conditions characterize the optimal allocation: the investment in physical capital and the oil depletion. In particular, the usual Euler equation holds for the capital investment decision:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = F'_{k,t+1} \tag{11}$$

where λ_t, the Lagrange multiplier on the feasibility constraint, is the social value of final output in period t. The optimality condition for oil reserves is given by

$$\beta \nu_{t+1} = \nu_t \tag{12}$$

where ν_t is the social value of oil reserves. Equation (12) is the Hotelling (1931) rule for this economy. Because output tomorrow (and consumption) can grow by either accumulating capital or reserves (i.e., by postponing extraction), the return on both assets must be the same so that there are no arbitrage opportunities. The combination of the equations (12) and (11) states this non-arbitrage condition holds at the optimum in terms of the resource rent, $\eta_t = \nu_t/\lambda_t$,

$$\frac{\eta_{t+1}}{\eta_t} = F'_{k,t+1} \tag{13}$$

At the intratemporal margin, the usual trade-off between leisure and consumption holds. That is, the marginal rate of substitution between consumption and labor equals the marginal rate of transformation

$$\frac{v'(N_t)}{u'(C_t)} = F'_{n,t} \tag{14}$$

Also, production efficiency requires that the marginal benefits, net of social costs, of employing labor and capital are equalized across productive sectors:

$$[F'_{E_2,t} - \phi\mu_t]F'_{2k,t} = F'_{k,t} \tag{15}$$

$$[F'_{E_2,t} - \phi\mu_t]F'_{2n,t} = F'_{n,t} \tag{16}$$

$$F'_{E_3,t}F'_{3k,t} = F'_{k,t} \tag{17}$$

$$F'_{E_3,t}F'_{3n,t} = F'_{n,t} \tag{18}$$

where μ_t is the social cost of carbon. The social cost of carbon comes from iterating forward on the optimality condition for the carbon stock, and it is equal to

$$\mu_t \equiv \sum_{h=0}^{\infty}\sum_{j}\varphi_j(1-\gamma_j)^h\beta^h\frac{\lambda_{t+h}}{\lambda_t}F'_{s,t+h} \tag{19}$$

The social cost of carbon measures the cost of the climate externality, which equals the output losses associated with burning an extra unit of oil in present value terms, given that we only consider damages to economic production. Efficient use of oil requires that the benefit of using an extra barrel in production equals its social cost.

$$F'_{E_1,t} = \eta_t + \mu_t \tag{20}$$

Finally, the transversality conditions for capital and oil reserves need to be satisfied. In particular, the stock of reserves or their social value approach zero at the planning horizon: $lim_{t\to\infty}\beta^t\nu_t R_t = 0$. The following section presents a decentralized environment that implements this optimal allocation with taxes.

4 Market Economy

In this section, we propose a decentralized economy with taxes. The set of tax instruments is said to be "complete" if it allows the government to affect the relevant economic decisions and also includes lump-sum taxes. Although it is easy to see that a Pigouvian carbon tax on carbon emissions alone would be enough to solve the climate externality, we allow for a complete set of tax instruments that includes carbon taxes, capital and labor income taxes, consumption taxes, energy taxes, and lump-sum taxes. Thus, all goods in the economy are subject to taxation. The goal is to explore the role that these different policy instruments can play in shaping climate policy when we later introduce restrictions on the instruments available to the government.

The final good's producer hires labor at a wage w_t, rents capital from households at rate r_t, and buys energy inputs from the energy firms at relative prices p_{it}. The problem of the firm is to choose the path of capital, employment, and energy use, $\{K_{0t}, N_{0t}, E_{1t}, E_{2t}, E_{3t}\}_{t=0}^{\infty}$, to maximize discounted profits given by

$$\Pi_0 = \sum_{t=0}^{\infty} q_t^0 [F(A_{0t}, S_t, N_{0t}, K_{0t}, E_{0t}) - r_t K_{0t} - w_t N_{0t} - \tau_t^e E_{0t} - \sum_{i=1}^{3} p_{it} E_{it}] \tag{21}$$

where τ_t^e is an energy tax, and energy is composed of oil, coal and renewables as defined in (2). Also, q_t^0 is the Arrow-Debreu price of one unit of consumption in period t in terms of consumption in period zero.

In the energy sector, a representative oil firm owns the stock of oil, operates the technology (3), and faces a carbon tax τ_t on the carbon content of oil extraction. The problem of the firm is to choose the path of oil extraction that maximizes the discounted profits given by

$$\Pi_1 = \sum_{t=0}^{\infty} q_t^0 (p_{1t} - \tau_t) E_{1t} \tag{22}$$

where p_{1t} is the price of oil in units of the consumption good, subject to the depletion equation (3) and the initial stock of oil reserves, R_0.

A representative firm in the coal sector ($i = 2$) operates the technology given by (4) and faces a carbon tax τ_t on the carbon content of coal production. The problem of the firm is to choose the inputs $\{N_{2t}, K_{2t}\}_{t=0}^{\infty}$ in order to maximize discounted profits given by

$$\Pi_2 = \sum_{t=0}^{\infty} q_t^0 [(p_{2t} - \phi\tau_t) E_{2t} - w_t N_{2t} - r_t K_{2t}] \tag{23}$$

Finally, the representative firm in the green sector ($j = 3$) operates the technology (4) and faces a per-unit tax equal to τ_t^r. Although it is natural to think about the tax on renewables as a subsidy, we define all instruments as taxes to keep notational consistency across sectors. As usual, a negative tax rate indicates a subsidy. The problem of the firm is to maximize discounted profits given by

$$\Pi_3 = \sum_{t=0}^{\infty} q_t^0 [(p_{3t} - \tau_t^r) F_3(A_{3t}, N_{3t}, K_{3t}) - w_t N_{3t} - r_t K_{3t}] \tag{24}$$

There is a continuum of mass one individuals, or a representative household, which derives utility from consumption and leisure. The representative household makes the capital investment decision and owns the firms. Consumers face a tax on consumption, labor income, and capital income. Therefore, households consume, work, and save subject to the following present value budget constraint

$$\sum_{t=0}^{\infty} q_t^0 [(1 + \tau_t^c) C_t + K_{t+1}] \leq \sum_{t=0}^{\infty} q_t^0 [(1 - \tau_t^k) r_t K_t + (1 - \tau_t^n) w_t N_t + T_t] + \Pi \tag{25}$$

where $\Pi = \sum_{j=0}^{1} \Pi_j$ are dividends from the firms, T_t is a lump-sum tax or rebate, and K_0 is the initial capital stock. The problem of the households is to choose a sequence $\{C_t, N_t, K_t\}_{t=0}^{\infty}$ to maximize (10) subject to (25), taking prices and taxes as given.

The government collects the tax revenue and rebates any surplus to households in a lump-sum fashion. Also, the government must finance an exogenous stream of spending. Thus, the government budget constraint is given by

$$\sum_{t=0}^{\infty} q_t^0 [\tau_t^n w_t N_t + \tau_t^k r_t K_t + \tau_t^c C_t + \tau_t (E_{1t} + \phi E_{2t}) + \tau_t^r E_{3t} + \tau_t^e E_{0t}] = \sum_{t=0}^{\infty} q_t^0 [T_t + G_t] \tag{26}$$

Definition 1 (Competitive equilibrium) *A competitive equilibrium given fiscal policy* $\{\tau_t^c, \tau_t^n, \tau_t^k, \tau_t^e, \tau_t^r, \tau_t, T_t, G_t\}_{t=0}^{\infty}$ *is a sequence of prices* $\{q_t^0, p_{it}, r_t, w_t\}_{t=0,i=1,2,3}^{\infty}$ *and an allocation* $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$ *such that: (i) given the fiscal policy and prices, the allocation solves the consumer's problem, maximizing (10) subject to (25), and the firms' problems, maximizing* Π_i *for i={0,1,2,3}; (ii) the government budget constraint (26) is satisfied; (iii) the carbon stock follows the carbon cycle (5); and (iv) prices clear markets.*

At an interior solution, profit maximization of the final good's producer implies that prices must satisfy

$$F'_{n,t} = w_t \tag{27}$$

$$F'_{k,t} = r_t \tag{28}$$

$$F'_{E_j,t} - \tau^e_t \alpha_{jt} = p_{jt} \tag{29}$$

for j=1,2,3 and $\alpha_{jt} = \partial E_{0t}/\partial E_{jt}$ from (2). Further, following the Hotelling rule, profit maximization for the oil extracting firm requires that the price of oil equals its social cost, $p_{1t} = \eta_t + \tau_t$ and that the return on oil extraction is the same across time so that

$$q^0_{t+1}(p_{1t+1} - \tau_{t+1}) = q^0_t(p_{1t} - \tau_t) \tag{30}$$

where the Arrow-prices satisfy

$$q^0_t = \beta^t \frac{u'(C_t)}{u'(C_0)} \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \tag{31}$$

and q^0_0 is normalized to 1. Plugging (31) and (29) for $j = 1$ into (30) we see that the return on oil is its marginal productivity net of the tax payment

$$\beta \frac{u'(C_{t+1})}{1+\tau^c_{t+1}}(F'_{E_1,t+1} - \tau^e_{t+1}\alpha_{1,t+1} - \tau_{t+1}) = \frac{u'(C_t)}{1+\tau^c_t}(F'_{E_1,t} - \tau^e_t \alpha_{1,t} - \tau_t) \tag{32}$$

In turn, profit maximization in the coal and renewable energy sectors implies that the following condition on prices must hold

$$(p_{2t} - \phi\tau_t)F'_{2k,t} = r_t \tag{33}$$

$$(p_{2t} - \phi\tau_t)F'_{2n,t} = w_t \tag{34}$$

$$(p_{3t} - \tau^r_t)F'_{3k,t} = r_t \tag{35}$$

$$(p_{3t} - \tau^r_t)F'_{3n,t} = w_t \tag{36}$$

On the consumer's side, the first order conditions for consumption and the capital stock imply a standard Euler equation

$$\beta \frac{u'(C_{t+1})}{1+\tau^c_{t+1}}[(1-\tau^k_{t+1})r_{t+1}] = \frac{u'(C_t)}{1+\tau^c_t} \tag{37}$$

Further, a no-arbitrage condition for the two assets arises from combining the equations (32) and (37) and establishes that the returns on oil and capital must be the same in equilibrium. Thus,

$$(1-\tau^k_{t+1})F'_{k,t+1} = \frac{F'_{E_1,t+1} - \tau^e_{t+1}\alpha_{1t+1} - \tau_{t+1}}{F'_{E_1,t} - \tau^e_t \alpha_{1t} - \tau_t} \tag{38}$$

Finally, the intratemporal trade-off between leisure and consumption implies that the marginal rate of substitution equals the relative prices so that

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1+\tau_t^c}{(1-\tau_t^n)w_t} \tag{39}$$

To summarize, the competitive equilibrium is fully characterized by two intertemporal conditions, equations (32) and (37), and the intratemporal conditions (33)-(36) and (39) together with the market clearing conditions (7)-(9), the transversality conditions for the stocks of capital and oil, and equilibrium prices satisfying (27)-(29) and (31).

In the next section, we explore what alternative set of taxes can implement the optimal allocation as the outcome of a competitive equilibrium.

5 Climate Policy

In this section, we show how explicit carbon pricing can be implemented implicitly using traditional taxes. We do so first in the standard Pigouvian setting, in which the climate externality is the only source of distortions. The government can return any revenue from carbon taxation in lump-sum transfers. We then rule out such transfers and add an exogenous stream of government spending that requires governments to raise revenue, financing spending in a distortionary manner.

5.1 Pigouvian taxes

The following proposition presents the main result of the paper. It establishes the equivalence between any sequence of carbon taxes with lump-sum transfers and an alternative decentralization which does not utilize the carbon tax at all. That is, the proposition establishes that, for any competitive equilibrium involving an explicit carbon tax, an equivalent competitive equilibrium with an implicit carbon tax exists. We emphasize that this carbon tax can follow any sequence. Importantly, it does not have to equal the social cost of carbon. A combination of the other taxes in our model can be just as effective as an explicit carbon tax in influencing the relevant economic decisions related to carbon emissions. The proof of this proposition is in the appendix.

Proposition 1 (Equivalence Result) *Let* $\Omega \equiv \{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$ *be a competitive equilibrium with an arbitrary fiscal policy* $\{\tau_t, T_t\}_{t=0}^{\infty}$ *with* $\tau_t = \tau_t^\star$ *for all* t*. Then* Ω

is also a competitive equilibrium with a fiscal policy given by

$$\tau_t^{e,Pigou} = \tau_t^\star \frac{\phi}{\alpha_{2t}} \quad ; \quad \tau_t^{r,Pigou} = -\tau_t^e \alpha_{3t} \tag{40}$$

$$\tau_t^{c,Pigou} = \frac{\tau_t^\star}{F_{E1,t} - \tau_t^\star}(1 - \phi\frac{\alpha_{1t}}{\alpha_{2t}}) \; ; \; \tau_{t+1}^{k,Pigou} = \frac{\tau_t^c - \tau_{t+1}^c}{1 + \tau_t^c} \quad ; \quad \tau_t^{n,Pigou} = -\tau_t^c$$

for every period t where $\alpha_{it} = \partial E_{0t}/\partial E_{it}$, $\tau_t = 0$, and any surplus rebated lump-sum through T_t.

In a competitive equilibrium with an explicit carbon tax ($\tau_t = \tau_t^\star$) emissions from oil and coal extraction are effectively regulated. With all other taxes set to zero, other key economic decisions, such as intratemporal decisions on consumption and labor, the capital savings margin, and renewable energy use, remain undistorted. The equivalence result shows that the fiscal policy of Proposition 1, composed of several taxes but no explicit carbon tax, can fully replicate the policy impact of carbon tax $\tau_t^\star$ on all relevant economic decisions, even without directly targeting the emissions margin.

In a competitive equilibrium with an implicit but no explicit carbon tax, the energy tax and the consumption tax are used to regulate emissions. The energy tax is set to match coal's carbon content according to the equivalence result in Proposition 1. Given that coal has the highest carbon content among energy sources, energy derived from renewables and oil extraction—both cleaner energy alternatives—is excessively taxed. Governments can implement a renewable subsidy with a decreasing consumption tax to undo this undesirable effect. The former directly targets and encourages renewable energy use, and the latter mitigates the excessive effects of the energy tax on oil extraction by discouraging future use in favor of current use. Oil use is brought back up to socially optimal level, but of course is lower than business-as-usual. Equation (32) shows this intertemporal effect of a consumption tax on the Hotelling rule.

In turn, the introduction of a consumption tax distorts the economy's saving rate. In this economy, two saving assets are present: oil and capital. While the policy effect of depleting oil reserves is desirable, given excessive energy taxation, an increase in the capital stock is not. Notice that a carbon tax does not distort the intertemporal wedge on capital investment. The introduction of a capital income tax preserves the undistorted nature of capital savings and maintains the non-arbitrage condition between the two assets. Lastly, the consumption tax also distorts the relative price relationship between consumption and labor. A labor income tax counteracts this distortion effectively.

Our result clearly illustrates the Principle of Targeting in Public Finance, which advocates for regulating economic activities through tools that directly affect the intended targets. Utilizing indirect taxes often leads to undesired distortions and runs the risk of resulting in inefficient outcomes. While Proposition 1 demonstrates that indirect taxation is still efficient, achieving this efficiency requires implementing a package of several taxes to replicate the complete impact of a single carbon tax. The multitude of taxes is necessary to undo the undesired distortions, which arise from indirect taxation of carbon emissions. The implicit pricing policy is more complex, as a result, than a straightforward, explicit carbon price. To reproduce an explicit carbon price, policymakers, however, only need knowledge of a handful of technological and carbon intensity coefficients.

Proposition 1 holds for any arbitrary carbon tax. The following proposition states the usual result that the socially optimal allocation can be decentralized by setting the carbon tax equal to the social cost of carbon, as described in equation (19). We draw on this result in a later section.

Proposition 2 (Optimal Climate Policy) *A competitive equilibrium Ω is socially optimal if*

$$\tau_t^\star = \mu_t \tag{41}$$

for all t.

With the externality associated with carbon emissions fully internalized, all other taxes become redundant and are set to zero (or alternatively the carbon tax is set to zero and all other taxes follow from Proposition 1 with $\tau^\star = \mu$), given that there are no government financing requirements or other externalities. For instance, the tax on renewable energy is rendered irrelevant as no externalities are associated with this particular productive sector.

The results in Proposition 1 and 2 illustrate how fiscal policy, even in the absence of explicit carbon taxes, effectively enforces carbon taxation in an implicit manner. Furthermore, pushing the extreme case of our model, these results highlight that certain certain fiscal policy combinations enable governments to forego implementing a carbon tax altogether while still achieving optimal climate policy.

In this section, we have denoted the set of optimal tax rates as “Pigou” to emphasize their role in solely internalizing a Pigouvian externality (Pigou, 1920). However, these taxes serve, in practice, as a vital source of government revenue in many countries. In the next section, we delve into implementing the socially optimal allocation within a Ramsey economy, where the government must generate revenue to fund its expenditures.

5.2 Ramsey taxes

We showed that, without a carbon tax, governments could rely on existing taxes to implement optimal climate policy. Governments, however, need to raise revenue to finance government spending using distortionary instruments. Therefore, it is important to explore taxes' ability to serve a double duty: curbing emissions and raising revenue.

We study this question taking the Ramsey approach to optimal taxation in that there is a stream of government spending and a tax system exogenously given. As before, we consider a wide range of taxes, but rule out lump-sum taxation. The overall optimal tax rates, in this case, display a combination of Pigouvian and Ramsey components. The "Pigouvian" part has been explored so far and captures the climate externality; the "Ramsey" part captures the government financing needs.

Given an exogenous stream of government spending, the Ramsey problem is to maximize social welfare, subject to two types of constraints. The first constraint is that taxes must finance the government spending when lump-sum taxes are not available; the second constraint is that taxes must induce an allocation that is a competitive equilibrium. Following the Ramsey tradition, the competitive equilibrium conditions are represented in the "Implementability constraint", which, together with the feasibility constraints, guarantees that the government's present value budget constraint holds. We show in the appendix that the implementability constraint for this economy takes the following form:

Proposition 3 (Implementability constraint) *Given the initial condition (K_0, R_0, S_{j0}), the allocation $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$ in a competitive equilibrium is fully characterized by the carbon dynamics (5), the market clearing conditions (7)-(9) and the following implementability constraint*

$$\sum_{t=0}^{\infty} \beta^t [u'(C_t)C_t - v'(N_t)N_t] = \frac{u'(C_0)}{1+\tau_0^c}[(F'_{k,0}(1-\tau_0^k)K_0 + (F'_{E1,0} - \tau_0^e \alpha_{1,0} - \tau_0)R_0] \qquad (42)$$

This implementability constraint differs from the typical one in that initial assets include the stock of oil reserves. As it is standard, we assume that taxation of the initial capital stock and the initial oil reserves is bounded above to avoid lump-sum taxation. All initial taxes are given.

Definition 2 (Ramsey allocation) *The* Ramsey allocation *is the solution to the Ramsey problem, which is to choose an allocation* $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$ *to maximize the welfare function (10) subject to the carbon cycle* (5), *the resource constraints* (7) − (9), *the implementability constraint (42), and the initial conditions* $\{K_0, R_0, S_{j0}, \tau_0^c, \tau_0^k, \tau_0^e, \tau_0\}$.

It will be useful to characterize the main results of this section against a business-as-usual benchmark. Such a business-as-usual economy corresponds to a Ramsey government that raises revenue using taxes, but does not seek any climate goal and takes the carbon stock dynamics (5) as given. In the following lemma, we label the optimal tax rates in a business-as-usual economy "Ramsey", as these taxes solely serve the revenue-raising motive. Let ϖ be the Lagrange multiplier on the implementability constraint, $EIS_t \equiv \frac{u'(C_t)}{-u''(C_t)C_t}$ the elasticity of intertemporal substitution, and $ELS_t \equiv \frac{v'(N_t)}{-v''(N_t)N_t}$ the elasticity of labor supply (Chari and Kehoe, 1999).

Lemma 1 (Ramsey taxes - Business as usual) *The Ramsey taxes in a business-as-usual economy are equal to*

$$\tau_t^{c,Ramsey} = \frac{\varpi(1 - 1/EIS_t)}{1 - \varpi(1 - 1/EIS_t)} \tag{43}$$

$$\tau_t^{n,Ramsey} = -\frac{\varpi(1 - 1/ELS_t)}{1 - \varpi(1 - 1/ELS_t)} \tag{44}$$

and $\tau_t^e = \tau_t^k = \tau_t^r = \tau_t = 0$ *for every period* $t \geq 1$.

The lemma reflects some well-known principles. A Ramsey government typically prefers to raise revenue with labor income taxes instead of capital income taxes. Consumption taxes are usually equivalent to capital income taxes and are redundant. However, in this economy, the Ramsey government uses consumption taxes instead of capital income taxes because there are two investment assets (oil and capital) and consumption taxes that drive a wedge in both investment decisions. Given taxes, the non-arbitrage condition between the two assets remains undistorted, which minimizes the distortions at the intertemporal margin. The manifestation of the Chamley-Judd result, that capital income taxation should be zero in the long run, in this economy is a constant consumption tax in the long-run. Non-constant consumption taxes distort much like capital income taxes in the standard neoclassical growth model with only one asset.

There are also special cases. In standard macro preferences, the elasticity of intertemporal substitution is constant and so is the Ramsey consumption tax. Moreover, the Ramsey consumption tax is zero if the elasticity equals one. In this case, the government must rely on labor income taxes and initial taxes on capital and oil reserves to meet its financing requirements.

This combination of Pigouvian and Ramsey problems follows the established additivity result of Sandmo (1975): the optimal tax rate is equal to the Pigouvian tax rate plus the Ramsey tax rate. The following proposition shows that a version of Sandmo's additivity results holds in this economy.

Proposition 4 (Ramsey taxes with a carbon tax) *Suppose that the Ramsey allocation is* $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$. *Then there exists a sequence of prices such that this allocation together with the prices constitute a competitive equilibrium with taxes equal to*

$$\tau_t = \mu_t$$

and

$$\tau_t^c = \tau_t^{c,Ramsey} \;\; ; \;\; \tau_t^n = \tau_t^{n,Ramsey}$$

and $\tau_t^e = \tau_t^r = \tau_t^k = 0$ *for* $t \geq 1$.

The result is the analog to Proposition 2 in a Ramsey setting. The proposition shows that the implementation of the Ramsey allocation requires taxes on labor income and consumption added to the Pigouvian carbon emissions tax. The tax formulas add the ones in Proposition 2 and Lemma 1. Of course, these formulas are endogenously defined in terms of the allocations and, hence, the actual numbers will be different.

The result closely relates to Barrage (2020) who shows that the optimal taxes in a climate-economy model with distortionary taxes include a carbon emissions tax coupled with capital and labor income taxes.[1] Because capital is the only asset in that paper, the Ramsey taxes include a capital income tax. The consumption tax in Proposition 3 is equivalent to the capital income tax in Barrage (2020). For standard macro preferences, we find that the consumption tax must be zero (or constant), similar to the findings in Barrage (2020) for the capital income tax. In this case, the Ramsey government raises revenue with labor income taxes and the Pigouvian carbon tax.

Barrage (2020) also studies constrained-efficient policy where the capital or labor income tax is exogenously fixed. Instead, we study the case of a carbon tax constrained to zero and

[1]Our economy differs from Barrage (2020) in that we include oil reserves as a saving asset and do not consider utility costs from climate change.

show again that it is possible to implement the optimal allocation through implicit carbon taxation. This result is remarkable because it shows that the tax system is "complete" in the sense that it contains enough independent taxes to affect all relevant wedges, including the ones that determine global carbon emissions in a setting where markets are incomplete, as there are no explicit carbon markets. The following proposition characterizes the result formally.

Proposition 5 (Ramsey taxes without a carbon tax) *Assume $\tau_t = 0$ for all t. Suppose that the Ramsey allocation is $\{C_t, N_{it}, E_{it}, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3}^{\infty}$. Then there exists a sequence of prices that, together with the allocation, constitute a competitive equilibrium with taxes equal to*

$$\tau_t^e = \tau_t^{e,Pigou} \;\; ; \;\; \tau_t^r = \tau_t^{r,Pigou} \;\; ; \;\; \tau_t^c \approx \tau_t^{c,Pigou} + \tau_t^{c,Ramsey}$$

$$\tau_{t+1}^k = \tau_{t+1}^{k,Pigou} \quad ; \quad \tau_t^n \approx \tau_t^{n,Pigou} + \tau_t^{n,Ramsey}$$

for $t \geq 1$.

The Ramsey taxes are now composed of two elements.[2] The first element is the Pigouvian tax rate and corresponds to the one characterized in Proposition 1. The second element is the Ramsey tax rate, which coincides with that in Lemma 1. While the optimal policy rule is to sum the revenue-raising and carbon-pricing taxes, actual tax rates again differ, since taxes are defined endogenously in terms of the allocations.

6 A quantitative exercise

In this section, we present a quantitative example. Our goal is not to perform a comprehensive quantitative exercise but to illustrate how traditional tax instruments can impose an implicit, first-best carbon tax. We adopt functional forms and parameter values to replicate an equilibrium carbon tax consistent with the estimations in Golosov et al. (2014). This allows us to discipline the result as the optimal taxes we find are equivalent to the carbon tax in that paper.

Assumption 1 *Suppose that utility is logarithmic, $u(C_t) = \log(C_t)$ and $v(N_t) = \frac{\varsigma}{1+1/\varepsilon} N_t^{1+1/\varepsilon}$, damage is multiplicative and exponential, $1 - x(S_t) = \exp(-\bar{\gamma}(S_t - \bar{S})$, final output production is unit-elastic with $\tilde{F}(A_{0t}, N_{0t}, K_{1t}, E_{0t}) = A_{0t} K_{0t}^{\alpha} E_{0t}^{\nu} N_{0t}^{1-\alpha-\nu}$, energy only requires labor*

[2] For ease of notation, we are using the approximation $(1 + x)(1 + y) \approx 1 + x + y$.

input, $F_i(A_{it}, N_{it}, K_{it}) = A_{it}N_{it}$ *for* $i = 2, 3$*, and the carbon cycle is given by one transitory and one permanent component,* $S_t = \bar{S} + \sum_{s=0}^{\infty}(1-d_s)E_{t-s}$ *with* $1 - d_s = \varphi_L + (1-\varphi_L)\varphi_0\varphi^s$.

We have also assumed that climate damages are a fraction of production, the energy composite has a constant elasticity of substitution, there is full depreciation of capital, and a geometric dissipation of carbon stocks—all features in line with Golosov et al. (2014).

Our calibration also follows that study for the relative factor shares in the aggregate production and energy sectors, the carbon cycle, and the damage function; e.g., we calibrate aggregate total factor productivity to a yearly output level of \$70trn and emissions to slightly above 8 GtC. The utility discount rate is set to 1.5% per year and preferences over consumption are logarithmic. Our model also allows for an endogenous leisure choice. Here we follow Barrage (2020) and calibrate to a Frisch elasticity of 0.78 and an initial share of time spent working to 0.227. Table 1 summarizes the parameter values.

Figure 1 plots the numerical results of our model for 2x2 cases of climate policy: with explicit or implicit carbon pricing and with or without lump-sum transfers. Ad-valorem taxes are shown on the left axis and per-unit taxes on the right axis. We convert the taxes on aggregate and renewable energy (τ_t^e and τ_t^r) from per-unit into ad-valorem taxes. Panel (a depicts the standard case, reported in Golosov et al. (2014), where the carbon price (black line) is a constant fraction of output. It starts at \$56 per tC in 2010 (on right axis on all panels) and then it remains fairly flat as there are no growth drivers in the baseline calibration. Since the carbon price is internalizing the only externality in that scenario, no other instruments is used (taxes are zero). An alternative decentralization with implicit carbon prices, and no carbon tax, is shown in panel (b). This policy involves nearly

Table 1: **Calibration**

φ	φ_L	φ_0	α	ν	β	ρ	ς	ε
0.0228	0.2	0.393	0.3	0.04	0.985^{10}	0.058	26.878	0.78
R_0	$\overline{S}$	S^P	S^T	$\bar{\gamma}$	$\kappa 1$	$\kappa 2$	$A_{2,0}$	$A_{3,0}$
253.8	581	699	118	$2.379\ 10^{-5}$	0.5429	0.1015	7693	1311
K_0	A_t	N_t						
128.92	397	1						

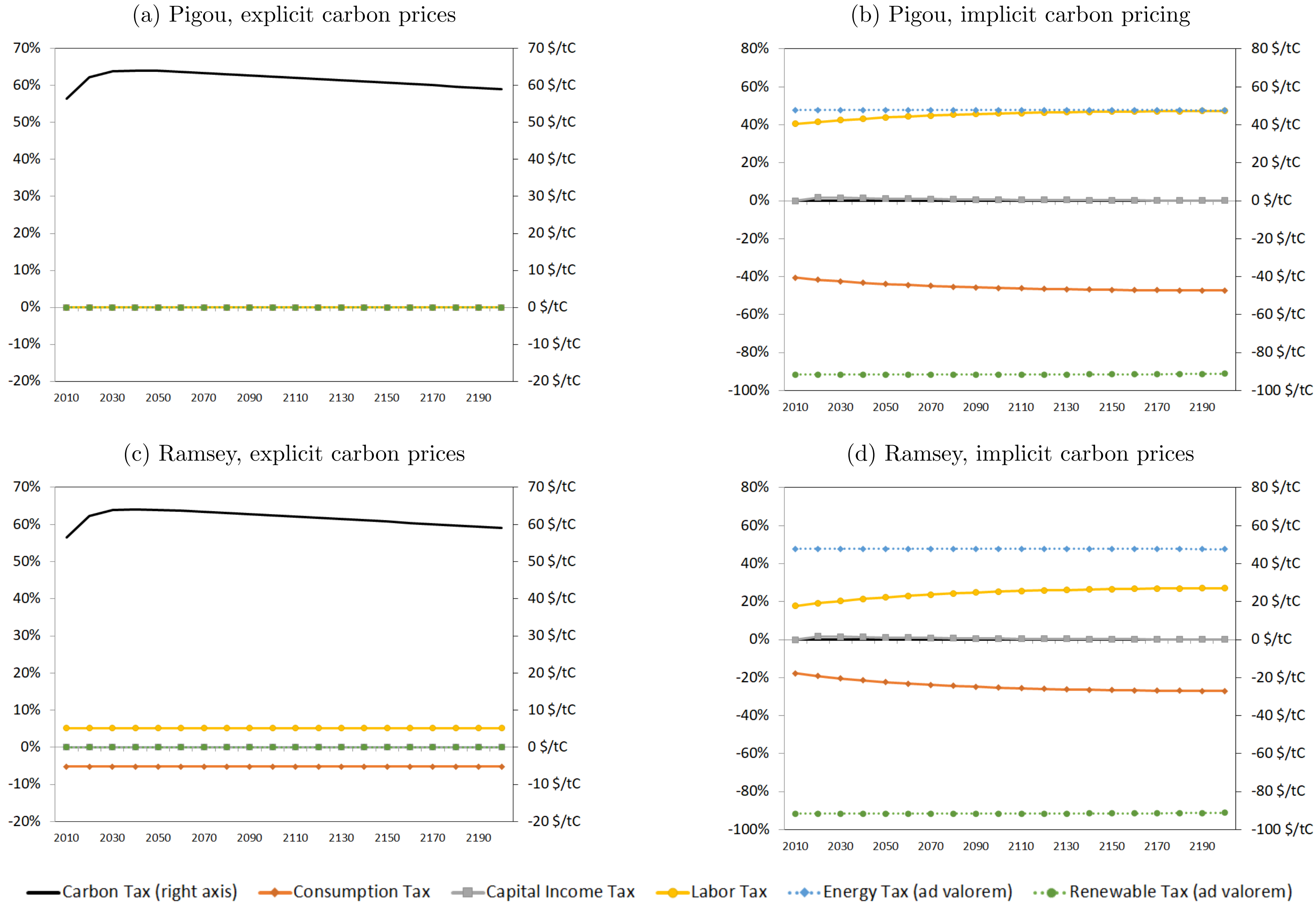

Figure 1: Optimal policies with explicit/implicit carbon pricing and with/without lump-sum transfers. Panel (a) "Pigou, explicit carbon pricing" displays the standard case of carbon pricing with transfers. An alternative decentralization is shown in panel (b) "Pigou, implicit carbon pricing" to the right. Panels (c) "Ramsey, explicit carbon pricing" and panel (d) "Ramsey, implicit carbon pricing" below illustrate climate policy when funds cannot rebated or financed using transfers.

constant energy tax (blue, dashed line) of about 50% to impose the cost of the climate externality. A general renewable subsidy (green, dashed line) of about 90% encourages carbon-free energy at the social optimal level and ensures that the energy tax only hits the supply of fossil fuels. The presence of scarce oil requires further corrective measures at the intertemporal margin. A decreasing consumption tax (orange line) encourages savings and, hence, encourages a delay in dissaving the economy's oil wealth. A decreasing capital income tax (gray line), starting at 1.7 %, is to avoid overaccumulation in physical capital, the economy's other asset. The policy mix attains its purpose of keeping reserves underground without distorting investment in physical capital. At the intratemporal margin, the consumption subsidy affects the relative price between consumption and leisure. A labor income tax (yellow line) mirroring the subsidy fixes this distortion.

Panel (c) depicts the case when the government cannot rely on lump-sum transfers to balance its budget in a Ramsey environment. The proceeds of carbon pricing are rebated using the combination of a small consumption subsidy and labor income tax to offset the subsidy's impact on households' leisure decisions. The effect is opposite when the government cannot rely on carbon pricing (panel (d)). Here, the total of fiscal receipts of climate policy is negative, i.e. the government needs to raise, not rebate, revenue to balance its budget. It does so by lowering and steepening the consumption subsidy and labor income tax to initial values of 18%. All other instruments (energy and capital income tax and renewable subsidy) remain unchanged to the Pigou setting of panel (b).

7 Extensions: Towards Net-Zero

Climate change in the model of section 2 causes gradual damages and the social cost of carbon emissions represents the discounted sum of all future marginal damages due to an extra emission. Global policy efforts, however, focus on a cap on temperature increase and a decarbonization of the world economy, e.g., the EU plans to achieve zero net emissions by 2050. In this section we extend our analysis by introducing two important elements of these policy efforts: negative emission technologies and a cap on cumulative emissions.

7.1 A Model with Carbon Capture

In this section, we extend the model economy to include a provider of atmospheric carbon capture. There are now five production units: the final good producers, the energy sectors, and a carbon capture producer, indexed by $i = \{0, 1, 2, 3, 4\}$. Capital, labour and productivity in each sector are denoted K_{it}, L_{it}, and A_{it}. All production functions exhibit constant returns to scale and satisfy the Inada conditions. The carbon capture technology uses capital and labor according to the function

$$Z_t = F_i(A_{it}, N_{it}, K_{it}) \quad \text{for i = 4} \tag{45}$$

Carbon capture reduces the stock of carbon in the atmosphere, S_t. As in the benchmark model, there are separate carbon stocks with varying carbon dynamics, S_{jt}, with $S_t = \sum_j S_{jt}$ and j arbitrary. Hence, carbon in container S_j evolves according to

$$S_{jt+1} = (1 - \gamma_j)S_{jt} + \varphi_j(E_{1t+1} + \phi E_{2t+1} - Z_{t+1}) \tag{46}$$

where $\gamma_j \in [0, 1]$ is the rate of carbon dissipation and $\varphi_j \in [0, 1]$ the share of emissions entering container j with $\sum_j \varphi_j = 1$. The economy starts with a stock of carbon S_{j0}. The parameter ϕ captures the relative carbon intensity of coal and oil use, again with coal being typically more polluting.

The feasibility constraints in this economy are now given by

$$C_t + K_{t+1} = Y_t \tag{47}$$

for every period t, together with

$$K_t = \sum_{i=0}^{4} K_{it} \tag{48}$$

$$\overline{N}_t \geq N_t = \sum_{i=0}^{4} N_{it} \tag{49}$$

for every period t, where $\overline{N}_t$ is the economy's labor endowment.

Socially Optimal Allocation. The *socially optimal allocation* is the path of consumption, labor, energy, carbon capture, capital, and carbon, $\{C_t, N_{it}, E_{it}, Z_t, K_{it}, S_{jt}\}_{t=0,i=0,1,2,3,4}^{\infty}$, that maximizes the welfare function (10) subject to the resource constraint (47)-(49), the carbon cycle (46) and the initial conditions K_0, R_0, and S_{j0}.

Optimality in carbon capture production requires ensuring that the benefits derived from employing capital and labor in this sector are equal to their alternative uses in other productive sectors. Formally, the following conditions must hold

$$\mu_t F'_{4,n} = F'_{n,t} \tag{50}$$

$$\mu_t F'_{4,k} = F'_{k,t} \tag{51}$$

Here, μ_t represents the marginal benefit of capturing carbon, which is quantified by the avoided climate damages captured in the social cost of carbon. On the other hand, the marginal cost (the right-hand side of equations (50) and (51)) corresponds to the cost of the inputs used in carbon capture production.

The socially optimal allocation satisfies the same optimality conditions as in Section 3, together with (50)-(51).

Market Equilibrium with Carbon Capture. In a market equilibrium, carbon capture firms operate the technology (45) and receive a subsidy, τ_t^z, per unit of production. Firms hire labor at a wage w and rent capital from households at rate r_t. The problem of the firm is to choose inputs $\{N_{4t}, K_{4t}\}_{t=0}^{\infty}$ to maximize discounted profits given by

$$\Pi_4 = \sum_{t=0}^{\infty} q_t^0 [\tau_t^z Z_t - w_t N_{4t} - r_t K_{4t}] \tag{52}$$

subject to (45). At an interior solution, profit maximization in the carbon capture sector implies that the following condition on prices must hold

$$\tau_t^z F'_{4n,t} = w_t \tag{53}$$

$$\tau_t^z F'_{4k,t} = r_t \tag{54}$$

To keep this section as a direct extension, we assume the absence of a carbon market, aligning the market structure in this section with that of the benchmark model. Consequently, the carbon capture sector operates as a separate and isolated sector which does not actively engage in the market. Firms produce if they perceive government support through τ_t^z.

It follows that a competitive equilibrium is fully characterized by the same intertemporal and intratemporal conditions as in section 4, with the additional market equilibrium for carbon capture producers, equations (53) and (54).

Implicit Carbon Prices with Carbon Capture. The next proposition extends the central results of the paper to an economy with an available carbon capture technology.

Given an arbitrary carbon price, $\tau_t^\star$, the implicit carbon prices characterized in Propositions 1 and 2 still implement the same market equilibrium when combined with a carbon capture subsidy aligned to that same carbon price.

Proposition 6 (Implicit carbon prices with carbon capture) *The socially optimal allocation can be decentralized with implicit carbon prices given by Propositions 1 and 2 and*

$$\tau_t^z = \tau_t^\star \tag{55}$$

for every period t. Any surplus is rebated lump-sum through T_t.

An alternative market structure is a carbon trading scheme, wherein carbon credits are tradable assets. Within this market, companies operating in the oil and coal sectors would have the opportunity to buy carbon credits, which they could use to counterbalance their emissions. In this context, the relevant carbon price would be determined based on net emissions, accounting for emissions after factoring in carbon capture credits.

Our choice to abstain from introducing a carbon trading market stems from the intention to maintain the focus of this section within the confines of a direct extension. Nevertheless, it is noteworthy that the findings seamlessly transition to the more intricate scenario of a carbon trading system. Furthermore, the consideration of a carbon capture subsidy arises primarily in the absence of a carbon tax when a carbon trading market is in place. If a carbon tax exists, the equilibrium price of carbon credits will align with the carbon tax rate, obviating the need for a targeted subsidy aimed solely at carbon capture initiatives.

7.2 A Cap on Cumulative Emissions

A further extension of the model encompasses the incorporation of a cap on cumulative emissions. In line with the works of Dietz and Venmans (2019) and van der Ploeg and Rezai (2021), the inclusion of an active constraint on cumulative emissions gives rise to a modified version of the social cost of carbon. This modified social cost of carbon is composed of two distinct components: the first component captures the marginal damages due to the externality, while the second component imposes Hotelling-type scarcity dynamics due to the exhaustible carbon budget.

A cap on cumulative emissions is easily introduced in our framework by assuming all emissions enter only one, permanent container ($\gamma_1 = 0$ and $\varphi_1 = 1$ in the evolution of atmospheric carbon follows the carbon cycle (46)) and by introducing an upper bound on the carbon stock, denoted by the inequality

$$S_t \leq \bar{S} \tag{56}$$

for all t, where $\bar{S}$ is the maximum allowable carbon stock. Let $\beta^t \vartheta_t$ be the Lagrange multiplier on this permanent atmospheric carbon, and let $\beta^t \hat{\omega}_t$ be the Lagrange multiplier on the upper bound constraint. The first order condition with respect to the carbon stock is

$$\vartheta_t = \lambda_t F'_{s,t} + \hat{\omega}_t + \beta \vartheta_{t+1} \tag{57}$$

where λ_t is the Lagrange multiplier on the resource feasibility constraint. Iterating forward on this expression, and expressing it in units dividing by λ_t, we get a modified version of the social cost of carbon

$$\tilde{\mu}_t \equiv \mu_t + \omega_t \tag{58}$$

where the first term captures marginal climate damages and is given by equation (19) with $\gamma_j = 0$ for all j, and the second term captures the Hotelling-type scarcity dynamics of the carbon budget and is given by

$$\omega_t \equiv \sum_{h=0}^{\infty} \beta^h \frac{\hat{\omega}_{t+h}}{\lambda_t} \tag{59}$$

This additional term is zero if the constraint (56) remains non-binding throughout. Conversely, consider the case where the constraint (56) becomes binding at some period T. In this case, the additional term remains small when the carbon budget—the difference between cumulative emissions and their cap—is big, and grows at the rate of interest

$$\frac{\omega_{t+1}}{\omega_t} = F'_{k,t+1}$$

As the budget draws its exhaustion, the additional term increases the cost of emitting significantly.

The central findings of the paper are untouched by this extension. We show in the appendix that only the social cost of carbon is modified by the cap, while all other optimality conditions remain unchanged. Because Proposition 1 holds for any arbitrary carbon price, it is easy to see that Proposition 2 retains its validity once the optimal carbon tax is updated to the new social cost of carbon at every t. Formally is implies,

Corollary 1 *With a cap on cumulative emissions, a competitive equilibrium* Ω *with taxes defined as in Proposition 1 is socially optimal if*

$$\tau_t^{\star} = \tilde{\mu}_t \tag{60}$$

for all t.

The proof of this corollary is in the appendix. It follows from the observation that all optimality conditions of the planning problem remain unaltered upon adding the constraint on cumulative emissions. The new constraint only affects the first-order condition related to the atmospheric carbon stock. Then, the modified social cost of carbon emerges through the forward iteration of this equation.

Furthermore, it is easy to verify that the main results of the paper remain applicable to the case of Ramsey taxation. The optimal carbon tax in Proposition 4 must be equated with the adjusted cost, $\tilde{\mu}_t$. The Ramsey taxes are unaltered as the optimality conditions that determine them are also unchanged. In turn, implicit carbon prices in traditional taxes implement the Ramsey allocation as long as the Pigouvian component of these taxes, outlined in Proposition 5, is linked to the optimal carbon price, i.e., $\tau_t^{\star} = \tilde{\mu}_t$.

8 Concluding Remarks

Implementing a global carbon price is the preferred solution to the climate externality, but pricing carbon has proved elusive, with a mere 23% of global emissions directly priced in 2023 (World Bank, 2023). However, fossil fuel users also confront price signals via implicit pricing measures, with fuel taxes dominating these signals (OECD, 2021). We study how climate and fiscal policy interact and derive conditions of correspondence between explicit and implicit carbon pricing measures. In our neoclassical growth model with a single capital stock, an energy composite of abundant coal, scarce oil, renewable energy, and a climate externality, a combination of energy, consumption, and income taxes can implement the effects of an explicit carbon tax. We also find that additivity holds, i.e., optimal tax rates are equal to the externality-correcting tax rate plus the revenue-raising tax rate. A simple quantitative exercise illustrates the alternative policy mix and compares it to an explicit carbon tax.

The equivalence between different taxes in a complete tax system is well understood. The tax equivalence to an optimal carbon tax, however, is not obvious, because markets are incomplete without a carbon price. There are relevant economic wedges not directly affected by any policy instruments in the absence of a carbon price, specifically those that determine global carbon emissions. Nevertheless, the results in this paper imply that traditional taxes can implement the first-best (climate) policy without carbon markets.

Our findings highlight the important issue of implicit carbon prices imposed by taxes usually not considered climate-related. Further research is, however, needed to understand the implicit pricing signals of arbitrary policy mixes, not just the specific policy mix of our equivalence results. While containing sectoral man-made capital stocks, a finite fossil resource, and an open-access climate state, our framework is still limited in its applicability. Numerical simulations of our results in finer CGE models with sectoral and intertemporal adjustment costs, endogenous technological progress, and limited substitutability are needed to capture the interactions between explicit and implicit carbon prices. Similarly, distributional resolution needs to be added to our analysis to properly motivate the politico-economic barriers to explicit carbon pricing and better understand intertemporal efficiency and equity trade-offs.

9 Mathematical Appendix

9.1 Characterization of the social optimum.

The Lagrangian for the social planner's problem is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{T} \beta^t \left(u(C_t) - v(N_t)\right) - \beta^t \lambda_t \left(K_{t+1} + C_t - F(S_t, A_{0t}, N_{0t}, K_{0t}, E_{0t})\right) \\ - \beta^t \nu_t \left(R_{t+1} - R_t + E_{1t}\right) + \sum_j \beta^t \vartheta_{jt} \left(S_{jt} - (1-\gamma_j) S_{jt-1} - \varphi_j (E_{1t} + \phi E_{2t})\right)\end{aligned}$$

with energy technologies $E_{0t} = [\kappa_1 E_{1t}^{\rho} + \kappa_2 E_{2t}^{\rho} + \kappa_3 E_{3t}^{\rho}]^{1/\rho}$ and $E_{it} = F_i(A_{it}, N_{it}, K_{it})$ for i = 2, 3 and the adding up constraints $N_{0t} = N_t - \sum_{i=1}^{3} N_{it}$, $K_{0t} = K_t - \sum_{i=1}^{3} K_{it}$, and $S_t = \sum_j S_{jt}$ and with λ_t, ν_t, and ϑ_{jt} the shadow values of capital and fossil fuel reserves and the j shadow cost of atmospheric carbon, respectively. The first-order conditions for K_{t+1} and R_{t+1} yield

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = F'_{k,t+1}$$

$$\beta \nu_{t+1} = \nu_t$$

which are equations (11) and (12) and which combine to (13) with $\eta_t \equiv \frac{\nu_t}{\lambda_t}$ the monetary scarcity rent on oil reserves. The first-order conditions for C_t and N_t yield

$$v'(N_t) = \lambda_t F'_{n,t}$$
$$u'(C_t) = \lambda_t$$

which combine to give equation (14). The first-order conditions for energy sectoral factors K_{it} and N_{it} yield, with $\mu_t \equiv \frac{1}{\lambda_t} \sum_j \varphi_j \vartheta_{jt}$ the monetary social cost of carbon,

$$F'_{k,t} = [F'_{E_2 t} - \phi \mu_t] F'_{2k,t}$$
$$F'_{n,t} = [F'_{E_2 t} - \phi \mu_t] F'_{2n,t}$$
$$F'_{k,t} = F'_{E_3,t} F'_{3k,t}$$
$$F'_{n,t} = F'_{E_3,t} F'_{3n,t}$$

which are equations (15)-(18). The first-order conditions for the jth component of atmospheric carbon, S_{t+1} yields

$$\vartheta_t = \lambda_t F'_{s,t} + \beta(1 - \gamma_j)\vartheta_{t+1}.$$

Integrating this expression forward and using the definition $\mu_t \equiv \frac{1}{\lambda_t} \sum_j \varphi_j \vartheta_{jt}$, we have equation (19). The first-order condition for E_{1t} gives

$$F'_{E1t} = \frac{\nu_t}{\lambda_t} + \frac{\sum_j \varphi_j \vartheta_{jt}}{\lambda_t} = \eta_t + \mu_t$$

which is equation (20). Combining (20) with (14), the no-arbitrage condition between the oil and capital stock can be expressed as

$$F'_{k,t+1} = \frac{F'_{E_1,t+1} - \mu_{t+1}}{F'_{E_1,t} - \mu_t} \tag{61}$$

In sum, the socially optimal allocation is fully characterized by equations (14), (15)-(20) and (61), with μ_t given by (19). Finally, optimality requires the transversality conditions for capital and oil reserves to hold.

9.2 Characterization of the social optimum with a cap on cumulative emissions.

The Lagrangian for the social planner's problem is

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t [(u(C_t) - v(N_t)) - \lambda_t (K_{t+1} + C_t - F(S_t, A_{0t}, N_{0t}, K_{0t}, E_{0t})) \\ - \nu_t (R_{t+1} - R_t + E_{1t}) + \vartheta_t (S_t - S_{t-1} - \varphi(E_{1t} + \phi E_{2t}) + \hat{\omega}_t(\bar{S} - S_t)]$$

Notice that the first order conditions for all economic variables remain the same, except for the first order conditions with respect to the carbon stock. The first-order conditions for the atmospheric carbon, S_t, now become

$$\vartheta_t = \lambda_t F'_{s,t} + \hat{\omega}_t + \beta \vartheta_{t+1}$$

Integrating this expression forward we get

$$\frac{\vartheta_t}{\lambda_t} = \sum_{h=0}^{\infty} \frac{\beta^h \lambda_{t+h} F'_{s,t+h}}{\lambda_t} + \sum_{h=0}^{\infty} \beta^h \frac{\hat{\omega}_{t+h}}{\lambda_t}$$

The first term is the expression for the social cost of carbon (19) with $\varphi_1 = 1$ and $\gamma_1 = 0$. Using the definition in (59), we obtain the modified social cost of carbon (58).

The second term is zero if constraint (56) never binds. Instead, suppose that (56) binds at some period T. Then, at any given point in time, we can compute the growth rate of the addition term in the social cost of carbon as

$$\frac{\omega_{t+1}}{\omega_t} = \frac{\lambda_t}{\lambda_{t+1}} \frac{\beta^{T-1}\hat{\omega_T} + \beta^T \hat{\omega_{T+1}} + \beta^{T+1}\hat{\omega_{T+2}} + ...}{\beta^T\hat{\omega_T} + \beta^{T+1}\hat{\omega_{T+1}} + \beta^{T+2}\hat{\omega_{T+2}} + ...}$$

$$\frac{\omega_{t+1}}{\omega_t} = \frac{\lambda_t}{\lambda_{t+1}} \frac{\beta}{\beta} \frac{\beta^{T-1}\hat{\omega_T} + \beta^T \hat{\omega_{T+1}} + \beta^{T+1}\hat{\omega_{T+2}} + ...}{\beta^T\hat{\omega_T} + \beta^{T+1}\hat{\omega_{T+1}} + \beta^{T+2}\hat{\omega_{T+2}} + ...}$$

$$\frac{\omega_{t+1}}{\omega_t} = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta}$$

$$\frac{\omega_{t+1}}{\omega_t} = F'_{k,t+1}$$

Hence, if the cap constraint (56) binds at some period T, the second term in the modified social cost of carbon grows at the rate of interest.

In sum, the socially optimal allocation with a cap on cumulative emissions is fully characterized by equations (14), (15)-(20) and (61), with $\tilde{\mu}_t$ given by (58). Finally, optimality requires the transversality conditions for capital and oil reserves to hold.

Proof of Proposition 1. By definition of a competitive equilibrium with taxes $\{\tau_t, T_t\}_{t=0}^{\infty}$, the allocation Ω satisfies the following system of equations

$$(F'_{E_1,t} - \phi\tau_t^\star)F'_{2k,t} = F'_{k,t} \tag{62}$$

$$(F'_{E_1,t} - \phi\tau_t^\star)F'_{2n,t} = F'_{n,t} \tag{63}$$

$$\beta u'(C_{t+1})[F'_{E_1,t+1} - \tau_{t+1}^\star] = u'(C_t)[F'_{E_1,t} - \tau_t^\star] \tag{64}$$

$$F'_{E_3,t}F'_{3k,t} = F'_{k,t} \tag{65}$$

$$F'_{E_3,t}F'_{3n,t} = F'_{n,t} \tag{66}$$

$$F'_{k,t+1} = \frac{F'_{E_1,t+1} - \tau_{t+1}^\star}{F'_{E_1,t} - \tau_t^\star} \tag{67}$$

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1}{F_{n,t}} \tag{68}$$

together with the market clearing conditions (7)-(9), the initial conditions and transversality conditions for the stocks of capital and oil and the carbon cycle equation (5). We need to show that a competitive equilibrium with fiscal policy $\{\tau_t^e, \tau_t^r, \tau_t^k, \tau_t^n, \tau_t^c, T_t\}_{t=0}^{\infty}$ as defined in Proposition 1, and $\tau_t = 0$, satisfies equations (62)-(68). Plug τ_t^e in equilibrium conditions (33)-(34) using (27)-(29) for equilibrium prices to get

$$(F'_{E_2,t} - \tau_t^e\alpha_{2t})F'_{2k,t} = F'_{k,t} \tag{69}$$

$$(F'_{E_2,t} - \tau_t^e\alpha_{2t})F'_{2n,t} = F'_{n,t} \tag{70}$$

Then, plug the optimal tax rate for τ_t^e to get

$$(F'_{E_2,t} - \tau_t^\star\frac{\phi}{\alpha_{2t}}\alpha_{2t})F'_{2k,t}(t) = F'_{k,t} \tag{71}$$

$$(F'_{E_2,t} - \tau_t^\star\frac{\phi}{\alpha_{2t}}\alpha_{2t})F'_{2n,t}(t) = F'_{n,t} \tag{72}$$

which coincides with (62) and (63).

Next, take equilibrium condition (37) and plug consumption and capital income taxes in to get

$$\beta\frac{u'(C_{t+1})}{1+\tau_{t+1}^c}[(1 - \frac{\tau_t^c - \tau_{t+1}^c}{1+\tau_t^c})r_{t+1}] = \frac{u'(C_t)}{1+\tau_t^c} \tag{73}$$

$$\beta\frac{u'(C_{t+1})}{1+\tau_{t+1}^c}[(\frac{1+\tau_t^c - \tau_t^c + \tau_{t+1}^c}{1+\tau_t^c})r_{t+1}] = \frac{u'(C_t)}{1+\tau_t^c} \tag{74}$$

Consumption taxes cancel out. Use (28) to replace for r_t to get

$$F'_{k,t+1} = \frac{u'(C_t)}{\beta u'(C_{t+1})} \tag{75}$$

and combine with (32) to obtain

$$F'_{k,t+1} = \frac{1+\tau^c_t}{1+\tau^c_{t+1}} \frac{F'_{E_1,t+1} - \tau^e_{t+1}\alpha_{1,t+1}}{F'_{E_1,t} - \tau^e_t \alpha_{1,t}} \tag{76}$$

From Proposition 1,

$$1+\tau^c_t = 1 + \frac{\tau^\star_t(1-\phi\frac{\alpha_{1t}}{\alpha_{2t}})}{F'_{E_1,t} - \tau^\star_t} = \frac{F'_{E_1,t} - \tau^\star_t + \tau^\star_t - \tau^\star_t \phi \frac{\alpha_{1t}}{\alpha_{2t}}}{F'_{E_1,t} - \tau^\star_t}$$

Therefore,

$$F'_{k,t+1} = \frac{F'_{E_1,t} - \tau^\star_t \phi\frac{\alpha_{1t}}{\alpha_{2t}}}{F'_{E_1,t} - \tau^\star_t} \frac{F'_{E_1,t+1} - \tau^\star_{t+1}}{F'_{E_1,t+1} - \tau^\star_{t+1}\phi\frac{\alpha_{1t+1}}{\alpha_{2t+1}}} \frac{F'_{E_1,t+1} - \tau^e_{t+1}\alpha_{1,t+1}}{F'_{E_1,t} - \tau^e_t\alpha_{1,t}} \tag{77}$$

Using that $\tau^e_t = \tau^\star_t \frac{\phi}{\alpha_{2t}}$ and canceling out terms, we get

$$F'_{k,t+1} = \frac{F'_{E_1,t+1} - \tau^\star_{t+1}}{F'_{E_1,t} - \tau^\star_t} \tag{78}$$

which coincides with (67), and together with (75), also (64) holds. To see that (65) and(66 hold, take equilibrium conditions 35 and 36 and substitute equilibrium prices using (27)-(29) to get

$$(F'_{E_3,t} - \tau^e_t\alpha_{3t} - \tau^r_t)F'_{3k,t}(t) = F'_{k,t} \tag{79}$$

$$(F'_{E_3,t} - \tau^e_t\alpha_{3t} - \tau^r_t)F'_{3n,t}(t) = F'_{n,t} \tag{80}$$

Plugging in τ^r_t from Proposition (1) we get (65) and (66). Finally, take (39) and plug in consumption and labor income tax rates to easily get (68). This completes the proof.

Proof of Proposition 2. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation when $\tau^\star_t = \mu_t$. A socially optimal allocation is fully characterized by the system of equations (14), (15)-(20) and (61), together with the feasibility conditions (7)-(9), the initial conditions and transversality conditions for the stocks of capital and oil and the carbon cycle equation (5). Due to the equivalence result established in Proposition 1, it is sufficient to show that the optimal allocation satisfies all equilibrium conditions with explicit carbon prices, summarized in equations (62)-(68). It is easy to see that (62) and (63) coincide with (15) and (16) if $\tau_t^\star = \mu_t$, and, also, (67) equals (61). All remaining optimality conditions are the same, by simple observation.

Proof of Proposition 3. At an interior solution, a competitive equilibrium allocation satisfies the intertemporal conditions (32) and (37), the intratemporal equation (34), the conditions on prices (27)-(29), (33-36) together with the carbon cycle dynamics (5), the feasibility constraint (7), the government budget balance (26), and the transversality condition for the capital stock. By Walras' law, if the consumer's budget constraint holds, then (26) holds as well. The proof consists on showing that all these equilibrium conditions can be summarized in an "implementability constraint" that uses (25) as the starting point. Rewrite (25) to get

$$\sum_{t=0}^{\infty} q_t^0[(1+\tau_t^c)C_t - w_t(1-\tau_t^n)N_t] = \sum_{t=0}^{\infty} q_t^0[(1-\tau_t^k)r_tK_t - K_{t+1}] + \Pi \tag{81}$$

Using (31) and (39), we have that

$$\sum_{t=0}^{\infty} \beta^t \frac{1+\tau_0^c}{u'(C_0)}[u'(C_t)C_t - v'(N_t)N_t] = \sum_{t=0}^{\infty} q_t^0[(1-\tau_t^k)r_tK_t - K_{t+1}] + \Pi \tag{82}$$

Notice that the right-hand side of (82) can be opened up to obtain

$$\sum_{t=0}^{\infty} \beta^t \frac{1+\tau_0^c}{u'(C_0)}[u'(C_t)C_t - v'(N_t)N_t] = q_0^0(1-\tau_0^k)r_0K_0 - q_0^0K_1 + q_1^0(1-\tau_1^k)r_1K_1 - q_1^0K_2 + .. + \Pi \tag{83}$$

where

$$\Pi = q_0^0[(p_{1,0}-\tau_0)(R_0-R_1)] + q_1^0[(p_{1,1}-\tau_1)(R_1-R_2)] + ...$$

and profits in sectors $j = 0, 2, 3$ are zero using the equilibrium condition on prices (27-29) and (33-36) in every period t. That is,

$$\sum_{t=0}^{\infty} \beta^t \frac{1+\tau_0^c}{u'(C_0)}[u'(C_t)C_t - v'(N_t)N_t] = (1-\tau_0^k)r_0K_0 - K_1\{1 - q_1^0(1-\tau_1^k)r_1\} - q_1^0K_2 + .. + \Pi \tag{84}$$

with

$$\Pi = (p_{1,0}-\tau_0)R_0 - R_1\{(p_{1,0}-\tau_0) + q_1^0(p_{1,1}-\tau_1)\} - (p_{1,1}-\tau_1)R_2) + ...$$

where the subsequent terms in between curly brackets in the right-hand side of the equation are zero from (37) and (32). Proceeding forward with the rest of the summands, and using the first order conditions with respect to capital and oil in every t, we get

$$\sum_{t=0}^{\infty} \beta^t \frac{1+\tau_0^c}{u'(C_0)}[u'(C_t)C_t - v'(N_t)N_t] = (1-\tau_0^k)r_0K_0 + (p_{1,0}-\tau_0)R_0 \tag{85}$$

where the value of the capital and resource stocks at $T \to \infty$ are zero by the transversality conditions. Further, use (28) and (29) to write down the implementability constraint only in terms of the allocation.

$$\sum_{t=0}^{\infty} \beta^t [u'(C_t)C_t - v'(N_t)N_t] = \frac{u'(C_0)}{1+\tau_0^c}[(1-\tau_0^k)F'_{k,0}K_0 + (F'_{E_1,0} - \tau_0^e \alpha_{1,0} - \tau_0)R_0] \tag{86}$$

which coincides with (42).

Proof of Lemma 1. The proof consists of showing that all conditions for an equilibrium are satisfied by the business-as-usual Ramsey allocation when taxes are set according to the lemma. For expositional ease redefine: $H_{ct} = 1/EIS_t$ and $H_{nt} = 1/ELS_t$. At an interior solution, the Ramsey allocation is characterized by the following system of equations for every period $t \geq 1$

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1-\varpi+\varpi H_{nt}}{1-\varpi+\varpi H_{ct}} \frac{1}{F'_{n,t}} \tag{87}$$

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})F'_{E_1,t+1} = u'(C_t)(1-\varpi+\varpi H_{ct})F'_{E_1,t} \tag{88}$$

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})F'_{k,t+1} = u'(C_t)(1-\varpi+\varpi H_{ct}) \tag{89}$$

together with the production efficiency conditions

$$F'_{E_2,t}F'_{2k,t} = F'_{k,t} \tag{90}$$

$$F'_{E_2,t}F'_{2n,t} = F'_{n,t} \tag{91}$$

$$F'_{E_3,t}F'_{3k,t} = F'_{k,t} \tag{92}$$

$$F'_{E_3,t}F'_{3n,t} = F'_{n,t} \tag{93}$$

and the carbon cycle constraint (5), the feasibility constraints $(7-9)$, the implementability constraint (42), and the initial conditions $\{K_0, R_0, S_0, \tau_0^c, \tau_0^k, \tau_0^e, \tau_0\}$. It is sufficient to show that (87-89) hold in a competitive equilibrium with taxes defined according to Lemma 1. Notice that the optimal consumption tax can be written as $1+\tau_t^c = \frac{1}{1-\varpi+\varpi H_{ct}}$. Plug this tax rate into (32) and (37), with $\tau_t = \tau_t^e = \tau_t^k = 0$, to get (88) and (89), respectively. Ramsey taxes on capital income, energy and carbon taxes are all zero in the business-as-usual economy. Also, plug the tax rates into (39) to get

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1-\varpi+\varpi H_{nt}}{(1-\varpi+\varpi H_{ct})F'_{n,t}} \tag{94}$$

which coincides with (87). It is easy to see that the Ramsey allocation satisfies the maximizing conditions (33-36) given the taxes. Equation (42) guarantees that the Ramsey allocation satisfies (25). Finally, the feasibility constraints (7-9) hold by definition of the Ramsey problem . This completes the proofs that all conditions for a competitive equilibrium are satisfied by the Ramsey allocation.

Proof of Proposition 4. The proof consists of showing that all conditions for an equilibrium are satisfied by the Ramsey allocation when taxes are set optimally. For expositional ease redefine: $H_{ct} = 1/EIS_t$ and $H_{nt} = 1/ELS_t$. At an interior solution, the Ramsey allocation is characterized by the following system of equations for every period $t \geq 1$

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1-\varpi+\varpi H_{nt}}{1-\varpi+\varpi H_{ct}} \frac{1}{F'_{n,t}} \tag{95}$$

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})[F'_{E_1,t+1}-\mu_{t+1}] = u'(C_t)(1-\varpi+\varpi H_{ct})[F'_{E_1,t}-\mu_t] \tag{96}$$

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})F'_{k,t+1} = u'(C_t)(1-\varpi+\varpi H_{ct}) \tag{97}$$

together with the production efficiency conditions

$$[F'_{E_2,t}-\phi\mu_t]F'_{2k,t} = F'_{k,t} \tag{98}$$

$$[F'_{E_2,t}-\phi\mu_t]F'_{2n,t} = F'_{n,t} \tag{99}$$

$$F'_{E_3,t}F'_{3k,t} = F'_{k,t} \tag{100}$$

$$F'_{E_3,t}F'_{3n,t} = F'_{n,t} \tag{101}$$

and the carbon cycle constraint (5), the feasibility constraints $(7-9)$, the implementability constraint (42), and the initial conditions $\{K_0, R_0, S_0, \tau_0^c, \tau_0^k, \tau_0^e, \tau_0\}$. Plug the taxes into (33) and (34)

$$[F'_{E_2,t}-\phi\mu_t]F'_{2k,t} = F'_{k,t} \tag{102}$$

$$[F'_{E_2,t}-\phi\mu_t]F'_{2n}(t) = F_{n,t} \tag{103}$$

which equal (98) and (99). The optimal consumption tax in the proposition can be written as $1+\tau_t^c = \frac{1}{1-\varpi+\varpi H_{ct}}$. Plug this expression and the one for the carbon tax, with $\tau_t^e = 0$ into (32) to get

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})[F'_{E_1,t+1}-\mu_{t+1}] = u'(C_t)(1-\varpi+\varpi H_{ct+1})[F'_{E_1,t}-\mu_t$$

which equals (96). It is straightforward to check that plugging taxes into (37) leads to (97). Also, plug the tax rates into (39) to get

$$\frac{u'(C_t)}{v'(N_t)} = \frac{1-\varpi+\varpi H_{nt}}{(1-\varpi+\varpi H_{ct})F'_{n,t}} \tag{104}$$

which coincides with (95). Finally, with zero taxes on energy and renewables (35-36) coincide with (100) and (101).

Finally, (42) guarantees that the Ramsey allocation satisfies (25), and (7) holds by definition of the Ramsey problem.

Proof of Proposition 5. The proof consists of showing that all conditions for an equilibrium are satisfied by the Ramsey allocation when taxes are set optimally. For expositional ease redefine: $H_{ct} = 1/EIS_t$ and $H_{nt} = 1/ELS_t$. At an interior solution, the Ramsey allocation is characterized by (95-101) for $t \geq 1$ and the carbon cycle constraint (5), the feasibility constraints $(7-9)$, the implementability constraint (42), and the initial conditions $\{K_0, R_0, S_0, \tau_0^c, \tau_0^k, \tau_0^e, \tau_0\}$. Plug the expression for τ_t^e into (33) and (34)

$$[F'_{E_2,t} - \frac{\phi\mu_t}{\alpha_{2t}}\alpha_{2t} - 0]F'_{2k,t} = F_{k,t} \tag{105}$$

$$[F'_{E_2,t} - \frac{\phi\mu_t}{\alpha_{2t}}\alpha_{2t} - 0]F'_{2n,t} = F_{n,t} \tag{106}$$

which equal (98) and (99).

Next, plug $(1+\tau_t^c)$ into (32) using the approximation $(1+\tau_t^{c,Pigou})(1+\tau_t^{c,Ramsey}) \approx 1+\tau_t^{c,Pigou}+\tau_t^{c,Ramsey}$. Hence,

$$\frac{\beta u'(C_{t+1})[F'_{E_1,t+1} - \phi\mu_{t+1}\frac{\alpha_{1t+1}}{\alpha_{2t+1}}]}{1+\frac{\mu_{t+1}}{F'_{E_1,t+1}-\mu_{t+1}}(1-\phi\frac{\alpha_{1t+1}}{\alpha_{2t+1}})\frac{1}{1-\varpi+\varpi H_{ct+1}} + \frac{\varpi(1-H_{ct+1})}{1-\varpi+\varpi H_{ct+1}}} = \frac{u'(C_t)[F'_{E_1,t} - \phi\mu_t\frac{\alpha_{1t}}{\alpha_{2t}}]}{1+\frac{\mu_t}{F'_{E_1,t}-\mu_t}(1-\phi\frac{\alpha_{1t}}{\alpha_{2t}})\frac{1}{1-\varpi+\varpi H_{ct}} + \frac{\varpi(1-H_{ct})}{1-\varpi+\varpi H_{ct}}}$$

After some simple algebra, the equation can be written as

$$\frac{\beta(1-\varpi+\varpi H_{ct+1})u'(C_{t+1})[F'_{E_1,t+1} - \phi\mu_{t+1}\frac{\alpha_{1t+1}}{\alpha_{2t+1}}]}{1+\frac{\mu_{t+1}}{F'_{E_1,t+1}-\mu_{t+1}}(1-\phi\frac{\alpha_{1t+1}}{\alpha_{2t+1}})} = \frac{(1-\varpi+\varpi H_{ct})u'(C_t)[F'_{E_1,t} - \phi\mu_t\frac{\alpha_{1t}}{\alpha_{2t}}]}{1+\frac{\mu_t}{F'_{E_1,t}-\mu_t}(1-\phi\frac{\alpha_{1t}}{\alpha_{2t}})}$$

This is

$$\beta(1-\varpi+\varpi H_{ct+1})u'(C_{t+1})[F'_{E_1,t+1} - \mu_{t+1}] = (1-\varpi+\varpi H_{ct})u'(C_t)[F'_{E_1,t} - \mu_t]$$

which equals (96). Next, plug taxes into into (37)

$$\beta\frac{u'(C_{t+1})}{1+\tau^c_{t+1}}[(1-\frac{\tau^{c,Pigou}_t-\tau^{c,Pigou}_{t+1}}{1+\tau^{c,Pigou}_t})F'_{k,t+1}]=\frac{u'(C_t)}{1+\tau^c_t} \tag{107}$$

and rewrite it as

$$\beta u'(C_{t+1})\frac{1+\tau^c_t}{1+\tau^c_{t+1}}\frac{1+\tau^{c,Pigou}_{t+1}}{1+\tau^{c,Pigou}_t}F'_{k,t+1}=u'(C_t) \tag{108}$$

We know that $1+\tau^c_t=(1+\tau^{c,Ramsey}_t)(1+\tau^{c,Pigou}_t)$. Therefore, we have

$$\beta u'(C_{t+1})\frac{1+\tau^{c,Ramsey}_t}{1+\tau^{c,Ramsey}_{t+1}}F'_{k,t+1}=u'(C_t) \tag{109}$$

Using (43), we then have

$$\beta u'(C_{t+1})(1-\varpi+\varpi H_{ct+1})F'_{k,t+1}=u'(C_t)(1-\varpi+\varpi H_{ct}) \tag{110}$$

which coincides with (97). The intratemporal condition with taxes is

$$\frac{u'(C_t)}{v'(N_t)}=\frac{1+\tau^c_t}{1-\tau^n_t}\frac{1}{F'_{n,t}} \tag{111}$$

We know that $1+\tau^c_t=(1+\tau^{c,Ramsey}_t)(1+\tau^{c,Pigou}_t)$ and also $\tau^{n,Pigou}_t=-\tau^{c,Pigou}_t$. And then, plugging the labor tax rate from the proposition, we have

$$\frac{u'(C_t)}{v'(N_t)}=\frac{(1+\tau^{c,Ramsey}_t)(1-\tau^{n,Pigou}_t)}{(1-\tau^{n,Pigou}_t)(1-\tau^{n,Ramsey}_t)}\frac{1}{F'_{n,t}} \tag{112}$$

Using (43)-(44), we then have

$$\frac{u'(C_t)}{v'(N_t)}=\frac{1-\varpi+\varpi H_{nt}}{1-\varpi+\varpi H_{ct}}\frac{1}{F'_{n,t}} \tag{113}$$

which equals (95). Finally, plug the tax rates into the equations (35-36) to get

$$(F'_{E3,t}-\tau^e_t\alpha_{3t}+\tau^e_t\alpha_{3t})F'_{3k,t}=F'_{k,t} \tag{114}$$

$$(F'_{E3,t}-\tau^e_t\alpha_{3t}+\tau^e_t\alpha_{3t})F'_{3n,t}=F_{n,t} \tag{115}$$

which coincides with (100) and (101).

Finally, (42) guarantees that the Ramsey allocation satisfies (25), and (7) holds by definition of the Ramsey problem.

Proof of Proposition 6. The proof consists of showing that all conditions for a competitive equilibrium are satisfied by the optimal allocation. A socially optimal allocation with carbon capture is fully characterized by the optimality conditions described in Section 3, together with the additional (50) and (51). Building upon the proof of Proposition 1 and 2, it remains to show that (50) and (51) hold in a competitive equilibrium when $\tau_t^z = \tau_t^\star$. To see this plug $\tau_t^\star$ into (53) and (54) and use (27) and (28) for equilibrium prices to get exactly (50) and (51), given that $\tau_t^\star = \mu_t$ by Proposition 2.

Proof of Corollary 1. The proof follows from the characterization derived in subsection 9.2. All optimality conditions remain the same in the social optimum with a cap on emissions, except for (19) which changes to (58). It follows that the proofs of Propositions 1 and 2 carry over with the only change of redefining μ_t to $\tilde{\mu}_t$.

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