Marriage and Employment Returns to Female Education

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Abstract

This paper develops a method to jointly estimate the marriage and employment returns to female education using a frictionless matching model with transferable utility. We propose indices that are independent of the marginal distributions of populations and unaffected by secular trends in education, marriage, and the female labor market. The returns and their differences can be estimated nonparametrically, allowing for partial identification of their sign without any distributional assumption. The estimation relies on widely available cross-sectional household surveys, allowing for evidence over time for the United States and about 20 other countries in different regions of the world. In the United States, we find that attending college and graduate school, while reducing the gain from marriage compared to singlehood from 1960 to 2000, increases marriage gain for women in recent years. Our estimations suggest that, in 2017, women with a graduate degree are indifferent between marrying a graduate degree husband and 18 percent increase in earnings (≈\$15,000 in 2023) while remaining single. Nonparametric estimations for the U.S. indicate that while the sign of the aggregated unconditional return indices are hardly identified without a distributional assumption, conditional on marital and employment statuses, the sign of the returns to education can often be identified. The cross-country evidence suggests that female education has a positive employment return almost everywhere, with a higher return to university education than high school. However, the marriage return to education is mostly negative or U-shaped across countries, suggesting that education can serve as a means to escape the social stigma of remaining single. Almost everywhere, conditional on being married and employed, female higher education leads to better jobs and spouses. More education has a concave relationship with spouse quality but a convex relationship with job quality.

JEL classifications: I26, J12, J16

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1 Introduction

In the past century, women's education increased significantly in all parts of the world, and in many countries, it has risen above men's education (Becker, Hubbard, and Murphy, 2010; Goldin, Katz, and Kuziemko, 2006). Acquiring education results in changing individuals' prospects in two markets: the labor market and the marriage market. While at the extensive margin, more schooling affects the gain from matching compared to remaining unmatched in each market (employment vs. non-employment and marriage vs. singlehood), at the intensive margin, it influences match quality (occupation and spouse types), and transfer (wage and marital surplus share). A key question in this regard is how large are the returns to education in each market and how to compare them. This paper contributes to answering this question methodologically and empirically.

Measuring return to education in the marriage market and comparing it with the return in the labor market is not straightforward. The main difficulty stems from the fact that, unlike the labor market, the transfers in the marriage market are not observable. Therefore, while labor return can be directly estimated from observed wage premiums, the marriage return must be indirectly estimated from observable match qualities and marriage patterns. An important challenge in measuring return without observing the surplus share of agents arises from secular changes in population supplies over time or across space. For instance, if the population of highly educated women increases but not that of men, the matching patterns by education will change. However, such changes can occur even under random matching, and it is necessary to differentiate between two components: the mechanical effect resulting from changes in the overall distribution of education levels and the effect caused by changes in the marriage return to education that reflect the benefits of marriage based on education levels.

In this paper, we present a new approach for estimation and comparison of marriage and employment returns to education, using a frictionless matching model with transferable utility. Our method focuses on match quality in both markets instead of transfer, which is unobservable in the marriage market, and incorporates earnings data from the labor market as additional information for estimating the model. The proposed method has several attractive features: first, it jointly estimates different margins of marriage and employment returns to education and enables comparison between them in the two markets. Second, we propose aggregate return indices independent of marginal distributions of the populations and unaffected by the secular trends in education, marriage, and the female labor market. Third, the returns and their differences can be estimated nonparametrically, allowing for partial identification of their sign without distributional assumption. Fourth, the estimation relies on widely available cross-sectional household surveys allowing for evidence over time and across countries.

The model extends the seminal work by Choo and Siow (2006) for the marriage market to a 2-to-1 matching framework such that women characterized by education and occupation match with firms and men in two bilateral matching markets. For matching decision, firms (men) are concerned about the marital (employment) status of the woman, but not the specific features of the spouse (job). The model provides a useful framework to measure marriage and employment returns to education conditional on marital and occupational statuses. These conditional returns are independent of marginal distributions of education levels, marriages, and jobs. We show that the proposed indices for marriage return, employment return, and their differences are partially identified and estimable using the 3-way table of observed matching frequencies in the marriage and job markets of women based on their education, marital, and working statuses. A remarkable feature of the model is that without imposing any parametric assumption about deterministic surplus and random components, one can determine not only the sign of the conditional return indices but also which one is bigger.

Using cross-sectional household data from the United States and about 20 other countries, we estimate the trend of marriage and employment returns to education across different regions. In the United states, female marriage return to college and graduate education compared to singlehood is significantly negative in 1960, but with an increasing trend it becomes positive after 2000. The extensive margin of employment return demonstrates a U-shaped relationship with increasing levels of education. Meanwhile, the intensive margin of the returns is positive at all levels and increases with education, particularly for employment return. Additionally, we find that black women in the United States exhibit comparatively less aversion to marry a spouse with a lower level of education or work in a lower-paying occupation than their white counterparts.

The estimated preference parameters in the overidentified model allows for evaluating the value of spouse education for employed women. The numbers suggest that, in 2017, relative to high school dropout women, while women with bachelor's degrees would pay around 18 percent of their annual earnings ($\approx$\$10,700 in 2023) to marry husbands with bachelor's or graduate degrees rather than remaining single, they would allocate about 14 percent of their annual earnings to refrain from marriage with a high school dropout man. The corresponding numbers for women with graduate degrees are about \$15,000 to marry a husband with the same educational background instead of remaining single and about \$7200 to refrain from marrying a high school dropout.

Our nonparametric estimations for the U.S. imply robust identification of the sign the returns, conditional on marital and employment statuses. However, only the intensive margin of the unconditional returns can be partially identified without making any distributional assumptions. This is due to the monotonic behavior of aggregate returns at their intensive margins, which remains stable across categories and over time. Conversely, the extensive margins of the returns display a non-monotonic behavior with increasing education level and across different groups, and it is not possible to make a robust inference for their aggregate measures without making distributional assumptions for random surplus terms.

The cross-country evidence shows that, at the extensive margin, while the employment return to female education has been mostly positive across countries, its marriage return is mostly negative or U-shaped suggesting that education can serve as a means to escape the social stigma of remaining single.

This evidence is consistent with psychological studies showing the existence of discrimination against single individuals which is more severe for women (e.g. Byrne and Carr, 2005). In contrast, the intensive margins of the returns to education are positive in almost all countries, but more education has an increasing and concave relationship with spouse quality and an increasing and convex relationship with job quality.

This paper contributes to the literature on different fronts. First, it develops a framework for the parametric and nonparametric estimations of the extensive and intensive margins of marriage and employment returns to female education and their comparison. In this regard, it is in line with the earlier contributions by Chiappori, Iyigun, and Weiss (2009), who theoretically model marriage and labor returns to education, and Chiappori, Salanié, and Weiss (2017) that estimates marriage college premium in a static model of frictionless matching with transferable utility. Chiappori, Dias, and Meghir (2018) extend this approach by introducing labor supply, consumption, and saving in a dynamic life-cycle model. While using dynamic models of marriage and labor supply (see also, Blundell, Costa Dias, Meghir, and Shaw, 2016; Goussé, Jacquemet, and Robin, 2017, among others) make perfect sense to analyze policy interventions with long-term effects through the marriage and labor markets, their unavoidable parametric restrictions limit their usefulness for the purpose of measurement and comparison of labor and marriage returns to education.[1] Generally, a nonparametric approach to gauge the returns to education and their differences is preferable to measurements mediated by particular modeling assumptions. This paper presents a method for measuring the returns to education by estimating the systematic matching surplus using data from a large market. This approach allows us to partially identify the sign of the returns and their differences without making any distributional assumptions about the random surplus term. Another advantage of the framework presented in this paper is measurements using cross-sectional data, which are available for numerous countries and give more statistical power in estimations than typical panel datasets.

Next, this paper contributes to the literature on the econometrics of frictionless matching markets. The seminal work of Choo and Siow (2006) (hereafter CS) develops a frictionless matching model under transferable utility to estimate the marriage surplus based on observable traits of married couples and singles. They assume that marriage surplus is composed of a deterministic part, depending only on observable traits, and a stochastic part, reflecting unobserved heterogeneity in preferences and traits. Their crucial identifying assumption is that the stochastic part is *separable* as the sum of male and female components, which only depend on the observable trait of their spouse.[2] This paper extends CS's separability framework into two bilateral markets and proposes conditional odds ratios as a measurement of association independent of marginal distributions. We also show that in the CS model, including the

[1] For example, Chiappori et al. (2018) assume that the marriage decision is made before the labor supply decision and the deterministic component of marriage surplus is monetary benefits coming from future behavior without any non-pecuniary elements.

[2] This observable trait is age in CS, education and human capital in Chiappori et al. (2017, 2018), and a vector of characteristics including education, height, BMI, health, and personality traits in Dupuy and Galichon (2014).

choices between categories mainly adds a constant term to the expected utilities that only depends on the number of categories and not on the conditional probabilities and the deterministic utilities. This result simplifies the estimation of CS model under alternative distributional assumptions by approximating expectation of maximum utility (Emax) with expectation of utilities plus a constant.

In another direction, the basic CS model, which is just-identified and assumes homoskedastic type-I extreme value distribution for unobserved heterogeneity, is extended in various ways in the later contributions. Chiappori et al. (2017) use a multi-market framework with a parametric restriction on the trend of marriage surplus over time to turn the just-identified model of CS into an over-identified model. Galichon and Salanie (2021) generalize the CS model to a parametric version with any distribution of unobserved heterogeneity that allows for including covariates and heteroskedasticity. The structural model of Chiappori et al. (2018) enables them to use additional information from later decisions on saving and labor supply to generate additional moments for empirical estimation. This paper contributes to this literature by exploiting information on earnings across occupations (when available) as over-identifying restrictions for discrete choices of women. To the best of our knowledge, this is a new extension of CS that this paper presents with a great application in disentangling two different returns to education.

On the empirical front, this paper adds to the gender economics literature, especially the role of education in women's empowerment. While previous literature on marriage return to education mainly focused on the U.S. and a few high-income countries, the method allows us to look into trends worldwide. To our knowledge, cross-country evidence on marriage return is not available in the previous literature, especially the findings that education can act as a tool to alleviate social stigma toward single women, in addition to its role in reducing discrimination in the labor market. Besides, our empirical evidence for the United States adds new content regarding the size and comparison of the two returns.

Finally, our model enables us to provide aggregate measures of sorting in the marriage and job markets and we contribute to the debate on trends in assortative matching within the U.S. marriage market over recent decades. While Greenwood, Guner, Kocharkov, and Santos (2014) and Chiappori et al. (2017) find an increase in assortativeness particularly among the highly educated, Eika, Mogstad, and Zafar (2019) have reported a decline among this group and an increase among those with less education. Consistent with Chiappori, Costa Dias, and Meghir (2020, 2021), our research shows that these differing trends can be attributed to differences in aggregation methods. To accurately measure sorting within marriage markets, it is important to use an index that is not affected by changes in the marginal distribution of discrete variables; otherwise, secular trends within populations may confound measurements. In this regard, we demonstrate that aggregate measurements of returns and sorting should be based on averaging the index conditional on categories rather than computing it for collapsed categories. The latter approach does not account for potential confounding variables and may lead to spurious conclusions (Simpson, 1951).

Before proceeding, some remarks are in order. Usually, the term *labor* return refers to the wage premium, but in this paper, we use the term *employment* return because the measurement is based on the discrete choices for employment and job status and not specifically the wage. This restriction also brings the advantage of the possibility of estimation without wage data, allowing for evidence using many available datasets that only report household members' occupations but not wage and income.[3] For the same reason, we abstract from men's return in the analysis because of their high labor participation rate and the fact that the return to education for men is mainly reflected in their wages rather than participation. This is in stark contrast to women whose participation decision is very sensitive to their education and household composition.

The rest of the paper is organized as follows. Section 2 outlines the theoretical framework and section 3 uses the theoretical framework to measure returns to education. Section 4 explains identification of the parameters and estimation strategy. Section 5 describes the data and section 6 presents empirical findings for the United States and other countries. Section 7 concludes.

2 Theoretical framework

The theoretical model is the extended version of CS in which the unobserved characteristics of individuals are random terms in the surplus function and their structure can be fully characterized by a separability condition. CS and the most of subsequent works (e.g. Chiappori et al., 2017; Galichon and Salanie, 2021) consider one matching market, but here we consider a joint decision in two bilateral and multi-dimensional matching markets.[4] The matching dimensions of the model are later linked to the main factors related to the research question: a woman's education, her working status, and the education of her partner.

This section first outlines our matching model and specifies the separability assumption. Then, we introduce conditional return indices and demonstrate how the separability assumption facilitates their sign-based identification. Lastly, we quantify the conditional returns by postulating a type-I extreme value distribution for unobservable terms.

2.1 Two bilateral matching markets under transferable utility

Suppose that women are competing in two bilateral matching markets: the job market to match with firms and the marriage market to match with men. In both markets, the agents play a frictionless matching game such that they either remain unmatched or match with a player in the opposite side of the market. The environment consists of a large number of players indexed by i, j, and k for women, jobs, and men, respectively. Consider sets $W, F, M \subset \mathbb{R}$, with densities $\mathcal{W}, \mathcal{F}$ and $\mathcal{M}$, as the population of women, firms and men. The two markets are linked as follows: in the job market, firm j, to evaluate matching with

[3] By focusing our analysis on women, we gain two other technical advantages over previous studies that included both men and women. Firstly, our approach eliminates the need to make assumptions about age differences between spouses and its evolution across different cohorts. Secondly, it allows us to more generally measure educational sorting by going beyond the measurement of homogamy (marrying someone of the same type) which only considers diagonal elements of the educational distribution matrix and ignores sorting outside of these elements.

[4] Chiappori, McCann, and Pass (2022) analyze multi-dimensional matching problems under a separable structure, but their framework is for one bilateral matching market. Here, we consider two interconnected markets that one side of each market which is the common node is bi-dimensional.

woman i, also considers her marital status k. Likewise, in the marriage market, man k is concerned about the occupational status j of the woman i. Thus, in both markets, women are bi-dimensional and the other side of the market is uni-dimensional such that (i, k) matches with j in the job market and (i, j) matches with k in the marriage market.

A matching in the job market is a probability measure $\mu(i, k; j)$, with marginals $\mathcal{W} \times \mathcal{M}$ and $\mathcal{F}$, that characterizes the matching probability of Mrs. $i \in W$ with marital status k and firm $j \in F$. Likewise, a matching in the marriage market is a probability measure $\nu(i, j; k)$, with marginals $\mathcal{W} \times \mathcal{F}$ and $\mathcal{M}$, that characterizes the matching probability of Mrs. $i \in W$ with employment status j and Mr. $k \in M$. To include unmatched players, μ is defined over $(W \times (M \cup \{\emptyset_M\}) \cup \{\emptyset_1\}) \times (F \cup \{\emptyset_F\})$, and ν is defined over $(W \times (F \cup \{\emptyset_F\}) \cup \{\emptyset_2\}) \times (M \cup \{\emptyset_M\})$, where $\emptyset_F$, $\emptyset_M$, $\emptyset_1$, and $\emptyset_2$ are dummy partners for non-working women, single women, unmatched firms, and single men, respectively. Formally,

$$\int_{j \in F \cup \{\emptyset_F\}} d\mu(i, k; j) = \mathcal{W} \times \mathcal{M}\ (i, k), \qquad \int_{(i,k) \in W \times (M \cup \{\emptyset_M\}) \cup \{\emptyset_1\}} d\mu(i, k; j) = \mathcal{F}(j)$$
$$\int_{k \in M \cup \{\emptyset_M\}} d\nu(i, j; k) = \mathcal{W} \times \mathcal{F}\ (i, j), \qquad \int_{(i,j) \in W \times (F \cup \{\emptyset_F\}) \cup \{\emptyset_2\}} d\nu(i, j; k) = \mathcal{M}(k)$$

In this framework, μ and ν determine who matches with whom, but they do not characterize how the gain from matching is divided between the matched partners. To determine allocation, we consider the transferable utility framework in the both markets. A potential match between (i, k) and j generates revenue $r(i, k; j)$ which is divided between the woman as wage w_{ik} and the firm as profit π_j. In the marriage market, a potential match between (i, j) and k generates a joint surplus $s(i, j; k)$, such that a well-chosen cardinalization of individual utilities adds up to that surplus. Let x_{ij} denote the payoff of a woman i with employment status j and v_k be the payoff of a man k in the marriage market. When the matching is probable, i.e., $\mu(i, k; j) > 0$ or $\nu(i, j; k) > 0$, the total gains and the players' payoff functions must satisfy the budget constraints

$$w_{ik} + \pi_j \leq r(i, k; j) \qquad x_{ij} + v_k \leq s(i, j; k) \tag{1}$$

Without loss of generality, we assume zero payoff for unmatched agents. Moreover, we assume that a woman's payoffs from the two markets build her overall utility in a quasi-linear form as

$$u = x + \Phi(w) \tag{2}$$

where Φ is a strictly increasing function that scales the transfer in the job market to the transfer marriage market.

A matching is *stable* in a market if in all matched pairs both players prefer being together to being with others and no matched player would rather be unmatched. Here, because the markets are interdependent, a matching is stable if and only if its payoff functions u_i, π_j and v_k satisfy

$$\forall (i,j,k) \in W \times F \times M, \qquad u_i + v_k \geq s(i,j;k) + \Phi\Big(r(i,k;j) - \pi_j\Big) \tag{3}$$

If condition (3) does not hold for any unmatched group of agents, it is desirable for all of them to leave their current partner(s), match together, and share the extra surplus $s(i,j;k) + \Phi\big(r(i,k;j) - \pi_j\big) - u_i - v_k > 0$. Conditions (1) to (3) imply that at the stable equilibrium, $u_i + v_k = s(i,j;k) + \Phi\big(r(i,k;j) - \pi_j\big)$ holds for any group of partners with positive matching probability.

2.2 Separable structure for unobservables

The population comprises a large number of women, firms, and men belonging to a small number of categories reflecting the type of women, jobs, and husbands, that are observable to the researcher. Let $I \in \{1, \ldots, N_I\}$ be the index of women's type, $J \in \{0, 1, \ldots, N_J\}$ be the job classification, and $K \in \{0, 1, \ldots, N_K\}$ be the index of men's type. Here, $J = 0$ is the null category corresponding to non-working women and $J \in \{1, \ldots, N_J\}$ corresponds to different occupations. Similarly, $K = 0$ corresponds to single women and $K \in \{1, \ldots, N_K\}$ corresponds to the husband's types of married women. We assume that the type rankings are ordinal for all I and for $J, K \geq 1$.

The total gain from matching in each market is composed of a deterministic part based on observable categories and a random component to reflect unobserved heterogeneity in traits and tastes. Typically, the role of unobservable factors of a woman's husband for her employer and vice versa is much less significant compared to the role unobservable traits and tastes of herself for them. Thus, we assume in the job (marriage) market, only the marriage (employment) status of the woman is important for firms (men) and not the specific features of the husband (job). Putting differently, although the marital status of women is important for filling a job position, the specific features of the husband (e.g. his physical attraction, cultural background) are not determinant factors. Similarly, in the marriage market, although men contemplate the employment status and occupation of a woman to match, they do not care about the specific features of the job (e.g. colleague types, workplace). Hence, the surplus in the two markets is written as follows:

$$r(i \in I, k \in K; j \in J) = R^{IJK} + \epsilon_{ij}^{K} \qquad s(i \in I, j \in J; k \in K) = S^{IJK} + \varepsilon_{ik}^{J} \tag{4}$$

Assumption 1 (Separability)**.** *The unobservable terms in surplus functions (4) are separable as:*

$$\epsilon_{ij}^{K} = \eta_i^{JK} + \beta_j^{IK} \qquad \varepsilon_{ik}^{J} = \lambda_i^{JK} + \gamma_k^{IJ}$$

such that $(\eta, \lambda)_i^{JK}, \beta_j^{IK}$, and γ_k^{IJ} are independent random variables.

From a technical point of view, separability rules out any interaction between partners' unobserved heterogeneity in total gain. Although being restrictive, there are several justification for the separability assumption summarized in Chiappori (2017, pp. 89-91). The great advantage of this assumption is reducing the complexity of a two-sided matching problem into a collection of one-sided problems (see Galichon and Salanie, 2021, for more details).

In the basic CS framework, separability is between the unobserved heterogeneity of the two mates. If we ignore the job market for a moment and matching is only between i and k, then separability implies $\varepsilon_{ik} = \lambda_i^K + \gamma_k^I$. In this setting, λ_i^K reflects both the preference of woman i on particular husband type K and specific qualities of i that are particularly attractive or unappealing for men in category K. Similarly, from man k perspective, the gain from matching depends on his partner's category I, and not more details. γ_k^I also captures the particular attractiveness of k for women in category I. Therefore, while separability allows for matching on unobservables, it rules out sorting based on only the unobserved characteristics on both sides of the market. Same interpretation is valid for ε_{ik}^J that also involves the observable category of employment J in the matching decision of the marriage market.

Proposition 1. *Under Assumptions 1, there exist $3 \times N_I \times N_J \times N_K$ numbers U^{IJK}, Π^{IJK}, and V^{IJK}, for $I = 1, \dots, N_I$, $J = 1, \dots, N_J$, $K = 1, \dots, N_K$, such that if at the stable matching woman $i \in I$ matches with job $j \in J$ and man $k \in K$, the individual utilities are:*

$$u_i^{JK} = U^{IJK} + \alpha_i^{JK}, \qquad \pi_j^{IK} = \Pi^{IJK} + \beta_j^{IK}, \qquad v_k^{IJ} = V^{IJK} + \gamma_k^{IJ}$$

where α_i^{JK} is independent of β_j^{IK} and γ_k^{IJ}.

Proof. Appendix A.1

2.3 Conditional returns and sign-based identification

Now, we define the conditional returns and their differences that lay the foundation for measuring aggregate indices, and show that the sign of conditional returns is the same as the sign of their corresponding conditional odds ratios. From now on, we assume that men and women are classified by their education, and jobs are categorized based on their skill requirements. Because there is no parametric assumption on the deterministic term of women's utilities U^{IJK}, we can assume that conditional on J and K, α_i has zero mean in each category I and skip it for computing conditional returns to female education. For education level I, the deterministic surplus of marriage $K \geq 1$ conditional on employment status J is $U^{IJK} - U^{IJ0}$. Therefore, conditional on spouse type K and employment status J, we can define the marriage return to attaining education I_2 compared to I_1 as the differences in their marriage surplus

$$r^m_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_2 J0} - (U^{I_1 JK} - U^{I_1 J0}), \qquad K \geq 1 \tag{5}$$

Similarly, the conditional employment return to education level I_2 compared to I_1 can be defined as

$$r^e_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_2 0K} - (U^{I_1 JK} - U^{I_1 0K}), \qquad J \geq 1 \tag{6}$$

In this framework, the conditional difference between marriage K and employment J returns to education level I_2 compared to education level I_1 becomes

$$\delta^{me}_{I_1 I_2 JK} = U^{I_2 0K} - U^{I_1 0K} - (U^{I_2 J0} - U^{I_1 J0}) \quad J, K \geq 1 \tag{7}$$

The first term is the surplus of higher education for a non-working woman married to husband K, and the second term is the surplus of higher education for a single woman employed as J. Therefore, conditional on spouse K and job J, the difference between marriage and employment returns to female education is equal to the difference in the higher education surplus of married non-working women and a single working women.

The indices defined in (5) and (6) gauge the extensive margins of return to education, i.e., how much higher education changes the gain from marriage compared to singlehood, and the gains from working compared to not working. At the intensive margin, education influences the quality of marriage and employment. In contrast to the extensive margins of the returns in which ordering is not important, for measuring the intensive margins of the returns, we need to measure how much higher education improves or worsens the quality of marriage and employment. In this regard, we define conditional returns at the intensive margin as[5]

$$r^s_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_2 J1} - (U^{I_1 JK} - U^{I_1 J1}) \qquad K \geq 2 \tag{8}$$

$$r^j_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_2 1K} - (U^{I_1 JK} - U^{I_1 1K}) \qquad J \geq 2 \tag{9}$$

Conditional on $J, K \geq 2$, $r^s_{I_1 I_2 JK}$ and $r^j_{I_1 I_2 JK}$ gauge the *better spouse* surplus and *better job* surplus of higher education compared to their bottom ranked categories, respectively. Similar to (7), we can define the difference between better spouse and better job surplus of higher education conditional on J and K as

$$\delta^{sj}_{I_1 I_2 JK} = U^{I_2 1K} - U^{I_2 J1} - (U^{I_1 1K} - U^{I_1 J1}) \qquad J, K \geq 2 \tag{10}$$

For conciseness, we use the following terminology to describe the extensive and intensive margins of return to education in the rest of the paper:

- *Marriage return* r^m: The extensive margin of marriage return to education compared to singlehood as in (5).

[5] When the quality of each classification is reflected in their index number ascendingly, we can also define intensive margin indices by measuring the difference w.r.t to one lower category of spouse or job instead the lowest category. In section 3.4, we discuss such measurements that are closely linked to sorting in the marriage and job markets.

- *Employment return* r^e: The extensive margin of employment return to education compared to non-working as in (6).
- *Spouse return* r^s: The intensive margin of marriage return to education measuring better spouse surplus as in (8).
- *Job return* r^e: the intensive margin of employment return to education measuring better job surplus as in (9).

Next, we link between the return indices and the empirical matching patterns. Let (IJK) be the element corresponding to row I, column J, and layer K of the $N_I \times (1 + N_J) \times (1 + N_K)$ table for the number of matched individuals in different combinations of education, occupation, and marriage categories. The below proposition shows how we can identify the sign of the above conditional return indices and their differences, using the empirical matching frequencies.

Proposition 2 (Partial identification with conditional log odds ratio)**.** *If α_i^{JK} is i.i.d. and its distribution functions $F_\alpha(\alpha_i^{JK})$ is strictly increasing with bounded and continuous derivative, then at the stable matching, we have*

$$
\begin{aligned}
r^m_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_2JK)(I_1J0)}{(I_2J0)(I_1JK)} \gtreqless 0 &\qquad r^s_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_2JK)(I_1J\;1)}{(I_2J\;1)(I_1JK)} \gtreqless 0 \\
r^e_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_2JK)(I_10K)}{(I_20K)(I_1JK)} \gtreqless 0 &\qquad r^j_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_2JK)(I_1\;1\;K)}{(I_2\;1\;K)(I_1JK)} \gtreqless 0 \\
\delta^{me}_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_20K)(I_1J0)}{(I_2J0)(I_10K)} \gtreqless 0 &\qquad \delta^{sj}_{I_1 I_2 JK} \gtreqless 0 \;&\Leftrightarrow\; \ln \frac{(I_2\;1\;K)(I_1J\;1)}{(I_2J\;1)(I_1\;1\;K)} \gtreqless 0
\end{aligned}
$$

Proof. Appendix A.2

This Proposition generalizes an attractive property of separable models of frictionless marriage markets. Graham (2011) shows that in a one-to-one matching framework under separability and i.i.d. feature for unobservables, the sign of the local degree of complementarity is identified. Proposition 2 shows that this sign-based identification is valid in two bilateral matching framework for the conditional returns indices, which have a form of local complementarity. This remarkable property of the model asserts that based on the matching patterns and with no further parametric assumption, we can determine not only the sign of conditional marriage and employment returns to education at different margins but also which one is bigger than the other.

2.4 Extreme value distribution and the conditional returns

To simplify the exposition and following the previous literature, we assume that the unobservables terms have type-I extreme value (Gumbel) distribution. This assumption is essentially based on McFadden's discrete choice model that gives a closed-form formula for conditional choice probabilities. The great advantage of type-I extreme value distribution is converting the local complementarity that has a difference-in-difference form to the log of ratio-of-ratio. Nevertheless, Galichon and Salanie (2021) show that with only separability assumption on unobservables, the equilibrium utilities of all categories are identifiable and the additive random utility matching model can be extended to other distributional assumptions. In section 4, we partly relax this assumption by using earnings data and adding heteroskedasticity in unobservable for estimations. Also, in Appendix B.1, we find that assuming a normal distribution for unobservables yields similar results for the United States.

Proposition 3. *If α_i^{JK} has type-I extreme value distribution, i.e., $F_\alpha(x) = e^{-e^{-x}}$, we have*

$$r^m_{I_1 I_2 JK} = \ln \frac{(I_2JK)(I_1J0)}{(I_2J0)(I_1JK)}, \qquad r^s_{I_1 I_2 JK} = \ln \frac{(I_2JK)(I_1J1)}{(I_2J1)(I_1JK)} \tag{11}$$

$$r^e_{I_1 I_2 JK} = \ln \frac{(I_2JK)(I_10K)}{(I_20K)(I_1JK)}, \qquad r^j_{I_1 I_2 JK} = \ln \frac{(I_2JK)(I_11K)}{(I_21K)(I_1JK)} \tag{12}$$

$$\delta^{me}_{I_1 I_2 JK} = \ln \frac{(I_20K)(I_1J0)}{(I_2J0)(I_10K)}, \qquad \delta^{sj}_{I_1 I_2 JK} = \ln \frac{(I_21K)(I_1J1)}{(I_2J1)(I_11K)} \tag{13}$$

Proof. Appendix A.3

The right-hand sides of (11) and (12) have similar units in terms of conditional odds ratios incorporating the relative importance of success/failure probabilities in terms of their level of magnitude. For example, if $K = 0$ denotes singlehood and $K = 1$ denotes marriage, conditional on employment status J, $\frac{(2J1)/(2J0)}{(1J1)/(1J0)}$ is the marriage odds ratio of education group $I = 2$ to $I = 1$. $r^m_{I_1 I_2 JK}$ and $r^e_{I_1 I_2 JK}$ are respectively measured in terms of the logs of marriage odds ratio and employment odds ratio of education level I_2 to I_1. The positive (negative) values for the returns is proportional to how much the respective odds increases (decreases) by higher education. The conditional difference in the two returns in (13) $\delta^{me}_{I_1 I_2 JK}$ also comprises two odds ratios and can be viewed as the log odds ratio of moving from a single working to a married non-working woman based on education I_2 to I_1. Similar arguments are valid for the intensive margin indices and their difference.

Notably, the conditional odds ratio does not depend on the marginal distributions of the discrete variables.[6] This property is important in our analysis because the marginal distributions of education, employment, and marriage can be very different across space and time. For measuring returns to female education in a specific time and location, we need a method that separates the interaction of female education with marriage and employment from the prevalence of female education, marriage, and employment, per se. The difference in prevalence can stem from factors out of the focus of analysis, such as the cost of education, the structure of labor demand, and marriage norms. With the same logic, Siow (2015) and Chiappori et al. (2020) use the log odds ratio as an index of marriage assortativeness that measures changes in sorting and not changes in marginal distributions of education for men and women. Also, Long and Ferrie (2013) use odds ratio to measure intergenerational occupational mobility irrespective of marginal distributions of occupation across two generations.

[6] If we re-weight rows, columns, and layers, by fixed vectors a^I, b^J, and c^K, the conditional odds ratios do not change.

Overall, while Proposition 2 introduces local odds ratio as a tool to determine the sign of the return, Proposition 3 state that under extreme value distributional assumption they help quantify the value of the returns.

3 Aggregating conditional returns to education

So far, our analysis of the return indices was conditional on J and K. In this section, we explore two strategies for aggregating the conditional returns. The first strategy, which is mostly used in the previous literature, computes the expected returns based on the difference-in-difference of expected utilites. Next, we introduce an alternative strategy which relies on computing expected conditional returns. The advantage of the latter strategy is its independence to marginal distribution of the populations which means that it rules out confounding factors. Regardless of the method, the aggregation applies to the two margins for the returns: The extensive margin in which education affects gains from marriage compared to singlehood and gains from working compared to not-working; The intensive margin in which education influences the quality of spouse and occupation.

3.1 The expected return indices

The interpretation of the conditional returns in (5) to (9) is such that if women i_1 and i_2 are randomly drawn from the large populations of education I_1 and I_2, respectively, and then they are both (forced to) match with categories J and K, how much the expected gain from marriage and working differ between them. However, at stable matching, women are not constrained to selecting specific categories, leading to a higher expected gain compared to the random category assignment. Therefore, one concern to aggregate the conditional returns is whether to include the choice between categories in aggregation or not and how it affects the measurement.

For this reason, we consider two averages for aggregating women utilities who are in education category I: $\bar{u}$ which is the expectation of U^{IJK} and $\overset{*}{u}$ which is the expected utility of women when they choose partner categories too. These aggregations can be unconditional or conditional on one of the two markets.

$$\begin{aligned}
\bar{u}_I &= E_I[U^{IJK}], & \overset{*}{u}_I &= E_I[\max_{J,K}\ U^{IJK} + \alpha_i^{JK}] \\
\bar{u}_{IJ} &= E_{IJ}[U^{IJK}], & \overset{*}{u}_{IJ} &= E_{IJ}[\max_{K}\ U^{IJK} + \alpha_i^{JK}] \\
\bar{u}_{IK} &= E_{IK}[U^{IJK}], & \overset{*}{u}_{IK} &= E_{IK}[\max_{J}\ U^{IJK} + \alpha_i^{JK}]
\end{aligned}$$

Proposition 4. *Let $F(\alpha)$ be the CDF of α_i^{JK} which is strictly increasing with bounded derivatives. The first order approximation at equal U^{IJK}s yields*

$$\overset{*}{u}_I - \bar{u}_I = (N_J+1)(N_K+1)\int_0^1 F^{-1}(x)\ x^{N_J N_K + N_J + N_K} dx$$

Proof. Appendix A.4

This proposition reveals a feature of transferable utility models of frictionless matching markets that, to the best of our knowledge, is not explored in the previous literature. It shows that including the choices between categories mainly adds a constant term to the expected utilities that only depends on the number of categories and the overall shape of the distribution function for unobservables.[7] Notably, under any distributional assumption, the constant term does not depend on the conditional probabilities and the deterministic utilities. This finding simplifies the estimation of expected utility in the CS model under distributional assumptions other than extreme value by approximating the expectation of maximum (Emax) of u_i with expectation of U^{IJK}.

A more practical result of Proposition 4 is that in computing the aggregate returns, using $\bar{u}$ or $\overset{*}{u}$ gives similar results, because their difference is a constant and independent of the U^{IJK}. As the return indices are in difference-in-difference form, the constant terms cancel out and the indices remain almost the same regardless of using $\bar{u}$ or $\overset{*}{u}$. Hence, under any distribution, almost all variation in aggregate returns is from the component that comes from matching within specific categories and not from the choice between different categories. In the rest of analysis, we mainly use $\bar{u}$ for computing the expected returns as it is the weighted average of U^{IJK} and simpler to compute.

3.1.1 The extensive margins of the expected returns

At the extensive margins, education affects the probabilities of marriage and labor participation for women. The former depends on how much higher education changes the average gain from marriage compared to singlehood, and the latter depends on the changes in gains from working compared to not working by higher education. The aggregate indices of the extensive margins of marriage return to education I_2 from I_1 for spouse type K and employment return to education I_2 from I_1 for occupation J are

$$\bar{r}^m_{I_1 I_2 K} = \bar{u}_{I_2 K} - \bar{u}_{I_2(K=0)} - \bar{u}_{I_1 K} + \bar{u}_{I_1(K=0)}, \qquad K \geq 1 \tag{14}$$

$$\bar{r}^e_{I_1 I_2 J} = \bar{u}_{I_2 J} - \bar{u}_{I_2(J=0)} - \bar{u}_{I_1 J} + \bar{u}_{I_1(J=0)}, \qquad J \geq 1 \tag{15}$$

The unconditional indices of marriage return and employment return to education I_2 compared to education I_1 are defined as follows

$$\bar{r}^m_{I_1 I_2} = \bar{u}^{K\geq 1}_{I_2} - \bar{u}^{K=0}_{I_2} - \bar{u}^{K\geq 1}_{I_1} + \bar{u}^{K=0}_{I_1} \tag{16}$$

$$\bar{r}^e_{I_1 I_2} = \bar{u}^{J\geq 1}_{I_2} - \bar{u}^{J=0}_{I_2} - \bar{u}^{J\geq 1}_{I_1} + \bar{u}^{J=0}_{I_1} \tag{17}$$

[7] In the particular case of extreme value distribution, Appendix A.4 shows that the difference is equal to the log number of categories to choose: $\overset{*}{u}_I - \bar{u}_I = \ln(N_J + 1) + \ln(N_K + 1)$

where the condition in the superscript is for taking expectation, like $\bar{u}_I^{K\geq 1} = E_I[U^{IJK}|K \geq 1]$.[8] Equations (16) and (17) measure the extensive margins of the returns for an average married woman ($\bar{u}_I^{K\geq 1}$) and an average employed woman ($\bar{u}_I^{J\geq 1}$).

Similarly, we can define $\bar{\delta}_I^r$ as the aggregate difference between marriage and employment returns to education level I_2 compared to the level I_1.

$$\bar{\delta}^{me}_{I_1,I_2} = \bar{u}_{I_2}^{J=0,K\geq 1} - \bar{u}_{I_1}^{J=0,K\geq 1} - \bar{u}_{I_2}^{J\geq 1,K=0} + \bar{u}_{I_1}^{J\geq 1,K=0} \tag{18}$$

The first two terms is the surplus of higher education for married non-working women averaged over different husbands and the rest is surplus of higher education for single working women averaged over different occupations. The aggregate difference between the marriage and employment returns to education is equal to the difference between them.

3.1.2 The intensive margins of the expected returns

At the intensive margin, education influences the quality of marriage and employment. In contrast to the extensive margins of the returns in which ordering is not important, for measuring the intensive margins of the returns, we need to measure how much higher education improves or worsens the quality of marriage and employment. Conditional on $J, K \geq 2$, $r^s_{I_1 I_2 JK}$ and $r^j_{I_1 I_2 JK}$ in Proposition 3 gauge the *better spouse* surplus and *better job* surplus of higher education, respectively, and $\delta^{sj}_{I_1 I_2 JK}$ is their difference. Then, similar to (16) and (17), the aggregate spouse and job return indices can be defined as follows

$$\bar{r}^s_{I_1 I_2} = \bar{u}_{I_2}^{K\geq 2} - \bar{u}_{I_2}^{K=1} - \bar{u}_{I_1}^{K\geq 2} + \bar{u}_{I_1}^{K=1} \tag{19}$$

$$\bar{r}^j_{I_1 I_2} = \bar{u}_{I_2}^{J\geq 2} - \bar{u}_{I_2}^{J=1} - \bar{u}_{I_1}^{J\geq 2} + \bar{u}_{I_1}^{J=1} \tag{20}$$

For married women, $\bar{r}^s_{I_1 I_2}$ gauges how much attaining education level I_2 compared to level I_1 affects the marriage surplus. In the same manner, for working women, $\bar{r}^j_{I_1 I_2}$ measures the increase in occupational surplus when the education level increases from level I_1 to level I_2. Similar to (18), we can also define $\bar{\delta}^{sj}_{I_1 I_2}$ as the average difference between these intensive margins of the married and employed women.

$$\bar{\delta}^{sj}_{I_1 I_2} = \bar{u}_{I_2}^{J=1,K\geq 2} - \bar{u}_{I_1}^{J=1,K\geq 2} - \bar{u}_{I_2}^{J\geq 2,K=1} + \bar{u}_{I_1}^{J\geq 2,K=1} \tag{21}$$

3.1.3 Collapsed classifications and expected returns under extreme value distribution

Proposition 3 indicates that under extreme value assumption, the conditional return indices are in form of conditional log odds ratios. An intuitive way to aggregate returns is to collapse classifications into

[8] We can similarly define $\overset{*}{r}^m$ and $\overset{*}{r}^e$ in which e.g. $\overset{*}{u}_I^{K\geq 1} = E_I[\max_{J,K\geq 1}\ u_i]$.

the groups of interest and computing unconditional log odds ratio. In particular, for measuring marriage (employment) return to female education one can collapse the three-dimensional patterns over working status (husband's education) and compute the log odds ratio of a 2×2 table. The below proposition shows that under extreme value assumption, the expected returns computed using $\overset{*}{u}_I$ are indeed the log odds ratio of the matching patterns collapsed over the classifications that are not of direct interest. For instance, $\overset{*}{r}^m_{I_1 I_2}$ represents the log of the odds ratio for a 2×2 table where I_1 and I_2 are the two rows. One column shows the overall population with $K = 0$, while the other column shows the overall population with a value of $K \geq 1$.

Proposition 5. *If α_i^{JK} has type-I extreme value distribution,*

$$\overset{*}{r}^m_{I_1 I_2} = \ln \frac{(I_2 + \ \geq 1)(I_1 \ + \ 0)}{(I_2 \ + \ 0)(I_1 + \ \geq 1)}, \qquad \overset{*}{r}^s_{I_1 I_2} = \ln \frac{(I_2 + \ \geq 2)(I_1 \ + \ 1)}{(I_2 \ + \ 1)(I_1 + \ \geq 2)}$$

$$\overset{*}{r}^e_{I_1 I_2} = \ln \frac{(I_2 \ \geq 1 +)(I_1 \, 0 +)}{(I_2 \, 0 +)(I_1 \ \geq 1 +)}, \qquad \overset{*}{r}^j_{I_1 I_2} = \ln \frac{(I_2 \ \geq 2 +)(I_1 \, 1 +)}{(I_2 \, 1 +)(I_1 \ \geq 2 +)}$$

$$\overset{*}{\delta}^{me}_{I_1 I_2} = \ln \frac{(I_2 \, 0 \ \geq 1)(I_1 \ \geq 1 \, 0)}{(I_2 \ \geq 1 \, 0)(I_1 \, 0 \ \geq 1)}, \qquad \overset{*}{\delta}^{sj}_{I_1 I_2} = \ln \frac{(I_2 \, 1 \ \geq 2)(I_1 \ \geq 2 \, 1)}{(I_2 \ \geq 2 \, 1)(I_1 \, 1 \ \geq 2)}$$

where the conditions for each index specify the range of values that are included in the summation and '+' means summation over all possible values of the index.

Proof. Appendix A.5

Although collapsing the population may seem natural to measure the returns, the association measured in the collapsed tables can differ substantially or even be in the opposite direction of conditional associations within each layer, a phenomenon known as Simpson's paradox (Simpson, 1951). For example, if we collapse the layers of the contingency table below, the log odds ratio turns negative, despite both layers having positive log odds ratios.

$$\text{layer 1:} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \qquad \text{layer 2:} \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \qquad \text{collapsed:} \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}$$

The Simpson's paradox refers to the fact that collapsing variables may influence the relationship between the variables being studied and if ignored the structure of the collapsed association becomes substantially different from the partial associations. In other words, by collapsing tables, we do not control for potentially confounding factors and the association structure among the variables of interest may not necessarily indicate their true interrelationships, but may instead reflect a confounded effect, which is the influence of the collapsing variable on the variables of interest. For this reason, we propose an alternative way for aggregation of the conditional return without collapsing the matching patterns.

3.2 Measuring returns free of marginal distributions

An important complication for measuring marriage and employment returns to education arises from the sensitivity to variations in the distributions of education and occupation. Changes in population supplies, the marriage rate, and labor participation rate, affect the probability of marriage and employment irrespective of the changes in the relative probabilities for different education levels. For the intensive margins, differences in the distribution of education among men and women and changes in occupational structure compounds the measurement of the returns.

An attractive feature of the conditional return indices are their independence to the marginal distribution of education, marriage, and employment. To elaborate this further, consider two different points in time or space that are only different in one-way secular trends of the three dimensions, such that

$$U_2^{IJK} = U_1^{IJK} + a^I + b^J + c^K \tag{22}$$

where a^I, b^J, and c^K are vectors of one-way fixed effects across education, occupation, and marriage groups, respectively. Then, from (5) and (6), we have $r^{m,1}_{I_1I_2JK} = r^{m,2}_{I_1I_2JK}$ and $r^{e,1}_{I_1I_2JK} = r^{e,2}_{I_1I_2JK}$. Next, consider two-way trends such that

$$U_2^{IJK} = U_1^{IJK} + A^{IJ} + B^{IK} + C^{JK} \tag{23}$$

where A^{IJ}, B^{IK}, and C^{JK} are two-way fixed effects across their respective dimensions. Here, using (5) and (6), we can simply show that $r^m_{I_1I_2JK}$ and $r^s_{I_1I_2JK}$ are independent of A^{IJ} and C^{JK}, and $r^e_{I_1I_2JK}$ and $r^j_{I_1I_2JK}$ are independent of B^{IJ} and C^{JK}. In other words, the conditional marriage and spouse returns only depend on the two-way education distribution of men and women, and the conditional employment and job returns only depend on the two-way distribution of women's education and occupation. Note that these properties are generally valid for the return indices without any distributional assumption for the unobservable terms.

This independence to marginal distributions is, however, not valid for the expected return indices defined above. To show this, we expand $\bar{r}^m_{I_1I_2}$ based on its elements. Same procedure can apply to other extensive and intensive margin indices.

$$\bar{r}^m_{I_1I_2} = \sum_{J=0}^{N_J}\sum_{K=1}^{N_K}\Big(\frac{P^{I_2JK}}{P^{I_2(+)(K\geq 1)}}U^{I_2JK} - \frac{P^{I_1JK}}{P^{I_1(+)(K\geq 1)}}U^{I_1JK}\Big) - \sum_{J=0}^{N_J}\Big(\frac{P^{I_2J0}}{P^{I_2(+)0}}U^{I_2J0} - \frac{P^{I_1J0}}{P^{I_1(+)0}}U^{I_1J0}\Big) \tag{24}$$

where $P^{IJK} = \frac{(IJK)}{\sum_L\sum_M(ILM)}$, and '(+)' and $(K \geq 1)$ in the super index mean summation over all possible values of J and over $K \geq 1$, respectively (i.e., $P^{I(+)(K\geq 1)} = \sum_{J=0}^{N_J}\sum_{K=1}^{N_K} P^{IJK}$ and so on). If U_1 and U_2 are linked as in (22) and (23), then one can show that $\bar{r}^m_{I_1I_2}$ is only independent to one-way fixed effects of women education a^I and not other marginal distributions. Same argument is valid for the expected return indices computed using $\overset{*}{u}$ in Proposition 5. For this reason, we build alternative return indices by directly aggregating conditional returns instead of difference-in-difference of expected utilities.

Because we are considering large matching markets, the matching patterns for two different types of women are statistically independent. This means that the probability of observing two random women one with education I_1 conditional on J_1K_1 and another with education I_2 conditional on J_2K_2 is $P^{I_1J_1K_1}P^{I_2J_2K_2}$. For the return indices conditional on I_1I_2JK as in (5) to (10), we have $J_1 = J_2 = J$ and $K_1 = K_2 = K$, thus the weights for conditional odds ratios in large matching markets becomes

$$P^{I_1I_2JK} = \frac{P^{I_1JK}P^{I_2JK}}{\sum_{L=0}^{N_J}\sum_{M=0}^{N_K} P^{I_1LM}P^{I_2LM}} = \frac{(I_1JK)(I_2JK)}{\sum_{L=0}^{N_J}\sum_{M=0}^{N_K}(I_1LM)(I_2LM)}$$

To generate aggregate "marginal-free" indices independent of marginal distributions as for conditional returns of I_1I_2JK, we can find the expectation of conditional returns using $P^{I_1I_2JK}$. The expected return indices can be written in the generic form $r_{I_1I_2} = \sum_J\sum_K \omega_{I_1I_2JK}\ r_{I_1I_2JK}$ where $\omega_{I_1I_2JK}$ is the proper weights based on $P^{I_1I_2JK}$ and summing to one. Then, the change in expected utilities between two points can be written as

$$r^2_{I_1I_2} - r^1_{I_1I_2} = \sum_J\sum_K(\omega^2_{I_1I_2JK} - \omega^1_{I_1I_2JK})\,\frac{r^1_{I_1I_2JK} + r^2_{I_1I_2JK}}{2} + \sum_J\sum_K \frac{\omega^1_{I_1I_2JK} + \omega^2_{I_1I_2JK}}{2}\,(r^2_{I_1I_2JK} - r^1_{I_1I_2JK})$$

where the first term depends on the change in weights and the second term is independent of that. In the same way, by considering a fixed weight $\hat{\omega}_{I_1I_2JK}$ equal to the average of $\omega_{I_1I_2JK}$ at all points, we can define $\hat{r}_{I_1I_2} = \sum_J\sum_K \hat{\omega}_{I_1I_2JK}\ r_{I_1I_2JK}$, and using that define the marginal-free return indices as follows

$$\hat{r}^m_{I_1I_2K} = \sum_{J=0}^{N_J} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(+)K}} r^m_{I_1I_2JK} \qquad \hat{r}^e_{I_1I_2J} = \sum_{K=0}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2J(+)}} r^e_{I_1I_2JK} \tag{25}$$

$$\hat{r}^m_{I_1I_2} = \sum_{J=0}^{N_J}\sum_{K=1}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(+)(K\geq 1)}} r^m_{I_1I_2JK} \qquad \hat{r}^s_{I_1I_2} = \sum_{J=0}^{N_J}\sum_{K=2}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(+)(K\geq 2)}} r^s_{I_1I_2JK} \tag{26}$$

$$\hat{r}^e_{I_1I_2} = \sum_{J=1}^{N_J}\sum_{K=0}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(J\geq 1)(+)}} r^e_{I_1I_2JK} \qquad \hat{r}^j_{I_1I_2} = \sum_{J=2}^{N_J}\sum_{K=0}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(J\geq 2)(+)}} r^j_{I_1I_2JK} \tag{27}$$

$$\hat{\delta}^{me}_{I_1I_2} = \sum_{J=1}^{N_J}\sum_{K=1}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(J\geq 1)(K\geq 1)}} \delta^{me}_{I_1I_2JK} \qquad \hat{\delta}^{sj}_{I_1I_2} = \sum_{J=2}^{N_J}\sum_{K=2}^{N_K} \frac{\bar{P}^{I_1I_2JK}}{\bar{P}^{I_1I_2(J\geq 2)(K\geq 2)}} \delta^{sj}_{I_1I_2JK} \tag{28}$$

where $\bar{P}^{I_1I_2JK}$ is the average of $P^{I_1I_2JK}$ over different points in time or space to be compared.

The aggregate $\hat{r}$ indices defined in (25) to (28), in contrast to the $\bar{r}$ indices defined in (14) to (21), are independent to one-way and two-way marginal distributions of population in the same way as conditional returns. Thus, they control for the confounding factors that might alter the direction of association in conditional and collapsed tables. In other words, $\hat{r}^m_{I_1I_2}$ and $\hat{r}^s_{I_1I_2}$ only depend on the association between education and marriage and they are independent of the changes in education×employment and employment×marriage joint trends. Similarly, $\hat{r}^e_{I_1I_2}$ and $\hat{r}^j_{I_1I_2}$ are determined by the association between education and employment regardless of the two-way trends of education×marriage and employment× marriage. We need to keep these properties in mind to interpret the estimations because they determine what the return indices measure and what they rule out. In the following, we explore different instances of change in marginal distributions and how $\bar{r}$ and $\hat{r}$ are affected by them.

First consider the tendency for singlehood increases among men but not among women. If this increase homogeneously happens across all educational groups, it only changes the one-way marginal distribution of the marriage group (c^K) and while changing $\bar{r}$, the $\hat{r}$ return indices are unaffected. But if the increase in singlehood rate is heterogeneous across educational groups, e.g. because of competition over educated men in the marriage market, the two-way distribution of education×marriage (B^{IK}) and subsequently marriage and spouse return indices are affected by this change. Similar analysis is valid for employment return. A homogeneous change in female labor demand across all occupations only affects one-way occupational distribution b^J and only $\bar{r}$ and not $\hat{r}$ is affected. But if female labor demand increases only for professional jobs that employ high educated women, then it changes the two-way education×occupation (A^{IK}) and thus employment and job return indices.

Next, we explore the cases in which the changes in the two-way marginal distributions alter $\bar{r}$ but have no impact on $\hat{r}$. For instance, suppose female education causes better labor market opportunities that reduces financial dependence on a husband and the pecuniary gains of marriage for women. While this channel may alter $\bar{r}^m$ and $\bar{r}^s$, it has no impact on $\hat{r}^m$ and $\hat{r}^s$ because $\hat{r}^m$ and $\hat{r}^s$ only measure the components of marriage return to education that are independent the interaction of education and marriage via employment. In a similar way, $\hat{r}^e$ and $\hat{r}^j$ are unaffected by the channel that higher education increases the chance of marrying an educated husband and having higher household income might discourage women from work. In short, $\hat{r}$ indices rule out not only the secular trends of education, employment, and marriage, but also any indirect effect on marriage and employment from each other. These instances are related to the impact of a change in A_{IK} (B_{IK}) on $\hat{r}^m$ and $\hat{r}^s$ ($\hat{r}^e$ and $\hat{r}^s$), but a change in C_{JK} also affects the two types of indices differently. In this regard, consider any changes in factors such as home production technology and childcare services that may affect the tradeoff between women's employment and marriage. While these channels may alter $\bar{r}^m$, $\bar{r}^s$, $\bar{r}^e$ and $\bar{r}^j$, they have no impact on $\hat{r}^m$, $\hat{r}^s$, $\hat{r}^e$ and $\hat{r}^j$ because they are independent of these trade-offs that only change the two-way distribution of employment×marriage.

In summary, while the $\bar{r}$ indices are in general dependent to marginal distributions and vulnerable to confounded effects, $\hat{r}$ return indices rule out not only all homogeneous changes across classifications but also the indirect effects through employment on marriage and vice versa, and the changes in the association between marriage and employment that are independent from education. $\hat{r}^m, \hat{r}^s$ only capture the underlying forces driving a change in the association between education and marriage types, and $\hat{r}^e, \hat{r}^j$ gauge the factors affecting the association between education and employment type.

3.3 Sign-based identification of aggregate returns

In this part, we explore the set identification of the aggregate indices without making any assumptions about the distribution of random surplus terms. The proposition below outlines the conditions under which the aggregate returns and their differences can be partially identified by their sign.

Proposition 6 (partial identification of aggregate return)**.** *If α_i^{JK} is i.i.d, and its distribution is strictly increasing with bounded and continuous derivative, at stable matching*

- *The sign of the expected return indices and their differences is identified with the sign of the log odds ratio of their corresponding 2×2 collapsed table listed in Proposition 5.*
- *If all conditional return elements of an aggregate return index are identified with the same sign, then the marginal-free aggregate index is identified with that sign.*
- *If one of the conditional return elements is identified with the opposite sign of some others, there exist distributions for difference in unobservable terms to make the aggregate marginal-free returns positive or negative.*

Proof. Appendix A.6

According to Proposition 5, under extreme value assumption the expected return indices are in the log odds ratio form. Proposition 6 shows that without any distributional assumption, the sign of the expected return is identified by the sign of the log odds ratios. However, section 3.1.3 highlights that the expected returns indices are not immune to confounding effects, which means that even if all conditional returns elements of an expected return index have the same sign, it may end up with an opposite sign. In this regard, the advantage of marginal-free indices is their independence to the confounding effects of the third factor on the association of the variables of interest.

3.4 Market assortativeness at the intensive margin

Our definitions of conditional returns in (16) to (21) are based on the comparison of utilities under two education levels by benchmarking the proper classification of marital or employment status. This is a convenient strategy for the purpose of measuring different returns to education and provides simple and intutive interpretation of the indices: At the extensive margin, it measures how much higher education affects the average gain from marriage (employment) compared to singlehood (non-employment). At the intensive margin, it measures how much higher education affects the average quality of spouse (job) conditional on being married (employed).

Still the framework introduced in section 2 allows for alternative indices at the intensive margin[9] to measure sorting by education in the marriage and job markets. Since we measure the quality of husband

[9]Note that, at the extensive margin the benchmark must always be singles and non-employed individuals, but to measure sorting, similar to spouse and job returns, we drop them. In this regard, the intensive margin indices can be found from extensive margin indices (e.g. $r^s_{I_1 I_2 JK} = r^m_{I_1 I_2 JK} - r^m_{I_1 I_2 J1}$ and $\theta^s_{I_1 I_2 JK} = r^m_{I_1 I_2 JK} - r^m_{I_1 I_2 JK-1}$) but not vice versa.

by his education, when the education levels are ascendingly ranked, we can define an index to capture the degree of educational assortative matching in the marriage market

$$\theta^s_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_1 JK} - U^{I_2 JK-1} + U^{I_1 JK-1} \tag{29}$$

and aggregate it using one of the methodologies described in sections 3.1 and 3.2. In this regard, when $\theta^s_{I_1 I_2}$ is increasing in I_2 for a fixed I_1, the marriage market is positively assortative in the sense that attaining more education increases the chance of marrying a husband with higher education. However, in contrast to marriage assortativeness indicators focusing on homogamy and the tendency to marry someone with the same education level, $\theta^s_{I_1 I_2}$ measures improvements in spousal quality anywhere in the distribution. For the same reason, various assortativeness indices in the literature (for a summary see Chiappori et al., 2020), are generally dependent to the size of diagonal elements of the two-way educational distribution of men and women. With an uneven distribution of education for the two population, diagonal elements can be small and matching with better spouse for educated women occurs in non-diagonal elements. The advantage of $\theta^s_{I_1 I_2}$ is capturing changes in the quality of spouse conditional on marriage over all elements of the population distribution. Similarly, we can define $\theta^j_{I_1 I_2}$ to reflect sorting in the job market that can be interpreted as the incremental change in the quality of job due to higher education, conditional on being employed:

$$\theta^j_{I_1 I_2 JK} = U^{I_2 JK} - U^{I_1 JK} - U^{I_2 J-1K} + U^{I_1 J-1K} \tag{30}$$

and aggregate it in the same way as the above job return indices.

4 Empirical methodology

The extreme value assumption, provides a useful formula $U^{IJK} = \ln \frac{(IJK)}{(I00)}$ for computing the deterministic parts of women's utilities by benchmarking U^{I00} (see Appendix A.3). Without the extreme value assumption, we can evaluate choice probabilities for alternative distributions of the unobserved heterogeneity by defining

$$\mathcal{P}^{IJK}(U^{F\cdot\cdot}) = \int_{-\infty}^{+\infty} \prod_{LM \neq JK} F_\alpha(U^{IJK} - U^{ILM} + \alpha_f^{JK}) f_\alpha(\alpha_f^{JK}) d\alpha_f^{JK} \tag{31}$$

From (43) in the Appendix A.2, the above function must equal the empirical probability patterns P^{IJK}. Note that because the sum of the probability vector is one, we must normalize one term in $U^{I\cdot\cdot}$, which is equivalent to finding the differences between U^{IJK} and $U^{I00} = 0$ by solving $\mathcal{P}^{IJK}(U^{I\cdot\cdot}) = P^{IJK}$. The advantage of the extreme value distribution is the closed-form solution as in Proposition 3, but for other distributions, we need to numerically compute the differences. After estimating the deterministic utilities, the indices introduced in section 3 that are linear combinations of them can be consequently computed.[10]

4.1 Over-identified estimation

The above procedure using only empirical matching patterns provides a just-identified estimation of the model not allowing for more parameters and performing statistical inference. In fact, the original CS framework is a nonparametric estimation of the deterministic surplus patterns, given a parametric assumption for the distribution of unobserved heterogeneity.

Previous studies turn CS model into an over-identified framework by considering multi markets (Chiappori et al., 2017), parametric surplus (Galichon and Salanie, 2021), and using future information of household decisions to recover the marriage surplus (Chiappori et al., 2018). Given that our model includes the decision in the labor market besides the marriage market, we can exploit earnings premium as the level of transfer to household from the job market and turn the just-identified CS structure to an over-identified model in a new manner in this literature.

When earnings data are available, one can add new over-identifying restrictions to the model by assuming that the difference between conditional deterministic gains of two distinct jobs for otherwise identical women is proportional to the difference in the average scaled earnings. From stability condition (3), for partners with positive matching probability, we have $u_i + v_k = s(i,j;k) + \Phi\big(r(i,k;j) - \pi_j\big)$. In deterministic terms, this equality becomes

$$U^{IJK} + V^{IJK} = S^{IJK} + \Phi(W^{IJK})$$

where $W^{IJK} = R^{IJK} - \Pi^{IJK}$ represents the average job's earnings of employed women in each category. We then postulate that given a fixed categories of I and K, the change in overall household payoff from different jobs of an employed woman is equal to the difference in their scaled earnings from the two jobs:

$$U^{IJK} - U^{IJ'K} + V^{IJK} - V^{IJ'K} = \Phi(W^{IJK}) - \Phi(W^{IJ'K}), \qquad J, J' \geq 1 \tag{32}$$

To establish over-identification restrictions from equation (32), we need to make two assumptions: the functional form of $\Phi(.)$ and how the extra wage is divided between the spouses. We consider a CRRA form for $\Phi(x) = x^{1-\phi}/(1-\phi)$ and estimate ϕ as a structural parameter of the model. Furthermore, we obtain sharing rules based on the just-identified structure of CS. If all players have type-I extreme value unobservable terms, because $U^{IJK} - U^{IJ'K} = V^{IJK} - V^{IJ'K} = \ln \frac{(IJK)}{(IJ'K)}$, the sharing rule is even and half of the additional transfer is allocated to married women. For single individuals, no sharing occurs, and the entire difference in job's earnings is received by the respective person. Thus, we specify the

[10] According to Proposition 4, for expected returns, we can approximate $\mathring{u}$ with $\bar{u}$ which is a linear function of U^{IJK}.

over-identification restrictions in (32) as follows

$$U^{IJK} - U^{IJ'K} = \rho_{IK}\big(\Phi(W^{IJK}) - \Phi(W^{IJ'K})\big), \qquad J, J' > 0 \tag{33}$$

$$\Phi(x) = \frac{x^{1-\phi}}{1-\phi} \qquad \rho_{IK} = \begin{cases} \frac{U^{IJK} - U^{IJ'K}}{U^{IJK} - U^{IJ'K} + V^{IJK} - V^{IJ'K}} & K \geq 1 \\ 1 & K = 0 \end{cases}$$

4.2 Estimating heteroskedastic model

In Proposition 3, we assumed that the distribution of unobserved heterogeneity is the same for all education and occupation classes. This assumption restricts variation in the relative importance of the observed and unobserved components across different groups. The wage data enables us to partially relax the simple extreme value distributional assumption and estimate a heteroskedastic model for unobserved terms in occupation of women such that their utility takes the form

$$u_i^{JK} = U^{IJK} + \sigma_I \alpha_i^{JK} \tag{34}$$

This structure changes the CDF to estimate the conditional probabilities of α_i to $F_\alpha(x) = e^{-e^{-\frac{x}{\sigma_I}}}$ and by changing the CDF in the proof of Proposition 3, we have

$$U^{IJK} = \sigma_I \ln \frac{(IJK)}{(I00)} \tag{35}$$

with U^{I00} normalized to zero.

As the scaling of the earnings surplus to marriage surplus can be heterogeneous for different women types, we can also add heteroskedasticity in the scaling function by assuming $\Phi(W^{IJK}) = \frac{(W^{IJK})^{1-\phi_I}}{1-\phi_I}$. Then, using the second order Taylor expansion, we can derive earnings moments (32) as

$$e(U^{IJK}, U^{IJ'K}, \phi_I) = \frac{\Delta_J U^{IJK}}{\rho_{IK}} (\bar{W}^{IK})^{\phi_I} + \frac{(\Delta_J W^{IJK})^2}{2\bar{W}^{IK}} \phi_I = \Delta_J W^{IJK}, \qquad J, J' > 0 \tag{36}$$

where $\bar{W}^{IK}$ is the average of W^{IJK} over J, $\Delta_J U^{IJK} = U^{IJK} - U^{IJ'K}$, and $\Delta_J W^{IJK} = W^{IJK} - W^{IJ'K}$. The second order approximation term which depends only on ϕ_I does not alter the estimations of U^{IJK} and only matters for the precision of ϕ_I.[11]

The earnings moments (36) are added to the population moments (35) making an over-identified system to estimate U^{IJK}. Because no earnings information is available for nonworking women, there are no additional moments for U^{I0K}. Hence, when earnings information is available, we have an over-identified model to estimate U^{IJK} for $J \neq 0$ and still a just-identified model to estimate U^{I0K}. In the heteroskedastic model to estimate $U^{IJK}, J \neq 0$, for each I, there are $N_J(N_K + 1) + 2$ parameters,

[11] We also iterate with higher orders of Taylor expansion, but while more computationally intensive the estimations of ϕ_I do not significantly change.

$N_J(N_K+1)$ moment equations as (35), and $(N_K+1)(N_J-1)$ moment equations as (36). Thus, the number of over-identifying restrictions of the heteroskedastic model is $N_I(N_K+1)(N_J-1)-2N_I$.

4.3 Minimum distance estimator

We can estimate the vector of parameters $h = (U^{IJK}, \sigma_I, \phi_I)$ by the below minimum distance estimator:

$$\min \; g^T(h) \times \Omega^{-1} \times g(h)$$

where

$$g(h) = \begin{bmatrix} \dfrac{U^{IJK}}{\sigma_I} - \ln \dfrac{(IJK)}{(I00)} \\ e(U^{IJK}, U^{I1K}, \phi_I) - (W^{IJK} - W^{I1K}) \end{bmatrix}$$

The weighting matrix Ω^{-1} is the inverse of the variance-covariance matrix of the empirical moments which is the optimal weighting based on the theory of the minimum distance estimator (MDE). Appendix A.7 explores how Ω is computed from data by assuming a multinomial distribution for the matching patterns and diagonal covariance structure for earnings. In the optimal MDE, the variance-covariance matrix of the parameters can be recovered from $\text{Var}(h) = (G^T \times \Omega^{-1} \times G)^{-1}$ where G is the derivative matrix of the vector of moment equations $g(h)$ with respect to the vector of structural parameters h. Finally, since all return indices are linear functions of U, their standard errors are computed by $\text{Var}(A \times h) = A \times \text{Var}(h) \times A^T$.

5 Data

According to Proposition 3, we can compute the conditional return indices using the 3-way discrete distribution of women's education, employment, and marriage.[12] This discrete distribution can be estimated using cross-sectional household censuses or representative surveys with information only on education and employment/occupation of household members. This is a great advantage of the model that makes the estimation feasible with widely available cross-sectional data, allowing for evidence over time and across space. When the datasets contain information about earnings of individuals, we can enhance the estimations from the just-identified framework to the over-identified framework as explained in section 4.

In this section, we describe different datasets that are used for estimation. The random samples of households are drawn from Integrated Public Use Microdata Series (IPUMS). To define marriage, we use the standard definition of IPUMS datasets which is based on the self-reported relationship to household head as "spouse". In addition, we only keep households with a female head or spouse aged between 35 to 50 in the sample, as this is the age range in which education is finished, and marriage and female labor force participation have stable rates (see Figure 1 of Chiappori et al., 2020). For the same reason,

[12] Although this proposition is based on type-I extreme value distribution, we can estimate the return indices for alternative distributions as we do for normal distribution in the Appendix B.1.

we drop the sample of unemployed women because they are not in the equilibrium of their job market status compared to out of labor force women. In the following, we first describe more details on the United States data, as it has the longest time span and the biggest sample size, and then we turn to cross-country data.

5.1 U.S. data

The U.S. data is drawn from IPUMS (Ruggles et al., 2020) and includes three sources: Census extracts for 1960, 1970, 1980, 1990, and 2000; American Community Survey (ACS) for 2001-2019; and Current Population Survey (CPS) for 1968-2022. As Census and ACS have bigger sample sizes, we used them as the main datasets for estimation and cross-check their trends with CPS. Also, as the annual sample of ACS generates some fluctuations in estimations, we use the IPUMS's 5-year ACS datasets for 2005-09, 2010-14, and 2015-19, and center them on their middle years (2007, 2012, 2017). Appendix Table **??** presents the lists of the U.S. datasets and their sample size.

The classifications of marriage and employment consist of a null category for single and non-working women who are not matched with a partner, and ranked groups for those who are matched. When dealing with ordinal discrete data, the selection of categories for ordered variables is based on various criteria. On the one hand, having more finely tuned categorizations can enhance statistical power for detecting associations (Agresti, 2010). On the other hand, to estimate models that rely on odds ratios, all elements of the contingency table must be non-zero, which can hinder the creation of detailed classifications for education and occupation.[13] In addition, for ordinal scales, unlike "interval scales", the absolute distances between categories are unknown and must be selected in a way that generates enough contrast between groups. Moreover, when selecting categorization for multiple variables with different characteristics (like spouse and job types), it is essential that the marginal distribution of the variable is similar in a given time or space. If one variable has finer categorization in the lower-tail of its distribution while the other has finer categorization in the upper-tail, the aggregate association measures are distorted by non homogeneity of the classifications of the two variables.[14]

Considering these criteria, in our main analysis, we split the U.S.'s detailed educational attainment code in IPUMS into five groups:

1. *Dropouts (D)*: those who have less than 12 years of education or have no high school qualification

2. *High school (H)*: those who finished high school

3. *Some college (C)*: those who attend 1 to 3 years of college, including Associate's degree

[13] Zero elements are quite likely when the sample size is small. For example, it is unlikely to find a working woman with a university degree marrying an illiterate man without work unless the sample size is big enough. A workaround for zero numbers in contingency table analysis is to change zeros to 0.5, but it is criticized because of adding fake data (see section 2.5.2 of Kateri, 2014).

[14] Note that the marginal-free return indices are independent of marginal distributions *given a classification* and not independent to changes in scales or merging categories.

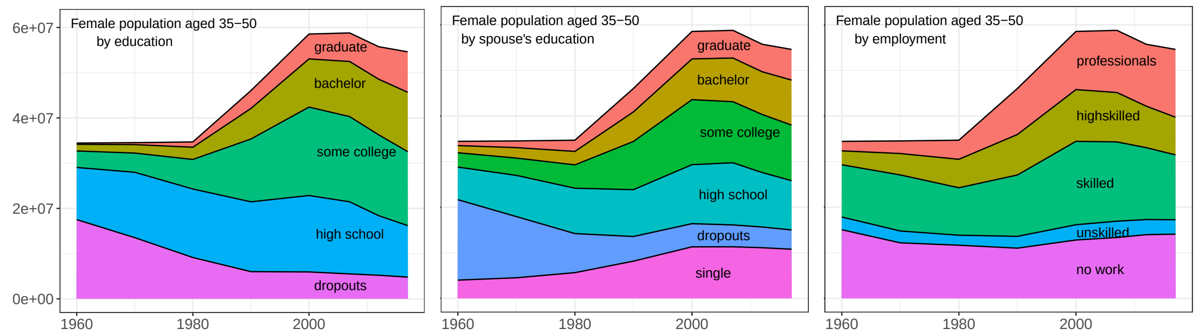

Figure 1: Female population aged 35-50 by education, marital, and employment status

4. *Bachelor (B)*: those who have bachelor's degree

5. *Graduate (G)*: those who have higher education than bachelor's degree

The occupation coding of the U.S. data is based on the SOC classification, and it is converted to ISCO based on the correspondence table provided by the Labor Bureau. We classify occupations into four groups based on the first digit ISCO coding system:

1. *Unskilled (U)*: elementary occupations (code 9)

2. *Skilled (S)*: skilled/semi-skilled workers (codes 0, 4 to 8)

3. *High skilled (H)*: technicians and associate professionals (code 3)

4. *Professional (P)*: managers, professionals (codes 1, 2)

Figure 1 displays the changes in population shares of women's education, their husband's education, and their job types. Between 1960 and 1990, we note a significant decrease in the number of individuals who dropped out of high school, along with a surge in the proportion of those with college degrees or higher, for both genders. Following this, the population shares remained relatively stable from 2000 onwards. The trend in job types reveals a substantial rise in the population of women employed in skilled, high skilled, and professional jobs between 1980 and 2000, with little variation in the share of each type before and after this period.

For each element of the population distribution in which women have a job, we compute inflation-adjusted mean and variance of yearly job's earnings by averaging IPUMS's *INCWAGE* variable that reports total pre-tax wage and salary income for the previous year.[15] This data will be used to estimate the over-identified model with heteroskedastic unobservables as described in section 4. With five education and four occupation categories, the model has 80 degrees of freedom.

Figure 2 illustrates the average earnings ratio for different job and spouse types to evaluate the adequacy of the classification in generating rankings and distinctions among categories. We observe

[15] We adjust the values by the annual consumer price index of the U.S. Bureau of Labor Statistics and the unit of earnings is in 1983 dollars.

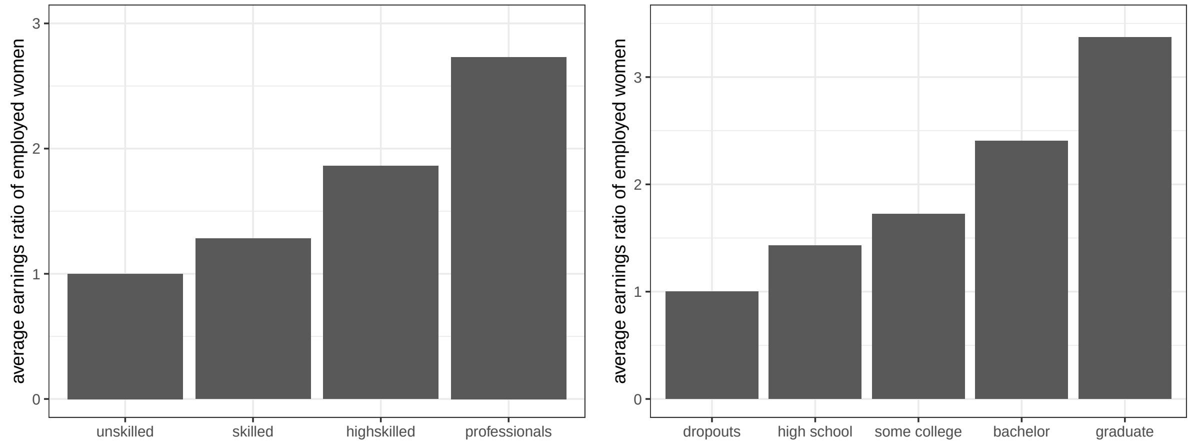

Figure 2: Average earnings ratio to the lowest rank by education and job classifications

increasing trends for both job and spouse classifications, and the top category exhibits similar numbers in terms of the earnings ratio.

5.2 Cross-country data

For cross-country comparisons, we mainly use the harmonized census data of IPUMS International (Minnesota Population Center, 2020). To have a large sample size, we focus on countries with more than 4 million population and more than 2% random sample of the census in the data. The exceptions are China, Italy after 2011, and Spain after 2005. Among available countries in IPUMS, some do not have occupational information (e.g., Pakistan, Bangladesh, Russia), in some, the individuals are not organized into households (e.g., U.K., Netherlands, Finland), and in many others, one of these problems exists for specific census rounds (e.g., Argentina, Indonesia, Ireland). For any of these reasons, the country or round is not included in estimations.

In the harmonized IPUMS datasets, the occupation code based on one digit of ISCO is available and enables us to separate jobs. However, because of sparse data of unskilled and professional occupations especially at university education in many countries, we merge unskilled to skilled and professional to high skilled jobs in the cross-country data. In addition, the harmonized educational coding is not similar to the IPUMS-USA, and we look into the national classification of each country to reconcile the cross-country and the United States classifications. Some college and graduate education are not separately coded in most countries, and thus in cross-country analysis, we merge college to graduate education and categorize them as university education. Finally, because wage and salary is not reported in most of the countries, we cannot estimate over-identified model and unlike U.S. estimations, all cross-country estimations are just-identified.[16]

[16]By comparing the over-identified result of the U.S. with its just-identified estimation and find qualitatively similar results. Thus, we expect that cross-country trends would remain similar if wage data was available.

6 Marriage and employment returns to female education

6.1 United States over time

We start presenting our empirical findings by discussing the evidence from the United States, which has the longest data span. After discussing the heteroskedasticity parameters, we present the trends for marriage return conditional on spouse education ($r^m_{I_1 I_2 K}$) and employment return conditional on job classification ($r^e_{I_1 I_2 J}$) and discuss the trade-offs between husband type and job earnings of employed women. Afterwards, we present the finding regarding both of the expected and marginal-free aggregate measures for the return to education and sorting in the markets over time. Finally, we proceed with a number of robustness tests. Unless states otherwise, the U.S. estimations are from heteroskedastic model with extreme value distribution for unobservables as described in section 4. The advantage of a distributional assumption like this to nonparametric sign-based identification is quantifying the returns and investigating their trends.

6.1.1 Preference parameters and hetroskedasticity

Table 1 presents the estimated curvature parameter ϕ under both homoskedastic and heteroskedastic assumptions. Given that the concavity of $\Phi(\cdot)$ is increasing in ϕ, higher levels of ϕ reflects more relative importance of transfer from marriage in the utility of women as defined in (2). The homoskedastic estimations in Table 1, indicate a rise in the average ϕ from 1960 to 1990, stabilizing around 0.7 thereafter. This value of ϕ suggests that, on average, the transfer from marriage is proportional to the root 3.33 of earnings in 1983 U.S. dollars. Across different levels of education, we observe an increasing pattern for ϕ_I in education I, indicating a greater valuation of surplus share from marriage among highly educated women.

In the Appendix, we present other parameters that are not of a central interest, along with optimization outputs. In particular, in Appendix Table B, the heteroskedasticity parameters σ_I show little variation around 1 suggesting that the unobserved heterogeneity of women in matching preference is not dispersed differently across various educational groups.

6.1.2 Conditional returns

Figure 3 demonstrates the marriage return to education I for different spouse types K as defined in (14). Because these indices are still conditional, there is little difference between $\bar{r}$ indices and $\hat{r}$ indices and we present the results only for $\hat{r}$. The numbers are measured in the log odds ratio, which tells us how much the log odds of marrying a husband K instead of remaining single are higher among women with education I than among women who dropped out of high school. We observe that marrying a higher educated husband always has positive marriage returns for women and the negative returns might only happen when they "marry down". Not all marrying down cases, however, have a negative marriage return

Table 1: Estimation of preference parameter ϕ for different educational categories. Standard errors are in parenthesis.

year	homoskedastic ϕ	heteroskedastic ϕ_I				
		dropouts	high school	some college	bachelor	graduate
1960	0.63	-0.225	0.604	0.675	0.681	0.629
	(0.071)	(7.471)	(0.276)	(0.134)	(0.091)	(0.106)
1970	0.663	0.563	0.552	0.356	0.666	0.73
	(0.056)	(0.57)	(0.454)	(1.323)	(0.094)	(0.056)
1980	0.676	0.378	0.541	0.633	0.706	0.709
	(0.061)	(3.982)	(0.632)	(0.234)	(0.098)	(0.069)
1990	0.705	0.348	0.644	0.713	0.73	0.721
	(0.06)	(6.169)	(0.312)	(0.143)	(0.091)	(0.084)
2000	0.691	0.392	0.586	0.69	0.721	0.731
	(0.081)	(5.406)	(0.654)	(0.181)	(0.119)	(0.097)
2007	0.714	0.471	0.659	0.722	0.744	0.74
	(0.065)	(2.297)	(0.307)	(0.133)	(0.1)	(0.087)
2012	0.711	0.48	0.606	0.727	0.745	0.737
	(0.069)	(2.118)	(0.56)	(0.138)	(0.104)	(0.087)
2017	0.717	0.508	0.656	0.732	0.754	0.732
	(0.071)	(1.913)	(0.397)	(0.128)	(0.109)	(0.101)

for women. In particular, women with a graduate degree who marry a bachelor's degree husband have significantly positive returns in all years. Also, for women with a bachelor degree who marry a husband with some college education, the marriage return is positive. This finding is consistent with Low (2023) that implies a non-monotonic relationship between female human capital and the quality of her husband because of the tradeoff between women's fertility and investment in human capital. For this reason, women at the very top of the human capital distribution marry down, on average, compared to women with lower human capital.

Another fact from Figure 3 is that in earlier years, women with a high school degree have significantly higher surplus share when marrying up to a bachelor's or a graduate degree but in recent years their marriage return is the same if they marry with a same degree husband or marry up. This suggests that the surplus of marrying up for them is reducing over time. In addition, having a graduate degree husband has a significantly positive marriage return at all education levels in 1960, but over time the surplus share of women with less education marrying a graduate husband reduces while for women with a graduate degree, it goes up. This pattern suggests that assortative matching produces higher gains at the top of the education distribution.

Figure 4 illustrates the employment return to different educational groups compared to dropouts for each occupation. The return to above college education is the highest conditional on professional jobs, and the log odds ratio of obtaining a professional job increases with educational attainment relative to dropouts. Moreover, the return to education is larger for high skilled than skilled jobs and for skilled than unskilled jobs. Specifically, the trend of the log odds ratios is slightly decreasing conditional on high skilled jobs and becomes more negative and decreasing for skilled and unskilled job. Notably, the return to education for unskilled jobs is negative for all education levels above high school, indicating

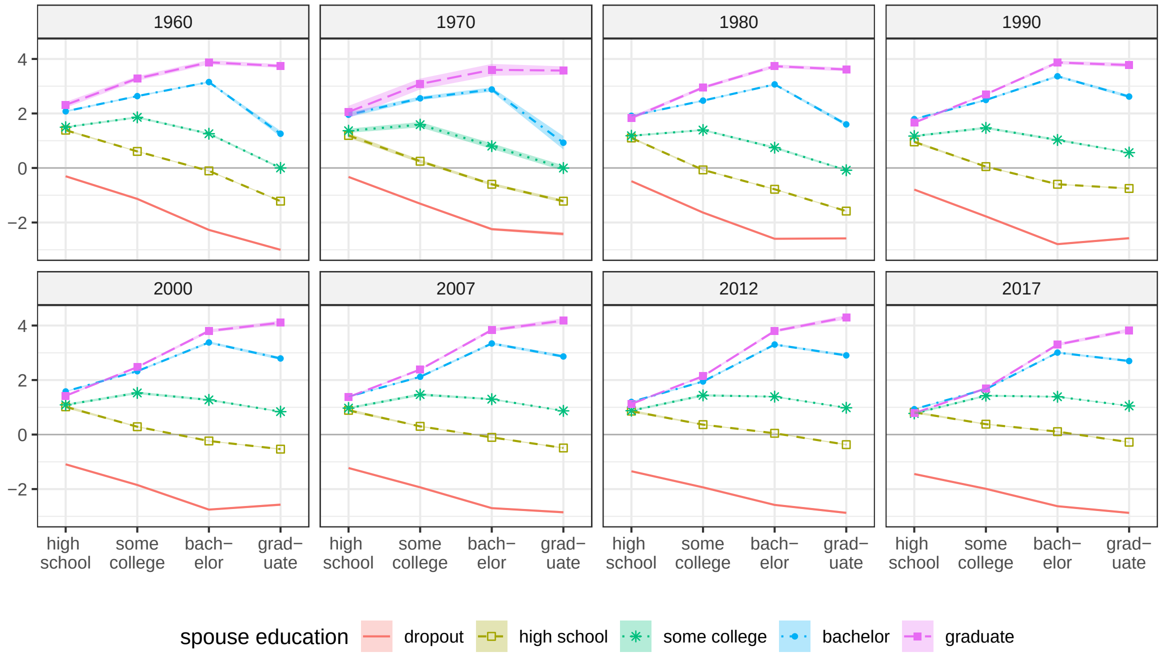

Figure 3: Marriage returns to female education conditional on spouse education over time in the U.S. The indices are $\hat{r}^m_{I_1 I_2 K}$ as defined in (14) for different spouse types K. The estimations are from the heteroskedastic model (34) and the shaded areas are their confidence intervals. Because the indices are conditional on K, estimation using marginal-free indices $\bar{r}^m_{I_1 I_2 K}$ are very similar and not reported for convenience.

that above high school educated women gain more by staying out of the labor force than by working as unskilled workers, compared to dropouts. A similar pattern is observed conditional on skilled jobs before 1980, but its return numbers are moving up after this period.

6.1.3 Earnings equivalence of spouse education for employed women

Using the estimated ϕ_I for different years, it is possible to rescale the return indices using the $\Phi^{-1}(\cdot)$ function and to interpret them in dollar terms. Given that all conditional return indices (equations (5) to (9)) are derived from difference-in-differences of deterministic utilities, we can approximate $\Phi^{-1}(r) \approx r/\Phi'(\bar{W})$, where $\bar{W}$ represents the average earnings corresponding to the index r. However, for conditional returns involving the utility of non-working women, the estimation of $\Phi^{-1}(r)$ is not possible.[17] This means that the conversion to earnings units is possible for all conditional job returns and conditional marriage and spouse returns involving *only employed women*. In contrast, neither the employment return nor marriage and spouse returns of non-working women can be converted to the earnings units. Following the methodology described in section 3, the conditional returns in dollar terms can be aggregated to unconditional returns. In particular, for employed women, we build the marriage return index conditional on spouse type $r^m_{I_1 I_2 K}$ in dollar terms by aggregating conditional return as (25). This dollar unit index reflects the equivalent compensation of marrying a spouse type for a single woman with education level I_2,

[17] From the inverse function theorem: $\Phi^{-1}(\Delta U) \approx \Phi^{-1\prime}(\bar{U})\Delta U = \Delta U/\Phi'(\Phi^{-1}(\bar{U}))$, but we only observe $\Phi^{-1}(\bar{U}) = \bar{W}$ for employed women. For non-working women, the utility is estimable up to an additive constant U^{I00} and its inverted values are unknown.

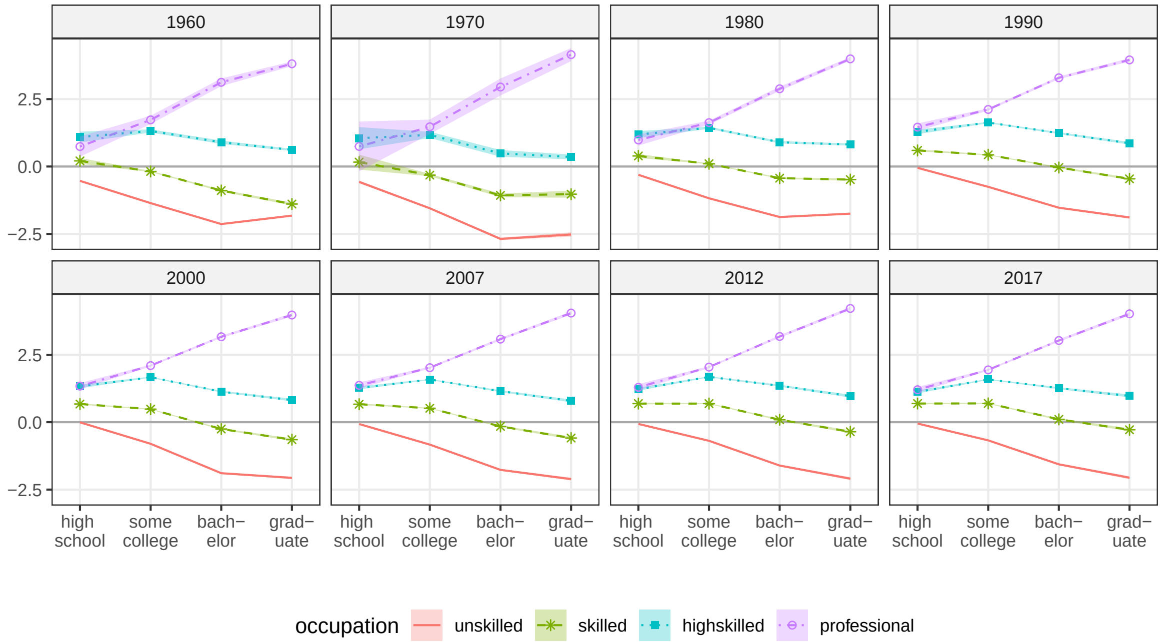

Figure 4: Employment returns to female education conditional on occupation type over time in the U.S. The estimated indices are $\hat{r}^e_{I_1 I_2 J}$ as defined in (15) for the two job types. The estimations are from the heteroskedastic model (34) and the shaded areas are their confidence intervals. Because the indices are conditional on J, estimation using marginal-free indices $\bar{r}^e_{I_1 I_2 J}$ are very similar and not reported for convenience.

relative to a single woman with education level I_1. Since $\bar{W}$ is measured in 1983 dollars, this compensation is also in 1983 prices.[18]

In Table 2, we report r^m_{1IK} in both 1983 USD and as a percentage of annual earnings of the corresponding educational cohort of women in 1960 and 2017. The provided estimates highlight the extra value of matching with different type of men for each type of women in comparison to dropout women. Consistent with Figure 3, the estimated numbers in Table 2 suggest that compared to dropouts, single women in all educational cohorts are willing to forgo a share of their earnings in exchange for a husbands with higher education than themselves. In contrast, the first row for each year indicates that marrying a dropout husband yields negative return for all educational groups. Notably, the most negative impact is for women with bachelor's degrees in 2017, who would pay approximately 14 percent of their earnings, in addition to what a dropout woman would pay, to remain single rather than marrying a husband without a high school education.

A husband with a high school education is more attractive for women with high school diplomas and some college education than dropout women and they would pay about 2-3 percent of their annual earnings for that. For women with bachelor's degrees, the preference for marrying a high school-educated husband is similar to that of dropout women, with the equivalent compensation around zero. However, women with graduate degrees would pay roughly 1.5 percent of their annual earnings, aside from the amount a dropout woman pays, to avoid marrying a husband with only a high school education. The compensation

[18]The 2023 USD is about 3 times of the 1983 USD.

Table 2: Estimation of equivalent annual enumeration of husband improvement from high school to bachelor across different educational cohorts of *employed* women. ΔW is measured in 1983 U.S. dollars.

year	husband type	equivalent annual compensation ΔW				percent of yearly earnings $\Delta W/W$			
	K	HS	SC	B	G	HS	SC	B	G
1960	dropout	-63	-329	-757	-695	-0.78	-3.66	-6.38	-4.18
	high school	232	222	8	-350	2.87	2.47	0.07	-2.10
	some college	248	625	455	63	3.08	6.95	3.83	0.38
	bachelor	328	804	1071	371	4.07	8.93	9.02	2.23
	graduate	336	972	1223	1029	4.17	10.80	10.30	6.19
2017	dropout	-528	-1522	-2700	-2468	-4.91	-11.52	-13.76	-9.05
	high school	333	282	1	-404	3.10	2.13	0.00	-1.48
	some college	317	1124	1461	1038	2.95	8.51	7.45	3.81
	bachelor	368	1238	3404	3148	3.42	9.37	17.35	11.55
	graduate	290	1150	3598	4948	2.70	8.71	18.34	18.15

for marrying a husband with some college education or higher is positive across all educational levels, indicating that, compared to dropout women, all other categories would willingly give up a portion of their earnings to enter such a marriage. The willingness to forgo earnings also holds for marriages with husbands holding bachelor's and graduate degrees with larger compensation values.

Converting the 2017 estimates to 2023 prices, the numbers suggest that relative to dropout women, women with bachelor's degrees would pay around \$4,400 yearly to marry a husband with some college education, and slightly over \$10,000 yearly to marry husbands with bachelor's or graduate degrees rather than remaining single. Moreover, in 2017, women with graduate degrees would annually spend around 18 percent of their earnings, which equates to approximately \$15,000 in 2023, to marry a husband with the same educational background instead of remaining single.

Table 2 also allows us to gauge the equivalent values for various types of husbands by calculating the differences between corresponding rows. For instance, the difference between the final and initial rows for each year indicates the worth of marrying a husband with a graduate degree instead of a high school dropout husband for each women's educational group, relative to dropout women. In terms of 2017 and 2023 prices, this number amounts to \$2,500 for high school-educated women, \$8,000 for women with some college education, \$18,900 for women with bachelor's degrees, and \$22,200 for women with graduate degrees.

To illustrate better the spouse return over time, in Table 3, we estimate the equivalent annual remuneration of transitioning the type of husband from high school degree to bachelor's degree over various years. The estimated numbers highlights how much women in each education category value men with a bachelor's degree vs. men with a high school degree compared to dropout women. Over time, the numbers show a widening utility gap between husbands with high school degrees and those with bachelor's degrees until 1990 that starts to shorten afterward. As we later see in Figure 5 this shift is primarily attributed to an increase in the return from high school dropout to high school, rather than a decrease in the return from high school dropout to bachelor's degree. Consistent with this trend, the highest

Table 3: Estimation of equivalent yearly compensation of husband change from high school to bachelor across different educational groups of *employed* women. ΔW is measured in 1983 U.S. dollars.

year	equivalent annual compensation ΔW				percent of yearly earnings $\Delta W/W$			
	HS	SC	B	G	HS	SC	B	G
1960	96	581	1063	721	1.2	6.46	8.95	4.34
1970	56	46	1187	1557	0.6	0.46	8.5	7.13
1980	73	605	1932	2096	0.83	6.21	16.92	11.7
1990	239	1413	3048	2981	2.41	11.84	19.59	13.83
2000	102	1044	2911	3570	0.96	8	16.2	15.18
2007	183	1255	3478	4147	1.65	9.21	18.37	15.91
2012	69	1100	3278	3893	0.67	8.62	17.54	14.95
2017	34	956	3403	3553	0.32	7.24	17.34	13.03

equivalent compensation, as a percent of annual earnings, materializes in the year 1990.

In Table 3, we also observe that women with a high school degree have the smallest equivalent annual remuneration which in most years is below \$100 and constitutes less than one percent of their annual earnings. These low numbers for high school women suggest two interrelated channels: first, their preference for highly educated husbands does not substantially different from that of high school dropouts; second, it potentially reflects a preference towards assortative mating within this group. A stronger indication of assortative mating emerges within the bachelor's degree holder category. Across all years, women with bachelor's degrees would exchange the highest percent of their income (often also the highest dollar values) for a partner with a similar educational background rather than a high school-educated partner.

6.1.4 Aggregate returns in the U.S.

In Figure 5, we present the trend of aggregate indices of extensive and intensive margins of the returns based on expected utilities as defined in (16) to (21) and expected conditional returns as defined in (26) to (28). This figure illustrates the change in returns between two adjacent education levels. With the five educational groups, there are four incremental steps for education to compare: dropouts to high school, high school to some college, some college to bachelor, and bachelor to graduate school.

The top row of Figure 5 shows the trend of marriage return. For dropout to high school, the expected return index and marginal-free index show different trends. While the expected marriage return is initially negative and then increasing up to 2000, the marginal-free index is positive from 1960 and declining after 1970. The two indices converge to the same values after 2000, showing that the marriage return declined over time for high school graduates reaching zero and negative values in recent years. For those who attended some college or obtained a bachelor's degree, the trends are similar for the two indices, except for 1960. Overall, starting from negative numbers, there is an increasing trend for the marriage return to college and bachelor's degree for women, with bachelor's degree surpassing college after 1990. For women with graduate-level education, at first, the marriage return is negative compared to those with lower levels of education. However, with an increasing trend, it becomes positive after 2010. These trends suggest that, on average, the relative gain from marriage at the top of the education distribution was far less for women 50 years ago in the U.S. Currently, this pattern is reversed, as evidenced by Figure A8 of Low (2023), which indicates that the spousal income gap between graduate- and college-educated women in the U.S. was negative up to 1990 but has become significantly positive since 2000. Our findings on the expected marriage return indices for women in the top row of Figure 5 are also fairly consistent with Figure 21 of Chiappori et al. (2017).

In contrast to marriage return, the second row of Figure 5 shows that the employment return has a similar trend in the two indices, implying that the confounding factors does not affect odds of employment significantly. The left graph suggests that although the women's employment return to education is initially not very different for high school educated women compared to dropouts, it becomes positive and statistically different from zero after 1980s. As shown in the second and third columns, over the period of study, the odds of female employment only slightly changes by attaining college education compared to high school. In contrast, graduate education has a significantly higher employment return in all years compared to all other groups. The log odds ratio of employment by graduate education is about 1 units higher than bachelor's degree in recent years. This implies that the odds of working for a woman with graduate education is about 3 times higher than a high school or college educated woman.

The third row of Figure 5 shows the difference between marriage and employment returns to education. We observe that the difference in marginal-free returns is negative for transition from dropout to high school after 1980. For some college it is initially negative but, after 2000, is not significantly different from high school. Similar pattern exist for transition from some college to bachelor's degree. For graduate education, in a sharp contrast, we observe the difference between the marriage and employment returns starts with a significantly negative log odds ratio. After 1990, it becomes increasing but still remains negative and significant.

The row four of the Figure 5 shows the aggregate spouse return to education conditional on being married. The positive and significant numbers for both indices suggests a monotonic relationship between education and spouse quality. The fifth row of Figure 5 illustrates the trend of the job return to education conditional on being employed. We observe that the relationship between education and the job return is not changing significantly over time for all education categories and it is monotonic across categories.

Although the intensive margins of marginal-free returns are positive in all years, they demonstrate two alternative patterns with higher levels of education. While the mean level of spouse return decreases when we move from the left graph to the right, the mean level of job return increases. This pattern suggests that more education has an increasing and concave relationship with spouse quality, but an increasing and convex relationship with job quality. Regarding this, we observe positive differences between spouse and job return for high school education and negative differences for graduate education after 1980. Such difference is not very different from zero for college education and bachelor degree.

In Figure 5, we generally find small standard errors of the return indices, except for cases with relatively low population, such as graduate degree women, and estimations for the year 1970 with a notably lower sample counts (Appendix Table A.2). The reason for this pattern is the variance of population moments (35) that, as described in Appendix A.7, is proportional to the inverse of the sample size of the corresponding educational cohorts. In this regard, we later observe higher standard errors in estimations using CPS data that characterizes with a smaller sample than Census/ACS data.

Figure 6 illustrates the trend of aggregate marginal-free returns in dollar units as a percentage of annual earnings. We report estimations using both homoskeastic ϕ and heteroskedstic ϕ_I as reported in Table 1. The confidence intervals are computed by assuming log-normal distribution for earnings and normal distribution for ϕ_I. Overall, we observe similar trends as in Figure 5. The absolute value variation of aggregate marriage return for employed women is less than 2 percent of their earnings. As evident in Table 2, this relatively low number compared to spouse return is due to the averaging over all types of husband. Conditional on the spouse type the return to marriage compared to singlehood is a much larger in either positive or negative numbers as percent of earnings. Because the estimated ϕ_I is increasing in I, using homoskedastic estimation of ϕ results in bigger numbers than heteroskedastic estimation at the bottom of educational distribution and in lower numbers for bachelor and above education.

6.1.5 Market sorting and expected vs. marginal-free indices

In Figure 7, we estimate the aggregate spouse and job sorting indices and their differences using the incremental measures of educational transition. The first row this Figure shows the aggregate spouse sorting by education conditional on being married. We observe two alternative trends for the expected and the marginal-free returns. While expected return is generally decreasing for all educational transitions, the marginal-free index is either constant or increasing over time. The positive and significant numbers for marginal-free index suggests a monotonic relationship between education and sorting in the marriage market, but the expected index shows values closer to zero and even a negative sorting of transition from high school to some college.

This marked contrast between the two indices contributes to the debate on the trend of assortative matching in the United States. While Greenwood et al. (2014) and Chiappori et al. (2017), find an increasing trend for educational sorting in the past decades, particularly among the highly educated, Eika et al. (2019) find a declining trend at the top. According to Chiappori et al. (2020, 2021), the likelihood ratio index used in Eika et al. (2019) is distorted by changes in marginal distribution of the population as the expected index $\bar{\theta}^s_{I_1 I_2}$, but the log odds ratio used in Chiappori et al. (2017) is marginal-free as $\hat{\theta}^s_{I_1 I_2}$. Thus, although the average data might show a decline of educational sorting in the U.S., after removing one-way and two-way confounding factors explained in section 3.2, we observe an increase in the past decades.

The second row of Figure 7 illustrates the trend of the job return to education conditional on being

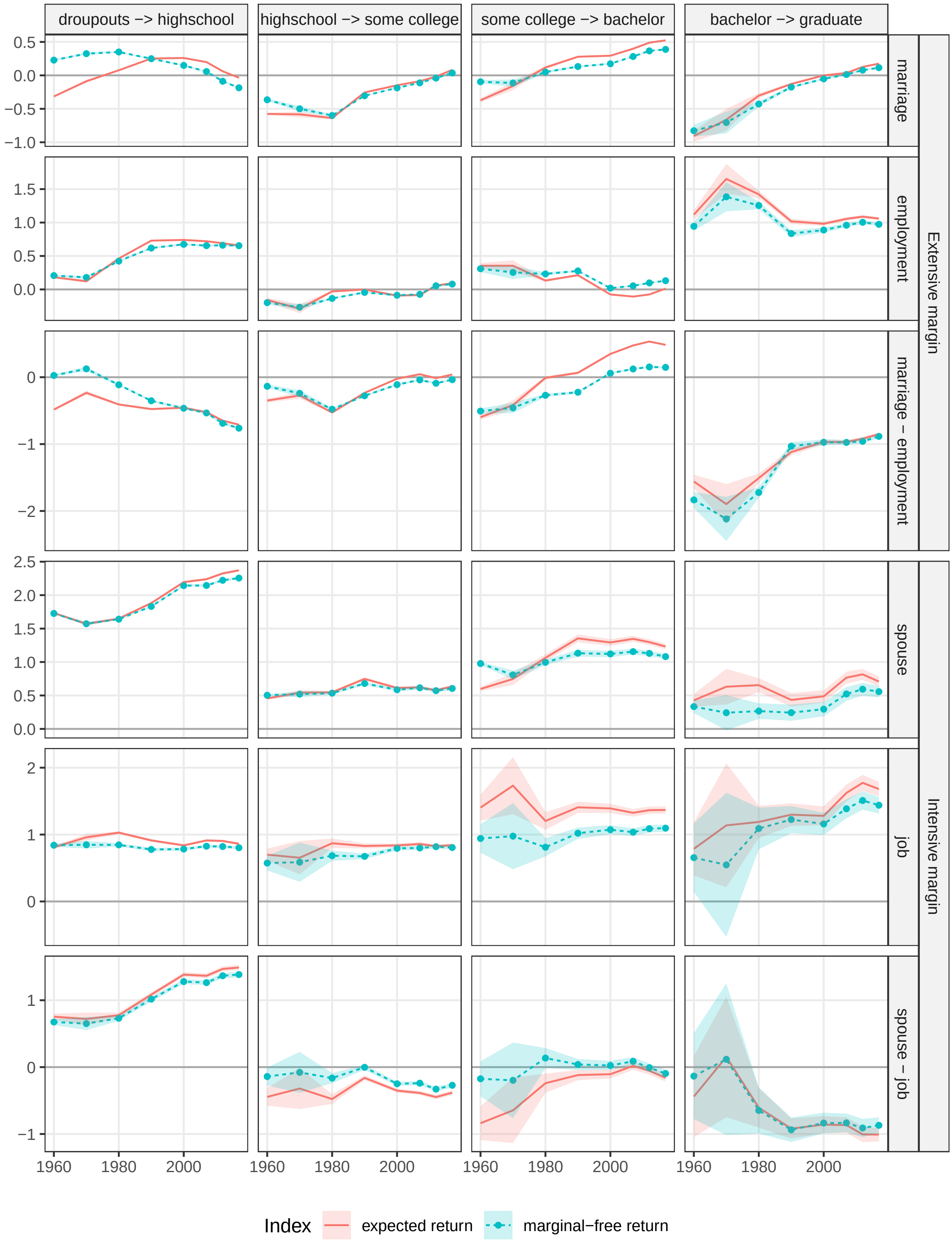

Figure 5: The trend of aggregate returns to female education and their differences in the U.S. The indices for the extensive margin are r^m, r^e, and δ^{me}. The indices for the intensive margin are r^s, r^j, and δ^{sj}. The red solid lines show estimations using $\bar{r}$ indices as defined in (16) to (21) and blue dashed lines are estimated by fixed weights as $\hat{r}$ in (26) to (28). All numbers are estimated using the heteroskedastic model (34) and the shaded areas are the confidence intervals as described in section 4.

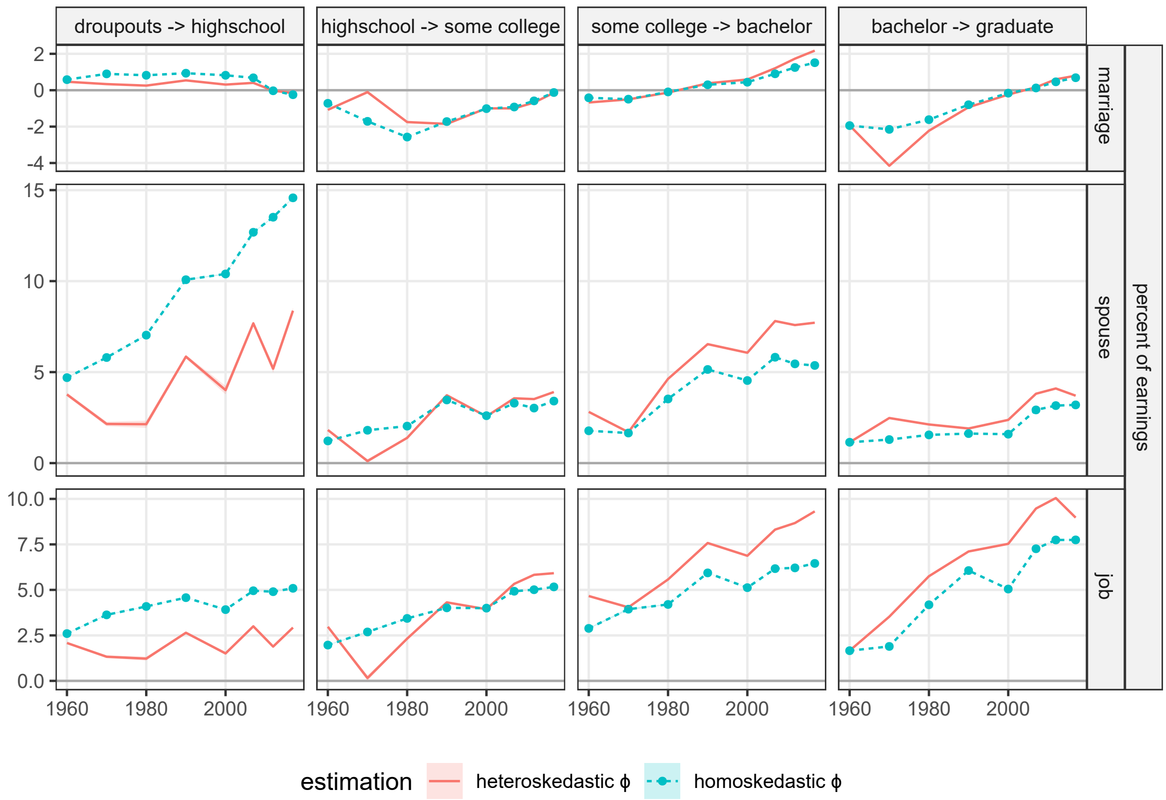

Figure 6: Marginal-free returns to female education as a percentage of annual earnings in the U.S. over time. The conversion to dollar value are based on homoskedastic and heteroskedastic estimations of ϕ_I in Table 1.

employed. We observe that the job sorting by education is not changing significantly over time for all education categories and it is monotonic across categories. Comparing the levels of expected and marginal-free indices implies that the confounding factors coming from variation in marginal distributions are very effective on the measurement of job sorting too especially at the lower end of educational distribution.

In Figure 5, we observe low deviations between the expected and marginal-free return indices as opposed to the stark differences between the expected and marginal-free sorting indices in Figure 7. The similarity between the two indices in Figure 5 is attributed to the fixed reference point, represented by the null or the lowest ranked category of marital or employment status, used in the computation of conditional components the aggregate returns. In contrast, the sorting indices are essentially the double difference of utilities between adjacent categories of spouse or job. Whether calculating first the conditional difference-in-difference and then their average, or conversely first the average elements and then the difference-in-difference, result in far less variation with a fixed benchmark for all components compared to components pertaining to neighboring groups. Therefore, the return indices are less vulnerable to population change and other confounding factors inducing change in marginal distributions than the sorting indices.

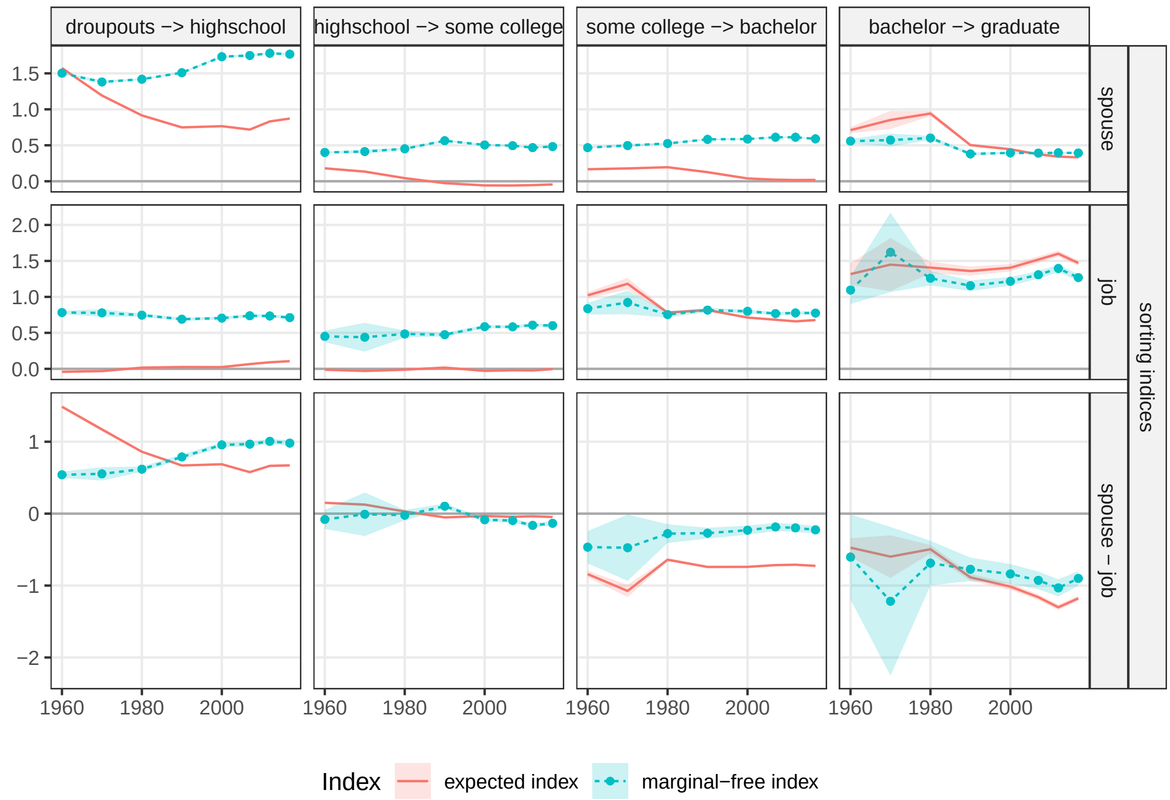

Figure 7: Marginal-free and expected sorting indices in the U.S. over time. The indices are the aggregation of spouse and job sorting defined as (29) and (30).

6.1.6 Robustness checks

In Appendix B, we present a series of robustness tests to verify the reliability of our U.S. estimations. First, in Appendix Figure A.1, we estimate the model using alternative classifications for education and occupation. Qualitatively, the levels and trends of the returns for the new classifications are combinations of those for the categories that are merged to create them. In the top panel, where we only change the classification of occupation, we observe little change in marriage and spouse return. For employment and job returns, the trends are similar, and the curves slightly shift upward or downward. The bottom panel shows the change in education classification, where both women and spouse types change. We observe that the trend of the returns for the merged classifications of education reflects the average of their split classifications.[19]

Second, in Appendix B.1, we change the distribution of unobservable terms from extreme value to normal distribution and do not observe a significant change in the results. Third, we cross-check the estimation of aggregate indices with CPS dataset and find similar trends. As explained above, the sampling error for low values in contingency tables increases the estimation error and generates large volatility in measures, as is visible in CPS estimations. For the same reason, estimating the return using CPS is infeasible for some rounds because its low sample size generates zeros cells. For those elements,

[19] In unreported graphs, we change only the spouse's education classification and find a similar trend as the top panel of Appendix Figure A.1, with the difference that the employment and job returns do not show any change, but the marriage and spouse return curves move upward or downward with similar trends over time.

we replace zero cells in CPS with 0.5 for estimations and use the average earnings for each education and job cohort to fill the missing values.

In summary, the heteroskedastic extreme value estimations for the U.S. imply that, getting a high school degree has a positive spouse and job returns, but in 1960-2017, the trends of its marriage and employment returns are decreasing and increasing, respectively. Attending college compared to high school has no employment return and negative marriage return, but it improve spouse and job qualities. Obtaining a bachelor's degree compared to some college has positive marriage and employment returns, on average, but its impact of spouse and job quality, is bigger. Going to graduate school had initially a significantly negative marriage return, but with an increasing trend its return has surpassed college education. The employment and job returns to graduate education is the highest. In addition, the intensive margins of the returns are increasing in education especially for the job return suggesting that conditional on marriage and employment, female higher education leads to better job and spouse. Overall, at the lower end of educational distribution, marriage return to education is higher than its employment return, but at the upper end the employment return is significantly bigger.

6.2 Sign-based identification of the indices in the U.S.

In this part, we investigate which findings for the U.S. are partially identified and can be generalized to any distributional assumption. According to Propositions 5 and 6, for the expected return indices the sign under any distributional assumption is the same as the sign under extreme value assumption. Therefore, the above findings regarding the sign of the aggregate expected returns and which one is bigger are valid for any other distributional assumptions. However, the expected return indices are vulnerable to confounding factors that change marginal distributions. For this reason, we focus on the identification of the sign of marginal-free indices.

According to Proposition 6 the sign of a marginal-free index can be identified under any distributional assumption if all of its conditional return elements are identified with the same sign, otherwise the aggregate index can get both signs based on proper choices for the distribution. To determine the sign of a conditional return, after computing the log odds ratio, we perform a one-sided Fisher's exact test to examine whether the sign of log odds ratio is significant or not. If the significance of the same sign is valid for all elements of a marginal-free index, it is identified with that sign. This generally means that, the more conditional elements an index aggregates, the less is the chance of its sign-based identification. For this reason, we first investigate the sign of the aggregate marginal-free indices conditional on either J or K and then unconditional returns.

With five ordinal education levels, there are ten pairs of low-high education groups to compare and numerous conditional returns across indices, years, marriage, and employment statuses. For better illustration, we present the summary of the findings for adjacent education levels in Figure 8 and the complete list in Appendix Figures A.4 and A.5. Figure 8 is comprised of multiple panels, with each panel representing an index denoted on the right. Rows depict incremental education levels for comparison, while columns show the marriage or employment status upon which the index is conditioned. Furthermore, each '+' and '–' corresponds to a positive or negative sign in a given year for that conditional index. Within each box, the years are sorted from top left (1960) to bottom right (2017). The fill colors in the figure are proportional to the number of positive or negative signs in the box.

The patterns of marriage return conditional on marital status in the top right panel reveal that positive/negative sign-based identification corresponds to marrying up/down. Specifically, conditional on a dropout husband, the sign of the marriage return is negative up to bachelor's degree, and unidentified for bachelor's to graduate education. This means that if the only available men to match are dropouts, an increase in women's education reduces their gain from marriage compared to staying single. Conditional on high school education for husbands, an increase in women's education from dropout to high school yields a positive marriage return but further education results in a negative marriage return. This pattern applies to some college and bachelor's degrees as well, where a marriage return is negative if the husband has lower education, but positive if the husband has higher education.

The second row's left panel demonstrates a similar pattern in employment return conditional on employment status. While conditional on unskilled jobs, all identified signs are negative, they are positive conditional on professional jobs. Additionally, conditional on skilled and high skilled job, the employment return is identified by positive sign up to high school and some college, respectively. However, for higher levels of education, both are identified by negative signs. Hence, we can conclude that the above findings regarding the conditional returns in Figures 3 and 4 are mostly valid even when the type-I extreme value assumption for unobservables is relaxed.

The top left panel of Figure 8 shows that conditional on employment status the sign of marriage return is mostly unidentified. Likewise, we observe that the sign of employment return is generally unidentified when conditioned on marital status. These results are not surprising since the primary determinants of the marriage return compared to remaining single (employment return compared to not working) is the type of spouse (job) that one has. Women with the same job may have different types of spouses (or vice versa), and thus the quality of the match conditional on the third factor can be either positive or negative. Consequently, it is difficult to determine the aggregate sign of marriage return based on job types and employment return based on spouse type. For the same reason, we cannot identify the sign of unconditional marriage and employment returns (as shown in Table 4).

The third row of Figure 8 displays the results obtained from sign-based identification of the difference between marriage and employment returns conditional on marriage or employment status. We find that marriage return is identified as bigger than employment return at the upper end of spouse distribution and the lower end of job distributions. Conversely, employment return is identified as begin greater for lower types of spouse and higher types of jobs.

At the intensive margin, for each education level, the sign of spouse return is positive conditional on marrying someone with the same or higher education level. The left panel of row four implies that conditional on being out of labor force and being married, college and bachelor education have positive spouse returns, but the sign for graduate education is either not identified or negative. This finding can be explained by the tradeoff between human capital and fertility at the upper-tail of educational distribution as argued by Low (2023).

The fifth row's left panel reveals a similar pattern for the job market as the spouse return: conditional on a job with higher skill requirement the sign of job return is positively identified. Also, the right panel of fifth row suggest that conditional on being single and working, the job return to education at all levels is identified with positive sign in some years, especially for college and bachelor education.

Table 4 shows the sign of the unconditional indices. As it is clear from the conditional returns, the sign of extensive margin of the returns is changing based on the categories of husbands and jobs. Thus, for no pair of education categories the sign of aggregate extensive margin return indices is identified. In comparison, at the intensive margin, the aggregate job returns are identified with a positive sign for dropouts and high school to college+ levels for many years. The sign of aggregate spouse return is only identified positive for transitions from dropouts to higher levels of education. Consistent with Figure 5, we find that the aggregate spouse return of obtaining a high school diploma is identified bigger than its job returns from 1990 onward.

Overall, we can draw strong inferences about marriage and spouse returns conditional on the husband's type, as well as employment and job returns conditional on job type, as the sign of the indices is identifiable under any distribution and remains consistent over time. In contrast, for unconditional returns, only the sign of spouse and job returns is identifiable for some education levels under any distributional assumption. This outcome can be attributed to the non-monotonic behavior of aggregate extensive margin returns by increasing education level and over time, as found in Figure 5. In comparison, we observe a monotonic behavior of the aggregate intensive margin returns that remains stable over time. Therefore, analyzing the return to education at the extensive margin without any distributional assumption can only be done locally and conditional on outcomes. For global measures that summarize the return over different outcomes, one needs to consider a distribution for the random surplus term.

6.2.1 Sorting indices

Figure 9 show the sign-based identification of aggregate sorting indices. The right panel shows that, for each education level, the sign of spouse sorting is positive conditional on marrying someone with the same education level. This provides strong evidence for assortative matching in the marriage market. We also find that spouse sorting is identified with positive sign for H→C conditional on above college education for husband. The spouse returns of D→H conditional on a graduate degree husband and B→G conditional on high school husband are either unidentified or negative. This means that if the distance between the education of the spouses are too much, the women are better off by marrying someone with

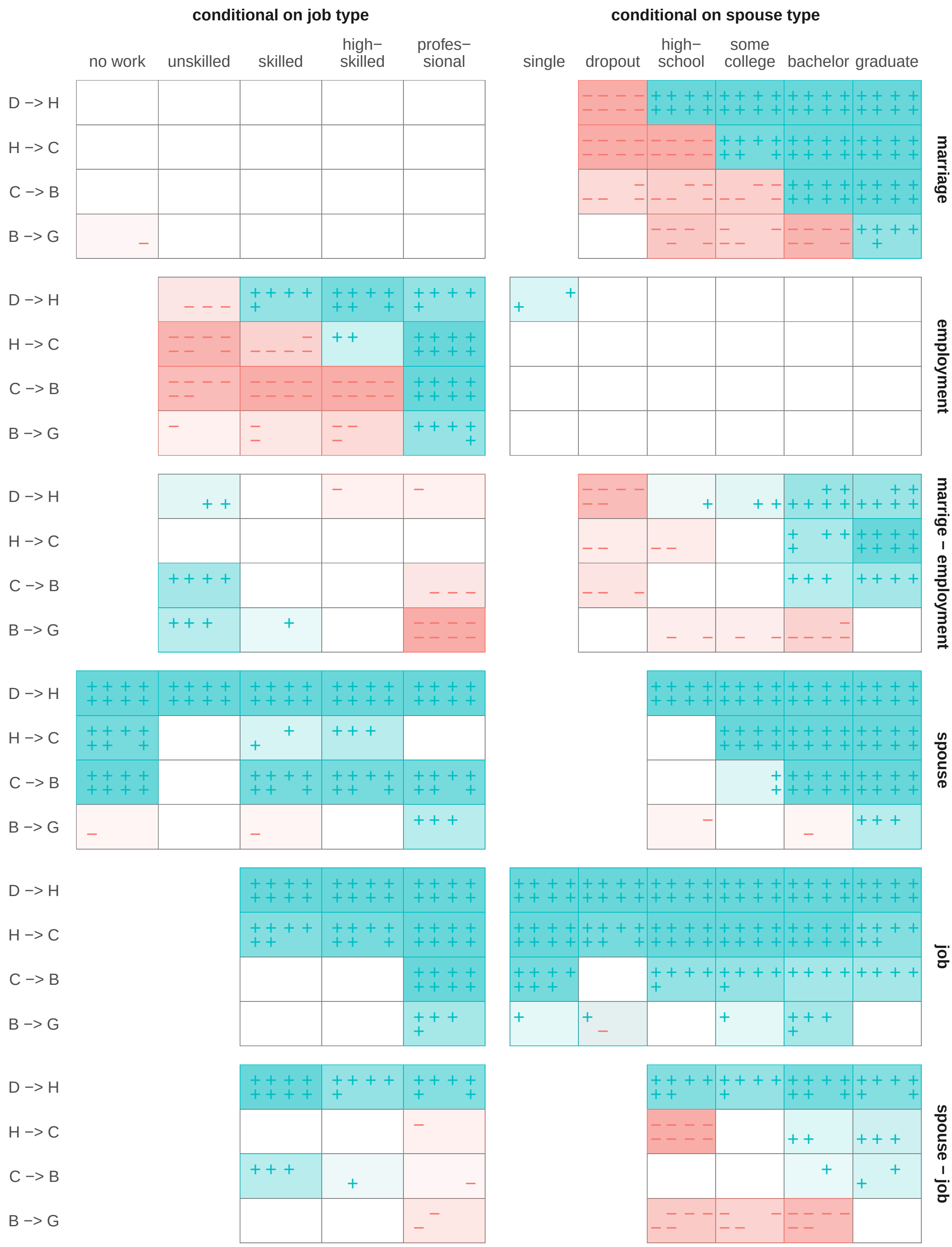

Figure 8: Sign-based identification of marginal-free returns conditional on marriage and employment status. The '+' and '–' mean that all conditional elements of the aggregate indices are significantly identified with and the same signs in one year of the data. The number of signs and fill color in each box show the number of years that the sign is identified. The order of signs within each box are based on years with 1960 in top left and 2017 in bottom right. Blanks means no sign-based identification is inferred. The inference for the sign of conditional odds ratio is based on one-sided Fisher's exact test. Abbreviation of education is D=dropout, H=high school, C=some college, B=bachelor and G=graduate school. The full list of elements and their sign-based identification is available in Appendix Figures A.4 and A.5.

Table 4: Sign-based identification of aggregate marginal-free returns. The '+' and '−' mean that all conditional elements of the aggregate indices are significantly identified with and the same signs. Blanks means no sign-based identification is inferred. The inference for the sign of conditional odds ratio is based on one-sided Fisher's exact test. Abbreviation of education is D=dropout, H=high school, C=some college, B=bachelor, and G=graduate school.

Aggregate returns																	
Education		marriage return								spouse return							
1	2	1960	1970	1980	1990	2000	2007	2012	2017	1960	1970	1980	1990	2000	2007	2012	2017
D	H									+	+	+	+	+	+	+	+
D	C									+	+	+	+	+	+	+	+
D	B									+	+	+	+	+	+	+	+
D	G												+	+	+	+	+
H	C																
H	B																
H	G																
C	B																
C	G																
B	G																

Education		employment return								job return							
1	2	1960	1970	1980	1990	2000	2007	2012	2017	1960	1970	1980	1990	2000	2007	2012	2017
D	H									+	+	+	+	+	+	+	+
D	C									+	+	+	+	+	+	+	+
D	B									+		+	+	+	+	+	+
D	G												+	+	+	+	+
H	C											+	+	+	+	+	+
H	B											+	+	+	+	+	+
H	G												+			+	+
C	B																
C	G																
B	G																

Education		marriage - employment return								spouse - job return							
1	2	1960	1970	1980	1990	2000	2007	2012	2017	1960	1970	1980	1990	2000	2007	2012	2017
D	H												+	+	+	+	+
D	C																
D	B																
D	G																
H	C																
H	B																
H	G																
C	B																
C	G																
B	G																

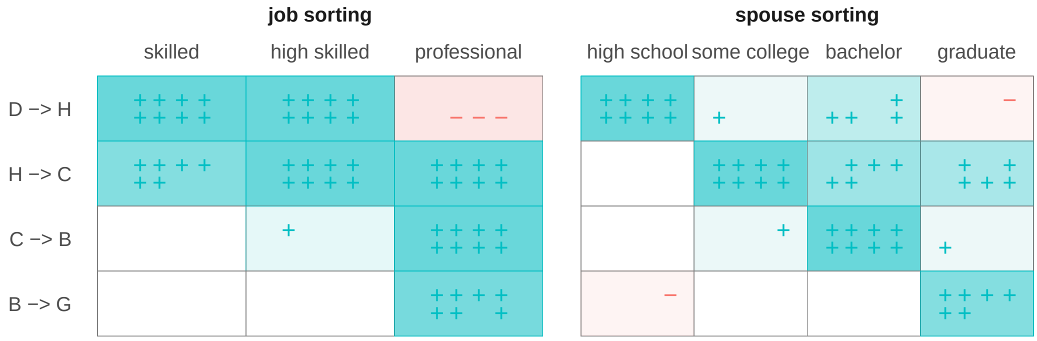

Figure 9: Sign-based identification of marginal-free sorting indices conditional on marriage and employment status. The explanations are the same as Figure 8.

closer education probably because of higher surplus share they receive.

The left panel of Figure 9 reveals a similar assortative matching in the job market. Because there is one less category for jobs than for education, the top two education classifications correspond with the top job classification. By combining B and G, we would create a square matrix with positively identified indices in the diagonal elements and negatively identified elements in the northeast. In other words, if a woman's education level is a close fit of the skill requirement of the job, more education results in a positive job sorting. However, if the education level is significantly different from the skill requirement of the job, such as having a high school education for professional jobs in recent years, women are better off pursuing a job with lower skill requirements.

6.3 Trends for white and black females in the U.S.

We can estimate the return indices for sub-populations with large-enough sample sizes, and Figures 10 and 11 present estimations for white and black populations in the United States. The top row of Figure 10 shows that in 1960 the educational distributions of black and white women are very unequal, such that the dropouts are more than 75% of black and less than 50% of white women. But over time the two population converge in educational attainment of women.

The middle graphs in Figure 10 show the marriage return to education over time for different spouse types separately for black and white women. Note that because we cannot compare the benchmark groups between the two populations, all indices reflect returns relative to dropouts of the same population. Therefore, we cannot judge about the absolute gains between the black and white women by these numbers. Having this in mind, we observe that marrying a husband with lower education has a higher return for black women at all education levels. In other words, the relative disutility of marrying down is much less for black women than white women. Moreover, in 1960, the marriage return of college+ educated black women compared to black women dropouts was much higher than college+ educated white women compared to white women dropouts. With the convergence of educational distribution across the two populations, this pattern is reversed over time and the marriage return of college+ education becomes higher for white women in recent years.

The bottom row of Figure 10 presents the employment return to education for different occupations among black and white women. The pattern of returns for unskilled and skilled jobs show less negative numbers for black women especially in earlier years. Therefore, we can conclude that the disutility of working in lower quality jobs compared to staying out of labor force is less in educated black women than educated white women relative to the low-educated women of their same race. We also observe that while the job return conditional on professional jobs was initially higher for black women, the two populations converge over time.

As for the aggregate effects at the extensive margins of the returns, Figure 11 shows that marriage return is similar before 2000, but afterward it is flat for blacks but decreasing for whites. There is no significant difference in employment return to high school education between the two populations. College education has higher marriage and employment returns for black women. Attaining bachelor's and graduate degree has initially a higher marriage return for black women, but with the increasing trend of the return for whites, the values converge in the two populations with close numbers in recent years. The employment return to graduate education is similar across the two populations too.

Regarding the aggregate spouse return, we observe either a decreasing or a flat trend for black population in comparison to the increasing trends of white population. This pattern is consistent with Chiappori et al. (2017) who find that while assortative matching has increased over time among the white population, it has hardly changed among blacks in the United States. The Figure also suggest that the job return trends are similar between the two populations

6.4 Other countries across time

Figures 12 and 13 show the trends of aggregate step-wise returns at the extensive and intensive margins, respectively, across countries in the sample. In the estimations, we estimate marginal-free indices using the world-wide average distribution and thus the returns are immune to structural difference in the marginal distributions of education and female employment across countries.[20] If we execute the estimation using the expected return indices, qualitatively, the trends are similar across countries.

Figure 12 shows that the marriage return to a high school diploma is mostly negative in other countries except for France and Ireland. The employment return to high school education is positive in different regions and university education has higher employment return than high school. The low employment return to university education is unique in the United States, and in all other countries in the sample, it is substantially positive in all available years. Moreover, the United States is also the only country in the sample where the marriage return has been higher than the employment return in some years. In other countries, education affects women's employment odds better than their marriage odds.

[20]For this reason, the estimated results for the U.S. are slightly shifted compared to the previous estimations.

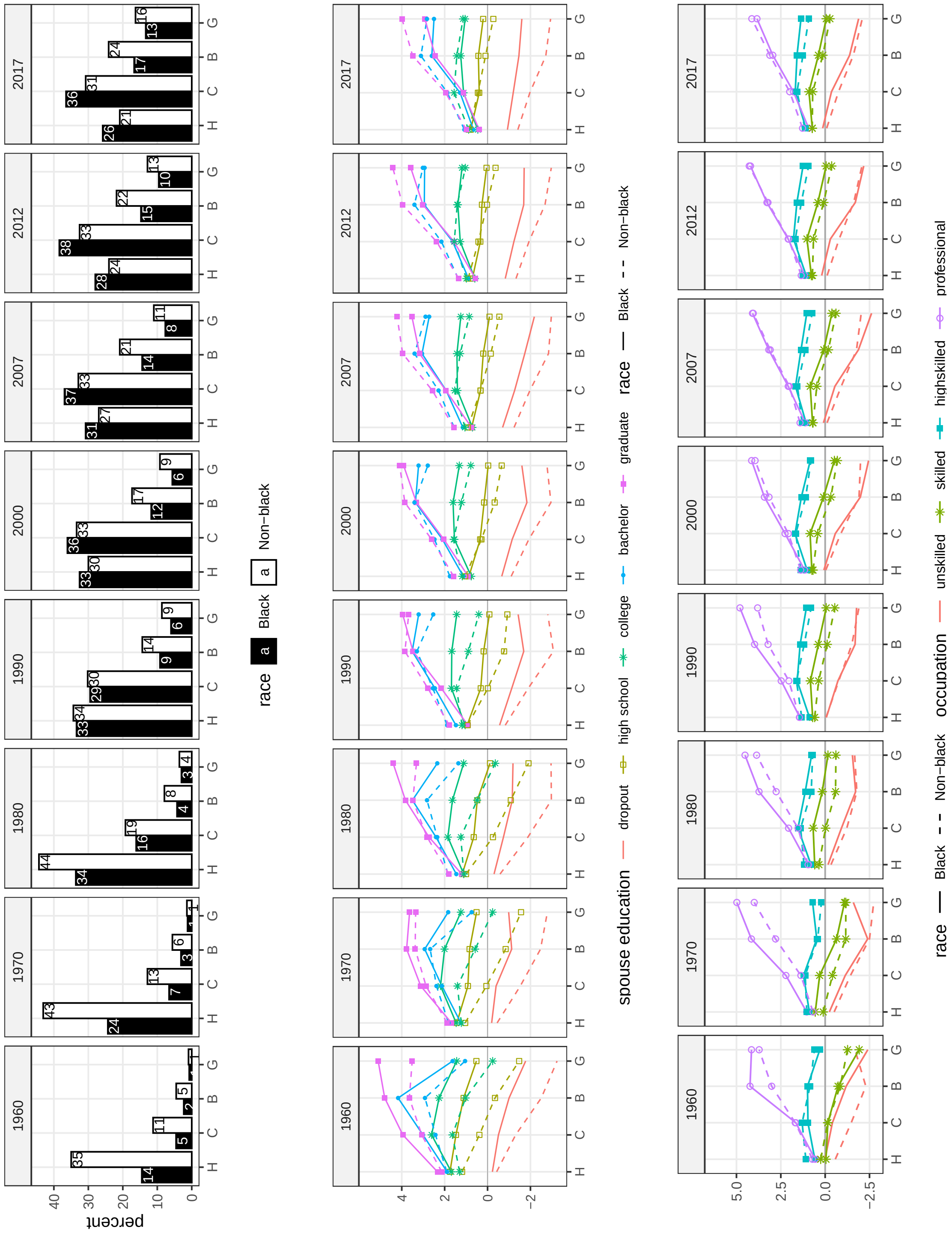

Figure 10: Marriage and employment returns to female education and the percent of educational groups by race in the U.S. The indices are $\hat{r}^{m}_{I_1 I_2 K}$ and $\hat{r}^{e}_{I_1 I_2 J}$ as defined in (14) to (15) and the estimations are from the heteroskedastic model (34). The solid and dashed lines show trends for black and white subsamples, respectively. The excluded group in showing percentage is high school dropout. Data is from census and 5-year samples of ACS.

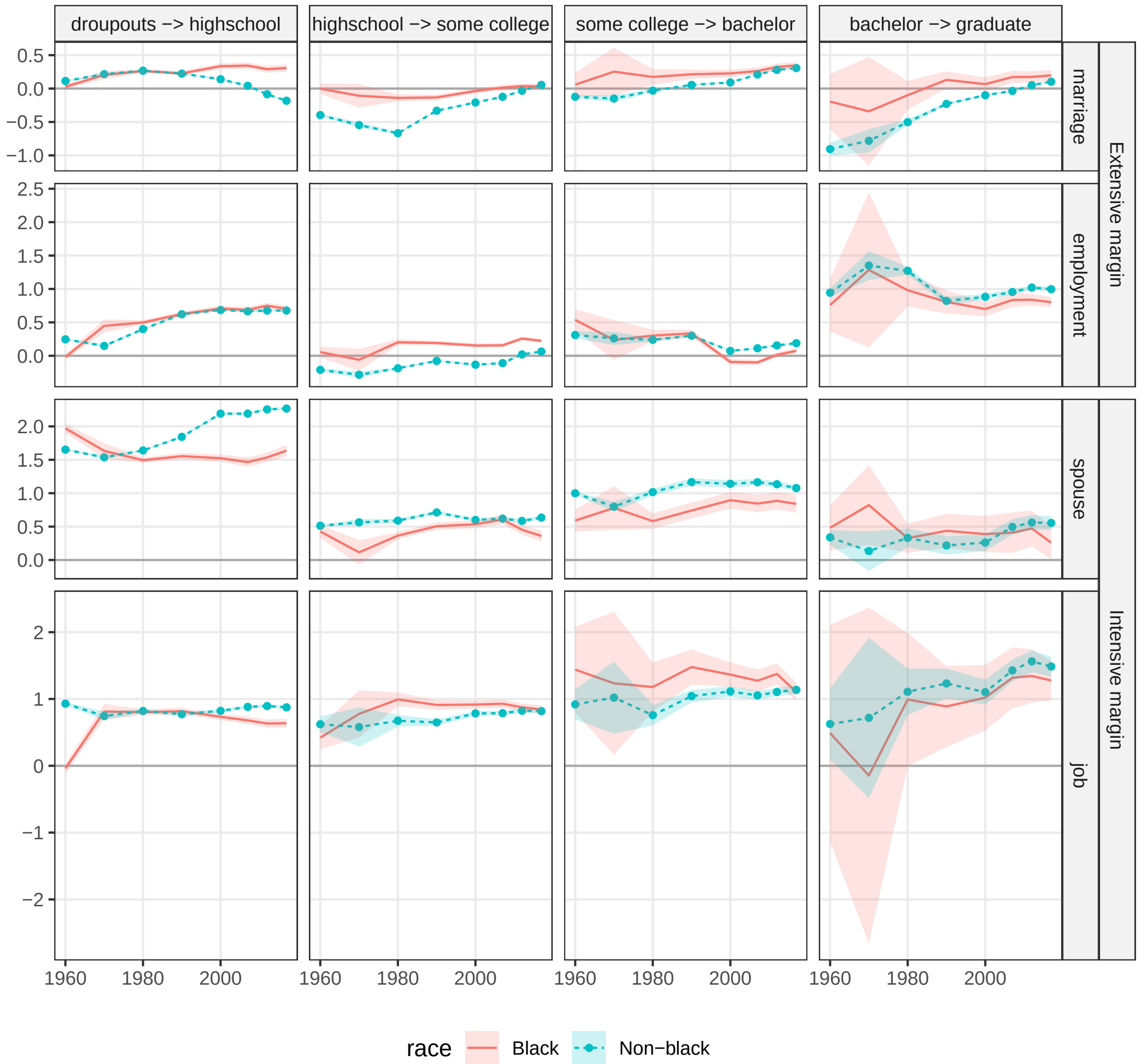

Figure 11: Marriage and employment returns to female education by race in the U.S. The estimated indices are marginal-free. Shaded area show the confidence interval. Data is from census and 5-year samples of ACS. Due to sparse wage data for black women in 1960 and 1970, the mean and variance of wage for some elements are estimated by the average of similar elements of the wage distribution.

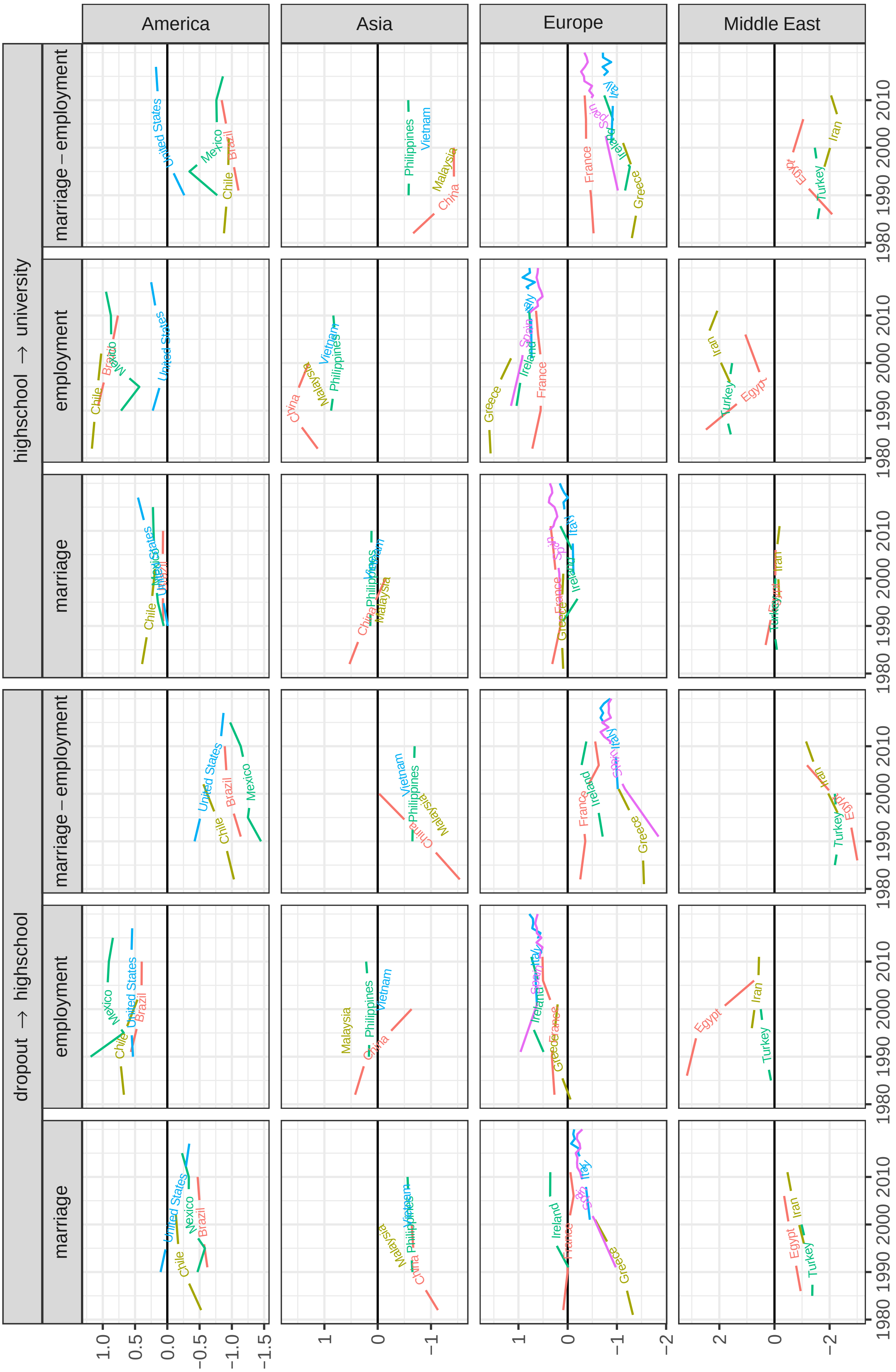

Figure 12: The marginal-free extensive margin of the returns across regions. The indices are marginal-free odds ratios $\hat{r}$ as in (26) to (28) estimated by world-wide fixed weights.

At the intensive margin, we observe that in almost all countries, both spouse and job returns to female education are positive. In particular, the left-hand side of Figure 13 shows that high school education increases the better spouse surplus substantially, and in many countries, its value is comparable to the better job surplus. Attending university compared to high school also have positive spouse and job returns, but the right-hand side of Figure 13 suggests that its job return is much higher than its spouse return. This evidence suggests that assortative matching of higher educated women with better spouses and better jobs exists almost everywhere, and at the top of educational distribution, the assortativeness is higher for jobs than spouse.

Figure 14 illustrates the marginal-free return indices averaged over time per country by considering dropout as the benchmark for comparison. At the top left graph, we observe that attaining education has an overall negative marriage return and while the trend is decreasing in some continues up to university education, in many others the trend flattens or becomes U-shaped for university education. Although many country-level factors can be important for determining the trends, a world-wide reason for the lower odds of marriage by moving from illiteracy or low-education to higher levels can be a phenomenon labeled as "singlism", that causes a negative stigma toward singles (e.g. DePaulo and Morris, 2006). Several psychological studies indicate that single people are perceived as not meeting sociocultural expectations and may experience negative social and psychological impacts. Singlism is more severe for women as they are more financially dependent on marriage than men[21] and may experience negative stereotypes related to their single status, such as being seen as unattractive or incomplete (Byrne and Carr, 2005; Sharp and Ganong, 2011). The negative marriage return to female education is equivalent to a reduction in the disutility of being single. Moreover, higher education can act as a signal of women's intellectual ability that can compensate for physical attractions, and reduces stigma costs by extending the social network of women to a wider circle with probably less traditional views toward singles. This is an area needing for more research.

In contrast to negative marriage return, the spouse return is positive, increasing, and concave in education allover the countries. Thus, conditional on marriage, education increases the quality of spouse especially in the middle of educational distribution and at the upper-end the gain increases at a lower pace. The right graphs of Figure 14 show that employment and job returns are increasing in education. While at the extensive margin the employment return is mainly concave or linear across countries, at the intensive margin the trend of job return to education has an increasing slope.

7 Conclusion

This paper extends the frictionless matching framework of Choo and Siow (2006) by assuming two bilateral markets and building a method for the joint estimation of marriage and employment returns to female

[21] Note that, according to section 3, the marginal-free marriage return index controls for employment status and thus the negative numbers for this index of marriage return is not due to the tradeoff between marriage and employment for women.

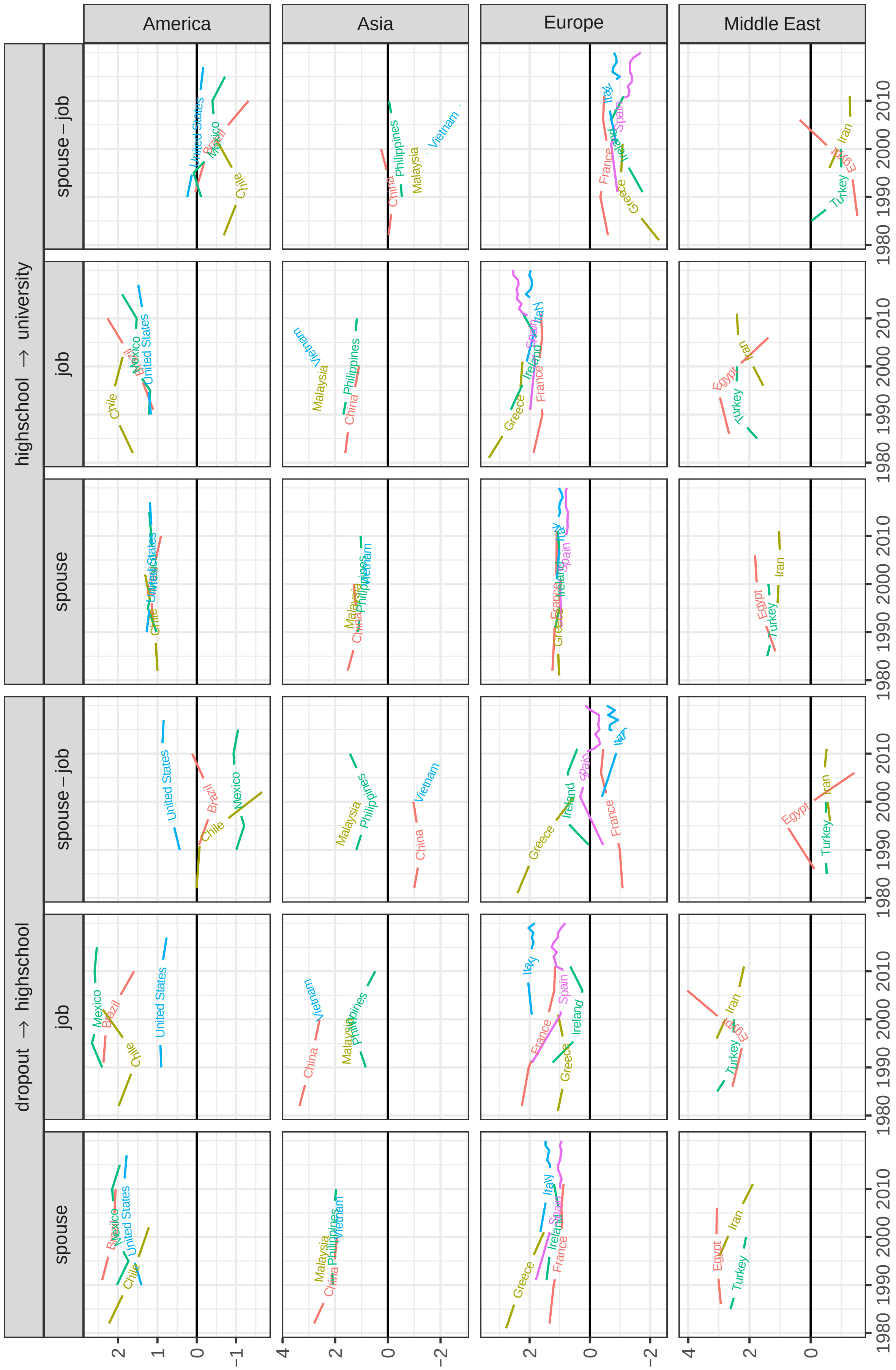

Figure 13: The marginal-free intensive margin of the returns across regions. The indices are marginal-free odds ratios $\hat{r}^s, \hat{r}^j, \hat{\delta}^{sj}$ as in (26) to (28) estimated by world-wide fixed weights.

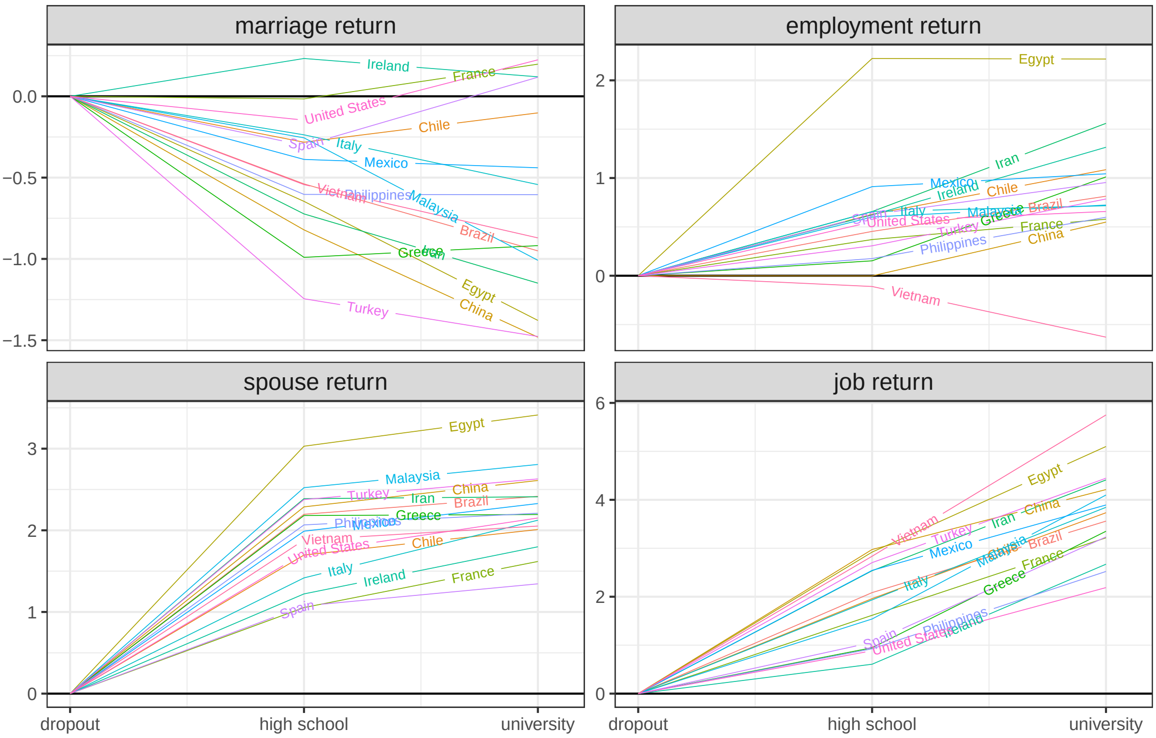

Figure 14: The marginal-free returns by education level across countries. The indices are estimated by world-wide fixed weights and they are averaged over time per country.

education. Our return indices measure the components of return to education that change the surplus of women regardless of not only the one-way distribution of variables but also their interaction via the other outcome. These components have two margins: an extensive margin reflecting the overall gain from marriage and employment compared to singlehood and non-participation, respectively, and an intensive margin that reflects the quality of match conditional on matching. A great advantage of this method is its low data requirement that allows for evidence over time and across space using cross-sectional household surveys.

Our findings for the United States suggest that attending college, compared to a high school degree, does not affect the odds of employment, but its marriage return has been positive and increasing over the past 20 years. In 1960, going to graduate school had a significantly negative marriage return, but it has been increasing in recent years and is now higher than the marriage return to college education. The employment return for graduate education is the highest at both margins. The findings for the intensive margin suggest that at the lower end of the educational distribution, the spouse return to education is higher than its job return, but at the upper end, the employment return is significantly greater. We also estimate return indices separately for black and white women in the U.S. and find that compared to high school dropouts of the same race, the disutility of marrying a lower educated husband and accepting a lower-paying employment is less for highly educated black women than white women.

Our nonparametric estimations for the U.S. suggest that even with no distributional assumption for unobservable preferences, the marriage return to education is identified with positive/negative sign when women marry up/down. We identify the employment return to education with a positive sign for professional occupations and a negative sign for unskilled jobs. Conditional on skilled and high skilled occupations, employment return is positive at the lower tail of education and negative at its upper tail.

While nonparametric estimations produce robust inference for the returns conditional on marital and employment statuses, only the intensive margin of unconditional return can be partially identified with no distributional assumption. The reason is the monotonic behavior of the aggregate returns at their intensive margins, which is stable over time. In comparison, the extensive margins of the returns behave in a non-monotonic pattern by increasing education level and one cannot make a robust inference for its aggregate measures without any distributional assumption for random surplus terms.

The cross-country evidence shows positive employment returns to female education with high values for university education. In contrast, high school education reduces the odds of marriage in most countries, and a plausible explanation for this pattern is that singlehood is much more difficult for women, both financially and socially, and female education can serve as a means to escape the social stigma of remaining single. This is in addition to its role as a factor in lowering discrimination in the labor market and an area for the future research. Lastly, our findings suggest that when women are both married and employed, as their education increases, their husband's quality improves at a decreasing rate but their job quality improves at an increasing rate.

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APPENDIX

A Mathematical Appendix

A.1 Proof of Proposition 1

Let $r(i,k;\emptyset_F)$, $s(i,j;\emptyset_M)$, $r(\emptyset_1;j)$ and $s(\emptyset_2;k)$, be the payoffs of non-working women, single women, unmatched firms, and single men, respectively. A stable match has these properties

- $s(i,j;k) + \Phi\big(r(i,k;j) - \pi_j\big) \leq u_i + v_k$ which holds with equality for the matched pairs
- $r(i,k;\emptyset_F) \leq r(i,k;j)$, $s(i,j;\emptyset_M) \leq s(i,j;k)$, $r(\emptyset_1;j) \leq r(i,k;j)$ and $s(\emptyset_2;k) \leq s(i,j;k)$.

which mean all matched individuals prefer being together to other matches, and no matched individual would rather be single.

From separability assumption we have

$$r(i,k;j) = R^{IJK} + \eta_i^{JK} + \beta_j^{IK}, \qquad s(i,j;k) = S^{IJK} + \lambda_i^{JK} + \gamma_k^{IJ} \tag{37}$$

Let $(i,j,k) \in (I,J,K)$ and $(i',j',k') \in (I,J,K)$ be two stable match pairs in the exact same categories. At the stable match, we should have

$$u_i + v_k = s(i,j;k) + \Phi\Big(r(i,k;j) - \pi_j\Big), \qquad u_{i'} + v_{k'} = s(i',j';k') + \Phi\Big(r(i',k';j') - \pi_{j'}\Big) \tag{38}$$

$$u_i + v_{k'} \geq s(i,j;k') + \Phi\Big(r(i,k';j) - \pi_j\Big), \qquad u_{i'} + v_k \geq s(i',j';k) + \Phi\Big(r(i',k;j') - \pi_{j'}\Big) \tag{39}$$

$$u_i + v_k \geq s(i,j';k) + \Phi\Big(r(i,k;j') - \pi_{j'}\Big), \qquad u_{i'} + v_{k'} \geq s(i',j;k') + \Phi\Big(r(i',k';j) - \pi_j\Big) \tag{40}$$

$$u_i + v_{k'} \geq s(i,j';k') + \Phi\Big(r(i,k';j') - \pi_{j'}\Big), \qquad u_{i'} + v_k \geq s(i',j;k) + \Phi\Big(r(i',k;j) - \pi_j\Big) \tag{41}$$

By combining (38) and (39), we obtain

$$\begin{aligned} s(i,j;k') - s(i,j;k) + &\Phi\Big(r(i,k';j) - \pi_j\Big) - \Phi\Big(r(i,k;j) - \pi_j\Big) \leq v_{k'} - v_k \\ &\leq s(i',j';k') - s(i',j';k) + \Phi\Big(r(i',k';j') - \pi_{j'}\Big) - \Phi\Big(r(i',k;j') - \pi_{j'}\Big) \end{aligned}$$

which from (37) and because $r(i,k';j) = r(i,k;j)$, $r(i',k';j') = r(i',k;j')$ yields

$$\gamma_{k'}^{IJ} - \gamma_k^{IJ} \leq v_{k'} - v_k \leq \gamma_{k'}^{IJ} - \gamma_k^{IJ} \quad \Rightarrow \quad v_{k'} - \gamma_{k'}^{IJ} = v_k - \gamma_k^{IJ} = V^{IJK}$$

This means that $v_k^{IJ} - \gamma_k^{IJ}$ does not depend on k and only on its category K. Similarly, by combining (38) and (40), we have

$$\beta_{j'}^{IK} - \beta_j^{IK} \leq \pi_{j'} - \pi_j \leq \beta_{j'}^{IK} - \beta_j^{IK} \quad \Rightarrow \quad \pi_{j'} - \beta_{j'}^{IK} = \pi_j - \beta_j^{IK} = \Pi^{IJK}$$

Consequently, we can write firms and men payoffs of matching as

$$\pi_i = \Pi^{IJK} + \beta_j^{IK}, \qquad\qquad v_k = V^{IJK} + \gamma_k^{IJ} \tag{42}$$

For finding u_i, we combine (38) and (40),

$$\begin{aligned} s(i',j;k) - s(i,j;k) &+ \Phi\Big(r(i',k;j) - \pi_j\Big) - \Phi\Big(r(i,k;j) - \pi_j\Big) \leq u_{i'} - u_i \\ &\leq s(i',j';k') - s(i,j';k') + \Phi\Big(r(i',k';j') - \pi_{j'}\Big) - \Phi\Big(r(i,k';j') - \pi_{j'}\Big) \end{aligned}$$

From (37) and (42), we have $r(i,k';j') - \pi_{j'} = r(i,k;j) - \pi_j = R^{IJK} - \Pi^{IJK} + \eta_i^{JK}$. Using approximation $\Delta\Phi(x) \approx \Phi'(x)\Delta x$ the above inequality becomes

$$\lambda_{i'}^{JK} - \lambda_i^{JK} + \Phi'\Big(R^{IJK} - \Pi^{IJK} + \eta_i^{JK}\Big)(\eta_{i'}^{JK} - \eta_i^{JK}) \leq u_{i'} - u_i \leq \lambda_{i'}^{JK} - \lambda_i^{JK} + \Phi'\Big(R^{IJK} - \Pi^{IJK} + \eta_i^{JK}\Big)(\eta_{i'}^{JK} - \eta_i^{JK})$$

Thus,

$$u_{i'} - \lambda_{i'}^{JK} - \Phi'\Big(R^{IJK} - \Pi^{IJK} + \eta_i^{JK}\Big)\eta_{i'}^{JK} = u_i - \lambda_i^{JK} - \Phi'\Big(R^{IJK} - \Pi^{IJK} + \eta_i^{JK}\Big)\eta_i^{JK} = U^{IJK}$$

by letting $\alpha_i^{JK} = \lambda_i^{JK} + \Phi'\Big(R^{IJK} - \Pi^{IJK} + \eta_i^{JK}\Big)\eta_i^{JK}$, we can write total payoffs of women from matching in both markets as

$$u_i = U^{IJK} + \alpha_i^{JK}$$

Note that since from Assumption 1, λ_i^{JK} and η_i^{JK} are independent of β_j^{IK} and γ_k^{IJ}, α_i^{JK} is independent of them too.

A.2 Proof of Proposition 2

We follow the proof of Theorem 4.1 of Graham (2011) and extends it in a three dimensional case. First, we show that the conditional choice probabilities are strictly increasing in the corresponding deterministic gain. Then we apply the sub-allocation feasibility condition in different 2×2 cases and show that the degree of complementarity are increasing in the conditional log odds ratios and is zero at random matching.

From the empirical matching pattern,

- $P^{IJK} = \frac{(IJK)}{\sum_L \sum_M (ILM)}$ is the probability of a women with education $i \in I$ has employment status J and marital status K,

Woman i chooses working status J and marital status K if and only if

$$u_i^{JK} \geq u_i^{J'K'} \quad \forall J' = 0, \ldots, N_J, \;\; K' = 0, \ldots, N_K$$

From Proposition 1, we can derive the empirical choice probabilities from the model as follows

$$
\begin{aligned}
P^{IJK} &= \Pr\{J, K = \arg\max u_i^{LM}\} \\
&= \Pr\{\alpha_i^{LM} \leq U^{IJK} - U^{ILM} + \alpha_i^{JK}, \forall L \neq J, M \neq K\} \\
&= \int_{-\infty}^{+\infty} \prod_{LM \neq JK} F_\alpha(U^{IJK} - U^{ILM} + \alpha_i^{JK}) f_\alpha(\alpha_i^{JK}) d\alpha_i^{JK}
\end{aligned} \tag{43}
$$

where $F_\alpha(.)$ and $f_\alpha(.)$ are respectively the CDF and PDF of α_i^{JK}.

Following Manski (1975), for all $JK \neq J'K'$

$$
\begin{aligned}
P^{IJK} - P^{IJ'K'} = \int_{-\infty}^{+\infty} \Bigg[& \prod_{LM \neq JK} F_\alpha(U^{IJK} - U^{ILM} + \alpha_i) \\
& - \prod_{LM \neq J'K'} F_\alpha(U^{IJ'K'} - U^{ILM} + \alpha_i) \Bigg] f_\alpha(\alpha_i) d\alpha_i
\end{aligned} \tag{44}
$$

and because F_α is strictly increasing, this gives

$$
U^{IJK} \gtreqless U^{IJ'K'} \qquad \Leftrightarrow \qquad P^{IJK} \gtreqless P^{IJ'K'} \tag{45}
$$

We can extend (45) to the conditional choice probabilities $\Pr\{u_i^{J'K'} < u_i^{JK}\} = \frac{P^{IJK}}{P^{IJK}+P^{IJ'K'}}$ by dividing the right-hand side inequality to $P^{IJK} + P^{IJ'K'}$ (which is positive)

$$
U^{IJK} \gtreqless U^{IJ'K'} \qquad \Leftrightarrow \qquad \Pr\{u_i^{J'K'} < u_i^{JK}\} \gtreqless \Pr\{u_i^{JK} < u_i^{J'K'}\} \tag{46}
$$

which states that the conditional choice probabilities are strictly increasing in the corresponding deterministic gain. We use this result in below.

Let $F_{\Delta\alpha}$ be the distribution function of the difference in α, then

$$
\Pr\{u_i^{J'K'} < u_i^{JK}\} = \Pr\{\alpha_i^{J'K'} - \alpha_i^{JK} < U^{IJK} - U^{IJ'K'}\} = F_{\Delta\alpha}(U^{IJK} - U^{IJ'K'})
$$

Now, consider the 2×2 sub-allocation contingency table with rows I, I' and columns $JK, J'K'$ as shown in Table A.1, where[22]

$$
S_{IJK} = \frac{(IJK)}{(IJK)+(IJ'K')+(I'JK)+(I'J'K')} \tag{47}
$$

$$
S_{I+} = \frac{(IJK)+(IJ'K')}{(IJK)+(IJ'K')+(I'JK)+(I'J'K')} \tag{48}
$$

$$
S_{+JK} = \frac{(IJK)+(I'JK)}{(IJK)+(IJ'K')+(I'JK)+(I'J'K')} \tag{49}
$$

Using this table, we can compute conditional choice probabilities as follows

[22]More precise notation would be $S_{IJK}^{I'J'K'}, S_{I+}^{I'J'K'}, S_{+JK}^{I'J'K'}$, but we skip the superscripts for simplicity.

	JK	$J'K'$	sum
I	S_{IJK}	$S_{I+} - S_{IJK}$	S_{I+}
I'	$S_{+JK} - S_{IJK}$	$1 - S_{I+} - S_{+JK} + S_{IJK}$	$1 - S_{I+}$
sum	S_{+JK}	$1 - S_{+JK}$	1

Table A.1: 2×2 sub-allocation contingency table with rows I, I' and columns $JK, J'K'$

$$\forall i \in I, \quad \Pr\{u_i^{J'K'} < u_i^{JK}\} = F_{\Delta\alpha}(U^{IJK} - U^{IJ'K'}) = \frac{S_{IJK}}{S_{I+}}$$
$$\forall i \in I', \quad \Pr\{u_i^{J'K'} < u_i^{JK}\} = F_{\Delta\alpha}(U^{I'JK} - U^{I'J'K'}) = \frac{S_{+JK} - S_{IJK}}{1 - S_{I+}}$$

The strict monotonicity of the conditional choice probabilities in (46) yields,

$$U^{IJK} - U^{IJ'K'} - (U^{I'JK} - U^{I'J'K'}) = F_{\Delta\alpha}^{-1}(\frac{S_{IJK}}{S_{I+}}) - F_{\Delta\alpha}^{-1}(\frac{S_{+JK} - S_{IJK}}{1 - S_{I+}}) \tag{50}$$

Exploiting the continuous and bounded derivative property, we can show that the derivative of the right-hand side of (50) w.r.t. S_{IJK} is positive

$$\frac{1}{S_{I+}} \frac{1}{f_{\Delta\alpha}(\frac{S_{IJK}}{S_{I+}})} + \frac{1}{1 - S_{I+}} \frac{1}{f_{\Delta\alpha}(\frac{S_{+JK} - S_{IJK}}{1 - S_{I+}})} > 0$$

Moreover, at random matching where $S_{IJK} = S_{I+}S_{+JK}$, we have $U^{IJK} + U^{I'J'K'} - U^{IJ'K'} - U^{I'JK} = 0$. Hence, being strictly increasing and crossing zero at $S_{IJK} = S_{I+}S_{+JK}$ yields

$$U^{IJK} + U^{I'J'K'} - U^{IJ'K'} - U^{I'JK} \gtreqless 0 \quad \Leftrightarrow \quad S_{IJK} \gtreqless S_{I+}S_{+JK} \tag{51}$$

and because ln is a strictly increasing operator, from (47) to (49), we get

$$U^{IJK} + U^{I'J'K'} - U^{IJ'K'} - U^{I'JK} \gtreqless 0 \quad \Leftrightarrow \quad \ln \frac{(IJK)(I'J'K')}{(IJ'K')(I'JK)} \gtreqless 0 \tag{52}$$

All of the conditional returns indices are as (52).

A.3 Proof of Proposition 3

From the assumption of type-I extreme value distribution for unobservables, (43) becomes

$$P^{IJK} = \int_{-\infty}^{+\infty} \prod_{LM \neq JK} e^{-e^{U^{ILM} - U^{IJK} - \alpha_i^{JK}}} e^{-\alpha_i^{JK} - e^{-\alpha_i^{JK}}} d\alpha_i^{JK} \tag{53}$$

Assume $\zeta_{LM} = e^{U^{ILM} - U^{IJK}}$, and $\chi = e^{-\alpha_i^{JK}} \to d\chi = -e^{-\alpha_i^{JK}} d\alpha_i^{JK}$

$$\begin{aligned}
P^{IJK} &= \int_0^{+\infty} \prod_{LM \neq JK} e^{-\chi\, \zeta_{LM}} e^{-\chi} d\chi = \int_0^{+\infty} e^{-\chi(1+\sum_{LM\neq JK}\zeta_{LM})} d\chi \\
&= \frac{1}{1+\sum_{LM\neq JK}\zeta_{LM}}
\end{aligned}$$

and since $\zeta_{LM} = \frac{e^{U^{ILM}}}{e^{U^{IJK}}}$, we get

$$P^{IJK} = \frac{e^{U^{IJK}}}{\sum_L \sum_M e^{U^{ILM}}} \tag{54}$$

By combining these with the conditional probabilities from the contingency table $P^{IJK} = \frac{(IJK)}{\sum_L \sum_M (ILM)}$, we have

$$U^{IJK} - U^{IJ'K'} = \ln \frac{P^{IJK}}{P^{IJ'K'}} = \ln \frac{(IJK)}{(IJ'K')} \tag{55}$$

Using (55), we can simply derive the return indices as stated in the proposition.

A.4 Proof of Proposition 4

Using Bayes rule of conditional probabilities, conditional on $i \in I$ matching with types JK, the expected utility of woman becomes:

$$\begin{aligned}
\mathring{u}_I^{JK} &= E[u_i^{JK} | J, K = \arg\max u_i^{LM}] \\
&= U^{IJK} + E[\alpha_i^{JK} | U^{IJK} + \alpha_i^{JK} > U^{ILM} + \alpha_i^{LM}, \forall L \neq J, M \neq K] \\
&= U^{IJK} + \frac{E[\alpha_i^{JK} \mathbb{1}(\alpha_i^{LM} < U^{IJK} - U^{ILM} + \alpha_i^{JK}, \forall L \neq J, M \neq K)]}{\Pr(\alpha_i^{LM} < U^{IJK} - U^{ILM} + \alpha_i^{JK}, \forall L \neq J, M \neq K)} \\
&= U^{IJK} + \frac{\int_{-\infty}^{+\infty} \alpha_i^{JK} \big[\prod_{LM\neq JK} F(U^{IJK} - U^{ILM} + \alpha_i^{JK})\big] f(\alpha_i^{JK}) d\alpha_i^{JK}}{\int_{-\infty}^{+\infty} \big[\prod_{LM\neq JK} F(U^{IJK} - U^{ILM} + \alpha_i^{JK})\big] f(\alpha_i^{JK}) d\alpha_i^{JK}}
\end{aligned} \tag{56}$$

and the unconditional expected utility when the choice between the categories is possible is

$$\mathring{u}_I = E[\max_{J,K}\ U^{IJK} + \alpha_i^{JK}] = \sum_{J=0}^{N_J} \sum_{K=0}^{N_K} \mathring{u}_I^{JK} \Pr\{J, K = \arg\max\ u_i^{JK}\} \tag{57}$$

Because $\bar{u}_I = \sum_{J=0}^{N_J} \sum_{K=0}^{N_K} P^{IJK} U^{IJK}$, combining (56) with (57) yields

$$\mathring{u}_I - \bar{u}_I = \sum_{J=0}^{N_J} \sum_{K=0}^{N_K} \int_{-\infty}^{+\infty} \alpha \big[\prod_{LM\neq JK} F(U^{IJK} - U^{ILM} + \alpha)\big] f(\alpha) d\alpha \tag{58}$$

To find the first order approximation, we define

$$\xi(U^{I\cdot\cdot}) = \sum_{J=0}^{N_J} \sum_{K=0}^{N_K} \xi^{IJK}(U^{I\cdot\cdot}) \quad \text{where} \quad \xi^{IJK}(U^{I\cdot\cdot}) = \int_{-\infty}^{+\infty} \alpha \big[\prod_{LM\neq JK} F(U^{IJK} - U^{ILM} + \alpha)\big] f(\alpha) d\alpha$$

For $LM \neq JK$

$$\frac{\partial\, \xi^{IJK}}{\partial\, U^{ILM}} = -\int_{-\infty}^{+\infty} \alpha f(U^{IJK} - U^{ILM} + \alpha)\big[\prod_{\substack{L'M' \neq JK \\ L'M' \neq LM}} F(U^{IJK} - U^{IL'M'} + \alpha)\big] f(\alpha) d\alpha$$

and

$$\frac{\partial\, \xi^{IJK}}{\partial\, U^{IJK}} = \int_{-\infty}^{+\infty} \alpha \sum_{LM \neq JK} \big[f(U^{IJK} - U^{ILM} + \alpha) \prod_{\substack{L'M' \neq JK \\ L'M' \neq LM}} F(U^{IJK} - U^{IL'M'} + \alpha)\big] f(\alpha) d\alpha$$

Then, we have

$$\frac{\partial\, \xi(U^{I\cdot\cdot})}{\partial\, U^{IJK}} = \frac{\partial\, \xi^{IJK}}{\partial\, U^{IJK}} + \sum_{LM \neq JK} \frac{\partial\, \xi^{ILM}}{\partial\, U^{IJK}}$$

When $\forall JK,\ U^{IJK} = U^{I00}$,

$$\xi^{IJK} = \int_{-\infty}^{+\infty} \alpha \Big(F(\alpha)\Big)^{(N_J+1)(N_K+1)-1} f(\alpha) d\alpha = \int_0^1 F^{-1}(x)\ x^{N_J N_K + N_J + N_K} dx$$

$$\frac{\partial\, \xi^{IJK}}{\partial\, U^{ILM}} = -\int_{-\infty}^{+\infty} \alpha \big(f(\alpha)\big)^2 \Big(F(\alpha)\Big)^{(N_J+1)(N_K+1)-2} d\alpha$$

$$\frac{\partial\, \xi^{IJK}}{\partial\, U^{IJK}} = (N_J N_K + N_J + N_K) \int_{-\infty}^{+\infty} \alpha \big(f(\alpha)\big)^2 \Big(F(\alpha)\Big)^{(N_J+1)(N_K+1)-2} d\alpha$$

which yields,

$$\xi(U^{I\cdot\cdot} = U^{I00}) = (N_J+1)(N_K+1) \int_0^1 F^{-1}(x)\ x^{N_J N_K + N_J + N_K} dx \quad \text{and} \quad \frac{\partial\, \xi(U^{I\cdot\cdot} = U^{I00})}{\partial\, U^{IJK}} = 0 \quad (59)$$

From (43), we can find each U^{IJK} and consequently $\mathring{u}_I - \bar{u}_I$ as functions of $P^{I00}, \ldots, P^{IN_J N_K}$. Let $\Xi(P^{I\cdot\cdot}) = \xi\big(U^{I\cdot\cdot}(P^{I\cdot\cdot})\big)$ be the difference as in (58). Then we compute the Taylor expansion of Ξ at $\forall JK,\ P^{IJK} = \frac{1}{(N_J+1)(N_K+1)}$. Because of symmetry in (43) at this point we also have $\forall JK,\ U^{IJK} = U^{I00}$. Thus, from (59), we obtain

$$\Xi(P^{I\cdot\cdot} = \frac{1}{(N_J+1)(N_K+1)}) = \xi(U^{I\cdot\cdot} = U^{I00}) = (N_J+1)(N_K+1) \int_0^1 F^{-1}(x)\ x^{N_J N_K + N_J + N_K} dx$$
$$\frac{\partial\, \Xi}{\partial P^{IJK}}(P^{I\cdot\cdot} = \frac{1}{(N_J+1)(N_K+1)}) = \sum_{LM} \frac{\partial \xi(U^{I\cdot\cdot} = U^{I00})}{\partial U^{ILM}} \frac{\partial U^{ILM}}{\partial P^{IJK}} = 0$$

Since $0 < P^{IJK} < 1$, in the higher order terms $(P^{IJK} - \frac{1}{(N_J+1)(N_K+1)})^n$ approaches zero and with bounded derivatives, we can neglect residual of expansion for $n \geq 2$. Thus, using the first order Taylor expansion at $\forall JK,\ P^{IJK} = \frac{1}{(N_J+1)(N_K+1)},\ U^{IJK} = U^{I00}$, we can approximate

$$\mathring{u}_I - \bar{u}_I \approx (N_J+1)(N_K+1) \int_0^1 F^{-1}(x)\ x^{N_J N_K + N_J + N_K} dx$$

To compute this when $F(\alpha) = e^{-e^{-x}}$, we use

$$\int_{-\infty}^{+\infty} x(e^{-x-ae^{-x}})dx = \int_{0}^{+\infty} -\ln u e^{-au} du = \frac{\ln a + \boldsymbol{\gamma}}{a} \tag{60}$$

where $\boldsymbol{\gamma} = \int_0^{+\infty} e^{-x} \ln x dx = -0.577121$ (Euler's constant). If we drop $\boldsymbol{\gamma}$ from $\overset{*}{u}_I$ then under type-I extreme value distribution

$$\overset{*}{u}_I - \bar{u}_I \approx \ln(N_J + 1) + \ln(N_K + 1)$$

A.5 Proof of Proposition 5

With type-I extreme value distribution, $\overset{*}{u}_I^{JK}$ can be computed using (56).

$$\overset{*}{u}_I^{JK} = U^{IJK} + \frac{\displaystyle\int_{-\infty}^{+\infty} \alpha e^{-\alpha-e^{-\alpha}} \prod_{LM \neq JK} e^{-e^{U^{ILM}-U^{IJK}-\alpha}} d\alpha}{\displaystyle\int_{-\infty}^{+\infty} e^{-\alpha-e^{-\alpha}} \prod_{LM \neq JK} e^{-e^{U^{ILM}-U^{IJK}-\alpha}} d\alpha} \tag{61}$$

The numerator is computed using (60) and with a same procedure as in Appendix A.3, the denominator becomes $e^{U^{IJK}} / \sum_L \sum_M e^{U^{ILM}}$. Thus,

$$\overset{*}{u}_I^{JK} = U^{IJK} + \ln(\sum_L \sum_M e^{U^{ILM}-U^{IJK}}) + \boldsymbol{\gamma} = \ln \sum_L \sum_M e^{U^{ILM}} + \boldsymbol{\gamma}$$

This is independent of JK and thus is equal to $\overset{*}{u}_I$. If we normalize $U^{I00} = 0$ and drop the Euler's constant term, we have

$$\overset{*}{u}_I = \ln \sum_{L=0}^{N_J} \sum_{M=0}^{N_K} e^{U^{ILM}} = \ln \frac{1}{P^{I00}} = \ln \frac{\sum_{L=0}^{N_J} \sum_{M=0}^{N_K} (ILM)}{(I00)} \tag{62}$$

In the same manner, by letting $Q_1^{IJK} = \frac{(IJK)}{\sum_L (IJL)}$ and $Q_2^{IJK} = \frac{(IJK)}{\sum_L (ILK)}$, we obtain

$$\overset{*}{u}_{IJ} = \ln \frac{1}{Q_1^{IJ0}} = \ln \frac{\sum_{L=0}^{N_J} (IJL)}{(IJ0)}, \qquad \overset{*}{u}_{IK} = \ln \frac{\sum_{L=0}^{N_K} (ILK)}{(I0K)} \tag{63}$$

Moreover, any restrictions on J or K in computing $\overset{*}{u}_I$ is reflected in the summations of (62) and assuming $(I00)$ is always among the choices, from (55) we have

$$E[\max_{\substack{J \in \mathcal{J} \\ K \in \mathcal{K}}} u_i] = \ln \sum_{L \in \mathcal{J}} \sum_{M \in \mathcal{K}} e^{U^{ILM}} = \ln \frac{\sum_{L \in \mathcal{J}} \sum_{M \in \mathcal{K}} (ILM)}{(I00)}$$

Computing each component of the expected return using this formula leads to the proposition equations.

A.6 Proof of Proposition 6

The proof for the first bullet point is in a same spirit as in Appendix A.2. We show the sign-based identification of $\bar{r}^m_{I_1 I_2}$ and the proof is similar for other indices. For the collapsed choices, we can define average utilities for woman i as

$$\bar{u}_i^{K\geq 1} = \bar{U}_I^{K\geq 1} + \alpha_i^{K\geq 1} \qquad \bar{u}_i^{K=0} = \bar{U}_I^{K=0} + \alpha_i^{K=0} \tag{64}$$

where

$$\bar{U}_I^{K\geq 1} = \sum_{J=0}^{N_J}\sum_{K=1}^{N_K} \frac{P^{IJK}}{P^{I(+)(K\geq 1)}} U^{IJK}, \quad \alpha_i^{K\geq 1} = \sum_{J=0}^{N_J}\sum_{K=1}^{N_K} \frac{P^{IJK}}{P^{I(+)(K\geq 1)}} \alpha_i^{JK},$$

$$\bar{U}_I^{K=0} = \sum_{J=0}^{N_J} \frac{P^{IJ0}}{P^{I(+)0}} U^{IJ0}, \qquad \alpha_i^{K=0} = \sum_{J=0}^{N_J} \frac{P^{IJ0}}{P^{I(+)0}} \alpha_i^{J0}$$

Using this notation, we have

$$\Pr\{\bar{u}_i^{K=0} \leq \bar{u}_i^{K\geq 1}\} = \Pr\{\alpha_i^{K=0} - \alpha_i^{K\geq 1} \leq \bar{U}_I^{K\geq 1} - \bar{U}_I^{K=0}\} = F_{\alpha^{K=0}-\alpha^{K\geq 1}}(\bar{U}_I^{K\geq 1} - \bar{U}_I^{K=0})$$

Considering the below 2×2 contingency table with rows I_2, I_1 and collapsed columns (J, K) as $(+,\ \geq 1)$ and $(+,\ =0)$,

	$+,\ \geq 1$	$+,\ =0$	sum
I_2	$(I_2\ +\ \geq 1)$	$(I_2\ +\ 0)$	$(I_2\ +\ +)$
I_1	$(+\ +\ \geq 1) - (I_2\ +\ \geq 1)$	$(+\ +\ +) - (I_2\ +\ +) - (+\ +\ \geq 1) + (I_2\ +\ \geq 1)$	$(+\ +\ +) - (I_2\ +\ +)$
sum	$(+\ +\ \geq 1)$	$(+\ +\ +) - (+\ +\ \geq 1)$	$(+\ +\ +)$

we can compute conditional probabilities as follows

$$F_{\alpha^{K=0}-\alpha^{K\geq 1}}(\bar{U}_{I_2}^{K\geq 1} - \bar{U}_{I_2}^{K=0}) = \frac{(I_2\ +\ \geq 1)}{(I_2\ +\ +)}$$

$$F_{\alpha^{K=0}-\alpha^{K\geq 1}}(\bar{U}_{I_1}^{K\geq 1} - \bar{U}_{I_1}^{K=0}) = \frac{(+\ +\ \geq 1) - (I_2\ +\ \geq 1)}{(+\ +\ +) - (I_2\ +\ +)}$$

From (64), because $E[\alpha_i^{K=0}] = E[\alpha_i^{K\geq 1}] = E[\alpha_i]$, we have $\bar{u}_I^{K\geq 1} - \bar{u}_I^{K=0} = \bar{U}_I^{K\geq 1} - \bar{U}_I^{K=0}$, therefore,

$$\bar{r}^m_{I_1 I_2} = F^{-1}_{\alpha^{K=0}-\alpha^{K\geq 1}}\left(\frac{(I_2\ +\ \geq 1)}{(I_2\ +\ +)}\right) - F^{-1}_{\alpha^{K=0}-\alpha^{K\geq 1}}\left(\frac{(+\ +\ \geq 1) - (I_2\ +\ \geq 1)}{(+\ +\ +) - (I_2\ +\ +)}\right)$$

Then, similar to Appendix A.2, under random matching $(I_2\ +\ \geq 1) = (I_2\ +\ +)(+\ +\ \geq 1)$, we have $\bar{r}^m_{I_1 I_2} = 0$ and

$$\frac{\partial\ \bar{r}^m_{I_1 I_2}}{\partial\ (I_2\ +\ \geq 1)} > 0$$

Thus,

$$(I_2\ +\ \geq 1) \gtreqless (I_2\ +\ +)(+\ +\ \geq 1) \quad \Leftrightarrow \quad \ln \frac{(I_2\ +\ \geq 1)(I_1\ +\ 0)}{(I_1\ +\ \geq 1)(I_2\ +\ 0)} \gtreqless 0 \quad \Leftrightarrow \quad \bar{r}^m_{I_1 I_2} \gtreqless 0$$

The second bullet point is straight forward as the aggregate marginal-free returns are the expectation of conditional return and the expectation of all positive (all negative) numbers is positive (negative).

For the third bullet point, we prove for $\hat{r}^m_{I_1 I_2}$ and the rest of indices are similar. From (50), we can write

$$r^m_{I_1 I_2 JK} = F^{-1}_{\Delta\alpha}(\frac{(I_2 JK)}{(I_2 JK) + (I_2 J0)}) - F^{-1}_{\Delta\alpha}(\frac{(I_1 JK)}{(I_1 JK) + (I_1 J0)})$$

Because inside the parenthesis is an increasing function of the numerator, we write

$$r^m_{I_1 I_2 JK} = g_2(JK) - g_1(JK)$$

where g_1 and g_2 are increasing functions of the value of elements $(I_1 JK)$ and $(I_2 JK)$, respectively. Without loss of generality, we assume that for a given g_1 and g_2, all conditional returns are positive except one, and the aggregate return is positive too. We then find an alternative distribution that turns the aggregate index to negative. Suppose

$$r^m_{I_1 I_2 JK} < 0 \text{ and } \forall J'K' \neq JK \; r^m_{I_1 I_2 J'K'} > 0 \tag{65}$$

Then, from (26)

$$r^m_{I_1 I_2} \gtreqless 0 \quad \Leftrightarrow \quad r^m_{I_1 I_2 JK} \gtreqless -\frac{1}{\bar{P}^{I_1 I_2 JK}} \sum_{LM \neq JK} \bar{P}^{I_1 I_2 LM} r^m_{I_1 I_2 LM} \tag{66}$$

The different sign of $r^m_{I_1 I_2 JK}$ from others means that at least one of its elements is the extremum. To see why, note that

$$r^m_{I_1 I_2 JK} < 0 \;\Rightarrow g_2(JK) - g_1(JK) < 0 \Rightarrow \text{ either } \; g_2(JK) < 0 \text{ or } \; g_1(JK) > 0$$

$$\forall J'K' \neq JK, \; r^m_{I_1 I_2 J'K'} > 0 \;\Rightarrow g_2(J'K') - g_1(J'K') > 0 \Rightarrow \text{ either } \; g_2(J'K') > 0 \text{ or } \; g_1(J'K') < 0$$

These conditions together yield

$$\forall J'K' \neq JK \text{ either } \; g_2(J'K') > g_2(JK) \text{ or } \; g_1(J'K') < g_1(JK)$$

If not, $r^m_{I_1 I_2 J'K'} - r^m_{I_1 I_2 JK} = g_2(J'K') - g_2(JK) - g_1(J'K') + g_1(JK) < 0$ which because g_1 and g_2 are continuous and increasing is a violation of assumption (65). With no loss of generality, assume

$\forall J'K' \neq JK,\ g_2(J'K') > g_2(JK)$, then let $\delta > 0$ be such that $\delta \leq \min[g_2(J'K')] - g_2(JK)$ and define

$$G_2(x) = \begin{cases} g_2(x) & \text{if } \ x \geq (I_2JK) + \delta \\ \big(g_2(x) - C\big)\frac{x-(I_2JK)}{\delta} + C & \text{if } \ (I_2JK) \leq x \leq (I_2JK) + \delta \\ g_2(x) + C & \text{if } \ x < (I_2JK) \end{cases}$$

where $C < g_2(JK)$ is a constant. By decreasing C one can make $r^m_{I_1I_2JK}$ more negative without changing $r^m_{I_1I_2J'K'}$. Thus, one can set a C that turns (66) to negative with an alternative distribution for the difference in i.i.d. unobservable terms as $G_2^{-1}\big(\frac{(I_2JK)(I_2J0)}{1-(I_2JK)}\big)$.

A.7 Covariance matrix estimation in over-identified model

Under type-I extreme value assumption for unobservable terms, the covariance of population moments becomes

$$\begin{aligned} \mathrm{Cov}(\frac{U^{IJK}}{\sigma_I}, \frac{U^{IJ'K'}}{\sigma_I}) &= \mathrm{Cov}(\ln P^{IJK} - \ln P^{I00},\ \ln P^{IJ'K'} - \ln P^{I00}) \\ &= \mathrm{Cov}(\ln P^{IJK},\ \ln P^{IJ'K'}) + \mathrm{Var}(\ln P^{I00}) - \mathrm{Cov}(\ln P^{IJK},\ \ln P^{I00}) - \mathrm{Cov}(\ln P^{IJ'K'},\ \ln P^{I00}) \end{aligned}$$

In the large markets the matching pattern of two different groups are independent and for $I_1 \neq I_2$

$$\mathrm{Cov}(\ln P^{I_1JK},\ \ln P^{I_2J'K'}) = 0$$

From the properties of the multinomial distribution, we have

$$\mathrm{Var}(P^{IJK}) = \frac{P^{IJK}(1-P^{IJK})}{\mathcal{I}_I} \qquad \mathrm{Cov}(P^{IJK}, P^{IJ'K'}) = -\frac{P^{IJK}P^{IJ'K'}}{\mathcal{I}_I}$$

where $\mathcal{I}_I$ is the total population of category I in the contingency table. Using $\mathrm{Cov}\big(\ln(x), \ln(y)\big) \approx \frac{\mathrm{Cov}(x,y)}{\mathrm{E}(x)\,\mathrm{E}(y)}$, we can approximate the above elements of covariance matrix by

$$\mathrm{Cov}(\ln P^{IJK}, \ln P^{I'J'K'}) = \mathbb{1}(I' = I)\frac{\mathbb{1}(J' = J \ \&\ K' = K) - P^{IJ'K'}}{\mathcal{I}_I P^{IJ'K'}}$$

Therefore,

$$\mathrm{Cov}\Big(\frac{U^{IJK}}{\sigma_I}, \frac{U^{IJ'K'}}{\sigma_I}\Big) = \begin{cases} \frac{1}{\mathcal{I}_I}(\frac{1}{P^{IJK}} + \frac{1}{P^{I00}}) & I = I', J = J', K = K' \\ \frac{1}{\mathcal{I}_I P^{I00}} & I = I', J \neq J' \text{ or } K \neq K' \\ 0 & I \neq I' \end{cases}$$

Moreover, with a diagonal covariance structure for earnings, we have

$$\text{Cov}(W^{IJK}, W^{I'J'K'}) = \mathbb{1}(I' = I \ \& \ J' = J \ \& \ K' = K)\,\text{Var}(W^{IJK})$$

For estimation, we benchmark $J' = 1$ for computing utility difference and $\Delta_J U^{IJK} = U^{IJK} - U^{I1K}$. Then, the covariance structure of earnings moments for $J, J' > 1$ becomes

$$\text{Cov}\left(e(U^{IJK}, U^{I1K}, \phi_I), e(U^{I'J'K'}, U^{I'1K'}, \phi_{I'})\right) = \begin{cases} \text{Var}(W^{IJK}) + \text{Var}(W^{I1K}) & I = I', J = J', K = K' \\ \text{Var}(W^{I1K}) & I = I', K = K', J \neq J' \\ 0 & I \neq I' \text{ or } K \neq K' \end{cases}$$

A.7.1 Estimation of standard errors

In estimation of standard errors, we build the derivative matrix G than includes the derivative of moment conditions w.r.t. U^{IJK}, σ_I and ϕ_I which are

$$\frac{\partial U^{IJK}/\sigma^I}{\partial U^{IJK}} = \frac{1}{\sigma^I}, \qquad \frac{\partial U^{IJK}/\sigma^I}{\partial \sigma^I} = -\frac{U^{IJK}}{(\sigma^I)^2}$$

$$\frac{\partial}{\partial U^{IJK}} e(U^{IJK}, U^{I1K}, \phi_I) = \frac{(\bar{W}^{IK})^{\phi_I}}{\rho_{IK}}, \qquad \frac{\partial}{\partial U^{I1K}} e(U^{IJK}, U^{I1K}, \phi_I) = -\frac{(\bar{W}^{IK})^{\phi_I}}{\rho_{IK}}$$

$$\frac{\partial}{\partial \phi} e(U^{IJ'K}, U^{I1K}, \phi_I) = \frac{\Delta_J U^{IJK} (\bar{W}^{IK})^{\phi_I} \ln \bar{W}^{IK}}{\rho_{IK}} + \frac{(\Delta_J W^{IJK})^2}{2\bar{W}^{IK}}$$

and since the return indices are linear functions of deterministic utilities, we compute their standard errors using $\text{Var}(A \times h) = A \times \text{Var}(h) \times A^T$. The standard errors of the dollar unit indices are estimated by Monte Carlo simulation of $\Phi^{-1}(r) \approx \frac{r}{\Phi'(W)} = rW^{\phi}$ assuming normal distribution for r and ϕ and log-normal distribution for W with 1000 sample and 1000 iterations. The confidence intervals are produced using Generalized confidence interval method.

B Additional Results and Robustness Tests

B.1 Relaxing extreme value distribution assumption for unobservables

Without the extreme value assumption of Proposition 3, using (31), we can evaluate choice probabilities for alternative distributions of the unobserved heterogeneity. To find numerical solutions, we consider the just-identified model with $U^{I00} = 0$.

Figure A.2 compares the trend of estimated coefficients using two alternative distributional assumptions for unobservable heterogeneity in the U.S. One is the type-I extreme value distribution another is the normal distribution with mean zero and variance $\pi^2/6$ (same as extreme value with CDF $e^{-e^{-x}}$). To estimate structural parameters using normal distribution, equation (31) is estimated for each $U^{I\cdot\cdot}$ using

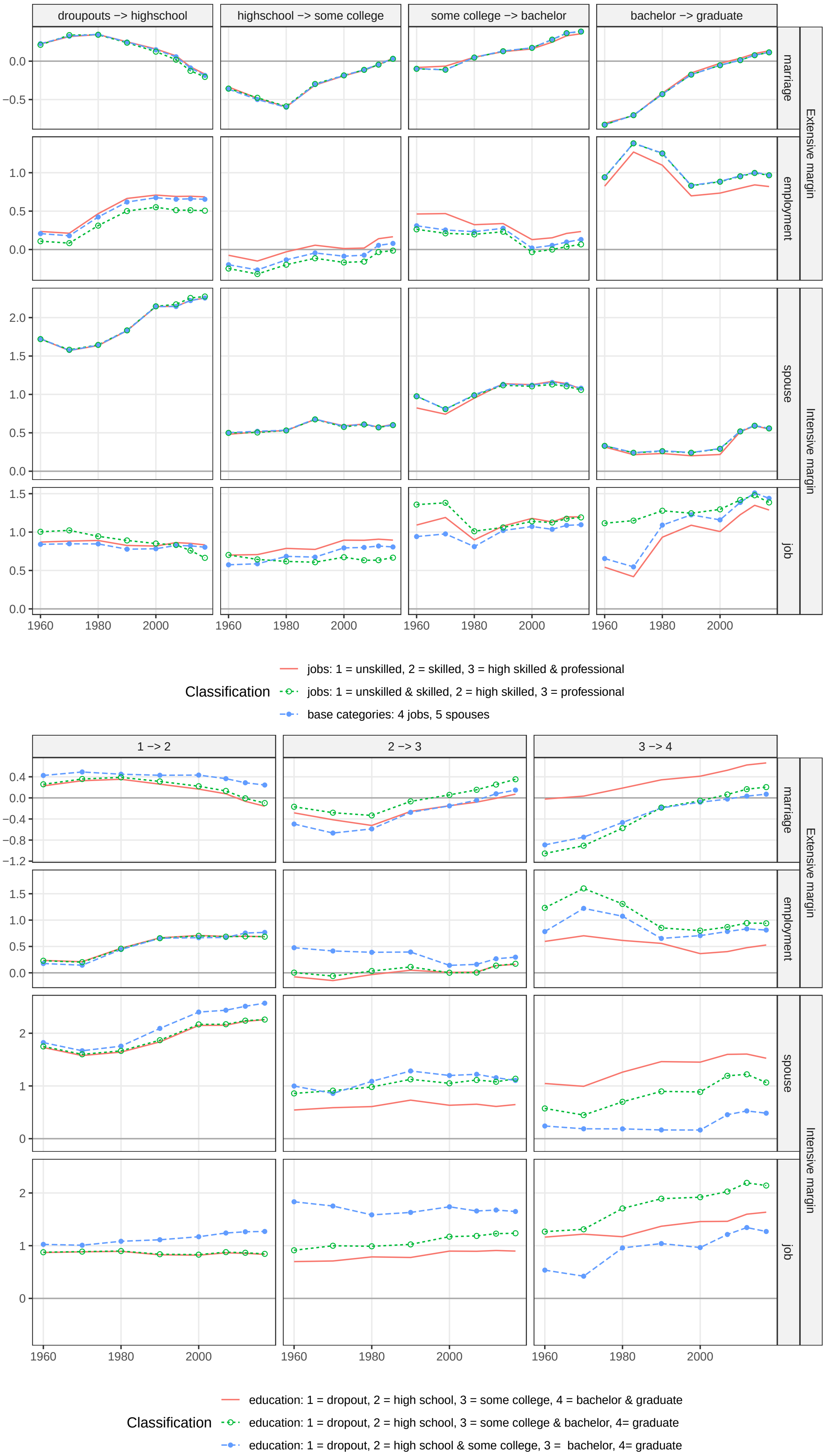

Figure A.1: Marginal-free returns to female education in the U.S. for different classifications of education and occupation.

Table A.2: Sample number of women between 35 and 50 across the U.S. datasets.

year	All	Black	CPS
1960	950,055	93,908	
1968			15,196
1969			15,223
1970	192,010	20,310	14,361
1971			14,227
1972			13,417
1973			12,892
1974			12,342
1975			11,972
1976			12,321
1977			14,322
1978			14,069
1979			13,911
1980	986,597	112,331	16,404
1981			16,491
1982			15,165
1983			15,610
1984			15,844
1985			16,277
1986			16,169
1987			16,349
1988			16,947
1989			15,855
1990	1,361,741	131,811	17,927
1991			18,386
1992			18,420
1993			18,764
1994			18,464
1995			18,447
1996			16,486
1997			16,831
1998			16,889
1999			17,098
2000	1,717,828	190,917	17,146
2001			29,513
2002			29,418
2003			29,198
2004			28,643
2005			27,855
2006			27,451
2007	1,756,493	182,793	27,033
2008			26,600
2009			26,367
2010			26,128
2011			25,149
2012	1,627,317	182,636	24,346
2013			24,199
2014			23,577
2015			23,166
2016			21,663
2017	1,554,248	160,644	21,683
2018			20,778
2019			20,726
2020			17,933
2021			18,738

Monte Carlo integration with 10,000 sample points. Overall, Figure A.2 shows that choosing normal distribution instead of extreme value does not significantly change the value of the returns and for the most part normal estimations lie between confidence interval of extreme value estimations.

B.2 US estimations using CPS data

Figure A.3 compares the aggregate indices for the U.S. estimated using CPS with the estimated indices using ACS and Census. We do not observe a significant difference between the two estimations. It should be noted that, because of the fewer number of observation in CPS, some elements of the contingency tables equal to zero in some years. We replace their number with 0.5 and their corresponding wage equal to the mean to be able to estimate.

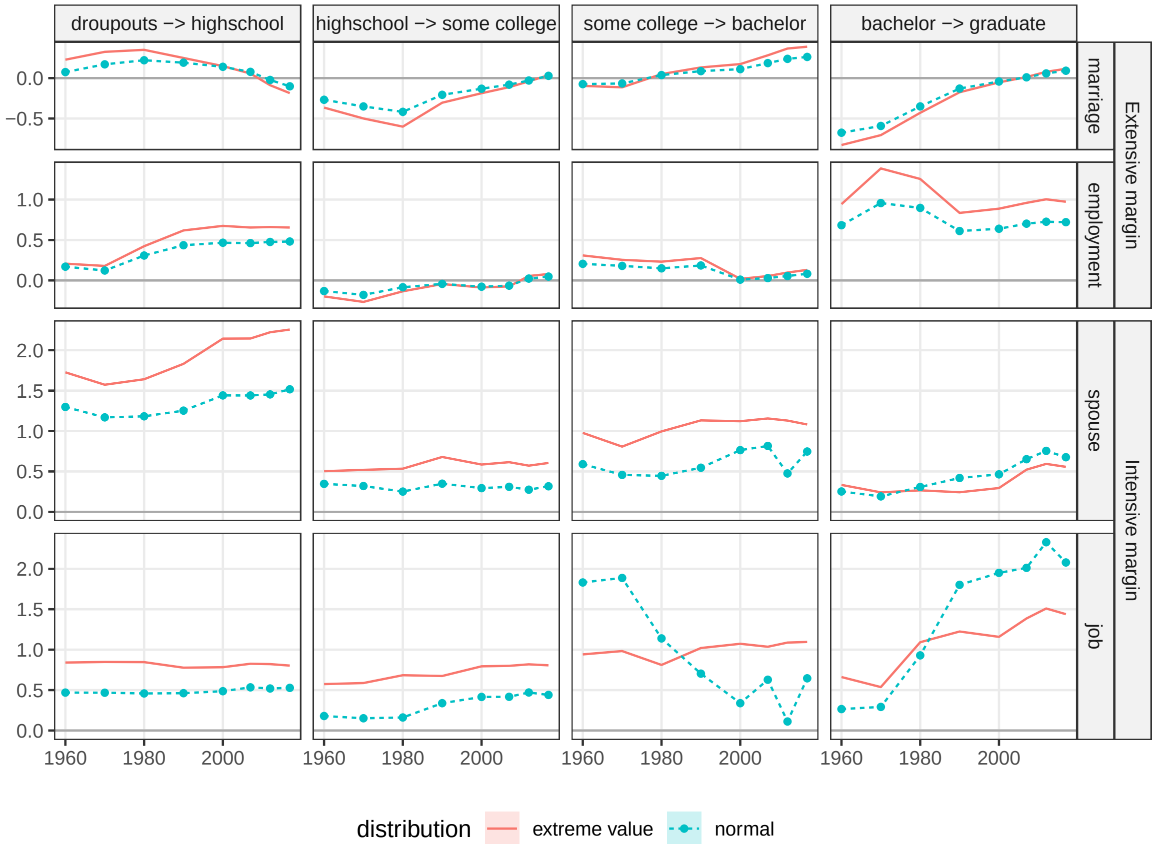

Figure A.2: Marriage and employment returns to female education in the United States using two distributional assumptions for unobservables α_i^{JK}: (1) extreme value distribution, (2) normal distribution. The indices are estimated by marginal-free returns as in (26) to (28).

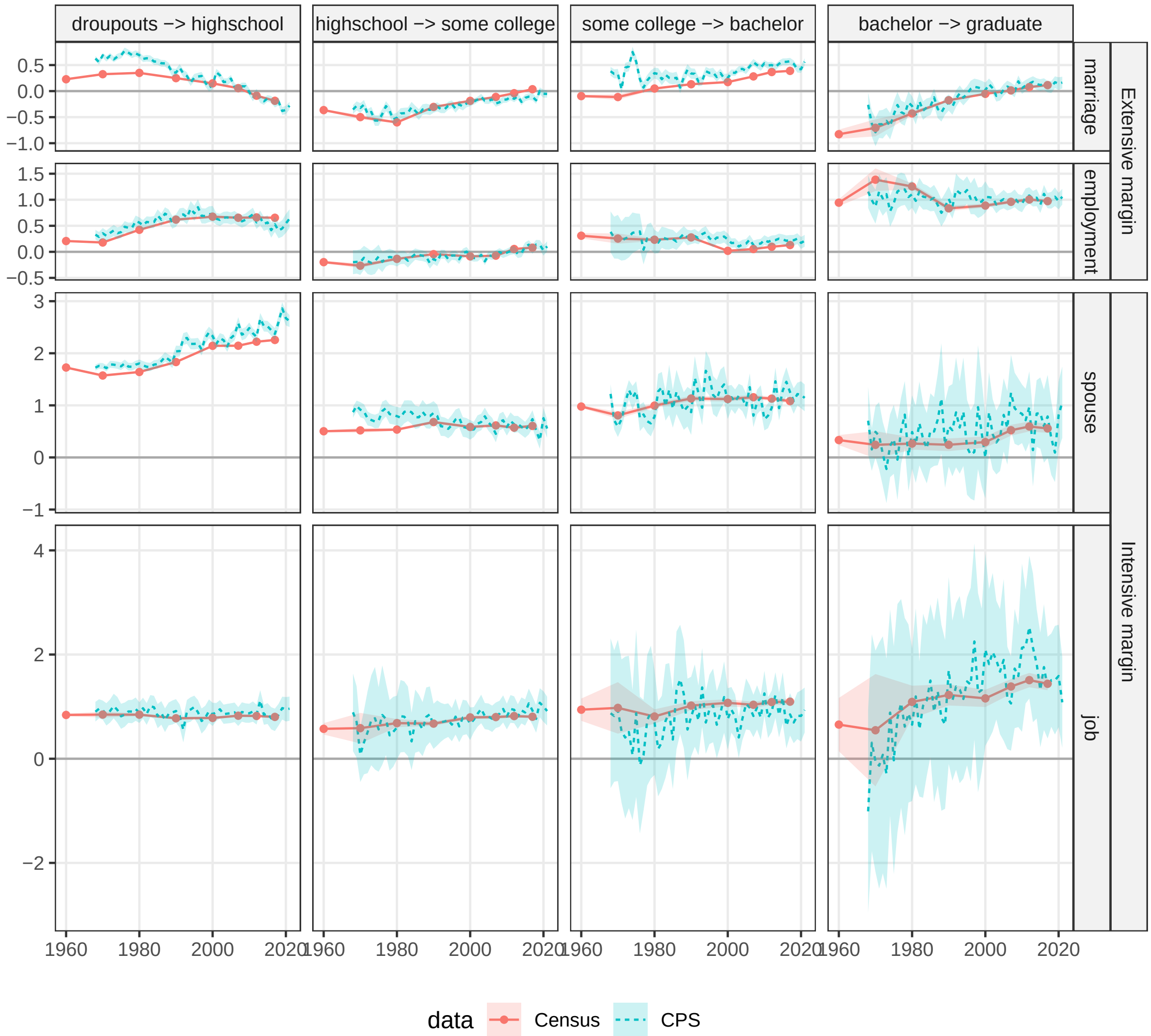

Figure A.3: Marginal-free marriage and employment returns to female education in the U.S. The lines show trends for different datasets and shaded area is confidence interval of estimations using Census+ACS. To avoid zero cell in CPS some college and bachelor are merged as college. Also, unskilled is merged into skilled and professionals into high skilled.

Figure A.4: Sign-based identification of marginal-free returns at the extensive margin conditional on marriage and employment status. The '+' and '–' mean that all conditional elements of the aggregate indices are significantly identified with and the same signs in one year of the data. The number of signs and fill color in each box show the number of years that the sign is identified. The order of signs within each box are based on years with 1960 in top left and 2017 in bottom right. Blanks means no sign-based identification is inferred. The inference for the sign of conditional odds ratio is based on one-sided Fisher's exact test. Abbreviation of education is D=dropout, H=high school, C=some college, B=bachelor and G=graduate school.

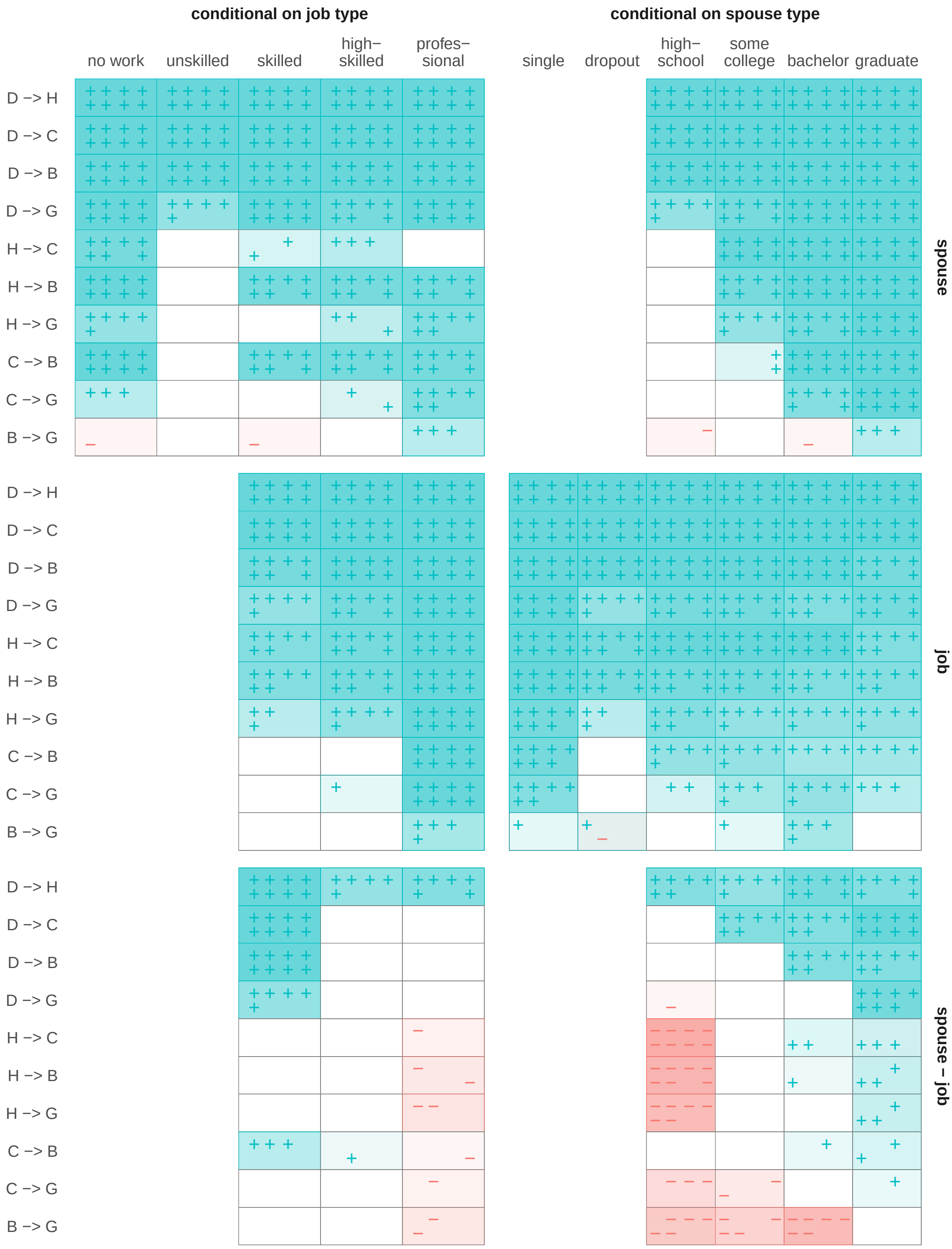

Figure A.5: Sign-based identification of marginal-free returns at the intensive margin conditional on marriage and employment status. The '+' and '–' mean that all conditional elements of the aggregate indices are significantly identified with and the same signs in one year of the data. The number of signs and fill color in each box show the number of years that the sign is identified. The order of signs within each box are based on years with 1960 in top left and 2017 in bottom right. Blanks means no sign-based identification is inferred. The inference for the sign of conditional odds ratio is based on one-sided Fisher's exact test. Abbreviation of education is D=dropout, H=high school, C=some college, B=bachelor and G=graduate school.

Table A.3: Heteroskedastic parameters' estimations for different educational categories.

year	dropouts	high school	some college	bachelor	graduate
1960	1.00000341	1.0000037	0.9999981	0.99998702	0.99994641
	(0.0022242)	(0.0041153)	(0.0193907)	(0.0191433)	(0.0292661)
1970	1.00000189	1.00000166	1.00001463	0.9999164	0.99973903
	(0.0069664)	(0.0095837)	(0.0543202)	(0.0324258)	(0.0675982)
1980	1.00000364	1.00000497	1.00000976	0.99999954	0.99999317
	(0.0042067)	(0.004297)	(0.0140189)	(0.0121614)	(0.0175412)
1990	1.00000711	1.00000838	1.00000614	1.0000107	1.00002763
	(0.005585)	(0.0053635)	(0.007329)	(0.0095618)	(0.0138766)
2000	1.00000625	1.00000995	1.00000993	1.00001012	1.00003972
	(0.0057754)	(0.0050663)	(0.0057479)	(0.008195)	(0.0116831)
2007	1.00000459	1.00000817	1.00000777	1.00000673	1.00002169
	(0.0064391)	(0.0051253)	(0.005674)	(0.0067893)	(0.0098856)
2012	1.00000352	1.00000717	1.00000673	1.00000783	1.00000883
	(0.0059234)	(0.0048995)	(0.0052113)	(0.0068158)	(0.0091886)
2017	1.0000032	1.00000544	1.00000564	1.00000746	1.00001727
	(0.0062077)	(0.0050387)	(0.0052376)	(0.0067993)	(0.008471)

Table A.4: Minimum distance estimations. The numbers are the optimized value of the distance after convergence. In the homoskedastic models there are 151 parameters and 84 degrees of freedom. The heteroskedastic model has 155 parameters and 80 degrees of freedom.

estimation	1960	1970	1980	1990	2000	2007	2012	2017
homoskedastic	9.657823	13.12075	5.558862	2.891519	2.736294	3.615094	3.594832	2.998494
heteroskedastic	8.475742	11.21744	4.620373	2.336934	2.105446	2.950242	2.875500	2.489238