Regular Stochastic Choice and Diminishing Marginal Utility

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Introduction

- Modelling of stochastic behaviour has received attention since the 19th century.
- We focus on one property of stochastic behaviour known as Regularity (Block & Marschak (1960)).
- Why? For now...
  - Regularity is "possibly the most well-known property of stochastic choice" (Cerreia-Vioglio et al., 2019).
- What?
  - Regularity posits that if an element is chosen from a menu with a certain probability, it is chosen with a weakly higher probability in all the sub-menus containing it.

Motivation

- ...Regularity is "possibly the most well-known property of stochastic choice" but...
- ...it does not have a natural behavioural interpretation.
  - ...is a necessary condition for Random Utility (Block & Marschak, 1960) models, many special cases of RUMs (Apesteguia et al. (2017), Manzini & Mariotti (2018), Gul & Pesendorfer (2006)), as well as Perturbed Utility models (Fudenberg et al., 2015).
  - ...its violations characterize randomization behaviour (Cerreia-Vioglio et al., 2019) as well as attention models (Cattaneo et al., 2020).
- The literature seems to consider Regularity as the stochastic analogue of Sen's α. However, this is a debated topic (Fishburn, 1978).

Our model and research question

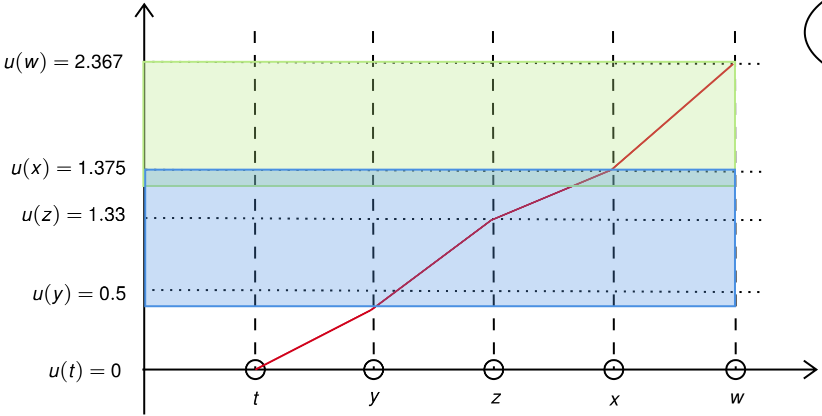
- We study a decision-maker who has a set of preferences, a set of consistency levels, and a probability distribution on pairs of them.
- This is a novel approach that has recently gained popularity (Dardanoni et al., 2022), (Masatlioglu & Filiz-Ozbay, 2022), (Petri, 2022).
- In a nutshell...
- ...every day, the DM's preferences and consistency levels are drawn.
- the DM selects a set of alternatives and randomizes.
- her choices across days are aggregated using the probability distribution on preferences and consistency levels.

RQ: We study which properties of the preferences and consistency levels characterize everyday *regular* choices, which in turn would guarantee that their aggregation is also *regular*.

Some preliminary observations

- **Observation:** Regularity can be satisfied by a highly inconsistent decision-maker and violated by a highly consistent one.
- Sen's α is only necessary for Regularity and needs a second property, denoted θ, to achieve the full characterization.
- From here, we study the properties of preferences and consistency levels induced by α and θ.

Diminishing marginal utility...



$c(A) = w, x$

$c(x, y, z) = x, y, z$

violation of Regularity

Preliminaries

- Let X be a finite set, and $\mathcal{X}$ be the set of all non-empty subsets of X.
- Let $c : \mathcal{X} \to \mathcal{X}$ be a choice correspondence, $c(A) \subseteq A$ for all $A \subseteq X$. Let CC be the set of all choice correspondences defined on $\mathcal{X}$.
- Let $p : X \times 2^X \to [0,1]$ with $p(x,A) = 0$ if $x \notin A$ and $\sum_{x \in A} p(x,A) = 1$ for all $A \subseteq X$. We call p a stochastic choice function.
- A tie-break rule π is a stochastic choice function, such that for all $x \in X$, and for all menus A: $p_{\pi|c}(x,A) = \pi(x, c(A))$.

Model

- A choice correspondence c is drawn with probability $\mu(c)$.
- Tie-break rules induce a stochastic choice function $p_{\pi|c}(x, A)$ that are aggregated by means of $\mu(c)$.

$$p(x, A) = \sum_{c \in \mathrm{CC}} \mu(c) \cdot p_{\pi|c}(x, A)$$

- For our main results, we focus on the uniform tie-break rules that induce the following $p_{u|c}$.

$$p_{u|c}(x, A) = \begin{cases} 0 & \text{if } x \notin c(A) \\ \frac{1}{|c(A)|} & \text{if } x \in c(A) \end{cases}$$

Regularity: preliminary observations

- **Definition (Regularity):** For all $x \in A \subseteq B$, $p(x, A) \geqslant p(x, B)$.
- **Observation – Aggregation:** If $p_{\pi|c}$ satisfy Regularity for all $c \in \text{CC}$ with $\mu(c) > 0$ then p satisfies Regularity.

Regularity: preliminary observations

- **Observation – Regularity is both fragile and resilient:**
  1. If $c \in \mathrm{CC}$ is rationalized by a weak order then the resulting $p_{u|c}$ satisfies Regularity.
  2. There exists a $c \in \mathrm{CC}$ rationalized by a simplest semiorder such that the resulting $p_{u|c}$ does not satisfy Regularity.
  3. There exists a $c \in \mathrm{CC}$ that cannot be rationalized by any binary relation such that the resulting $p_{u|c}$ satisfies Regularity.

Regularity: characterization

- **Definition: (Sen's α)**: for all $x \in A \subseteq B$, $x \in c(B) \Rightarrow x \in c(A)$.
- **Observation:** Regularity implies Property α, but the converse is not true.

$$\left[\begin{array}{c|cccc} A & \{x,y,z\} & \{x,y\} & \{y,z\} & \{x,z\} \\ c(A) & x & x,y & y & x \\ p_{u|c}(A) & (1,0,0) & (0.5,0.5) & (1,0) & (1,0) \end{array}\right]$$

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- **Definition: (Property θ)**: $A \subseteq B$ and $c(A) \cap c(B) \neq \varnothing$ imply $|c(A)| \leqslant |c(B)|$.
- The intuition behind Property θ is simple. As Regularity requires choice probabilities to decrease in set inclusion, the stochastic choice function becomes more uniform as more elements are added to the set.

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- **Proposition:** $p_{u|c}$ satisfies Regularity if and only if c satisfies Property α and Property θ.

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- **Proposition:** $p_{u|c}$ satisfies Regularity if and only if c satisfies Property α and Property θ.
- **Question:** What are the behavioural implications of Property α and θ?
  - To answer this question, we need to look at representations for the choice correspondences.

Deep and decision utility

- We rely on generalized threshold models [GTM] (Aleskerov et al., 2007).
- The DM is characterized by a utility function $u : X \to \mathbb{R}^+$ and a non-negative symmetric threshold $\varepsilon : X \times X \times \mathcal{X} \to \mathbb{R}^+$.
- The utility function u is the deep utility while the decision utility is:

$$f(x,y,A) = \frac{u(x)}{\varepsilon(x,y,A)}$$

- The couple (u,ε) maps into choices as follows: for each set A, $c(A) = \{x \in A : \nexists y \in A : f(y,x,A) - f(x,y,A) > 1\}$.

"High" consistency levels

- **Observation:** The following are equivalent:
  1. The choices are represented by a semiorder.
  2. Deep and decision utility coincides.
  3. ε is a constant threshold.
  4. The trace is transitive and complete.

"High" consistency levels - diminishing marginal utility

- **Definition (standard concavity):** A function $u : \mathbb{Z} \rightarrow \mathbb{R}$ is **concave** if $u(x+1) - u(x-1) \leqslant 2u(x)$.

Theorem

Property θ is satisfied only if there exists a constant threshold representation with a globally concave utility function.

"High" consistency levels - diminishing marginal utility

- **Definition (strong concavity, similar to boundeness):** There exists a $k \geqslant 3$ such that for all $i > k$, $u(x_i) - u(x_k) \leqslant \varepsilon$.

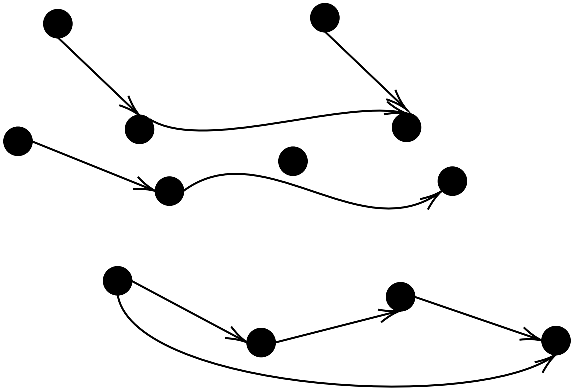
Theorem

Let c be rationalized by a semiorder. Property θ is satisfied if and only if there exists a constant threshold representation with a strongly concave utility function.

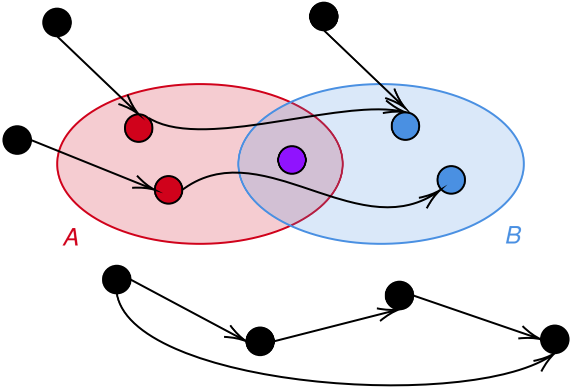
"Medium" consistency levels (α, γ - binariness)

- **Observation (transitivity):** If there exists an acyclic relation that rationalizes the choices then Property θ implies the relation is also transitive.
- However, how do we define concavity here?

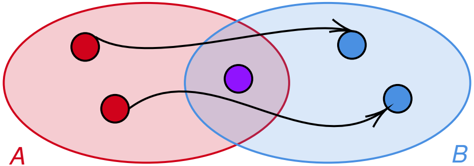
"Medium" consistency levels - antichains



"Medium" consistency levels - antichains



"Medium" consistency levels - order of antichains



- **Definition (strong concavity):** For all antichains A, B, if $A \gtrdot B$ and $A \cap B \neq \emptyset$ then $|A| \geqslant |B|$.

Theorem

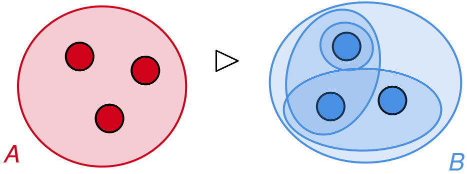
Let c be rationalized by a partial order. Property θ is satisfied if and only if Strong Concavity holds.

"Low" consistency levels (α) + (*outcast*)

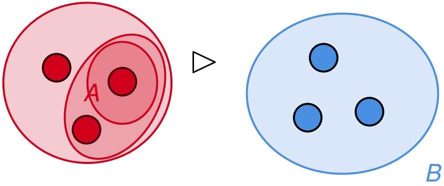
- **Definition (revealed hyper-relation):** For every two disjoint sets $A, B \subseteq X$, $A \rhd B$ if $B \cap c(A \cup B) = \varnothing$.
- **Definition (outcast):** For all $A, B \subseteq X$, $c(B) \subseteq A \subseteq B$ implies $c(A) \subseteq c(B)$.

"Low" consistency levels - monotonicity

Up-Monotonicity



Down-Monotonicity



"Low" consistency levels (α) + (*outcast*)

- **Observation (monotonicity):**

  1. The Outcast condition is satisfied if and only if $\rhd$ satisfies up-monotonicity.
  2. Property α is satisfied if and only if $\rhd$ satisfies down-monotonicity.

"Low" consistency levels (α) + (*outcast*)

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  2. Property α is satisfied if and only if $\rhd$ satisfies down-monotonicity.
- **Observation ($\rhd$ is monotonic):** If Property α is satisfied then Property θ implies the Outcast condition.

"Low" consistency levels - diminishing marginal utility

- **Definition (strong concavity):** For all antichains A, B of $\rhd$, if $A \gtrdot B$ and $A \cap B \neq \emptyset$ then $|A| \geqslant |B|$.

Theorem

The revealed preference relation $\rhd$ satisfies monotonicity and strong concavity if and only if Property α and Property θ are satisfied.

Conclusion

- We provide a behavioural story to answer the question: "why Regularity is so ubiquitous?"
- We propose diminishing marginal utility as a possible explanation.
- This explanation holds regardless of the consistency level of the DM; namely, outside the standard assumptions of transitive and complete preferences.
- Incidentally, we provide several results that connect different parts of the theoretical literature.

Related Literature

- Regularity: Block & Marschak (1960), McClellon (2015), Apesteguia et al. (2017), Manzini & Mariotti (2018), Cerreia-Vioglio et al. (2018), Cattaneo et al. (2020).
- Deterministic and stochastic choice theory: seminal papers are Fishburn (1973) and Fishburn (1978); more recent contributions: Dasgupta & Pattanaik (2007), Fudenberg et al. (2015) Horan (2021), Cerreia-Vioglio et al. (2021), and Ok & Tserenjigmid (2022).
- Stochastic models with heterogeneous preferences and consistency levels: , Dardanoni et al. (2020), Dardanoni et al. (2022), Petri (2022), and Masatlioglu & Filiz-Ozbay (2022).
- Representations of choice correspondence. Great reviews: M.A.Aizerman (1985) and Aleskerov et al. (2007). Old and more recent relevant contributions: Luce (1956), Scott & Suppes (1958), Rabinovitch (1977), Tyson (2008), Masatlioglu et al. (2012), Lleras et al. (2017), Frick (2016).

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