Rational Inattention via Ignorance Equivalence

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joint with
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August 29, 2023

Introduction

Main Contribution

- We propose the novel concept of the **ignorance equivalent** as a parsimonious summary of the RI decision problem and its properties.
  - Ability to learn is *'equivalent'* to access to a larger menu
  - Learning analogue of the certainty equivalent for lotteries

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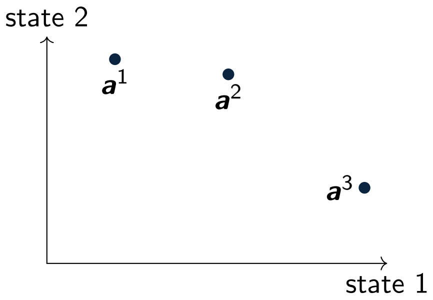
  - Ability to learn is *'equivalent'* to access to a larger menu
  - Learning analogue of the certainty equivalent for lotteries
- The equivalence between learning ability and fictional payoffs simplifies analysis, including in multi-player settings.

Model Setup

- An agent is choosing an action from a finite menu $\mathcal{A}$.
- Action $\boldsymbol{a} \in \mathcal{A} \subseteq \mathbb{R}^I$ yields utility a_i in state $i \in \mathcal{I} = 1, ..., I$.
- The agent has a prior belief $\boldsymbol{\pi} \in \Delta\mathcal{I}$ over states.

- **Payoffs, states and actions**



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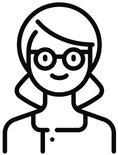
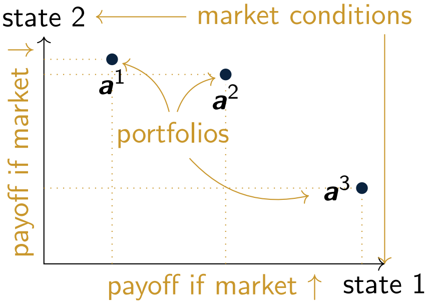


Investor Example



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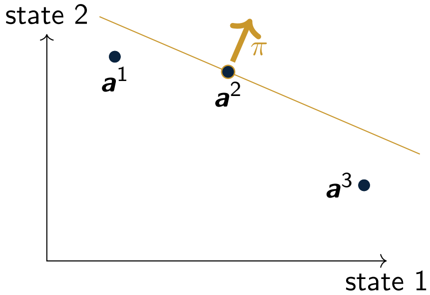

- **No information**
  - Agent maximizes expected utility under prior belief $\boldsymbol{\pi}$.

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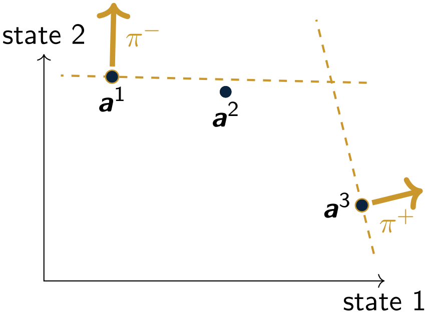
- **Incomplete information**
  - Agent observes signal draw s,
  - updates belief to $\boldsymbol{\pi}^s$,
  - maximizes expected utility under this belief.



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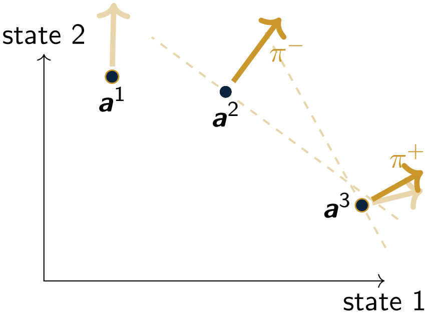
- **Rational Inattention**
  - Agent chooses *which* costly signal to draw,
  - ... and proceeds as before.



Welfare: $W(\mathcal{A}, \boldsymbol{\pi}, c) =$ E[consumption utility] - [cost of signal].

Learning Strategies

- Agent can choose *any* **learning strategy** $\mathcal{S} = \langle \mathcal{A}, \boldsymbol{q} \rangle$, where $q_i(\boldsymbol{a})$ denotes the likelihood of taking action $\boldsymbol{a}$ conditional on state i.

Notation: $c(\mathcal{S}, \boldsymbol{\pi})$ cost of signal $\mathcal{S}$ under prior belief $\boldsymbol{\pi}$ | UPS : uniformly posterior separable

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- Cost $c(\mathcal{S}, \boldsymbol{\pi})$ satisfies five intuitive properties, formal statements
which are shared by all smooth & prior-concave UPS costs.

  Prominent examples:
  - Mutual Information *(Sims JME'03)*,
  - some Tsallis costs *(Caplin-Dean-Leahy '19)*
  - Total Information *(Bloedel-Zhong '20)*, which subsumes
    - Wald costs *(Morris-Strack '19)*, Fisher Information *(Hébert-Woodford '20)*.

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- the agent is willing to commit to *never* implement α,

$$W(\mathcal{A}, \pi, c) \geq W(\mathcal{A} \cup \{\alpha\}, \pi, c).$$

  Agent is willing to forgo learning opportunities that arise when α *is added to* $\mathcal{A}$.

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Since larger menus are always better,

$$\boxed{\{\alpha\} \sim \mathcal{A} \sim \mathcal{A} \cup \{\alpha\}}$$

in terms of the agent's preference over menus given cost c and prior π.

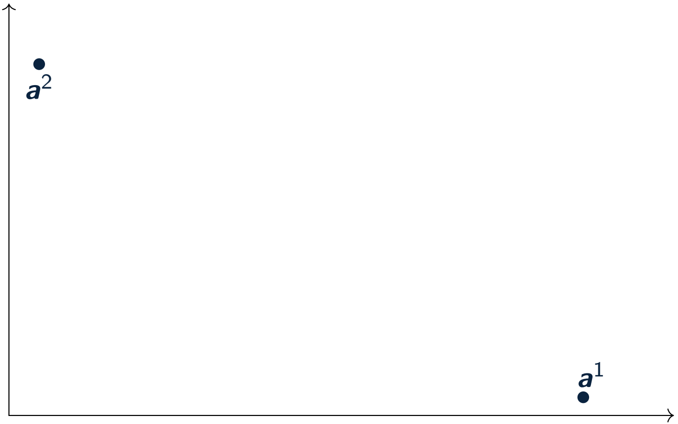
Existence and Uniqueness

Theorem 1

Each RI problem $(\mathcal{A}, \boldsymbol{\pi}, c)$ admits a unique ignorance equivalent $\boldsymbol{\alpha}$.
The ignorance equivalent can be constructed from any optimal signal $\mathcal{S}$.

Ignorance equivalence: $W(\{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c) = W(\mathcal{A}, \boldsymbol{\pi}, c) = W(\mathcal{A} \cup \{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c)$.

Locating the Ignorance Equivalent



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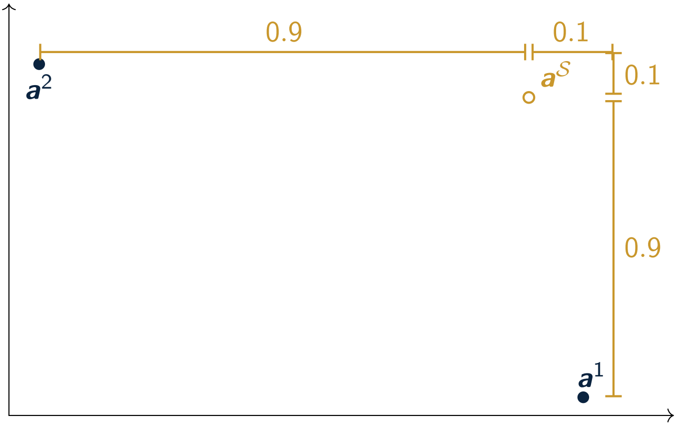

- Strategy $\mathcal{S}$ always implements the high-payoff action with $q_i(\boldsymbol{a}^i) = 0.9$.
- In each state i, the expected consumption payoff is $a_i^{\mathcal{S}} = \sum_{\boldsymbol{a} \in \mathcal{A}} q_i(\boldsymbol{a}) a_i$.

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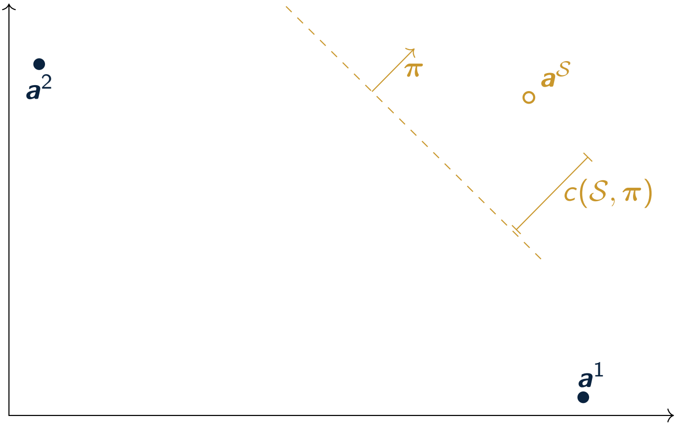

- Following $\mathcal{S}$ yields net utility $\boldsymbol{\pi} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\pi})$ under prior $\boldsymbol{\pi}$.

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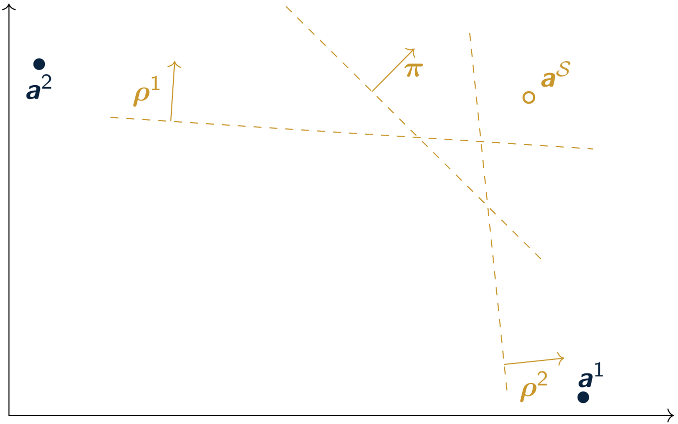

- The same construction shows net utility from $\mathcal{S}$ under any belief $\boldsymbol{\rho}$.

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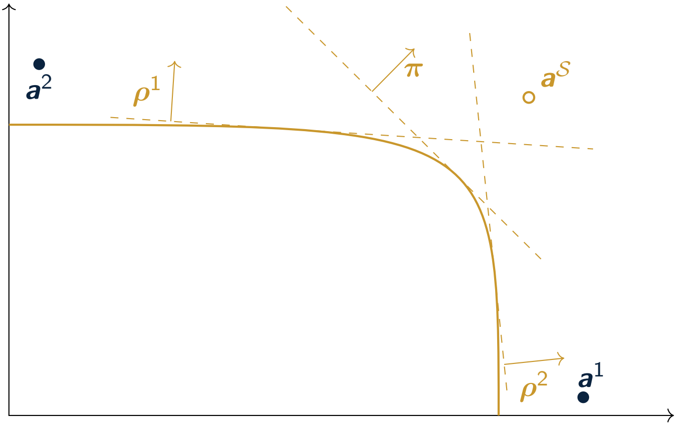

- Dominance: $\boldsymbol{x} \precsim \mathcal{S} \iff \boldsymbol{\rho} \cdot \boldsymbol{x} \leq \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho})\ \forall \rho.$

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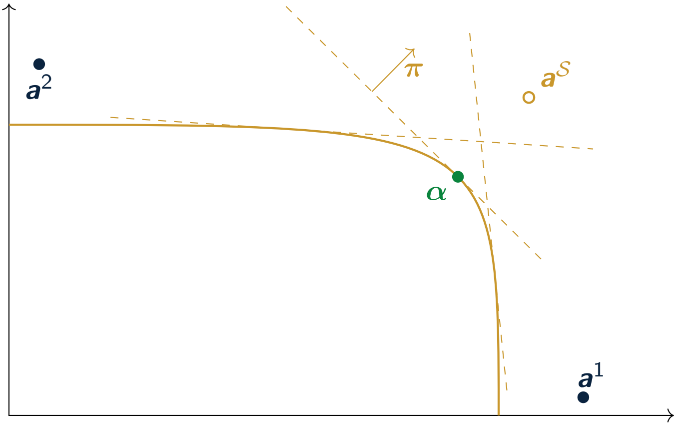
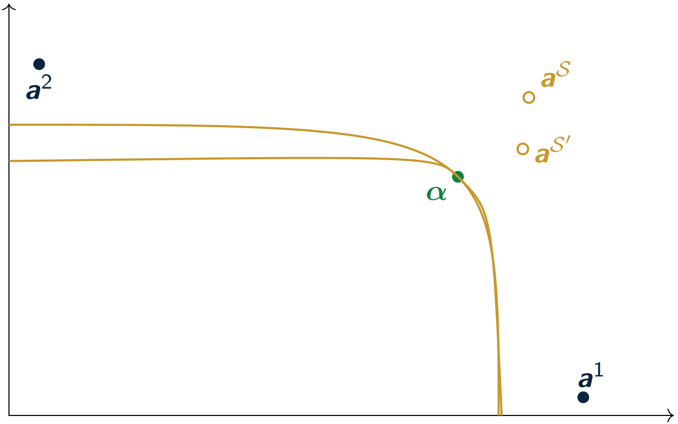


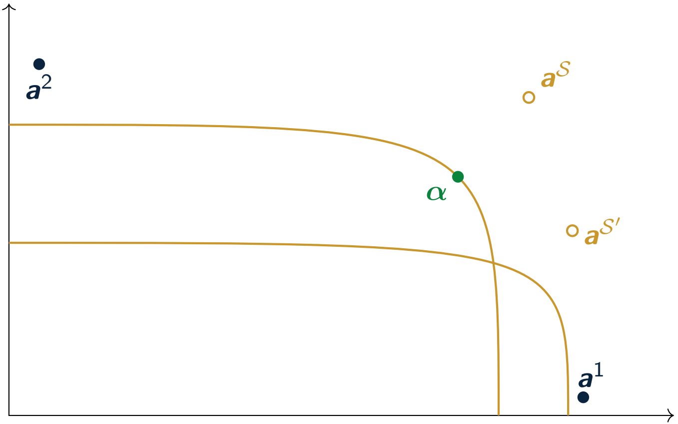
- Unconditional implementation of the ignorance equivalent α
  - is just as good as the optimal $\mathcal{S}$ under the prior.
  - is no better than $\mathcal{S}$ under *any* belief.

Identifying optimal signals



- Optimal signals are exactly those that beat $\boldsymbol{\alpha}$ under any belief.

A suboptimal signal



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Properties of the Ignorance Equivalent

The ignorance equivalent α of RI problem $(\mathcal{A}, \boldsymbol{\pi}, c)$ ...

- characterizes all optimal learning strategies w/o reference to beliefs.

  Corollary 1: If α is also the ignorance equivalent of $(\mathcal{A}, \boldsymbol{\pi}', c)$, then the two RI problems have the same set of optimal learning strategies.

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⋆ is sufficient to identify which menu additions $\boldsymbol{a}^+ \in \mathbb{R}^I$ are welfare enhancing, $W(\mathcal{A} \cup \{\boldsymbol{a}^+\}, \boldsymbol{\pi}, c) > W(\mathcal{A}, \boldsymbol{\pi}, c)$.

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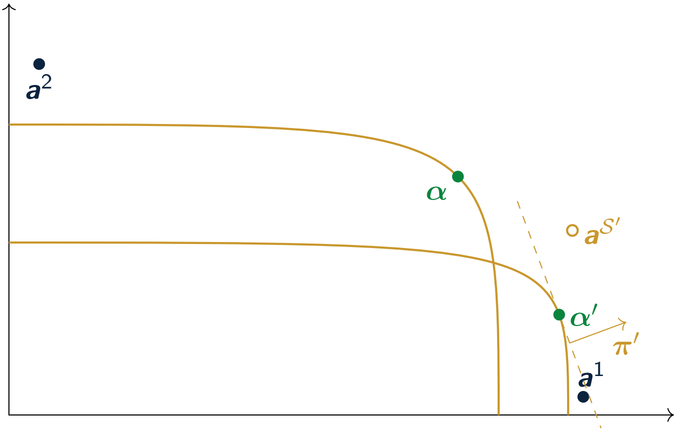
- does not distort learning under *any* prior.

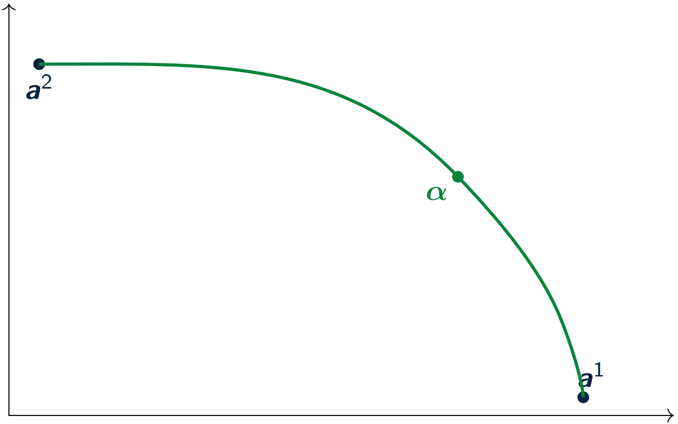
Corollary 3: If learning strategy $\langle \mathcal{A}, \boldsymbol{q} \rangle$ is optimal in RI problem $(\mathcal{A}, \boldsymbol{\pi}', c)$, then it is also optimal in $(\mathcal{A} \cup \{\alpha\}, \boldsymbol{\pi}', c)$.

Self-selection Property



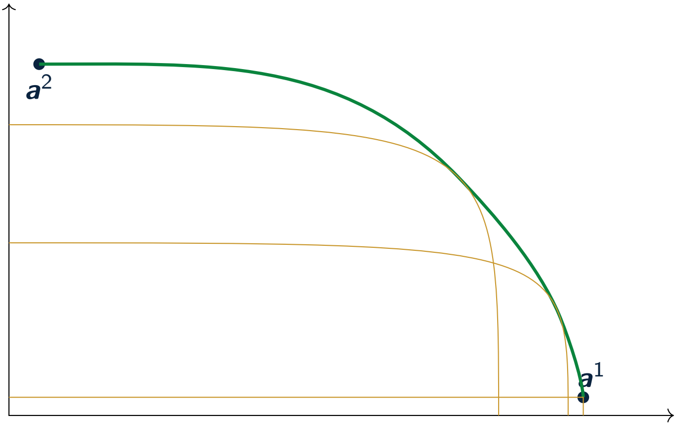
- Stopping at α' is no better than continuing with $\mathcal{S}'$.

Learning-Proof Menu



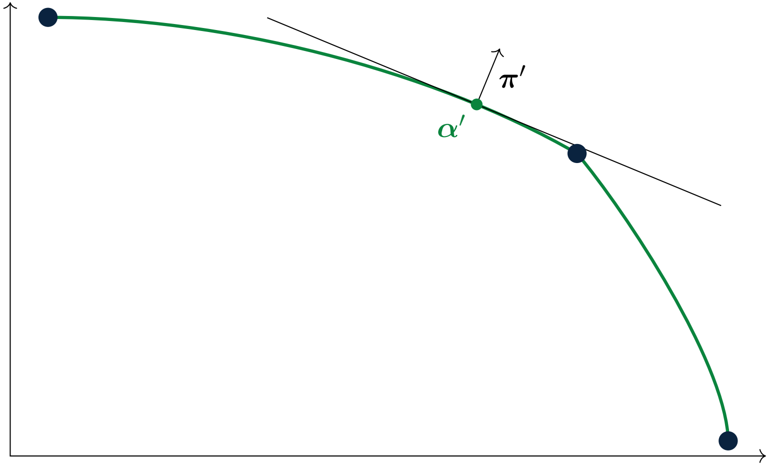
- Together, ignorance equivalents form the **Learning-Proof menu** $\bar{\mathcal{A}}$.

Learning-Proof Menu



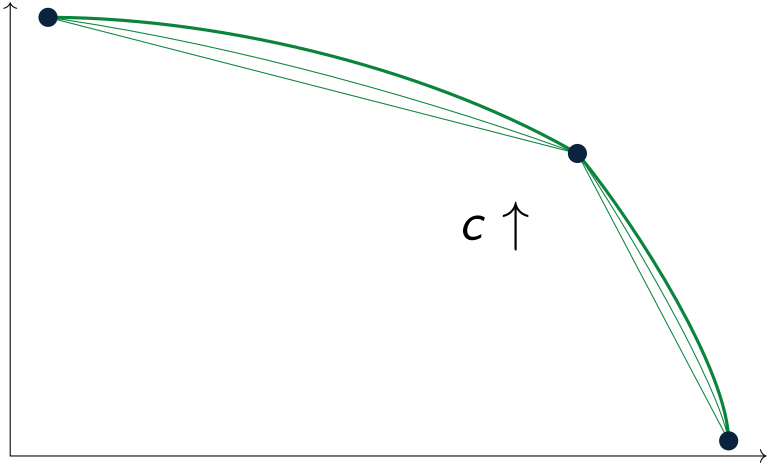
- The learning-proof menu can also be constructed from the signals.

Learning-Proof Menu



The Learning-Proof Menu is a **EU representation** of the RI problem:
Ability to learn is *as if* agent has access to $\bar{\mathcal{A}}$ rather than $\mathcal{A}$.

Learning-Proof Menu



For high costs, $\bar{\mathcal{A}}$ approaches the boundary of the convex hull.

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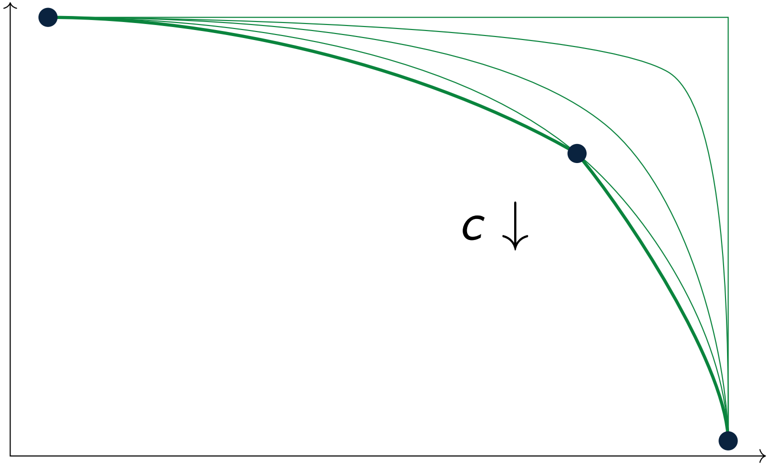

For low costs, $\bar{\mathcal{A}}$ approaches the boundary of the hypercube.

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Certainty equivalent:

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Ignorance equivalent:

- Condenses the entire menu $\mathcal{A}$ into one payoff vector α.
- Retains enough detail to capture learning opportunities.
- Useful for comparisons across signals, menus, and beliefs.

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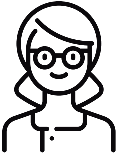

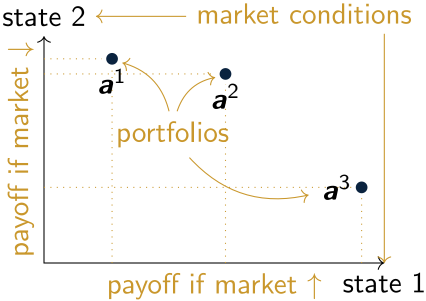

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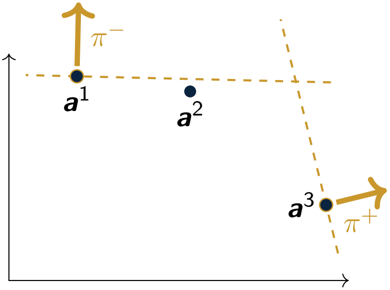

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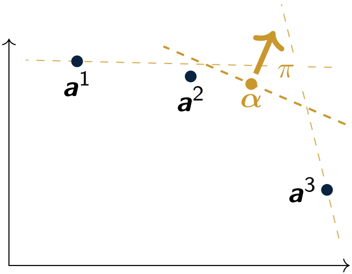
- α describes a portfolio that
  - is purchased unconditionally
  - makes her no better off.



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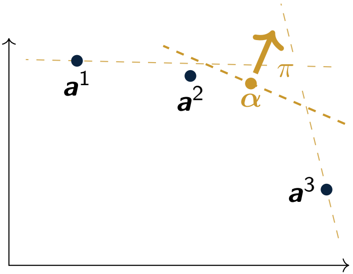

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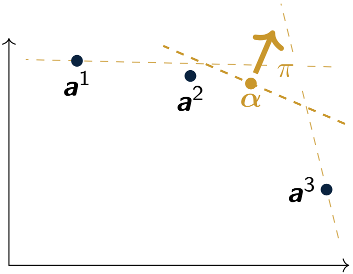

- α is what he wants to offer!
  - Unconditional acceptance.
  - Avoids adverse selection.
  - Reveals no free information.
  - Socially optimal.

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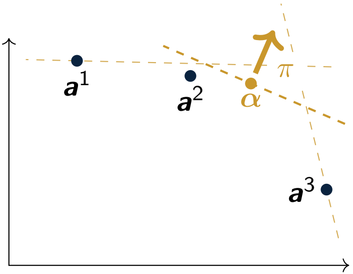

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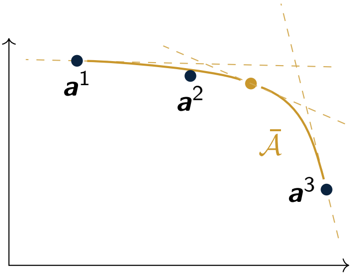
  - The manager can offer $\bar{\mathcal{A}}$.



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- If agents can access different menus and have comparative advantages in learning (technology acquisition example), the first best typically involves learning by multiple agents (teams problem).
  - *We construct a PBE that achieves the first best through repeated trades.*

visuals

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Appendix

Admissible cost functions

back

Notation:

- Signal $\mathcal{S} = \langle S, \boldsymbol{q} \rangle$ returns $s \in S$ with probability $q_i(s)$ in state i.
- $c(\mathcal{S}, \boldsymbol{\rho}) \in [0, \infty)$ denotes the cost of that signal under belief $\boldsymbol{\rho}$.

Admissible cost functions

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We impose five conditions on c:

1. The cost function is continuous.
   - $\forall S, \forall \hat{c} \geq 0$, the pre-image $\left\{(\boldsymbol{q}, \boldsymbol{\pi}) \in (\Delta S)^{\mathcal{I}} \times \Delta \mathcal{I} \mid c(\langle S, \boldsymbol{q} \rangle, \boldsymbol{\pi}) \gtrless \hat{c}\right\}$ is open.

Admissible cost functions

back

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We impose five conditions on c:

1. The cost function is continuous.
2. The agent can freely dispose of information.
   - $c(\cdot, \boldsymbol{\pi})$ is non-decreasing in the Blackwell order $\forall \boldsymbol{\pi}$.
   - $c(\mathcal{S}, \cdot)$ is weakly concave in the prior $\forall \mathcal{S}$.

Admissible cost functions

back

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We impose five conditions on c:

1. The cost function is continuous.
2. The agent can freely dispose of information.
3. Ties are broken through learning:
   - $\forall \boldsymbol{\pi} \in \Delta\mathcal{I}, \forall \boldsymbol{a} \in \mathbb{R}^I$ with $\boldsymbol{\pi} \cdot \boldsymbol{a} = 0$ and $\boldsymbol{\pi} \cdot |\boldsymbol{a}| > 0$, $\exists \mathcal{S} = \langle \{0, 1\}, \boldsymbol{q} \rangle$ such that $c(\mathcal{S}, \boldsymbol{\pi}) < \sum_{i \in \mathcal{I}} \pi_i q_i(1) a_i$.

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We impose five conditions on c:

1. The cost function is continuous.
2. The agent can freely dispose of information.
3. Ties are broken through learning:
4. Sequential information acquisition brings no cost savings.
   - For any contingency plan

     draw $\langle S, \boldsymbol{q} \rangle \longrightarrow$ observe $s \longrightarrow$ update belief to $\boldsymbol{\pi}^s \longrightarrow$ draw $\langle S^s, q^s \rangle$,

     the one-shot implementation

     $$\tilde{\mathcal{S}} = \langle S \times \textstyle\bigcup_{s \in S} S^s, \tilde{\boldsymbol{q}} \rangle \text{ with } \tilde{q}_i(s, \tilde{s}) = q_i(s) q_i^s(\tilde{s})$$

     is no more costly in expectation,

     $$c(\tilde{\mathcal{S}}, \boldsymbol{\pi}) \leq c(\langle S, \boldsymbol{q} \rangle, \boldsymbol{\pi}) + \textstyle\sum_{s \in S} (\boldsymbol{\pi} \cdot \boldsymbol{q}(s)) \, c(\langle S^s, q^s \rangle, \boldsymbol{\pi}^s).$$

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⋆ Sequential information acquisition brings no extra costs.

- As above, but with the opposite inequality,

$$c(\tilde{\mathcal{S}}, \boldsymbol{\pi}) \leq c(\langle S, \boldsymbol{q} \rangle, \boldsymbol{\pi}) + \sum_{s \in S} (\boldsymbol{\pi} \cdot \boldsymbol{q}(s))\, c(\langle S^s, q^s \rangle, \boldsymbol{\pi}^s).$$

Optimal Allocation: Setup

back

Task: Allocate a single opportunity to one RI agent $k \in \{1, ..., K\}$.

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Game Setup:

- All agents share a common prior $\boldsymbol{\pi}^0$ about the state i.
- Initially, the opportunity rests with agent 1.
  It remains transferable as long as it has not been executed.
- Agents can learn at any time
  - each according to a (possibly distinct) cost function c^k,
  - regardless of whether they currently own the opportunity,
  - without 'executing' the opportunity.

Optimal Allocation: Questions

back

First-best allocation: Consider a social planner who

- has access to all actions in $\mathcal{A}^P = \bigcup_{k=1}^{K} \mathcal{A}^k$.

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Trade: A trade between agents k and ℓ at *terms* $\boldsymbol{t} \in \mathbb{R}^I$

- requires the agreement of both agents.
- means that agent k releases the opportunity to agent ℓ, who in turn pays the former t_i once the state i realizes.

Optimal Allocation: Example

back

- *Teams.* A firm buys a new technology that will affect many stakeholders. Some workers are uniquely qualified to learn about specific characteristics of the technology. Can they achieve the optimal sequence of cost-benefit investigations across all workers?

Comparative Advantage

back

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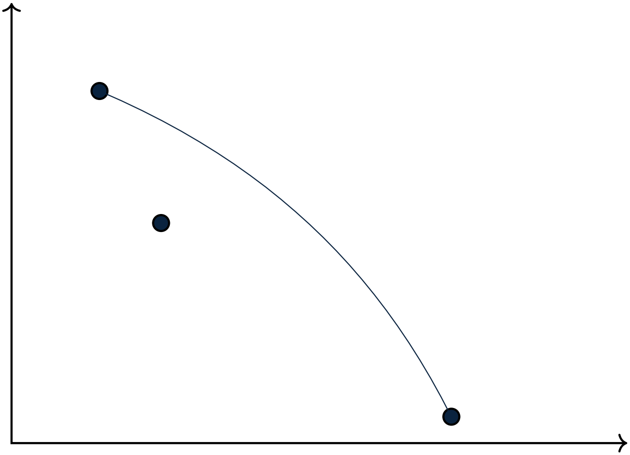
  - All agents are willing to learn or execute if and only if it is socially efficient for them to do so.

Comparative Advantage: Visual

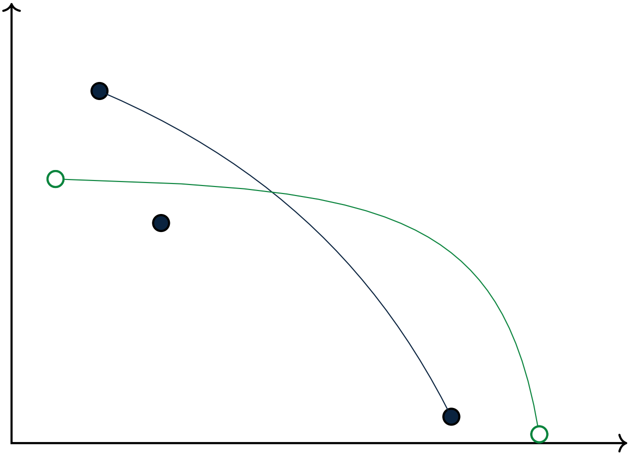
back



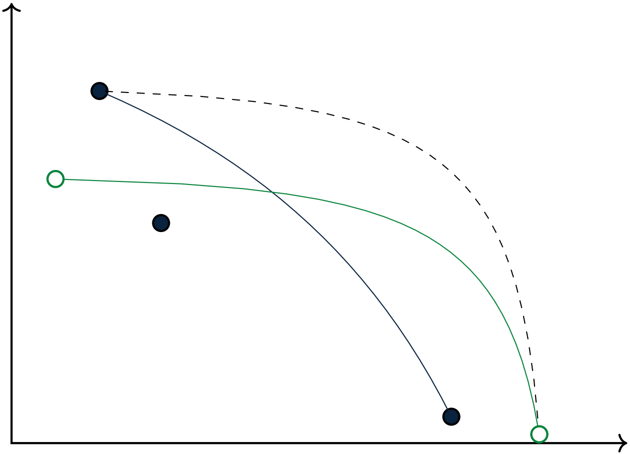
Payoff possibilities for agent 1

Comparative Advantage: Visual

back



Payoff possibilities for agent 2



Payoff possibilities for social planner

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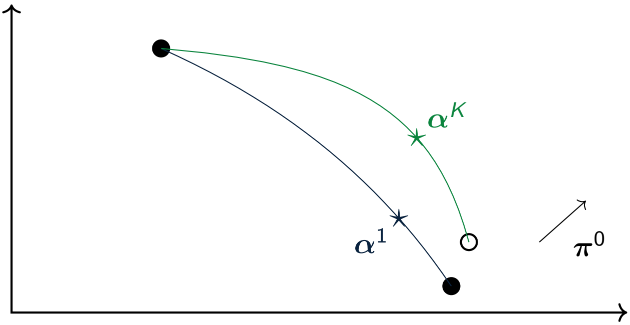
  - **Theorem 3:** $\boldsymbol{t}$ adds welfare to $\mathcal{A}^K \iff \boldsymbol{t}$ adds welfare to $\{\alpha^K\}$
  - Agent K's ignorance equivalent α^K imposes an upper bound on $\boldsymbol{t}$.

Absolute Advantage: Visuals

back



$$\mathcal{A}^1 = \{\bullet, \bullet\}, \mathcal{A}^K = \{\bullet, \bullet, \circ\}$$

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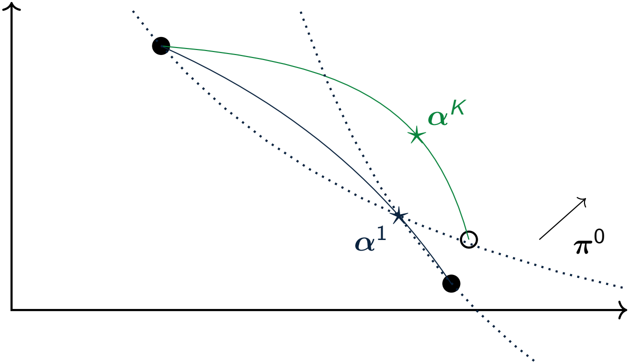

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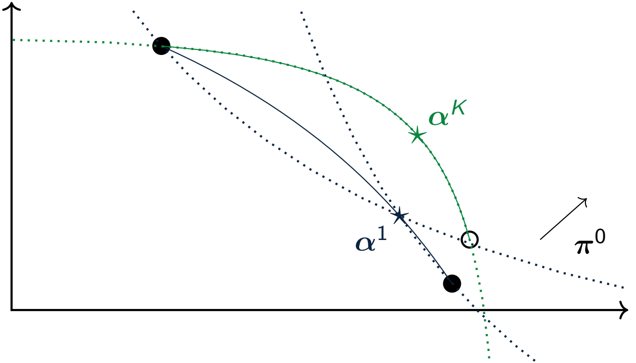

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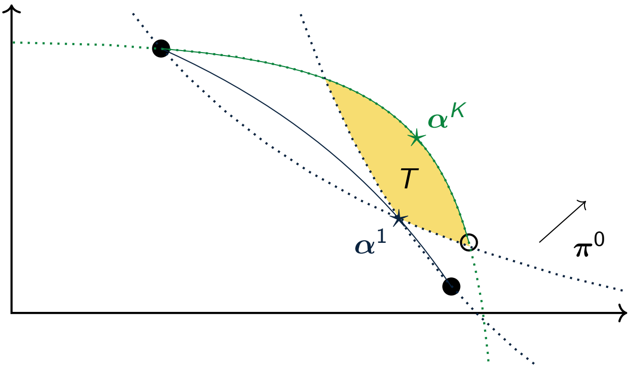

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Absolute Advantage: Visuals

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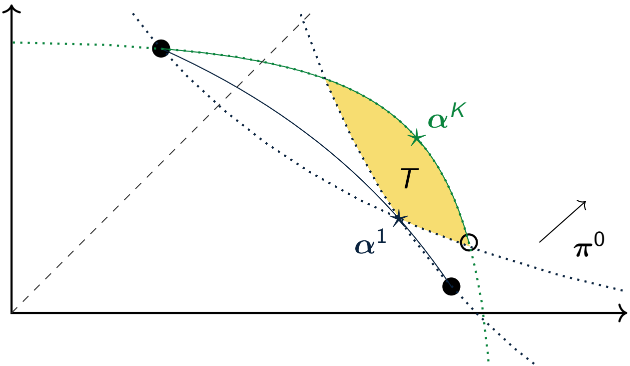

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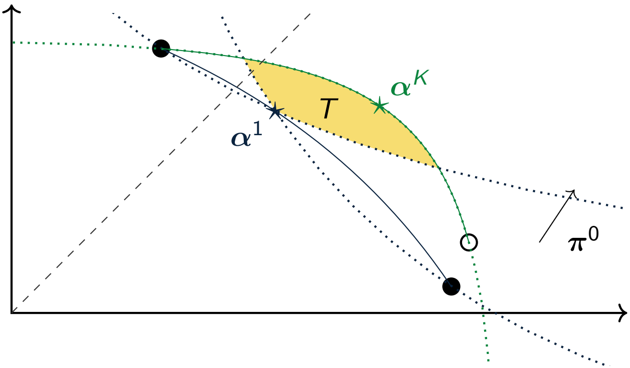

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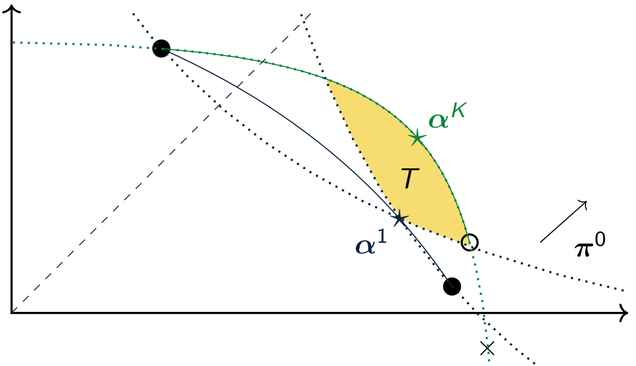

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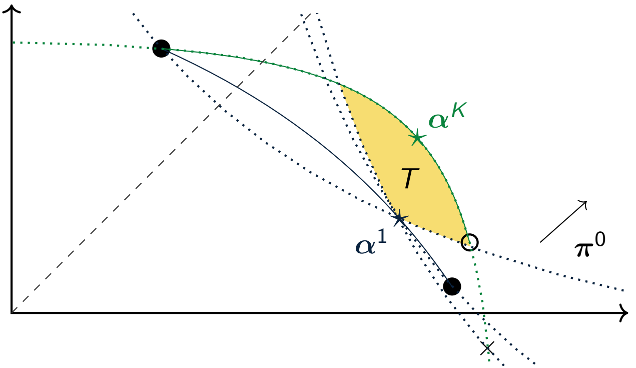

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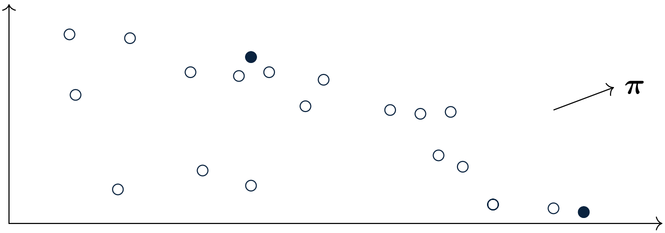
RI patterns of behavior

- Agents consider only actions in a (small) *consideration set*.

  Caplin-Dean-Leahy (REStud'18), Jung-Kim-Matějka-Sims (REStud'19)



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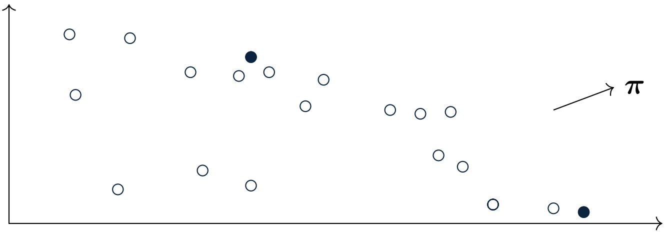

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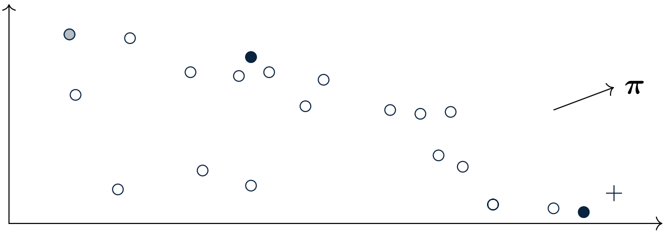

RI patterns of behavior

- Agents consider only actions in a (small) *consideration set*.

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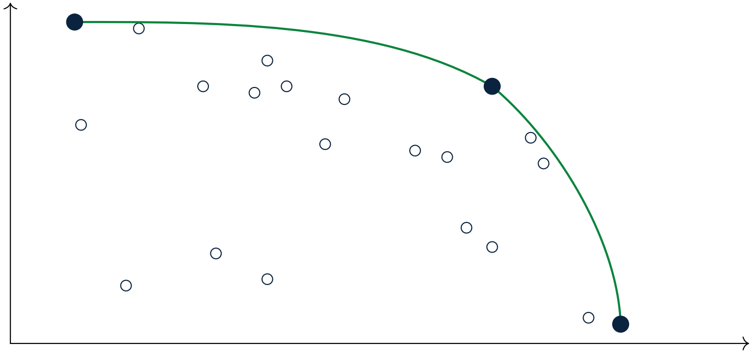
    Crémer&Khalil (AER'92), Yoder (JPE'22), Yang (REStud'20), Gentzkow&Kamenica (AER'14), Matyskova&Montes ('21), Porcher ('20)

Anchor Actions



Theorem

Anchors $\boldsymbol{a} \in \mathcal{A} \cap \bar{\mathcal{A}}$ *form a 'latent' consideration set:*

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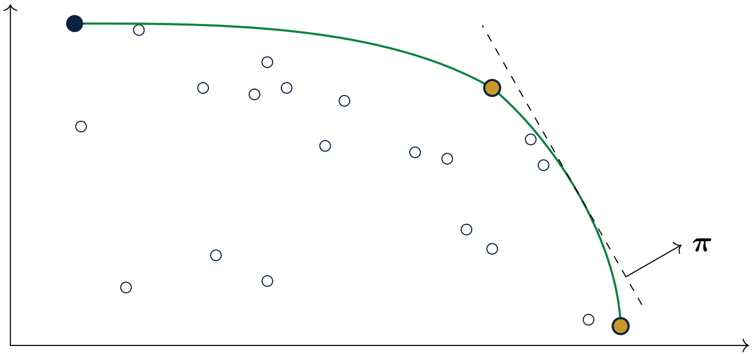

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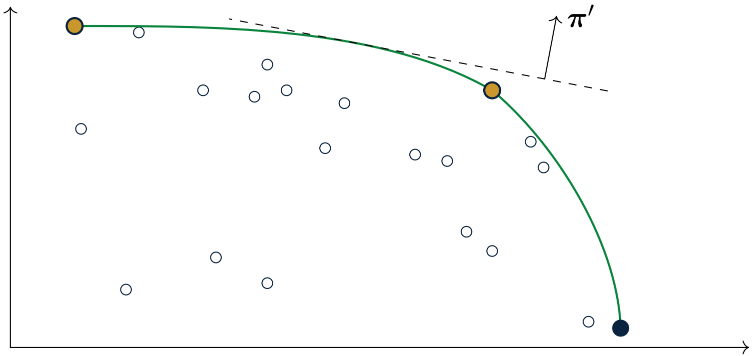

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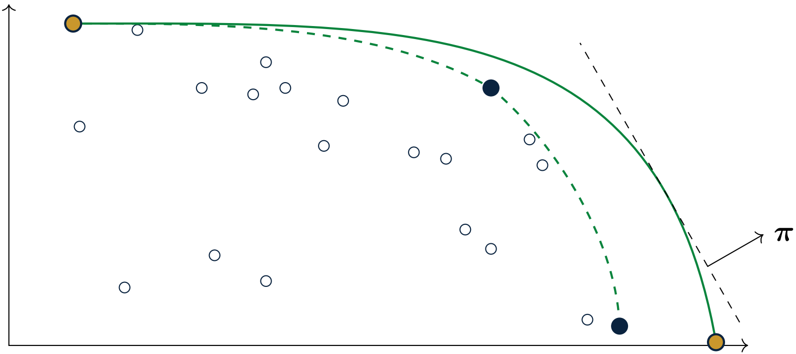

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- *Menu expansion can activate any of them for a fixed prior* $\boldsymbol{\pi}$.

details

Menu Expansion ⋆

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Consider an RI problem $(\mathcal{A}, \boldsymbol{\pi}, c)$.

Question: What happens if we add action $\boldsymbol{a}^{+} \in \mathbb{R}^{I}$ to the menu $\mathcal{A}$?

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... and the 'right' complement $\boldsymbol{a}^+$ can activate any anchor action.

Theorem (Activation of anchor actions)

For any anchor action $\boldsymbol{a} \in \mathcal{A} \cap \bar{\mathcal{A}}$, *there exists* $\boldsymbol{a}^+ \in \mathbb{R}^I$ *such that*

$$p(\boldsymbol{a}) > 0 \text{ in RI problem } (\mathcal{A} \cup \boldsymbol{a}^+, \boldsymbol{\pi}, c).$$

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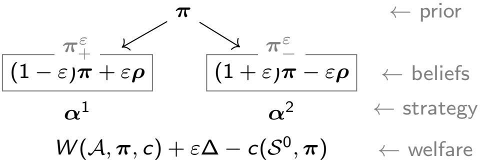

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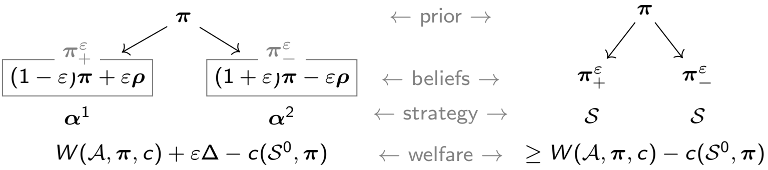

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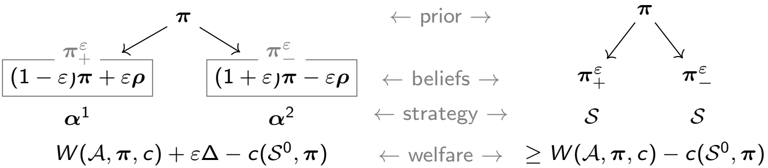

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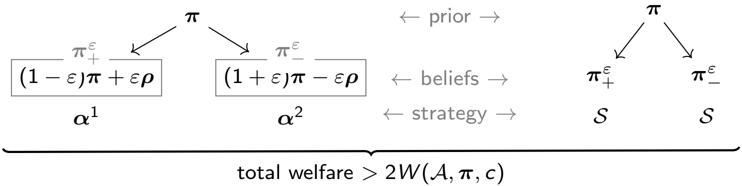

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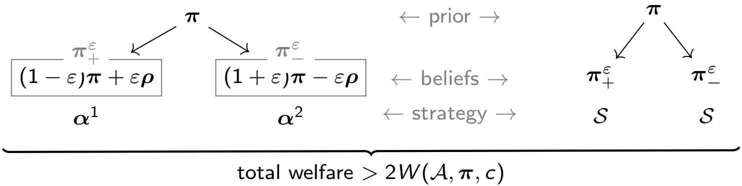

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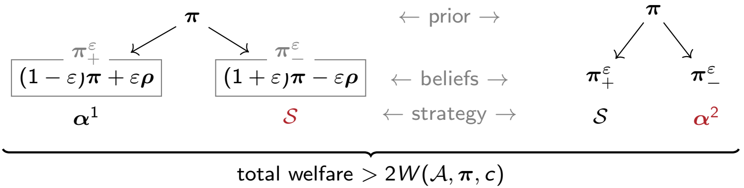

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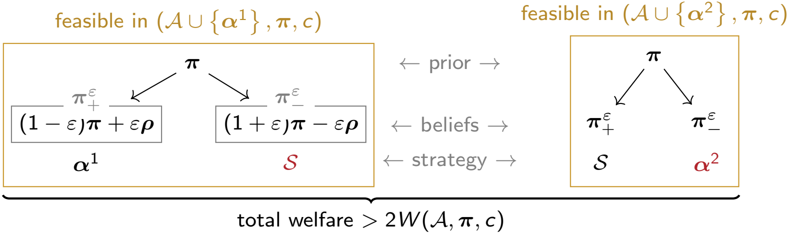

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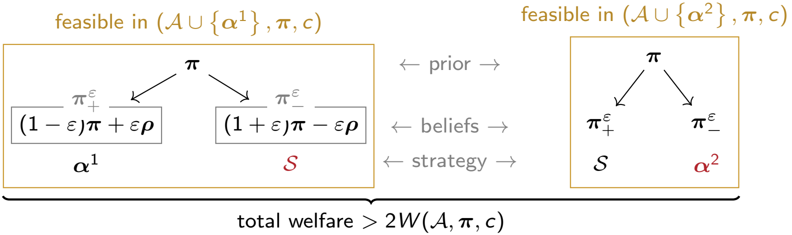

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Proof Sketch: Necessity of Dominance

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- Suppose $\boldsymbol{\alpha}$ beats $\mathcal{S}$ under some posterior, $\boldsymbol{\rho} \cdot \boldsymbol{\alpha} = \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho}) + \Delta$ with $\Delta > 0$.

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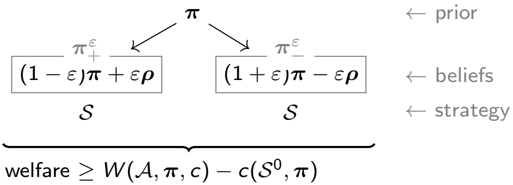

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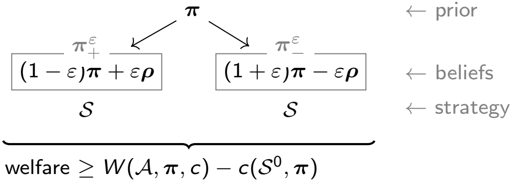

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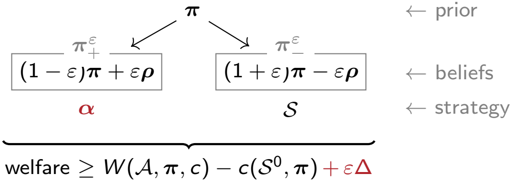

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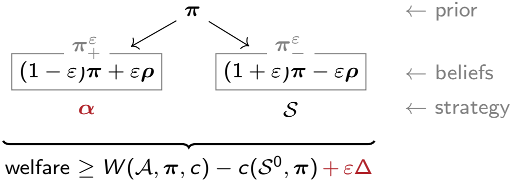
- By ②, $\boldsymbol{\alpha}$ beats $\mathcal{S}$ by at least $\varepsilon\Delta$ at $\boldsymbol{\pi}_{+}^{\varepsilon} = (1-\varepsilon)\boldsymbol{\pi} + \varepsilon\boldsymbol{\rho}$.
- Pick a signal $\mathcal{S}^0$ that updates beliefs towards or away from $\boldsymbol{\rho}$ with equal probability.



- Implement $\boldsymbol{\alpha}$ when advantageous.
- By ③, welfare is $> W(\mathcal{A}, \boldsymbol{\pi}, c)$ for small ε.

Dominance: $\boldsymbol{\alpha} \precsim \mathcal{S} \iff \boldsymbol{\rho} \cdot \boldsymbol{\alpha} \leq \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho})\ \forall \boldsymbol{\rho}$.

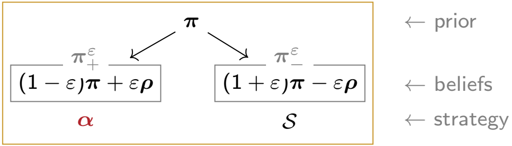
Ignorance Equivalence: $W(\{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c) = \quad W(\mathcal{A}, \boldsymbol{\pi}, c) = W(\mathcal{A} \cup \{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c)$.

Proof Sketch: Necessity of Dominance

back

- Suppose $\boldsymbol{\alpha}$ beats $\mathcal{S}$ under some posterior, $\boldsymbol{\rho} \cdot \boldsymbol{\alpha} = \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho}) + \Delta$ with $\Delta > 0$.
- By 2, $\boldsymbol{\alpha}$ beats $\mathcal{S}$ by at least $\varepsilon\Delta$ at $\boldsymbol{\pi}_+^{\varepsilon} = (1-\varepsilon)\boldsymbol{\pi} + \varepsilon\boldsymbol{\rho}$.
- Pick a signal $\mathcal{S}^0$ that updates beliefs towards or away from $\boldsymbol{\rho}$ with equal probability.

feasible in $(\mathcal{A} \cup \{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c)$



welfare $\geq W(\mathcal{A}, \boldsymbol{\pi}, c) - c(\mathcal{S}^0, \boldsymbol{\pi}) + \varepsilon\Delta$

- Implement $\boldsymbol{\alpha}$ when advantageous.
- By 3, welfare is $> W(\mathcal{A}, \boldsymbol{\pi}, c)$ for small ε.

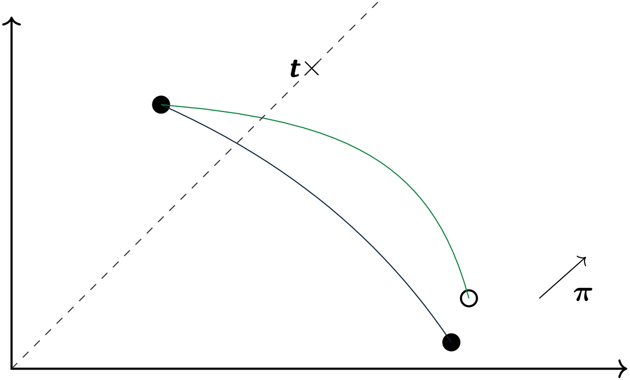
Dominance: $\boldsymbol{\alpha} \precsim \mathcal{S} \iff \boldsymbol{\rho} \cdot \boldsymbol{\alpha} \leq \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho})\ \forall \boldsymbol{\rho}$.

Ignorance Equivalence: $W(\{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c) = W(\mathcal{A}, \boldsymbol{\pi}, c) = W(\mathcal{A} \cup \{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c)$.

Unverifiable states (2)

back

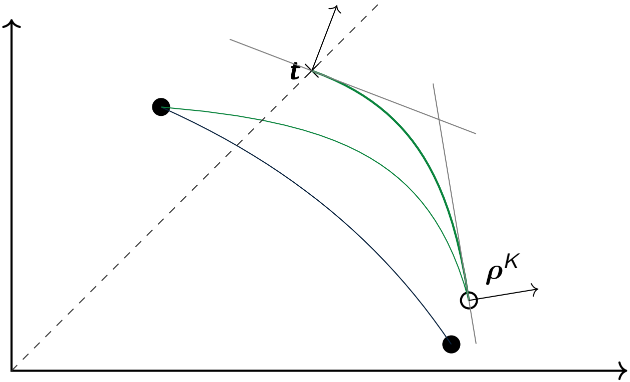
For a particular transfer $\boldsymbol{t}$,



Unverifiable states (2)

back

For a particular transfer $\boldsymbol{t}$,

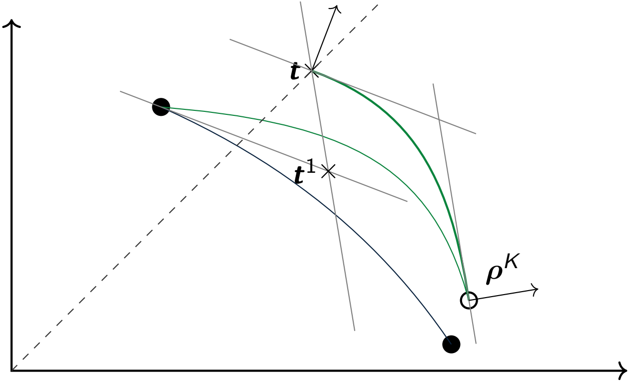


The learning-proof menu $\overline{\mathcal{A}^K \cup \{\boldsymbol{t}\}}$ determines under which posterior $\boldsymbol{\rho}^K$ Agent K accepts.

Unverifiable states (2)

back

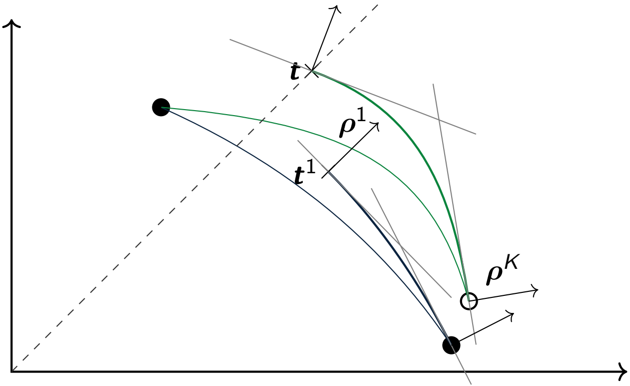
For a particular transfer $\boldsymbol{t}$,



Given Agent K's strategy, offer $\boldsymbol{t}$ is payoff-equivalent to certain trade at $\boldsymbol{t}^1$.

Unverifiable states (2)

For a particular transfer $\boldsymbol{t}$,

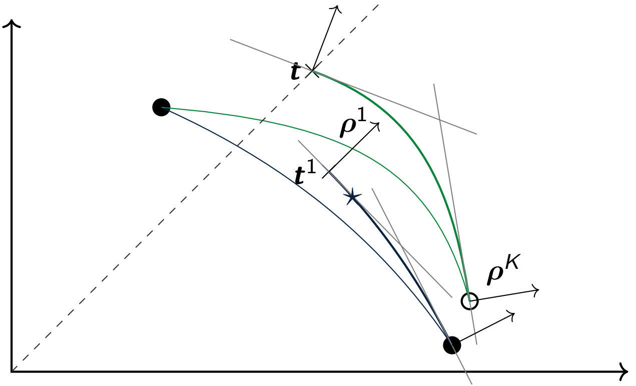


The learning-proof menu $\overline{\mathcal{A}^1 \cup \{\boldsymbol{t}^1\}}$ determines under which posterior $\boldsymbol{\rho}^1$ Agent 1 offers.

Unverifiable states (2)

back

For a particular transfer $\boldsymbol{t}$,

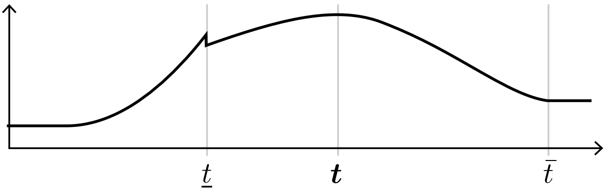


The ignorance equivalent of $(\mathcal{A}^1 \cup \{\boldsymbol{t}^1\}, \boldsymbol{\pi}, c^1)$ determines Agent 1's payoff.

Unverifiable states (3)

back

Using this construction, we can determine Agent 1's payoff for any constant transfer,



where

- $\underline{t}$ maximal transfer that Agent K accepts unconditionally,
- $\bar{t}$ maximal transfer that Agent K rejects unconditionally,
- $\boldsymbol{t}$ the one plotted previously, apparently optimal.

Unverifiable states (solution)

back

Finding: The equilibrium TIOLI offer from Agent 1 involves partial trade and pre-trade learning by both.