Proxy Variables and Feedback Effects in Decision Making

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Decisions affect Data

- We often only have access to proxies of the true variables we want to consider
- Tradition in Psychology and Economics of modelling agents as ‘Flawed Statisticians’ (eg ‘What you see is all there is’, Kahneman (2011))

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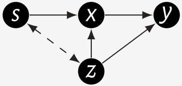
- We often only have access to proxies of the true variables we want to consider
- Tradition in Psychology and Economics of modelling agents as 'Flawed Statisticians' (eg 'What you see is all there is', Kahneman (2011))

- I propose a framework in which decision makers use **proxy variables** to form beliefs
  - The decision makers assume that the proxies are identical to the true variables
  - Feedback effects: Interplay between data and choices, so I define an equilibrium

A Recipe for Belief Formation

Four Variables: s - circumstance, z -signal, x - action, y - outcome
vNM utility: $u : Y \times X \times S \to \mathbb{R}$
Causal Structure:



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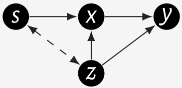

Joint Density: $p(y, x, z, s) = p(s, z)\sigma(x|s, z)p(y|x, z)$

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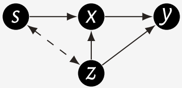

Joint Density: $p(y, x, z, s) = p(s, z)\sigma(x|s, z)p(y|x, z)$

With perfect measurement of (y, x, z, s), beliefs about $y|x, z, s$ are:

$$p(y|x, z, s) = \frac{p(y, x, z, s)}{p(x, z, s)} = p(y|x, z)$$

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We assume that DM can only access measurements $(z^\bullet, x^\bullet, y^\bullet) \in Z \times X \times Y$

There is a *Proxy Mapping*:
$\pi : Z \times X \times Y \to \Delta(Z \times X \times Y)$

This induces a density over proxies:
$p_\pi(y^\bullet, x^\bullet, z^\bullet) = \int_{Y\times X\times Z} \pi(y^\bullet, x^\bullet, z^\bullet|y, x, z)p(y, x, z)d\mu$

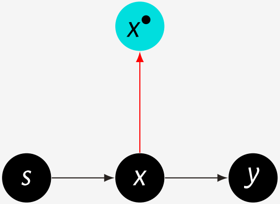
A Recipe for Belief Formation

Four Variables: s - circumstance, z -signal, x - action, y - outcome
True Joint Density: $p(y, x, z, s) = p(s, z)\sigma(x|s, z)p(y|x, z)$
Proxy Joint Density:
$p_\pi(y^\bullet, x^\bullet, z^\bullet) = \int_{Y\times X\times Z} \pi(y^\bullet, x^\bullet, z^\bullet|y, x, z)p(y, x, z)d\mu$

This is then used to form beliefs about $y|x, z$:

$$p_\pi(y^\bullet = y|x^\bullet = x, z^\bullet = z) = \frac{p_\pi(y^\bullet = y, x^\bullet = x, z^\bullet = z)}{p_\pi(x^\bullet = x, z^\bullet = z)}$$

Example: Police and Thieves



Symmetric measurement error:

$$x^\bullet = x + \epsilon$$

Variables:

y	crime level
x	police numbers
s	cost of crime
$x^\bullet$	reported police numbers (proxy)

Rational Expectations/Perfect Measurement Benchmark

A representative DM who knows the true conditional distribution $p(y|x, z)$ can maximize expected utility:

$$max_{x \in X} \mathbb{E}[U(x, z, s)] \equiv \int_Y u(y, x, s) p(y|x, z) d\mu(y) \tag{1}$$

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- The DM perceives action x as affecting outcomes y through:

$$p_{\pi}(y^{\bullet} = y | x^{\bullet} = x, z^{\bullet} = z; \sigma)$$

- They maximize perceived expected utility:

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$$p_\pi(y^\bullet = y | x^\bullet = x, z^\bullet = z; \sigma)$$

- They maximize perceived expected utility:

$$\max_{x \in X} V(x, z, s; \sigma) \equiv \int_Y u(y = y^\bullet, x, s) p_\pi(y^\bullet | x^\bullet = x, z^\bullet = z; \sigma) d\mu(y^\bullet) \tag{2}$$

Equilibrium Concept

Definition 1

For every $s \in S, z \in Z$ and strategy σ, define the following set:

$$X(s, z; \sigma) \equiv$$

$$\{x \in X : x \notin \arg\max \int_{Y^\bullet} u(y = y^\bullet, x, s) p_\pi(y^\bullet | x^\bullet = x, z^\bullet = z; \sigma) d\mu(y^\bullet)\}$$

An ϵ-**Proxy Equilibrium** is a full-support strategy σ^*_ϵ such that for every interval $I \subseteq X(s, z; \sigma^*_\epsilon)$, we have $\sigma^*_\epsilon(I|s, z) < \epsilon$

Definition 2

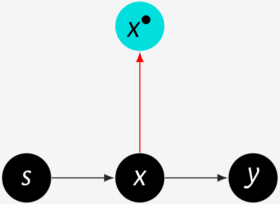
A **Proxy Equilibrium** is the limit of ϵ- Proxy Equilibria as $\epsilon \to 0$.

Equilibrium Existence

Proposition

Assume all variable spaces are finite.
Then a Proxy Equilibrium exists.

Example: Police and Thieves



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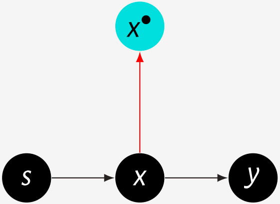

Symmetric measurement error:

$$x^\bullet = x + \epsilon$$

$$u(y, x, s) = -s \cdot y - \frac{1}{2}x^2$$

$$y = \beta x, \beta < 0$$
$$s \sim \mathcal{N}(\mu_s, \sigma_s^2)$$
$$\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

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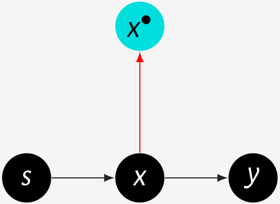

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Rational expectations benchmark: DM learns s and solves

$$max_{x \in X}(-s \cdot \beta x - \frac{1}{2}x^2)$$

This gives solution

$$x = -s\beta$$

Example: Police and Thieves

A linear Proxy Equilibrium is a strategy such that $x(s) = \theta_0 + \theta_1 s$ for some $(\theta_0, \theta_1) \in \mathbb{R}^2$

Proposition

1. *There is always a linear Proxy Equilibrium in which the municipal leader never changes police numbers, with best response $x^{nv}(s) = 0$.*

Example: Police and Thieves

A linear Proxy Equilibrium is a strategy such that $x(s) = \theta_0 + \theta_1 s$ for some $(\theta_0, \theta_1) \in \mathbb{R}^2$

Proposition

1. *There is always a linear Proxy Equilibrium in which the municipal leader never changes police numbers, with best response* $x^{nv}(s) = 0$.
2. *In addition, if* $|\beta| \geq 2\frac{\sigma_\epsilon}{\sigma_s}$*, then there exist two additional linear Proxy Equilibria, with best response:*

$$x^{-}(s) = (-\frac{1}{2}\beta - \frac{1}{2}\sqrt{\beta^2 - 4\frac{\sigma_\epsilon^2}{\sigma_s^2}})s$$

$$x^{+}(s) = (-\frac{1}{2}\beta + \frac{1}{2}\sqrt{\beta^2 - 4\frac{\sigma_\epsilon^2}{\sigma_s^2}})s$$

The Rest of the Paper

Two general results:

1. Characterization of strategies that can be supported as equilibria even when we have an arbitrarily close to perfect measurement
2. If strategies are full support, then close to perfect measurement $\longrightarrow$ close to rational expectations

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An application to Market entry in which:

- We always have excessive entry in Proxy Equilibrium
- Greater proxy 'noise' leads to greater extent of excessive entry (not true without equilibrium effects!)

Literature

- **Behavioural Equilibrium Concepts:** Jehiel (2005), Eyster and Rabin (2005), Esponda and Pouzo (2016), Spiegler (2016), Spiegler (2021)
- **Dynamic Misspecified Learning**: Heidhues et al. (2018), Bohren and Hauser (2021), Fudenberg et al. (2021), Frick et al. (2020)
- **Selection Effects/Data Limitations:** Jehiel (2018), Esponda and Pouzo (2017), Spiegler (2017), Fudenberg et al. (2022)
- **Overconfidence/Over-precision:** Moore and Healy (2008), Ortoleva and Snowberg (2015), Scheinkman and Xiong (2003), Daniel et al. (1998)

Almost Perfect Proxies

The total variation distance between probability measures Q_1 and Q_2 on measure space $(\Omega, \mathcal{A})$ is:

$$TV(Q_1, Q_2) = \sup_{A \in \mathcal{A}} |Q_1(A) - Q_2(A)| \tag{3}$$

Let π_δ be the *perfect measurement* proxy mapping

Definition 3

We say the proxy mapping π is **strongly** η**-close to perfect** if for $\eta > 0$ we have that:

$$\sup_{(y,x,z) \in Y \times X \times Z} TV(\pi(.|y, x, z), \pi_\delta(.|y, x, z)) < \eta \tag{4}$$

Characterization of Proxy Equilibria

Assume $Y \times X \times Z \times S$ is finite

Definition 4

A strategy $\sigma^* : S \times Z \to \Delta(X)$ is **potentially implementable** if at every $z \in Z$, the following two conditions hold.

(1) For any action $x \in supp\{\sigma^*(.|z)\}$ there exists an $s \in S$ such that, for every $x' \in supp\{\sigma^*(.|z)\}$:

$$\sum_{y \in Y} u(y, x, s) p(y|x, z) \geq \sum_{y \in Y} u(y, x', s) p(y|x', z) \quad (5)$$

Characterization of Proxy Equilibria

Assume $Y \times X \times Z \times S$ is finite

Definition 4

A strategy $\sigma^* : S \times Z \to \Delta(X)$ is **potentially implementable** if at every $z \in Z$, the following two conditions hold.

(2) For every action $x^{ns} \notin supp\{\sigma^*(.|z)\}$, there exists a full-support conditional distribution $q : X \times Z \to \Delta(Y)$ such that for any $s \in S$ and $x^s \in supp\{\sigma^*(.|s, z)\}$ we have that:

$$\sum_{y \in Y} u(y, x^s, s) p(y|x^s, z) \geq \sum_{y \in Y} u(y, x^{ns}, s) q(y|x^{ns}, z) \quad (5)$$

Characterization of Proxy Equilibria

Proposition

Let $Y \times X \times Z \times S$ *be finite and* $supp\{p(.|x,z)\} = Y$ *for every* $(x,z) \in X \times Z$*.*

Then for all small enough $\eta > 0$, $\sigma^* : S \times Z \to \Delta(X)$ *is a Proxy Equilibrium under some proxy mapping that is strongly* η*-close to perfect if and only if it is a* ***potentially implementable strategy****.*

Convergence to Perfect Measurement Benchmark

Let P be a probability measure over the true variables $Y \times X \times S \times Z$
Let P_π be a probability measure over the proxy variables induced by proxy mapping π

Definition 5

Given $\eta > 0$, we say the proxy mapping π is η**-close to perfect** given the distribution over true variables P if we have that:

$$TV(P, P_\pi) < \eta \tag{6}$$

Full Support Assumption

Assumption 1

The distribution F over variables in $Y \times X \times Z$ is said to satisfy the ***full support assumption*** *if it admits a density $f(\tilde{y}, \tilde{x}, \tilde{z})$ such that $f(\tilde{x}, \tilde{z}) > 0$ for every realization $(\tilde{x}, \tilde{z}) \in X \times Z$.*

Convergence to Perfect Measurement Benchmark

Proposition

Assume the ***full support assumption*** *holds for the true distribution* P.

Then for almost every $(y, x, z) \in Y \times X \times Z$, *for any* $\epsilon > 0$, *there exists an* $\eta > 0$ *such that if the proxy mapping* π *is* η***-close to perfect*** *given true distribution* P *and induces a distribution over the proxy variables that satisfies the* ***full support assumption***, *then* $|p_\pi(y^\bullet = y | x^\bullet = x, z^\bullet = z) - p(y|x, z)| < \epsilon$.

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