Prospect Equality: A Force of Redistribution

Xiangyu Qu

EEA-ESEM, Collective Decision Making and Social Choice

August 31, 2023
Redistribution Puzzle

Meltzer and Richard [1981], claimed that as income inequality increases, a society will prefer policies supporting greater redistribution to counter excessive income disparities.

Empirical Evidence is ambiguous:

- Meltzer and Richard [1983], Borge and Rattsø [2004]

- Alesina and Glaeser [2004] observed the opposite pattern:
  - Western European countries have lower levels of objective income inequality than the US but demand a higher redistribution policy.
Inequality by countries

Figure: *

OECD data
Tax Rates by countries

Top marginal tax rate, by country

UK 92%
US 72%
France 62%
Germany 53%

2017 supertax 75%

Sources: PwC, Piketty (2014)
Recent Evidence

Perceived inequality deviates from objective inequality: Page and Goldstein [2016], Kuhn [2020]

Perceived inequality—not the objective one—correlates strongly with demand for redistribution: Gimpelson and Treisman [2018]

Determinants of perceived inequality

- Objective Inequality
- **Prospect** Equality
  - US believes **efforts** determine incomes.
  - West Europe believes **luck** determines incomes.
Emmanuel Macron: “France is not like other countries. The sense of injustice is more intense than elsewhere.”
This paper

I ask
  • which elements are most relevant to perceived inequality?

I suggest
  • a novel model of perceived inequality.

I provide
  • behavioral foundation for such model.

I demonstrate
  • the redistribution puzzle is compatible with my model.
The Setup

- a society contains $n \geq 2$ individuals
- $x = (x_1, \ldots, x_n) \in X \subset \mathbb{R}_+^n$: an income allocation
- $\mathcal{A}$: the set of nonempty subsets of $X$
  - prospect set $A \in \mathcal{A}$
  - $\mathbb{D}$: set of pairs $(x, A)$.
- $\preceq \subset \mathbb{D} \times \mathbb{D}$
The Model

Definition

A function $J : \mathbb{D} \to \mathbb{R}$ is an index of the *perception of inequality* if there exists an index of objective inequality $I$ such that for $(x, A) \in \mathbb{D}$,

$$J(x, A) = |I(x) - \theta \min_{y \in A} I(y)|,$$

where $0 \leq \theta \leq 1$. In particular, we say $J$ is a *Gini index* of the perception of inequality if $I$ is an objective Gini coefficient defined as follows: for all $x \in X$,

$$I_g(x) = \frac{\sum_{1 \leq i < j \leq n} |x_i - x_j|}{n^2 \mu(x)}.$$

(1)  

(2)
Objective vs Perceived Inequality

Example

- A society consists of two individuals.
- Income profiles: \( x = (7, 3), \ y = (5, 5), \ x' = (9, 1) \) and \( y' = (8, 2) \).
- Gini indices are \( I_g(x) = 0.4 \), \( I_g(y) = 0 \), \( I_g(x') = 0.8 \) and \( I_g(y') = 0.6 \).
- Consider two alternatives \((x, \{x, y\})\) and \((x', \{x', y'\})\).
- Let \( \theta = 0.8 \).

\[
J(x, \{x, y\}) = I_g(x) - 0.8 \times I_g(y) = 0.4 > 0.32 = I_g(x') - 0.8 \times I_g(y') = J(x', \{x', y'\}).
\]

Hence, the perceived inequality from \( J(x, \{x, y\}) > J(x', \{x', y'\}) \).
Voting on Redistribution

Voter $i$ has prospect inequality preferences over $\mathbb{D}$ if $u_i : \mathbb{D} \rightarrow \mathbb{R}$ has the following form:

$$u_i(x, A) = x_i - \delta \cdot |I_g(x) - \theta \min_{y \in A} I_g(y)|. \quad (3)$$

where scalars $0 \leq \delta, \theta \leq 1$.

- $\delta = 0$: self-interest voters. (Meltzer and Richard [1981])
- $\theta = 0$: inequality averse voters. (Tyran and Sausgruber [2006], Dhami and al Nowaihi [2010])
Assumptions

• voter \( i \) belongs to either rich or poor class: \( n_r + n_p = n \)

• Income allocation \( x = (x_r, x_p) \), where \( x_r > x_p \)

• Prospect equality:
  ▶ low: pretax income allocation \( x \) with \( n_{rl} + n_{pl} \) voters.
  ▶ high: perfectly equal allocation \( x^* \) with \( n_{rh} + n_{ph} \) voters.
Pre-tax utilities

1). The pretax utility of rich voters with low prospects is

\[ u_{rl}(x) = x_r - \delta |I_g(x) - I_g(x)| = x_r \]

(2). The pretax utility of rich voters with high prospects is

\[ u_{rh}(x) = x_r - \delta |I_g(x) - I_g(x^*)| = x_r - \delta I_g(x). \]

(3). The pretax utility of poor voters with low prospects is

\[ u_{pl}(x) = x_p \]

(4). The pretax utility of poor voters with high prospects is

\[ u_{ph}(x) = x_p - \delta I_g(x). \]
Tax Scheme

- uniform redistribution policy: $0 < t \leq 1$
- total collected tax: $(n_r x_r + n_p x_p) t$
- transfer $b = \frac{(n_r x_r + n_p x_p) t}{n}$
- $x(t)$ after-tax income allocation with tax rate $t$
- For $q \in (0, 1]$, $q$-majority voting rule: the number of voters who vote for policy $t$ must be greater than $qn$ for the policy to be accepted.
After-tax utilities

1’). The after-tax utility of rich voters with low prospects is

$$u_{r\ell}(x(t)) = (1 - t)x_r + b - \delta|I_g(x(t)) - I_g(x)|$$

Therefore, a rich voter with a low prospect will vote for tax policy $t$ if and only if

$$\delta < \frac{b - tx_r}{I_g(x) - I_g(x(t))}.$$ 

Since $b - tx_r < 0$ and $\delta \geq 0$, a rich voter with a low prospect will never vote for redistribution.

(2’). The after-tax utility of rich voters with high prospects is

$$u_{rh}(x(t)) = (1 - t)x_r + b - \delta I_g(x(t))$$

Therefore, a rich voter with a high prospect will vote for tax policy $t$ if and only if

$$\delta > \frac{tx_r - b}{I_g(x) - I_g(x(t))}.$$ 

Rich voter with a high prospect is sufficiently sensitive to perceive inequality, then she will vote for tax policy $t$.  

After-tax utilities

(3'). The after-tax utility of poor voters with low prospects is

\[ u_{pl}(x(t)) = (1 - t)x_p + b - \delta |I_g(x) - I_g(x(t))| \]

Therefore, a poor voter with a low prospect will vote for tax policy \( t \) if and only if

\[ \delta < \frac{b - tx_p}{I_g(x) - I_g(x(t))}. \]

Contrary to the above case, if a poor voter with a low prospect is overly sensitive to the perception of inequality, then she will not vote for redistribution.

(4'). The after-tax utility of poor voters with high prospects is

\[ u_{ph}(x(t)) = (1 - t)x_p + b - \delta I_g(x(t)). \]

It is immediately clear that a poor voter with a high prospect will always vote for redistribution.
Proposition

Consider a tax policy $t \in (0, 1]$.

(i) If $n_p < n_r$ and $\delta \in \left(\frac{tx_r - b}{I_g(x) - I_g(x(t))}, \frac{b - tx_p}{I_g(x) - I_g(x(t))}\right)$, then tax policy $t$ is accepted iff $n_{rh} + n_p > qn$.

(ii) If $n_p < n_r$ and $\delta > \frac{b - tx_p}{I_g(x) - I_g(x(t))}$, then tax policy $t$ is accepted iff $n_{rh} + n_{ph} > qn$.

(iii) If $n_p > n_r$ and $\delta > \frac{tx_r - b}{I_g(x) - I_g(x(t))}$, then tax policy $t$ is accepted iff $n_{rh} + n_{ph} > qn$.

(iv) If $n_p > n_r$ and $\delta < \frac{b - tx_p}{I_g(x) - I_g(x(t))}$, then tax policy $t$ is accepted iff $n_p > qn$.
The Model

Definition

A function $J : \mathbb{D} \rightarrow \mathbb{R}$ is an index of the perception of inequality if there exists an index of objective inequality $I$ such that for $(x, A) \in \mathbb{D}$,

$$J(x, A) = |I(x) - \theta \min_{y \in A} I(y)|,$$

(4)

where $0 \leq \theta \leq 1$. In particular, we say $J$ is a Gini index of the perception of inequality if $I$ is an objective Gini coefficient defined as follows: for all $x \in X$,

$$I_g(x) = \frac{\sum_{1 \leq i < j \leq n} |x_i - x_j|}{n^2 \mu(x)}.$$  

(5)
Axiom 1. \textit{(Weak order.)} $\succeq$ is complete and transitive.

Axiom 2. \textit{(Continuity.)} For all $(x, A) \in \mathcal{D}$, the sets \{(y, B) \in \mathcal{D} : (x, A) \succsim (y, B)\} and \{(y, B) \in \mathcal{D} : (y, B) \succsim (x, A)\} are closed.
Let \( \tilde{x} \) be the income distribution obtained from \( x \) by rearranging the incomes in an increasing order, i.e., \( \{x_1, \ldots, x_n\} = \{\tilde{x}_1, \ldots, \tilde{x}_n\} \) and \( \tilde{x}_1 \leq \ldots \leq \tilde{x}_n \).

**Definition**

If \( n \geq 3 \) and \( x \in X \), then the function \( L_x \), for \( p \in [0, 1] \) and \( k = 0, 1, \ldots, n \), defined by

\[
L_x(p) = \frac{1}{n \mu(x)} \sum_{i=1}^{k} \tilde{x}_i \quad \text{if} \quad \frac{k}{n} \leq p < \frac{k+1}{n}
\]

is called the *Lorenz measure* associated with \( x \), and its graph is referred to as the corresponding Lorenz curve.

**Axiom 3.** *(Lorenz principle.)* If \( x \) Lorenz dominates \( y \), then \( (x, \{x\}) \gtrless (y, \{y\}) \). For perfect equality, \( x^* \) and all \( y \in X \), \( (x^*, \{x^*\}) \gtrless (x^*, \{y\}) \) and \( (x^*, \{x^*\}) \gtrless (y, \{x^*\}) \).
Axiom

\((x, \{y\})\) is underprospect if \((x^*, \{y\}) \succsim (x, \{x^*\})\); overprospect if \((x, \{x^*\}) \succsim (x^*, \{y\})\); and ideal prospect if \((x, \{x^*\}) \sim (x^*, \{y\})\).

Axiom 4 (Monotonicity.) For all ideal prospect alternatives \((x, \{y\})\),

(i) if \((x, \{x\}) \succsim (x', \{x'\}) \succsim (x'', \{x''\})\), then 
\((x', \{y\}) \succsim (x'', \{y\})\);

(ii) if \((y', \{y'\}) \succsim (y'', \{y''\}) \succsim (y, \{y\})\), then 
\((x, \{y''\}) \succsim (x, \{y\})\).
Axiom

For $c > 0$, let

$$X_c = \{x \in X : \mu(x) = c\}$$

be the set of income profiles wherein each profile has the same average income $c$, and define

$$\tilde{X}_c = \{x \in X_c : x_1 \leq x_2 \leq \cdots \leq x_n\}$$

Axiom 5. \textbf{(Order-preserving Independence)} For $c > 0$ and $x, x', y, y', z, z' \in \tilde{X}_c$, if $(x, \{x'\}), (y, \{y'\})$ and $(z, \{z'\})$ are all underprospect (or all overprospect), then $(x, \{x'\}) \succsim (y, \{y'\})$ implies $(\alpha x + (1 - \alpha)z, \{\alpha x' + (1 - \alpha)z'\}) \succsim (\alpha y + (1 - \alpha)z, \{\alpha y + (1 - \alpha)z'\})$ for all $\alpha \in [0, 1]$. 

Axiom

For \( x \in X \) and \( 1 \leq i, j \leq n \), we say \( i \) precedes \( j \) in \( x \) if \( x_i \leq x_j \) and there is no \( 1 \leq k \leq n \) such that \( x_i < x_k < x_j \).

Axiom 6. \((\text{Ben Porath-Gilboa Transfer Principle.})\) For \( c > 0 \), take any \( x, y, x', y' \in X_c \) and \( 1 \leq i, j \leq n \). If

(a) \( i \) precedes \( j \) in \( x, y, x', y' \)
(b) \( x_i = x'_i + s, \ x_j = x'_j - s \) and \( y_i = y'_i + s, \ y_j = y'_j - s \) for some \( s > 0 \)
(c) \( x_k = x'_k \) and \( y_k = y'_k \) for \( k \notin \{i, j\} \)

are satisfied, then

\((x, \{x\}) \succeq (y, \{y\})\) if and only if \((x', \{x'\}) \succeq (y', \{y'\})\).
Axiom

A prospect set $A$ dominates $B$ if for each $x \in A$ there exists $y \in B$ such that 
$$(x, \{x\}) \succsim (y, \{y\}).$$
We say prospect sets $A$ and $B$ are equivalent if $A$ dominates $B$ and $B$ dominates $A$.

**Axiom 7 (Equivalence.)** For all $x$ and $A, B$, if $A$ and $B$ are equivalent, then 
$$(x, A) \sim (x, B).$$
Characterization

Theorem

An individual preference relation $\succeq$ satisfies Axioms 1-7 if and only if there exists $J$ as in eqs (1) and (2) that represents $\succeq$. 
Conclusion

- It is widely observed that perceived, not objective, inequality is positively correlated to redistribution policy.
- This paper formally construct and characterize a model to reflect perceived inequality of voters.
- I demonstrate that the proposed model can well explain redistribution puzzle under a voting scheme.