Prospect Equality: A Force of Redistribution

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Redistribution Puzzle

Meltzer and Richard [1981], claimed that as income inequality increases, a society will prefer policies supporting greater redistribution to counter excessive income disparities.

Empirical Evidence is ambiguous:

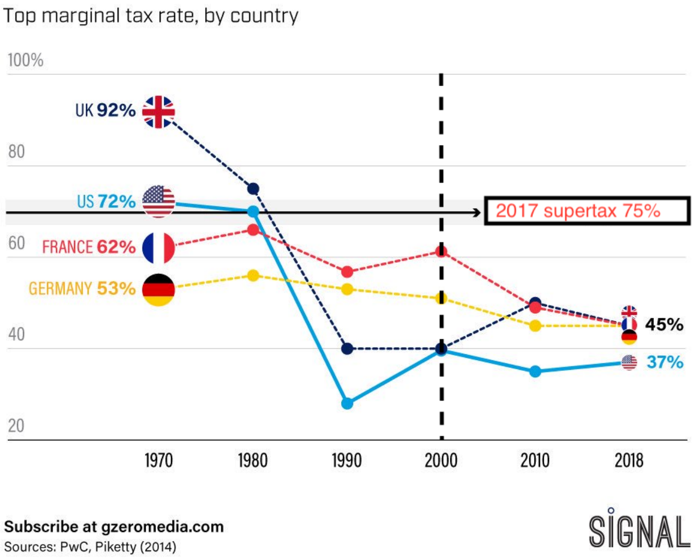
- Meltzer and Richard [1983], Borge and Rattsø [2004]
- Alesina and Glaeser [2004] observed the opposite pattern:
  - Western European countries have lower levels of objective income inequality than the US but demand a higher redistribution policy.

Inequality by countries

Figure: *

OECD data

Tax Rates by countries



Recent Evidence

Perceived inequality deviates from objective inequality: Page and Goldstein [2016], Kuhn [2020]

Perceived inequality—not the objective one—correlates strongly with demand for redistribution: Gimpelson and Treisman [2018]

Determinants of perceived inequality

- Objective Inequality
- Prospect Equality
  - US believes efforts determine incomes.
  - West Europe believes luck determines incomes.

Prospect Equality

Emmanuel Macron: "France is not like other countries. The sense of injustice is more intense than elsewhere."

This paper

I ask

- which elements are most relevant to perceived inequality?

I suggest

- a novel model of perceived inequality.

I provide

- behavioral foundation for such model.

I demonstrate

- the redistribution puzzle is compatible with my model.

The Setup

- a society contains $n \geq 2$ *individuals*
- $x = (x_1, \ldots, x_n) \in X \subset \mathbb{R}^n_+$: an *income* allocation
- $\mathscr{A}$: the set of nonempty subsets of X
  - *prospect* set $A \in \mathscr{A}$
  - $\mathbb{D}$: set of pairs (x, A).
- $\succsim \subset \mathbb{D} \times \mathbb{D}$

The Model

Definition

A function $J : \mathbb{D} \rightarrow \mathbb{R}$ is an index of the *perception of inequality* if there exists an index of objective inequality I such that for $(x, A) \in \mathbb{D}$,

$$J(x, A) = |I(x) - \theta \min_{y \in A} I(y)|, \tag{1}$$

where $0 \leq \theta \leq 1$. In particular, we say J is a *Gini index* of the perception of inequality if I is an objective Gini coefficient defined as follows: for all $x \in X$,

$$I_g(x) = \frac{\sum_{1 \leq i < j \leq n} |x_i - x_j|}{n^2 \mu(x)}. \tag{2}$$

Objective vs Perceived Inequality

Example

- A society consists of two individuals.
- Income profiles: $x = (7, 3)$, $y = (5, 5)$, $x' = (9, 1)$ and $y' = (8, 2)$.
- Gini indices are $I_g(x) = 0.4$, $I_g(y) = 0$, $I_g(x') = 0.8$ and $I_g(y') = 0.6$.
- Consider two alternatives $(x, \{x, y\})$ and $(x', \{x', y'\})$.
- Let $\theta = 0.8$.

$$J(x, \{x, y\}) = I_g(x) - 0.8 \times I_g(y) = 0.4 > 0.32 = I_g(x') - 0.8 \times I_g(y') = J(x', \{x', y'\}).$$

Hence, the perceived inequality from

$$J(x, \{x, y\}) > J(x', \{x', y'\}).$$

Voting on Redistribution

Voter i has prospect inequality preferences over $\mathbb{D}$ if $u_i : \mathbb{D} \to \mathbb{R}$ has the following form:

$$u_i(x, A) = x_i - \delta \cdot \left| I_g(x) - \theta \min_{y \in A} I_g(y) \right|. \tag{3}$$

where scalars $0 \leq \delta, \theta \leq 1$.

- $\delta = 0$: self-interest voters. (Meltzer and Richard [1981])
- $\theta = 0$: inequality averse voters. (Tyran and Sausgruber [2006], Dhami and al Nowaihi [2010])

Assumptions

- voter i belongs to either *rich* or *poor* class: $n_r + n_p = n$
- Income allocation $x = (x_r, x_p)$, where $x_r > x_p$
- Prospect equality:
  - low: pretax income allocation x with $n_{r\ell} + n_{p\ell}$ voters.
  - high: perfectly equal allocation x^* with $n_{rh} + n_{ph}$ voters.

Pre-tax utilities

1). The pretax utility of rich voters with low prospects is

$$u_{r\ell}(x) = x_r - \delta \left| I_g(x) - I_g(x) \right| = x_r$$

(2). The pretax utility of rich voters with high prospects is

$$u_{rh}(x) = x_r - \delta \left| I_g(x) - I_g(x^*) \right| = x_r - \delta I_g(x).$$

(3). The pretax utility of poor voters with low prospects is

$$u_{p\ell}(x) = x_p$$

(4). The pretax utility of poor voters with high prospects is

$$u_{ph}(x) = x_p - \delta I_g(x).$$

Tax Scheme

- uniform redistribution policy: $0 < t \leq 1$
- total collected tax: $(n_r x_r + n_p x_p)t$
- transfer $b = \frac{(n_r x_r + n_p x_p)t}{n}$
- $x(t)$ after-tax income allocation with tax rate t
- For $q \in (0, 1]$, q-majority voting rule: the number of voters who vote for policy t must be greater than qn for the policy to be accepted.

After-tax utilities

1'). The after-tax utility of rich voters with low prospects is

$$u_{r\ell}(x(t)) = (1-t)x_r + b - \delta\Big|I_g(x(t)) - I_g(x)\Big|$$

Therefore, a rich voter with a low prospect will vote for tax policy t if and only if

$$\delta < \frac{b - tx_r}{I_g(x) - I_g(x(t))}.$$

Since $b - tx_r < 0$ and $\delta \geq 0$, a rich voter with a low prospect will never vote for redistribution.

(2'). The after-tax utility of rich voters with high prospects is

$$u_{rh}(x(t)) = (1-t)x_r + b - \delta I_g(x(t))$$

Therefore, a rich voter with a high prospect will vote for tax policy t if and only if

$$\delta > \frac{tx_r - b}{I_g(x) - I_g(x(t))}.$$

Rich voter with a high prospect is sufficiently sensitive to perceive inequality, then she will vote for tax policy t.

After-tax utilities

(3'). The after-tax utility of poor voters with low prospects is

$$u_{p\ell}(x(t)) = (1-t)x_p + b - \delta\Big|I_g(x) - I_g(x(t))\Big|$$

Therefore, a poor voter with a low prospect will vote for tax policy t if and only if

$$\delta < \frac{b - tx_p}{I_g(x) - I_g(x(t))}.$$

Contrary to the above case, if a poor voter with a low prospect is overly sensitive to the perception of inequality, then she will not vote for redistribution.

(4'). The after-tax utility of poor voters with high prospects is

$$u_{ph}(x(t)) = (1-t)x_p + b - \delta I_g(x(t)).$$

It is immediately clear that a poor voter with a high prospect will always vote for redistribution.

Result

Proposition

Consider a tax policy $t \in (0, 1]$.

(i) *If* $n_p < n_r$ *and* $\delta \in (\frac{tx_r - b}{I_g(x) - I_g(x(t))}, \frac{b - tx_p}{I_g(x) - I_g(x(t))})$, *then tax policy* t *is accepted iff* $n_{rh} + n_p > qn$.

(ii) *If* $n_p < n_r$ *and* $\delta > \frac{b - tx_p}{I_g(x) - I_g(x(t))}$, *then tax policy* t *is accepted iff* $n_{rh} + n_{ph} > qn$.

(iii) *If* $n_p > n_r$ *and* $\delta > \frac{tx_r - b}{I_g(x) - I_g(x(t))}$, *then tax policy* t *is accepted iff* $n_{rh} + n_{ph} > qn$.

(iv) *If* $n_p > n_r$ *and* $\delta < \frac{b - tx_p}{I_g(x) - I_g(x(t))}$, *then tax policy* t *is accepted iff* $n_p > qn$.

The Model

Definition

A function $J : \mathbb{D} \to \mathbb{R}$ is an index of the *perception of inequality* if there exists an index of objective inequality I such that for $(x, A) \in \mathbb{D}$,

$$J(x, A) = |I(x) - \theta \min_{y \in A} I(y)|, \tag{4}$$

where $0 \leq \theta \leq 1$. In particular, we say J is a *Gini index* of the perception of inequality if I is an objective Gini coefficient defined as follows: for all $x \in X$,

$$I_g(x) = \frac{\sum_{1 \leq i < j \leq n} |x_i - x_j|}{n^2 \mu(x)}. \tag{5}$$

Axiom

Axiom 1. (*Weak order.*) $\succsim$ is complete and transitive.

Axiom 2. (*Continuity.*) For all $(x, A) \in \mathbb{D}$, the sets $\{(y, B) \in \mathbb{D} : (x, A) \succsim (y, B)\}$ and $\{(y, B) \in \mathbb{D} : (y, B) \succsim (x, A)\}$ are closed.

Let $\tilde{x}$ be the income distribution obtained from x by rearranging the incomes in an increasing order, *i.e.*, $\{x_1, \ldots, x_n\} = \{\tilde{x}_1, \ldots, \tilde{x}_n\}$ and $\tilde{x}_1 \leq \ldots \leq \tilde{x}_n$.

Definition

If $n \geq 3$ and $x \in X$, then the function L_x, for $p \in [0,1]$ and $k = 0, 1, \ldots, n$, defined by

$$L_x(p) = \frac{1}{n\mu(x)} \sum_{i=1}^{k} \tilde{x}_i \qquad \text{if } \frac{k}{n} \leq p < \frac{k+1}{n}$$

is called the *Lorenz measure* associated with x, and its graph is referred to as the corresponding Lorenz curve.

Axiom 3. (*Lorenz principle.*) If x Lorenz dominates y, then $(x, \{x\}) \succsim (y, \{y\})$. For perfect equality, x^* and all $y \in X$, $(x^*, \{x^*\}) \succsim (x^*, \{y\})$ and $(x^*, \{x^*\}) \succsim (y, \{x^*\})$.

Axiom

$(x, \{y\})$ is *underprospect* if $(x^*, \{y\}) \succsim (x, \{x^*\})$; *overprospect* if $(x, \{x^*\}) \succsim (x^*, \{y\})$; and *ideal prospect* if $(x, \{x^*\}) \sim (x^*, \{y\})$.

Axiom 4 (Monotonicity.) For all ideal prospect alternatives $(x, \{y\})$,

(i) if $(x, \{x\}) \succsim (x', \{x'\}) \succsim (x'', \{x''\})$, then $(x', \{y\}) \succsim (x'', \{y\})$;

(ii) if $(y', \{y'\}) \succsim (y'', \{y''\}) \succsim (y, \{y\})$, then $(x, \{y''\}) \succsim (x, \{y'\})$.

Axiom

For $c > 0$, let

$$X_c = \{x \in X : \mu(x) = c\}$$

be the set of income profiles wherein each profile has the same average income c, and define

$$\tilde{X}_c = \{x \in X_c : x_1 \leq x_2 \leq \cdots \leq x_n\}$$

Axiom 5. (*Order-preserving Independence*) For $c > 0$ and $x, x', y, y', z, z' \in \tilde{X}_c$, if $(x, \{x'\}), (y, \{y'\})$ and $(z, \{z'\})$ are all underprospect (or all overprospect), then $(x, \{x'\}) \succsim (y, \{y'\})$ implies $(\alpha x + (1-\alpha)z, \{\alpha x' + (1-\alpha)z'\}) \succsim (\alpha y + (1-\alpha)z, \{\alpha y + (1-\alpha)z'\})$ for all $\alpha \in [0, 1]$.

Axiom

For $x \in X$ and $1 \leq i, j \leq n$, we say i precedes j in x if $x_i \leq x_j$ and there is no $1 \leq k \leq n$ such that $x_i < x_k < x_j$.

Axiom 6. (*Ben Porath-Gilboa Transfer Principle.*) For $c > 0$, take any $x, y, x', y' \in X_c$ and $1 \leq i, j \leq n$. If

(a) i precedes j in x, y, x', y'

(b) $x_i = x'_i + s$, $x_j = x'_j - s$ and $y_i = y'_i + s$, $y_j = y'_j - s$ for some $s > 0$

(c) $x_k = x'_k$ and $y_k = y'_k$ for $k \notin \{i, j\}$

are satisfied, then

$$(x, \{x\}) \succsim (y, \{y\}) \text{ if and only if } (x', \{x'\}) \succsim (y', \{y'\}).$$

Axiom

A prospect set A dominates B if for each $x \in A$ there exists $y \in B$ such that $(x, \{x\}) \succsim (y, \{y\})$. We say prospect sets A and B are *equivalent* if A dominates B and B dominates A.

Axiom 7 (*Equivalence.*) For all x and A, B, if A and B are equivalent, then $(x, A) \sim (x, B)$.

Characterization

Theorem

An individual preference relation $\succsim$ satisfies Axioms 1-7 if and only if there exists J as in eqs (1) and (2) that represents $\succsim$.

Conclusion

- It is widely observed that perceived, not objective, inequality is positively correlated to redistribution policy.
- This paper formally construct and characterize a model to reflect perceived inequality of voters.
- I demonstrate that the proposed model can well explain redistribution puzzle under a voting scheme.