Reallocation dynamics in production networks with heterogeneous elasticities

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Motivation

- Production side of the economy has shown to be more and more connected (Acemoglu & Azar, [2020])
- Firms and sectors are very intertwined and it has some major implications on shock propagation, with more or less vulnerabilities : COVID-19, Ukrainian conflict...

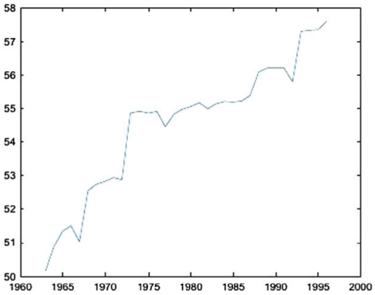

FIGURE 1.—Evolution of U.S. input–output linkages. Average degree (number of suppliers) from the U.S. summary input–output tables, 1963–1996. We use the harmonized commodity-by-commodity matrix, which is available for 61 industries during this period.

- The ability of firms to find a supplier and substitute inputs determines the resilience of supply chains, and in turn shapes the downstream propagation of supply shocks along the production network
- This puts stress on the idea that **reallocation patterns** are essential in our understanding of shock propagation
- Production networks modelling has shifted from Cobb-Douglas to CES modelling to take these mechanisms into account

Motivation : two challenges (the U-IES)

- Models use a unique input elasticity of substitution for ALL sectors ! (henceforth U-IES hypothesis)
- This is a strong restriction on sectors' ability to substitute
- And it might considerably affect propagation patterns

Are sectors' substitution abilities really identical ?

Motivation : two challenges (quantification of reallocation channels)

- There is very few empirical quantification of the reallocation channels in the literature
- Nevertheless, industrial policies can play a major role in counterfeiting these effects when they're unfavourable for the economy

What is the contribution of reallocation mechanisms in shock transmission ?

Main results

- Intermediate input elasticities of substitution are heterogeneous across industries
- The H-IES model yields more variability in aggregate fluctuations as compared to that in U-IES models
- The contribution of reallocation mechanisms to shock propagation is quantitatively important
- Sectoral shocks are the main drivers of aggregate fluctuations

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A CES technology is a function with :

$$\left(\sum_j \phi_j^{\frac{1}{\sigma}} C_{jt}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where ϕ_j are weights (e.g: demand share), C_{jt} are the goods and σ is the elasticity of substitution. When :

- $\sigma > 1$, goods are "gross substitutes" : a higher price in a sector increases demand for other products
- $\sigma < 1$, goods are "gross complements" : a higher price in a sector decreases demand for other products
- $\sigma = 1$, the function is Cobb-Douglas

The agent

Representative household maximizing its consumption basket :

$$max_{(C_{jt})_j}\ C_t = D_t^{\frac{1}{\sigma-1}} \left(\sum_j \phi_j^{\frac{1}{\sigma}} C_{jt}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

under simplified budget constraint:

$$C_t = w_t$$

where w_t is the real wage

Firms maximize their profit:

$$max_{Y_{jt},L_{jt},(M_{jit})_i} \; p_{jt}Y_{jt} - w_{jt}L_{jt} - \sum_{i=1}^{N} p_{it}M_{jit}$$

$$s.t: \;\; Y_{jt} = \xi_j Z_t A_{jt} L_{jt}^{\beta_j} (\sum_{i=1}^{N} \gamma_{ij}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}} \qquad for \;\; t = 0, 1...$$

with Y_{jt} the output of sector j, L_{jt} the labor use,M_{jit} the input use of good i for sector j's output, Z_t the common state of technologyand A_{jt} the sectoral state of technology. We add the resource constraint :

$$Y_{jt} = C_{jt} + \sum_{i=1}^{N} M_{ijt}$$

There are three types of shocks in our model :

$$(\textit{Sectoral Supply}) \quad : \ \log(A_{jt}) = \rho_j \log(A_{j(t-1)}) + \epsilon_{jt}$$

$$(\textit{Aggregate Supply}) \quad : \ \log(Z_t) = \rho_Z \log(Z_{t-1}) + \epsilon_{Zt}$$

$$(\textit{Aggregate Demand}) \quad : \ \log(D_t) = \rho_D \log(D_{t-1}) + \epsilon_{Dt}$$

avec :

$$\epsilon_{jt} \overset{i.i.d}{\sim} \mathcal{N}(0, \upsilon_j^2)$$

$$\epsilon_{Zt} \overset{i.i.d}{\sim} \mathcal{N}(0, \Upsilon_Z^2)$$

$$\epsilon_{Dt} \overset{i.i.d}{\sim} \mathcal{N}(0, \Upsilon_D^2)$$

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Estimated parameters

In our model, we estimate :

- The standard deviations of shocks $(\upsilon_j)_j, \Upsilon_Z, \Upsilon_D$
- The persistence coefficients $(\rho_j)_j$, ρ_Z and ρ_D
- The elasticities of substitution for production $(\sigma_j)_j$ and consumption σ

The sectors

AGR :	Agriculture, forestry, fishing and hunting
MIN :	Mining
UTI :	Utilities
CON :	Construction
MAN :	Manufacturing
WHO :	Wholesale trade
RET :	Retail trade
TRA :	Transportation and warehousing
INF :	Information
FIN :	Finance, insurance, real estate, rental and leasing
BUS :	Professional and business services
EDU :	Educational services, health care, and social assistance
ART :	Arts. entertainment, recreation, accommodation and food services
OTH :	Other services, except government
GOV :	Government

Table: Description of the 15 industries

Methodology & Calibration

- We use bayesian estimation under Dynare, combining prior knowledge on the estimates distribution and data
- For the prior distribution, we use specifications close to the literature
- For the calibration of the other parameters, we follow a methodology close to Atalay (2021)

Data

- The data used as observables are the sectoral outputs (Y_j) provided by the BEA and the Personal Consumption Expenditure index (C) given by the FRED database between 1948Q1 and 2020Q4
- As the data are annual, we have quarterlyised them using a temporal disaggregation method à la Chow-Lin, which gives 291 estimation points
- These data were also filtered to remove trends and obtain stationary observables

Sectoral Estimates

Parameter		Prior distribution			Posterior distribution	
		Type	Mean	SD	Mean	[5%;95%]
Input Elasticities	σ_{AGR}	$\mathcal{IG}$	0.9	2	1.11	[0.76 , 1.45]
	σ_{MIN}	$\mathcal{IG}$	0.9	2	1.60	[1.19 , 1.99]
	σ_{UTI}	$\mathcal{IG}$	0.9	2	0.31	[0.19 , 0.42]
	σ_{CON}	$\mathcal{IG}$	0.9	2	0.29	[0.19 , 0.38]
	σ_{MAN}	$\mathcal{IG}$	0.9	2	0.21	[0.16 , 0.26]
	σ_{WHO}	$\mathcal{IG}$	0.9	2	0.39	[0.22 , 0.56]
	σ_{RET}	$\mathcal{IG}$	0.9	2	0.76	[0.25 , 1.34]
	σ_{TRA}	$\mathcal{IG}$	0.9	2	0.33	[0.20 , 0.46]
	σ_{INF}	$\mathcal{IG}$	0.9	2	0.27	[0.18 , 0.35]
	σ_{FIN}	$\mathcal{IG}$	0.9	2	0.26	[0.19 , 0.32]
	σ_{BUS}	$\mathcal{IG}$	0.9	2	0.26	[0.18 , 0.33]
	σ_{EDU}	$\mathcal{IG}$	0.9	2	0.72	[0.24 , 1.23]
	σ_{ART}	$\mathcal{IG}$	0.9	2	1.73	[0.44 , 2.84]
	σ_{OTH}	$\mathcal{IG}$	0.9	2	0.63	[0.23 , 1.08]
	σ_{GOV}	$\mathcal{IG}$	0.9	2	2.98	[2.00 , 3.93]

Notes: $\mathcal{B}$ denotes the Beta and $\mathcal{IG}$ the Inverse Gamma (type 1) distribution.

Table: Results of posterior estimation for sectoral parameters

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Proposition

The first-order effect of a productivity shock in sector i on the log output of sector j around the steady state can be decomposed into three complementary channels as follows:

$$\frac{\partial log Y_j}{\partial \epsilon_i}|_{\epsilon=0} = TOT_{ji} = DE_{ji} + CR_{ji} + IR_{ji}, \tag{1}$$

where each channel is given by:

$$DE_{ji} = l_{ji}$$

$$CR_{ji} = \frac{1}{\lambda_j}(1-\sigma)\left[\sum_{r=1}^{N} \phi_r l_{rj} l_{ri} - (\sum_{r=1}^{N} \phi_r l_{ri})(\sum_{r=1}^{N} \phi_r l_{rj})\right]$$

$$IR_{ji} = \frac{1}{\lambda_j}\sum_{k=1}^{N} \lambda_k(1-\sigma_k)\left[\sum_{r=1}^{N} \gamma_{kr} l_{rj} l_{ri} - \frac{1}{1-\beta_k}(\sum_{r=1}^{N} \gamma_{kr} l_{ri})(\sum_{r=1}^{N} \gamma_{kr} l_{rj})\right].$$

How to read a multiplier matrix

Shocks from MIN

A 1% TFP shock from CON translates in a 0,06% output variation of AGR

$$
\begin{pmatrix}
1.48 & 0.16 & 0.11 & 0.06 & 1.03 & 0.30 & 0.03 & 0.19 & 0.08 & 0.71 & 0.53 & 0.00 & 0.06 & 0.05 & 0.02 \\
0.07 & 1.38 & 0.11 & 0.08 & 0.83 & 0.17 & 0.02 & 0.18 & 0.09 & 0.67 & 0.68 & 0.00 & 0.06 & 0.04 & 0.02 \\
0.06 & 0.43 & 1.20 & 0.08 & 0.69 & 0.15 & 0.03 & 0.25 & 0.10 & 0.58 & 0.65 & 0.00 & 0.07 & 0.05 & 0.03 \\
0.07 & 0.14 & 0.06 & 1.03 & 0.91 & 0.17 & 0.09 & 0.13 & 0.07 & 0.41 & 0.46 & 0.00 & 0.04 & 0.04 & 0.01 \\
0.17 & 0.28 & 0.09 & 0.05 & 2.23 & 0.23 & 0.02 & 0.17 & 0.07 & 0.46 & 0.52 & 0.00 & 0.05 & 0.04 & 0.02 \\
0.03 & 0.07 & 0.07 & 0.04 & 0.40 & 1.12 & 0.01 & 0.17 & 0.10 & 0.57 & 0.68 & 0.00 & 0.06 & 0.06 & 0.04 \\
0.03 & 0.07 & 0.09 & 0.05 & 0.36 & 0.09 & 1.02 & 0.17 & 0.10 & 0.60 & 0.59 & 0.01 & 0.05 & 0.05 & 0.02 \\
0.05 & 0.10 & 0.09 & 0.05 & 0.61 & 0.12 & 0.03 & 1.31 & 0.08 & 0.58 & 0.50 & 0.00 & 0.06 & 0.06 & 0.03 \\
0.04 & 0.08 & 0.07 & 0.04 & 0.54 & 0.11 & 0.02 & 0.12 & 1.43 & 0.54 & 0.74 & 0.00 & 0.11 & 0.05 & 0.02 \\
0.04 & 0.08 & 0.12 & 0.13 & 0.43 & 0.09 & 0.02 & 0.11 & 0.13 & 2.17 & 0.76 & 0.00 & 0.10 & 0.06 & 0.03 \\
0.03 & 0.05 & 0.05 & 0.03 & 0.32 & 0.07 & 0.01 & 0.08 & 0.10 & 0.47 & 1.59 & 0.00 & 0.07 & 0.04 & 0.01 \\
0.03 & 0.06 & 0.05 & 0.03 & 0.41 & 0.09 & 0.01 & 0.07 & 0.07 & 0.47 & 0.43 & 1.02 & 0.06 & 0.04 & 0.01 \\
0.05 & 0.09 & 0.10 & 0.04 & 0.49 & 0.10 & 0.03 & 0.10 & 0.09 & 0.57 & 0.61 & 0.00 & 1.09 & 0.05 & 0.02 \\
0.04 & 0.07 & 0.06 & 0.05 & 0.48 & 0.10 & 0.03 & 0.09 & 0.09 & 0.59 & 0.46 & 0.01 & 0.05 & 1.06 & 0.02 \\
0.04 & 0.09 & 0.05 & 0.06 & 0.50 & 0.09 & 0.01 & 0.09 & 0.09 & 0.32 & 0.39 & 0.01 & 0.04 & 0.03 & 1.01
\end{pmatrix}
$$

Impacts on GOV

Recipients

Transmitters

Figure: Downstream multiplier matrix for the US economy

Heatmap example with the Input reallocation channel

We only show the Input reallocation channel heatmap *IR* here :

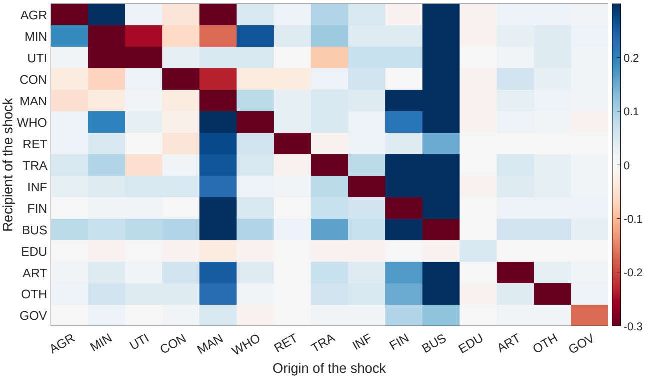

Figure: Heatmap of input reallocation multiplier for the US economy

- Blue means amplification, red means dampening
- Overall, *IR* channel dampens the shock propagation due to complementarity on average

Remember that :

$$TOT_{ji} = DE_{ji} + CR_{ji} + IR_{ji}$$

To understand the extent to which reallocation channels CR and IR matter, we compute the percentage error of prediction of the idiosyncratic shock multiplier under Cobb-Douglas against CES :

$$\varepsilon_{ji}^{(I)} = \frac{|DE_{ji} - TOT_{ji}|}{TOT_{ji}}, \tag{2}$$

This first type error shows how important are reallocation channels (the higher $\varepsilon^{(I)}$, the more important reallocation is).

Why does reallocation matter ?

First type errors $\varepsilon^{(I)}$ are represented with the following heatmap :

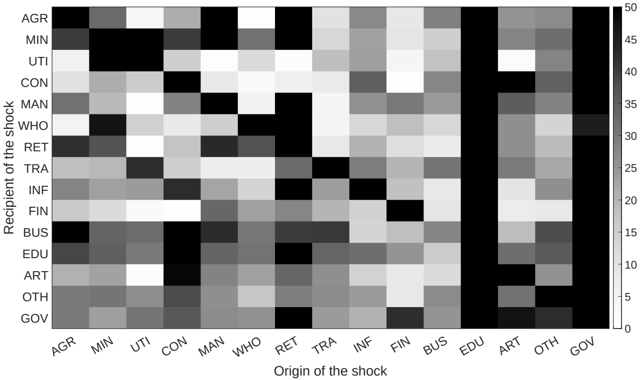

Figure: Relative percentage loss ε_{ij} for predicting idiosyncratic shock multipliers for Cobb-Douglas against CES substitutions

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Reason 1 : elasticities are indeed dispersed

The estimated elasticites in our model are dispersed and relatively far from 0. Common estimate used in the literature for calibration comes from Atalay (2017) : $\sigma = 0.1$.

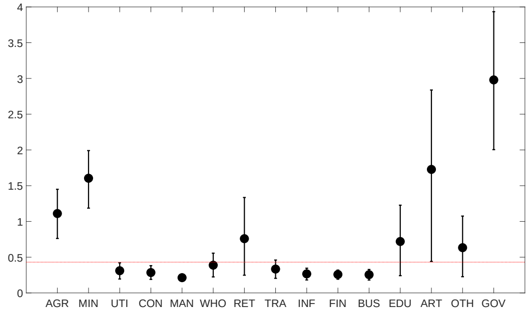

Figure: Dispersion of estimated elasticities of substitution

Reason 2 : the Input reallocation channel is strongly affected

Remember that the only channel which is elasticity-dependent is IR :

$$IR_{ji} = \frac{1}{\lambda_j} \sum_{k=1}^{N} \lambda_k (1-\sigma_k) \left[\sum_{r=1}^{N} \gamma_{kr} I_{rj} I_{ri} - \frac{1}{1-\beta_k} (\sum_{r=1}^{N} \gamma_{kr} I_{ri}) (\sum_{r=1}^{N} \gamma_{kr} I_{rj}) \right].$$

Question is : how does H-IES vs U-IES affect this channel ? We answer by computing the second type (II) of relative errors of prediction of the idiosyncratic shock multiplier is given by:

$$\varepsilon_{ji}^{(II)} = \frac{|IR_{ji}|_{\sigma_j=0.43} - IR_{ji}|}{IR_{ji}}, \tag{3}$$

Reason 2 : the Input reallocation channel is strongly affected

The second type errors on the Input Reallocation channel are very high :

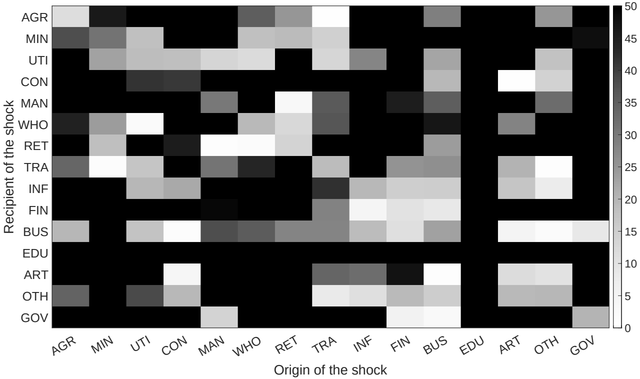

Figure: Relative percentage difference $\varepsilon_{ij}^{(II)}$ of Input Reallocation multipliers (IR) between U-IES and H-IES hypothesis

Reason 3 : recessions (as well as booms) are underestimated

- Remember that the Input Reallocation channel under H-IES dampens shock propagation overall due to complementarity
- But U-IES pushes the production network towards even more complementarity
- The result is an underestimation of aggregate volatility !

Reason 3 : recessions (as well as booms) are underestimated

We feed both models with the same sequence of shocks (here, estimated recessive shocks) and we compare the aggregate output response :

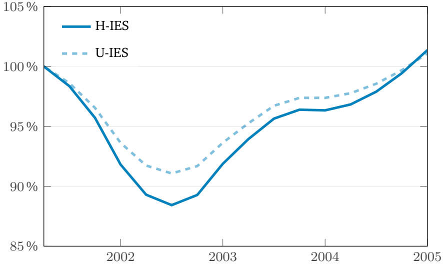

Figure: (a) Dotcom crisis

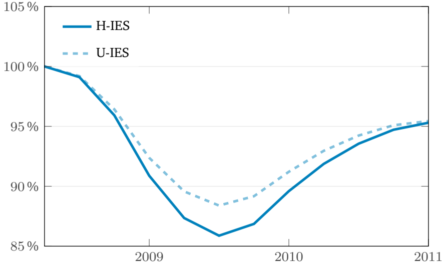

Figure: (b) Subprime crisis

Figure: Counterfactual output with H-IES and U-IES models (the U-IES case corresponds to $\sigma_j = 0.43$)

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Main take-away

- First bayesian estimation of a multi-sectoral model with heterogeneous CES elasticities
- Sectors exhibit heterogeneous elasticities of substitution, and it matters !
- Theoretical decomposition of shock propagation with quantitative estimation of the different channels
- Input reallocation channel is crucial in aggregate volatility and is strongly affected by H-IES vs U-IES hypothesis
- Sectoral shocks are the main drivers of aggregate volatility

Model comparison

Model type	No substitution ($\sigma = 1,\ \sigma_j = 1$)	Partial substitution ($\sigma \neq 1,\ \sigma_j = 1$)	Partial substitution ($\sigma = 1,\ \sigma_j \neq 1$)	Full substitution ($\sigma \neq 1,\ \sigma_j = 0.43$)	Full substitution ($\sigma \neq 1,\ \sigma_j \neq 1$)
Prior probability	0.2	0.2	0.2	0.2	0.2
Log marginal data density	16240.62	16383.95	16622.51	16649.18	16691.27
Bayes ratio	1	3.45e+62	7.95e+165	4.23e+177	7.36e+195
Posterior model probability	0.00	0.00	0.00	0.00	1.00

Table: The comparison of prior and posterior model probabilities with different specifications (with parameters taken at their posterior mode).