Model 00000 Quantitative Exercises

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Composition Effects in OTC Transaction Costs

Mariano J. Palleja

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Inventory Cost and Liquidity in Over the Counter Markets

Post '08 crisis regulations affected OTC markets (Volcker Rule, Basel III).

Dealer's inventory costs increased.

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- ▶ To quantify the effect, researchers distinguish between trading mechanisms:
 - Principal: Customers exchange immediately against dealers' inventories.
 - Agency: Customers wait for dealers to find them a counterparty.

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- Corporate bond evidence
 - \uparrow principal cost & \downarrow principal share.

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- Agency: Customers wait for dealers to find them a counterparty.

Corporate bond evidence

- ▶ \uparrow principal cost & \downarrow principal share.
- ! Customers' **speed-cost trade-off** affects the trading mechanism choice.
 - Endogenous migration away from principal after a regulation change:
 - \implies Composition bias: samples pre and post regulation differ.

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This Paper:

I build and estimate a quantitative search model to address:

1. What determines customers' trading mechanism choice?



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This Paper:

I build and estimate a quantitative search model to address:

2. How such mechanism choice affects transaction costs measures?



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This Paper:

I build and estimate a quantitative search model to address:

3. What if inventory costs increase?



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This Paper:

I build and estimate a quantitative search model to address:

4. What is the size and sign of the composition effect?



Contribution

1. Search literature of OTC markets.

Duffie, Gârleanu and Pedersen (2005), Lagos and Rocheteau (2009), Weill (2020)

- + Endogenous trading mechanism as a customer choice.
- ✓ I study optimal mechanism choice as a function of key parameters.
- 2. Models of dealers' trading mechanism choice.

Tse and Xu (2017); Cimon and Garriot (2019); An (2020); An and Zheng (2022); Saar et. al. (2023).

- + Customers' trading mechanism choice.
- $\checkmark~$ I incorporate the endogenous response of customers to shocks.
- + Non-degenerate distribution of transaction costs and volume traded.
- $\checkmark~$ I compute observable and counterfactual cost resembling empirical measures.
- 3. Empirical literature of OTC market liquidity.

Bao, O'Hara, and Zhou (2018), Bessembinder, Jacobsen and Venkataraman (2018), Dick-Nielsen and Rossi

(2019), Goldstein and Hotchkiss (2020), O'Hara and Zhou (2021), Kargar et.al. (2021), Choi, Huh and Shin

(2023), Rapp and Waibel (2023)

- + Model of endogenous trading mechanism choice.
- ✓ I quantify the composition bias when market conditions change.

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Model

Lagos and Rocheteau (2009):

- Continuous time and infinitely lived agents.
- Single asset in fixed supply, which is traded OTC.
- Customers have quasi-linear flow utility $u_i(a) + d$:
 - *d* is the net consumption of *numeraire* good, produced by customers.
 - a is the asset holding.
 - $u_i(a) = \epsilon_i u(a)$, where ϵ_i is a time-varying preference.

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Two trading mechanisms:

- At random time, customer contact dealers and choose:
 - 1. Principal: immediate exchange paying bargained fee.
 - 2. Agency: delayed exchange paying bargained fee.

Dealers execute orders in a frictionless market, at price p.

- 1. Principal: immediate access paying $\theta p |a' a|$
- 2. Agency: delayed access at random time.

Model details

- ▶ $\uparrow |a' a| \implies \uparrow Mg$ trading surplus.
- ▶ Principal costs are linear: as $\uparrow |a^P a|$, speed benefit > speed costs.
 - \implies More "desperate" customers pay principal premium.



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Transaction Costs are increasing in trading surplus.

- Bilateral (Nash) bargain: cost split trading surplus.
- Principal trades pay premium $\propto \theta p |a^P a|$.



Counterfactual Spreads and Composition Effect

1. Compute empirical measures \mathcal{S}^P and \mathcal{S}^A as vol weighted avg spreads.



Counterfactual Spreads and Composition Effect

- 1. Compute empirical measures \mathcal{S}^{P} and \mathcal{S}^{A} as vol weighted avg spreads.
- 2. Increase parameter θ and compute counterfactuals \tilde{S}^P and \tilde{S}^A using only non-migrant trades.



Counterfactual Spreads and Composition Effect

- 1. Compute empirical measures \mathcal{S}^P and \mathcal{S}^A as vol weighted avg spreads.
- 2. Increase parameter θ and compute counterfactuals \tilde{S}^P and \tilde{S}^A using only non-migrant trades.



3. Compute Composition Effect (CE) in each mechanism as:

$$\textit{CE} \equiv (\Delta \mathcal{S} - \Delta ilde{\mathcal{S}}) / \Delta \mathcal{S}$$



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Baseline Calibration

Unit of time
$$= 1$$
 month $\mid u_i(a) = \epsilon_i imes rac{a^{1-\sigma}}{1-\sigma}$

Parameter	Description	Value		
- Normalization-				
A	Asset supply	1		
ϵ_i	Preference shifter	$\left\{\frac{i-1}{l-1}\right\}_{i=1}^{20}$		
- External calibration-				
r	Discount rate	0.5%		
π_i	Preference shifter distribution	1//		
η	Dealer's bargain power	0.95		
- GMM calibration-				
α	Contact with dealer rate	9.15		
δ	Preference shock rate	2.59		
β	Agency execution rate	1.00		
θ	Inventory cost	0.89 bp		
σ	CRRA coeff.	2.73		

 θ Discussion

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GMM Calibration

I estimate

$$\hat{\upsilon} = \arg\min_{\upsilon \in \Upsilon} [(m(\upsilon) - m_s) \oslash m_s]' W[(m(\upsilon) - m_s) \oslash m_s]$$

where $\upsilon = [\alpha, \delta, \beta, \theta, \sigma]$, $m = [S^P, S^A, T, \gamma^P, \gamma^A]$, and W = I.

Moment	Empirical			Theoretical
	p50 (<i>m₅</i>)	p25	p75	
\mathcal{S}^{P} , Principal Vol Weighted Spread	9.12	5.87	14.20	10.29
$\mathcal{S}^{\mathcal{A}}$, Agency Vol Weighted Spread	5.00	2.56	8.73	4.04
${\mathcal T}$, Monthly Turnover	3.27	2.28	4.61	3.47
	$\hat{\gamma} (m_s)$	$\hat{\gamma} - s.e.$	$\hat{\gamma} + s.e.$	
γ^P , Principal Spread-Size slope	1.45	1.33	1.58	1.31
$\gamma^{\mathcal{A}}$, Agency Spread-Size slope	0.61	0.50	0.73	0.69

Sample moments computed from TRACE 2016-2019, using IG bonds with at least 10 observations in all variables used. Percentiles represent the cross section of bond level computed variables. n=2829 bonds.

Model 00000 Quantitative Exercises

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Inventory costs increase: customers migrate away from principal.

heta: 0.1 bp
ightarrow 0.89 bp



- 1. Principal trading migrate towards agency.
- 2. Migration is not random: stronger when closer to optimal positions.

Model 00000

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The rise in principal costs are overestimated in $\approx 1/3$.



heta : 0.1*bp* ightarrow 0.89*bp*

- Turnover decreases 4.6%, and agency share increases 2.4%.
- $\Delta S^P = 0.76 bp$ and $\Delta \tilde{S}^P = 0.51 bp$: $\implies CE^P = 32.2\%$
- $\Delta S^A = 0.24 bp$ and $\Delta \tilde{S}^A = 0.24 bp$: $\implies CE^A = -1.2\%$

Conclusion

OTC transaction cost measures are subject to a composition bias:

- Trading mechanisms are endogenous.
- Choice is a function of each customer' speed-cost trade off.

I develop a model to account for it:

- Secondary market with search frictions.
- Immediate principal and delayed agency trading.
- Speed-cost trade-off defines terms of trade of each customer
- I build counterfactual measure to quantify the composition bias:
 - Inventory Cost: 32.2% in principal, -1.2% in agency.
- Results suggest that policies affecting dealer's inventory costs had a smaller negative impact over market liquidity than previously thought.

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08 Financial Crisis increased Principal Trading Costs

Basel III (finalized in 2013 in US)

- Liquidity Coverage Ratio (LCR): "high-quality" assets in proportion to any borrowing with term 30 days or less.
- Net Stable Funding Ratio (NSFR): fund assets that mature at various terms less than one year with financing that has at least a matching term.
- Revised Capital Adequacy Ratio (CAR): larger minimum of equity and reserves as a percentage of risk-weighted assets.
- Leverage Ratio (LR), maintain a quantity of stock and cash equal to at least 3% (5% for G-SIBs) of assets.

Volcker Rule (full compliance by Jul 2015)

- Prohibits banks from engaging in proprietary trading of risky securities.
 - Market making is excepted, but the distinction is blurry.
 - Reports of measures as proxies for the underlying trading motive.



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Customer's Path



Shocks:



- α: contact with dealers.
- β: execution of agency trade.

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Customer's Value Function: contact dealers and choose mechanism.



$$V_{i_0}(a) = \mathbb{E}_{i_0} \Big[\underbrace{\int_{0}^{\tau_{\alpha}} e^{-rs} u_{i_s}(a) ds}_{\text{utility of holding } a} + e^{-r\tau_{\alpha}} \max \Big\{ \underbrace{V_{i_{\alpha}}^{P}(a)}_{\text{principal}}, \underbrace{V_{i_{\alpha}}^{A}(a)}_{\text{agency}} \Big\} \Big]$$

- τ_α: time it takes to contact a dealer.
- *i_s*: preference type at time *t* = *s*.
- $u_i(a)$: ut. function of customer $\{i, a\}$.
- E over:
 - 1. next contact with dealers \rightarrow Poisson rate α .
 - 2. preference shocks \rightarrow Poisson rate δ .
 - 3. execution of agency trade \rightarrow Poisson rate β .

Principal Choice: customers pay ϕ^P to trade immediately.



immediate trade

- $a_{i_{\alpha}}^{P}$: optimal principal asset holdings of customer $\{i_{\alpha}, a\}$.
- p: inter-dealer price.
- $\phi_{i\alpha}^P$: fee charged in the principal trade.

Agency choice: customers pay ϕ^A and wait to trade.



- τ_β: time it takes to execute agency trades.
- ▶ $a_{i_{\beta}}^{A}$: optimal agency asset holdings of customer $\{i_{\beta}, a\}$. Chosen at execution.
- $\phi^A_{i_{\alpha}}$: fee charged when agency. Arranged at contact with dealers.

Dealer's Value Function: principal intermediation is costly.

Dealers pay inventory costs to intermediate on principal:

$$W_t = \mathbb{E}\Big[e^{-r(\tau_{\alpha})}\int \Phi_{i_{\alpha}}(a)dH_{t+\tau_{\alpha}} + W(t+\tau_{\alpha})\Big],$$

$$\Phi_i(\mathbf{a}) = \begin{cases} \phi_i^P - \theta p | a_i^P - a | & \text{if principal,} \\ e^{-r(T_\beta - T_\alpha)} \phi_i^A & \text{if agency,} \end{cases}$$

where

- H_t: distribution of customers at time t.
- ▶ $\theta \in [0, \frac{r}{r+\beta})$ is the constant marginal inventory cost per dollar traded.

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Transaction Costs as functions of liquidity needs.

Nash Bargain where dealers hold η power

Principal Problem: Immediate and costly execution

$$\phi_i^{P}(\mathbf{a}) = \eta \Big[\underbrace{V_i(\mathbf{a}_i^{P}) - p(\mathbf{a}_i^{P} - \mathbf{a}) - V_i(\mathbf{a})}_{\text{Customer's Surplus}} \Big] + (1 - \eta) \Big[\underbrace{\theta p | \mathbf{a}_i^{P} - \mathbf{a}|}_{\text{Invetory Cost}} \Big]$$

Agency Problem: Delayed and non costly execution

$$\mathbb{E}[e^{-r\tau_{\beta}}]\phi_{i_{\alpha}}^{A}(\mathbf{a}) = \eta \Big[\underbrace{\mathbb{E}_{i_{\alpha}}\Big[\int_{0}^{\tau_{\beta}} e^{-rs} u_{i_{\alpha+s}}(\mathbf{a})ds + e^{-r\tau_{\beta}}\left(V_{i_{\beta}}(a_{i_{\beta}}^{A}) - p[a_{i_{\beta}}^{A} - \mathbf{a}]\right)\Big] - V_{i_{\alpha}}(\mathbf{a})}_{\text{Customer's Surplus}}\Big]$$

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- Both principal and agency fees are increasing in a consumers' surplus.
- Principal trades pay premium fee $(1 \eta)\theta p |a_i^P a|$.

Optimal Trading Mechanism: A speed-cost trade-off

Indifference Condition:

$$V_i(a_i^P) - p(a_i^P-a) - p heta|a_i^P-a| = ar{U}_i^eta(a) + \hat{eta}[ar{V}_i^A - p(ar{a}_i^A-a)]$$

Assume $a_i^P > a$:



• The larger the distance $|a_i^P - a|$, the bigger the marginal trading surplus.

▶ Principal costs are linear: as $\uparrow |a_i^P - a|$, speed benefit outweighs premium costs.

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Flow Bellman Equation

Analytical expressions for expectations

$$V_i(\mathbf{a}) = \bar{U}_i^{\kappa}(\mathbf{a}) + \hat{\kappa} \big[(1 - \hat{\delta}) \max \left\{ V_i^{\mathcal{P}}(\mathbf{a}), V_i^{\mathcal{A}}(\mathbf{a}) \right\} + \hat{\delta} \sum_j \pi_j \max \left\{ V_j^{\mathcal{P}}(\mathbf{a}), V_j^{\mathcal{A}}(\mathbf{a}) \right\} \big]$$

where

$$\begin{split} V_i^P(a) &= V_i(a_i^P) - p(a_i^P - a) - p\theta |a_i^P - a|, \\ V_i^A(a) &= \bar{U}_i^\beta(a) + \hat{\beta}[\bar{V}_i^A - p(\bar{a}_i^A - a)], \\ \bar{U}_i^\nu(a) &= \left[(1 - \hat{\delta}_\nu) u_i(a) + \hat{\delta}_\nu \sum_j \pi_j u_j(a) \right] \frac{1}{r + \nu}, \\ \bar{V}_i^A &= (1 - \hat{\delta}_\beta) V_i(a_i^A) + \hat{\delta}_\beta \sum_j \pi_j V_j(a_j^A), \\ \bar{a}_i^A &= (1 - \hat{\delta}_\beta) a_i^A + \hat{\delta}_\beta \sum_j \pi_j a_j^A, \\ \hat{\kappa} &= \frac{\kappa}{r + \kappa}, \quad \hat{\beta} &= \frac{\beta}{r + \beta}, \quad \hat{\delta}_\nu = \frac{\delta}{r + \delta + \kappa}, \quad \nu = [\kappa, \beta] \quad \kappa = \alpha(1 - \eta). \end{split}$$

Steady State Distribution

Define n_[a,i,ω] as the mass of customers with:

- ▶ $a \in A^*$: Asset holdings
- $i \in \{1 : I\}$: Preference shocks
- $\omega \in {\omega_1, \omega_2}$: Waiting for dealer (ω_1) or for execution (ω_2)
- Flow across states:

$$\begin{array}{ll} \text{Contact dealer at rate } \alpha : \begin{cases} n_{[a,i,\omega_1]} \to n_{[a',i,\omega_1]} & \forall \{a,i\} & \text{if principal} \\ n_{[a,i,\omega_1]} \to n_{[a,i,\omega_2]} & \forall \{a,i\} & \text{if agency} \end{cases}$$

$$\begin{array}{ll} \text{Pref. shock at rate } \delta : & n_{[a,i,\omega]} \to n_{[a,j,\omega]} & \forall \{a,\omega\} \end{array}$$

$$\begin{array}{ll} \text{Execution shock at rate } \beta : & n_{[a,i,\omega_2]} \to n_{[a',i,\omega_2]} & \forall \{i\} \end{cases}$$

Shocks + Policy Functions $\rightarrow T_{[3I \times I \times 2]}$.

$$n = \lim_{k \to \infty} n_0 T^k$$

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Inflow-Outflow Equations

$$\begin{split} n_{[a_{i}^{P,b},i,\omega_{1}]} &: \quad \delta\pi_{i}\sum_{j\neq i}n_{[a_{i}^{P,b},j,\omega_{1}]} + \alpha\sum_{a\in Buy_{i}^{P}}n_{[a,i,\omega_{1}]} = n_{[a_{i}^{P,b},i,\omega_{1}]}[\delta[1-\pi_{i}] + \alpha\mathbf{1}_{[a_{i}^{P,b}\notin NoT_{i}^{P}]}] \\ n_{[a_{i}^{P,s},i,\omega_{1}]} &: \quad \delta\pi_{i}\sum_{j\neq i}n_{[a_{i}^{P,s},j,\omega_{1}]} + \alpha\sum_{a\in Sell_{i}^{P}}n_{[a,i,\omega_{1}]} = n_{[a_{i}^{P,s},i,\omega_{1}]}[\delta[1-\pi_{i}] + \alpha\mathbf{1}_{[a_{i}^{P,s}\notin NoT_{i}^{P}]}] \\ n_{[a_{i}^{A},i,\omega_{1}]} &: \quad \delta\pi_{i}\sum_{j\neq i}n_{[a_{i}^{A},j,\omega_{1}]} + \beta\sum_{a\in\mathcal{A}^{*}}n_{[a,i,\omega_{2}]} = n_{[a_{i}^{A},i,\omega_{1}]}[\delta[1-\pi_{i}] + \alpha\mathbf{1}_{[a_{i}^{A}\notin NoT_{i}^{P}]}] \\ n_{[a,i,\omega_{1}]} &: \quad \delta\pi_{i}\sum_{j\neq i}n_{[a_{j},j,\omega_{1}]} = n_{[a_{j},i,\omega_{1}]}[\delta[1-\pi_{i}] + \alpha\mathbf{1}_{[a_{j}\notin NoT_{i}^{P}]}], \quad a\in\cup_{j\neq i}\{a_{j}^{P,b},a_{j}^{P,s},a_{j}^{A}\} \\ n_{[a,i,\omega_{2}]} &: \quad \delta\pi_{i}\sum_{j\neq i}n_{[a_{i},j,\omega_{2}]} + \alpha n_{[a_{i},i,\omega_{1}]}\mathbf{1}_{[a_{i}\in\Gamma_{i}^{A}]} = n_{[a_{i},i,\omega_{2}]}[\delta[1-\pi_{i}] + \beta], \quad a\in\mathcal{A}^{*} \end{split}$$

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Steady State Equilibrium

The steady state equilibrium is defined as:

- 1. Optimal asset holdings $\{a_i^P(a), a_i^A\}_{i=1}^I$.
- 2. Fees $\{\phi_i^P(a), \phi_i^A(a)\}_{i=1}^I$.
- 3. Trading mechanism sets $\{\Gamma_i^P, \Gamma_i^A\}_{i=1}^I$ where $\Gamma = \{Buy, Sell, NoT\}$.
- 4. Stationary distribution $n_{[a,i,\omega]}$.
- 5. Inter-dealer price p.

Such that

- 1. Optimal assets maximize consumer trading surplus.
- 2. Fees maximize Nash products.
- 3. Sets $\{\Gamma_i^P, \Gamma_i^A\}_{i=1}^I$ are defined using thresholds satisfying the indifference conditions.
- 4. Distribution $n_{[a,i,\omega]}$ satisfies inflow-outflow equations.
- 5. Price satisfy $\sum_{j=1}^{2} \sum_{i=1}^{\prime} \sum_{a \in \mathcal{A}^*} an_{[a,i,\omega_j]} = A$.

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Extra Slides

Solution Method

- 1. Set an initial guess for the equilibrium price p.
 - 1.1 Set an asset holdings grid and an initial guess for $V_i(a)$
 - 1.2 Compute optimal asset holdings $\{a_i^P(a), a_i^A\}_{i=1}^I$ using eq. (4) and eq. (6).
 - 1.3 Compute trading mechanism choice for each pair $\{i, a\}$, using indifference condition.
 - 1.4 Fix $\{a_i^P(a), a_i^A\}_{i=1}^l$, and iterate *h* times the following steps:
 - 1.4.1 Update $V_i(a)$ using eq. (1).
 - 1.4.2 Compute trading mechanism choice for each pair $\{i, a\}$, using indifference condition
 - 1.5 Update $V_i(a)$ using eq. (1) until convergence with initial guess of step (a).
- 2. Define trading mechanism sets $\{\Gamma_i^P, \Gamma_i^A\}_{i=1}^I$ using thresholds.
- 3. Compute transition matrix T using inflow-outflow equations.
- 4. Set vector n_0 and obtain $n = \lim_{k \to K} n_0 T^k$, with K sufficiently large to reach convergence.
- 5. Compute total demand and update *p* until excess demand in market clearing equations converges towards zero.

Note: Our Bellman operator is a contraction mapping with modulus $\hat{\kappa}$ and operates in a complete normed vector space

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Discussion on Inventory Costs calibration

Inventory Costs θ :

- Suppose we want to capture the regulations-induced inventory costs.
- ▶ Greenwood et. al. (2017), Duffie (2018), Fed stress test (2019): Leverage Ratio Requirement as most important constraint for U.S. banks → LR: hold extra capital when including assets in inventory: 3% to 5%/
- LR cost = $p[a' a][e^{zm} 1]x\%$, where bank face x% of capital requirement and z% opportunity costs for such capital, and offload position after *m* days.
- Model cost = $2\theta p[a' a]$. $\implies \theta = [e^{zm} 1]x\%/2$
- Take z = r as the opportunity cost.
- Goldstein and Hotchkiss (2020), TRACE 02-11, m = 10.6 days.
- ► During sample period, 2016-2019, x% = 5% for GSIB banks. $\implies \theta = 0.44b.p.$.

My estimated $\hat{\theta} = 0.89 b. p.$, so arguably adding other cost on top of LR.

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Empirical moments details I

Data Sources

► TRACE Academic: US dealers corporate bond transactions.

- Dealers with anonymous identifiers.
- 2016m1 2019m12.
- Standard filters: error cleaning + literature basics ¹.
- IG Bonds
- FISD (bond characteristics)

Principal-Agency classification.

- Keep only customer-dealer trades.
- Agency: trades that share the same dealer-bond executed within a 15 min.
 - ▶ ≥ 50% vol if partial match.
 - Competing trades sorted by time distance and volume.
- Principal trades: non-agency trades.

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¹Among the most significant filters, I follow the literature and drop preferred, convertible or exchangeable, yankee bonds, bonds with sinking fund provision, variable coupon, with time to maturity $\langle -1 \rangle$ year for issued $\langle 2 \rangle$ months $| = \langle 0 \rangle \langle 0 \rangle$

Empirical moments details II

1) S, Vol Weighted Spreads

- Remove micro trades (≤\$100k)
- For each trade, compute Choi, Huh and Shin (2023)'s Spread1:

$$s_{i,b,d} = Q \times (\frac{p_{i,b,d} - p_{b,d}^{DD}}{p_{b,d}^{DD}}) \quad , \quad p_{b,d}^{DD} = \frac{\sum_{i \in DD_{b,d}} vol_{b,d,i}^{DD} p_{b,d,i}^{DD}}{\sum_{i \in DD_{b,d}} vol_{b,d,i}^{DD}}$$

where i=trade, b=bond, d=day, Q = 1 (-1) if customer buys (sells).

$$S_b^P = \sum_{i,d} (s_{i,b,d} \times vol_{i,b,d}^P) / \sum_{i,d} vol_{i,b,d}^P \\ S_b^A = \sum_{i,d} (s_{i,b,d} \times vol_{i,b,d}^A) / \sum_{i,d} vol_{i,b,d}^A$$

2) \mathcal{T} , Monthly Turnover

- $k_b =$ numbers of days between offering and maturity, within the period sample.
- iao_b = the average amount outstanding of bond during k_b days.

•
$$\mathcal{T}_b = \left(\sum_i vol_{i,b}/iao_b\right)/(k_b/30.5)$$
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Empirical moments details III

- 3) γ , Spread-Size slopes
 - ► $s_{i,d,b} = \alpha + \beta FE + \gamma (vol_{i,d,b}^{P}/iao_{b}) + \epsilon_{i,d,b}$, with FE = [dealer, bond, day].
 - $\hat{\gamma}^P$ and $\hat{\gamma}^A$ are OLS estimates over corresponding subsamples.
 - SE clustered by bond-day.

Dependent Variable:	Transact Principal	ion Cost (bp) Agency
Trade size (pp)	1.45*** (0.13)	0.61*** (0.12)
Dealer FE	Yes	Yes
Bond FE	Yes	Yes
Day FE	Yes	Yes
Observations	1,505,133	97,305
Adjusted R ²	0.108	-0.023

Clustered (Bond & Day) standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

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Theoretical moments details

1) S, Vol Weighted Spreads

$$\begin{split} \mathcal{S}^{P} &= \sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{P}} \frac{n_{[a,i,\omega_{1}]} |a_{i}^{P} - a|}{\sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{P}} n_{[a,i,\omega_{1}]} |a_{i}^{P} - a|} \frac{\phi_{a,i}^{P}}{|a_{i}^{P} - a|p} \\ \mathcal{S}^{A} &= \sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{A}} \frac{n_{[a,i,\omega_{1}]} rav_{a,i}}{\sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{A}} n_{[a,i,\omega_{1}]} rav_{a,i}} \frac{\phi_{a,i}^{A}}{rav_{[a,i]}p} \end{split}$$

where realized agency volume $rav_{a,i} = (1 - \hat{\delta})|a_i^A - a| + \hat{\delta}\sum_{j \in \mathcal{I}} \pi_j |a_j^A - a|$

2) \mathcal{T} , Monthly Turnover

$$\mathcal{T} = \sum_{i \in \mathcal{I}} \alpha \Big[\sum_{\mathbf{a} \in \Gamma_i^P} n_{[\mathbf{a}, i, \omega_1]} | \mathbf{a}_i^P - \mathbf{a} | + \sum_{\mathbf{a} \in \Gamma_i^A} n_{[\mathbf{a}, i, \omega_1]} r_{\mathbf{a} \mathbf{v}_{\mathbf{a}, i}} \Big]$$

3) γ , Spread-Size slopes

$$\hat{\gamma}^P = \frac{\operatorname{cov}(\phi^P/(|a^P - a|p), |a^P - a|)}{\operatorname{var}(|a^P - a|)} \quad , \quad \hat{\gamma}^A = \frac{\operatorname{cov}(\phi^A/(\operatorname{rav} * p), \operatorname{rav})}{\operatorname{var}(\operatorname{rav})}$$

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Matching % Agency Volume vs Spread ratio

- Assume trading costs are an increasing linear function in speed valuation.
- Assume mass of traders is uniformly distributed across speed valuation line.
- Unique threshold split principal and agency trades.

 \implies Max spread ratio = 2, achieved when % agency volume \rightarrow 100%.



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Moments Choice Discussion I

Moments' relevance for the paper's goal

- The main goal of the paper is to characterize the Composition Effect, which is determined by:
 - Migration of trades.
 - Differential spreads paid by migrants and non migrants.
- ▶ In the model migration occurs when trading mechanism thresholds change.
- Migrants are thus located in the extreme of the trading size distribution conditional on preference type.

 \implies matching the slope of spreads on trading size informs of the differential of spreads paid by migrant and non migrants.

Moments as sources of identification

 All parameters affect prices and quantities in the model (whether directly or through GE effects)

 \implies Moments chosen cover both prices, quantities, and the relation among them

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Moments Choice Discussion II

Theoretical moments as parameters change around \hat{v}



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Extra Slides

Spreads per dollar:
$$\frac{\phi_i(a)}{|a'-a|} \frac{10000}{p}$$



Spread Decomposition: Principal Trades

$$\mathcal{S}^{P} = \sum_{i \in \mathcal{I}} \sum_{\mathbf{a} \in \Gamma_{i}^{P}} \underbrace{\frac{n_{[\mathbf{a}, i, \omega_{1}]} |\mathbf{a}_{i}^{P} - \mathbf{a}|}{\sum_{i \in \mathcal{I}} \sum_{\mathbf{a} \in \Gamma_{i}^{P}} n_{[\mathbf{a}, i, \omega_{1}]} |\mathbf{a}_{i}^{P} - \mathbf{a}|}_{\text{steady state vol weight}} \underbrace{\frac{\phi_{\mathbf{a}, i}^{P}}{|\mathbf{a}_{i}^{P} - \mathbf{a}|p}}_{\text{fee per dollar}}$$

Spread Decomposition: Consider change in parameter $q \in \{0,1\}$

$$S^{P}(q = 0) = S^{P,0}_{P^{0},P^{1}} \times w^{P,0}_{P^{0},P^{1}} + S^{P,0}_{P^{0},A^{1}} \times w^{P,0}_{P^{0},A^{1}} + S^{P,0}_{P^{0},NT^{1}} \times w^{P,0}_{P^{0},NT^{1}}$$

$$S^{P}(q = 1) = S^{P,1}_{P^{0},P^{1}} \times w^{P,1}_{P^{0},P^{1}} + S^{P,1}_{A^{0},P^{1}} \times w^{P,1}_{A^{0},P^{1}} \times w^{P,0}_{A^{0},P^{1}} + S^{P,0}_{N^{0},P^{1}} \times w^{P,0}_{N^{0},P^{1}} \times w^{P,0}_{P^{0},P^{1}} \times w^{P,0}_{P^{0},NT^{1}} \times w^{P,0}_{P^{0},NT$$

Spread Decomposition: Agency Trades

$$S^{A} = \sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{A}} \frac{n_{[a,i,\omega_{1}]} rav_{a,i}}{\sum_{i \in \mathcal{I}} \sum_{a \in \Gamma_{i}^{A}} n_{[a,i,\omega_{1}]} rav_{a,i}} \frac{\phi_{a,i}^{A}}{rav_{[a,i]}p}$$

where $rav_{a,i}$ accounts for realized agency volume:

$$\mathsf{rav}_{\mathsf{a},i} = (1-\hat{\delta})|\mathsf{a}_i^{\mathcal{A}}-\mathsf{a}| + \hat{\delta}\sum_{j\in\mathcal{I}}\pi_j|\mathsf{a}_j^{\mathcal{A}}-\mathsf{a}|$$

Spread Decomposition:

$$\begin{split} \Delta \mathcal{S}^{\mathcal{A}} &= \underbrace{\mathcal{S}^{\mathcal{A},1}_{\mathcal{A}^{0},\mathcal{A}^{1}} \times w^{\mathcal{A},1}_{\mathcal{A}^{0},\mathcal{A}^{1}} - \mathcal{S}^{\mathcal{A},0}_{\mathcal{A}^{0},\mathcal{A}^{1}} \times w^{\mathcal{A},1}_{\mathcal{A}^{0},\mathcal{A}^{1}}}_{\text{ongoing agency traders}} \\ &+ \underbrace{\mathcal{S}^{\mathcal{A},1}_{\mathcal{P}^{0},\mathcal{A}^{1}} \times w^{\mathcal{A},1}_{\mathcal{P}^{0},\mathcal{A}^{1}}}_{\text{principal} \to \text{ agency}} + \underbrace{\mathcal{S}^{\mathcal{A},1}_{\mathcal{N}^{T0},\mathcal{A}^{1}} \times w^{\mathcal{A},1}_{\mathcal{N}^{T0},\mathcal{A}^{1}}}_{\text{no traders} \to \text{ agency}} \\ &- \underbrace{\mathcal{S}^{\mathcal{A},0}_{\mathcal{A}^{0},\mathcal{P}^{1}} \times w^{\mathcal{A},0}_{\mathcal{A}^{0},\mathcal{P}^{1}}}_{\text{agency} \to \text{principal}} - \underbrace{\mathcal{S}^{\mathcal{A},0}_{\mathcal{A}^{0},\mathcal{N}^{-1}} \times w^{\mathcal{A},0}_{\mathcal{A}^{0},\mathcal{N}^{-1}}}_{\text{agency} \to \text{ no traders}} \end{split}$$

Counterfactual Measures

Composition-free spread under parametrization $q \in \{0, 1\}$:

Only those customer who would not migrate when q changes.

$$egin{aligned} & ilde{\mathcal{S}}^{P}(q)\equiv \mathcal{S}^{P,q}_{P^{0},P^{1}}, \ & ilde{\mathcal{S}}^{\mathcal{A}}(q)\equiv \mathcal{S}^{\mathcal{A},q}_{\mathcal{A}^{0},\mathcal{A}^{1}}. \end{aligned}$$

Composition-free spread changes:

Change in spread fixing the set of customers to those non-migrants.

$$\begin{split} \Delta \tilde{\mathcal{S}}^{P} &\equiv \mathcal{S}^{P,1}_{P^{0},P^{1}} - \mathcal{S}^{P,0}_{P^{0},P^{1}}, \\ \Delta \tilde{\mathcal{S}}^{A} &\equiv \mathcal{S}^{A,1}_{A^{0},A^{1}} - \mathcal{S}^{A,0}_{A^{0},A^{1}}, \end{split}$$

Composition effect bias:

Percentage difference between avg and composition-free measures.

$$\begin{split} CE^{P} &\equiv (\Delta \mathcal{S}^{P} - \Delta \tilde{\mathcal{S}}^{P}) / \Delta \mathcal{S}^{P}, \\ CE^{A} &\equiv (\Delta \mathcal{S}^{A} - \Delta \tilde{\mathcal{S}}^{A}) / \Delta \mathcal{S}^{A}. \end{split}$$

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Extra Slides

Execution speed increase: customers migrate towards agency.

 $\beta: \mathbf{1} \rightarrow \mathbf{3}$



- 1. Principal trades migrate towards agency.
- 2. Non-random migration.

Extra Slides

The rise in principal cost is mostly explained by the composition effect.

 $\beta: \mathbf{1} \to \mathbf{3}$



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Turnover increases and agency share decreases.

- $\Delta S^P = 0.65 bp$ and $\Delta \tilde{S}^P = 0.07 bp$: $\implies CE^P = 89.5\%$
- $\Delta S^A = 2.40 bp$ and $\Delta \tilde{S}^A = 2.42 bp$: $\implies CE^A = -1.03\%$

Quantitative Exercises Robustness Checks

I compute the composition effect (CE) in both quantitative exercises using:

• Alternative preference distribution, $\pi_i \sim Beta(\lambda, \lambda)$

• Alternative dealer's baragin power η .

			Composition Effect					
			λ			η		
		0.2	1	5	0.91	0.95	0.99	
$\Delta \theta$	CE ^P CE ^A	18.49 -0.20	32.19 -1.19	28.65 0.42	25.99 0.50	32.19 -1.19	34.58 -16.78	
Δeta	CE ^P CE ^A	79.64 -1.14	89.54 -1.03	101.38 0.26	74.71 -1.09	89.54 -1.03	105.18 -4.08	

 $CE^{P} \equiv (\Delta S^{P} - \Delta \tilde{S}^{P}) / \Delta S^{P}$, $CE^{A} \equiv (\Delta S^{A} - \Delta \tilde{S}^{A}) / \Delta S^{A}$

